



NUSYM24 XIIth International
Symposium on Nuclear Symmetry
Energy, Caen, September 9-14, 2024

Equation of state in dense nuclear matter controlled by nuclear data

Flow data,
Giant Monopole Resonance.

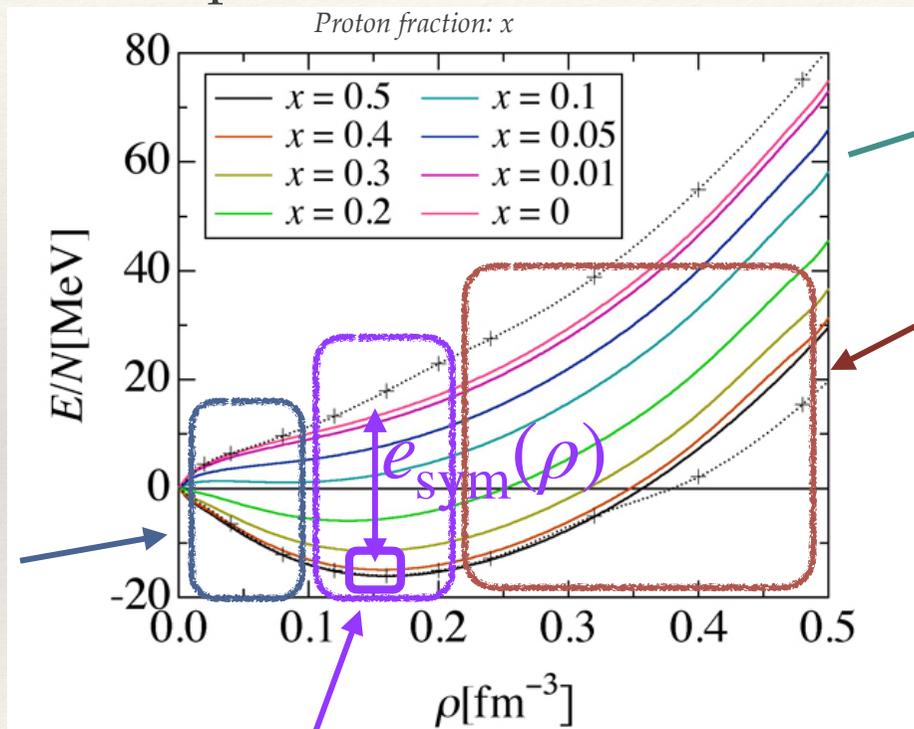
Jérôme Margueron, IRL-NPA, CNRS & MSU, FRIB, East Lansing, USA.

In collaboration with R. Kumar (post-doc), E. Khan (IJCLab),
and P. Danielewicz, W. Lynch, B. Tsang (FRIB).



Known and unknown of the nuclear EOS

From a prediction to a measurement:



pQCD

Predictions

Nuclear empirical parameters (NEP):

$$n_{sat} = 0.155 \pm 0.005 \text{ fm}^{-3} \quad E_{sym} = 32 \pm 2 \text{ MeV}$$

$$E_{sat} = -15.8 \pm 0.4 \text{ MeV} \quad L_{sym} = 60 \pm 15 \text{ MeV}$$

$$K_{sat} = 240 \pm 20 \text{ MeV}$$

Controlled by low density QCD modeling or nuclear physics

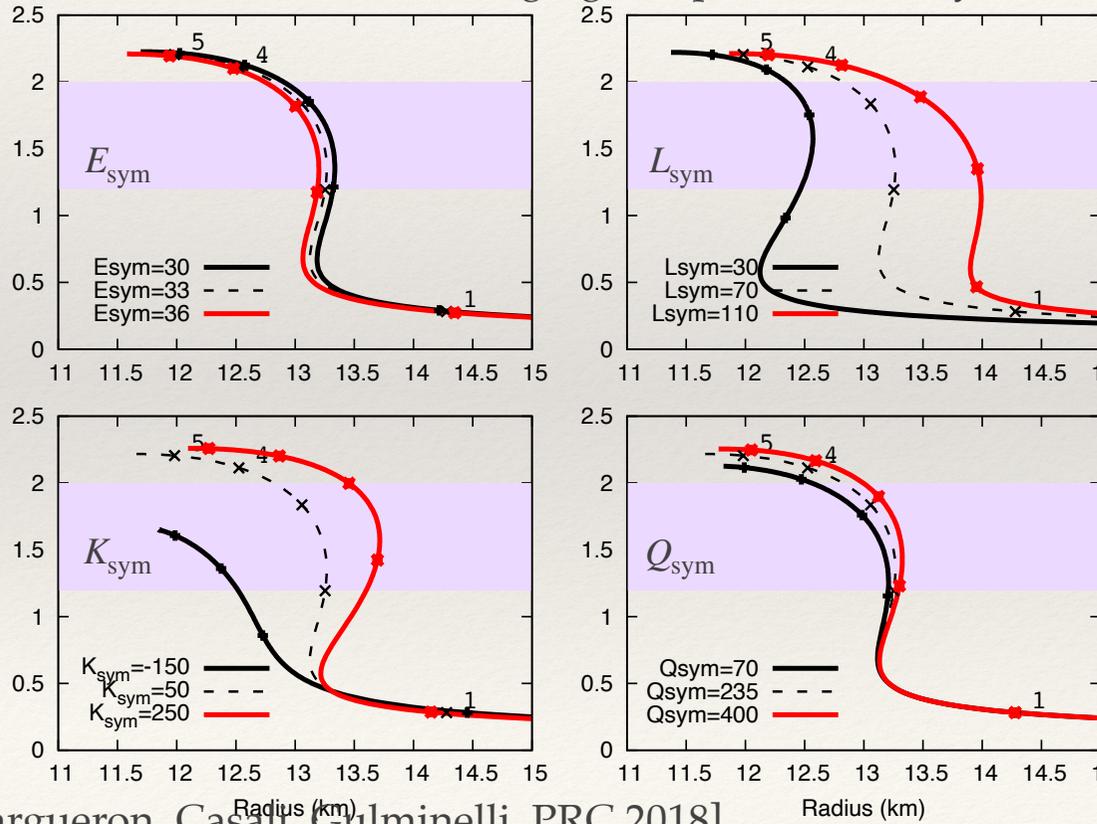
Saturation density n_{sat} & energy E_{sat} , E_{sym} , ...

Controlled by nuclear experiments.

