$B_s \to \mu^+\mu^-\gamma$ at large q^2 from lattice QCD

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Motivations

Why $B_s \to \mu^+\mu^-\gamma$ at large q^2 ?

- The $B_s \to \mu^+\mu^-\gamma$ decay allows for a new test of the SM predictions in $b \to s$ FCNC transitions.

- Despite the $\mathcal{O}(\alpha_{\rm em})$-suppression w.r.t. the widely studied $B_s \to \mu^+\mu^-$, removal of helicity-suppression makes the two decay rates comparable in magnitude.

- At very high $\sqrt{q^2}$ = invariant mass of the $\mu^+\mu^-$, the contributions from penguin operators appearing in the weak effective-theory, which are difficult to compute on the lattice, are suppressed [Guadagnoli, Reboud, Zwicky, JHEP '17] ✓.

In this talk I will present the first, ($\simeq$) first-principles lattice QCD calculation of the $B_s \to \mu^+\mu^-\gamma$ decay rate for $q^2 \gtrsim (4.2\ \mathrm{GeV})^2$.

The effective weak-Hamiltonian

The low-energy effective theory describing the $b \to s$ transition, neglecting doubly Cabibbo-suppressed terms, is

$$\mathcal{H}_{\text{eff}}^{b\to s} = 2\sqrt{2} G_F V_{tb} V_{ts}^* \left[\sum_{i=1,2} C_i(\mu) \mathcal{O}_i^c + \sum_{i=3}^{6} C_i(\mu) \mathcal{O}_i + \frac{\alpha_{\text{em}}}{4\pi} \sum_{i=7}^{10} C_i(\mu) \mathcal{O}_i \right]$$

current-current: $\mathcal{O}_1^c = \left(\bar{s}_i \gamma^\mu P_L c_j\right)\left(\bar{c}_j \gamma^\mu P_L b_i\right)$, $\mathcal{O}_2^c = \left(\bar{s} \gamma^\mu P_L c\right)\left(\bar{c} \gamma^\mu P_L b\right)$,

ph./chromo. penguins: $\mathcal{O}_7 = -\frac{m_b}{e} \bar{s} \sigma^{\mu\nu} F_{\mu\nu} P_R b$, $\mathcal{O}_8 = -\frac{g_s m_b}{4\pi\alpha_{\text{em}}} \bar{s} \sigma^{\mu\nu} G_{\mu\nu} P_R b$,

semileptonic: $\mathcal{O}_9 = \left(\bar{s} \gamma^\mu P_L b\right)\left(\bar{\mu} \gamma_\mu \mu\right)$, $\mathcal{O}_{10} = \left(\bar{s} \gamma^\mu P_L b\right)\left(\bar{\mu} \gamma_\mu \gamma^5 \mu\right)$

- The amplitude $\mathcal{A}$ is the sandwich of $\mathcal{H}_{\text{eff}}^{b\to s}$ between initial and final states

$$\mathcal{A}[\bar{B}_s \to \mu^+\mu^-\gamma] = \langle \gamma(\boldsymbol{k}, \varepsilon) \mu^+(p_1) \mu^-(p_2) | - \mathcal{H}_{\text{eff}}^{b\to s} | \bar{B}_s(\boldsymbol{p}) \rangle_{\text{QCD+QED}} \ ,$$

- To lowest-order in $\mathcal{O}(\alpha_{\text{em}})$ [Beneke et al, EPJC 2011]:

$$\mathcal{A}[\bar{B}_s \to \mu^+\mu^-\gamma] = -e \frac{\alpha_{\text{em}}}{\sqrt{2}\pi} V_{tb} V_{ts}^* \varepsilon_\mu^* \left[\sum_{i=1}^{9} C_i \overbrace{H_i^{\mu\nu}}^{\text{NP}-\text{QCD}} L_{V\nu} + C_{10} \left(\overbrace{H_{10}^{\mu\nu}}^{\text{NP}-\text{QCD}} L_{A\nu} - \overbrace{\frac{i}{2} f_{B_s} L_A^{\mu\nu} p_\nu}^{\text{PT}-\text{contribution}} \right) \right]$$

The local form factors and penguin operators

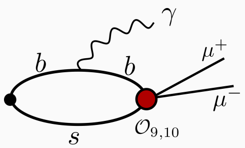
The non-perturbative information is encoded in the hadronic tensors $H_i^{\mu\nu}$, which can be grouped in three categories:

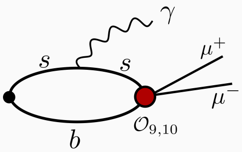
Contributions from semileptonic operators:

$$H_9^{\mu\nu}(p,k) = H_{10}^{\mu\nu}(p,k) = i\int d^4y\ e^{iky}\ \hat{\mathrm{T}}\langle 0|\left[\bar{s}\gamma^\nu P_L b\right](0) J^\mu_{\mathrm{em}}(y)|\bar{B}_s(\boldsymbol{p})\rangle$$

$$= -i\left[g^{\mu\nu}(k\cdot q) - q^\mu k^\nu\right]\frac{F_A}{2m_{B_s}} + \varepsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma \frac{F_V}{2m_{B_s}}$$

- Parametrized by vector and axial form factors $F_V(x_\gamma)$ and $F_A(x_\gamma)$ $[x_\gamma \equiv 2E_\gamma/m_{B_s}]$. E_γ is the photon energy in the rest-frame of the $\bar{B}_s$.





The local form factors and penguin operators

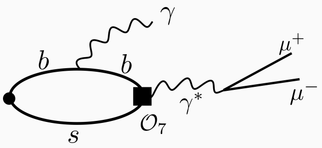
The non-perturbative information is encoded in the hadronic tensors $H_i^{\mu\nu}$, which can be grouped in three categories:

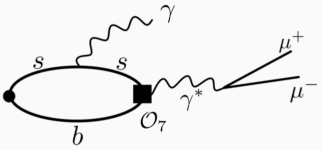
Contributions from photon-penguin operator (A-type):

$$H_{7A}^{\mu\nu}(p,k) = i\frac{2m_b}{q^2}\int d^4y\; e^{iky}\; \hat{\mathrm{T}}\langle 0|\left[-i\bar{s}\sigma^{\nu\rho}q_\rho P_R b\right](0)J_{\mathrm{em}}^{\mu}(y)|\bar{B}_s(\boldsymbol{p})\rangle$$

$$= -i\left[g^{\mu\nu}(k\cdot q) - q^\mu k^\nu\right]\frac{F_{TA}m_b}{q^2} + \varepsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma\frac{F_{TV}m_b}{q^2}$$

- Parametrized by tensor and axial-tensor form factors $F_{TV}(x_\gamma)$ and $F_{TA}(x_\gamma)$. Algebraic constraint: $F_{TV}(1) = F_{TA}(1)$.





