Structure-Dependent QED Corrections in Exclusive Leptonic B-Decays

European Research Council
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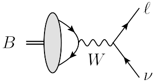
"Workshop on Radiative Leptonic B Decays"
Marseille - Feb 28, 2024

Cluster of Excellence
PRISMA+
Precision Physics, Fundamental Interactions and Structure of Matter

Why $B \to \ell\nu$?



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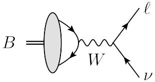

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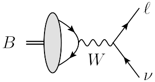

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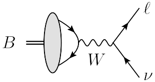

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- **Without** QED, hadronic effects are **well-understood**, with QCD uncertainties $\lesssim 1\%$

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|B(p)\rangle = i\,f_B\,p^\mu\,, \qquad f_B = 189.4 \pm 1.4\,\text{MeV}$$

[FNAL/MILC 1712.09262]

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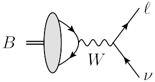

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- In the **exclusive** channel, QED corrections can be **sizeable**, competing with **QCD** uncertainties!
  $\longrightarrow$ A precise prediction is needed!

QED Corrections at High and Low Scales

- Electromagnetic corrections are sensitive to the **lepton mass** and the restriction on **additional radiation**, yielding large (double) logarithmic corrections

$$\alpha_{\mathrm{EM}} \log^{(2)} \frac{m_\ell^2}{m_B^2}\,, \qquad\qquad \alpha_{\mathrm{EM}} \log \frac{E_\gamma^2}{m_B^2}\,.$$

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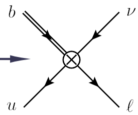

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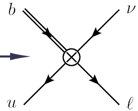
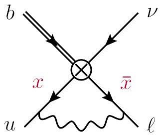
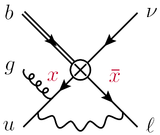


  - Radiation **softer** than Λ_{QCD} sees the meson as a **point-like** object. $\rightarrow$ description as a Yukawa theory.

QED Corrections at Intermediate Scales

- **Energetic** virtual photons with $E_\gamma \sim m_b$ and $\vec{p}_\gamma \parallel \vec{p}_\ell$ can **recoil** against the light **spectator** quark and transfer momentum between the lepton and the spectator.





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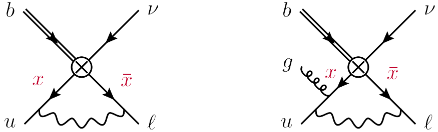

- The light partons are then **displaced** along the **lightcone**, and the hadronic currents become **non-local**. The corresponding hadronic matrix elements

$$\langle 0|\bar{q}(z_-)\dots h_v(0)|B\rangle\,, \qquad \langle 0|\bar{q}(z_-)\dots G_{\mu\nu}(y_-)\dots h_v(0)|B\rangle\,,$$

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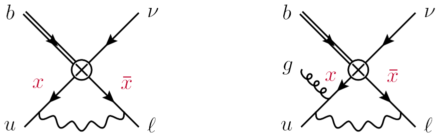

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- In the case of QED, due to the **external states** being **charged**, these distributions are **no longer process-universal** because QED is sensitive to the directions of the charged final state. $\longrightarrow$ see Martin's talk tomorrow!

A Multiscale Problem

Beyond leading order in QED, the process $B \to \ell\,\nu$ depends on a variety of both **static** and **dynamical** scales (here discussing only $\ell = \mu$)

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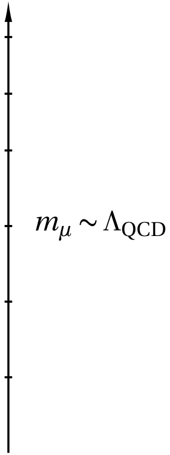

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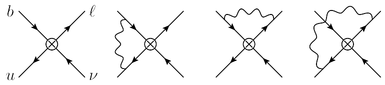

$m_\mu \sim \Lambda_{\mathrm{QCD}}$ description of meson by Fock states $|b\bar{q}\rangle + |b\bar{q}g\rangle + \ldots$ [Beneke et al (*2019*), JHEP10 232]

