Toward Extracting B LCDAs from Data

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based on works with

- *2203.15679* – Thorsten Feldmann & Philip Lüghausen
- *2210.09832* – Bastian Kubis, Stephan Kürten, Marvin Zanke

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Motivation

- goal: extract as much information on leading-twist $\overline{B}$ light-cone distribution amplitude (LCDA) $\phi_+(\omega)$ from information on $\overline{B} \to \gamma^{(*)}$ form factors
  - $B \to \gamma$ form factors are function of single kinematic variable E_γ
  - collinear factorization of FFs at large $E_\gamma \sim M_B/2$

    [Korchemsky et al. hep-ph/9911427; many developments since; see Martin's talk]
  - dominant sensitivity to "inverse moment" of leading LCDA ϕ_+

$$L_0 = \int_0^\infty \frac{d\omega}{\omega} \phi_+(\omega)$$

- collinear factorization suffers from soft contributions even at $E_\gamma = M_B/2$, i.e., maximal E_γ in $B \to \gamma \ell^- \overline{\nu}$ decay
  - QCD sum rule approach w/ hard-to-quantify systematic unc. [Braun, Khodjamirian 1210.4453]
  - Photon LCDA sum rule suffers from similar problems [Wang, Shen 1803.06667]
  - remains systematic theory barrier to cleanly extract ϕ_+ properties

Possible Solution?

- Several groups suggested investigating similar process with off-shell photon

[Wang 1606.03080; Bharucha et al. 2102.03193; Beneke et al. 2102.10060; Ivanov, Melikhov 2107.07247]

$$\overline{B}(p) \to \gamma^{(*)}(q)[\to \ell'^{+}\ell'^{-}]\ell^{-}(k_1)\overline{\nu}(k_2) \qquad \ell \neq \ell'$$

- soft contributions are expected to increase due to resonant enhancement at $q^2 > 0$
- however: apparent suppression of soft contributions at $q^2 < 0$ or $k^2 < 0$
  - not an "unphysical process"
  - form factors at $q^2 < 0$ describe $\ell'^{-}\overline{B} \to \ell'^{-}\ell^{-}\overline{\nu}$ scattering

Suggested Strategy

- collect FF data (experimental, lattice) at $q^2, k^2 \geq 0$
- derive dispersion relations to extrapolate FFs to a fixed value of $q_0^2 < 0$ and/or $k_0^2 < 0$ 1st part
- compare extrapolated results with collinear factorization predictions to extract information on B-meson LCDAs 2nd part

Dispersion Relations for $B \to \gamma^*$ Form Factors

Aim

1. project hadronic Lorentz tensor onto a *basis* of 4 scalar-valued hadronic form factors $F_i(q^2, k^2)$
2. determine dispersion relations that describe evolution of the F_i in both q^2 (weak momentum transfer) and k^2 (e.m. momentum transfer)

Requirements: choice of basis must be free of "artificial" singularities (i.e., of kinematic origin)

- some suspicious results in the literature [Ivanov,Melikhov 2107.07247 v1&v2]
- collinear factorization results known for 3 out of 4 FFs [Beneke et al. 2102.10060]

Ward Identity and all that ...

using currents $J^\mu_{\text{had}} = \overline{u}\gamma^\mu(1-\gamma_5)b$ and $J^\mu_{\text{lep}} = \overline{\ell}\gamma^\mu(1-\gamma_5)\nu$, define

$$Q_B T^{\mu\nu}_{\text{had}} = \int dx \exp^{iqx} \langle 0|T\{J^\mu_{\text{e.m.}}(x) J^\nu_{\text{had}}(0)\}|B^-\rangle$$

$$\langle \ell^- \overline{\nu}_\ell \gamma^* | J_{\text{lep},\nu}(0) J_{\text{had}}{}^\nu(0)|B^-\rangle = e Q_B \varepsilon^*_\mu \left[T^{\mu\nu}_{\text{had}} + T^{\mu\nu}_{\text{FSR}}\right] \left[\overline{u}\gamma_\nu(1-\gamma_5)v\right]$$

