Radiative corrections in chiral perturbation theory
Applications to (semi-)leptonic kaon and pion decays

Marc Knecht

Centre de Physique Théorique UMR7332,
CNRS Luminy Case 907, 13288 Marseille cedex 09 - France
knecht@cpt.univ-mrs.fr

Workshop on radiative leptonic B decays – Campus de Luminy, Feb. 28 - March 1, 2024

OUTLINE

- Introduction
- ChPT in a nutshell
- QED corrections in pion and kaon decays: the ChPT point of view
- Two case studies or radiative corrections in real life
- Concluding remarks

Introduction

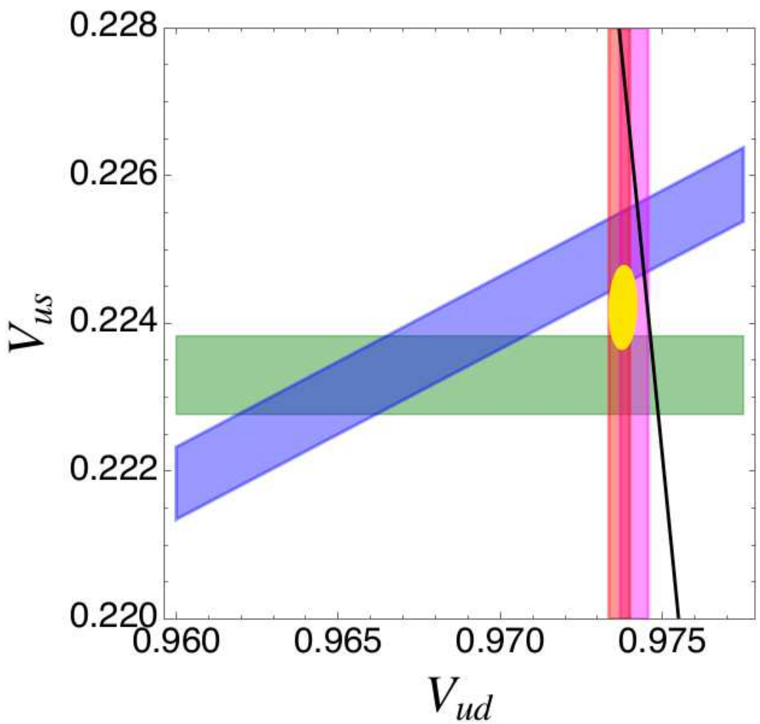
Precision measurements of decays of K, D, B mesons, or hadronic decay modes of the τ lepton, allow to put constraints on physics beyond the standard model (tests of lepton flavour universality or of CKM unitarity, CP violation, admixture of right-handed currents,...)

Precision measurements of decays of K, D, B mesons, or hadronic decay modes of the τ lepton, allow to put constraints on physics beyond the standard model (tests of lepton flavour universality or of CKM unitarity, CP violation, admixture of right-handed currents,...)

This is in particular true for rare decays of the kaons (e.g. $K \to \pi\nu\bar{\nu}$), but also for more "traditional" decays modes, like the semi-leptonic or non-leptonic ones

Precision measurements of decays of K, D, B mesons, or hadronic decay modes of the τ lepton, allow to put constraints on physics beyond the standard model (tests of lepton flavour universality or of CKM unitarity, CP violation, admixture of right-handed currents,...)

This is in particular true for rare decays of the kaons (e.g. $K \to \pi\nu\bar{\nu}$), but also for more "traditional" decays modes, like the semi-leptonic or non-leptonic ones



V. Cirigliano, A. Crivellin, M. Hoferichter, M. Moulson, Phys. Lett. B 838, 137748 (2023)

$$\Delta^{(1)}_{\rm CKM} = |V^{\beta}_{ud}|^2 + |V^{K_{\ell 3}}_{us}|^2 - 1 = -0.00176(56) \quad [-3.1\sigma]$$
$$\Delta^{(2)}_{\rm CKM} = |V^{\beta}_{ud}|^2 + |V^{K_{\ell 3}/K_{\ell 2};\beta}_{us}|^2 - 1 = -0.00098(58) \quad [-1.7\sigma]$$
$$\Delta^{(3)}_{\rm CKM} = |V^{K_{\ell 3}/K_{\ell 2};K_{\ell 3}}_{ud}|^2 + |V^{K_{\ell 3}}_{us}|^2 - 1 = -0.0164(63) \quad [-2.6\sigma]$$

Precision measurements of decays of K, D, B mesons, or hadronic decay modes of the τ lepton, allow to put constraints on physics beyond the standard model (tests of lepton flavour universality or of CKM unitarity, CP violation, admixture of right-handed currents,...)

This is in particular true for rare decays of the kaons (e.g. $K \to \pi\nu\bar{\nu}$), but also for more "traditional" decays modes, like the semi-leptonic or non-leptonic ones

In addition, these also provide information on low-energy strong interactions (e.g. decay constants, structure of form factors, $\pi\pi$ scattering lengths,...), that in turn allow to test predictions or to determine non-perturbative parameters (low-energy constants in the case of kaons) that occur also in other processes

$K^{\pm} \to \pi^+\pi^- e^{\pm}\nu$

- Geneva-Saclay high-statistics experiment: $3 \cdot 10^4$ events, a_0 at 20%

L. Rosselet et al., Phys. Rev. D 15, 574 (1977)

- BNL-E865: $4 \cdot 10^5$ events

S. Pislak et al., Phys. Rev. 67, 072004 (2003) [Phys. Rev. 81, 119903 (2010)] [hep-ex/0301040]

- NA48/2: $1.1 \cdot 10^6$ events, a_0 at 6%

J. R. Batley et al., Eur. Phys. J. C 70, 635 (2010)

The experimental values of the two S-wave scattering lengths

$$a_0 = 0.222(14) \qquad a_2 = -0.0432(97)$$

compare quite well with the prediction from two-loop chiral perturbation theory

$$a_0 = 0.220(5) \qquad a_2 = -0.0444(10)$$

G. Colangelo, J. Gasser, H. Leutwyler, Nucl. Phys. B 603, 125 (2001)

$K^\pm \to \pi^+\pi^- e^\pm \nu$

- Geneva-Saclay high-statistics experiment: $3 \cdot 10^4$ events, a_0 at 20%

L. Rosselet et al., Phys. Rev. D 15, 574 (1977)

- BNL-E865: $4 \cdot 10^5$ events

S. Pislak et al., Phys. Rev. 67, 072004 (2003) [Phys. Rev. 81, 119903 (2010)] [hep-ex/0301040]

- NA48/2: $1.1 \cdot 10^6$ events, a_0 at 6%

J. R. Batley et al., Eur. Phys. J. C 70, 635 (2010)

The experimental values of the two S-wave scattering lengths

$$a_0 = 0.222(14) \qquad a_2 = -0.0432(97)$$

compare quite well with the prediction from two-loop chiral perturbation theory

$$a_0 = 0.220(5) \qquad a_2 = -0.0444(10)$$

G. Colangelo, J. Gasser, H. Leutwyler, Nucl. Phys. B 603, 125 (2001)

But taking isospin corrections ($m_u \neq m_d$ and $M_\pi \neq M_{\pi^0}$) into account turns out to be crucial in order to reach this agreement

J. Gasser, PoS KAON , 033 (2008), arXiv:0710.3048 [hep-ph]

Precision measurements of decays of K, D, B mesons, or hadronic decay modes of the τ lepton, allow to put constraints on physics beyond the standard model (tests of lepton flavour universality or of CKM unitarity, CP violation, admixture of right-handed currents,...)

