

# Comparison of heavy-ion transport models in a box: Collision integral in the presence of momentum dependent interactions

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# Past TMEP investigations

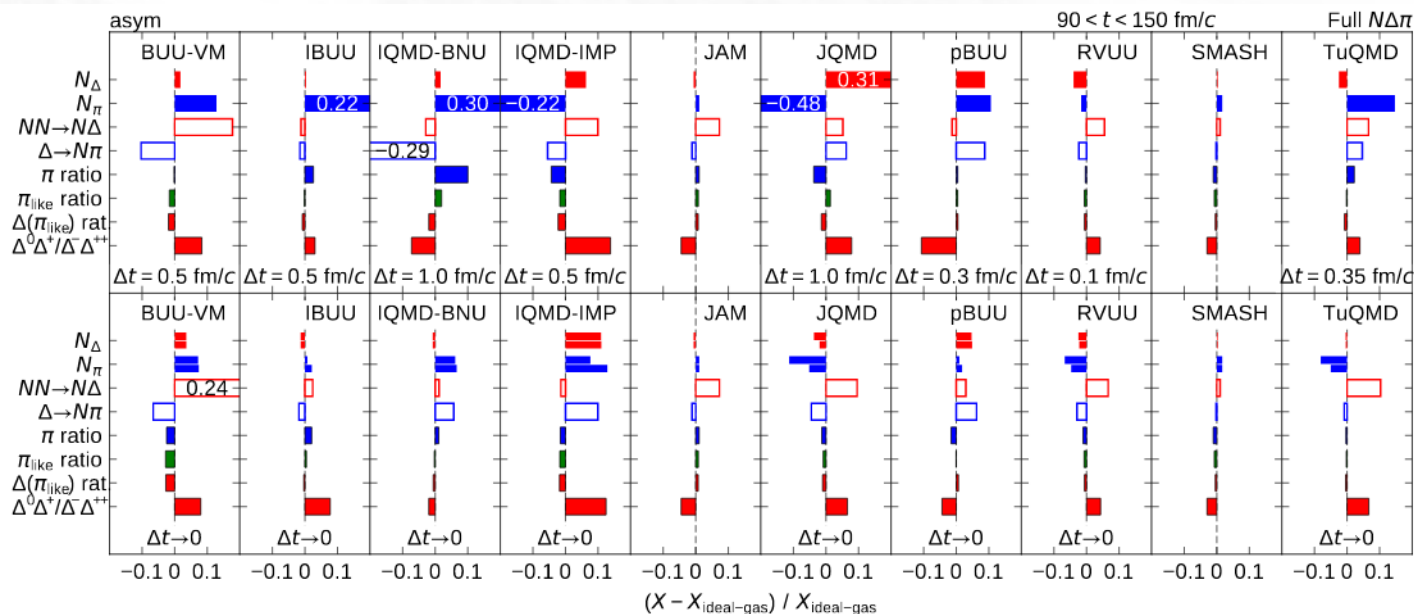
## Publications:

- 1) J. Xu et al., "Understanding transport simulations of heavy-ion collisions at 100A and 400A MeV: Comparison of heavy-ion transport codes under controlled conditions, Phys.Rev.C 93, 044609 (2016)
  - first assessment of differences among transport model predictions
- 2) Y.X. Zhang et al., "Comparison of heavy-ion transport simulations: Collision integral in a box", Phys.Rev.C 97, 034625 (2018)
  - comparison of various approaches for solving the collision integral for elastic collisions in a box; study of the effectiveness of different Pauli blocking algorithm in preserving the Fermi-Dirac character of nucleonic distributions
- 3) A. Ono et al., "Comparison of heavy-ion transport simulations: Collision integral with pions and  $\Delta$  resonances in a box", Phys. Rev. C 100, 044617 (2019)
  - no mean-field, no Pauli blocking; comparison to exact results for equilibrium quantities and rate equation
- 4) M. Colonna et al., "Comparison of heavy-ion transport simulations: Mean-field dynamics in a box", Phys.Rev.C 104, 024603 (2021)
  - mean-field (momentum independent) response for isospin symmetric nuclear matter (first sound propagation)
- 5) H. Wolter et al., "Transport model comparison studies of intermediate-energy heavy-ion collisions", Prog. Part. Nucl. Phys. 125, 193264 (2022)
  - progress report + model (code) descriptions (26 of the most widely used transport models)
- 6) J. Xu et al., "Understanding pion production from transport simulations of heavy-ion collisions at 270A MeV", PRC 109, 044609 (2024)
  - pion production in HIC; common initialization, all ingredients of the transport model have been included; momentum independent interaction

# Milestones of past TMEP investigations

A. Ono et al., “Comparison of heavy-ion transport simulations: Collision integral with pions and  $\Delta$  resonances in a box”  
 Phys. Rev. C 100, 044617 (2019)

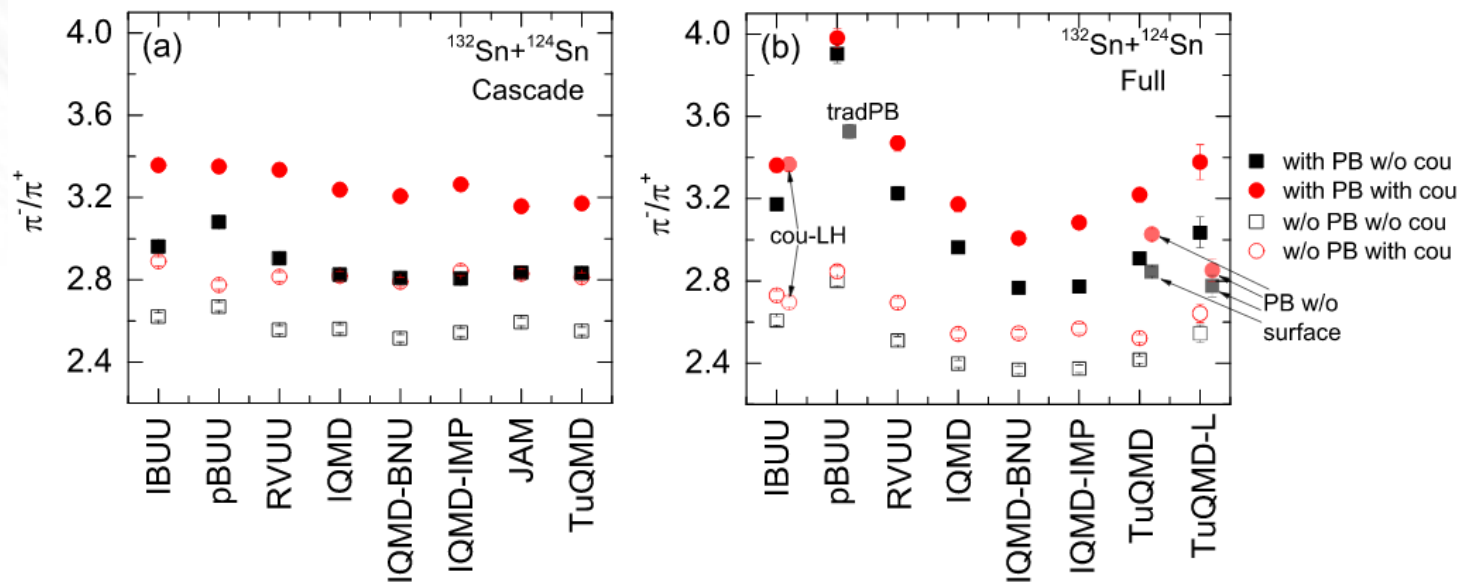
Conclusion: “The uncertainty in the transport-code predictions of the  $\pi^-/\pi^+$  ratio, after letting the existing  $\Delta$  resonances decay, is found to be within a few percent for the system initialized at  $n/p=1.5$ .”



# Milestones of past TMEP investigations

J. Xu et al., “Understanding pion production from transport simulations of heavy-ion collisions at 270A MeV”, PRC 109, 044609 (2024)

**Conclusion:** “...we find a convergence of the codes in the final charged pion yield ratio to less than 15%, while the uncertainty is expected to be reduced to about 4% if the same or similar ingredients, i.e., an improved Pauli blocking and calculation of the non-linear term in the mean-field potential, are incorporated in each code.”