EoS and neutron stars observations

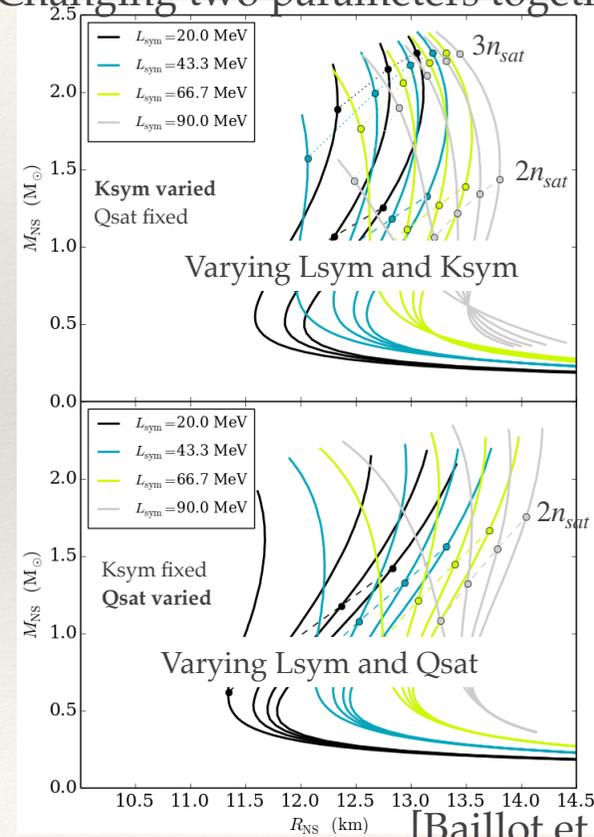
Impact of changing the NEP on the MR relation of neutron stars

Isovector channel Changing one parameter only



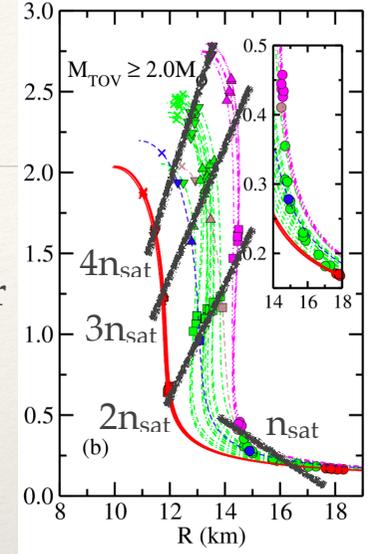
[Margueron, Casali, Gulminelli, PRC 2018]

Changing two parameters together



[Baillot et al., ApJ 2020]

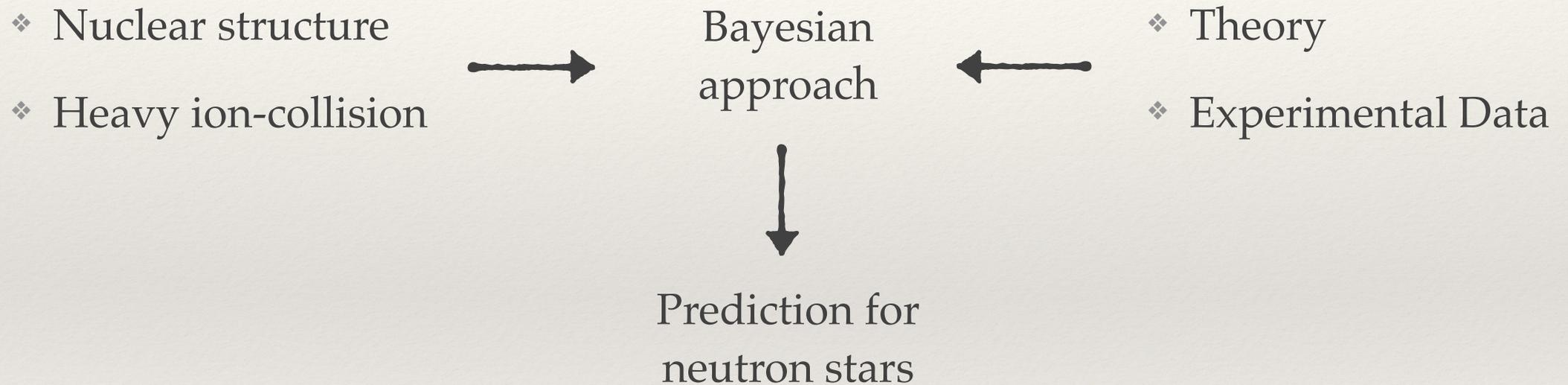
Carlson+ PRC 2023



Questions:

- Neutron star crust contribution?
- Phase transition in the inner core?

A multi-physics approach based on Bayesian statistics

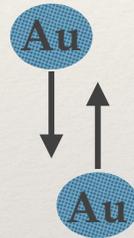


Nuclear heavy-ion collision

Overview of the dynamics for a Au+Au collision:

Initial condition:

Head-on collision
($b=6$ fm).