This is in particular true for rare decays of the kaons (e.g. $K \to \pi\nu\bar{\nu}$), but also for more "traditional" decays modes, like the semi-leptonic or non-leptonic ones

In addition, they also provide information on low-energy strong interactions (e.g. decay constants, structure of form factors, $\pi\pi$ scattering lengths,...), that in turn allow to test predictions or to determine non-perturbative parameters (low-energy constants in the case of kaons) that occur also in other processes

$\longrightarrow$ quantitative control over radiative, or more generally, isospin-breaking (IB) corrections has become mandatory

Precision measurements of decays of K, D, B mesons, or hadronic decay modes of the τ lepton, allow to put constraints on physics beyond the standard model (tests of lepton flavour universality or of CKM unitarity, CP violation, admixture of right-handed currents,...)

This is in particular true for rare decays of the kaons (e.g. $K \to \pi\nu\bar{\nu}$), but also for more "traditional" decays modes, like the semi-leptonic or non-leptonic ones

In addition, they also provide information on low-energy strong interactions (e.g. decay constants, structure of form factors, $\pi\pi$ scattering lengths,...), that in turn allow to test predictions or to determine non-perturbative parameters (low-energy constants in the case of kaons) that occur also in other processes

$\longrightarrow$ quantitative control over radiative, or more generally, isospin-breaking (IB) corrections has become mandatory

note: $M_{\pi^0} \neq M_{\pi^\pm}$ is an electromagnetic effect!

ChPT in a nutshell

For $\mu \ll \Lambda_{\rm had} \sim 1{\rm GeV}$ (where kaon physics takes place), the relevant degrees of freedom are no longer quarks, but the lightest pseudoscalar mesons that become the Goldstone bosons of the spontaneous breaking of chiral symmetry in the limit of massless light quarks $m_{u,d,s} \to 0$

For $\mu \ll \Lambda_{\rm had} \sim 1{\rm GeV}$ (where kaon physics takes place), the relevant degrees of freedom are no longer quarks, but the lightest pseudoscalar mesons that become the Goldstone bosons of the spontaneous breaking of chiral symmetry in the limit of massless light quarks $m_{u,d,s} \to 0$

$\longrightarrow$ construct an effective lagrangian that describes the interactions among these pseudoscalar mesons in a systematic low-energy expansion, taking into account all the constraints that follow from the spontaneously broken chiral $SU(3)_L \times SU(3)_R$ chiral symmetry

S. Weinberg, Physica A 96, 327 (1979)

J. Gasser, H. Leutwyler, Annals Phys. 158, 142 (1984); Nucl. Phys. B 250, 465 (1985)

For $\mu \ll \Lambda_{\rm had} \sim 1{\rm GeV}$ (where kaon physics takes place), the relevant degrees of freedom are no longer quarks, but the lightest pseudoscalar mesons that become the Goldstone bosons of the spontaneous breaking of chiral symmetry in the limit of massless light quarks $m_{u,d,s} \to 0$

$\longrightarrow$ construct an effective lagrangian that describes the interactions among these pseudoscalar mesons in a systematic low-energy expansion, taking into account all the constraints that follow from the spontaneously broken chiral $SU(3)_L \times SU(3)_R$ chiral symmetry

S. Weinberg, Physica A 96, 327 (1979)

J. Gasser, H. Leutwyler, Annals Phys. 158, 142 (1984); Nucl. Phys. B 250, 465 (1985)

$\longrightarrow$ systematic expansion in powers of $p/\Lambda_{\rm had}$ (with $m_q \sim p^2$, i.e. $M_\pi^2 \sim p^2$)

$\longrightarrow$ starts at $\mathcal{O}(p^2)$, power counting consistent with loop expansion

$$\mathcal{L}^{\rm str}(2) = \frac{F_0^2}{4}\langle \partial^\mu U^\dagger \partial_\mu U \rangle - \frac{<\bar{q}q>}{2}\langle \mathcal{M}(U + U^\dagger)\rangle + \cdots$$

$$\mathcal{M} = {\rm diag}(m_u, m_d, m_s)$$

For $\mu \ll \Lambda_{\rm had} \sim 1{\rm GeV}$ (where kaon physics takes place), the relevant degrees of freedom are no longer quarks, but the lightest pseudoscalar mesons that become the Goldstone bosons of the spontaneous breaking of chiral symmetry in the limit of massless light quarks $m_{u,d,s} \to 0$

$\longrightarrow$ construct an effective lagrangian that describes the interactions among these pseudoscalar mesons in a systematic low-energy expansion, taking into account all the constraints that follow from the spontaneously broken chiral $SU(3)_L \times SU(3)_R$ chiral symmetry

S. Weinberg, Physica A 96, 327 (1979)

J. Gasser, H. Leutwyler, Annals Phys. 158, 142 (1984); Nucl. Phys. B 250, 465 (1985)

has been extended to NNLO and even to NNNLO...

$$\mathcal{L}^{\rm str} = \mathcal{L}_2^{\rm str}(2) + \mathcal{L}_4^{\rm str}(10+0) + \mathcal{L}_6^{\rm str}(90+23) + \mathcal{L}_8^{\rm str}(1233+705) + \cdots$$

J. Gasser, H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

J. Bijnens, G. Colangelo, G. Ecker, JHEP 02, 020 (1999); Annals Phys. 280, 100 (2000)

J. Bijnens, L. Girlanda, P. Talavera, Eur. Phys. J. C 23, 539 (2002)

T. Ebertshaüser, H. W. Fearing, S. Scherer, Phys. Rev. D 65, 054033 (2002)

J. Bijnens, N. Hermansson-Truedsson, S. Wang, JHEP 01 (2019)

J. Bijnens, N. Hermansson-Truedsson, J. Ruiz-Vidal, JHEP 01 (2024)

Loops with mesons prodice divergences that are absorbed by the counterterms (low-energy constants)

For $\mu \ll \Lambda_{\rm had} \sim 1{\rm GeV}$ (where kaon physics takes place), the relevant degrees of freedom are no longer quarks, but the lightest pseudoscalar mesons that become the Goldstone bosons of the spontaneous breaking of chiral symmetry in the limit of massless light quarks $m_{u,d,s} \to 0$