# Participating Codes

Code	Type	Correspondents	References
dcQMD	QMD	D. Cozma	[1,2,3]
RVUU	BUU	Z. Zhang, C.M. Ko	[4,5,6]
sJAM	QMD	N. Ikeno, A. Ono	[7,8,9]

## References:

1. M.D. Cozma, Phys. Lett. B 754, 166 (2016)
2. M.D. Cozma, Phys. Rev. C 95, 014601 (2017)
3. M.D. Cozma, M.B. Tsang, Eur. J. Phys. A 57, 309 (2021)
4. T. Song, C.M. Ko, Phys. Rev. C 91, 014901, (2015)
5. C. M. Ko and Q. Li, Phys. Rev. C 37, 2270 (1988)
6. C. M. Ko and G.-Q. Li, Journal of Physics G: Nuclear and Particle Physics 22, 1673 (1996)
7. N. Ikeno, A. Ono, Y. Nara, and A. Ohnishi, Phys. Rev. C 93, 044612 (2016)
8. Y. Nara, N. Otuka, A. Ohnishi, K. Niita, and S. Chiba, Phys.Rev. C 61, 024901 (1999)
9. N. Ikeno and A. Ono, arXiv:2307:02395 (accepted Phys. Rev. C)

# Goals

- 1) Better understand the role of momentum and/or isospin asymmetry dependent interactions in generating threshold shifts in inelastic processes
- 2) Underline the role of energy conservation in ensuring consistency with thermodynamical models (already explored in Z.Zhang, C.M. Ko, Phys.Rev.C 97, 014610 (2018))
- 3) Assess the magnitude of differences between transport and thermal model results and investigate possible approaches to reduce such differences. Ultimately, differences between transport models using different parametrizations for momentum dependent interactions could be more easily (efficiently) understood using thermal models.
- 4) Propose benchmarks that any code should satisfy

# Homework Description

- 1) casc\_vacmass:** - no mean field; vacuum masses; no Pauli blocking (check A. Ono et al., PRC 100, 044617 (2019))
  - initialization uses vacuum masses (Boltzmann  $T=60$  MeV)
  - results: dcQMD, RVUU, sJAM (practically the same)
- 2) casc\_effmass:** - no mean field; effective masses; no Pauli blocking ( $m_N^*=0.7m_N$ ;  $m_\Delta^*=0.950$  GeV)
  - initialization uses effective masses (Boltzmann  $T=60$  MeV)
  - results: dcQMD, RVUU (practically the same)
- 3) full\_mdi:** - mean field (compressibility modulus  $K_0=230$  MeV and effective nucleon mass  $m^*=0.70$ ); no Pauli blocking no threshold effects
  - initialization uses effective masses (Boltzmann  $T=60$  MeV)
  - results: dcQMD, RVUU (practically the same)
- 4) full\_mdi\_th:** - mean field (compressibility modulus  $K_0=230$  MeV and effective nucleon mass  $m^*=0.70$ ); no Pauli blocking threshold effects included
  - initialization uses effective masses (Boltzmann  $T=60$  MeV)
  - results: dcQMD, RVUU, sJAM

Included Reactions in the Collision Term: 1)  $NN \leftrightarrow NN$  and  $NN \leftrightarrow N\Delta$  (PHASE I :  $\delta=0.0$ ; PHASE III:  $\delta=0.2$ )  
2)  $NN \leftrightarrow NN$ ,  $NN \leftrightarrow N\Delta$  and  $\Delta \leftrightarrow N\pi$  (PHASE II:  $\delta=0.0$ ; PHASE IV:  $\delta=0.2$ )

$\delta=0.2$ :  $\Delta m_{np}^*/m_N = -0.33\delta$ ;  $S_0=32$  MeV;  $L=60$  MeV

# Interaction Terms

**dcQMD:** 
$$\frac{E}{N}(\rho, \beta, \mathbf{x}, \mathbf{y}) = \frac{1}{2} A_1 u + \frac{1}{2} A_2(\mathbf{x}, \mathbf{y}) u \beta^2 + \frac{B u^\sigma}{\sigma + 1} (1 - \mathbf{x} \beta^2) + \frac{D u^2}{3} (1 - \mathbf{y} \beta^2)$$
 inspired by Gogny type interaction  
Das et al., PRC67, 034611 (2003)

$$\frac{+1}{u \rho_0^2} \sum_{\tau, \tau'} C_{\tau\tau'} \int \int d^3 p d^3 p' \frac{f_\tau(p, p') f_{\tau'}(p, p')}{1 + (\vec{p} - \vec{p}')^2 / \Lambda^2} \quad u = \frac{\rho}{\rho_0}$$

**RVUU:** relativistic mean-field model NL $\rho\delta$  T. Song, C.M. Ko, Phys. Rev. C 91, 014901, (2015)

**sJAM:** 
$$U(p) = \sqrt{(M + \Sigma_s)^2 + p^2} + \Sigma_0 - \sqrt{M^2 + p^2}$$

**Requirement:** fluctuations (particularly in density) suppressed to the maximum extent possible

**Note:** each code author has provided her/his own thermal model results

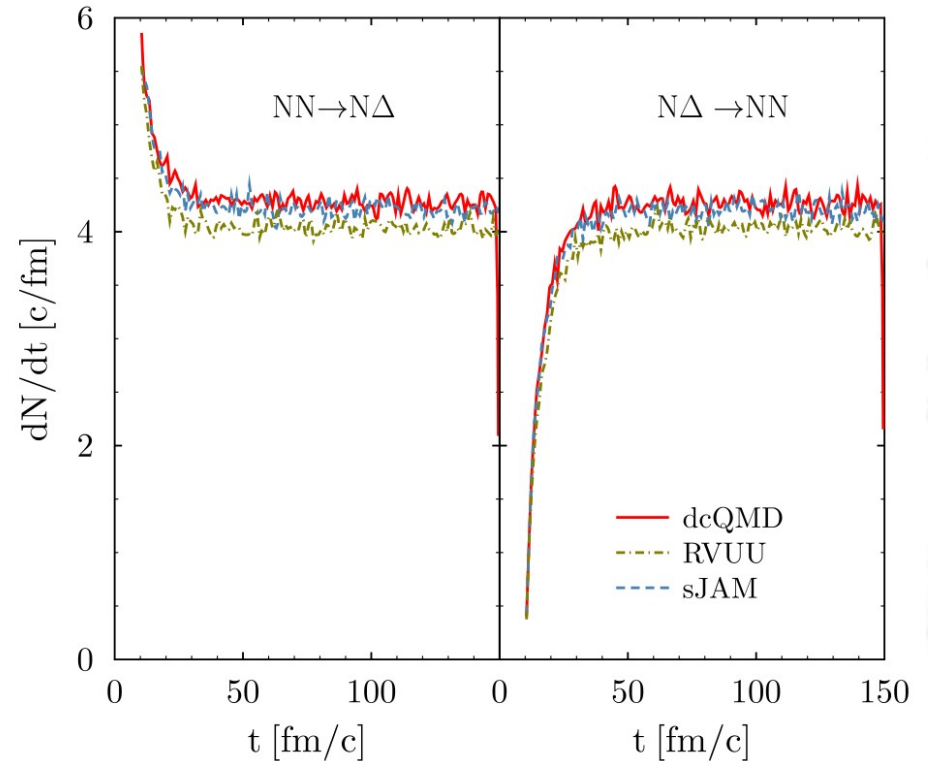


# Available Results

- Initial momentum distribution of nucleons
- Final ( $t=150$  fm/c) momentum distributions of nucleons,  $\Delta(1232)$ , pions
- Final ( $t=150$  fm/c) mass distribution of  $\Delta(1232)$
- Time evolution of  $\Delta(1232)$  and pion multiplicities
- Time evolution of inelastic collision rates
- Invariant mass distribution of collision rates

more `casc_vacmass` results are available in  
A. Ono et al., PRC 100, 044617 (2019)

## `casc_vacmass`



# Isospin Symmetric Matter

## N- $\Delta$ System

**Box size:** 20 fm

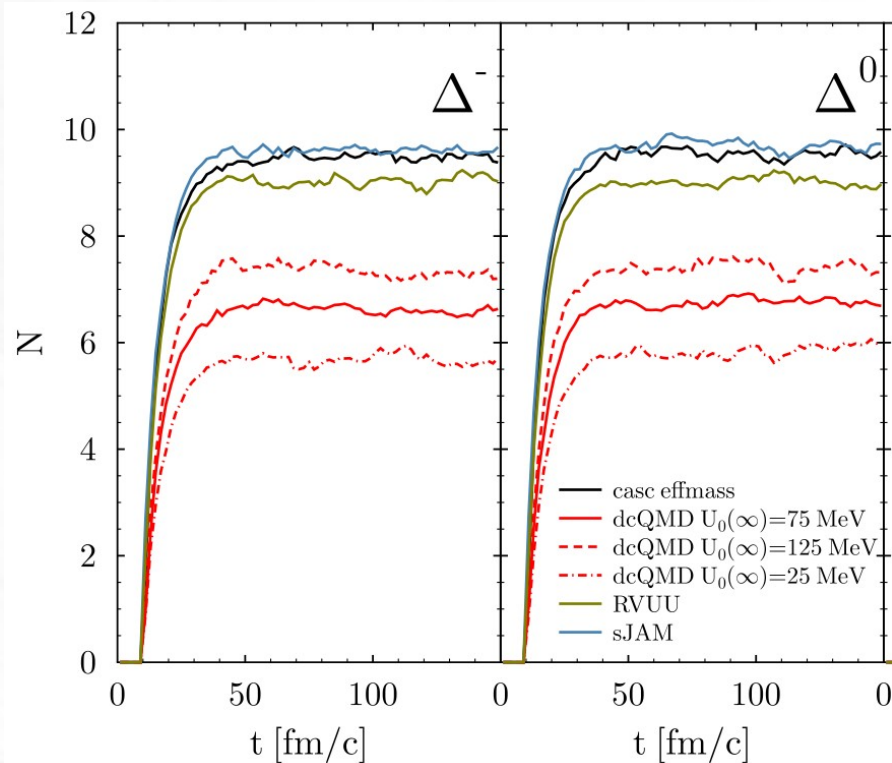
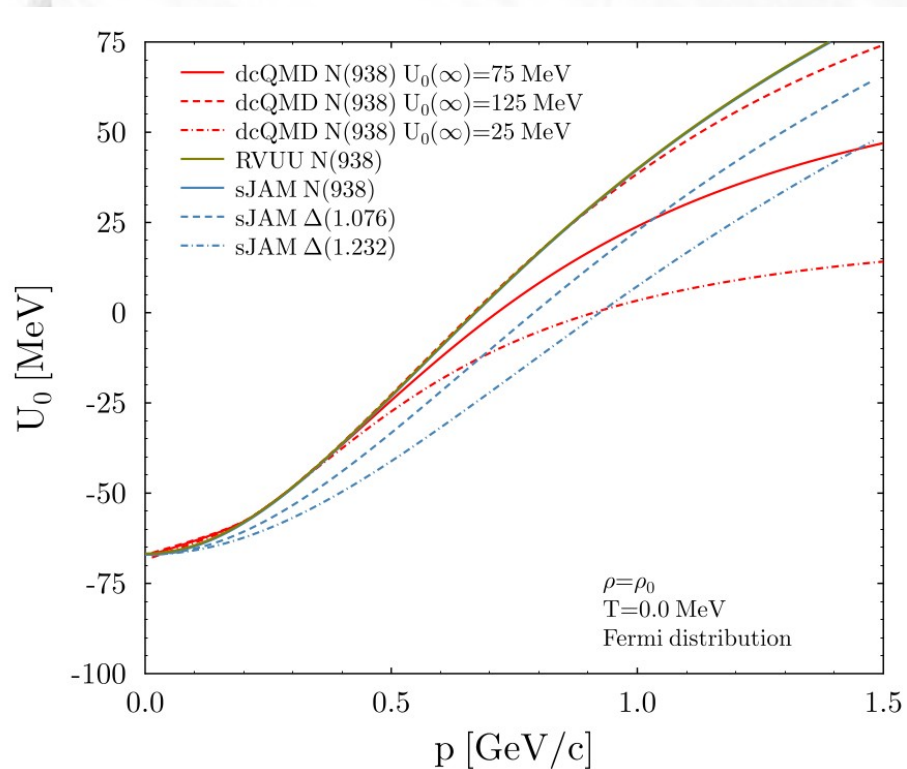
**Initial Temperature:** 60 MeV