The local form factors and penguin operators

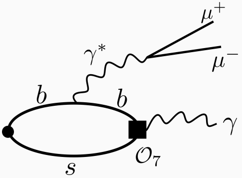
The non-perturbative information is encoded in the hadronic tensors $H_i^{\mu\nu}$, which can be grouped in three categories:

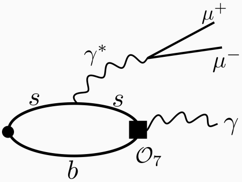
Contributions from photon-penguin operator (B-type):

$$H_{7B}^{\mu\nu}(p,k) = i\frac{2m_b}{q^2}\int d^4y\; e^{iqy}\; \hat{\mathrm{T}}\langle 0|\left[-i\bar{s}\sigma^{\mu\rho}k_\rho P_R b\right](0) J_{\mathrm{em}}^{\nu}(y)|\bar{B}_s(\boldsymbol{p})\rangle$$

$$= -i\left[g^{\mu\nu}(k\cdot q) - q^\mu k^\nu\right]\frac{\bar{F}_{TA} m_b}{q^2} + \varepsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma \frac{\bar{F}_{TV} m_b}{q^2}$$

- Parametrized by a single form factor $\bar{F}_T(x_\gamma) = \bar{F}_{TV}(x_\gamma) = \bar{F}_{TA}(x_\gamma)$.





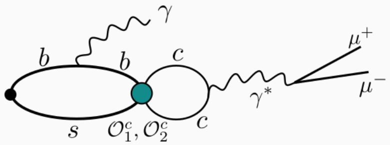
The local form factors and penguin operators

The non-perturbative information is encoded in the hadronic tensors $H_i^{\mu\nu}$, which can be grouped in three categories:

Contributions from four-quark and chromomagnetic operators:

$$H^{\mu\nu}_{i=1-6,8}(p,k) = \frac{(4\pi)^2}{q^2}\int d^4y\; d^4x\; e^{iky}e^{iqx}\;\hat{\mathrm{T}}\langle 0|J^\mu_{\rm em}(y)J^\nu_{\rm em}(x)\mathcal{O}_i(0)|\bar{B}_s(\boldsymbol{p})\rangle$$

- In the high-q^2 region, they are formally of higher-order in the $1/m_b$ expansion [Guadagnoli, Reboud, Zwicky, JHEP '17].
- We did not compute them, but have future plans to do so.
- In the evaluation of the branching fractions we only included a phenomenological description of the allegedly dominant contribution from the following charming-penguin diagram:



This contribution is dominated by vector $c\bar{c}$ resonances. Some of them overlap with the q^2 region we consider. A description of our parameterization will come later.

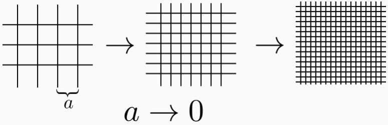
The local form factors on the lattice

We computed on the lattice the local form factors F_V, F_A, F_{TV}, F_{TA} and $\bar{F}_T$ for

$$x_\gamma \in [0.1 : 0.4] \implies 4.16 \text{ GeV} < \sqrt{q^2} < 5.1 \text{ GeV}$$

Two main sources of systematics on the lattice, which must be controlled:

- Continuum-limit extrapolation $(a \to 0)$...

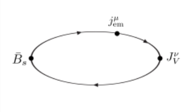


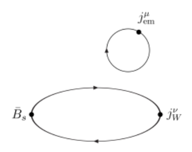
- ...which we handle by simulating at four values of the lattice spacing $a \in [0.057 : 0.09]$ fm using configurations produced by the ETM Collaboration.
- Extrapolation to the physical B_s meson mass, which we handle by simulating at five different values of the heavy-strange meson mass $m_{H_s} \in [m_{D_s} : 2m_{D_s}]$...
- ...and then performing the extrapolation $m_{H_s} \to m_{B_s}$ via pole-like+HQET scaling relations. On current lattices in fact we cannot simulate directly the B_s meson, which is too heavy.

Sketch of the lattice calculation of the form factors

Our lattice input is (for simplicity I discuss here only the case of F_V):

$$B_V^{\mu\nu}(t, x_\gamma) = \int \mathrm{d}t_y\, \mathrm{d}^3y\, \mathrm{d}^3x\, e^{E_\gamma t_y}\, e^{-i\boldsymbol{k}\boldsymbol{y}}\, \hat{\mathrm{T}}\langle 0|\underbrace{J_V^\nu}_{\bar{s}\gamma^\nu b}(t,0) J_{\mathrm{em}}^\mu(t_y, \boldsymbol{y}) \phi_{B_s}^\dagger(0, \boldsymbol{x})|0\rangle$$





We neglect the quark disconnected diagram. It vanishes exactly in the $\mathrm{SU}(3)$-symmetric limit and for $m_c \to \infty$. This is the electroquenched approximation.

- $\phi_{B_s}^\dagger$ is an interpolating operator having the quantum numbers to create a $\bar{B}_s$.
- After amputating external states one has

$$R_V^{\mu\nu}(t, x_\gamma) \equiv \frac{2m_{B_s}}{e^{-t(m_{B_s} - E_\gamma)}\, \langle \bar{B}_s(\mathbf{0})|\phi_{B_s}^\dagger(0)|0\rangle} B_V^{\mu\nu}(t, x_\gamma)$$

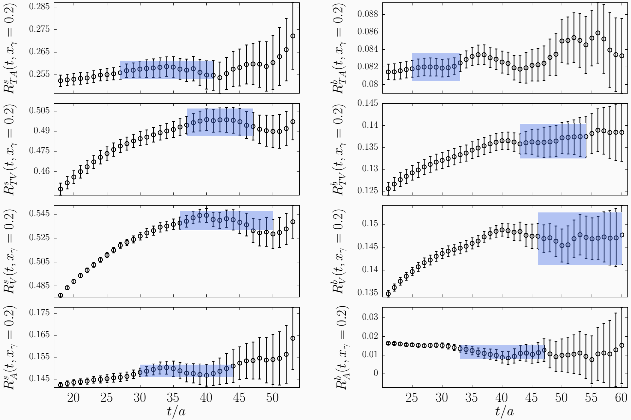
- We always inject photon momentum $\boldsymbol{k}$ in lattice direction $\hat{z}$. In this setup:

$$R_V(t, x_\gamma) \equiv \frac{1}{k_z} R_V^{12}(t, x_\gamma) \xrightarrow[0 \ll t \ll T/2]{} F_V(x_\gamma) \qquad \checkmark$$

- Similar estimators for F_A, F_{TV} and F_{TA}. $\bar{F}_T$ analysis more complex [Backup].

Extraction of the form factors from lattice data

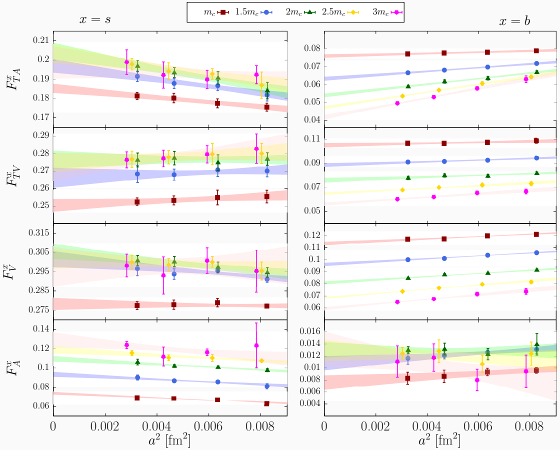
Illustrative example on the finest lattice spacing $a \sim 0.057$ fm for $x_\gamma = 0.2$ and $m_h/m_c = 2$.

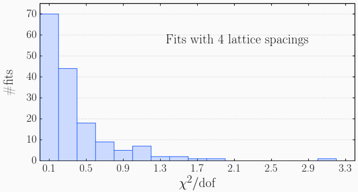


- We analyze separately the two contributions corresponding to the emission of the real photon from the strange or the heavy quark.
- $x_\gamma = 2E_\gamma/m_{H_s}$ kept fixed increasing the heavy-meson mass ($E_\gamma \propto m_{H_s}$).

