Radiative leptonic decays with an energetic / virtual photon

M. Beneke (TU München)

Workshop on radiative leptonic B decays,
Marseille, February 28 - March 1, 2024

MB, J. Rohrwild, 1110.3228
MB, V.M. Braun, Y. Ji, Y.B. Wei, 1804.04962
MB, C. Bobeth, Y-m. Wang, 2008.12494
MB, Böer, Rigatos, Vos, 2102.10060

Three reasons to study $B \to \gamma\ell\nu, \gamma\ell^+\ell^-, \ell\bar{\nu}_\ell\ell^{(\prime)}\bar{\ell}^{(\prime)}$ (at large $E_\gamma \gg \Lambda_{\rm QCD}$)

- Radiative electroweak FCNC decay $B \to \gamma\ell^+\ell^-$ less hadronic than $K^{(*)}\ell^+\ell^-$ $\leftarrow$ unfortunately not true

- For a measurement of λ_B and the leading-twist B-meson LCDA.
  Appear in almost all exclusive B decays in LP in the heavy-quark expansion (spectator scattering)

$$iF_{\rm stat}(\mu)\Phi_{B+}(\omega,\mu) = \frac{1}{2\pi}\int dt\, e^{it\omega}\, \langle 0|(\bar{q}_s Y_s)(tn_-)\not{n}_-\gamma_5(Y_s^\dagger h_v)(0)|\bar{B}_v\rangle_\mu$$

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\,\Phi_{B+}(\omega,\mu),$$

- Factorization theory beyond LP

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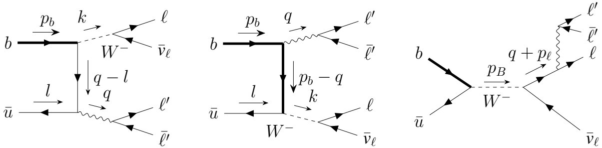
- Factorization theory beyond LP

- For their own $\leftarrow$ this workshop

Basic physics

$$T^{\mu\nu} = \int d^4x\, e^{iqx} \langle 0|\mathrm{T}\{j^\mu_{\rm em}(x)(\bar{u}\gamma^\nu(1-\gamma_5)b)(0)\}|B^-\rangle$$



(a) (b) (c)

(a) Short-distance $x \sim 1/m_b$ heavy quark-W^* vertex
Light-cone expansion of $j_{\rm em}(x) j_{\rm weak}(0)$, $x^2 \sim 1/(m_b\Lambda) \ll 1/\Lambda^2$
Even when $q^2 = 0$, as long as $n_+q \sim m_b$ (or $\gg \Lambda$) $[n_+q = 2E_\gamma$ for $q^2 = 0]$.

(b), (c) Always, short-distance, f_B

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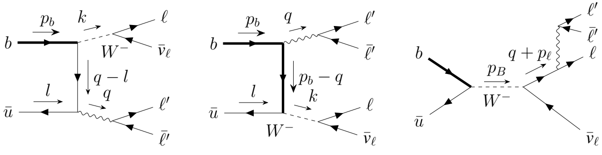

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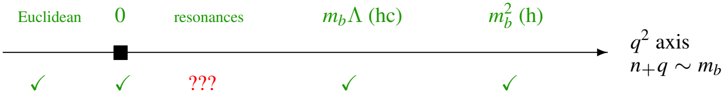


Basic physics

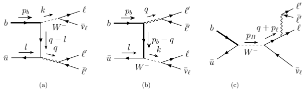
radiative-semileptonic ($\gamma\ell\nu$)	• LP ✓ • main issue are *soft* power corrections
four-lepton ($\ell\nu\ell^{(\prime)}\ell^{(\prime)}$)	• time-like photon virtuality $\rightarrow$ resonances • longitudinal form factor • local and global parton-hadron duality violation
radiative-electroweak FCNC ($\gamma\ell\ell$)	• when real photon from FCNC weak current, time-like virtual photon $\rightarrow$ resonances .... [as above] • four-quark operators, charmonium resonances • CP asymmetries (very small for B_s)
double-radiative ($\gamma\gamma$)	• as above, without resonances