**Initial Composition:** 640 N + 640 P ( $\rho=\rho_0$ )

**Requirements:** fluctuations in density removed

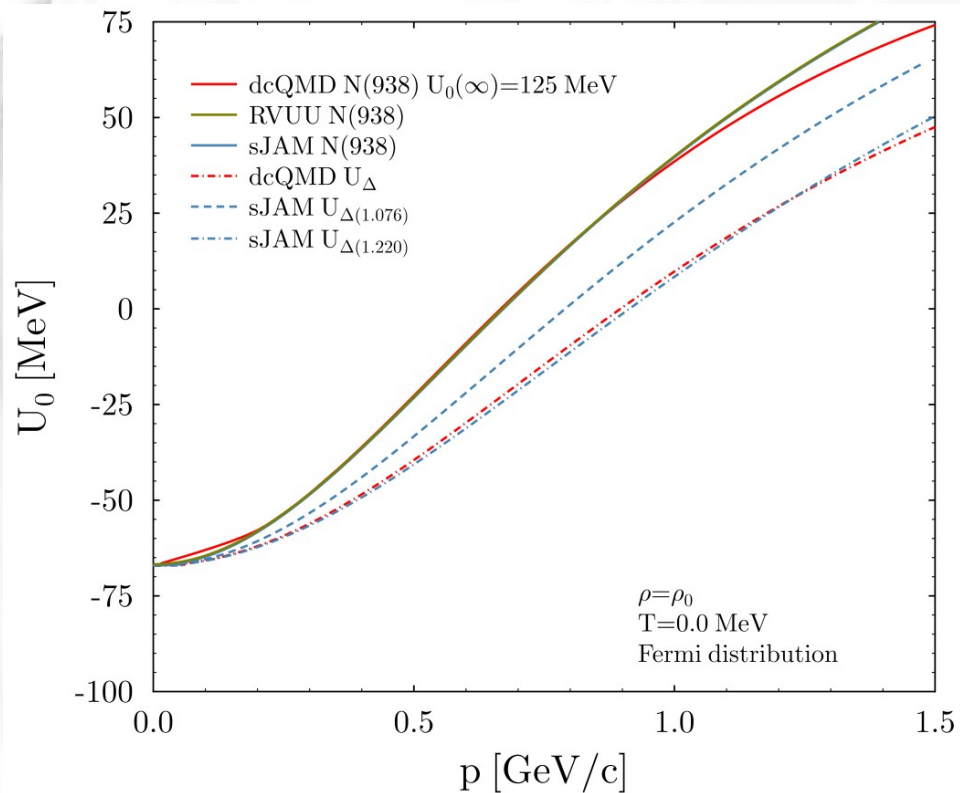
# Full\_mdi\_th (Equilibrium multiplicities)