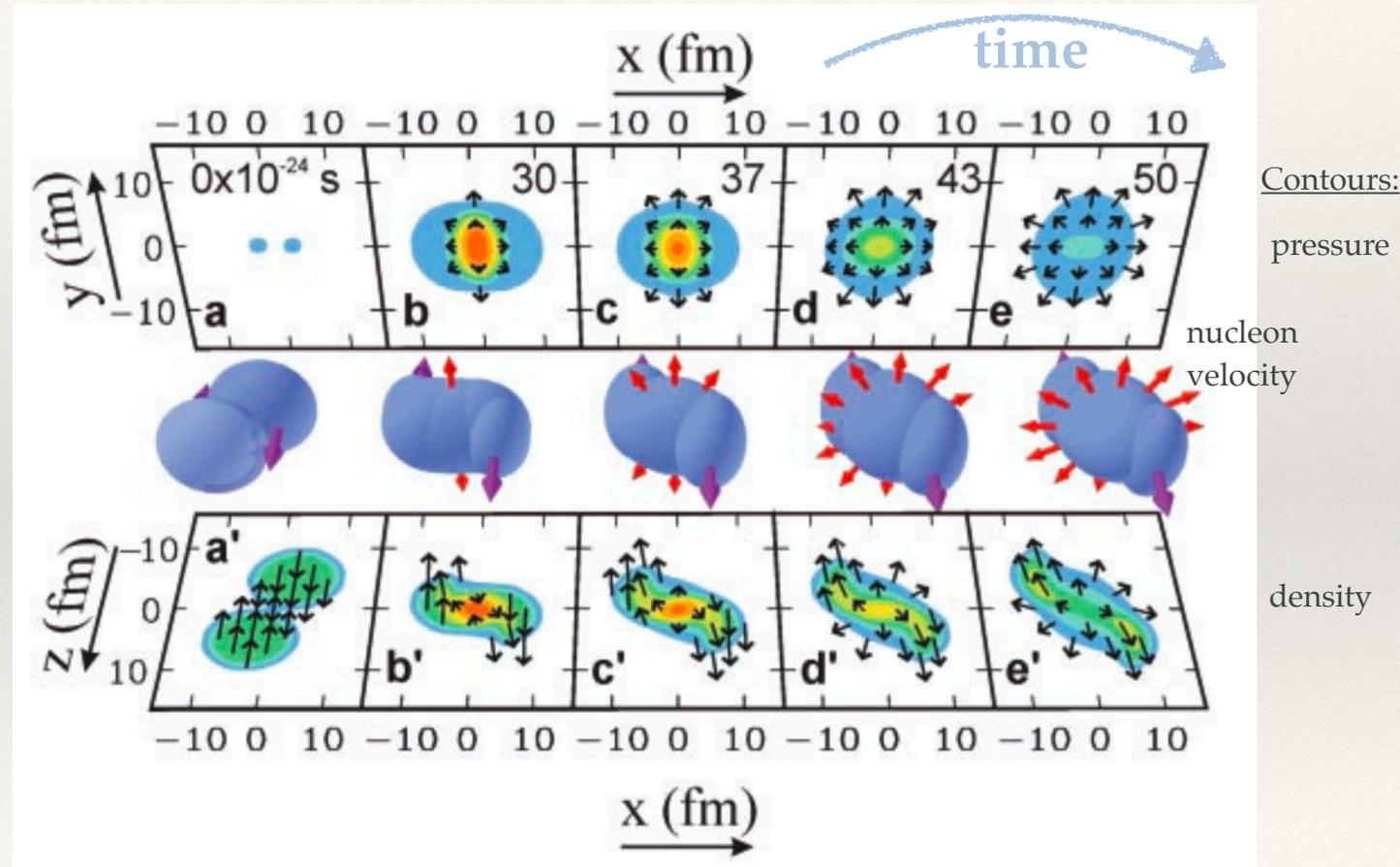


Particle distribution:

$$\frac{1}{v_0} \frac{dN}{u_T du_T dy_0 d\phi} =$$

$$1 + 2v_1 \cos \phi + 2v_2 \cos 2\phi + \dots$$

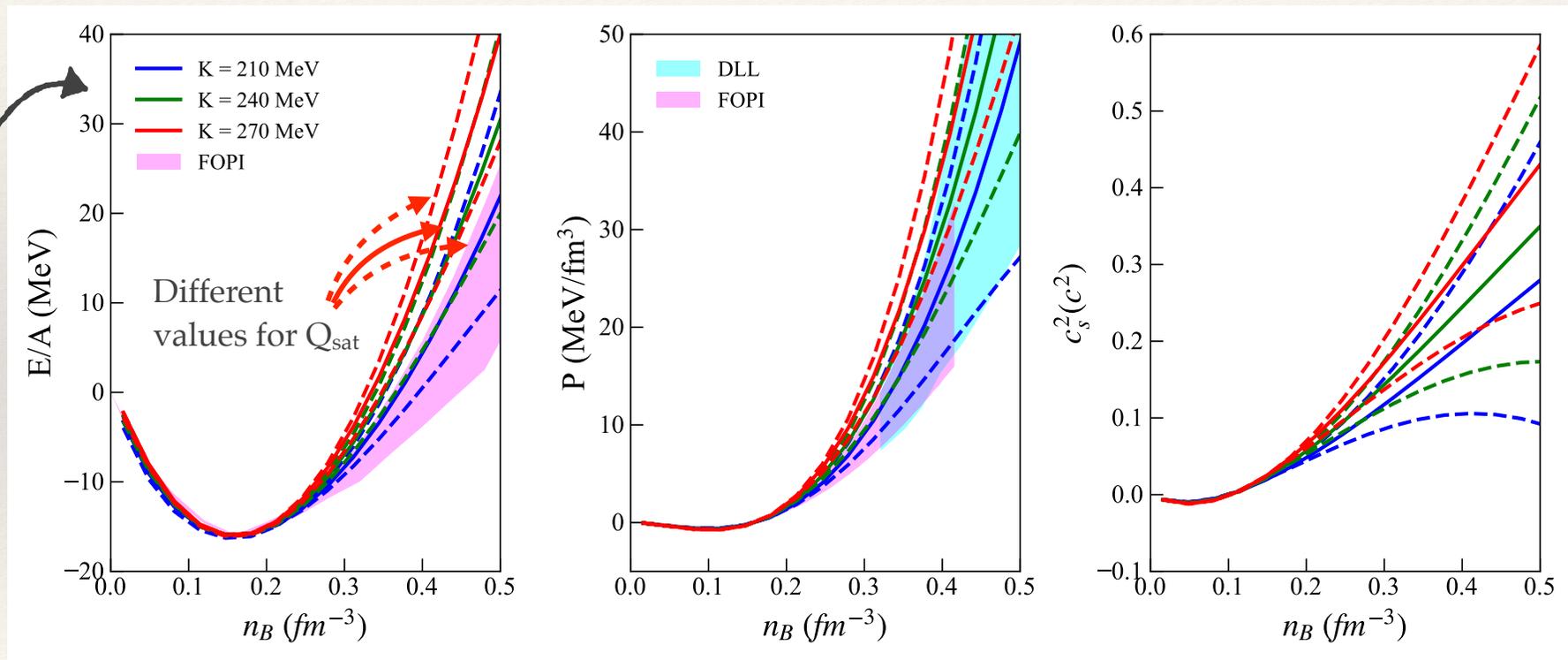
↙ directed flow
↙ elliptic flow



Danielewicz, Lacey, Lynch, Science 2002

Equation of state in symmetric matter

Controlled by n_{sat} , E_{sat} , K_{sat} , Q_{sat} .



K_{sat} is fixed by GMR results (prior).
 $K_{\text{sat}} = 240 \pm 30$ MeV

→ 9 prior EoSs varying K_{sat} and Q_{sat} .

