# The Equation of State of Nuclear Matter from Collective Flows in Intermediate Energy Heavy-Ion Collisions: An Update

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# Overview

**Motivation**

**Model Details**

dcQMD – interaction parametrization
Threshold effects
Medium modification of cross-section
Final state treatment

**Study of EoS, Effective Masses, σ* using Nucleonic Obs**

Framework
Impact of Different Observables
Model Dependence
Constraints

**Perspectives**

**Summary & Conclusions**

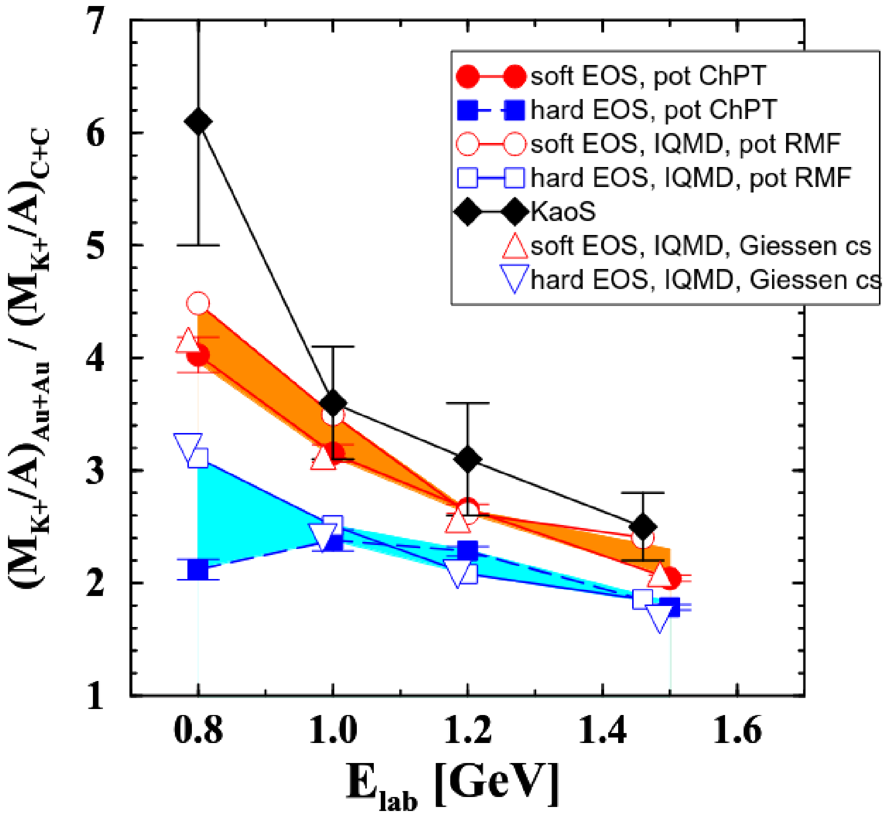
for details see D.C. arXiv:2407.16411

# Rare probes in HIC

intermediate energy HIC collisions provide the opportunity to study three different aspects of the in-medium NN interaction:
- in-medium cross-sections
- momentum dependence of optical potential
- density dependence of EoS of cold nuclear matter

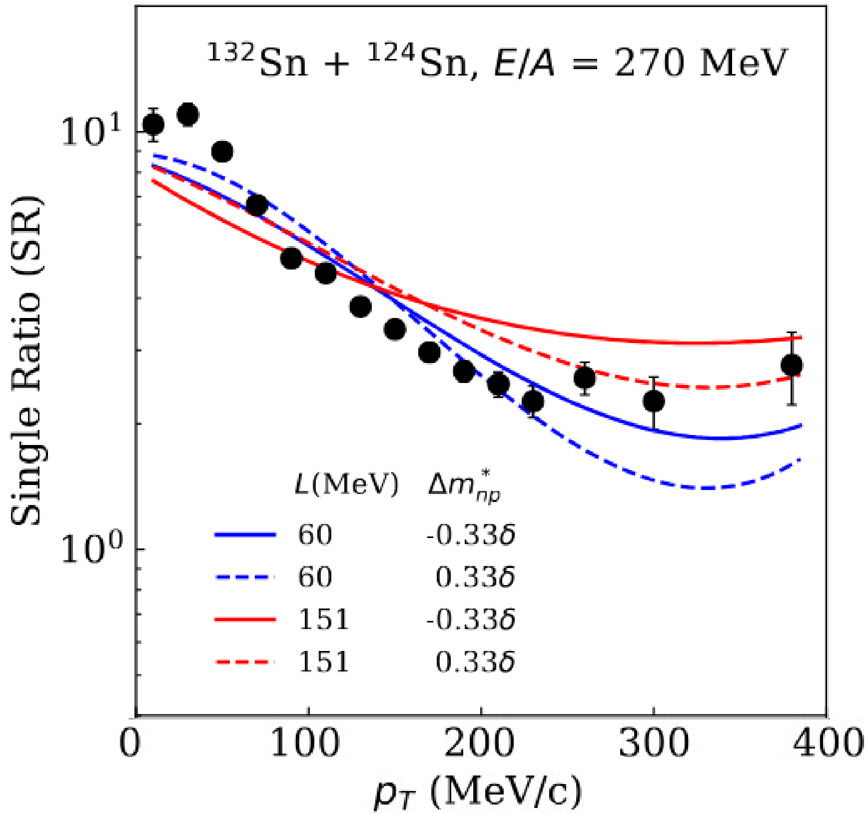
a divide-and-conquer approach has been often employed

$K^+$ ratio (KaOS Coll): EoS of SNM




C. Fuchs, PPNP 56, 1 (2006)

$\pi^-/\pi^+$ ratio (SπRIT Coll): symmetry energy



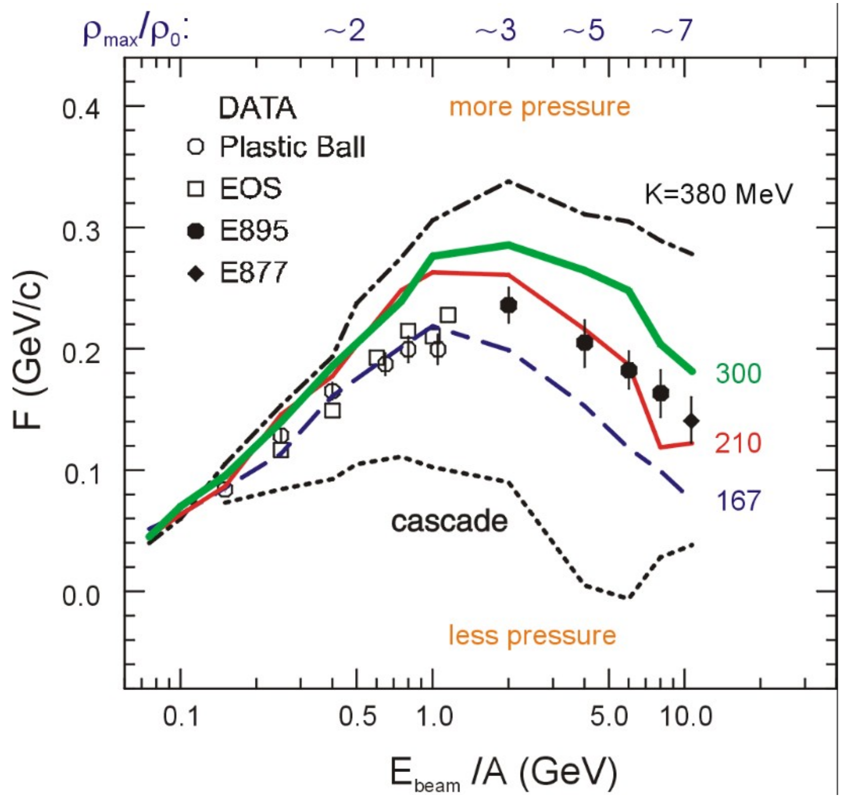

J. Estee et al. PRL 126, 162701 (2021)

# Nucleonic Observables

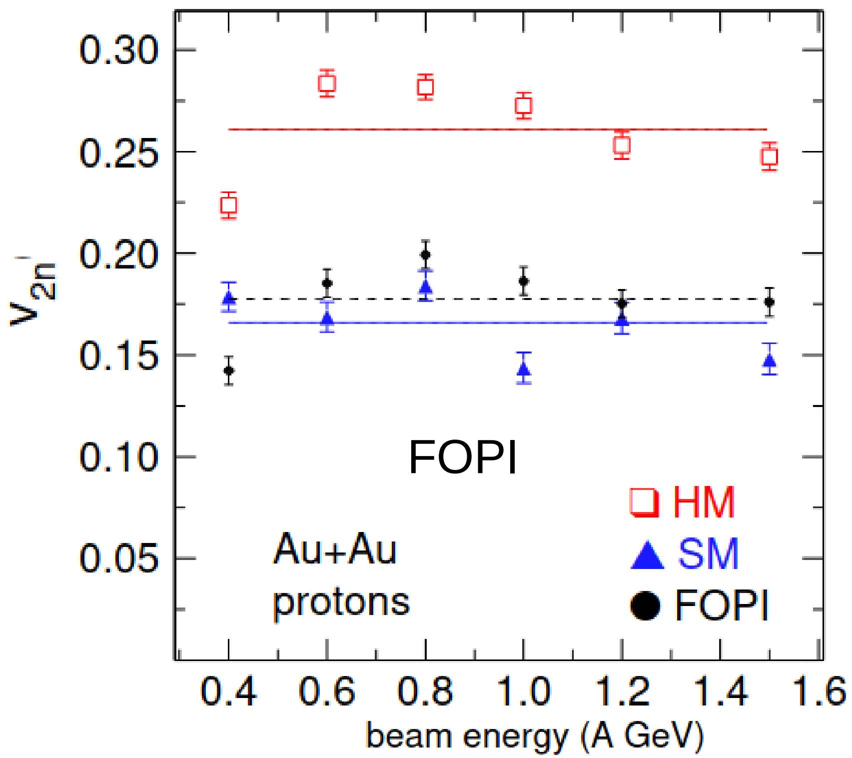
extensive database of experimental data for nucleonic observables for intermediate energy HIC (FOPI, HADES, SπRIT, KaOS and others)

have not been used to their full potential

approach similar to that of Danielewicz et al. (Science 298, 2002), potential residual biases avoided by simultaneously determining in-medium cross-sections, optical potential and EoS




Danielewicz et al., Science 298, 1592 (2002)




Le Fevre et al., NPA 945, 112 (2016)