Continuum limit extrapolation

We perform the continuum-limit extrapolation at fixed m_{H_s} and x_γ





We performed a total of 160 continuum-limit extrapolations.

$\Longleftarrow$ Example for $x_\gamma = 0.4$.

Systematic errors evaluated performing fits using only the three finest lattice spacings.

Results obtained using three or four lattice spacings combined using AIC.

Extrapolation to the physical B_s meson mass (I)

After continuum extrapolation, the most delicate task is to extrapolate the form factors, computed for $m_{H_s} \in [m_{D_s} : 2m_{D_s}]$, to $m_{B_s} \sim 5.367$ GeV.

- Elegant scaling laws were derived in the limit of large photon energies E_γ and large m_{H_s} [Beneke et al, EPJC 2011, JHEP 2020]. Up to $\mathcal{O}(E_\gamma^{-1}, m_{H_s}^{-1})$ one has

$$\frac{F_V(x_\gamma, m_{H_s})}{f_{H_s}} = \frac{|q_s|}{x_\gamma}\left(\frac{R(E_\gamma,\mu)}{\lambda_B(\mu)} + \xi(x_\gamma, m_{H_s}) + \frac{1}{m_{H_s}x_\gamma} + \frac{|q_b|}{|q_s|}\frac{1}{m_h}\right)$$

$$\frac{F_A(x_\gamma, m_{H_s})}{f_{H_s}} = \frac{|q_s|}{x_\gamma}\left(\frac{R(E_\gamma,\mu)}{\lambda_B(\mu)} + \xi(x_\gamma, m_{H_s}) - \frac{1}{m_{H_s}x_\gamma} - \frac{|q_b|}{|q_s|}\frac{1}{m_h}\right)$$

$$\frac{F_{TV}(x_\gamma, m_{H_s},\mu)}{f_{H_s}} = \frac{|q_s|}{x_\gamma}\left(\frac{R_T(E_\gamma,\mu)}{\lambda_B(\mu)} + \xi(x_\gamma, m_{H_s}) + \frac{1-x_\gamma}{m_{H_s}x_\gamma} + \frac{|q_b|}{|q_s|}\frac{1}{m_{H_s}}\right)$$

$$\frac{F_{TA}(x_\gamma, m_{H_s},\mu)}{f_{H_s}} = \frac{|q_s|}{x_\gamma}\left(\frac{R_T(E_\gamma,\mu)}{\lambda_B(\mu)} + \xi(x_\gamma, m_{H_s}) - \frac{1-x_\gamma}{m_{H_s}x_\gamma} + \frac{|q_b|}{|q_s|}\frac{1}{m_{H_s}}\right)$$

- λ_B is 1st inverse-moment of B_s LCDA. R, R_T are radiative corrections. ξ is a power-suppressed term $\propto 1/E_\gamma, 1/m_{H_s}$, f_{H_s} the decay constant of H_s meson.
- Photon emission from b ($\propto |q_b|$) power-suppressed w.r.t. to emission from s.
- Tensor form factors are scale and scheme dependent. On the lattice we obtained them in $\overline{\mathrm{MS}}$ scheme at $\mu = 5$ GeV.

Extrapolation to the physical B_s meson mass (II)

- The scaling relations discussed above are only valid for very energetic photons.
- While we have $E_\gamma \propto m_{H_s}$, for small $x_\gamma = 2E_\gamma/m_{H_s}$ and not very large m_{H_s}, there are sizable corrections to the previous relations.
- Assuming vector-meson-dominance (VMD) one has $(W = \{V, A, TV, TA\})$

$$\frac{F_W(x_\gamma, m_{H_s})}{f_{H_s}} \propto \frac{1}{\sqrt{r_W^2 + \frac{x_\gamma^2}{4}} + \frac{x_\gamma}{2} - 1} + \mathcal{O}(\frac{1}{E_\gamma}, \frac{1}{m_{H_s}})$$

$$r_V = r_{TV} = \frac{m_{H_s^*}}{m_{H_s}} \; , \qquad r_A = r_{TA} = \frac{m_{H_{s1}}}{m_{H_s}}$$

- H_s^* and H_{s1} are respectively the ground state $J^P = 1^-$ and $J^P = 1^+$ mesons, made of an heavy quark and a strange anti-quark.
- In the static limit $m_{H_s} \to \infty$ one has $r_W = 1$ and, for non-zero x_γ, the LO scaling laws $F_W \propto f_{H_s}/x_\gamma$ are recovered.
- However, away from the static limit and for small(ish) x_γ the quasi-pole structure generates large corrections to the LO scaling laws...

Extrapolation to the physical B_s meson mass (III)

Making use of the HQET scaling laws:

$$m^2_{\bar{H}^*_s} - m^2_{\bar{H}_s} = 2\lambda_2 + \mathcal{O}\left(\frac{1}{m_h}\right) , \qquad \lambda_2 \simeq 0.24 \text{ GeV}^2$$

$$m_{\bar{H}_{s1}} - m_{\bar{H}_s} = \Lambda_1 + \mathcal{O}\left(\frac{1}{m_h}\right) , \qquad \Lambda_1 \simeq 0.5 \text{ GeV}$$

the denominator in the VMD Ansatz becomes

$$r_{V/TV} = \frac{m_{H^*_s}}{m_{H_s}} \simeq 1 + \frac{\lambda_2}{m^2_{H_s}} \implies \sqrt{r^2_{V/TV} + \frac{x^2_\gamma}{4}} + \frac{x_\gamma}{2} - 1 \simeq \frac{\lambda_2}{m^2_{H_s}} + \frac{x_\gamma}{2} + \dots$$

$$r_{A/TA} = \frac{m_{H_{s1}}}{m_{H_s}} \simeq 1 + \frac{\Lambda_1}{m_{H_s}} \implies \sqrt{r^2_{A/TA} + \frac{x^2_\gamma}{4}} + \frac{x_\gamma}{2} - 1 \simeq \frac{\Lambda_1}{m_{H_s}} + \frac{x_\gamma}{2} + \dots$$

If $x_\gamma \ll 2\lambda_2/m^2_{H_s}$ ($x_\gamma \ll 2\Lambda_1/m_{H_s}$), the presence of a quasi-pole generates an enhancement of $F_{V/TV}$ ($F_{A/TA}$) of order $\mathcal{O}(m^2_{H_s})$ ($\mathcal{O}(m_{H_s})$).

To extrapolate to the physical B_s we cook up a phenomenological fit Ansatz which combines the scaling laws valid for very hard photons, with the quasi-pole correction due to resonance contributions.