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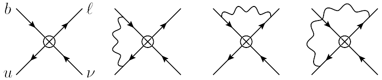

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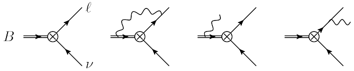

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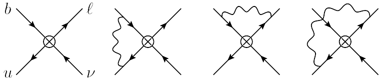

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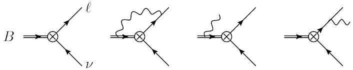

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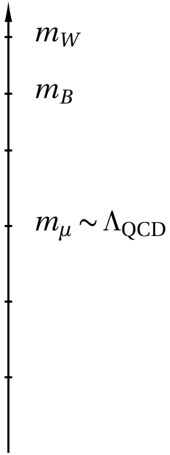

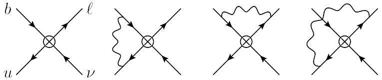

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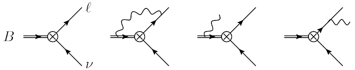

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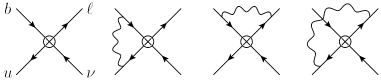

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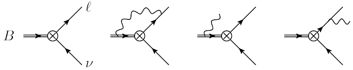

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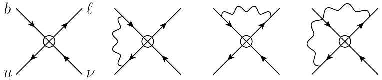

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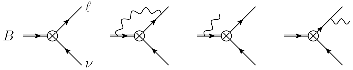

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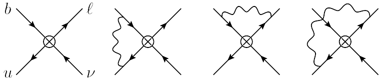

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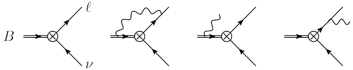

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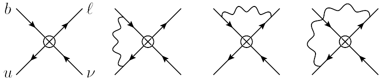

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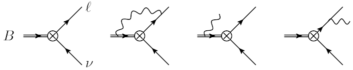

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→ **Factorization** of scales requires a number of different **EFT** constructions

(LEFT, HQET, $\text{SCET}_{\text{I}_+,\text{II}}$, bHLET, ...)

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- The **virtual** corrections above Λ_{QCD} are described by a **SCET** factorization formula. Recall: chirality-suppressed decay $\Rightarrow$ subleading-power in SCET.

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i\mathscr{A}^{\text{virt}}_{B\to\ell\nu} = &-\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_b} K_A(m_\ell) \bar{u}(p_\ell) P_L v(p_\nu) \\
&\cdot \left[H_A(m_b) S_A + \int d\omega \int_0^1 dx\, H_B(m_b, x) \cdot J_B(m_b\omega, x) \cdot S_B(\omega) \right]
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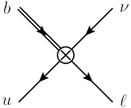


$S_A = \langle 0|\bar{q}(0)\not{n}\, P_L\, h_v(0)|B\rangle$

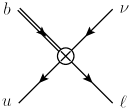
local matrix element $\rightarrow$ HQET decay constant $F_B(\mu)$

Factorization of Virtual Corrections - Endpoint Divergences

- The **virtual** corrections above Λ_{QCD} are described by a **SCET** factorization formula. Recall: chirality-suppressed decay $\Rightarrow$ subleading-power in SCET.
- In SCET, operators with **multiple collinear fields** are distributions in the **momentum fraction** of the shared direction, matrix elements are **convolution integrals**:

$$i\mathscr{A}^{\mathrm{virt}}_{B\to\ell\nu} = -\frac{4G_F}{\sqrt{2}} K_{\mathrm{EW}}(\mu) V_{ub} \frac{m_\ell}{m_b} K_A(m_\ell)\bar{u}(p_\ell) P_L v(p_\nu)$$

$$\cdot\left[\boxed{H_A(m_b) S_A} + \int d\omega \int_0^1 dx \,\boxed{H_B(m_b, x)\cdot J_B(m_b\omega, x)\cdot S_B(\omega)}\right]$$