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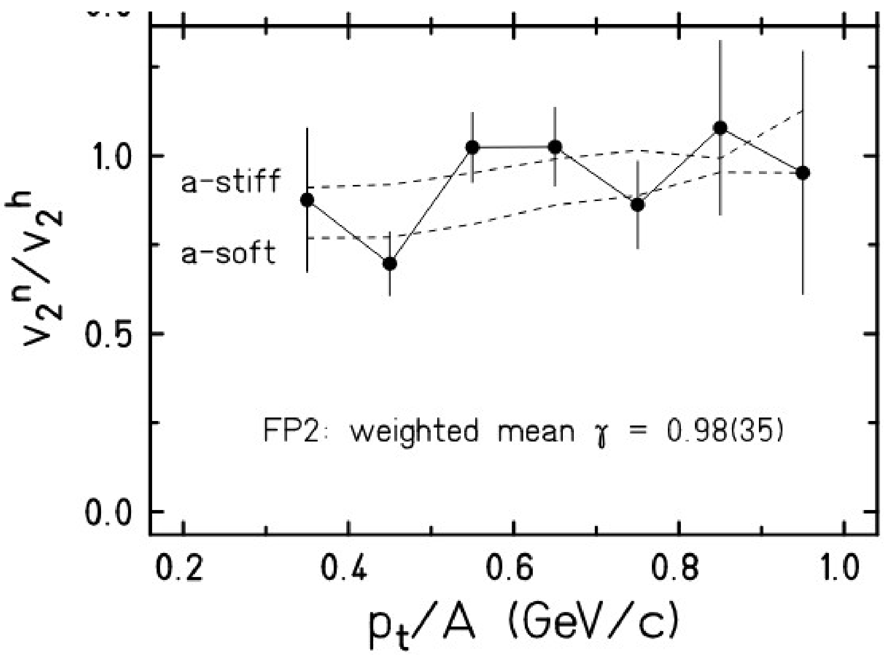
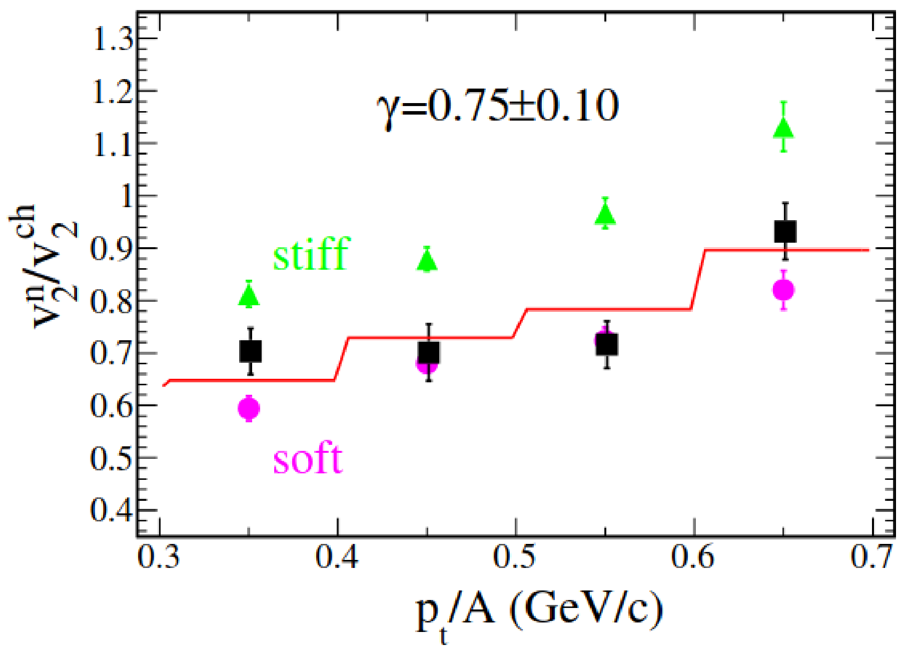
FOPI-LAND




P. Russotto et al., PLB 697, 471 (2011)

ASYEOS




P. Russotto et al., PRC 94, 034608 (2016)

SπRIT Collaboration: C.Y.Tsang et al. PLB 853, 138661 (2024) (see also talks of M.Kurata-Nishimura, B. Tsang)

# Model Details

dcQMD transport model: newest versions EPJA 57, 309 (2021)
arXiv:2407.16411

an upgraded version of TuQMD, see H. Wolter et al.

Prog.Part.Nucl.Phys. 125, 103962 (2022)

# Interaction (nucleonic component)

momentum dependent potential **MDI2** -generalization of MDI of Das, Das Gupta, Gale, Li PRC67, 034611 (2003)

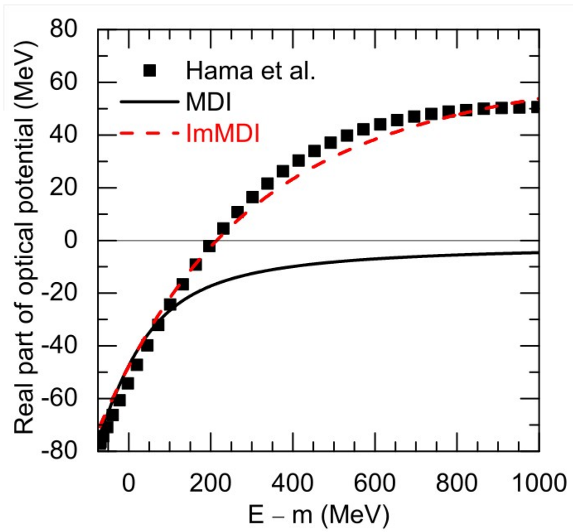
$$\frac{E}{N}(\rho,\beta,x,y)=\frac{1}{2}A_1 u+\frac{1}{2}A_2(x,y)u\beta^2+\frac{Bu^{\sigma}}{\sigma+1}(1-x\beta^2)+\frac{Du^2}{3}(1-y\beta^2)$$

$$+\frac{1}{u\rho_0^2}\sum_{\tau,\tau'}C_{\tau\tau'}\int\int d^3p\,d^3p'\,\frac{f_\tau(p,p')f_{\tau'}(p,p')}{1+(\vec{p}-\vec{p}\,')^2/\Lambda^2}$$

$$A_2(x,y)=A_2^0+\frac{2xB}{\sigma+1}\bar{u}^{\sigma-1}+\frac{2yD}{3}\bar{u} \qquad u=\frac{\rho}{\rho_0}$$

Fit:
$U_\infty$,$K_0$,$J_0$,$m^*$ -isoscalar
$S(\tilde{u})$,$L$,$K_{sym}$,$\delta m_{isv}$ -isovector

momentum dependent part: similar with that of J. Xu et al. PRC 91, 014611 (2015)
(see also C. Hartnack, J. Aichelin PRC 49, 2801 (1994) )
used previously to test model dependence: flow ratio PRC 88, 44912 (2013)
pion multiplicity ratio PLB 753, 166 (2016)
independent part: extra term (vary L vs. $K_{sym}$ and also $J_0$ vs. K independently)




from J. Xu et al. PRC 91, 014611 (2015)

| Input | | Parameters | |
|---|---|---|---|
| $\rho_0$ [fm$^{-3}$] | 0.16 | $\Lambda$ [MeV] | 708.001 |
| $E_B$ [MeV] | -16.0 | $C_l$ [MeV] | -13.183 |
| $m_s^*/m$ | 0.70 | $C_u$ [MeV] | -140.405 |
| $\delta^*_{n-p}$ ($\rho_0$, $\beta$ =0.5) | 0.165 | $B$ [MeV] | 137.305 |
| $K_0$ [MeV] | 245.0 | $\sigma$ | 1.2516 |
| $J_0$ [MeV] | -350.0 | $\tilde{A}_l$ [MeV] | -130.495 |
| $\tilde{\rho}$ [fm$^{-3}$] | 0.10 | $\tilde{A}_u$ [MeV] | -8.828 |
| S($\tilde{\rho}$) [MeV] | 25.4 | $D$ [MeV] | 7.357 |

# Threshold Effects (dcQMD)

- direct consequence of imposing (total) energy conservation in the medium

$$\sqrt{p_1^2+m_1^2}+U(p_1)+\sqrt{p_2^2+m_2^2}+U(p_2)=\sqrt{p'^2_1+m'^2_1}+U(p'_1)+\sqrt{p'^2_2+m'^2_2}+U(p'_2)$$

- only few transport models below 1 AGeV:

RBUU: G. Ferini et al. PRL 97, 202301 (2006), RVUU: T. Song, C.M. Ko PRC 91, 014901 (2015);
χBUU: Z. Zhang et al, PRC 98, 054614 (2018), AMD+JAM: N. Ikeno et al., PRC 108, 044601 (2023)

- required for thermodynamical consistency of the model

Z.Zhang et al, PRC 97, 014610 (2018)

- reactions: NN↔NN, NN↔NR, R↔Nπ (R↔Nππ not corrected)

- assumptions (dcQMD): - two-body collisions are part of N-body one
  - in-medium two-body collisions modeled as a succession of bare (vacuum-like) collisions followed/preceded by energy exchanges with the fireball, while momentum is conserved
  - reaction with highest probability: corresponds to the one which included the bare collision of highest probability

Example: elastic NN

$$\boxed{\frac{d\sigma^{(med)}}{d\Omega} = (2\pi)^4 \frac{m_1^* m_2^*}{k_i^* \sqrt{s_i^*}} |M_{fi}^{(med)}(\rho,\delta,\{\tau\})|^2 \frac{k_f^* m_{1'}^* m_{2'}^*}{\sqrt{s_f^*}}}$$

$$|M_{fi}^{(med)}(\rho,\delta,\{\tau\})|^2 = \frac{1}{2}(|M_{fi}^{(vac)}(\tilde{s}_i)|^2 + |M_{fi}^{(vac)}(\tilde{s}_f)|^2) \longleftarrow \sqrt{\tilde{s}_{i,f}} - 2\,m_N = \sqrt{s_{i,f}^*} - \sqrt{s_{th}^*} + U_{i,f} - U_{th}$$

Introduced in TuQMD/dcQMD in DC, PLB 753, 166 (2016)

# Collision Term

**Elastic baryon-baryon collisions**

-modified Cugnon parametrization to accurately describe elastic cross-sections at low impact energy (<100 MeV) but also total cross-sections above pion production threshold J. Cugnon et al., NIMB 111, 215 (1996)