The global fit Ansatz

We extrapolate to the physical B_s through a combined fit of the form factors [$z = 1/m_{H_s}$, fit parameters are in red]:

$$\frac{F_V(x_\gamma, z)}{f_{H_s}} = \frac{|q_s|}{x_\gamma} \frac{1}{1 + C_V \frac{2z^2}{x_\gamma}} \left(K + (1+\delta_z)\frac{z}{x_\gamma} + \frac{1}{z^{-1} - \Lambda_H} + A_m z + A_{x_\gamma} \frac{z}{x_\gamma} \right)$$

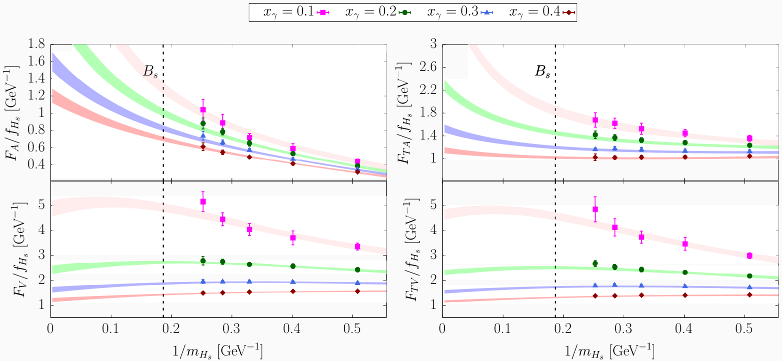
$$\frac{F_A(x_\gamma, z)}{f_{H_s}} = \frac{|q_s|}{x_\gamma} \frac{1}{1 + C_A \frac{2z}{x_\gamma}} \left(K - (1+\delta_z)\frac{z}{x_\gamma} - \frac{1}{z^{-1} - \Lambda_H} + A_m z + (A_{x_\gamma} + 2KC_A) \frac{z}{x_\gamma} \right)$$

$$\frac{F_{TV}(x_\gamma, z)}{f_{H_s}} = \frac{|q_s|}{x_\gamma} \frac{1 + 2C_V z^2}{1 + C_V \frac{2z^2}{x_\gamma}} \left(K_T + (A_m^T + 1)z + A_{x_\gamma}^T \frac{z}{x_\gamma} + (1+\delta_z')z\frac{1 - x_\gamma}{x_\gamma} \right)$$

$$\frac{F_{TA}(x_\gamma, z)}{f_{H_s}} = \frac{|q_s|}{x_\gamma} \frac{1 + 2C_A^T z}{1 + C_A^T \frac{2z}{x_\gamma}} \left(K_T + (A_m^T + 1)z + A_{x_\gamma}^T \frac{z}{x_\gamma} - (1+\delta_z' - 2K_T C_A^T)z\frac{1 - x_\gamma}{x_\gamma} \right)$$

- Fit structure takes into account constraints from the scaling laws valid at large E_γ and m_{H_s}, and contains the resonance corrections (relevant at small x_γ).
- We included in the fit also NNLO $1/E_\gamma^2$, $1/m_{H_s}^2$ corrections.
- Some of the constraints appearing in the large energy/mass EFT have been relaxed as they are valid neglecting $\mathcal{O}(m_s)$ and radiative corrections to the power-suppressed terms.

The form factors at the physical point $m_{B_s} \simeq 5.367$ GeV



- Observed steeper m_{H_s}-dependence of the form factors at small x_γ ✓. [Determination of f_{H_s} and f_{B_s} in backup].
- We performed more than 500 fits, by including or not some of the fit parameters from previous global fit Ansatz, and imposing or not $K = K_T$ and $C_A = C_A^T$.
- Different fits combined using AIC or by including in the final average (and with a uniform weight) only those fits having $\chi^2/dof < 1.4$ (the two strategies give consistent results, second criterion used to give final numbers).

Fit parameters

Pole parameters:

$$C_V = (0.57(3)\ \text{GeV})^2\ , \qquad C_A = 0.70(7)\ \text{GeV}\ , \qquad C_A^T = 0.77(4)\ \text{GeV}$$

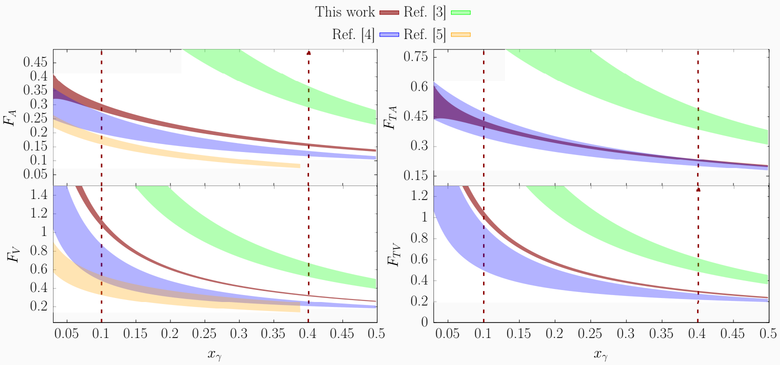
Expectations from pure VMD:

$$C_V^{\text{VMD}} = \lambda_2 \simeq (0.5\ \text{GeV})^2, \qquad C_A^{\text{VMD}} = C_A^{T,\text{VMD}} = \Lambda_1 \simeq 0.5\ \text{GeV}$$

- In vector channels, where VMD is expected to be a reasonable approximation, substantial agreement between C_V and C_V^{VMD}.
- In the axial channels, VMD does not work very well: many resonances of masses $m_{\text{res}} \sim m_{H_s} + \mathcal{O}(\Lambda_{\text{QCD}})\dots$
- ... which is the reason why for F_A and F_{TA} two different parameters C_A, C_A^T have been introduced. C_A and C_A^T of order $\mathcal{O}(\Lambda_{QCD})$, as expected.
- For K and K_T we obtain:

$$K = 1.46(10)\ \text{GeV}^{-1}\ , \qquad K_T = 1.39(6)\ \text{GeV}^{-1}$$

Comparison with previous calculations

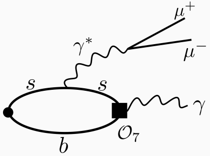


- Ref. [3] = Janowski, Pullin , Zwicky , JHEP '21 , light-cone sum rules.
- Ref. [4] = Kozachuk, Melikhov, Nikitin , PRD '18 , relativistic dispersion relations.
- Ref. [5] = Guadagnoli, Normand, Simula, Vittorio, JHEP '23, VMD/Lattice.

With a few exceptions, our results for the form factors differ significantly from the earlier estimates (which also differ from each other).

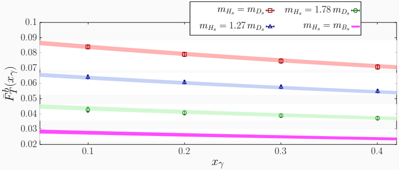
A quick look at the (small) form factor $\bar{F}_T$

The lattice determination of the form factor $\bar{F}_T$ is hindered by the presence of problems of analytic continuation when γ^* is emitted by a strange quark.

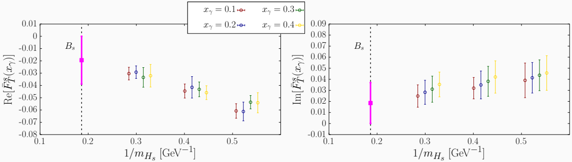


For $q^2 \gtrsim m_\phi^2$ the relevant Minkowskian correlation functions needed to evaluate $\bar{F}_T$ cannot be analytically continued to Euclidean spacetime. $\bar{F}_T$ also develops an immaginary part.

Recently, we have developed a new method, based on spectral reconstruction techniques, which allows to circumvent this problem [Frezzotti et al., PRD '23]. [Backup]



$\Longleftarrow$ b-quark contribution not affected by this issue. Mass extrapolation carried out using VMD-inspired Ansatz assuming Υ-resonances dominance.



$\Longleftarrow$ s-quark contribution develops imag. part. Current uncertainty on $\bar{F}_T^s$ is 100%. OK, given that $\bar{F}_T \ll F_{TA}, F_{TV}$.