## Momentum dependence of the interaction

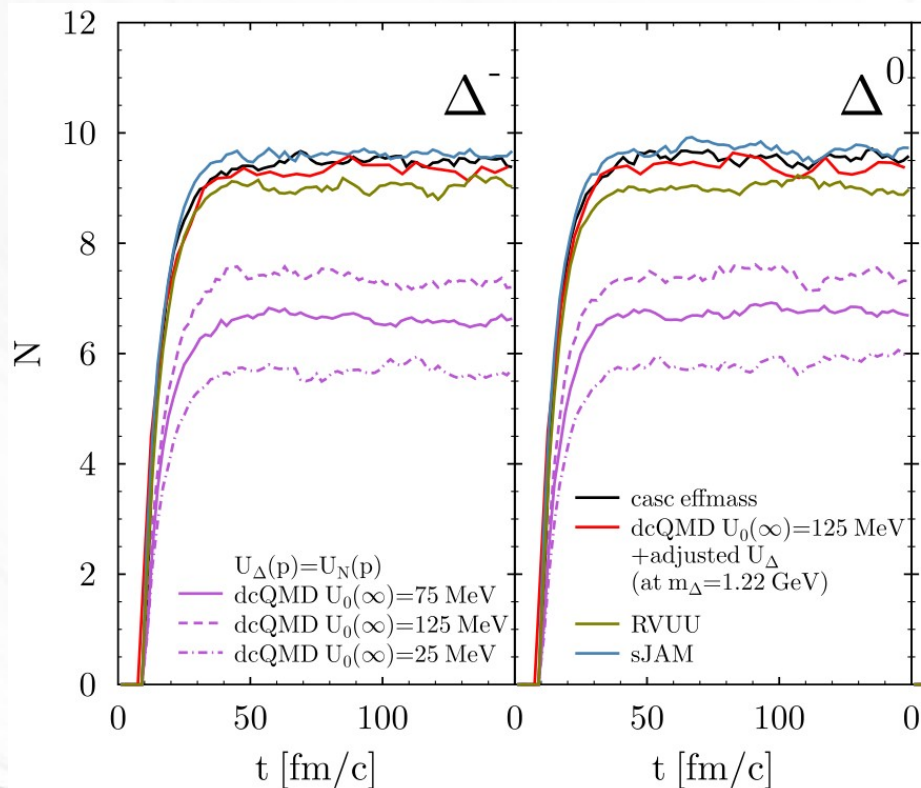


# Full\_mdi\_th (Equilibrium multiplicities)

## Momentum dependence of the interaction

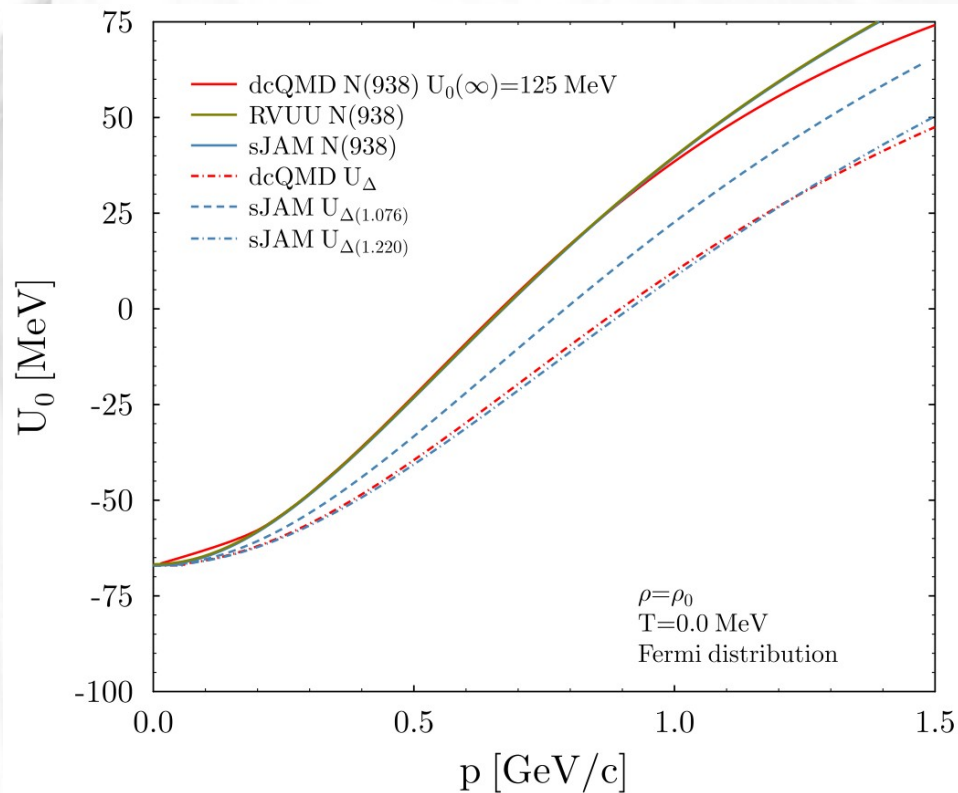


## Multiplicities

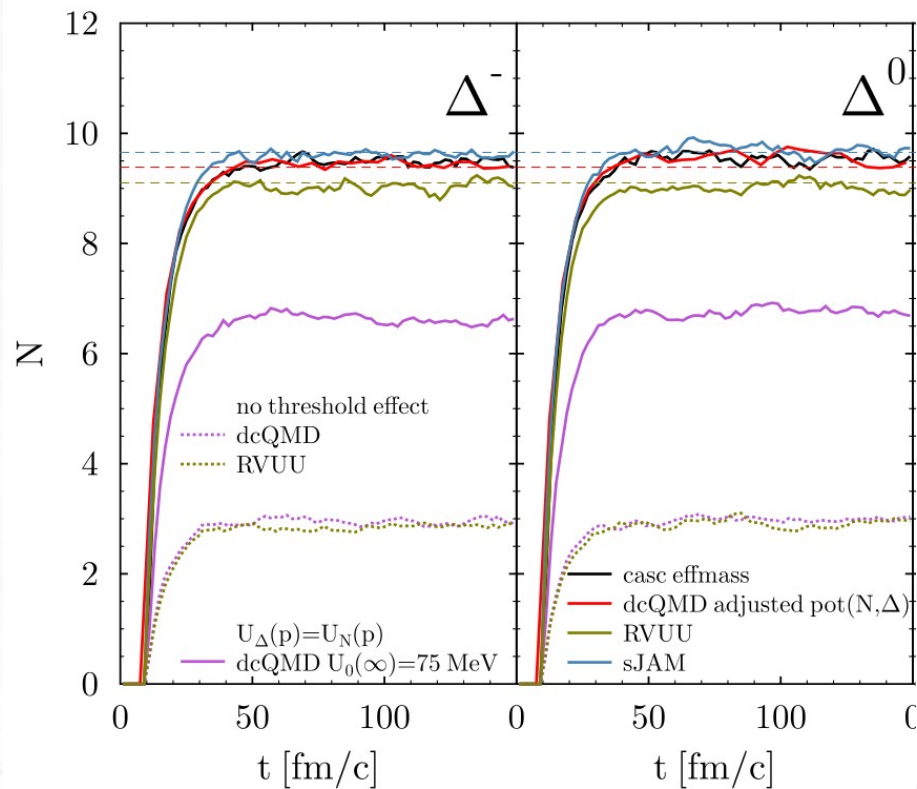


# Full\_mdi\_th (Equilibrium multiplicities)

## Momentum dependence of the interaction

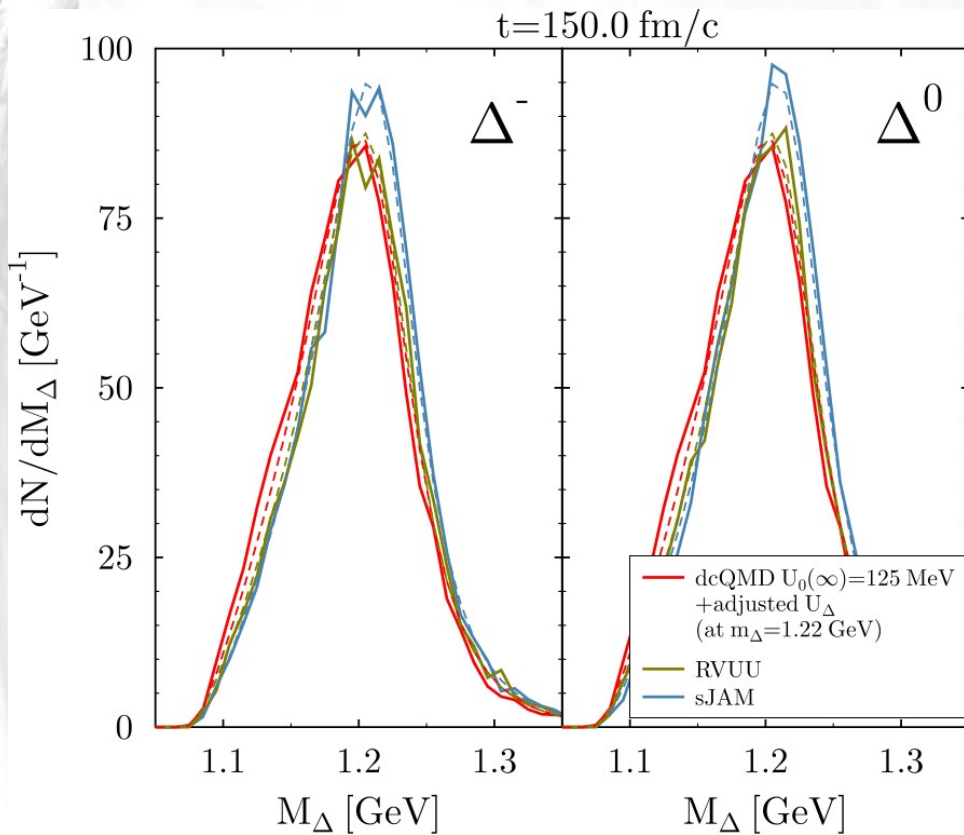


## Multiplicities

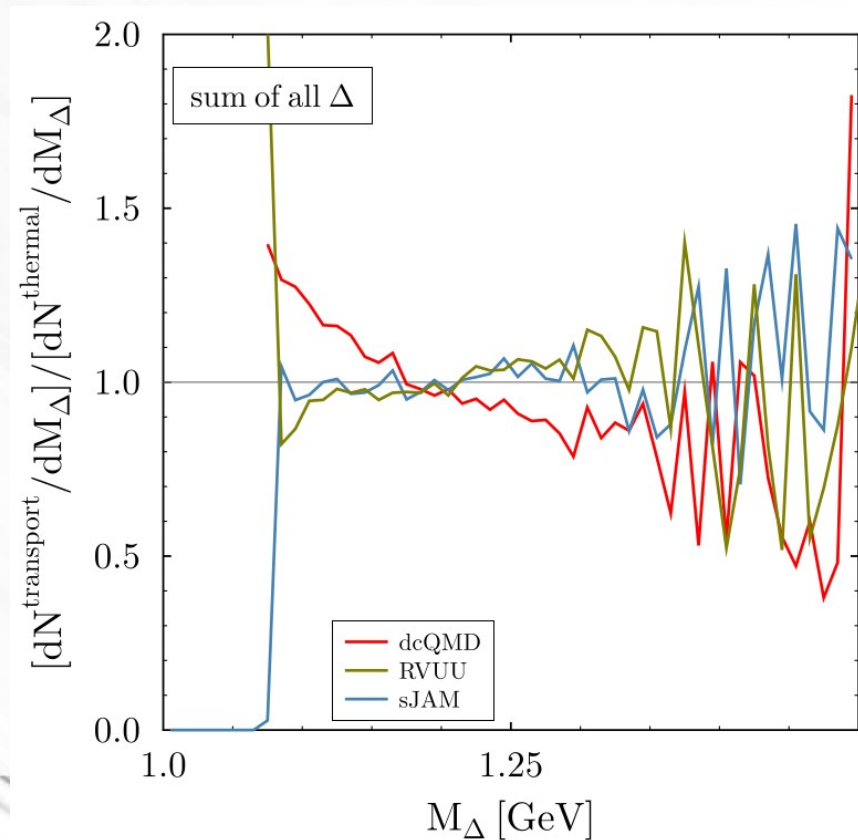