**In-medium modification factor**

- collision criterion based on effective masses determined using EoM (consistency with the $dt \rightarrow 0$ fm/c limit)
- in-medium modification of elastic cross-sections

$$\sigma^{med} = f(\rho, \delta,)\, \sigma^{vac}_{mod}$$

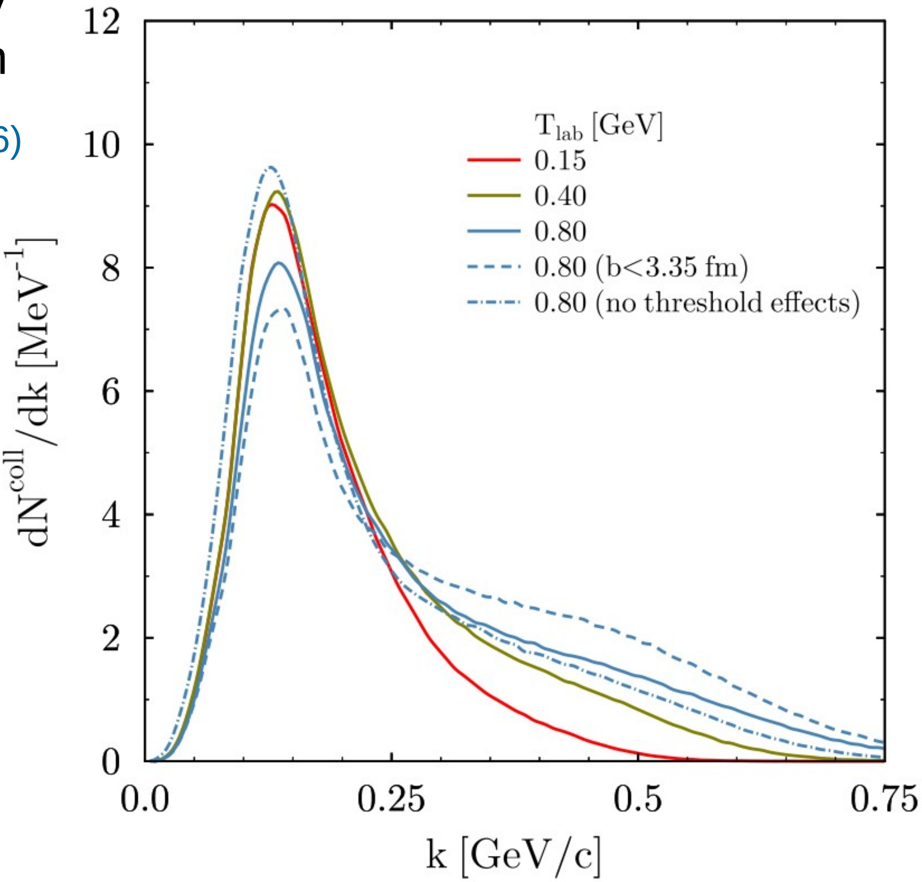
$$f(\rho, \delta) = \exp[\alpha\rho/\rho_0 + \beta_1 \delta\rho/\rho_0 + \beta_2(\tau_1 + \tau_2)\delta\rho/\rho_0]$$

$\sigma^{vac}_{mod}$ – flux and phase-space factors computed using effective masses

B.A. Li et al. PRC72, 064611 (2005)

$f(\rho,\delta)$ – accounts for medium modifications of transition matrix due to departure from the quasi-particle picture

C. Fuchs et al. PRC 64, 024003 (2001)




**Resonance production:** OBE model

S.Huber et al., NPA 573, 587 (1994)

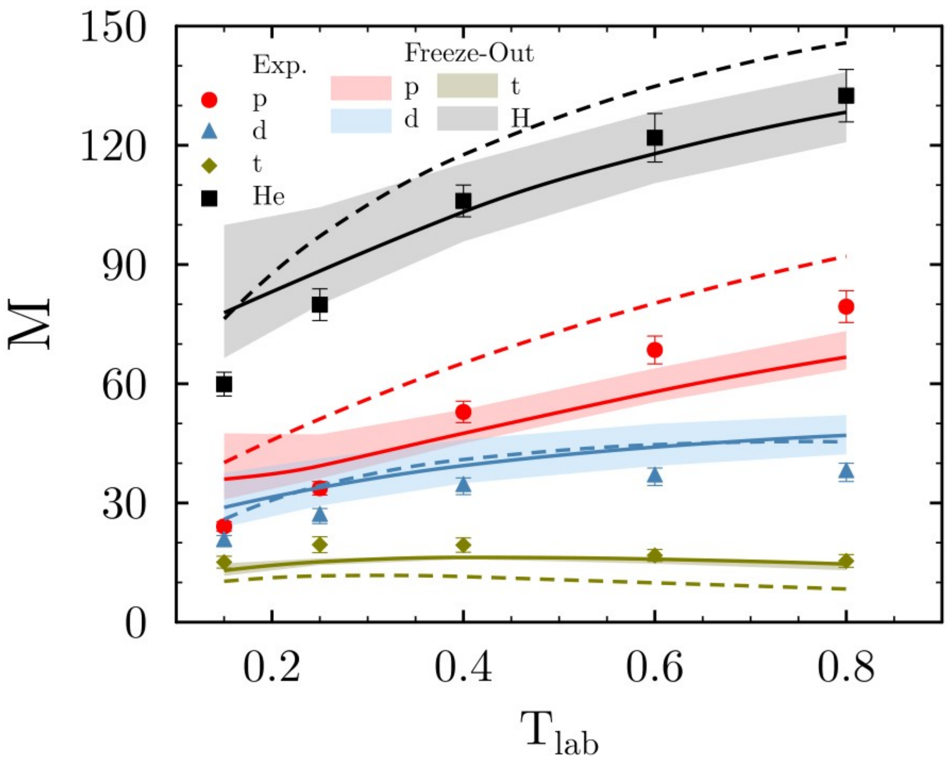
**Isospin dependent Pauli blocking** (including a modified surface term)

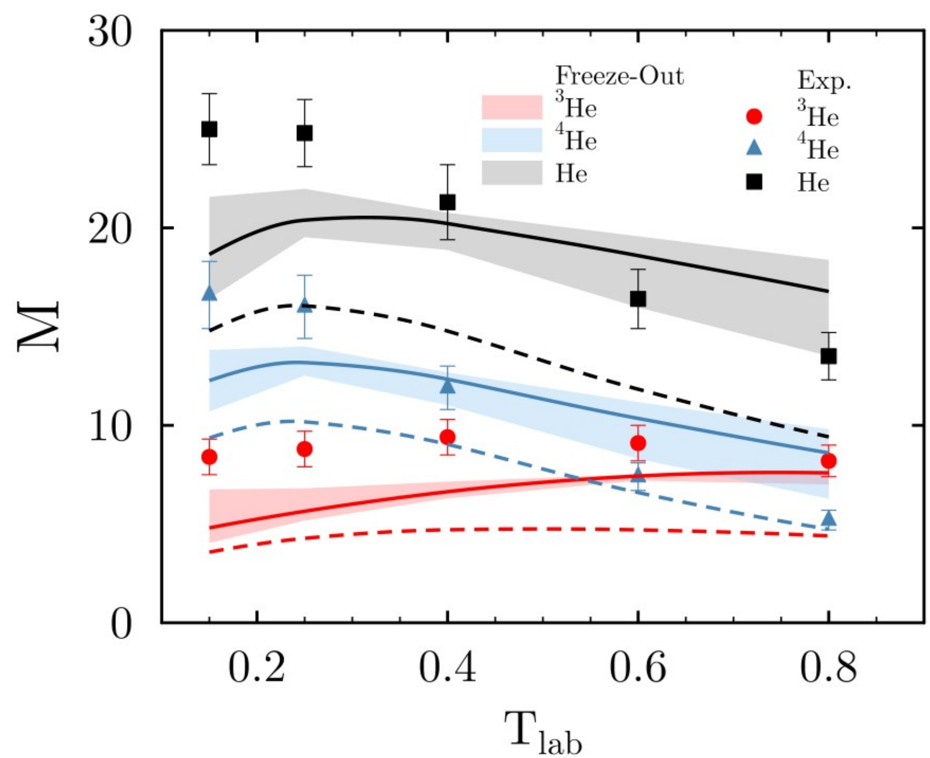
# Cluster multiplicities

- **final state:** MST algorithm all stable clusters (τ > 1ms ) with A≤15, 23 additional A>15 (B,C,N,O)
- coalescence algorithm applied at local freeze-out time

- transport model parameters determined from a fit of $v_1$,$v_2$ and varxz experimental data
- δr=3.0-4.0 fm, δp=0.2-0.3 GeV/c

Hydrogen central ($b_0$<0.15)AuAu HICs (FOPI) Helium




dashed curves: results with the coalescence model applied at final time (150 fm/c)

# Nucleonic Observables

1) FOPI data set

- theoretical flow data away from mid-rapidity plagued by contamination from heavier clusters (most visible for proton $v_1(p_T)$ but also $v_1(y)$)
- exp data up 0.8 GeV/nucleon (impact of nucleon resonances on dynamics is small)

| Label | Observables | Reference |
|---|---|---|
| FOPI1 | $v_1(y)$, $\|y\| \leq 0.5$ for Z=1,2 clusters; M5 centrality<br>$^{197}$Au+$^{197}$Au, $^{129}$Xe+CsI, $^{58}$Ni+$^{58}$Ni (0.25, 0.40) | A. Andronic et al.<br>PRC 67, 034907(2003) |
| FOPI2 | $v_1(y)$, $\|y\| \leq 0.5$ for p, d, A=3 and $\alpha$ clusters; $0.25 \leq b_0 \leq 0.45$<br>$^{197}$Au+$^{197}$Au (0.25, 0.40, 0.60, 0.80)<br>$v_2(y)$, $\|y\| \leq 0.5$ for p, d and $\alpha$ clusters; $0.25 \leq b_0 \leq 0.45$<br>$^{197}$Au+$^{197}$Au (0.15, 0.25, 0.40, 0.60, 0.80)<br>$v_2(p_T)$, $\|y\| \leq 0.4$ for p, d and t clusters; $0.25 \leq b_0 \leq 0.45$<br>$^{197}$Au+$^{197}$Au (0.15, 0.25, 0.40, 0.60, 0.80) | W. Reisdorf et al.<br>NPA 876, 1 (2012) |
| varxz | varxz for p,d,t clusters; $b_0 \leq 0.15$<br>$^{40}$Ca+$^{40}$Ca (0.4, 1.0), $^{58}$Ni+$^{58}$Ni (0.15, 0.25), $^{96}$Ru+$^{96}$Ru (0.4, 1.0)<br>$^{96}$Zr+$^{96}$Zr (0.4), $^{129}$Xe+CsI (0.15, 0.25)<br>$^{197}$Au+$^{197}$Au (0.15, 0.25, 0.4, 0.6, 0.8) | W. Reisdorf et al.<br>NPA 848, 366 (2010) |
| spectra | longitudinal and transverse rapidity spectra for protons in $Z \leq 3$ clusters<br>$^{40}$Ca+$^{40}$Ca (0.4), $b_0 \leq 0.15$, $\|y_{L,T}/y_P\| \leq 1.25$ | W. Reisdorf et al.<br>PRL 92, 232301 (2004) |

results using above date set only: D.C. arXiv: 2407.16411

# Nucleonic Observables

2) FOPI-LAND, ASYEOS – dedicated symmetry energy measurement

3) SPIRIT – dedicated effective masses and symmetry energy campaign

| Label | Observables | Reference |
|---|---|---|
| FOPI-LAND | $v_2(p_T)$, 0.25$\leq$ y $\leq$ 0.75 for n,Z=1; b $\leq$ 7.5 fm<br>$^{197}$Au+$^{197}$Au (0.40) | P. Russotto et al.<br>PLB 697, 471 (2011) |
| ASYEOS | $v_2(p_T)$, 0.3$\leq$ y $\leq$ 0.7 for n,Z>0; b $\leq$ 7.5 fm<br>$^{197}$Au+$^{197}$Au (0.40) | P. Russotto et al.<br>PRC 94, 034608 (2016) |
| SPIRIT | varxz for p,d,t clusters; $b \leq 1.6$ fm<br>$^{112}$Sn+$^{124}$Sn (0.27)<br>$v_1(p_T), v_1(y), v_2(y)$ for p in Z=1,2 clusters; $3.6 \leq b \leq 6.3$ fm<br>$^{132}$Sn+$^{124}$Sn (0.27)<br>$v_1(p_T), v_1(y), v_2(y)$ for p in Z=1,2 clusters; $4.0 \leq b \leq 6.0$ fm<br>$^{108}$Sn+$^{112}$Sn (0.27) | C.Y. Tsang et al.<br>PLB 853, 138661 (2024) |