From the form factors to the branching fractions

The differential branching fraction for $\bar{B}_s \to \mu^+\mu^-\gamma$ can be decomposed as a sum of three terms

$$\frac{\mathrm{d}\mathcal{B}}{dx_\gamma} = \frac{\mathrm{d}\mathcal{B}_{\mathrm{PT}}}{dx_\gamma} + \frac{\mathrm{d}\mathcal{B}_{\mathrm{INT}}}{dx_\gamma} + \frac{\mathrm{d}\mathcal{B}_{\mathrm{SD}}}{dx_\gamma} \qquad \left[q^2 = m_{B_s}^2(1-x_\gamma)\right]$$

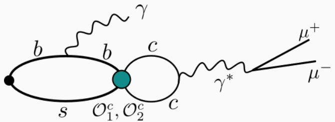
- $\mathrm{d}\mathcal{B}_{\mathrm{PT}}/\mathrm{dx}_\gamma$ is the point-like contribution ($\propto f_{B_s}^2$).
- It suffers from an IR-divergence ($\mathrm{d}\mathcal{B}/\mathrm{dx}_\gamma \propto 1/\mathrm{x}_\gamma$ at small x_γ), which is then cancelled by the virtual-photon correction to $\bar{B}_s \to \mu^+\mu^-$ through the Block-Nordsieck mechanism.
- $\mathrm{d}\mathcal{B}_{\mathrm{INT}}/\mathrm{dx}_\gamma$ is the interference contribution and depends linearly on the form factors.
- $\mathrm{d}\mathcal{B}_{\mathrm{SD}}/\mathrm{dx}_\gamma$ is the structure-dependent contribution and is quadratic in the form factors.

Both the interference and structure-dependent contributions are infrared finite.

Adding contributions from penguin operators

We did not compute from first-principles the contributions from four-quark and chromomagnetic operators $\mathcal{O}_{i=1-6,8}$.

- It is expected that among these contributions the dominant one in $\bar{B}_s \to \mu^+\mu^-\gamma$ at $q^2 > (4.2\ \mathrm{GeV})^2$ is the charming-penguin diagram stemming from $\mathcal{O}_{1-2}$ due to $J^P = 1^-$ charmonium resonances.



This contribution can be included as a shift of the Wilson coefficient C_9:

$$C_9 \to C_9^{\mathrm{eff}}(q^2) = C_9 - \Delta C_9(q^2)$$

$\delta_V = |k_V| - 1 = 0$ holds in the factorization approximation.

In analogy with previous works [Guadagnoli et al, JHEP '17, '23] we model $\Delta C_9(q^2)$ as

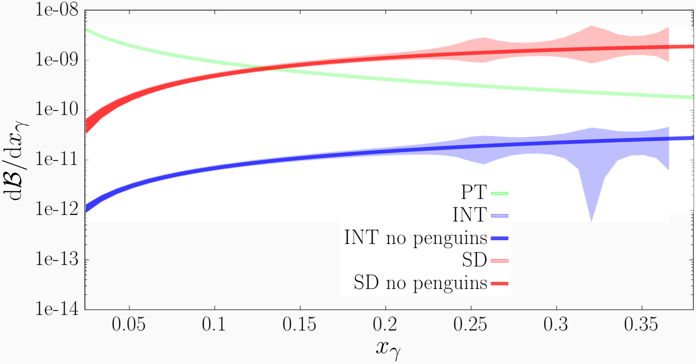
$$\Delta C_9(q^2) = \frac{9\pi}{\alpha_{\mathrm{em}}^2}\,\bar{C} \sum_V |k_V| e^{i\delta_V} \frac{m_V\, B(V \to \mu^+\mu^-)\,\Gamma_V}{q^2 - m_V^2 + i m_V \Gamma_V}$$

$$\bar{C} = C_1 + C_2/3 \simeq -0.2$$

$V_{c\bar{c}}$	$M_{V_{c\bar{c}}}$ [GeV]	Γ [MeV]	$\mathcal{B}(V_{c\bar{c}} \to \mu^+\mu^-)$
J/ψ	3.096900(6)	0.0926(17)	0.05961(33)
$\Psi(2S)$	3.68610(6)	0.294(8)	$8.0(6) \cdot 10^{-3}$
$\Psi(3770)$	3.7737(4)	27.2(1.0)	$^*9.6(7) \cdot 10^{-6}$
$\Psi(4040)$	4.039(1)	80(10)	$^*1.07(16) \cdot 10^{-5}$
$\Psi(4160)$	4.191(5)	70(10)	$^*6.9(3.3) \cdot 10^{-6}$
$\Psi(4230)$	4.2225(24)	48(8)	$3.2(2.9) \cdot 10^{-5}$
$\Psi(4415)$	4.421(4)	62(20)	$2(1) \cdot 10^{-5}$
$\Psi(4660)$	4.630(6)	72^{+14}_{-12}	not seen

We assume uniformly distributed phases $\delta_V \in [0, 2\pi]$ and $|k_V| = 1.75(75)$.

The differential branching fractions

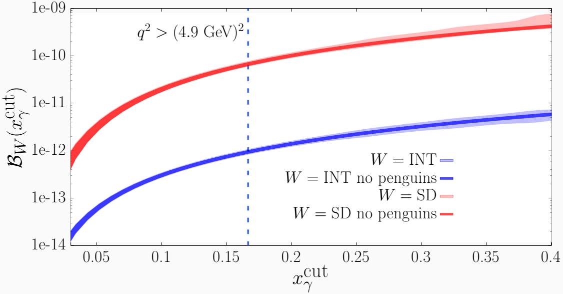


- For $x_\gamma \gtrsim 0.15$, the SD is dominant over the PT contribution.
- For $x_\gamma \gtrsim 0.2$, charming-penguin uncertainties become dominant, due to the presence of charmonium states which overlap with the $x_\gamma-$region considered.
- INT contribution is always about two orders of magnitude smaller than SD.

The branching fractions

$$\mathcal{B}(x_\gamma^{\rm cut}) = \int_0^{x_\gamma^{\rm cut}} {\rm d}x_\gamma \, \frac{{\rm d}\mathcal{B}}{{\rm d}x_\gamma} \qquad x_\gamma^{\rm cut} \equiv 1 - \frac{q^2_{\rm cut}}{m^2_{B_s}}$$

- $E_\gamma^{\rm cut} = x_\gamma^{\rm cut} m_{B_s}/2$ is the upper-bound on the measured photon energy.

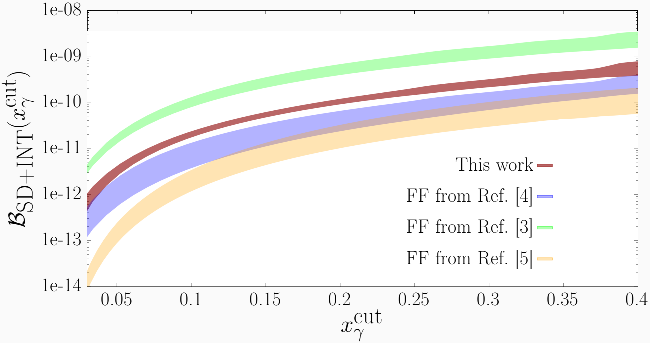


- SD contribution dominated by vector form factor F_V. Tensor form-factor contributions suppressed by small Wilson coefficient $C_7 \ll C_9, C_{10}$.
- At $x_\gamma^{\rm cut} \sim 0.4$ our estimate of charming-penguins uncertainties is around 30%.