$B^- \to \ell \bar{\nu}_\ell \ell^{(\prime)} \bar{\ell}^{(\prime)}$

2102.10060, with P. Böer, P. Rigatos and K.K.Vos

Other work employing factorization methods:
Bharucha, Kindra, Mahajan 2102.03193; Wang, Wang, Wei 2111.11811
Resonance / hadronic models and dispersion relations:
Danilina, Nikitin, 2017 + 2309.11164; Danilina, Nikitin, Toms, 1911.03670; Ivanov, Melikhov, 2107.07247, 2204.2792; Kürten, Zanke, Kubis, van Dyk, 2210.09832

Form factor decomposition



$$T^{\mu\nu} = F_1 g^{\mu\nu} + F_2 \epsilon^{\mu\nu\alpha\beta} k_\alpha q_\beta + F_3 k^\mu q^\nu + F_4 q^\mu k^\nu + F_5 k^\mu k^\nu + F_6 q^\mu q^\nu \qquad F_i = F_i(k^2, q^2)$$

$6 \to 4$ [Ward identity] $\to 3$ [massless lepton limit]

$$= (g^{\mu\nu} v \cdot q - v^\mu q^\nu)\, \hat{F}_{A_\perp} + i\,\epsilon^{\mu\nu\alpha\beta} v_\alpha q_\beta F_V - \hat{F}_{A_\parallel} v^\mu q^\nu + (q^\mu, k^\nu) \text{ terms}$$

Include final-state emission in $F_{A_\perp}$, $F_{A_\parallel}$. Redefine $F_{A_\parallel}$ to correspond to longitudinally polarized virtual photon and *vanishes as* $\mathcal{O}(q^2)$ *as* $q^2 \to 0$

For non-identical lepton flavours

$$\frac{d^2\text{Br}\left(B^- \to \ell\,\bar\nu_\ell\,\ell'\bar\ell'\right)}{dq^2\,dk^2} = \frac{\tau_B G_F^2 |V_{ub}|^2 \alpha_{\text{em}}^2}{2^8 3^2 \pi^3 m_B^5} \frac{\sqrt{\lambda}}{q^2} \sqrt{1 - \frac{4m_{\ell'}^2}{q^2}} \left(1 - \frac{m_\ell^2}{k^2}\right)$$
$$\times \left(8k^2 \left(m_B^2 + q^2 - k^2\right)^2 |F_{A_\perp}|^2 + 8k^2 \lambda\, |F_V|^2 + \frac{\lambda^2}{q^2} |F_{A_\parallel}|^2\right)$$

Keep lepton mass in phase-space.

Factorization

$$\text{QCD} \quad \overset{\text{remove h}}{\longrightarrow} \quad \text{SCET}_\text{I} \quad \overset{\text{remove hc}}{\longrightarrow} \quad \text{SCET}_\text{II}$$

Accuracy: $\mathcal{O}(\alpha_s)$ at LP, $\mathcal{O}(\alpha_s^0)$ at NLP Λ/n_+q

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Accuracy: $\mathcal{O}(\alpha_s)$ at LP, $\mathcal{O}(\alpha_s^0)$ at NLP Λ/n_+q

Leading power (rigorous, all orders)

$$T^{\mu\nu}(p,q) = 2\, C_V^{(A0)} \int d^4x\, e^{iqx} \langle 0|T\left\{ j^{\mu}_{q,\text{SCET}_\text{I}}(x), [\bar{q}_\text{hc}\gamma_\perp^\nu P_L h_v](0)\right\}|B_v^-\rangle$$

Only $F_L = (F_V + F_{A_\perp})/2 \neq 0$ at LP due to helicity conservation for $n_+q \gg \Lambda$.
$F_R^\text{LP} = F_{A_\parallel}^\text{LP} = 0$.