# Theoretical Simulations

## Model parameters:

| | | |
|---|---|---|
| $m^*$ | [0.6, 0.9] | isoscalar effective mass |
| $V_\infty$ | [25, 125] MeV | isoscalar potential $p \to \infty$ |
| $K_0$ | [ 165, 355 ] MeV | compressibility modulus |
| $\alpha$ | [-0.4, 0.8] | in-medium $\sigma_{NN}$, $\delta=0.0$ |
| $\Delta m^*_{np}$ | [-0.25,0.25] at ($\rho=\rho_0$, $\delta=0.5$) | n-p effective mass diff. |
| L | [15, 145 ] MeV | slope symmetry energy |
| $\beta_1$ | [-0.5,3.5] | in-medium $\sigma_{NN}$, $\delta<>0.0$ |
| $\beta_2$ | [-1.5, 2.5] | $\sigma_{nn}<>\sigma_{pp}$, $\delta<>0.0$ |

**Enforced correlations:**

$J_0=-600+(K_0-165)*3.125$

$K_{sym}=-488+L*6.728$

in units of [MeV]

Input:

$E/N(\rho_0)=-16.0$ MeV

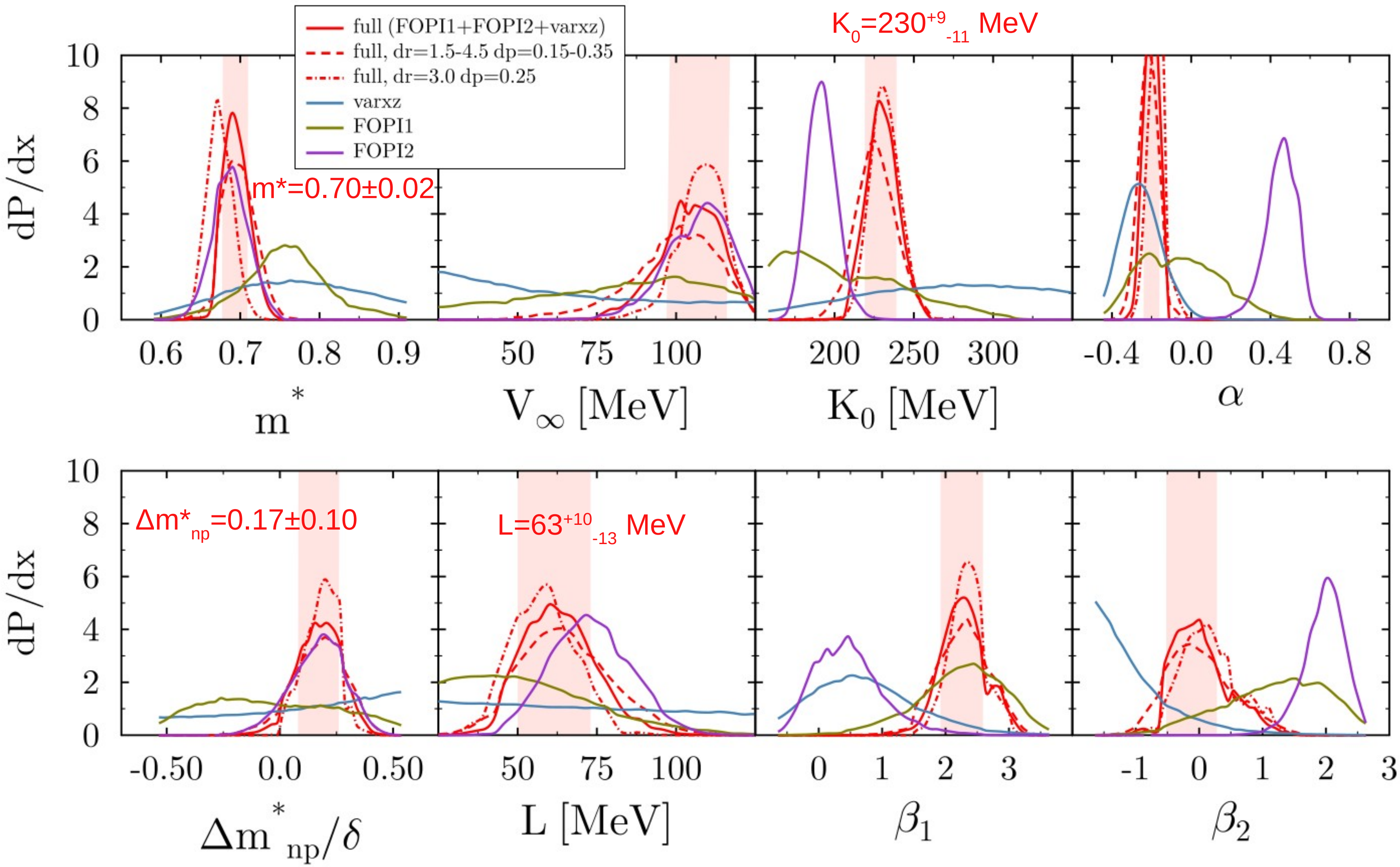
$S(0.62\rho_0)=25.5$ MeV

properties of doubly-magic nuclei (binding energies, rms charge radii, single particle energies)
Brown, PRL 111, 232502 (2013)

Model uncertainty: statistical+ systematical ($\delta r=3.0$-$4.0$ fm, $\delta p=0.2$-$0.3$ GeV/c ) added in quadrature

Model emulator: sum of monomials of degree ≤ 2; checked robustness LOO-CV method

# Constraints



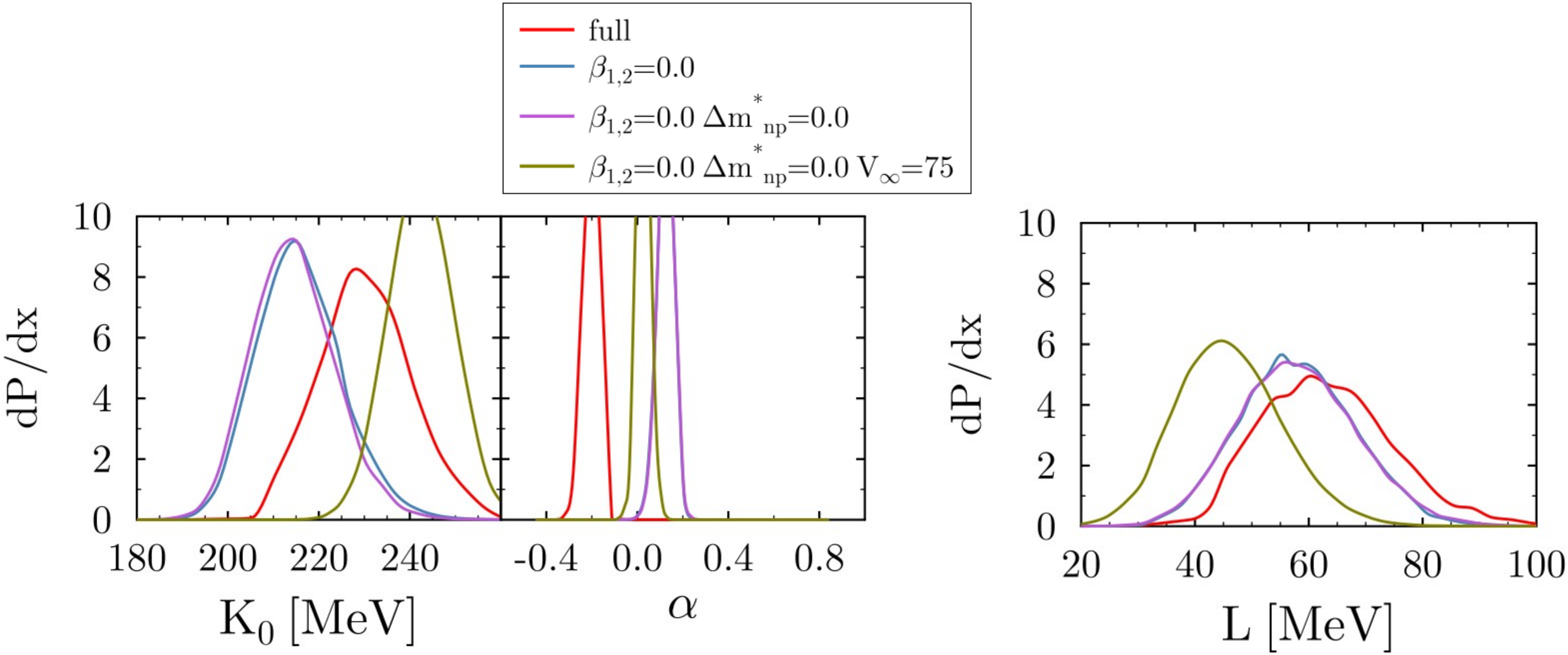

- exp data for isospin symmetric systems crucial for breaking degeneracy of isospin dep. and isospin indep. in-medium effects on cross-sections