- only sum of tensors fulfil Ward identity

$$q_\mu \left[T^{\mu\nu}_{\text{had}} + T^{\mu\nu}_{\text{FSR}}\right] = 0 = q_\mu \left[T^{\mu\nu}_{\text{had,h.}} + T^{\mu\nu}_{\text{had,inh.}} + T^{\mu\nu}_{\text{FSR}}\right] = q_\mu \left[T^{\mu\nu}_{\text{had,h.}}\right]$$

- T_{FSR} is known, T_{had} is described in terms of $\overline{B} \to \gamma^*$ form factors
- split hadronic tensor in homogeneous and inhomogeneous parts
  - inhomogeneous part chosen such that $T_{\text{had,inh.}} + T_{\text{FSR}}$ fulfils Ward identity in sum
  - homogeneous part $T_{\text{had,h.}}$ fulfils Ward identity on its own

Issues and Approach

issues:

- Ward identity fixes $T_{\text{had,inh.}}$ only incompletely
  - finding: different choices of $T_{\text{had,inh.}}$ can induce kinematic singularities in homogeneous part despite restoring the Ward identity!

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- Ward identity fixes $T_{\text{had,inh.}}$ only incompletely
  - finding: different choices of $T_{\text{had,inh.}}$ can induce kinematic singularities in homogeneous part despite restoring the Ward identity!
- for purely electromagnetic form factors, Bardeen-Tung-Tarrach (BTT) procedure yields FF basis w/o kinematic singularities [Bardeen, Tung 1968; Tarrach 1975]
  - not applicable: $B^- \to \gamma^*$ FFs describe both e.m. and weak currents

our approach [Kubis, Kürten, DvD, Zanke 2210.09832]

1. clarify parametrization of $T_{\text{had,inh.}}$; check against explicit Chiral Perturbation Theory results for $K \to \gamma^* \ell^- \overline{\nu}$ FFs
2. adapt BTT procedure for use in combined e.m.- and weak-current induced FFs $\to$ describe $T_{\text{had,h.}}$ in terms of FFs free of kinematic singularities

Step 1: Generic Form for Inhomogeneity

1. Ward identity is satisfied for inhomogeneities of form:

$$T^{\mu\nu}_{\text{had,inhom}} = -f_B\left[a g^{\mu\nu} + b\frac{k^\mu k^\nu}{k\cdot q} + c\frac{k^\mu q^\nu}{k\cdot q} + (1-b)\frac{q^\mu k^\nu}{q^2} + (1-a-c)\frac{q^\mu q^\nu}{q^2}\right],$$

2. in the literature

Label	a	b	c	$T^{\mu\nu}_{\text{H,inhom.}}(k,q)$
$\mathcal{A}$	1	$\frac{2(k\cdot q)}{2(k\cdot q)+q^2}$	0	$-f_B\left[g^{\mu\nu} + \frac{(2k^\mu+q^\mu)k^\nu}{2(k\cdot q)+q^2}\right]$
$\mathcal{B}$	0	$\frac{k\cdot q}{k\cdot q+q^2}$	$\frac{k\cdot q}{k\cdot q+q^2}$	$-f_B\frac{(k+q)^\mu(k+q)^\nu}{k\cdot q+q^2}$
$\mathcal{C}$	0	1	1	$-f_B\frac{k^\mu(k+q)^\nu}{k\cdot q}$
$\mathcal{D}$	0	0	0	$-f_B\frac{q^\mu(k+q)^\nu}{q^2}$

3. difference between any of these choices is homogeneous but shuffles singularities from inhomogeneous part into homogeneous part or vice versa

4. we identify only one choice ($\mathcal{A}$) which leads to $1/[k^2 - M_B^2]$ pole only in the pseudoscalar form factor
   compatible with ChPT results in $K \to \gamma^*$ form factors

[Bijnens et al. hep-ph/9209261]

1. the analogue to the QED Ward identity for the weak current reads

$$k_\nu T^{\mu\nu}_{\text{had,hom}} = T^{\mu}_{\text{had,P,hom}}$$

   - $T_{\text{had,P}}$ is obtained from T_{had} by replacing $J_{\text{had}} \to J_{\text{had,P}} = (m_b + m_u)[\overline{u}\gamma_5 b]$
   - ditto for the homogeneous parts