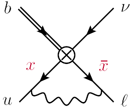


$$S_A = \langle 0|\bar{q}(0)\not{n}\, P_L\, h_v(0)|B\rangle$$

local matrix element $\longrightarrow$ HQET decay constant $F_B(\mu)$

$$S_B(\omega) = \int ds\;\; e^{i\omega s}\langle 0|\bar{q}(sn)\not{n}\, P_L\, h_v(0)|B\rangle$$

energetic-photon exchange $\longrightarrow$ LCDAs



Factorization of Virtual Corrections - Endpoint Divergences

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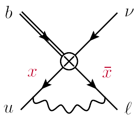

- At $x = 0$ and $\bar{x} = 0$, one **collinear** field has anomalously **low energy** and the mode **separation** between **soft** and **collinear** modes **fails**.

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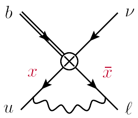

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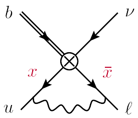

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- This rearrangement begins with **identifying** the small-x region of the **second term** with a contribution to the **first** one, this is called **refactorization**.

[Liu, Neubert (2003.03393); Liu et al (2009.04456, 2009.06779, 2112.00018); Beneke et al (2008.04943, 2205.04479)]
[Bell et al (2205.06021); Feldmann et al (2211.04209); Hurth, Szafron (2312.10450)]

Refactorization-Based Subtraction

$$i\,\mathscr{A}^{\text{virt}}_{B\to\ell\nu} = -\frac{4G_F}{\sqrt{2}} K_{\text{EW}}(\mu) V_{ub} \frac{m_\ell}{m_b} K_A(m_\ell) \bar{u}(p_\ell) P_L v(p_\nu)$$

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endpoint-divergent

$$S_A = \langle 0 | O_A | B \rangle$$

$$O_A = \bar{q}_s \not{n} P_L h_{v_B} Y_n^{(\ell)\dagger}$$

$$J_B = -Q_\ell Q_u \frac{\alpha}{2\pi} \frac{e^{\epsilon\gamma_E}\Gamma(\epsilon)}{1-\epsilon} \left(\frac{\mu^2}{m_b\,\omega\,x\,\bar{x}} \right)^{\epsilon} \left(\frac{1}{x} + 1 - 2\epsilon \right)$$

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subtract small-x

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$$
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$$

$$
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$$

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Virtual Corrections Summary

- We can now write the **virtual** amplitude:

$$\mathscr{A}^{\text{virtual}}_{B\to\ell\bar{\nu}} = i\sqrt{2}G_F K_{\text{EW}}(\mu)V_{ub}\,\frac{m_\ell}{m_b}\sqrt{m_B}\,F(\mu,m_b)\;\bar{u}(p_\ell)P_L\,v(p_\nu)\left[\mathscr{M}_{2p}(\mu)+\mathscr{M}_{3p}(\mu)\right]$$

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with

$$\begin{aligned}
\mathscr{M}_{2p}(\mu) = 1 + \frac{C_F\alpha_s}{4\pi}\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2} - 2\right] + \frac{\alpha}{4\pi}\Bigg\{ Q_b^2\left[\frac{3}{2}\ln\frac{m_b^2}{\mu^2} - 2\right] - Q_\ell Q_b\left[\frac{1}{2}\ln^2\frac{m_b^2}{\mu^2} + 2\ln\frac{m_b^2}{\mu^2} - 3\ln\frac{m_\ell^2}{\mu^2} + 1 + \frac{5\pi^2}{12}\right] \\
+2Q_\ell Q_u \int_0^\infty d\omega\, \phi_-^B(\omega) \ln\frac{m_b\,\omega}{\mu^2} + Q_\ell^2\left[\frac{1}{\epsilon_{\text{IR}}}\left(\ln\frac{m_B^2}{m_\ell^2} - 2\right) + \frac{1}{2}\ln^2\frac{m_\ell^2}{\mu^2} - \frac{1}{2}\ln\frac{m_\ell^2}{\mu^2} + 2 + \frac{5\pi^2}{12}\right]\Bigg\} \\
\mathscr{M}_{3p}(\mu) = \frac{\alpha}{\pi} Q_\ell Q_u \int_0^\infty d\omega \int_0^\infty d\omega_g\, \phi_{3g}^B(\omega, \omega_g)\left[\frac{1}{\omega_g}\ln\left(1 + \frac{\omega_g}{\omega}\right) - \frac{1}{\omega + \omega_g}\right]
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[Cornella, MK, Neubert, 2022]