# Isospin asymmetry dependent $\sigma_{NN}$

$$\sigma^{med} = f(\rho,\delta,)\,\sigma^{vac}_{mod}$$

$$f(\rho,\delta) = \exp[\alpha\rho/\rho_0 + \beta_1\,\delta\rho/\rho_0 + \beta_2(\tau_1+\tau_2)\,\delta\rho/\rho_0]$$

in-medium modification of the transition amplitude




- setting $\Delta m^*_{np}$=-0.5δ leads to an impact on isoscalar m* at 1σ level

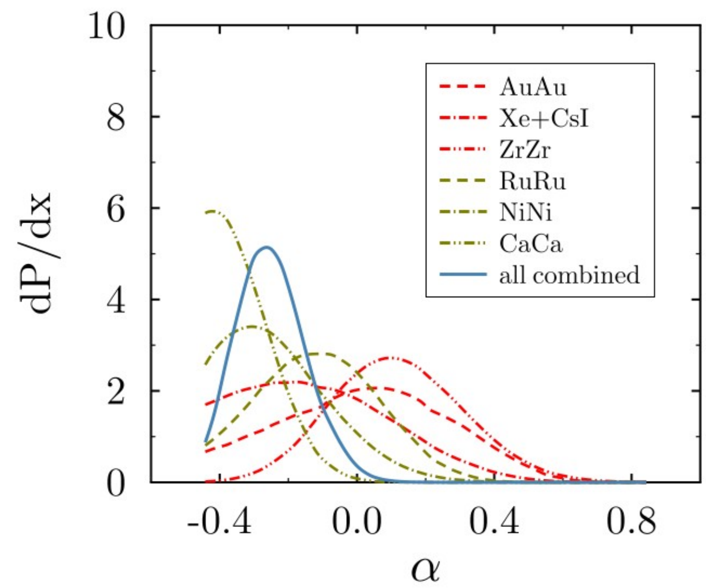
# Relevance of Isospin Symmetric Systems

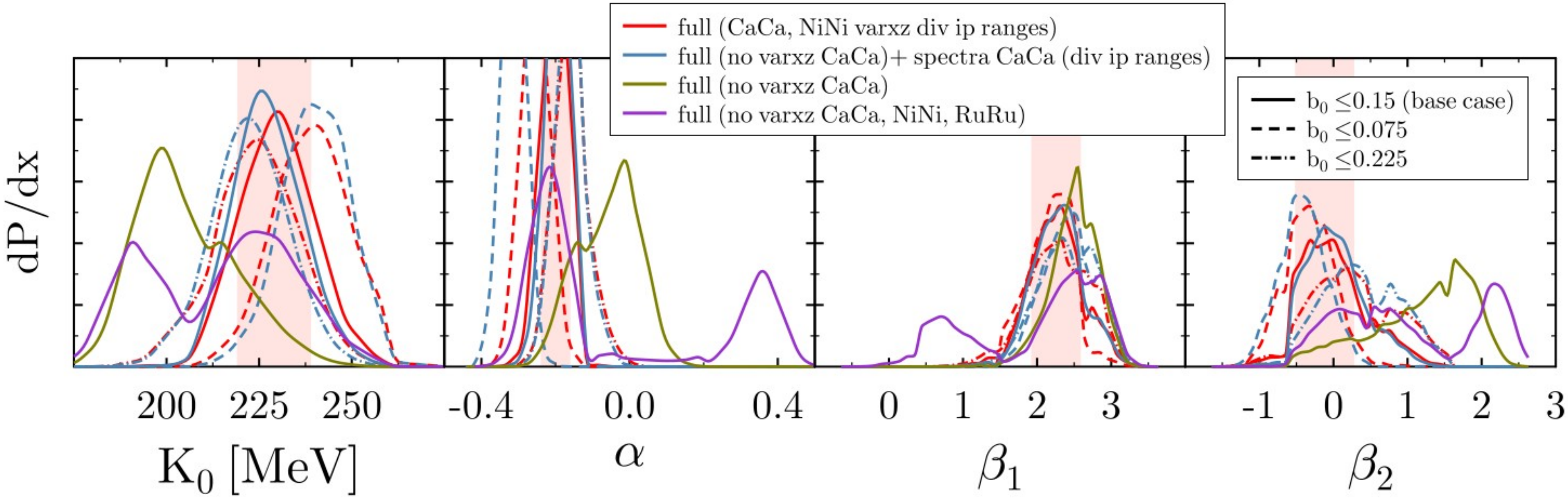
- isospin symmetric systems favor stronger in-medium reduction of cross-sections

- Pauli blocking algorithm less efficient for light system (~92% for AuAu vs. ~85% for CaCa)