Bayesian approach

Model: IQMD [Aichelin, PR 1991;
Hartnack, EPJA 1998]

$E_{inc} = 400\text{-}1500 \text{ MeV} / u$

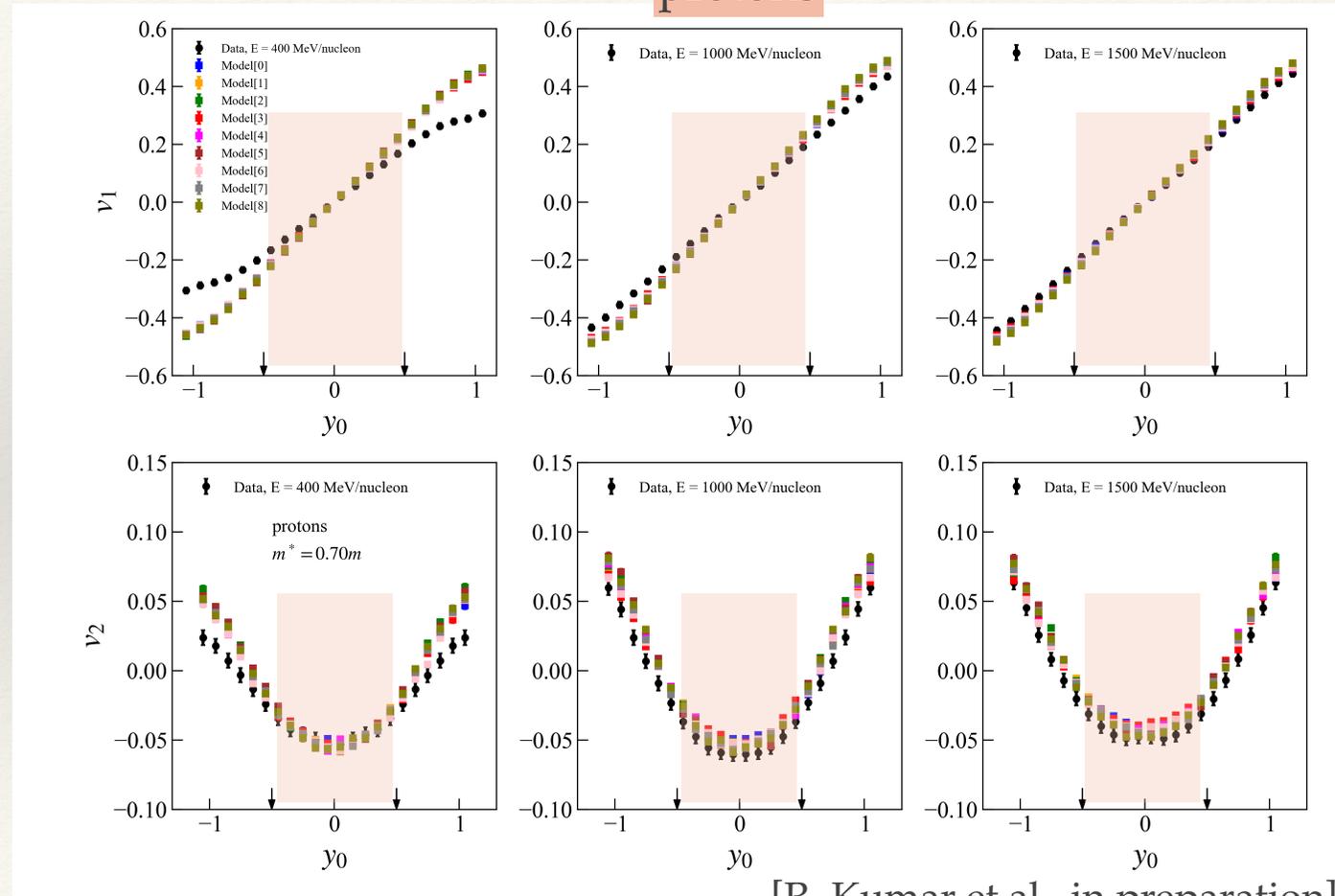
Each EoS is assessed by its ability
to reproduce experimental data:

$$P_{EoS} \propto \exp\left(-\frac{1}{2}\chi^2\right)$$

with

$$\begin{cases} \chi^2 = \frac{1}{N_{dof}} \Delta_{explmod}^T \Sigma^{-1} \Delta_{explmod} \\ \Delta_{explmod} = O(exp) - O(model) \\ \text{FOPI data} \end{cases}$$

protons



[R. Kumar et al., in preparation]

Bayesian approach

deuterons

Model: IQMD [Aichelin, PR 1991;
Hartnack, EPJA 1998]