$\longrightarrow$ construct an effective lagrangian that describes the interactions among these pseudoscalar mesons in a systematic low-energy expansion, taking into account all the constraints that follow from the spontaneously broken chiral $SU(3)_L \times SU(3)_R$ chiral symmetry

S. Weinberg, Physica A 96, 327 (1979)

J. Gasser, H. Leutwyler, Annals Phys. 158, 142 (1984); Nucl. Phys. B 250, 465 (1985)

has been extended to NNLO and even to NNNLO...

$$\mathcal{L}^{\rm str} = \mathcal{L}_2^{\rm str}(2) + \mathcal{L}_4^{\rm str}(10+0) + \mathcal{L}_6^{\rm str}(90+23) + \mathcal{L}_8^{\rm str}(1233+705) + \cdots$$

J. Gasser, H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

J. Bijnens, G. Colangelo, G. Ecker, JHEP 02, 020 (1999); Annals Phys. 280, 100 (2000)

J. Bijnens, L. Girlanda, P. Talavera, Eur. Phys. J. C 23, 539 (2002)

T. Ebertshaüser, H. W. Fearing, S. Scherer, Phys. Rev. D 65, 054033 (2002)

J. Bijnens, N. Hermansson-Truedsson, S. Wang, JHEP 01 (2019)

J. Bijnens, N. Hermansson-Truedsson, J. Ruiz-Vidal, JHEP 01 (2024)

- allows to describe strong interactions among mesons at low-energies

For $\mu \ll \Lambda_{\text{had}} \sim 1\text{GeV}$ (where kaon physics takes place), the relevant degrees of freedom are no longer quarks, but the lightest pseudoscalar mesons that become the Goldstone bosons of the spontaneous breaking of chiral symmetry in the limit of massless light quarks $m_{u,d,s} \to 0$

$\longrightarrow$ construct an effective lagrangian that describes the interactions among these pseudoscalar mesons in a systematic low-energy expansion, taking into account all the constraints that follow from the spontaneously broken chiral $SU(3)_L \times SU(3)_R$ chiral symmetry

S. Weinberg, Physica A 96, 327 (1979)

J. Gasser, H. Leutwyler, Annals Phys. 158, 142 (1984); Nucl. Phys. B 250, 465 (1985)

has been extended to NNLO and even to NNNLO...

$$\mathcal{L}^{\text{str}} = \mathcal{L}_2^{\text{str}}(2) + \mathcal{L}_4^{\text{str}}(10+0) + \mathcal{L}_6^{\text{str}}(90+23) + \mathcal{L}_8^{\text{str}}(1233+705) + \cdots$$

J. Gasser, H. Leutwyler, Nucl. Phys. B 250, 465 (1985)

J. Bijnens, G. Colangelo, G. Ecker, JHEP 02, 020 (1999); Annals Phys. 280, 100 (2000)

J. Bijnens, L. Girlanda, P. Talavera, Eur. Phys. J. C 23, 539 (2002)

T. Ebertshaüser, H. W. Fearing, S. Scherer, Phys. Rev. D 65, 054033 (2002)

J. Bijnens, N. Hermansson-Truedsson, S. Wang, JHEP 01 (2019)

J. Bijnens, N. Hermansson-Truedsson, J. Ruiz-Vidal, JHEP 01 (2024)

- what about semi-leptonic decays?

- Semi-leptonic transitions

At the quark level:

$$\mathcal{L}_{\text{eff}}^{SL} = -\frac{G_{\text{F}}}{\sqrt{2}} \left[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\right] \left\{V_{ud}\left[\bar{u}\gamma^\mu(1-\gamma_5)d\right] + V_{us}\left[\bar{u}\gamma^\mu(1-\gamma_5)s\right]\right\} + \text{h. c.}$$

- No QCD corrections in $\mathcal{L}_{\text{eff}}^{SL}$

- factorized form

- the description of semi-leptonic decays amounts to the evaluation of the relevant form factors:

$\langle 0 \vert [\bar{d}\gamma^\mu\gamma_5 u](0) \vert \pi^+\rangle$	$[\pi_{\ell 2}]$	$\langle 0 \vert [\bar{s}\gamma^\mu\gamma_5 u](0) \vert K^+\rangle$	$[K_{\ell 2}]$
$\langle \pi^0 \vert [\bar{d}\gamma^\mu u](0) \vert \pi^+\rangle$	$[\pi_\beta]$		
$\langle \pi^0 \vert [\bar{s}\gamma^\mu\gamma_5 u](0) \vert K^+\rangle$	$[K_{\ell 3}^+]$	$\langle \pi^- \vert [\bar{s}\gamma^\mu\gamma_5 u](0) \vert K^0\rangle$	$[K_{\ell 3}^0]$
$\langle \pi\pi \vert [\bar{s}\gamma^\mu\gamma_5 u](0) \vert K\rangle$		$\langle \pi\pi \vert [\bar{s}\gamma^\mu u](0) \vert K\rangle$	$[K_{\ell 4}]$
$\langle \pi\pi\pi \vert [\bar{s}\gamma^\mu u](0) \vert K\rangle$		$\langle \pi\pi\pi \vert [\bar{s}\gamma^\mu\gamma_5 u](0) \vert K\rangle$	$[K_{e5}]$

- Semi-leptonic transitions

At the quark level:

$$\mathcal{L}_{\text{eff}}^{SL} = -\frac{G_{\text{F}}}{\sqrt{2}} \left[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\right] \{V_{ud}\,[\bar{u}\gamma^\mu(1-\gamma_5)d] + V_{us}\,[\bar{u}\gamma^\mu(1-\gamma_5)s]\} + \text{h. c.}$$

- No QCD corrections in $\mathcal{L}_{\text{eff}}^{SL}$

- factorized form

- the description of semi-leptonic decays amounts to the evaluation of the relevant form factors:

$$\langle 0|[\bar{d}\gamma^\mu\gamma_5 u](0)|\pi^+\rangle \quad [\pi_{\ell 2}] \qquad \langle 0|[\bar{s}\gamma^\mu\gamma_5 u](0)|K^+\rangle \quad [K_{\ell 2}]$$

$$\langle \pi^0|[\bar{d}\gamma^\mu u](0)|\pi^+\rangle \quad [\pi_\beta]$$

$$\langle \pi^0|[\bar{s}\gamma^\mu\gamma_5 u](0)|K^+\rangle \quad [K_{\ell 3}^+] \qquad \langle \pi^-|[\bar{s}\gamma^\mu\gamma_5 u](0)|K^0\rangle \quad [K_{\ell 3}^0]$$

$$\langle \pi\pi|[\bar{s}\gamma^\mu\gamma_5 u](0)|K\rangle \qquad \langle \pi\pi|[\bar{s}\gamma^\mu u](0)|K\rangle \quad [K_{\ell 4}]$$

$$\langle \pi\pi\pi|[\bar{s}\gamma^\mu u](0)|K\rangle \qquad \langle \pi\pi\pi|[\bar{s}\gamma^\mu\gamma_5 u](0)|K\rangle \quad [K_{e5}]$$