# Full\_mdi\_th (Mass Distribution)

Mass distribution at equilibrium



Quantitative comparison with thermal models



# N- $\Delta$ - $\pi$ System

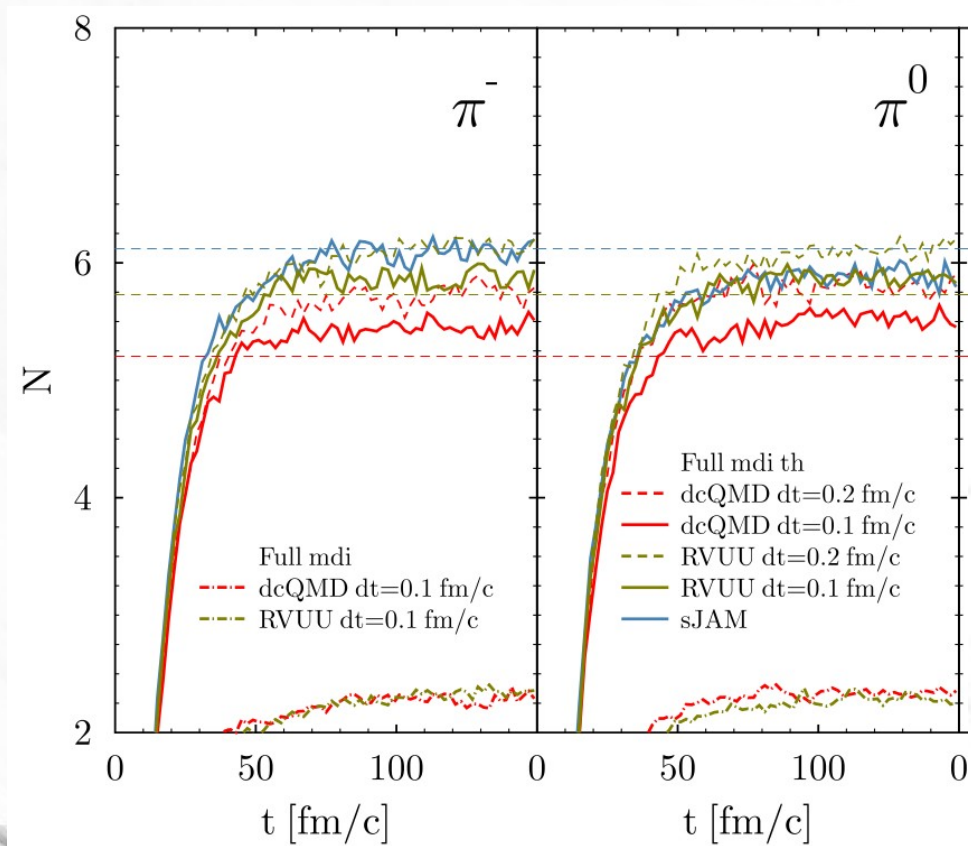
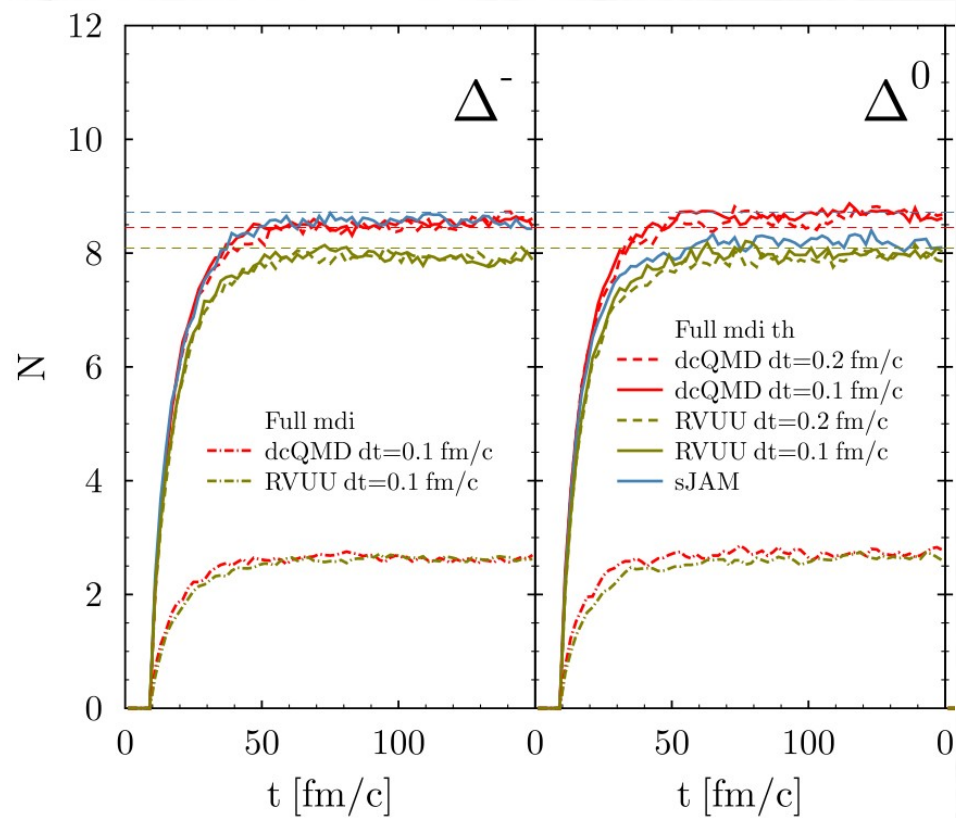
**Box size:** 20 fm

**Initial Temperature:** 60 MeV

**Initial Composition:** 640 N + 640 P ( $\rho=\rho_0$ )

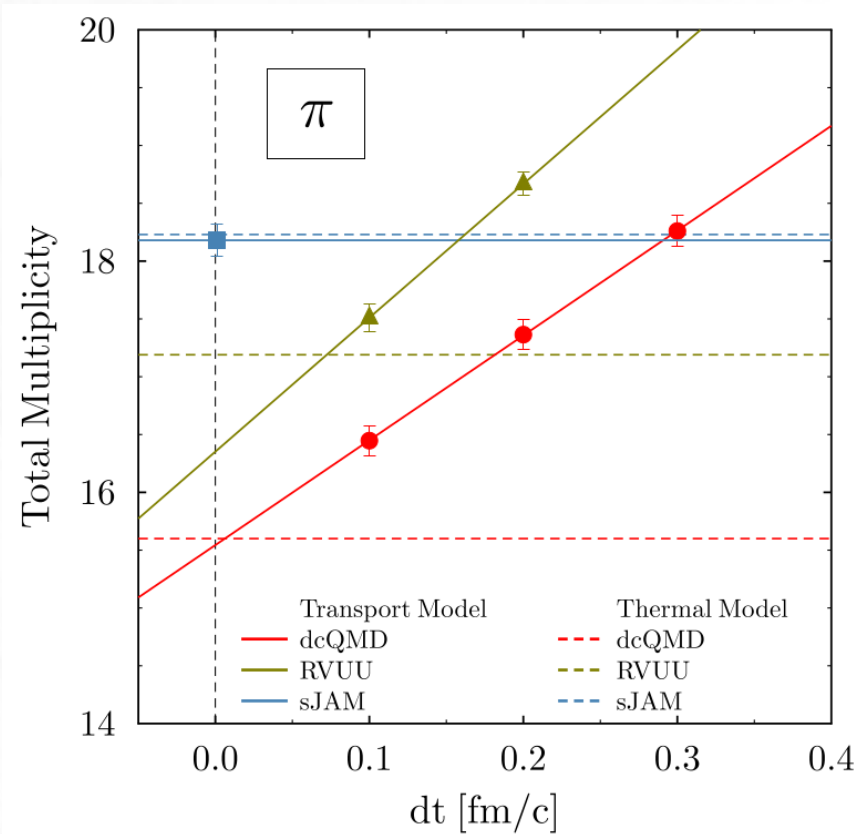
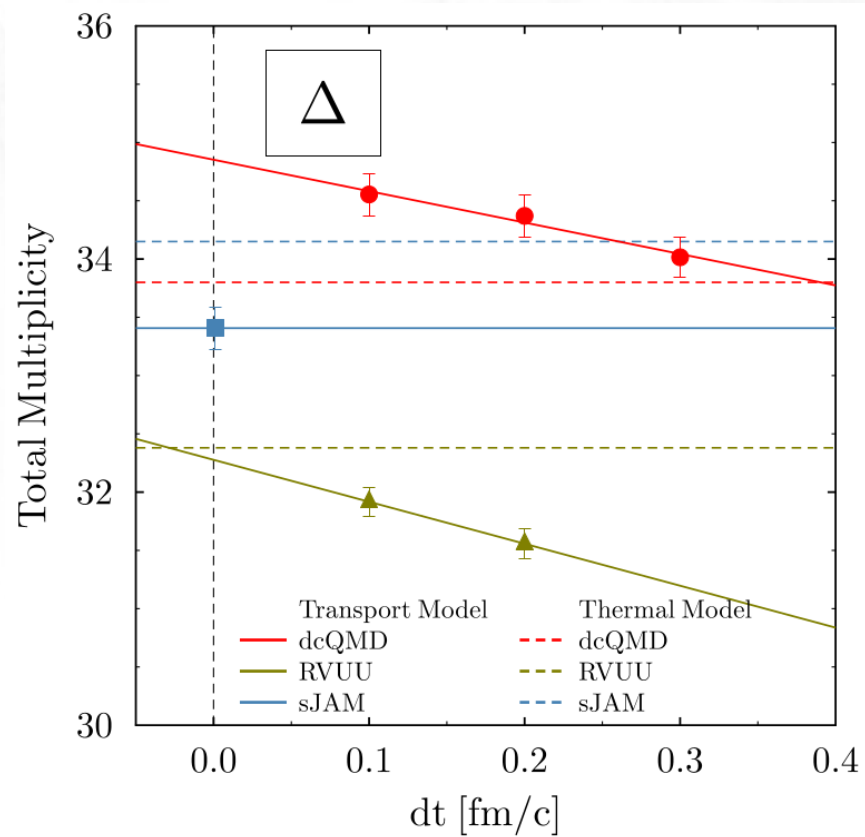
**Requirements:** fluctuations in density removed