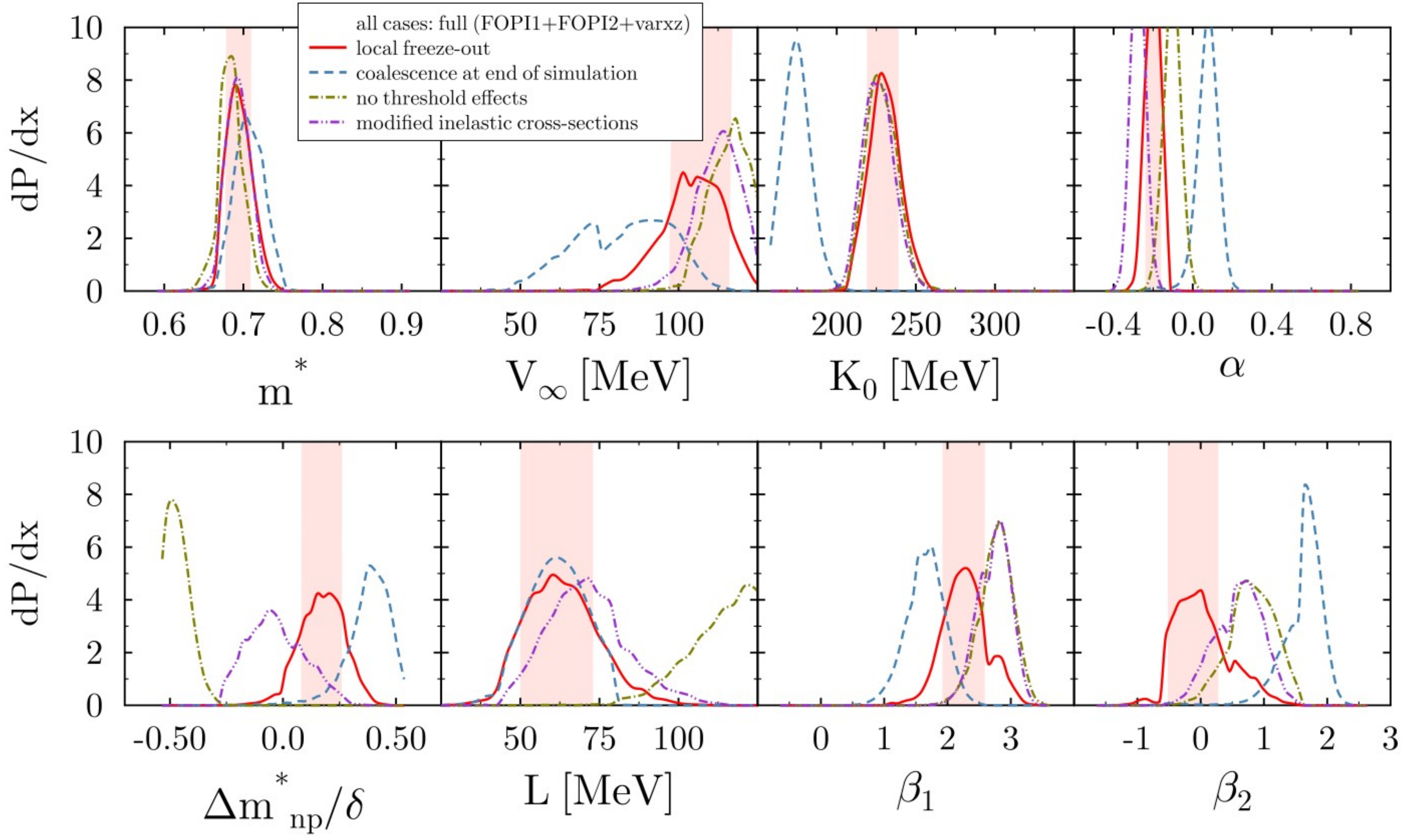
- observed extra reduction ~25%

- Z=1,2 $v_1$ for NiNi crucial in breaking degeneracy between α and $\beta_1$

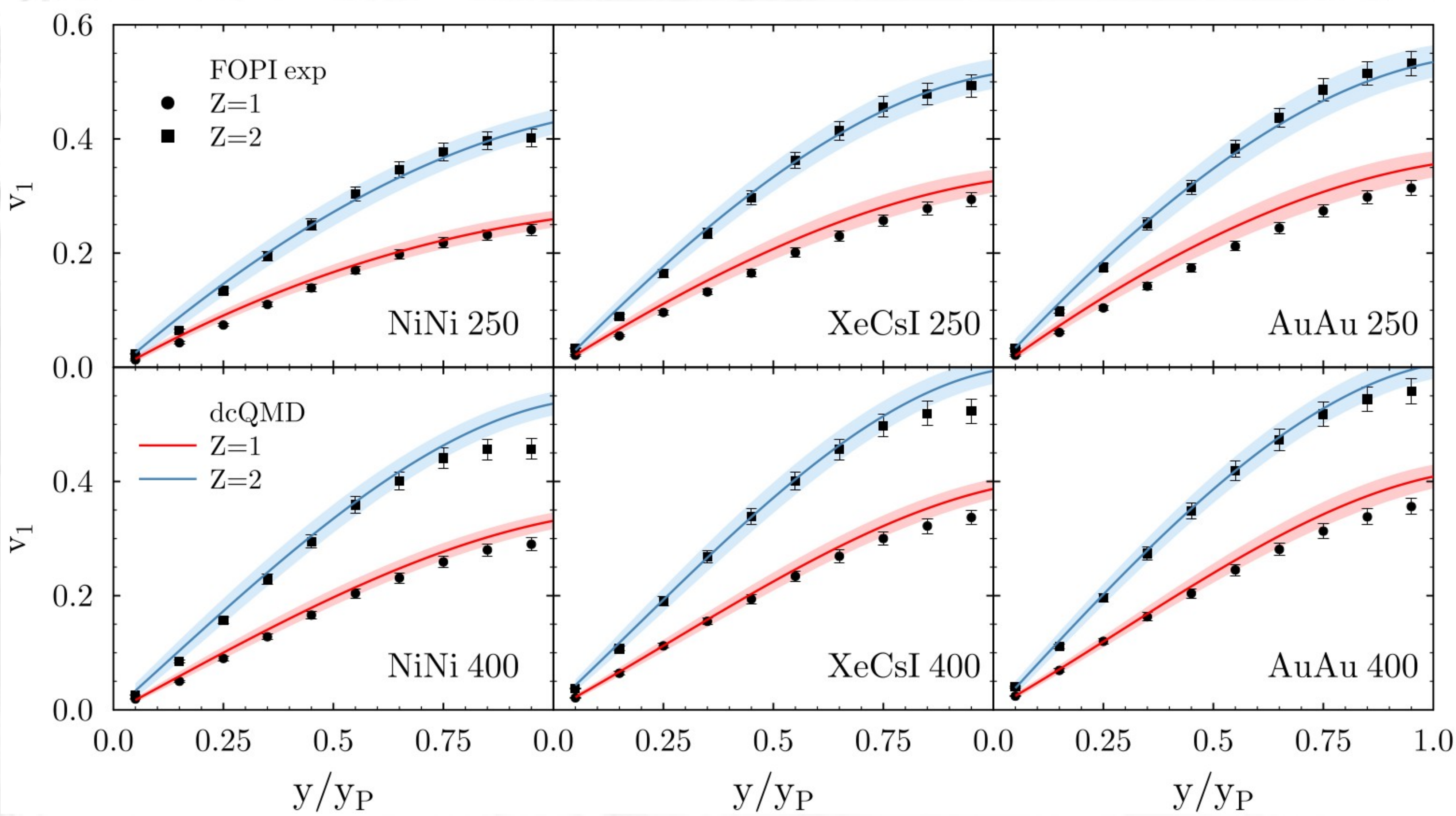


# Threshold Effect and Inelastic Channels



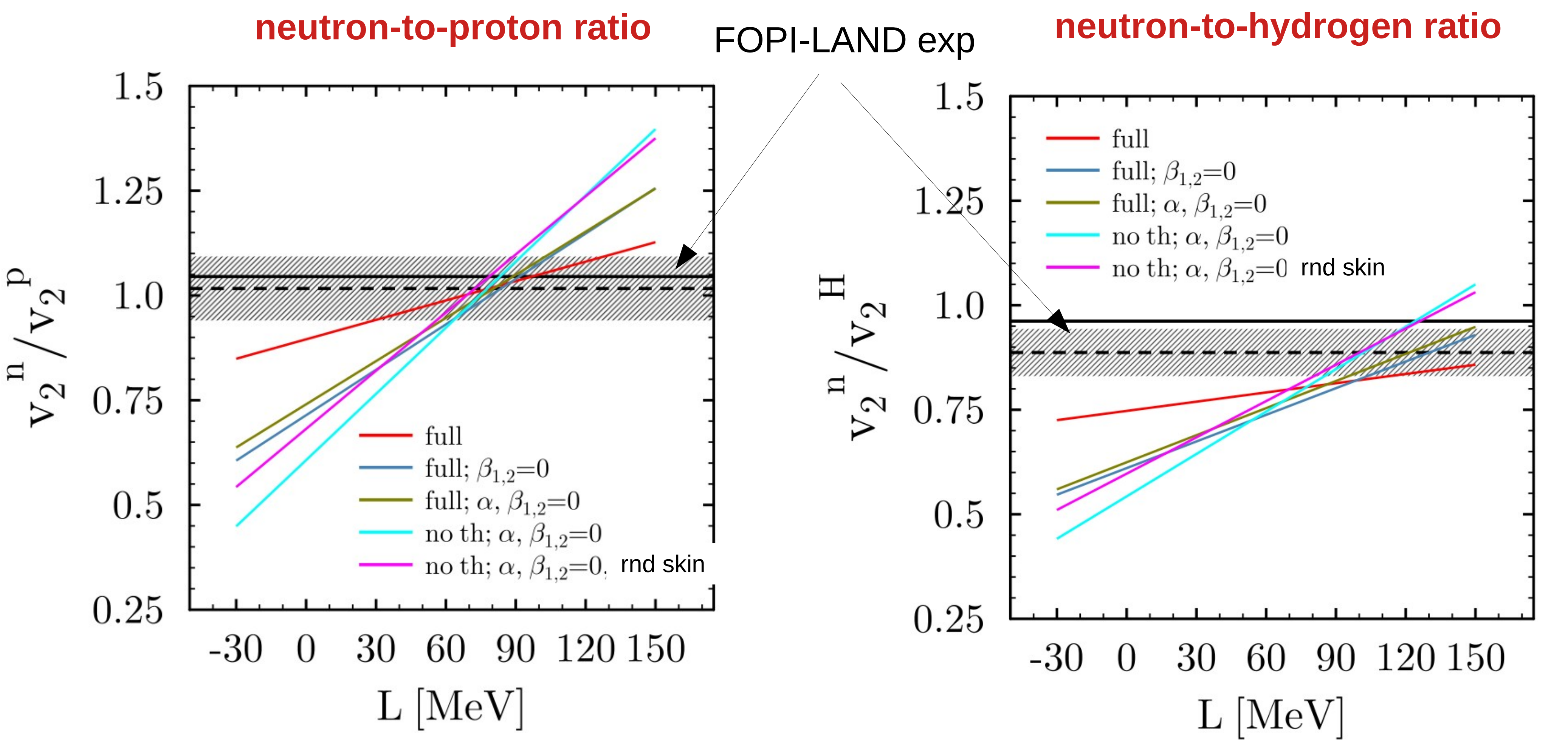
# FOPI1: $v_1(y)$




- M5 centrality
- only data with $y/y_P<0.5$ included in the fit

Full comparison to experimental data set: see D.C. arXiv:2407.16411

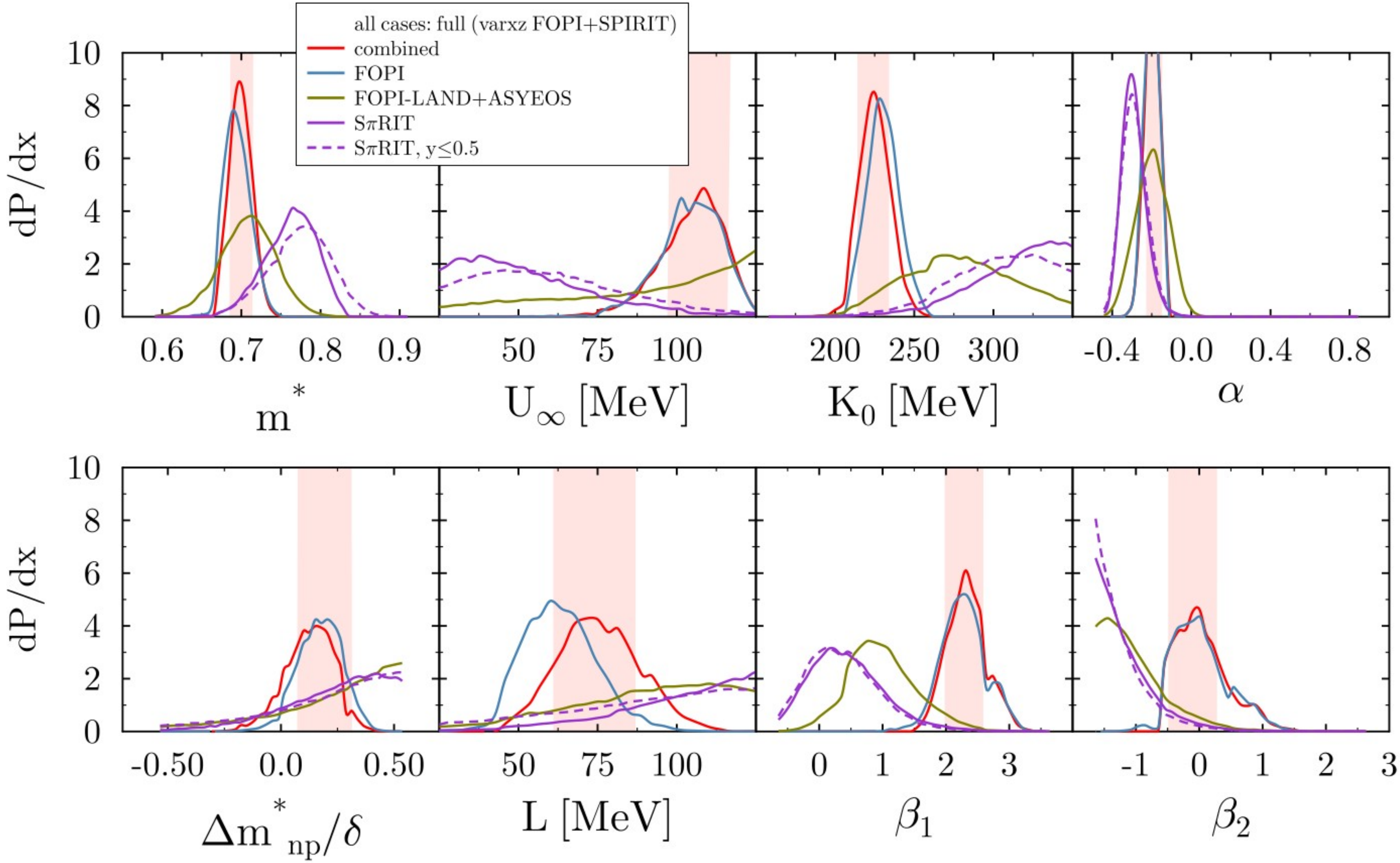
# FOPI-LAND Experiment Revisited




- identical results for neutron-to-charged particles ratio (ASYEOS experiment)

- future ASYEOS2 measurement (2025) aims at determining n/p with greater accuracy