$$F_L^\text{LP} = C_V^{(A0)}(\mu)\, \frac{Q_u F_B(\mu) m_B}{n_+q} \int_0^\infty d\omega\, \underbrace{\phi_+^B(\omega;\mu)}_{\text{B-LCDA}} \times \underbrace{\frac{J(n_+q, q^2, \omega; \mu)}{\omega - n_-q - i0^+}}_{\text{Generates rescattering phase}} \qquad [n_-q = q^2/n_+q]$$

Complex q^2-dependent inverse moment of the B-LCDA:

$$\frac{1}{\lambda_B^+(n_-q)} \equiv \int_0^\infty d\omega\, \frac{\phi_+^B(\omega)}{\omega - n_-q - i0^+}$$

Beyond leading power (murky, follows 2008.12494, $B \to \gamma\ell\bar{\ell}$)

$$F_L^{\rm NLP} = \underbrace{\xi(q^2, v\cdot q)}_{\text{set to } -r_{\rm LP}\,\times\, F_L^{\rm LP}} + \frac{Q_\ell f_B}{2v\cdot q}\,,$$

$$F_R^{\rm NLP} = \frac{F_B}{n_+q}\,\frac{m_B Q_u}{n_+q}\left(1+\frac{n_-q}{\lambda_B^+(n_-q)}\right) - \frac{F_B m_B Q_b}{q^2-2m_b v\cdot q} - \frac{Q_\ell f_B}{2v\cdot q}$$

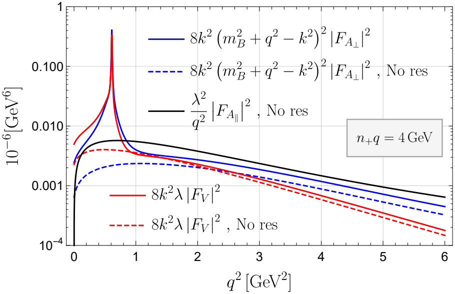
$$\tilde{F}_{A_\parallel}^{\rm NLP} = \frac{4F_B m_B Q_u}{n_+q}\,\frac{n_-q}{n_+q}\left(\frac{1}{\lambda_B^+(n_-q)}-\frac{1}{\lambda_B^-(n_-q)}\right) - \frac{2F_B Q_u}{n_+q}\left(1+\frac{n_-q}{\lambda_B^+(n_-q)}\right)$$
$$+\frac{2F_B m_b Q_b}{2v\cdot q m_b - q^2} - \frac{2f_B Q_\ell}{2v\cdot q} + \underbrace{\xi'(q^2, v\cdot q)}_{\text{set to } 0}\,,$$

Resonances in the $q^2 \sim \Lambda^2$ region are technically $(\Lambda/n_+q)^2$ and can be added without double counting. They dominate any q^2-bin which contains them due to global *parton-hadron duality violation*.

$$F_{L(R)}^{\rm res} = \sum_{V=\rho^0,\,\omega} c_{\rm V}\,\frac{f_V m_V}{m_V^2-q^2-im_V\Gamma_V}\,\frac{1}{2}\left(\frac{2m_B}{m_B+m_V}V^{B\to V}(k^2) \pm \frac{m_B+m_V}{v\cdot q}A_1^{B\to V}(k^2)\right)$$

$$F_{A_\parallel}^{\rm res} \to 0 \qquad \text{[should be improved]}$$

Di-lepton invariant mass spectrum



Left: Contribution to the q^2 distribution from separate FFs, $n_+q = 4\,\mathrm{GeV}$.

- Longitudinal polarization dominates except at very small q^2

$$\frac{d^2\mathrm{Br}^{(F_{A_\parallel})}}{dq^2\,dk^2} \Big/ \frac{d^2\mathrm{Br}^{(F_L)}}{dq^2\,dk^2} \overset{\approx}{\underset{q^2\to 0}{}} \frac{27\pi^2 q^2}{4m_B^2}$$

- π^2 from rescattering phase!

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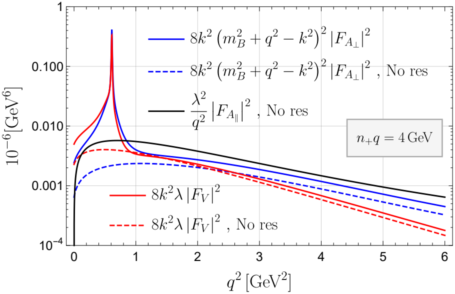

Left: Contribution to the q^2 distribution from separate FFs, $n_+q = 4\,\mathrm{GeV}$.