$\longrightarrow$ can be obtained from $\mathcal{L}^{\text{str}}$

QED corrections in pion and kaon decays:

the ChPT point of view

Adding electromagnetic interactions requires to include the photon as a low-energy degree of freedom

$$\partial_\mu U \longrightarrow \partial_\mu U - ieA_\mu[Q, U] \quad Q = \mathrm{diag}(2/3, -1/3, -1/3) \quad eQ \sim p$$

Adding electromagnetic interactions requires to include the photon as a low-energy degree of freedom

$$\partial_\mu U \longrightarrow \partial_\mu U - ieA_\mu[Q, U] \quad Q = \mathrm{diag}(2/3, -1/3, -1/3) \quad eQ \sim p$$

Loops with photons will generate new divergences

$\longrightarrow$ additional counterterms are required

Adding electromagnetic interactions requires to include the photon as a low-energy degree of freedom

$$\partial_\mu U \longrightarrow \partial_\mu U - ieA_\mu[Q,U] \quad Q = \mathrm{diag}(2/3,-1/3,-1/3) \quad eQ \sim p$$

Loops with photons will generate new divergences

$\longrightarrow$ additional counterterms are required

$$\mathcal{L}^{\mathrm{str;EM}} = \mathcal{L}_2^{\mathrm{str;EM}}(1) + \mathcal{L}_4^{\mathrm{str;EM}}(13+0) + \cdots$$

$$\mathcal{L}_2^{\mathrm{str;EM}} = e^2 C \langle QU^\dagger QU \rangle \qquad \mathcal{L}_4^{\mathrm{str;EM}}(13+0) = \sum_{i=1}^{13} K_i \mathcal{O}_i^{\mathrm{str;EM}}$$

G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B 321, 311 (1989)

R. Urech, Nucl. Phys. B 433, 234 (1995)

H. Neufeld, H. Rupertsberger, Z. Phys. C 71, 131 (1996)

Adding electromagnetic interactions requires to include the photon as a low-energy degree of freedom

$$\partial_\mu U \longrightarrow \partial_\mu U - ieA_\mu[Q,U] \quad Q = \mathrm{diag}(2/3,-1/3,-1/3) \quad eQ \sim p$$

Loops with photons will generate new divergences

$\longrightarrow$ additional counterterms are required

$$\mathcal{L}^{\mathrm{str;EM}} = \mathcal{L}_2^{\mathrm{str;EM}}(1) + \mathcal{L}_4^{\mathrm{str;EM}}(13+0) + \cdots$$

$$\mathcal{L}_2^{\mathrm{str;EM}} = e^2 C\langle QU^\dagger QU\rangle \qquad \mathcal{L}_4^{\mathrm{str;EM}}(13+0) = \sum_{i=1}^{13} K_i \mathcal{O}_i^{\mathrm{str;EM}}$$

G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B 321, 311 (1989)

R. Urech, Nucl. Phys. B 433, 234 (1995)

H. Neufeld, H. Rupertsberger, Z. Phys. C 71, 131 (1996)

- allows to describe radiative corrections to strong interactions among mesons at low-energies, e.g. $\pi\pi \to \pi\pi$, $\pi K \to \pi K$,...

Adding electromagnetic interactions requires to include the photon as a low-energy degree of freedom

$$\partial_\mu U \longrightarrow \partial_\mu U - ieA_\mu[Q,U] \quad Q = \mathrm{diag}(2/3,-1/3,-1/3) \quad eQ \sim p$$

Loops with photons will generate new divergences

$\longrightarrow$ additional counterterms are required

$$\mathcal{L}^{\mathrm{str;EM}} = \mathcal{L}_2^{\mathrm{str;EM}}(1) + \mathcal{L}_4^{\mathrm{str;EM}}(13+0) + \cdots$$

$$\mathcal{L}_2^{\mathrm{str;EM}} = e^2 C\langle QU^\dagger QU\rangle \qquad \mathcal{L}_4^{\mathrm{str;EM}}(13+0) = \sum_{i=1}^{13} K_i \mathcal{O}_i^{\mathrm{str;EM}}$$

G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B 321, 311 (1989)

R. Urech, Nucl. Phys. B 433, 234 (1995)

H. Neufeld, H. Rupertsberger, Z. Phys. C 71, 131 (1996)

- allows to describe radiative corrections to strong interactions among mesons at low-energies, e.g. $\pi\pi \to \pi\pi$, $\pi K \to \pi K$,...

- What about semi-leptonic decays?

In the presence of electromagnetism, the semi-leptonic interactions do no longer factorize

photons can be exchanged between the quark current and the leptonic current

In the presence of electromagnetism, the semi-leptonic interactions do no longer factorize

photons can be exchanged between the quark current and the leptonic current

$\longrightarrow$ loops involving at the same time mesons, photons **and** leptons

In the presence of electromagnetism, the semi-leptonic interactions do no longer factorize

⟶ loops involving at the same time mesons, photons **and** leptons

⟶ need to also include the light leptons in the low-energy EFT

$$\partial_\mu U \longrightarrow \partial_\mu U - ieA_\mu[Q,U] + iU\sum_\ell(\bar{\ell}\gamma_\mu\nu_{\ell L}Q_{\rm w} + \overline{\nu_L}\gamma_\mu\ell Q_{\rm w}^\dagger) \quad Q_{\rm w} = -2\sqrt{2}G_F\begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{L}^{\rm lept} = \mathcal{L}_2^{\rm lept}(0) + \mathcal{L}_4^{\rm lept}(5) + \cdots \qquad \mathcal{L}_2^{\rm lept}(0) = \sum_\ell[(\bar{\ell}(i\partial\!\!\!/ + eA\!\!\!/ - m_\ell)\ell + \overline{\nu_{\ell L}}i\partial\!\!\!/\nu_{\ell L}]$$

$$\mathcal{L}_4^{\rm lept} = \sum_{i=1}^{5} X_i\mathcal{O}_i^{\rm lept}$$

M. K., H. Neufeld, H. Rupertsberger, P. Talavera, Eur. Phys. J. C 12, 469 (2000)

Crucial issue: determination of low-energy constants

• K_i

- identify the corresponding QCD correlators (two-, three- and four-point functions), in the chiral limit, convoluted with the free photon propagator

- study their short-distance behaviour

- write spectral sum rules

- saturate with lowest-lying narrow-width resonances

B. Moussallam, Nucl. Phys. B 504, 391 (1997) [hep-ph/9701400]

B. Ananthanarayan, B. Moussallam, JHEP06, 047 (2004) [hep-ph/0405206]

Analogous to the DGMLY sum-rule for C

$$C = -\frac{1}{16\pi^2}\frac{3}{2\pi}\int_0^\infty ds\, s\, \ln\frac{s}{\mu^2}\left[\rho_{VV}(s) - \rho_{AA}(s)\right]$$

T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low and J. E. Young, Phys. Rev. Lett. 18, 759 (1967)

B. Moussallam, Eur. Phys. J. C 6, 681 (1999) [hep-ph/9804271]

does not depend on the scale μ^2 of the logarithm because of the 2nd Weinberg sum rule!