# Combined Constraint

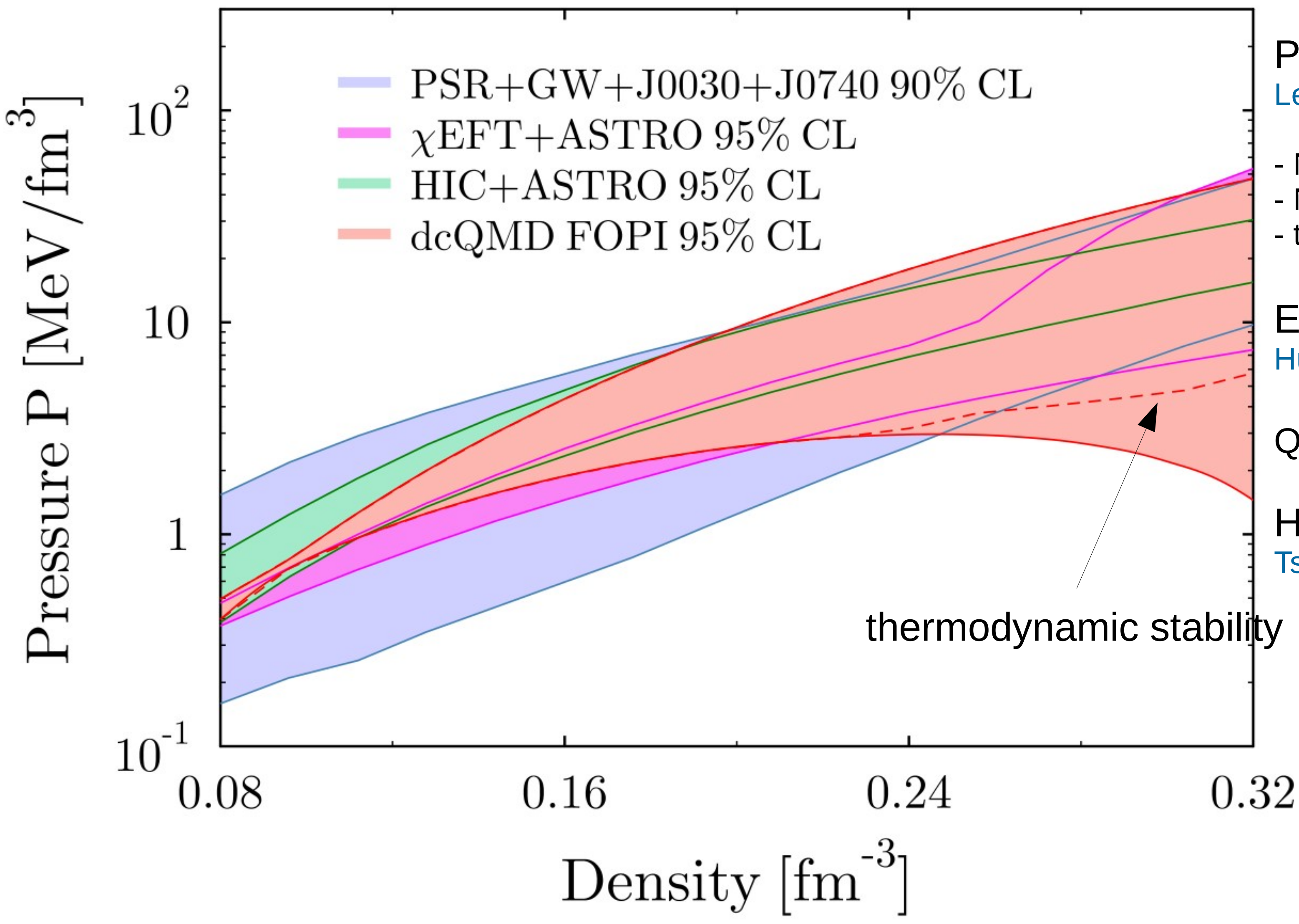



| | FOPI | combined |
|---|---|---|
| $K_0$ [MeV] | 230 ± 10 | 225 ± 10 |
| L [MeV] | 63 ±10 | 74 ± 13 |

# Pressure

Neutron star matter (δ=0.93)

Input: $S(0.62\rho_0)$=25.5 MeV




Pure Astro:
Legred et al PRD 104, 063003

- NS radii
- NS maximum mass
- tidal deformatibility (GW)

EFT:
Huth et al Nature 606, 276

QMC -2 and 3 body forces ($N^2LO$)

HIC:
Tsang et al Nat. Astron. 8, 328

SNM: - elliptic flow
- GMR

ANM: - dipole polarizability,
- neutron skin
- nuclear masses
- iso-diff, n/p ratio, flow ratio, pion ratio

Probed densities: see D.C. arXiv:2407.16411

# Perspectives

**Experimental side:** ASYEOS (2025) $v_2^n/v_2^p$: AuAu 0.25, 0.40, 0.80 GeV/nucleon

HADES (2025?) nucleonic flows, pions: AuAu 0.40, 0.60, 0.80 GeV/nucl

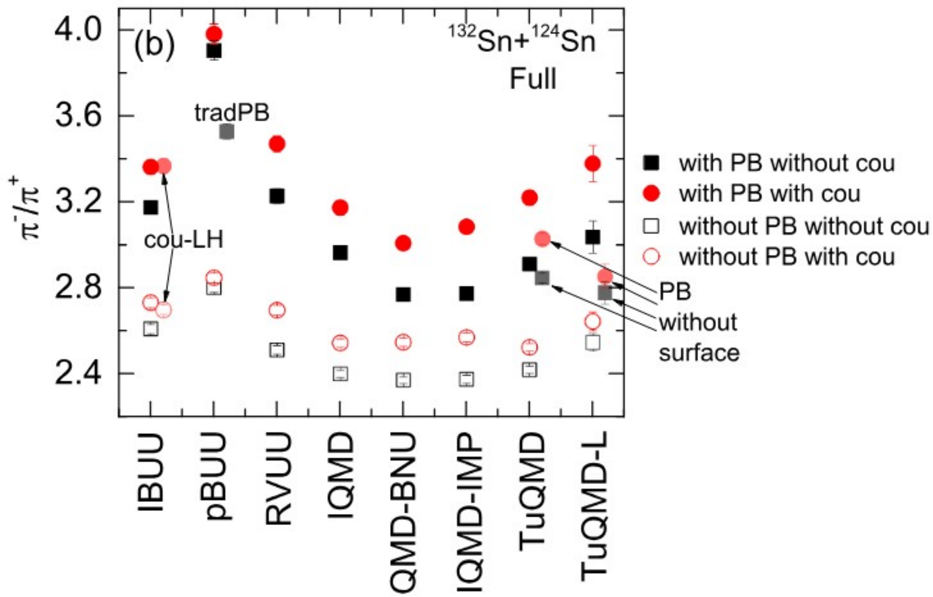
SπRIT (2024): nucleonic, pionic observables Xe+Sn 0.34 GeV/nucl

**Theoretical side:** - make use of lower $p_T$ part of pion spectra (SπRIT): non-resonant pion production channels

- access higher densities: FOPI and HADES data above 1 GeV/nucleon: treat two-pion production channels consistently

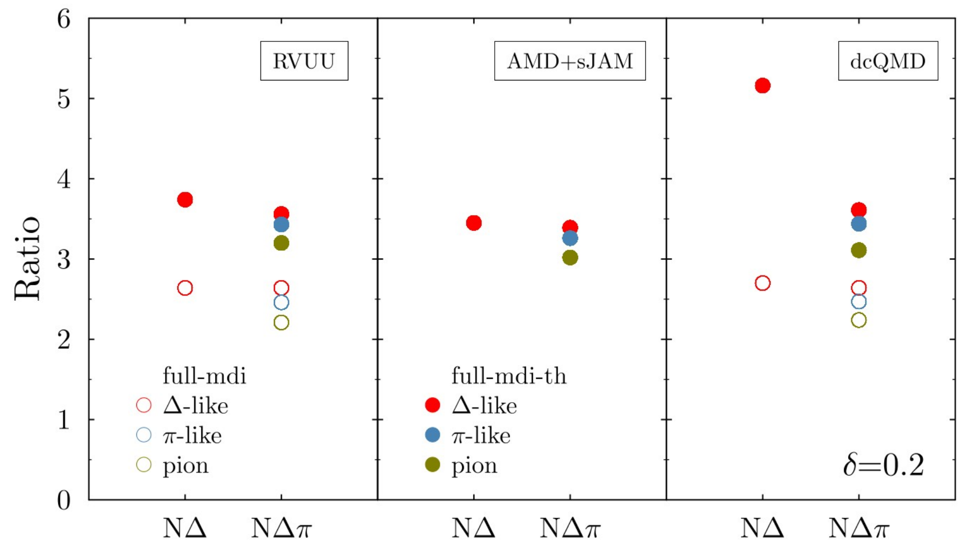
- assesment of systematic uncertainty of transport model predictions (TMEP)

**HIC with MII**



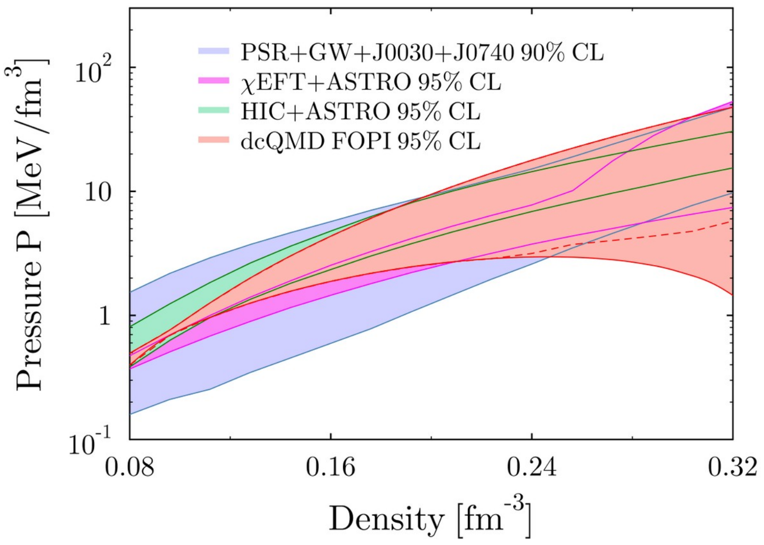

J. Xu et al. PRC 109, 044609 (2024)

**Collision integral with MDI in a box**




D.C. et al. (TMEP), in preparation

# Summary & Conclusions




**Study of EoS, effective masses and σ* using nucleonic observables** in AuAu collisions of intermediate impact energy (0.15-0.80 GeV/nucleon)

- in-medium effects on elastic collisions that depend on density, isospin asymmetry and isospin projection
- clusterization algorithm applied at local freeze-out time
- systematic uncertainty due to coalescence parameters;
- constraints extracted from FOPI experimental data for $v_1(y)$, $v_2(y)$, $v_2(p_T)$ and varxz
- model dependence: threshold effects and isospin asymmetry dependence of σ* have significant impact

**Comparison to a study of SE using pionic observables**
- full agreement (in contradiction with the situation ~10 years ago)

**68% CL Result**

$m^*=0.70 \pm 0.02$   $V_\infty=104 \pm 8$ MeV

$\Delta m^*_{np}=(0.17 \pm 0.10)\delta$

$K_0=230 \pm 10$ MeV

$L=63 \pm 12$ MeV

**Perspectives:** - remove imposed correlation between $L_0 \leftrightarrow J_0$ and $L \leftrightarrow K_{sym}$; allow for a variation of $E/N(\rho_0)$ and $S(0.1\ fm^{-3})$.
- include explicit cluster degrees of freedom to be able to use experimental data sets to their full potential
- improve model to use more accurate part of pion spectra and access higher densities with HIC above 1.0 GeV/nucleon
- estimate model dependence (TMEP)

# Probed Density (Free Protons)

- observables are functionals of the EoS
- sensitivity: functional derivatives w.r.t to EoS or d EoS/ dρ, etc.

$$\frac{d\,Obs}{d\,EoS}=\frac{\lim_{\epsilon\to 0} Obs\left[\frac{dEoS}{d\rho}(\rho)+\epsilon\,\delta(\rho-\tilde{\rho})\right]-Obs\left[\frac{dEoS}{d\rho}(\rho)-\epsilon\,\delta(\rho-\tilde{\rho})\right]}{2\epsilon}$$