Bottom: Dependence of F_L on λ_B at $n_+q = 4\,\mathrm{GeV}$.

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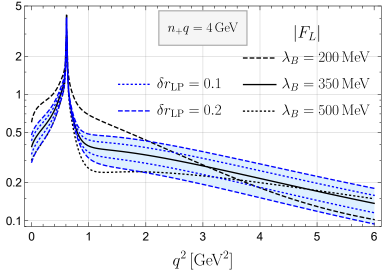


Binned branching fractions (non-identical leptons)

Decay	q^2 bin [GeV^2]	LP LO	LP NLO	NLP loc	NLP $+\xi$	Total +res	Uncertainty $\mu_{h,hc}$	Uncertainty $r_{\rm LP}$	Uncertainty λ_B	Uncertainty tot
$\mu^-\mu^+e^-\bar\nu_e$	$[4m_\mu^2, 0.96]$	0.58	0.51	0.70	0.48	1.57	$^{+0.02}_{-0.02}$	$^{+0.35}_{-0.29}$	$^{+1.33}_{-0.40}$	$^{+1.37}_{-0.49}$
	$[4m_\mu^2, 6]$	0.76	0.66	0.98	0.67	1.78	$^{+0.02}_{-0.02}$	$^{+0.43}_{-0.35}$	$^{+1.46}_{-0.47}$	$^{+1.52}_{-0.58}$
	$[1, 6]$	0.18	0.14	0.26	0.18	0.20	$^{+0.00}_{-0.00}$	$^{+0.08}_{-0.06}$	$^{+0.11}_{-0.06}$	$^{+0.14}_{-0.08}$
	$[1.5, 6]$	0.10	0.08	0.15	0.10	0.11	$^{+0.00}_{-0.00}$	$^{+0.05}_{-0.04}$	$^{+0.03}_{-0.03}$	$^{+0.06}_{-0.05}$
	$[2, 6]$	0.062	0.042	0.090	0.062	0.068	$^{+0.001}_{-0.001}$	$^{+0.030}_{-0.022}$	$^{+0.002}_{-0.012}$	$^{+0.030}_{-0.025}$
$e^-e^+\mu^-\bar\nu_\mu$	$[q^2_{\rm min}, 0.96]$	1.23	1.04	1.23	0.81	2.28	$^{+0.03}_{-0.04}$	$^{+0.66}_{-0.53}$	$^{+2.40}_{-0.67}$	$^{+2.49}_{-0.86}$
	$[1, 6]$	0.18	0.14	0.26	0.18	0.20	$^{+0.00}_{-0.00}$	$^{+0.08}_{-0.06}$	$^{+0.11}_{-0.06}$	$^{+0.14}_{-0.08}$

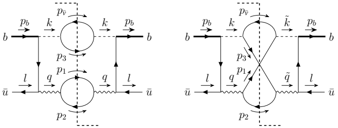
- BR in units of 10^{-8}. Cut $n_+q > 3$ GeV requires measurement of k^2.
- NLP is sizable.
- Rate drops by factor 10 - 20 when bin exlcudes the resonance region.
- Difference between electrons and muons in total rate from phase-space.
- Sensitivity to B-LCDA decreases with $q^2_{\rm min}$ of bin.