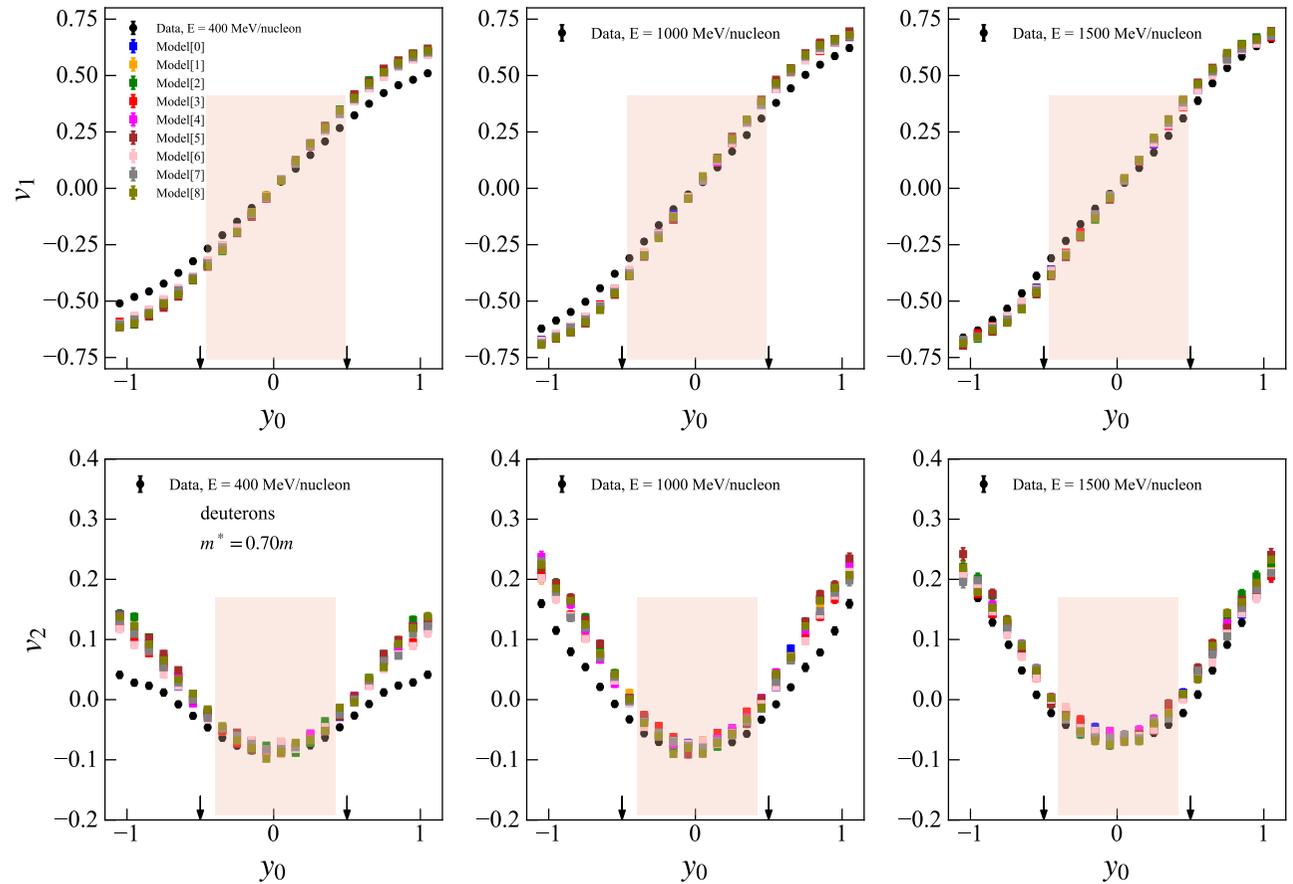
$E_{inc} = 400-1500 \text{ MeV} / u$

Each EoS is assessed by its ability
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[R. Kumar et al., in preparation]

Bayesian approach

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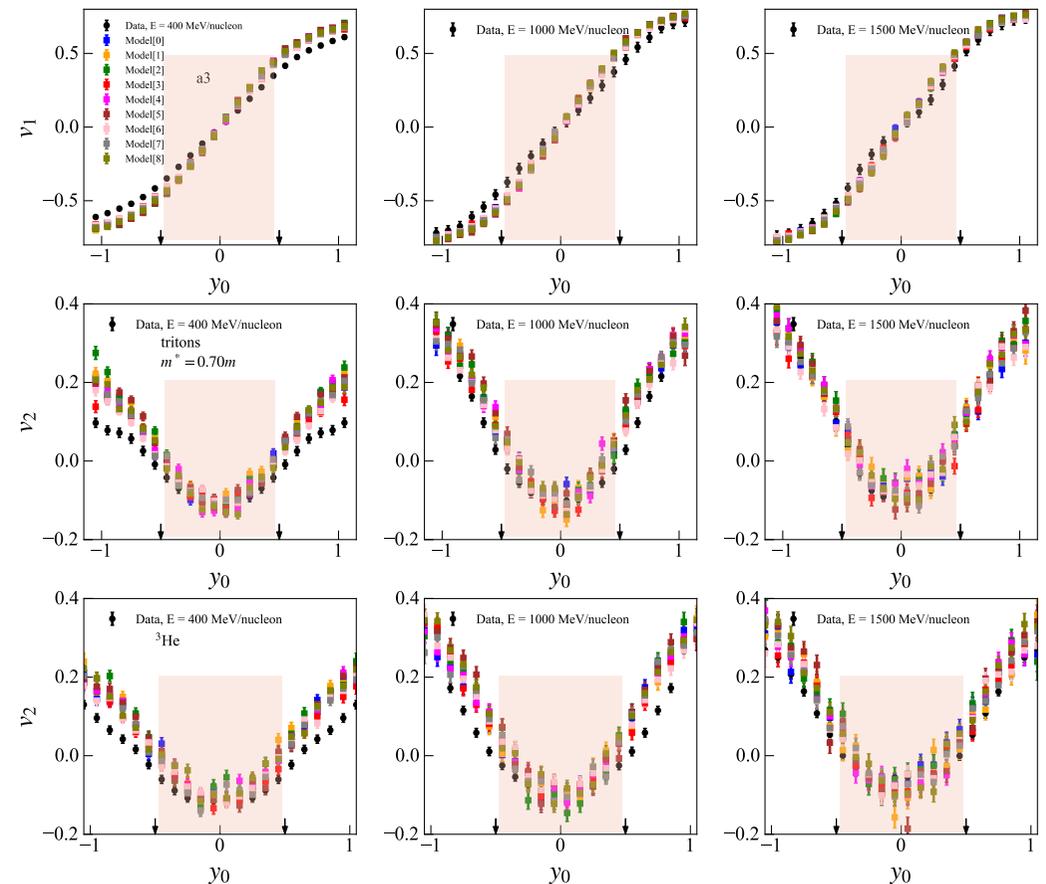
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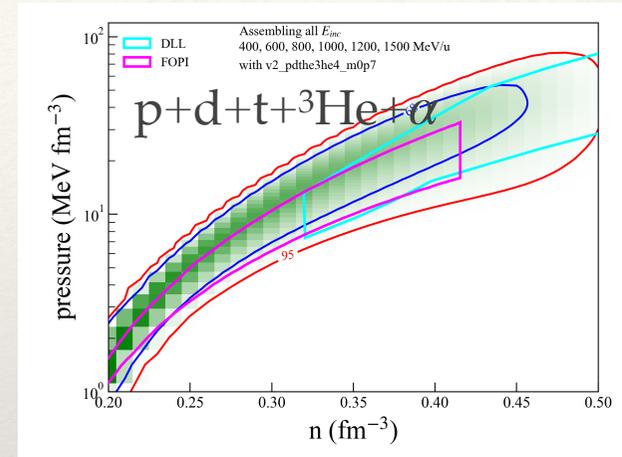
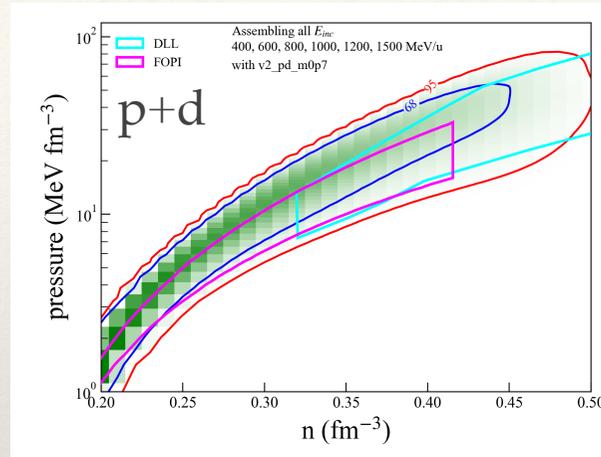
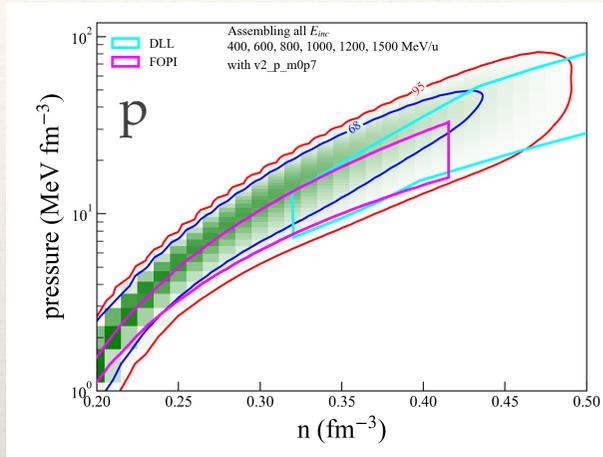
Triton, ${}^3\text{He}$