2. split the homogeneous tensor into two parts:

$$T^{\mu\nu}_{\text{had,hom}} = \tilde{T}^{\mu\nu}_{\text{had,hom}} + \frac{k^\nu}{k^2} T^{\mu}_{\text{had,P,hom}} \qquad k_\nu \tilde{T}^{\mu\nu}_{\text{had,hom}} = 0$$

3. apply the regular BTT procedure to $\tilde{T}^{\mu\nu}_{\text{had,hom}}$ and $T^{\mu}_{\text{had,P,hom}}$

Result

$$T^{\mu\nu}_{\text{had,hom.}} = \frac{1}{M_B}\left[(k\cdot q)g^{\mu\nu} - k^\mu q^\nu\right]F_1 + \frac{1}{M_B}\left[\frac{q^2}{k^2}k^\mu k^\nu - \frac{k\cdot q}{k^2}q^\mu k^\nu + q^\mu q^\nu - q^2 g^{\mu\nu}\right]F_2$$
$$+ \frac{1}{M_B}\left[\frac{k\cdot q}{k^2}q^\mu k^\nu - \frac{q^2}{k^2}k^\mu k^\nu\right]F_3 + \frac{i}{M_B}\varepsilon^{\mu\nu\rho\sigma}k_\rho q_\sigma F_4$$

well-defined spin/parity quantum numbers:

- F_1, F_2: axialvector form factors; k^2 poles are $J^P = 1^+$, e.g., the B_1
- F_3: pseudoscalar form factor; k^2 poles are $J^P = 0^-$, e.g., the B
- F_4: vector form factor; k^2 poles are $J^P = 1^-$, e.g., the B^*

► diagonalizes the rate in the limit $m_\ell \to 0$, i.e., $d\Gamma/dq2 \not\ni F_i F_j^* + F_i^* F_j$ (except for FSR terms)

Systematic Parametrization of $\phi_+(\omega)$

Definition

matrix element of light-cone operator in HQET normalized to the matrix element of the corresponding local operator [Grozin hep-ph/9607366]

$$\tilde{\phi}_+(\tau;\mu) = \frac{\langle 0|\overline{q}(\tau n)\,[\tau n, 0]\,\not{n}\gamma_5\,h_v(0)|B(v)\rangle}{\langle 0|\overline{q}(0)\,\not{n}\gamma_5\,h_v(0)|B(v)\rangle}\,.$$

with $n^2 = 0$

Fourier transform commonly used:

$$\phi_+(\omega) = \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi}\,\tilde{\phi}_+(\tau)$$

more LCDAs exist, but let's start somewhere "easy"

State of the Art

- ϕ_+ is modelled, typically as an exponential ansatz

- ϕ_+ develops a non-exponential “radiative tail” under RG evolution

  [Lee, Neubert hep-ph/0509350]

  - some models account for this, chiefly the one from the same paper by Lee & Neubert (pin the tail on an exponential model)

- further models inspired by π LCDA / experience in modelling $\gamma^* \to \pi\gamma$

  [Beneke et al. 1804.04962]

Properties of ϕ_+ / $\tilde{\phi}_+$

P1 $\tilde{\phi}_+(\tau)$ is analytic in the half plane $\text{Im}\,\tau < 0$
P2 $\tilde{\phi}_+(\tau)$ is analytic on the real axis, except for a (log.) singular point at $\tau = 0$.
$\Rightarrow$ $\tilde{\phi}_+(\tau)$ is integrable on the real axis and

$$\int_{-\infty}^{+\infty} d\tau \phi_+(\tau) = 0$$

P3 $\tilde{\phi}_+(\tau - i\varepsilon)$ converges toward $\tilde{\phi}_+(\tau)$ almost everwhere for $\tau \in \mathbb{R}$

P1 to P3 imply that the Fourier transform

$$\phi_+(\omega) = \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} \tilde{\phi}_+(\tau)$$

exists and

$$\int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} \left|\tilde{\phi}_+(\tau)\right|^2 = \int_0^{\infty} d\omega \left|\phi_+(\omega)\right|^2 < \infty$$