Comparison with current LHCb upper-bound for $x_\gamma^{\rm cut} \sim 0.166$.

$$\mathcal{B}_{\rm SD}^{\rm LHCb}(0.166) < 2 \times 10^{-9}\,, \qquad \mathcal{B}_{\rm SD}(0.166) = 6.9(9) \times 10^{-11} \qquad \text{[This work]}$$

Comparison with previous works



- Ref. [3] = Janowski, Pullin , Zwicky , JHEP '21 , light-cone sum rules.
- Ref. [4] = Kozachuk, Melikhov, Nikitin , PRD '18 , relativistic dispersion relations.
- Ref. [5] = Guadagnoli, Normand, Simula, Vittorio, JHEP '23, VMD/Lattice.

Differences with earlier estimates can be traced back to the fact that our determination of F_V (which gives the dominant contribution to the branching) is larger (smaller) than the one of Refs. [4-5] (Ref. [3]) by a factor of about $1.5 - 2$.

Conclusions

- We have presented a first-principles lattice calculation of the form factors F_V, F_A, F_{TV}, F_{TA} entering the $\bar{B}_s \to \mu^+\mu^-\gamma$ decay, in the electroquenched approximation.

- Systematic errors have been controlled thanks to the use of gauge configurations produced by the ETM Collaboration, which correspond to four values of the lattice spacing $a \in [0.057 : 0.09]$ fm, and through the use of five different heavy-strange masses $m_{H_s} \in [m_{D_s} : 2m_{D_s}]$.

- Presently our result for the branching fractions have uncertainties ranging from $\sim 15\%$ at $\sqrt{q^2_{\rm cut}} = 4.9$ GeV to $\sim 30\%$ at $\sqrt{q^2_{\rm cut}} = 4.2$ GeV.

- At small $q^2_{\rm cut}$ uncertainty dominated by the charming-penguins which we included using a phenomenological parameterization.

Outlook:

- Evaluate electro-unquenching effects.

- Evaluate charming-penguins contributions from first-principles.

- Simulate on finer lattice spacings to be able to reach higher m_{H_s} and reduce the impact of the mass-extrapolation.

Thank you for the attention!

Backup

Determination of f_{H_s}

We determined the decay constant corresponding to the five simulated values of the heavy-strange mass m_{H_s} on the same ensembles used to determine the form factors.

- f_{H_s} determined using two different estimators, which only differ by $\mathcal{O}(a^2)$ cut-off effects.
- 1st estimator: f_{H_s} determined from mesonic pseudoscalar two-point correlation function (std method). We refer to this determination as $f^{\mathrm{2pt}}_{H_s}$.
- 2nd estimator: from the zero-momentum correlation function:

$$\int \mathrm{d}^4 y \; \hat{T}\langle 0|J^i_{\mathrm{em}}(y)J^i_A(0)|\bar{H}_s(\mathbf{0})\rangle \propto f_{H_s}$$

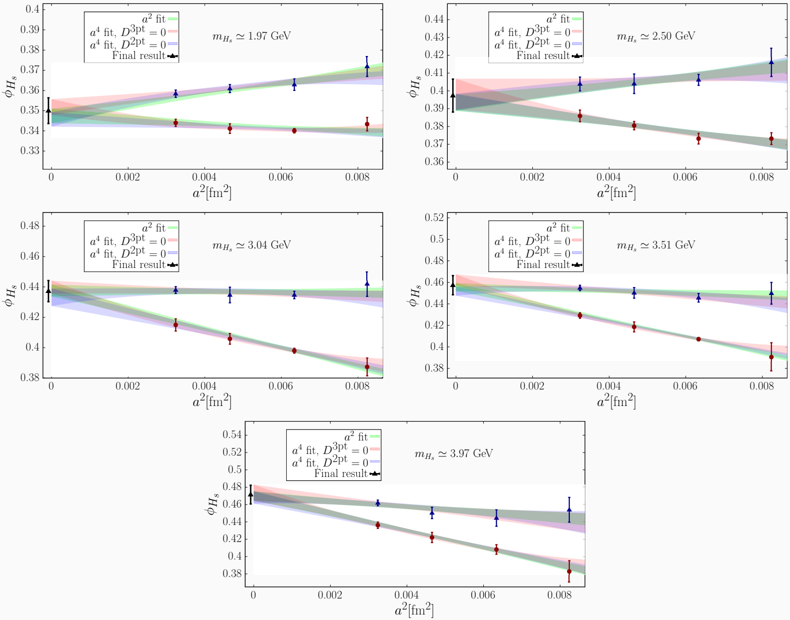
- $J^\nu_A = \bar{s}\gamma^\nu\gamma_5 h$ is the axial current. We refer to this determination as $f^{\mathrm{3pt}}_{H_s}$.

Combined continuum-extrapolation of $f^{\mathrm{2pt}}_{H_s}$ and $f^{\mathrm{3pt}}_{H_s}$ using the Ansatz:

$$\phi^{\mathrm{2pt}}_{H_s} \equiv f^{\mathrm{2pt}}_{H_s}\sqrt{m_{H_s}} = A + B^{\mathrm{2pt}}a^2 + D^{\mathrm{2pt}}a^4$$

$$\phi^{\mathrm{3pt}}_{H_s} \equiv f^{\mathrm{3pt}}_{H_s}\sqrt{m_{H_s}} = A + B^{\mathrm{3pt}}a^2 + D^{\mathrm{3pt}}a^4$$

Continuum-limit extrapolation of $\phi_{H_s} = f_{H_s}\sqrt{m_{H_s}}$

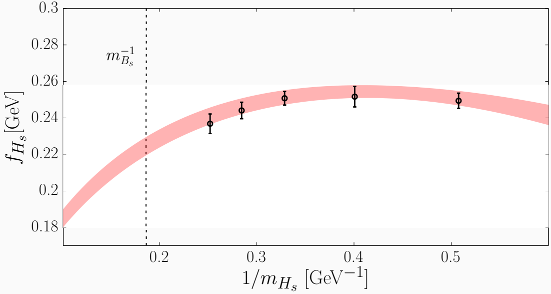


Extrapolation to the physical B_s mass

To extrapolate to the physical B_s mass, we employed the following HQET Ansatz

$$\phi(m_{H_s}) = \underbrace{C_{\gamma^0\gamma^5}(m_h, m_h)}_{\text{HQET/QCD matching}} \; \exp\underbrace{\left\{\int_0^{\alpha_s(m_h)} \frac{\gamma_{\tilde{J}}(\alpha_s)}{2\beta(\alpha_s)} \frac{d\alpha_s}{\alpha_s}\right\}}_{\text{HQET-evolutor}} \left(A + \frac{B}{m_{H_s}}\right)$$

- A and B are free fit parameters.
- m_h should be identified with the pole mass m_h^{pole} (notoriously affected by renormalon ambiguities). We used in place of the pole mass the meson mass: $m_{H_s} - m_h^{\mathrm{pole}} \simeq \mathcal{O}(\Lambda_{\mathrm{QCD}})$.