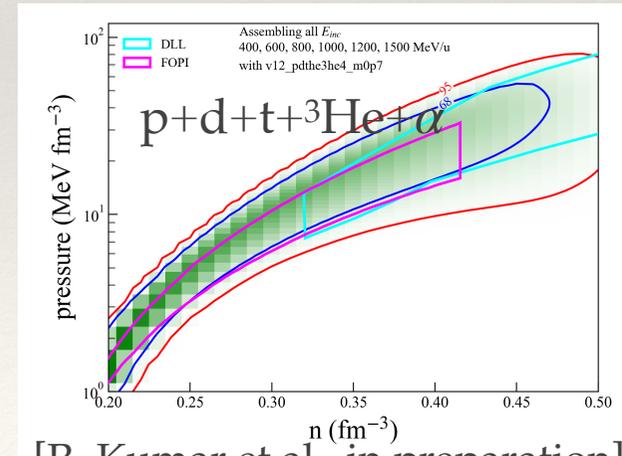
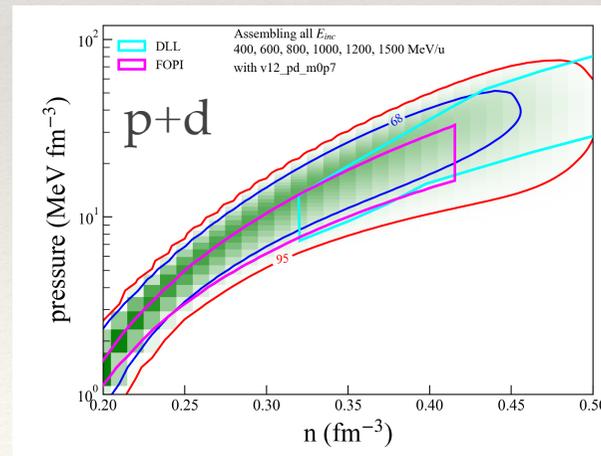
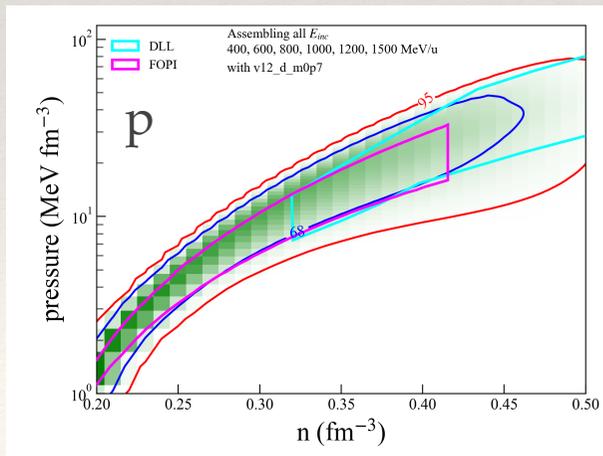
[R. Kumar et al., in preparation]

EoS prediction: P versus n

v_2 only



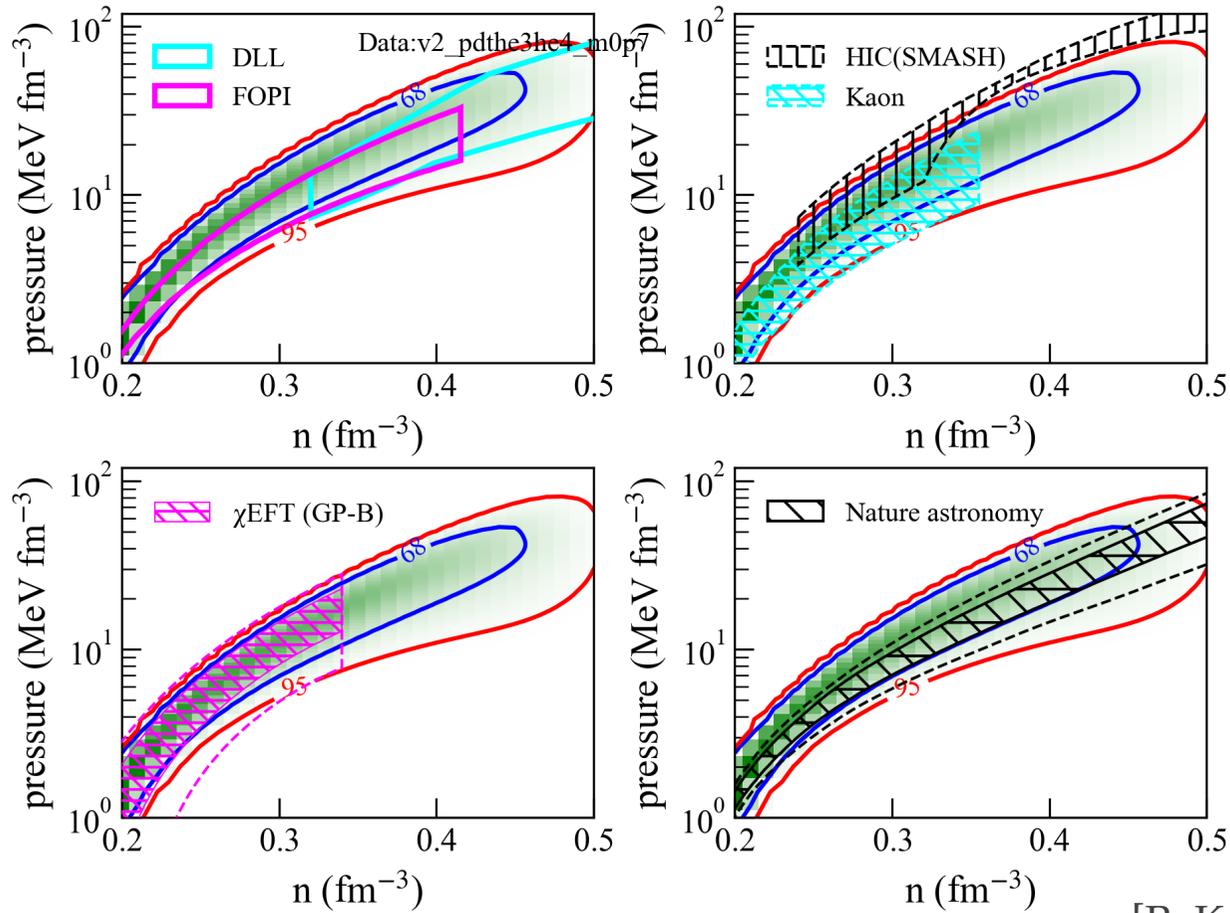
v_1 & v_2



[R. Kumar et al., in preparation]