S. Weinberg, Phys. Rev. Lett. 18, 507 (1967)

Crucial issue: determination of low-energy constants

- X_i

- two-step matching procedure:

i) compute radiative corrections to $\bar{q}q' \to \ell\nu$ in the SM and in the four-fermion theory

ii) match the radiatively corrected four-fermion theory to the chiral lagrangian, by identifying the QCD correlators (convoluted with the free photon propagator) that describe the X_i's
Saturate the resulting spectral sum rules with lowest-lying resonance states

S. Descotes-Genon, B. Moussallam, Eur. Phys. J. C 42, 403 (2005) [hep-ph/0505077]

Applications to many examples (non-exhaustive list)

- $\pi \to \ell\nu_\ell(\gamma)$ and $K \to \ell\nu_\ell(\gamma)$ M. K., H. Neufeld, H. Rupertsberger, P. Talavera, Eur. Phys. J. C 12, 469 (2000)
  V. Cirigliano, I. Rosell, JHEP 0710, 005 (2007)
  J. Gasser, G. R. S. Zarnauskas, Phys. Lett. B 693, 122 (2010)
  V. Cirigliano, H. Neufeld, Phys. Lett. B 700, 7 (2011)

- $K \to \pi\ell\nu_\ell(\gamma)$ V. Cirigliano, M. K., H. Neufeld, H. Rupertsberger and P. Talavera, Eur. Phys. J. C 23, 121 (2002)
  A. Kastner, H. Neufeld, Eur. Phys. J. C 57, 541 (2008)
  V. Cirigliano, M. Giannotti, H. Neufeld, JHEP 0811, 006 (2008)
  J. Gasser, B. Kubis, N. Paver, M. Verbeni, Eur. Phys. J. C 40, 205 (2005)

- $\pi^+ \to \pi^0 e\nu_e$ V. Cirigliano, M. K., H. Neufeld, H. Pichl, Eur. Phys. J. C 27, 255 (2003)

- $K^+ \to \pi^+\pi^-\ell\nu_\ell$ V. Cuplov, PhD thesis (2004); V. Cuplov, A. Nehme, hep-ph/0311274
  A. Nehme, Nucl. Phys. B 682, 289 (2004)
  P. Stoffer, Eur. Phys. J. C 74, 2749 (2014)

- $K^+ \to \pi^0\pi^0\ell\nu_\ell$ V. Bernard, S. Descotes-Genon and M. K., Eur. Phys. J. C **75**, 145 (2015)

- $\ldots$ V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J. Portolés, Rev. Mod. Phys. 84, 399 (2012)

Applications to many examples (non-exhaustive list)

– $\pi \to \ell\nu_\ell(\gamma)$ and $K \to \ell\nu_\ell(\gamma)$ M. K., H. Neufeld, H. Rupertsberger, P. Talavera, Eur. Phys. J. C 12, 469 (2000)
V. Cirigliano, I. Rosell, JHEP 0710, 005 (2007)
J. Gasser, G. R. S. Zarnauskas, Phys. Lett. B 693, 122 (2010)
V. Cirigliano, H. Neufeld, Phys. Lett. B 700, 7 (2011)

– $K \to \pi\ell\nu_\ell(\gamma)$ V. Cirigliano, M. K., H. Neufeld, H. Rupertsberger and P. Talavera, Eur. Phys. J. C 23, 121 (2002)
A. Kastner, H. Neufeld, Eur. Phys. J. C 57, 541 (2008)
V. Cirigliano, M. Giannotti, H. Neufeld, JHEP 0811, 006 (2008)
J. Gasser, B. Kubis, N. Paver, M. Verbeni, Eur. Phys. J. C 40, 205 (2005)

– $\pi^+ \to \pi^0 e\nu_e$ V. Cirigliano, M. K., H. Neufeld, H. Pichl, Eur. Phys. J. C 27, 255 (2003)

– $K^+ \to \pi^+\pi^-\ell\nu_\ell$ V. Cuplov, PhD thesis (2004); V. Cuplov, A. Nehme, hep-ph/0311274
A. Nehme, Nucl. Phys. B 682, 289 (2004)
P. Stoffer, Eur. Phys. J. C 74, 2749 (2014)

– $K^+ \to \pi^0\pi^0\ell\nu_\ell$ V. Bernard, S. Descotes-Genon and M. K., Eur. Phys. J. C **75**, 145 (2015)

– ... V. Cirigliano, G. Ecker, H. Neufeld, A. Pich, J. Portolés, Rev. Mod. Phys. 84, 399 (2012)

Two case studies: K_{e4}^{00} and π_β

NA48/2 has measured the two $K_{e4}^{\pm}$ channels:

K_{e4}^{+-} [i.e. $K^{\pm} \to \pi^+\pi^- e^{\pm}\nu_e$], about 10^6 events

J. R. Batley et al. [NA48/2 Coll.], Phys. Lett. B 715, 105 (2012)

K_{e4}^{00} [i.e. $K^{\pm} \to \pi^0\pi^0 e^{\pm}\nu_e$], about $6.5 \cdot 10^4$ events (unitarity cusp in $M_{\pi^0\pi^0}$ seen)

J. R. Batley et al. [NA48/2 Coll.], JHEP 1408, 159 (2014)

In the isospin limit, there is a form factor common to the two matrix elements, whose normalization f_s can thus be measured in both decay distribution

$$|V_{us}|f_s[K_{e4}^{+-}] = 1.285 \pm 0.001_{\rm stat} \pm 0.004_{\rm syst} \pm 0.005_{\rm ext},$$

$$(1+\delta_{EM})|V_{us}|f_s[K_{e4}^{00}] = 1.369 \pm 0.003_{\rm stat} \pm 0.006_{\rm syst} \pm 0.009_{\rm ext}$$

i.e.

$$(1+\delta_{EM})\frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = 1.065 \pm 0.010$$

where δ_{EM} is an unspecified coefficient supposed to account for unknown radiative corrections

Can one understand this 6.5% effect in terms of isospin breaking?

$\longrightarrow$ need to understand how radiative corrections were treated in the K_{e4}^{+-} mode...