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[Cornella, MK, Neubert, 2022]

- The “generalized decay constant” $F(\mu,\Lambda)$ is an **unknown non-perturbative object**, which can **hopefully** be obtained from the **lattice**.

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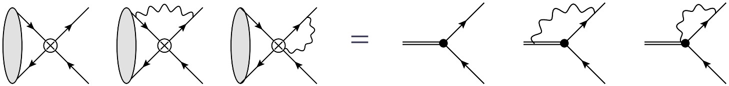
- Matching to the resulting low-energy EFT by taking hadronic matrix elements:

$$\left\langle \ell\ \nu \middle| \mathscr{L}_{\rm SCET\,II \otimes HQET} \middle| B \right\rangle = \langle \ell\ \nu | \mathscr{L}_{\rm bHLET \otimes HSET} | B \rangle$$



The Low-Energy Theory

- At first glance, the low-energy theory is a **theory of only Wilson lines**, since the interactions of the B and the muon with ultrasoft and ultrasoft-collinear photons can be **decoupled**:

$$Y_v^{(\mathrm{us,usc})}(x) = \mathscr{P} \exp\left\{ i e \int_{-\infty}^{0} ds\, v \cdot A_{\mathrm{us,usc}}(x + s v) \right\}$$

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$$S(E_s,\mu) = \int\!\!\int d\omega_{\mathrm{us}}\, d\omega_{\mathrm{usc}}\, \theta\left(\frac{E_s}{2} - \omega_{\mathrm{us}} - \omega_{\mathrm{usc}} \right) W_{\mathrm{us}}(\omega_{\mathrm{us}},\mu) W_{\mathrm{usc}}(\omega_{\mathrm{usc}},\mu)$$

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- Integration and renormalization carried out in **Laplace space** to yield resummation of **soft logarithms**.

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(usoft + usoft-collinear)

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Fermi theory

hard loops

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Fermi theory

hard loops

hard-collinear loops

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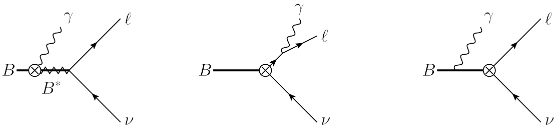

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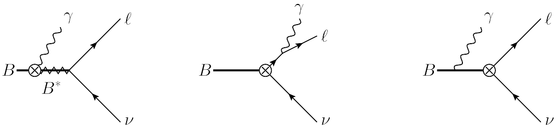

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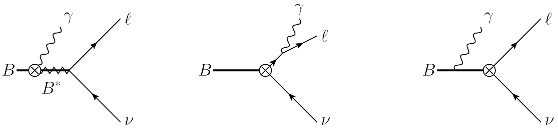

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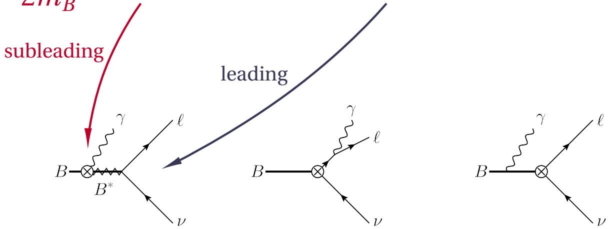