Idea: 2-Norm

define a "2-norm" for ϕ_+

$$\chi \equiv \int_{-\infty}^{\infty} \frac{d\tau}{2\pi} \left|\tilde{\phi}_+(\tau)\right|^2 |1 + i\omega_0\tau|^2$$

which is finite but unknown

ω_0: intrinsic hadronic scale

- find parametrization for ϕ_+ that diagonalizes χ, i.e.:

$$\phi_+(\tau) = \frac{1}{\omega_0} \sum_n a_n f_n(\tau) \qquad \text{so that} \qquad \omega_0\chi \sim \sum_n |a_n|^2$$

- if χ were known, then coefficients would be bounded: $-\sqrt{\chi} < a_n < \sqrt{\chi}$

Idea: Conformal Map

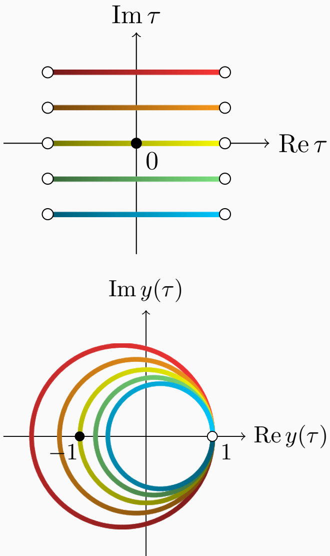
- map lower halfplane in τ to interior of unit circle in y

$$\tau \mapsto y(\tau) = \frac{i\omega_0\tau - 1}{i\omega_0\tau + 1}$$

- account for weight $|1 + i\omega_0\tau|^2$ in the norm and Jacobian of the map
- Taylor expand analytical remainder of ϕ_+ around $y = 0$ ($\tau = -i/\omega_0$)

$$\phi_+(\tau) = \frac{1}{(1 + i\omega_0\tau)\,(1 + i\omega_0\tau)} \sum_n a_n y^n(\tau)$$

$$\chi = \frac{1}{2\omega_0} \sum_n |a_n|^2$$



Consequences

- generalisation of an existing model
  - replicate by keeping only the $n = 0$ term
- even truncated parametrisation can replicate features of the radiative tail
  - "[*it*] *generalizes the Grozin-Neubert relations* [...] *to one-loop accuracy* [...]"
  - Lee-Neubert model is reasonably replicated with truncation at 3rd order
- "pathological" models can only be replicated with some difficulty
  - example: free parton model

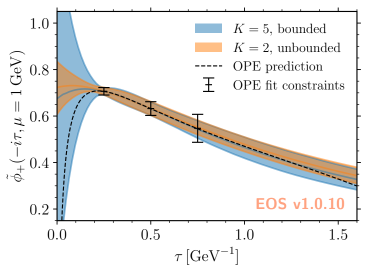
- several way to account for RG evolution with the parametrization
  - preferred and numerically fastest: evolve coefficients a_n, evolution matrix is only dependent on scales and ω_0
- inverse moment

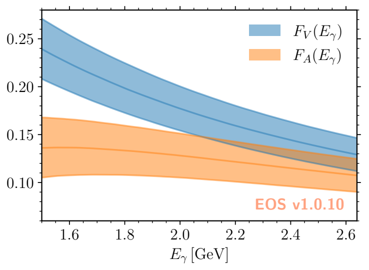
$$L_0 = \frac{1}{\lambda_B} = \int_0^\infty \frac{d\omega}{\omega}\phi_+(\omega) = \frac{1}{\omega_0}\sum_{n=0}^{\infty}\frac{a_{2n}}{2n+1}$$

- collinear factorization result for $B^- \to \gamma$ form factors, available in EOS

Pheno Outlook

- global fit to all available information on ϕ_+:
  - $B \to \gamma^{(*)}$ FF from exp / Lattice QCD; short-distance expansion of $\phi_+(\tau)$
- adhoc assumption so far: value of $\chi \to$ Lattice QCD?
- right: preliminary plots from sensitvity study, using ficticious inputs in underconstrained fit





Conclusion

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- theory work onging, need to get rid of soft contributions to increase sensitivity to ϕ_+ in collinear factorization results for $B \to \gamma^{(*)}$
  - one approach: extrapolate to spacelike momentum transfer q^2 / k^2
- part 1: revisit dispersion relations to ensure that extrapolation works
  - clarified some issues in the literature
- part 2: go beyond adhoc modelling of ϕ_+ with systematic parametrisation
  - Lattice QCD input on 2-norm of ϕ_+ would be very helpful