Treatment of radiative corrections in the data analyses:

K_{e4}^{00}: no radiative corrections whatsoever applied (hence the factor $\delta_{\rm EM}$!)

K_{e4}^{+-}:

– Sommerfeld-Gamow-Sakharov factors applied to each pair of charged legs

– Corrections induced by emission of real photons treated with PHOTOS

Z. Wąs et al., Comp. Phys. Comm. 79, 291 (1994); Eur. Phys. J. C 45, 97 (2006); C 51, 569 (2007);
Q.-J. Xu, Z. Wąs, Chin. Phys. C 34, 889 (2010)

– PHOTOS also implements (1 loop QED) w.f.r. on the external charged legs and virtual photon exchanges between charged external legs [$\longrightarrow$ no IR divergences], based on

Y. M. Bystritskiy, S. R. Gevorkian, E. A. Kuraev, Eur. Phys. J. C 67, 47 (2009)

– All structure-dependent corrections are discarded (gauge invariant truncation)

– UV divergences not treated

$$\left(|V_{us}|^2 G_F^2\right)^{\text{bare}} \left(1 - \frac{9}{4}\frac{\alpha}{\pi} \ln \frac{\Lambda^2}{M_\pi^2}\right) = |V_{us}|^2 G_F^2 \quad [K_{e4}^{+-}]$$

– UV divergences not treated

$$\left(|V_{us}|^2 G_F^2\right)^{\mathrm{bare}} \left(1 - \frac{9}{4}\frac{\alpha}{\pi} \ln \frac{\Lambda^2}{M_\pi^2}\right) = |V_{us}|^2 G_F^2 \quad [K_{e4}^{+-}]$$

– but

$$\left(|V_{us}|^2 G_F^2\right)^{\mathrm{bare}} \left(1 - \frac{3}{4}\frac{\alpha}{\pi} \ln \frac{\Lambda^2}{M_\pi^2}\right) = |V_{us}|^2 G_F^2 \quad [K_{e4}^{00}]$$

– UV divergences not treated

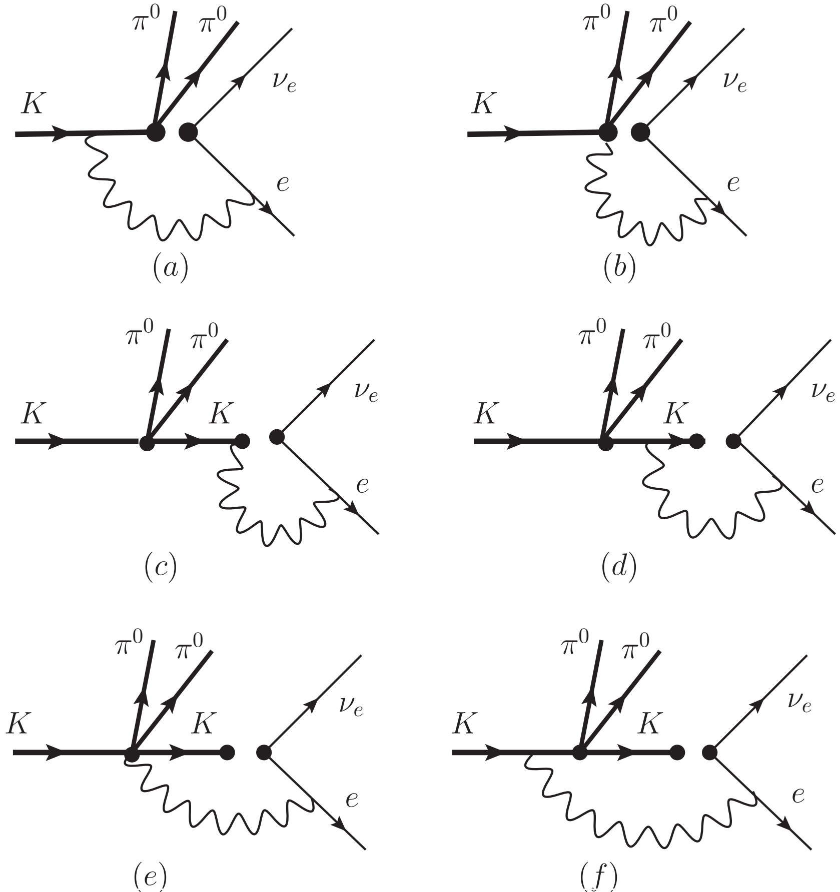
$$\left(|V_{us}|^2 G_F^2\right)^{\rm bare}\left(1-\frac{9}{4}\frac{\alpha}{\pi}\ln\frac{\Lambda^2}{M_\pi^2}\right)=|V_{us}|^2G_F^2\qquad [K_{e4}^{+-}]$$

– but

$$\left(|V_{us}|^2 G_F^2\right)^{\rm bare}\left(1-\frac{3}{4}\frac{\alpha}{\pi}\ln\frac{\Lambda^2}{M_\pi^2}\right)=|V_{us}|^2G_F^2\qquad [K_{e4}^{00}]$$

– correct treatment is to include the counterterms K_i and X_i and to renormalize the form factors

$$\left(1-\frac{9}{8}\frac{\alpha}{\pi}\ln\frac{\Lambda^2}{M_\pi^2}\right)f_s^{\rm bare}[K_{e4}^{+-}]=f_s[K_{e4}^{+-}]\qquad\qquad \left(1-\frac{3}{8}\frac{\alpha}{\pi}\ln\frac{\Lambda^2}{M_\pi^2}\right)f_s^{\rm bare}[K_{e4}^{00}]=f_s[K_{e4}^{00}]$$



Non factorizable radiative corrections

Besides w.f. factors of QED, only diagram (a) is considered in a PHOTOS-like treatment of radiative corrections [diagrams (b), (c), and (d) vanish for $m_e \to 0$]

Adding the diagrams for the emission of a soft photon, one obtains

$$\Gamma^{\text{tot}} = \Gamma(K_{e4}^{00}) + \Gamma^{\text{soft}}(K_{e4\gamma}^{00}) = \Gamma_0(K_{e4}^{00}) \times (1 + 2\delta_{EM}) \quad \text{with } \delta_{EM} = 0.018$$

$$\longrightarrow \quad \frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]} = 1.047 \pm 0.010$$

V. Bernard, S. Descotes-Genon, M. K., Eur. Phys. J. C 75, 145 (2015) can one understand the origin of the remaining $\sim 4.5\%$ effect?

can one understand the origin of the remaining $\sim 4.5\%$ effect?