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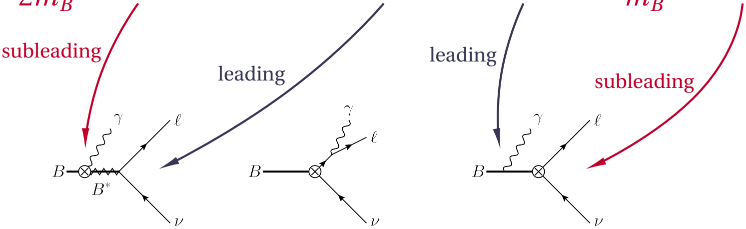

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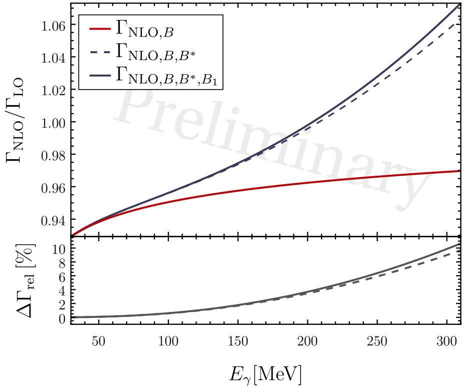
An Interplay of Suppressions



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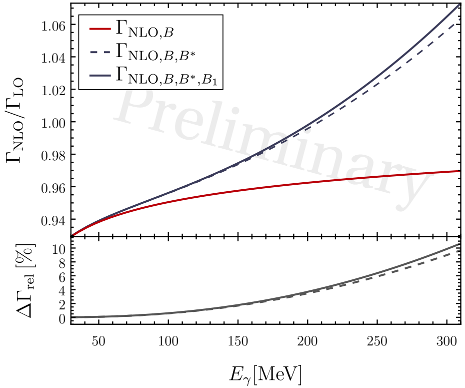

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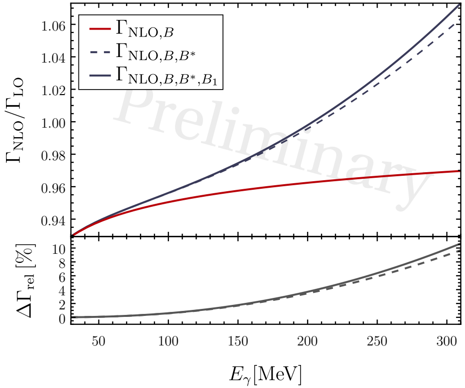

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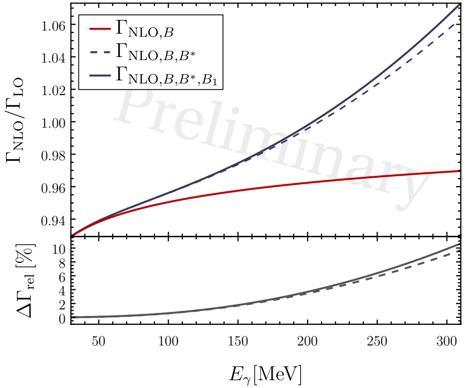

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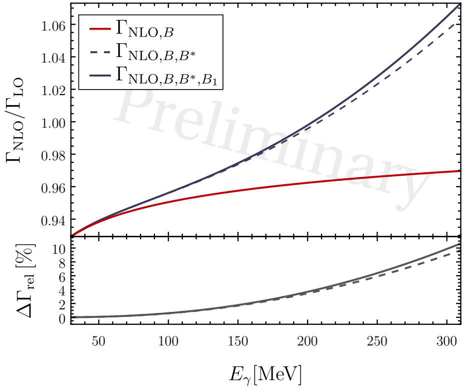

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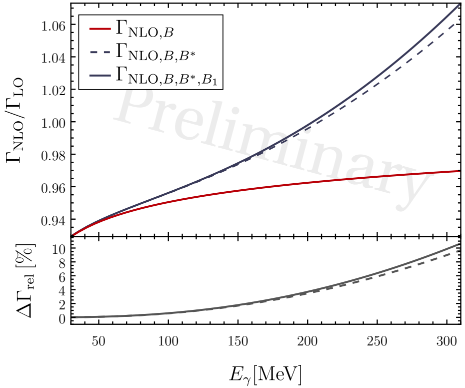

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