Comparison to other results

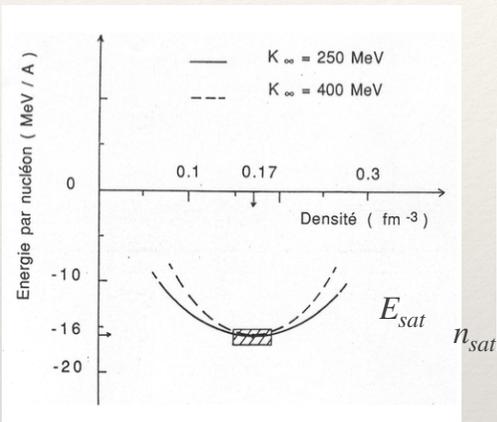
v_2 only
 $p+d+t+{}^3\text{He}+\alpha$



[R. Kumar et al., in preparation]

Analysis of Giant Monopole Resonances

Density dependence of the energy around n_{sat}



Measured in different nuclei

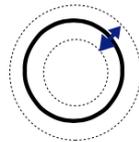
Compressible liquid-drop:

$$e(n) \approx e(n_{sat}) + \frac{1}{2} K_{sat} x^2 + \dots \text{ with } x = (n - n_{sat}) / (3n_{sat}).$$

Incompressibility

How is incompressibility measured?

α scattering on nuclei
→ monopolar compression



Extracted

vibration frequency: $\hbar\omega = \hbar \sqrt{\frac{K_A}{mr_0^2} A^{-1/3}}$

$$K_A = K_{sat} + K_s A^{-1/3} + K_{sym} \left(\frac{N-Z}{N+Z} \right)^2 + K_{coul} \frac{Z^2}{A^{4/3}}$$

Uniform matter

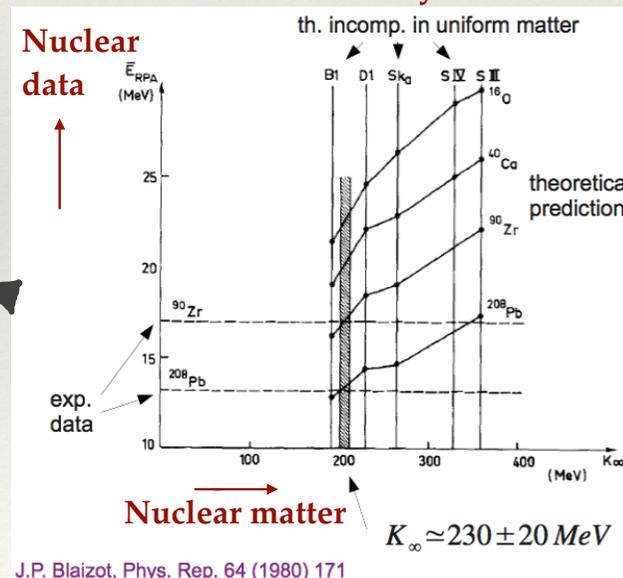
Finite size effects

Present estimation:

$$K_{sat} = 240 \pm 20 \text{ MeV}$$

[Garg & Colo, 2018]

Correlation analysis

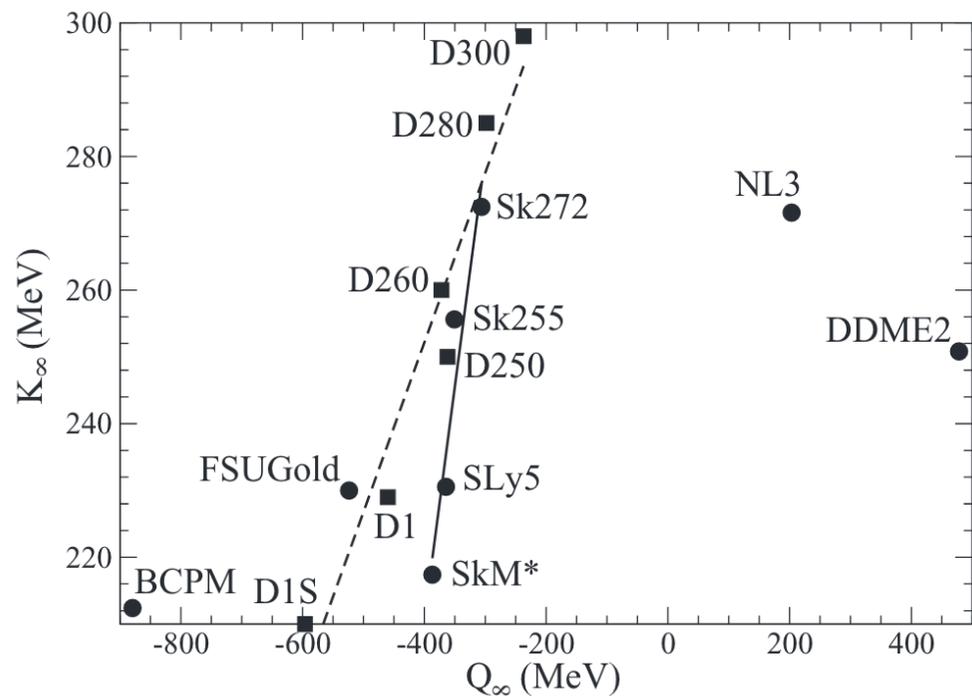


Model dependence?
Underestimation of the uncertainty?