$\longrightarrow$ isospin breaking in the quark masses

$$\left.\frac{f_s[K_{e4}^{00}]}{f_s[K_{e4}^{+-}]}\right|_{\rm LO} = \left(1 + \frac{3}{2R}\right) = 1.039 \pm 0.002$$

V. Cuplov, PhD Thesis (2004); A. Nehme, Nucl. Phys. B 682, 289 (2004)

$$R = \frac{m_s - m_{ud}}{m_d - m_u} = 38.2(1.1)(0.8)(1.4)$$

Z. Fodor et al. [BMW Coll.], Phys. Rev. Lett. 117, 082001 (2016)

$\pi^{\pm} \to \pi^0 e^{\pm} \nu$

Possible source of information on $|V_{ud}|$

Many advantages:

- pure vector transition (like super-allowed Fermi transitions, but unlike neutron β decay)

- no problem with nuclear transition matrix elements in evaluation of radiative corrections (like neutron β decay, but unlike super-allowed Fermi transitions)

- protected from first-order isospin breaking corrections (Ademollo-Gatto-Sirlin theorem)

$\pi^{\pm} \to \pi^0 e^{\pm} \nu$

Possible source of information on $|V_{ud}|$

Many advantages:

- pure vector transition (like super-allowed Fermi transitions, but unlike neutron β decay)

- no problem with nuclear transition matrix elements in evaluation of radiative corrections (like neutron β decay, but unlike super-allowed Fermi transitions)

- protected from first-order isospin breaking corrections (Ademollo-Gatto-Sirlin theorem)

- can be evaluated in two-flavour low-energy EFT: $f_+^{\pi_\beta}(0) = 1.0046(5)$

V. Cirigliano, M. K., H. Neufeld, H. Pichl, Eur. Phys. J. C 27, 255 (2003) [hep-ph/0209226]

$\pi^{\pm} \to \pi^0 e^{\pm} \nu$

Possible source of information on $|V_{ud}|$

Many advantages:

- pure vector transition (like super-allowed Fermi transitions, but unlike neutron β decay)

- no problem with nuclear transition matrix elements in evaluation of radiative corrections (like neutron β decay, but unlike super-allowed Fermi transitions)

- protected from first-order isospin breaking corrections (Ademollo-Gatto-Sirlin theorem)

- can be evaluated in two-flavour low-energy EFT: $f_+^{\pi_\beta}(0) = 1.0046(5)$

V. Cirigliano, M. K., H. Neufeld, H. Pichl, Eur. Phys. J. C 27, 255 (2003) [hep-ph/0209226]

$\longrightarrow$ cleanest way to extract V_{ud}

Serious drawback: $\Gamma_{\pi_\beta}/\Gamma_{\rm tot} \sim 1 \cdot 10^{-8}$

PIBETA exp. at PSI: $\Gamma_{\pi_\beta}/\Gamma_{\rm tot} = [1.036 \pm 0.004_{\rm stat} \pm 0.004_{syst} \pm 0.003_{\pi_{e2}}] \cdot 10^{-8}$

$\sim 10^6 \pi^+/{\rm sec}, 6.4 \cdot 10^4$ events $\longrightarrow V_{ud}^{\rm PIBETA} = 0.9739(30)$

D. Počanić et al. (PIBETA Coll.), Phys. Rev. Lett. 93, 181803 (2004) [hep-ex/0312030]

$\pi^{\pm} \to \pi^0 e^{\pm} \nu$

Possible source of information on $|V_{ud}|$

Many advantages:

- pure vector transition (like super-allowed Fermi transitions, but unlike neutron β decay)

- no problem with nuclear transition matrix elements in evaluation of radiative corrections (like neutron β decay, but unlike super-allowed Fermi transitions)

- protected from first-order isospin breaking corrections (Ademollo-Gatto-Sirlin theorem)

- can be evaluated in two-flavour low-energy EFT: $f_+^{\pi_\beta}(0) = 1.0046(5)$

V. Cirigliano, M. K., H. Neufeld, H. Pichl, Eur. Phys. J. C 27, 255 (2003) [hep-ph/0209226]

$\longrightarrow$ cleanest way to extract V_{ud}

Serious drawback: $\Gamma_{\pi_\beta}/\Gamma_{\rm tot} \sim 1 \cdot 10^{-8}$

PIBETA exp. at PSI: $\Gamma_{\pi_\beta}/\Gamma_{\rm tot} = [1.036 \pm 0.004_{\rm stat} \pm 0.004_{syst} \pm 0.003_{\pi_{e2}}] \cdot 10^{-8}$

$\sim 10^6 \pi^+/{\rm sec},\ 6.4 \cdot 10^4$ events $\longrightarrow V_{ud}^{\rm PIBETA} = 0.9739(30)$

D. Počanić et al. (PIBETA Coll.), Phys. Rev. Lett. 93, 181803 (2004) [hep-ex/0312030]

PIONEER proposal at PSI could deliver $\sim 7 \cdot 10^5$ ($\sim 7 \cdot 10^6$) events during phase II (III)

W. Altmannshofer *et al.* [PIONEER], [arXiv:2203.01981 [hep-ex]]

Concluding remarks

High precision reached by the data concerning non-leptonic and semi-leptonic decay modes of the kaons has made the treatment of isospin-breaking effects ($m_u \neq m_d$ and $\alpha \neq 0$) unavoidable

High precision reached by the data concerning non-leptonic and semi-leptonic decay modes of the kaons has made the treatment of isospin-breaking effects ($m_u \neq m_d$ and $\alpha \neq 0$) unavoidable

A lot of activity has been going on, extending the scope of the low-energy EFT in order to meet this necessity (inclusion of photons, leptons). Only a fraction of the many applications has been mentioned here

High precision reached by the data concerning non-leptonic and semi-leptonic decay modes of the kaons has made the treatment of isospin-breaking effects ($m_u \neq m_d$ and $\alpha \neq 0$) unavoidable

A lot of activity has been going on, extending the scope of the low-energy EFT in order to meet this necessity (inclusion of photons, leptons). Only a fraction of the many applications has been mentioned here

The issue of additional low-energy constants has been dealt with in a rather satisfactory manner, but these phenomenological estimates may not be sufficient in order to match the experimental precision in the future

High precision reached by the data concerning non-leptonic and semi-leptonic decay modes of the kaons has made the treatment of isospin-breaking effects ($m_u \neq m_d$ and $\alpha \neq 0$) unavoidable

A lot of activity has been going on, extending the scope of the low-energy EFT in order to meet this necessity (inclusion of photons, leptons). Only a fraction of the many applications has been mentioned here

The issue of additional low-energy constants has been dealt with in a rather satisfactory manner, but these phenomenological estimates may not be sufficient in order to match the experimental precision in the future

The effects due to $M_\pi \neq M_{\pi^0}$ are important (e.g. for K_{e4}).
ChPT at NLO is not always sufficient.
$\longrightarrow$ This issue can be dealt with through more elaborate/adapted approaches, like NREFT, dispersive representations,...