Binned branching fractions (identical leptons)

Decay	$q^2_{\rm low}$ bin $[\mathrm{GeV}^2]$	LP LO	LP NLO	NLP loc	NLP $+\xi$	Total $+$res	Uncertainty $\mu_{h,hc}$	$r_{\rm LP}$	λ_B	tot
$\mu^-\mu^+\mu^-\bar\nu_\mu$	$[4m_\mu^2, 0.96]$	0.58	0.51	0.71	0.49	1.54 (1.77)	$^{+0.02}_{-0.02}$	$^{+0.35}_{-0.29}$	$^{+1.29}_{-0.39}$	$^{+1.34}_{-0.48}$
	$[4m_\mu^2, 6]$	0.74	0.64	0.97	0.67	1.75 (2.00)	$^{+0.02}_{-0.02}$	$^{+0.42}_{-0.34}$	$^{+1.40}_{-0.45}$	$^{+1.46}_{-0.56}$
	$[1, 6]$	0.15	0.11	0.25	0.17	0.19 (0.21)	$^{+0.00}_{-0.00}$	$^{+0.07}_{-0.05}$	$^{+0.10}_{-0.05}$	$^{+0.12}_{-0.06}$
	$[1.5, 6]$	0.08	0.06	0.14	0.10	0.11 (0.11)	$^{+0.01}_{-0.01}$	$^{+0.04}_{-0.03}$	$^{+0.03}_{-0.02}$	$^{+0.05}_{-0.04}$
	$[2, 6]$	0.04	0.03	0.08	0.06	0.06 (0.07)	$^{+0.00}_{-0.00}$	$^{-0.02}_{-0.02}$	$^{+0.00}_{-0.01}$	$^{+0.03}_{-0.02}$
$e^-e^+e^-\bar\nu_e$	$[q^2_{\rm min}, 0.96]$	1.22	1.03	1.23	0.80	2.23 (2.57)	$^{+0.04}_{-0.06}$	$^{+0.65}_{-0.53}$	$^{+2.33}_{-0.65}$	$^{+2.42}_{-0.82}$
	$[1, 6]$	0.15	0.12	0.25	0.18	0.20 (0.22)	$^{+0.00}_{-0.00}$	$^{+0.07}_{-0.05}$	$^{+0.10}_{-0.05}$	$^{+0.12}_{-0.07}$

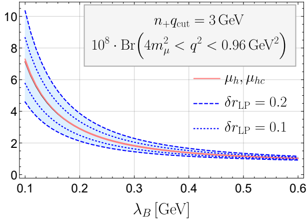
- BR in units of 10^{-8}.
- Identify the invariant masses of the two $\ell^-\ell^+$ pairings as $q^2_{\rm low} < q^2_{\rm high}$. Require $n_+ q_{\rm low} > 3$ GeV and $n_+ q_{\rm high} > 3$ GeV for both by measuring $k^2_{\rm low}$ and $k^2_{\rm high}$, respectively. ($k^2_{\rm low}$ is not necessarily lower than $k^2_{\rm high}$.)
- Can only be implemented numerically on the theory calculation:

$$\mathrm{Br}\left(B^- \to \ell\,\bar\nu_\ell\,\ell\bar\ell\right) = \mathrm{Br}\left(B^- \to \ell\,\bar\nu_\ell\,\ell'\bar\ell'\right) + \mathrm{Br}_{\rm int}\left(B^- \to \ell\,\bar\nu_\ell\,\ell\bar\ell\right)$$

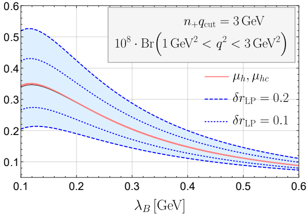


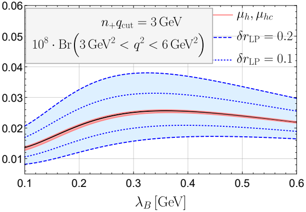
- Numerically the interference term is at most a few percent.

Sensitivity to λ_B and the B-LCDA



$$\frac{1}{\lambda_B^+(n_- q)} \equiv \int_0^\infty d\omega \, \frac{\phi_+^B(\omega)}{\omega - \frac{q^2}{n_+ q} - i0^+}$$





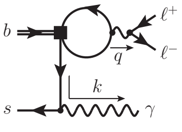
Small-q^2 bin has similar sensitivity as $B^- \to \gamma \ell \bar{\nu}_\ell$, but depends on resonance contribution. Higher q^2 retains some sensitivity, gradually decreasing.