# Full\_mdi\_th (Equilibrium multiplicities)

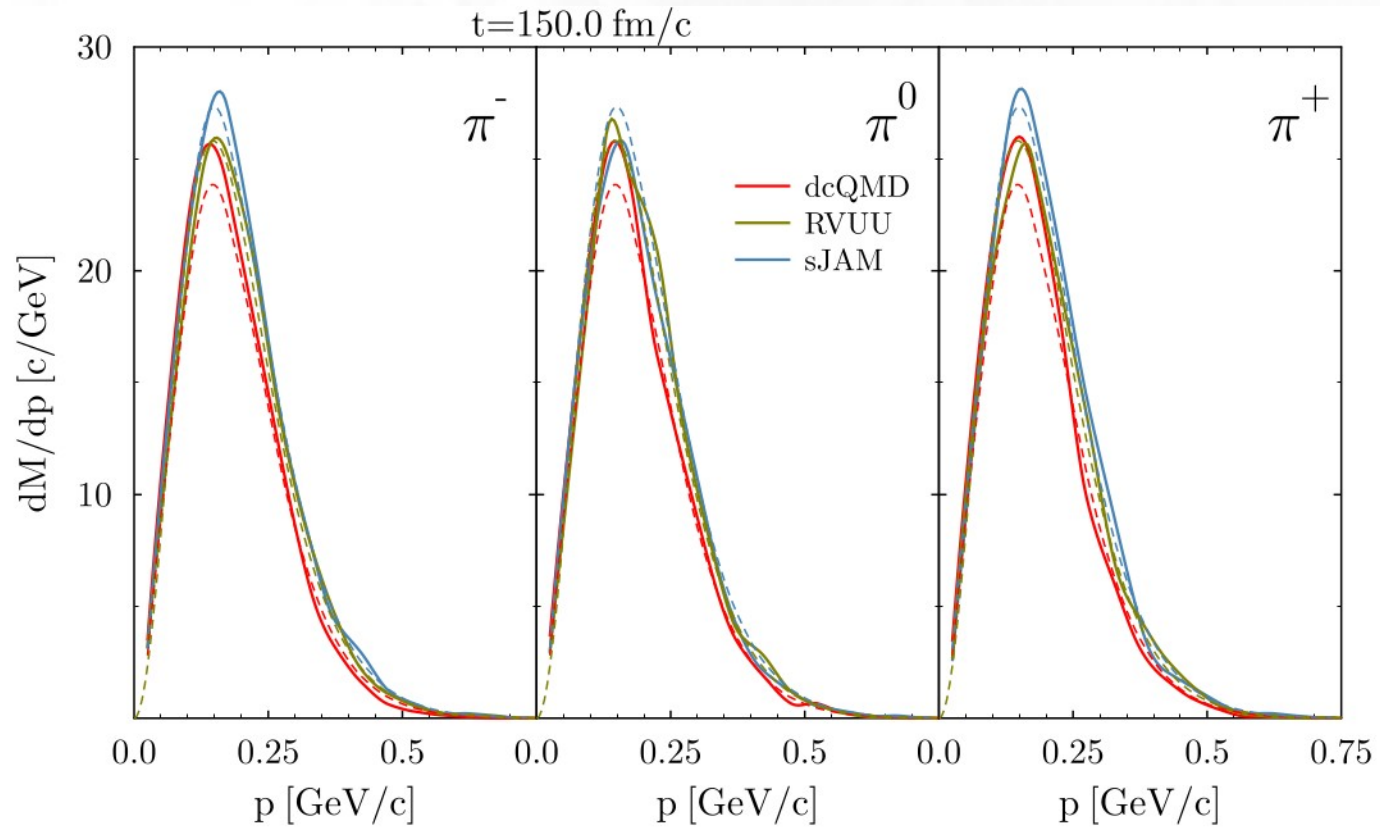




# Full\_mdi\_th (Time Step Dependence)



# Full\_mdi\_th (Pion Momentum Distribution)



# Isospin Asymmetric Matter

**Box size:** 20 fm

**Initial Temperature:** 60 MeV

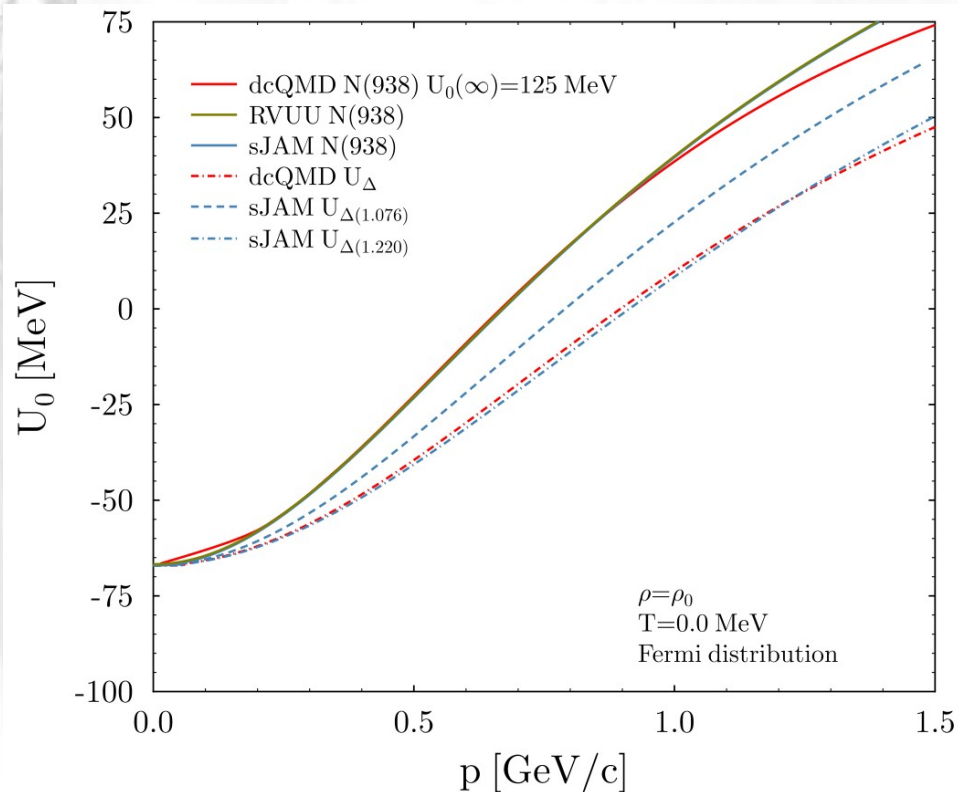
**Initial Composition:** 768 N + 512 P ( $\rho=\rho_0; \delta=0.2$ )

**Requirements:** fluctuations in density removed

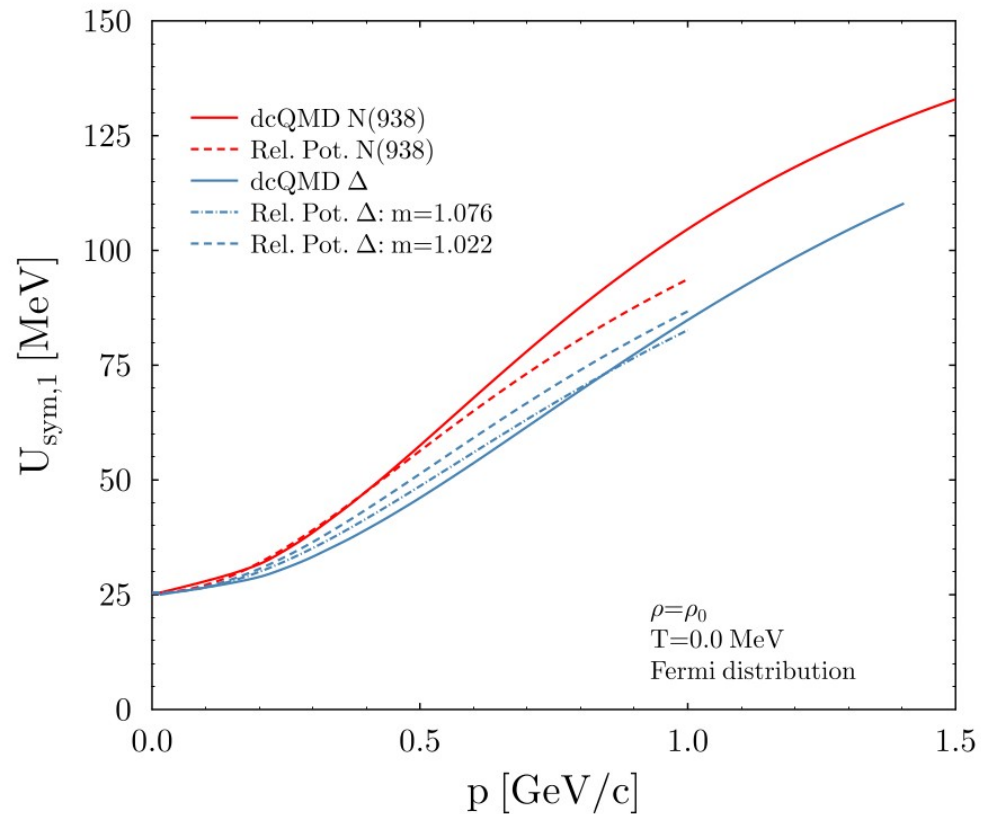
# Momentum dependence of the interaction

at saturation density and  $\delta=0.2$  (where context imposes that)