Some general remarks

Radiative corrections to total decay rates are typically at the level of a few %

$$\Gamma = \Gamma_0 \left[1 + \alpha \frac{\Delta\Gamma}{\Gamma_0}\right] \qquad \alpha \frac{\Delta\Gamma}{\Gamma_0} \sim \pm(1-3)\%$$

Some general remarks

Radiative corrections to total decay rates are typically at the level of a few %

$$\Gamma = \Gamma_0 \left[1 + \alpha \frac{\Delta\Gamma}{\Gamma_0} \right] \qquad \alpha \frac{\Delta\Gamma}{\Gamma_0} \sim \pm(1-3)\%$$

Radiative corrections to differential decay rates can, locally, be more important, e.g. $\sim \pm 10\%$

$$\frac{d^2\Gamma}{dxdy} = \frac{d^2\Gamma_0}{dxdy} \left[1 + \alpha\delta(x,y) \right] \qquad \alpha\delta(x,y) \sim \pm(1-10)\%$$

[cf. also situations where there are experimental cuts...]

Some general remarks

Radiative corrections to total decay rates are typically at the level of a few %

$$\Gamma = \Gamma_0 \left[1 + \alpha \frac{\Delta\Gamma}{\Gamma_0}\right] \qquad \alpha \frac{\Delta\Gamma}{\Gamma_0} \sim \pm(1-3)\%$$

Radiative corrections to differential decay rates can, locally, be more important, e.g. $\sim \pm 10\%$

$$\frac{d^2\Gamma}{dxdy} = \frac{d^2\Gamma_0}{dxdy}\left[1 + \alpha\delta(x,y)\right] \qquad \alpha\delta(x,y) \sim \pm(1-10)\%$$

[cf. also situations where there are experimental cuts...]

Emission of soft photons can sometimes lift the helicity suppression:
for instance in $B \to \mu\nu_\mu$

$$\left(\frac{M_B}{m_\mu}\right)^2 \times \alpha$$ is not small...

D. Bećirević, B. Haas, E. Kou, Phys. Lett. B 681, 257 (2009)

Some general remarks

Radiative corrections will, in general, generate:

- UV divergences

Some general remarks

Radiative corrections will, in general, generate:

- UV divergences

$\longrightarrow$ necessity to work within a well-defined theoretical framework

Some general remarks

Radiative corrections will, in general, generate:

- UV divergences

$\longrightarrow$ necessity to work within a well-defined theoretical framework

- IR divergences

Some general remarks

Radiative corrections will, in general, generate:

- UV divergences

$\longrightarrow$ necessity to work within a well-defined theoretical framework

- IR divergences

$\longrightarrow$ produced by w.f. corrections, photon-exchange vertex diagrams

$\longrightarrow$ necessity to construct IR-safe observables (in practice, include soft-photon emission)

R. Zwicky, Symmetry 13, 2036 (2021)

Some general remarks

Radiative corrections will, in general, generate:

- UV divergences

$\longrightarrow$ necessity to work within a well-defined theoretical framework

- IR divergences

$\longrightarrow$ produced by w.f. corrections, photon-exchange vertex diagrams
$\longrightarrow$ necessity to construct IR-safe observables (in practice, include soft-photon emission)

R. Zwicky, Symmetry 13, 2036 (2021)

- Coulomb singularities

Some general remarks

Radiative corrections will, in general, generate:

- UV divergences

$\longrightarrow$ necessity to work within a well-defined theoretical framework

- IR divergences

$\longrightarrow$ produced by w.f. corrections, photon-exchange vertex diagrams

$\longrightarrow$ necessity to construct IR-safe observables (in practice, include soft-photon emission)

R. Zwicky, Symmetry 13, 2036 (2021)

- Coulomb singularities

$\longrightarrow$ long-range interaction dominates for small relative velocities

Some general remarks

Radiative corrections will, in general, generate:

- UV divergences

$\longrightarrow$ necessity to work within a well-defined theoretical framework

- IR divergences

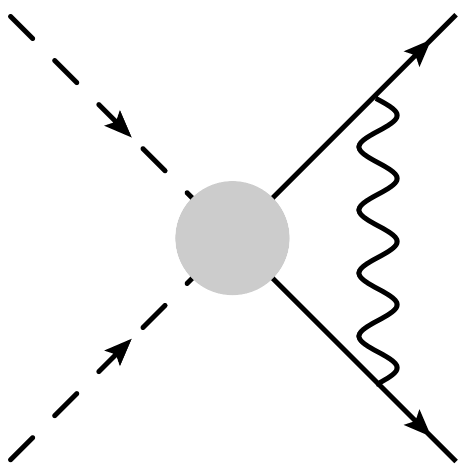
$\longrightarrow$ produced by w.f. corrections, photon-exchange vertex diagrams

$\longrightarrow$ necessity to construct IR-safe observables (in practice, include soft-photon emission)

R. Zwicky, Symmetry 13, 2036 (2021)

- Coulomb singularities

$\longrightarrow$ long-range interaction dominates for small relative velocities



$$\sim \mathcal{M}_0 \times \frac{(-1)}{2} e_1 e_2 \frac{\pi\alpha}{v_{12}((p_1+p_2)^2)}$$

$$v_{12}((p_1+p_2)^2) \equiv \frac{\lambda^{1/2}((p_1+p_2)^2, m_1^2, m_2^2)}{(p_1+p_2)^2 - m_1^2 - m_2^2}$$

Some general remarks

Radiative corrections will, in general, generate:

- UV divergences

$\longrightarrow$ necessity to work within a well-defined theoretical framework

- IR divergences

$\longrightarrow$ produced by w.f. corrections, photon-exchange vertex diagrams

$\longrightarrow$ necessity to construct IR-safe observables (in practice, include soft-photon emission)

R. Zwicky, Symmetry 13, 2036 (2021)

- Coulomb singularities

$\longrightarrow$ long-range interaction dominates for small relative velocities

$\longrightarrow$ apply Sommerfeld-Gamow-Sakharov resummation factors

$$|\mathcal{M}_0|^2 \left[1 + \frac{(-1)}{2}\, e_1 e_2\, \frac{2\pi\alpha}{v_{12}((p_1+p_2)^2)}\right]$$

$\longrightarrow$

$$|\mathcal{M}_0|^2 T(\eta_{12}), \quad T(\eta) = \frac{\eta}{1-e^{-\eta}} = 1 + \frac{\eta}{2} + \cdots, \quad \eta_{12} \equiv -e_1 e_2 \frac{2\pi\alpha}{v_{12}}$$

Most of the time, radiative corrections are small, knowing them at 10% or even 20% relative precision is usually sufficient

G. Martinelli, talk at KAON2016

Interesting prospects from lattice QCD (at least for kaons)

G. Anzivino et al., Workshop summary – Kaons@CERN 2023 [arXiv:2311.02923 [hep-ph]]

There are many interesting situations where low-energy effective theory does not apply (hadronic tau decays, semi-leptonic decays of B and D mesons)...

...for each situation the appropriate framework must be found

Thanks for your attention!