$B_s \to \mu^+\mu^-\gamma$

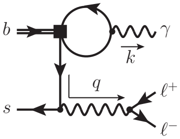
2008.12494, with C. Bobeth and Y. Wang

Basic features of $B_s \to \mu^+\mu^-\gamma$

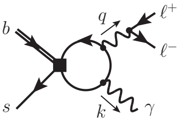
- Require an energetic photon, $E_\gamma > 1.5\,\mathrm{GeV} \sim m_B/2$
- Very rare, branching fraction $10^{-10} - 10^{-8}$ depending on the $q^2 = m^2_{\mu^+\mu^-}$ bin. Not yet observed.



A-type



B-type



NLP

- Theoretically shares features with $B \to \ell\nu\gamma$ ($\to$ B-LCDA at LP) and $B \to K^{(*)}\ell\ell$ (charmonium resonances, stay below $q^2 = 6\,\mathrm{GeV}^2$)
- Standard SCET calculation, except for light-meson resonances in the B-type contribution. Accuracy: $\mathcal{O}(\alpha_s)$ at LP, $\mathcal{O}(\alpha_s^0)$ at NLP Λ/E_γ

Structure of the theoretical result

LP amplitude

$$\overline{\mathcal{A}}_{\mathrm{type}-A} = ie\,\frac{\alpha_{\mathrm{em}}}{4\pi}\,\mathcal{N}_{\mathrm{ew}}\,\epsilon^{\star}_{\mu}\left\{\left(V_9^{\mathrm{eff}}(q^2) + \frac{2\,\overline{m}_b\,m_{B_q}}{q^2}\,V_7^{\mathrm{eff}}(q^2)\right)L_{V,\nu} + V_{10}^{\mathrm{eff}}(q^2)L_{A,\nu}\right\}\mathcal{T}^{\mu\nu}(k)$$

$$\overline{\mathcal{A}}_{\mathrm{type}-B} = ie\,\frac{\alpha_{\mathrm{em}}}{4\pi}\,\mathcal{N}_{\mathrm{ew}}\,\epsilon^{\star}_{\mu}\,\frac{4\,\overline{m}_b E_\gamma}{q^2}\,V_7^{\mathrm{eff}}(k^2=0)L_{V,\nu}\,\mathcal{T}^{\mu\nu}(q)$$

$$V_7^{\mathrm{eff}}(q^2) = C_7^{\mathrm{eff}}\,C_{T_1}^{(A0)}(q^2) + \ldots$$

$$V_9^{\mathrm{eff}}(q^2) = C_9^{\mathrm{eff}}(q^2)\,C_V^{(A0)}(q^2) + \ldots \qquad \text{[same as for } B\to X_s\ell\ell]$$

$$V_{10}^{\mathrm{eff}}(q^2) = C_{10}\,C_V^{(A0)}(q^2) + \ldots$$

$\mathcal{T}^{\mu\nu}(r)$ = $\mathrm{SCET_I}$ correlation function of electromagnetic and flavour-changing current [same as for $B\to X_s\ell\ell$]

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$$V_{10}^{\text{eff}}(q^2) = C_{10}\,C_V^{(A0)}(q^2) + \ldots$$

$\mathcal{T}^{\mu\nu}(r)$ = SCET_I correlation function of electromagnetic and flavour-changing current [same as for $B \to X_s\ell\ell$]

Resonance amplitude [Do no show other NLP contributions]

$$\overline{\mathcal{A}}_{\text{res}} = -ie\,\frac{\alpha_{\text{em}}}{4\pi}\,\mathcal{N}_{\text{ew}}\,\epsilon^{\star}_{\mu}\,(g_\perp^{\mu\nu} + i\varepsilon_\perp^{\mu\nu})\,\frac{m_{B_q}}{2}\,\frac{4\overline{m}_b E_\gamma}{q^2}\,V_7^{\text{eff}}(0)L_{V,\nu}\,\frac{c_V f_V m_V T_1^{B_q\to V}(0)}{m_V^2 - i m_V\Gamma_V - q^2}$$