We obtain: $f_{B_s} = 224.5\,(5.0)$ MeV FLAG average: 230.3 (1.3) MeV

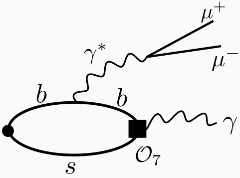
Determination of the form factor $\bar{F}_T$

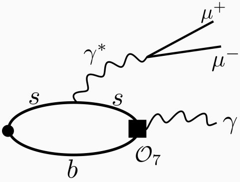
The form factor $\bar{F}_T$, is the smallest of all the form factors (and barely relevant within present accuracy). It can be computed from the knowledge of the following hadronic tensor

$$H^{\mu\nu}_{\bar{T}}(p,k) = i\int d^4x\; e^{i(p-k)x}\;\hat{\mathrm{T}}\langle 0|J^{\nu}_{\bar{T}}(0)J^{\mu}_{\mathrm{em}}(x)|\bar{B}_s(\mathbf{0})\rangle = -\varepsilon^{\mu\nu\rho\sigma}k_\rho p_\sigma\,\frac{\bar{F}_T}{m_{B_s}}$$

where (Z_T is the renormalization constant of tensor current)

$$J^{\nu}_{\bar{T}} = -iZ_T(\mu)\bar{s}\,\sigma^{\nu\rho}\,b\,\frac{k_\rho}{m_{B_s}}$$



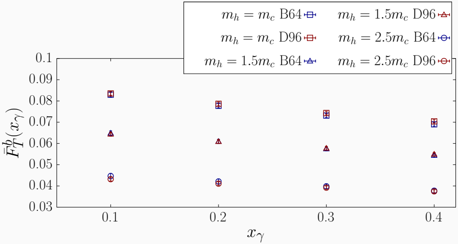


- When the virtual photon γ^* is emitted by a strange quark, the presence of $J^P = 1^-$ $s\bar{s}$ intermediate states forbid the analytic continuation of the relevant correlation functions from Minkowskian to Euclidean spacetime (where we perform MC simulations).

The $b-$quark contribution to $\bar{F}_T$

Let us start discussing the simpler contribution $\bar{F}_T^b$, due to the emission of γ^* from a b-quark.

- In this case the calculation proceeds as in the case of the other form factors F_W, $W = \{V, A, TV, TA\}$, i.e. the hadronic tensor $H^{\mu\nu}_{\bar{T}_b}$ can be directly evaluated from Euclidean spacetime simulations.
- We performed simulations for three value of the heavy-strange meson mass $m_{H_s} \in [m_{D_s} : 1.8 m_{D_s}]$ (or in terms of the heavy quark mass m_h for $m_h/m_c = 1, 1.5, 2.5$), and two values of the lattice spacings (the two gauge ensembles are called B64 and D96). Very small cut-off effects observed.



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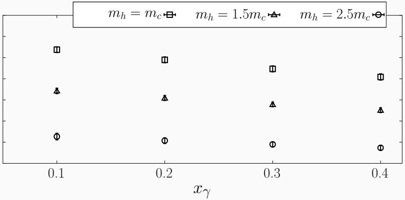


after continuum-limit extrapolation

Mass extrapolation of $\bar{F}_T^b$ (I)

The extrapolation of $\bar{F}_T^b(x_\gamma)$ to the physical mass $m_{B_s} = 5.367$ GeV is carried out using a VMD inspired Ansatz.

- $\bar{F}_T^b$ is expected to be dominated by $J^P = 1^-$ $b\bar{b}$ resonance contributions (e.g. $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\ldots$), which can be approximated as stable states.
- Using an unphysical heavy quark mass $m_h < m_b$ these states will be fictitious $h\bar{h}$, $J^P = 1^-$, intermediate states.
- The contribution to $\bar{F}_T^b$ of a given resonance "n" of mass m_n and electromagnetic decay constant f_n is given by

$$\bar{F}_{T,n}^b(x_\gamma) = \frac{q_b\, f_n\, m_n\, g_n^+(0)}{E_n(E_n + E_\gamma - m_{H_s})} + \text{regular terms}$$

where $E_n = \sqrt{m_n^2 + E_\gamma^2}$ and (η is the polarization of the vector resonance)

$$\langle n(-\boldsymbol{k}, \eta)|\, \bar{s}\sigma^{\mu\nu} h\, |\bar{H}_s(\mathbf{0})\rangle = i\eta_\beta^* \epsilon^{\mu\nu\beta\gamma} g_n^+(p_\gamma^2)(p + q_n)_\gamma + \ldots$$

with $q_n = (E_n, -\boldsymbol{k})$, $p_\gamma = p - q_n$.

Mass extrapolation of $\bar{F}_T^b$ (II)

In the heavy-quark limit the following scaling laws hold

$$f_n \propto \frac{1}{\sqrt{m_h}} + \ldots \propto \frac{1}{\sqrt{m_{H_s}}} + \ldots \,, \qquad \frac{m_n}{m_{H_s}} = 2 + \frac{\Lambda_T^n}{m_{H_s}} + \ldots$$

- $\Lambda_T^n \simeq \mathcal{O}(\Lambda_{\rm QCD})$ and ellipses indicate NLO terms in the heavy-quark expansion.
- Using these relations $\bar{F}_{T,n}^b$ can be approximated by

$$\bar{F}_{T,n}^b(x_\gamma) = \frac{q_b}{m_{H_s}} \frac{f_n\, g_n^+(0)}{1 + \frac{x_\gamma}{2} + \frac{\Lambda_T^n}{m_{H_s}}} \left(1 + \mathcal{O}\left(x_\gamma, \frac{\Lambda_{\rm QCD}}{m_{H_s}}\right)\right)$$

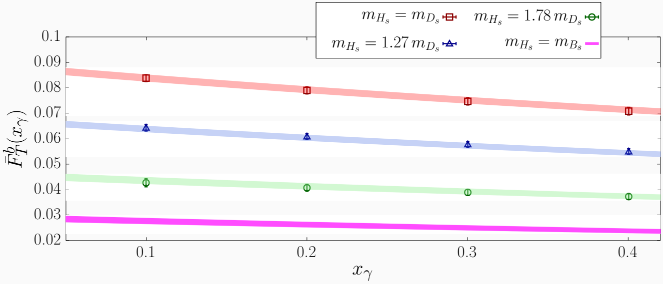
- Our strategy is to replace the tower of resonance contributions, with a single effective-pole

$$\bar{F}_T^b(x_\gamma, m_{H_s}) = \frac{1}{m_{H_s}} \frac{A + B\, x_\gamma}{1 + \frac{x_\gamma}{2} + \frac{\Lambda_T}{m_{H_s}}}$$

- A , B and Λ_T are free-fit parameters. Our Ansatz assumes $g_n^+ \propto \sqrt{m_{H_s}}$, which is consistent with our data.

Final results for $\bar{F}_T^b$

We have performed a global fit of the x_γ- and m_{H_s}-dependence of our lattice data, using the Ansatz in the previous slide.



- Our VMD-inspired Ansatz (which contains only 3 free-parameters) perfectly captures the x_γ and m_{H_s} dependence of the data.
- The magenta band corresponds to the extrapolated results at $m_{B_s} = 5.367$ GeV. Effective-pole located at $2m_{H_s} + \Lambda_T \simeq 10.4(1)$ GeV.
- As anticipated, this contribution turns out to be one order of magnitude suppressed w.r.t. F_{TV} and F_{TA}.