[Khan, JM, Vidaña PRL 2012, PRC 2013]

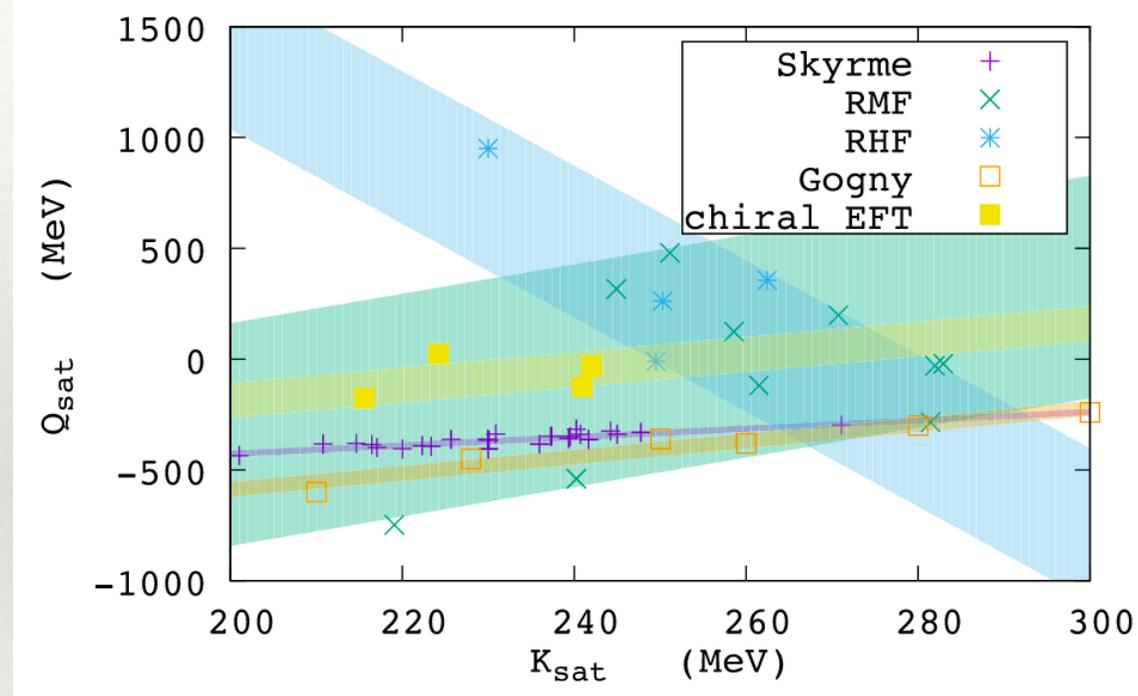
[Grams+ PRC 2022]

Exploration of the $K_{\text{sat}}-Q_{\text{sat}}$ diagram



[Khan & Margueron, PRC 88 (2013)]

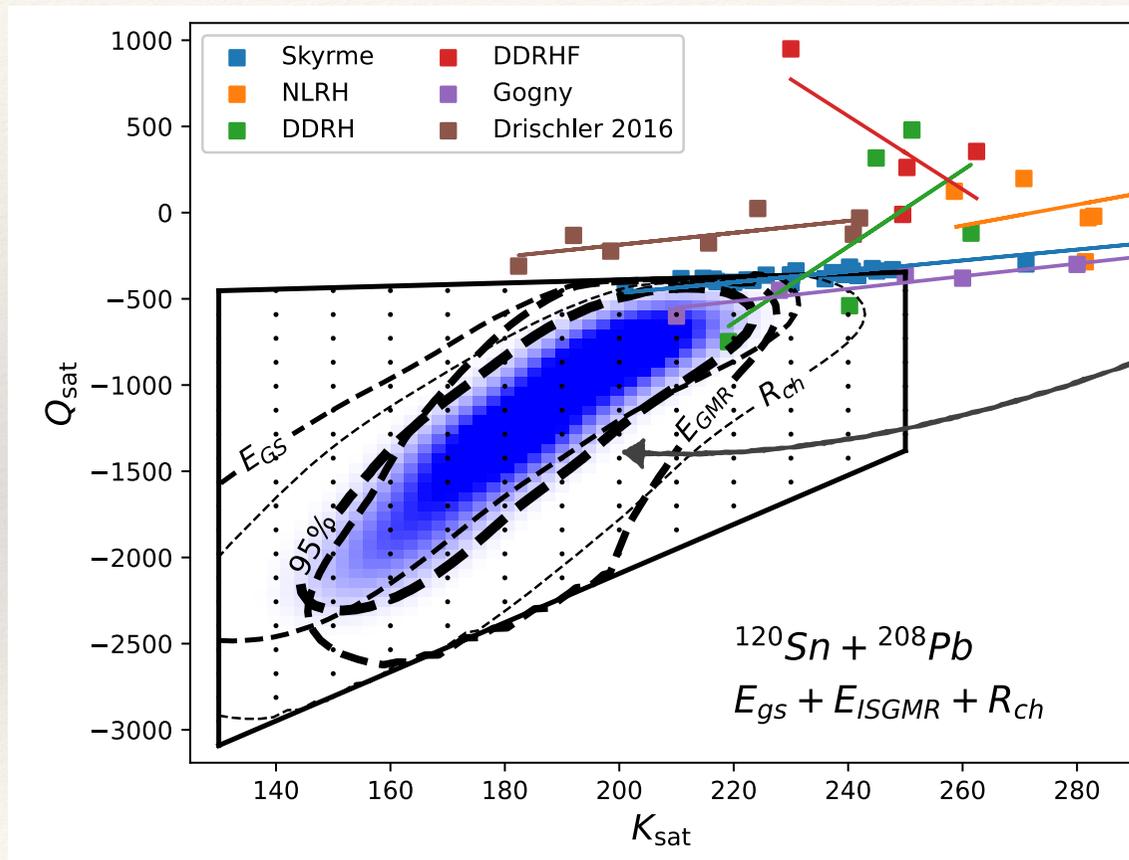
Artificial correlation between K_{sat} and Q_{sat} , due to the lack of flexibility of Skyrme EDF.



[Margueron, Casali, Gulminelli, PRC 97 (2018)]

Spurious correlations between K_{sat} and Q_{sat} exists also for other models.

Exploration of the $K_{\text{sat}}-Q_{\text{sat}}$ diagram compatible with the GMR



This region of the parameter space is also compatible with GMR + ground state energy + charge radius.

—> The value of K_{sat} can be lower than 200 MeV, provided Q_{sat} is reduced as well.

[Margueron & Khan, in preparation]

Conclusions

The nuclear incompressibility may be reproduced by EDF models with low values of K_{sat} , provided Q_{sat} is reduced.

HIC Flow Data (FOPI) may constrain the nuclear matter equation of state, especially K_{sat} and Q_{sat} .

Additional questions w.r.t. model dependencies should be explored more systematically.

The present analysis shows that the Bayesian approach could be applied to compare directly experimental data and model predictions to infer the EoS.



In the **future**, more data and more consistencies between different measurements shall be explored.