Corresponds to $B_s \to V[\to \mu^+\mu^-]\gamma$
Resonances $\phi(1020)$, $\phi(1680)$, $\phi(2170)$ with widths 4.249(12), 150(50), 104(20) MeV

Global duality violation and form factors

- The resonance contribution to the differential branching fraction is formally $\mathcal{O}(\Lambda_{\rm QCD}^2/m_b^2)$ but dominates any q^2 bin, in which it is contained, if its width is small [MB, Buchalla, Neubert, Sachrajda, 2009]

$$R \equiv \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \, \frac{d\Gamma_{\rm res}}{dq^2} \Bigg/ \int_{q^2_{\rm min}}^{q^2_{\rm max}} dq^2 \, \frac{d\Gamma_{\rm LP}^{\rm type-\mathit{B}}}{dq^2} \approx 4\pi \left(\frac{c_V \lambda_{B_q} T_1^{B_q\to V}(0)}{Q_q F_{B_q}}\right)^2 \times \frac{f_V^2}{m_V \Gamma_V} \times \frac{1}{\ln \frac{q^2_{\rm max}}{q^2_{\rm min}}}$$

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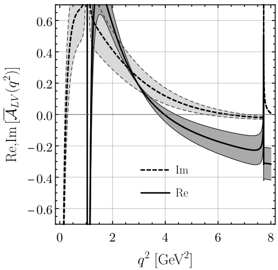
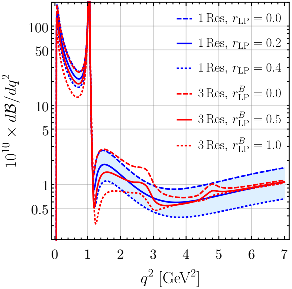
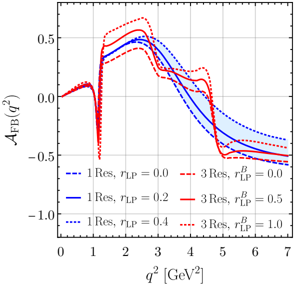
- Left-handed photon vectorial amplitude:



Zero of real part implies forward-backward asymmetry $\propto \cos\theta_\ell$, but its observation requires B tagging $\rightarrow$ not observable at LHCb.

Rate predictions for $B_s \to \gamma\mu^+\mu^-$





q^2 bin [GeV^2]	LP LO	LP NLO	NLP loc	NLP loc + A	NLP all	uncertainty of "NLP all" $\mu_{h,hc}$	λ_{B_q}, $\widehat{\sigma}^{(q)}_{B_1}$	$r_{\rm LP}$	total
$B_s \to \gamma\mu\bar{\mu}$									
$[4m_\mu^2,\ 6.0]$	2.32	2.96	3.81	4.03	12.43	$^{+0.11}_{-0.56}$	$^{+3.56}_{-1.42}$	$^{+1.39}_{-1.19}$	$^{+3.83}_{-1.93}$
$[2.0,\ 6.0]$	0.40	0.34	0.31	0.36	0.30	$^{+0.01}_{-0.04}$	$^{+0.21}_{-0.08}$	$^{+0.14}_{-0.11}$	$^{+0.25}_{-0.14}$
$[3.0,\ 6.0]$	0.30	0.22	0.19	0.22	0.21	$^{+0.01}_{-0.03}$	$^{+0.18}_{-0.07}$	$^{+0.10}_{-0.08}$	$^{+0.20}_{-0.10}$
$[4.0,\ 6.0]$	0.22	0.15	0.12	0.15	0.15	$^{+0.01}_{-0.02}$	$^{+0.14}_{-0.05}$	$^{+0.07}_{-0.05}$	$^{+0.16}_{-0.08}$
$[4m_\mu^2,\ 8.64]$	2.77	3.24	4.05	4.34	12.74	$^{+0.14}_{-0.60}$	$^{+3.85}_{-1.50}$	$^{+1.54}_{-1.31}$	$^{+4.15}_{-2.08}$

Bins above $q^2 > 2\,\mathrm{GeV}^2$ are theoretically on more solid ground but have branching fractions below 10^{-9}.