The strange-quark contribution $\bar{F}_T^s$

The hadronic tensor $H^{\mu\nu}_{\bar{T}_s}$ cannot be analytically continued to Euclidean spacetime
[$J^s_{\rm em} = q_s \bar{s}\gamma^\mu s$, $\hat{H}$ is the Hamiltonian]

$$H^{\mu\nu}_{\bar{T}_s}(p,k) = i\int_{-\infty}^{\infty} dt\, e^{i(m_{B_s}-E_\gamma)t}\,\langle 0|J^\nu_{\bar{T}}(0)\,J^s_{\rm em}(0,-\boldsymbol{k})|\bar{B}_s(\mathbf{0})\rangle$$

$$= \langle 0|J^\nu_{\bar{T}}(0)\frac{1}{\hat{H}-E_\gamma-i\varepsilon}J^{s,\mu}_{\rm em}(0,-\boldsymbol{k})|\bar{B}_s(0)\rangle$$

$$+ \langle 0|J^{s,\mu}_{\rm em}(0,-\boldsymbol{k})\frac{1}{\hat{H}+E_\gamma-m_{B_s}-i\varepsilon}J^\nu_{\bar{T}}(0)|\bar{B}_s(0)\rangle = H^{\mu\nu}_{\bar{T}_s,1}(p,k) + H^{\mu\nu}_{\bar{T}_s,2}(p,k)$$

- Analytic continuation $t \to -it$ possible only if the following positivity-conditions are met

$$\langle n|\hat{H}-E_\gamma|n\rangle > 0, \qquad \langle n|\hat{H}+E_\gamma-m_{B_s}|n\rangle > 0$$

- $|n\rangle$ is any of the intermediate-states that can propagate between the electromagnetic and tensor currents.
- The second condition is equivalent to $q^2 < m_n^2$ (m_n is the rest-energy of the intermediate state $|n\rangle$)...
- ...which is violated because the smallest m_n here is $2m_K$. In the case of the $b-$quark this is instead m_Υ. The first condition is instead always satisfied.

The spectral-density representation

The main idea for circumventing the problem of analytic continuation is to consider the spectral-density representation of the hadronic tensor $[E = m_{B_s} - E_\gamma]$

$$H^{\mu\nu}_{\bar{T}_s,2}(E,\boldsymbol{k}) = \lim_{\varepsilon\to 0^+} \int_{E^*}^{\infty} \frac{dE'}{2\pi} \frac{\rho^{\mu\nu}(E',\boldsymbol{k})}{E' - E - i\varepsilon} = \mathrm{PV} \int_{E^*}^{\infty} \frac{dE'}{2\pi} \frac{\rho^{\mu\nu}(E',\boldsymbol{k})}{E' - E} + \frac{i}{2}\rho^{\mu\nu}(E,\boldsymbol{k})$$

- The spectral-density $\rho^{\mu\nu}$ is related to the Euclidean correlation function $C^{\mu\nu}(t,\boldsymbol{k})$, which we can directly compute on the lattice, via

$$\underbrace{C^{\mu\nu}(t,\boldsymbol{k})}_{\text{lattice input}} = \int_{E^*}^{\infty} \frac{dE'}{2\pi} e^{-E't} \rho^{\mu\nu}(E',\boldsymbol{k})$$

- Unfortunately, determining $\rho^{\mu\nu}$ from $C^{\mu\nu}(t,\boldsymbol{k})$, which is computed on the lattice at a discrete set of times and with a finite accuracy, is not possible (inverse Laplace transform problem).

- The regularized quantity that we can evaluate, exploiting the Hansen-Lupo-Tantalo method [PRD 99 '19], is a smeared version of the hadronic tensor, obtained by considering non-zero values of the Feynman's ε

$$H^{\mu\nu}_{\bar{T}_s,2}(E,\boldsymbol{k};\varepsilon) = \int_{E^*}^{\infty} \frac{dE'}{2\pi} \frac{\rho^{\mu\nu}(E',\boldsymbol{k})}{E' - E - i\varepsilon}$$

The smeared form factor

The evaluation of the hadronic tensor at finite ε leads to a smeared form factor $\bar{F}_T^s(x_\gamma;\varepsilon)$. In the limit of vanishing ε one has

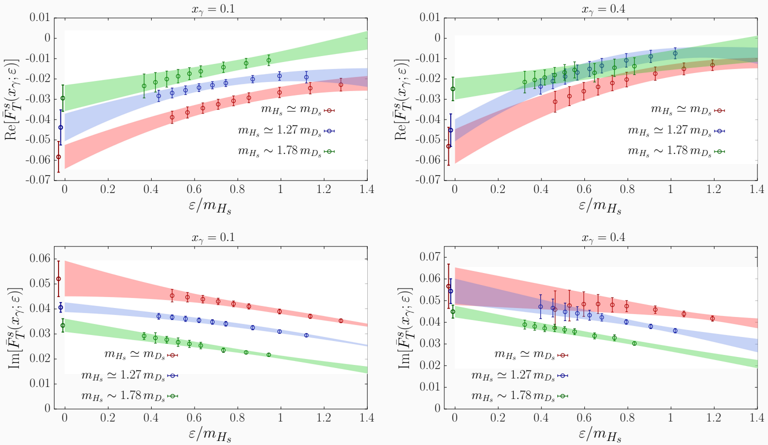
$$\lim_{\varepsilon\to 0^+} \bar{F}_T^s(x_\gamma;\varepsilon) = \bar{F}_T^s(x_\gamma)$$

- As we have shown in [Frezzotti et al. PRD 108 '23], the corrections to the vanishing ε limit are of the form

$$\bar{F}_T^s(x_\gamma;\varepsilon) = \bar{F}_T^s(x_\gamma) + A_1\,\varepsilon + A_2\varepsilon^2 + \mathcal{O}(\varepsilon^3)$$

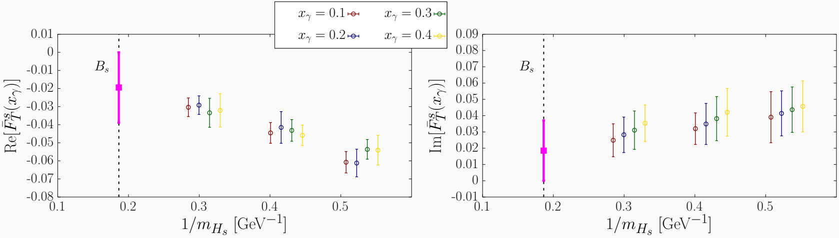
- The onset of the polynomial regime depends on the typical size $\Delta(E)$ of the interval around E on which the hadronic tensor is significantly varying, and one needs $\varepsilon \ll \Delta(E)$.
- We evaluated $\bar{F}_T(x_\gamma;\varepsilon)$ for several values of $\varepsilon/m_{H_s} \in [0.4:1.3]$, and then performed a polynomial extrapolation in ε.

The vanishing-ε extrapolation



Both the real and imaginary part of the smeared form factor $\bar{F}_T^s(x_\gamma;\varepsilon)$ show an almost linear behaviour at small ε. Besides the polynomial extrapolations, we have performed additional model-dependent, non-polynomial, extrapolations, to have a conservative estimate of the possible systematics associated to the vanishing-ε limit.

$\bar{F}_T^s$ at the physical mass $m_{B_s} \simeq 5.367$ GeV



- Very small x_γ dependence observed.

- To have a conservative error estimate, we take the results at the largest simulated mass $m_{H_s} \simeq 1.78\, m_{D_s}$ as a bound for the value of the form factor at the physical point, $m_{H_s} = m_{B_s}$.