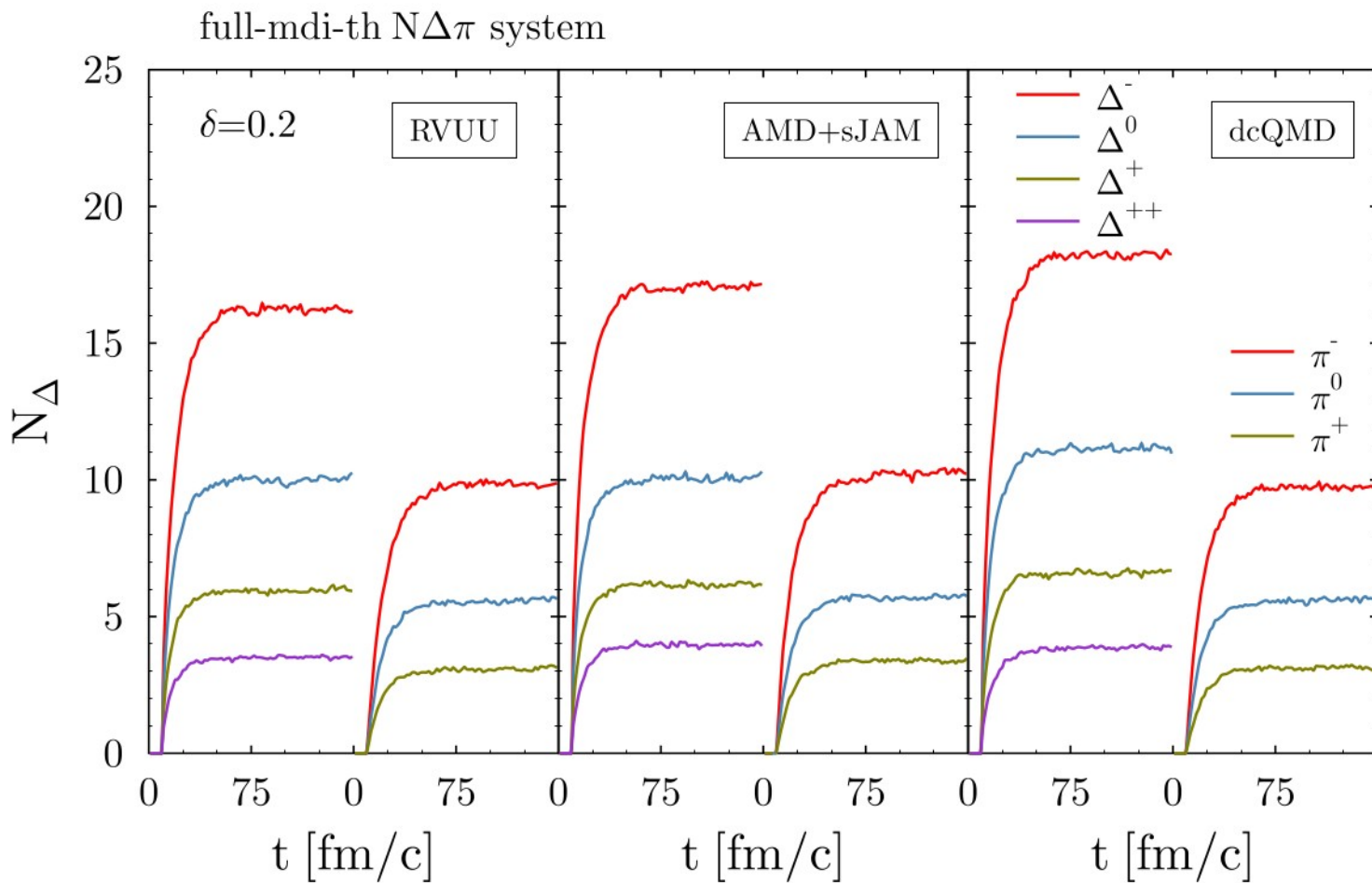
**isoscalar**



**isovector**



# Full\_mdi\_th: Equilibrium Multiplicities



# Full\_mdi\_th: Ratios

$$R_{\Delta-like} = \frac{N_{\Delta^-} + \frac{1}{3}N_{\Delta^0}}{N_{\Delta^{++}} + \frac{1}{3}N_{\Delta^+}},$$

$$R_{\pi-like} = \frac{N_{\pi^-} + N_{\Delta^-} + \frac{1}{3}N_{\Delta^0}}{N_{\pi^+} + N_{\Delta^{++}} + \frac{1}{3}N_{\Delta^+}}$$

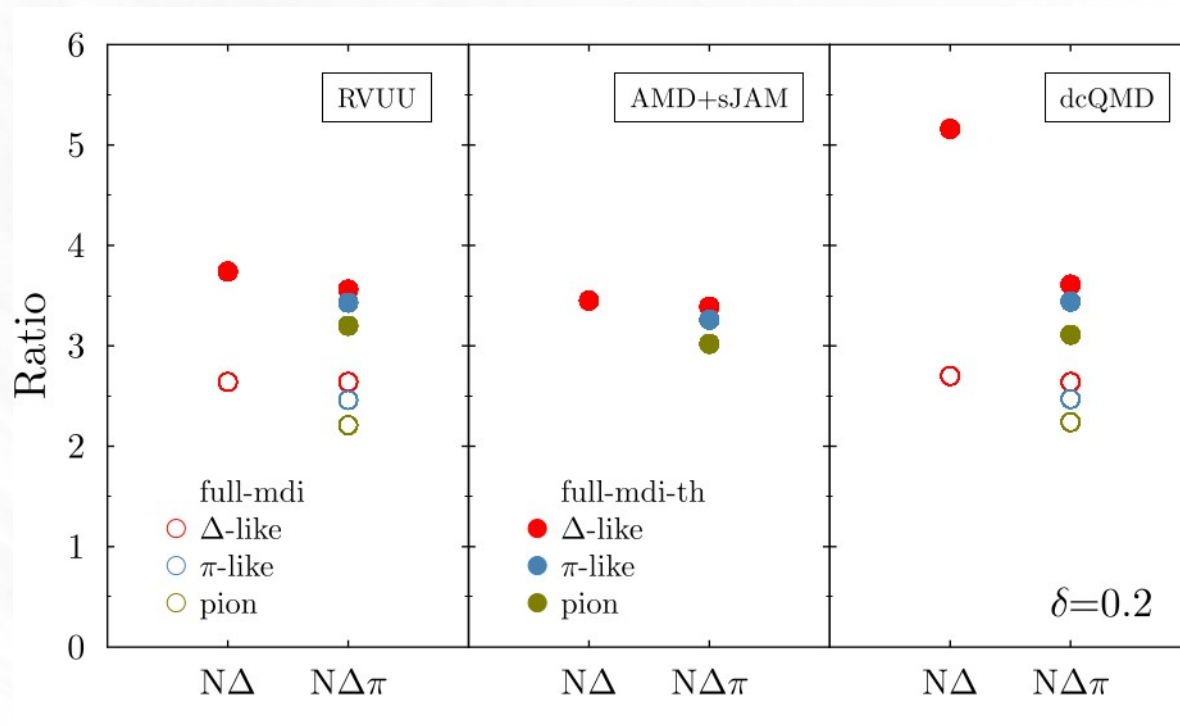
$$R_{\pi} = \frac{N_{\pi^-}}{N_{\pi^+}}.$$

$$U_{\Delta^-} = U_n = U_{is} + U_{iv}$$

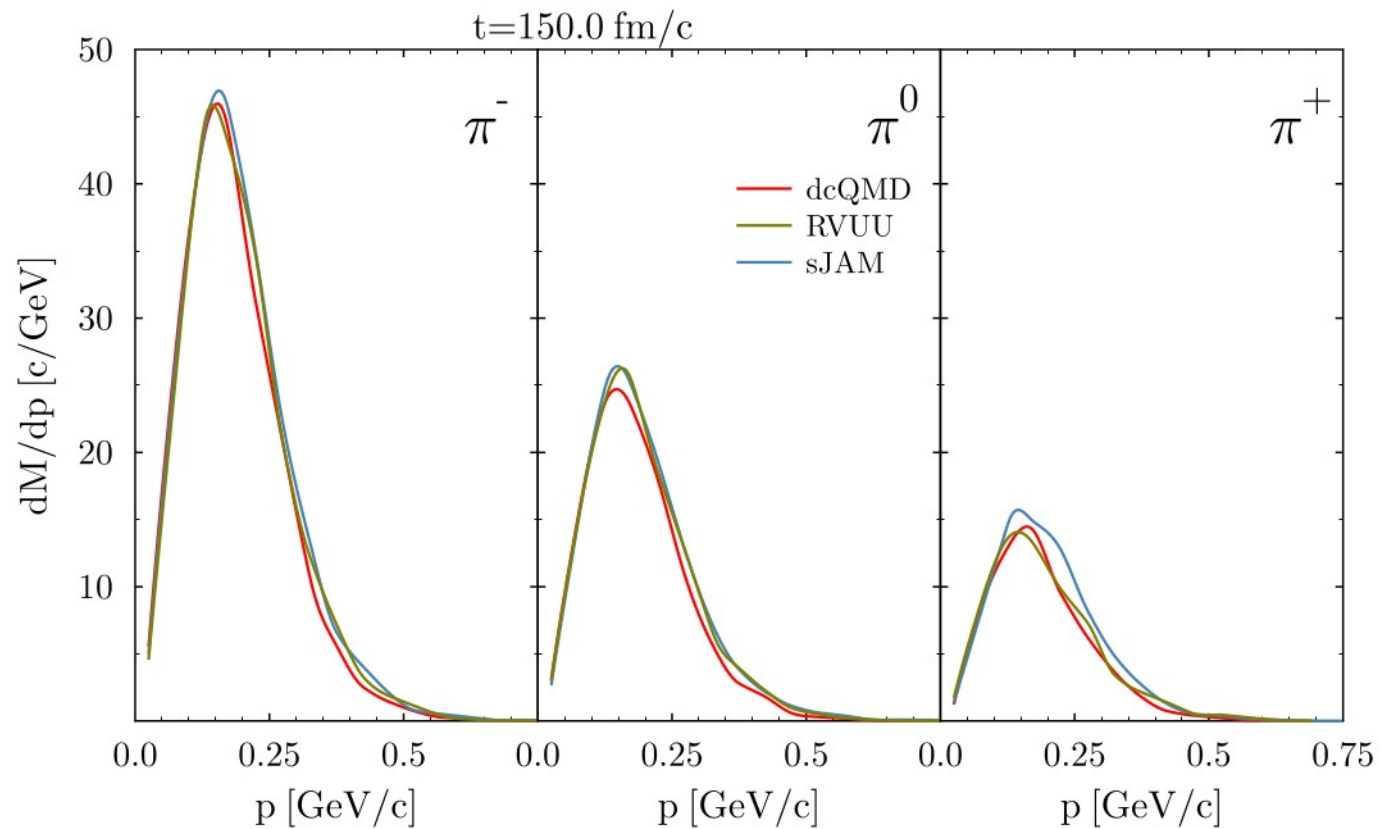
$$U_{\Delta^0} = \frac{2}{3}U_n + \frac{1}{3}U_p = U_{is} + \frac{1}{3}U_{iv}$$

$$U_{\Delta^+} = \frac{1}{3}U_n + \frac{2}{3}U_p = U_{is} - \frac{1}{3}U_{iv}$$

$$U_{\Delta^{++}} = U_p = U_{is} - U_{iv}$$

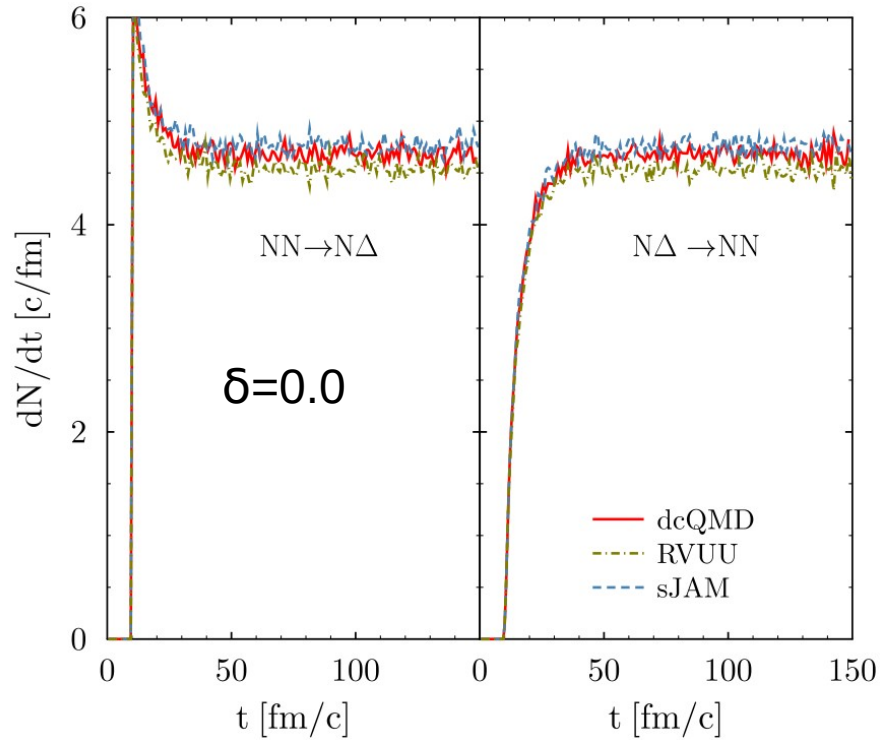


# Pion Momentum Distribution

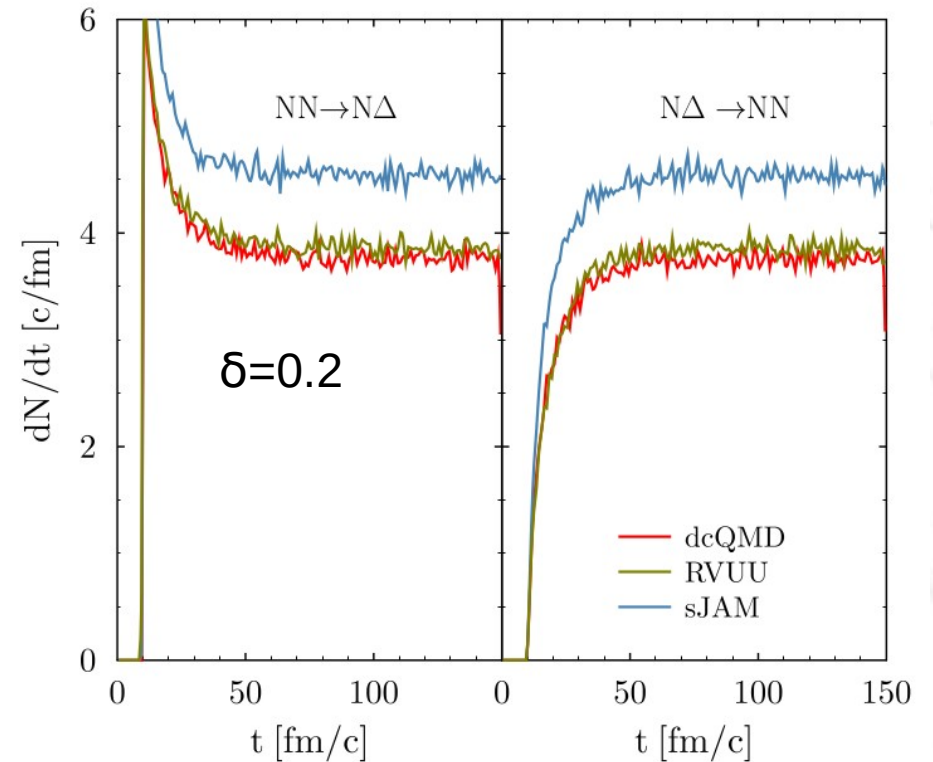


# Reaction Rates

Time dependence of collision rates



related to the choice of effective masses ??

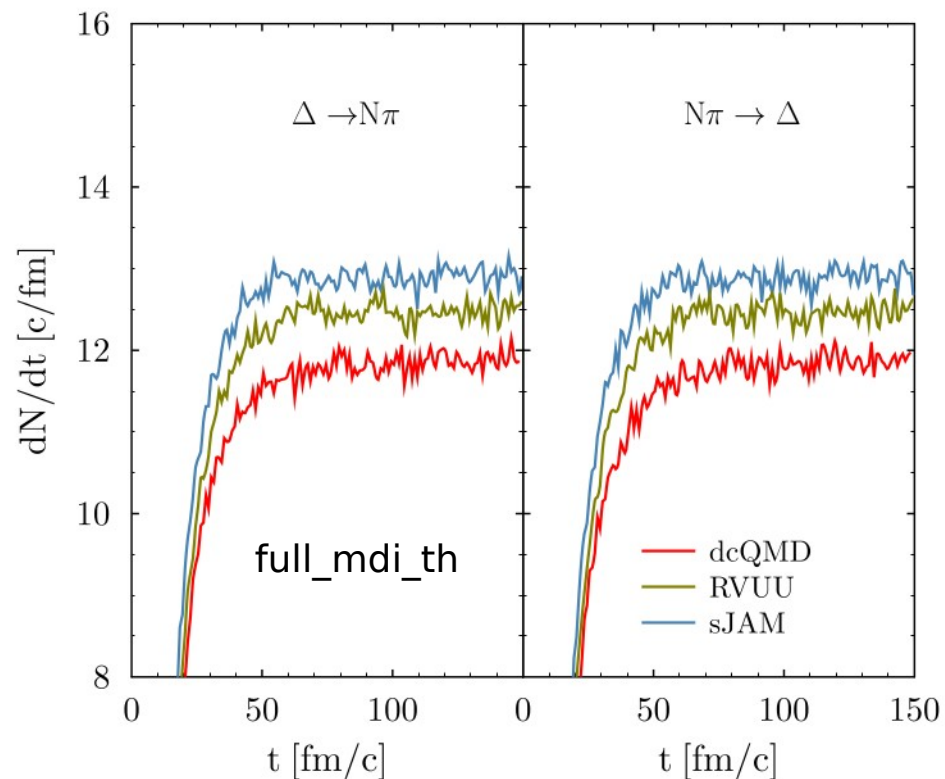
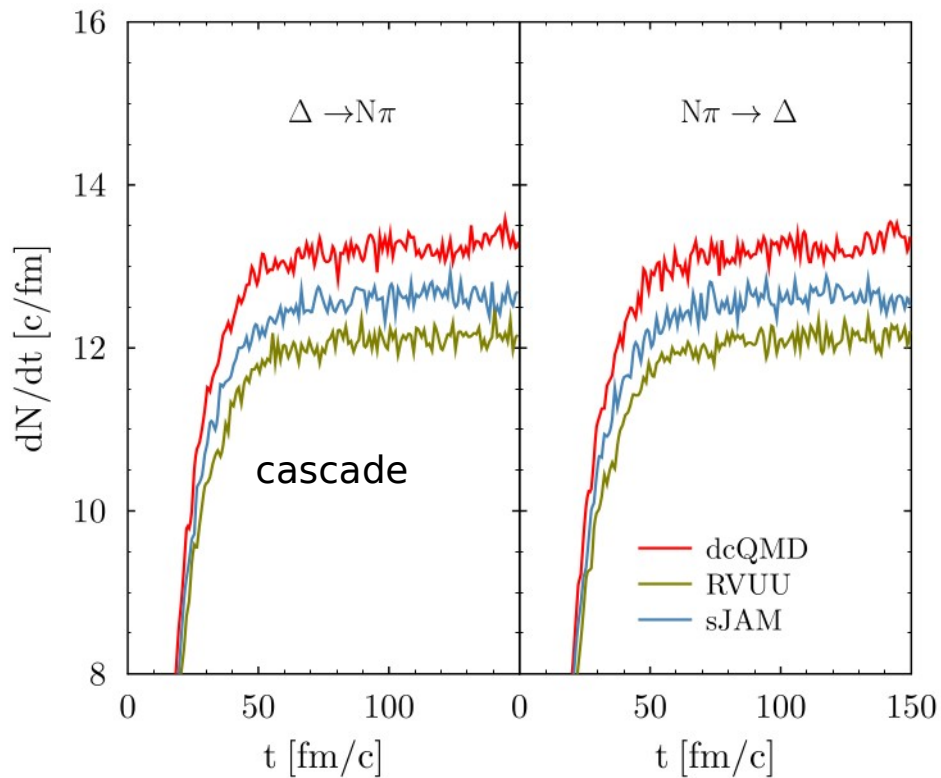




# Reaction Rates

## Time dependence of collision rates

exact results expected to shed light on the source of differences



# Summary

most studied quantities reveal a good agreement (5%) among transport models ( $\delta=0.0; 0.2$ )

## Exceptions:

- $\Delta^{++}, \Delta^+$  multiplicities: dcQMD ( $\delta=0.2$ )
- $\Delta$  mass spectra: dcQMD ( $\delta=0.0, 0.2$ )
- collision rates  $NN \leftrightarrow N\Delta$ : dcQMD, RVUU versus sJAM
- collision rates  $\Delta \leftrightarrow N\pi$  : magnitude similar to cascade calculations

## Possible Developments:

- investigate the impact of density fluctuations
- sensitivity to symmetry energy and effective masses