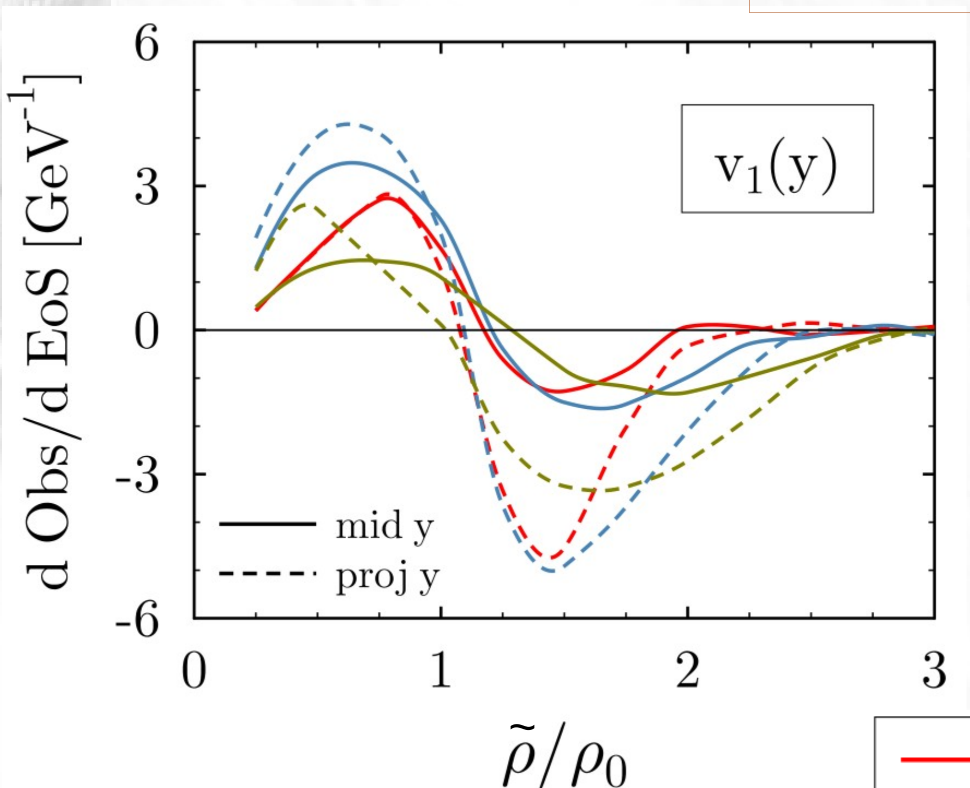
$$\Delta\langle H_{L_0}\rangle = \sum_{i=n,p} \frac{\varepsilon}{2}\left[Erf\left(\frac{\tilde{u}}{\eta}\right) - Erf\left(\frac{\tilde{u}-u_i}{\eta}\right)\right] \quad (1$$

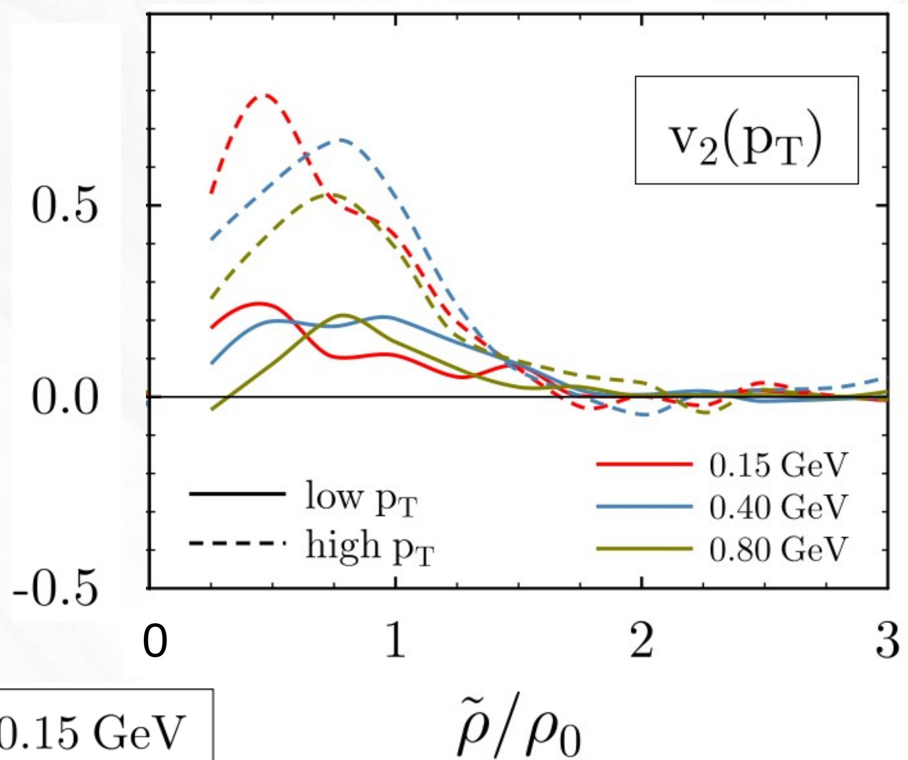
$$\Delta\langle H_L\rangle = \sum_{i=n,p} \frac{\varepsilon}{2}\,\tilde{\tau}_i\,\delta_i\left[Erf\left(\frac{\tilde{u}}{\eta}\right) - Erf\left(\frac{\tilde{u}-u_i}{\eta}\right)\right]$$

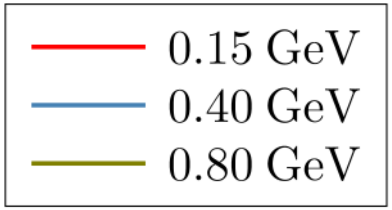
**Isoscalar EoS**

AuAu 3.35<b<6.0 fm

**Symmetry Energy**



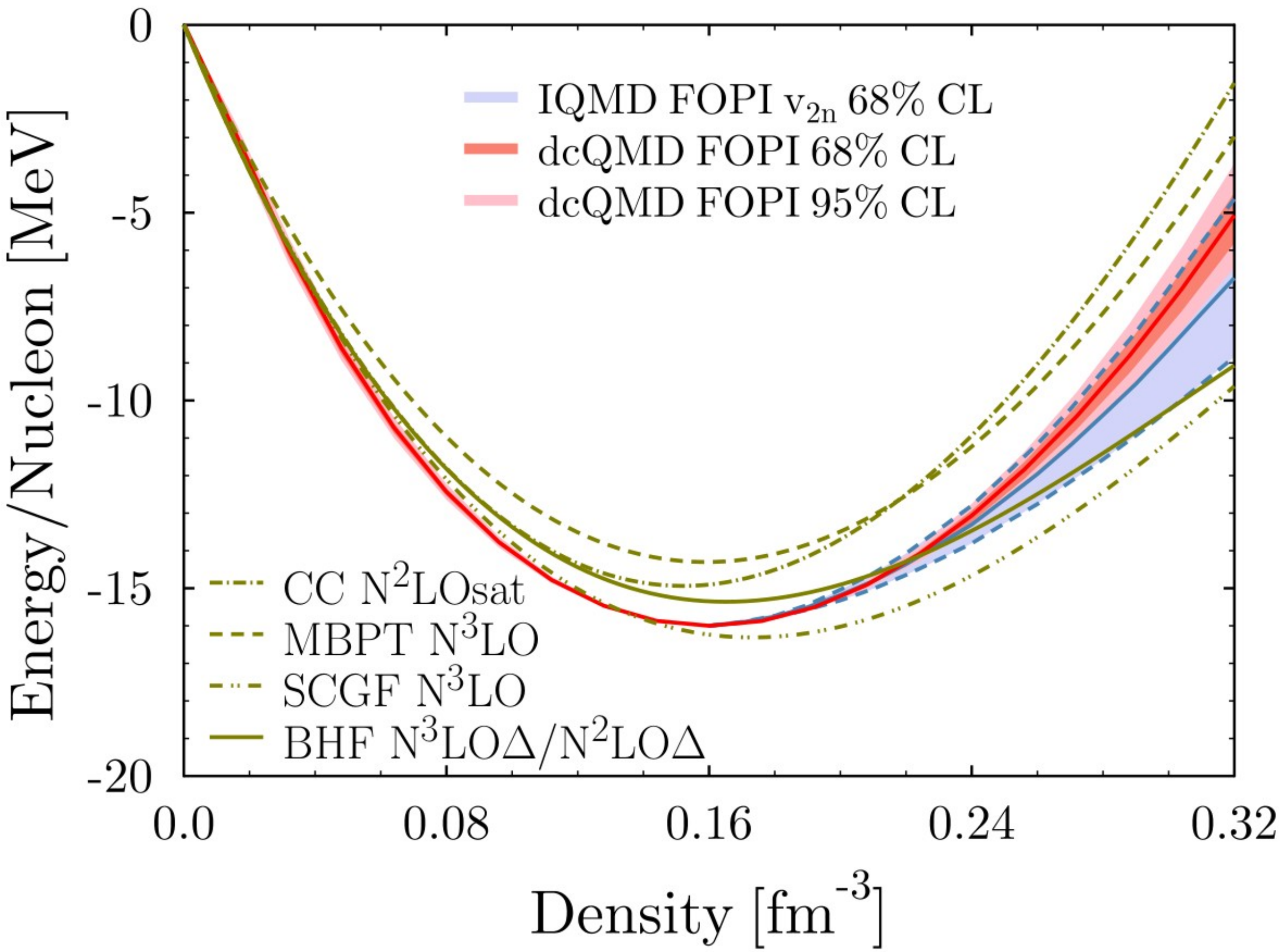




mid y: $0.0<y/y_P<0.5$
proj y: $0.5<y/y_P<1.0$

low $p_T$: $0.4<p_T/p_P<1.2$
high $p_T$: $1.2 <p_T/p_P<2.0$

# EoS of Symmetric Matter




$K_0$=230 ± 10 MeV 68%CL

IQMD result:

A.Le Fevre et al., NPA 945, 112 (2016)

$K_0$=190 ± 30 MeV 68%CL

Microscopic calculations:

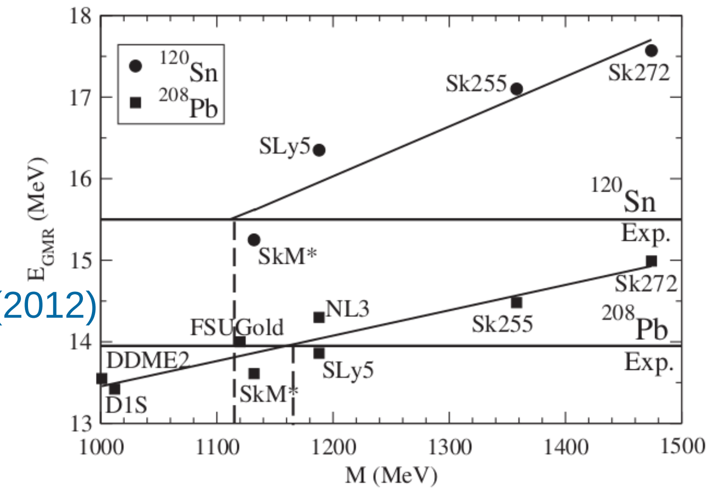
1. A. Ekstrom et al., PRC 91, 051301 (2015)
2. C. Drischler et al., PRC 102, 054315 (2020)
3. A. Carbone, PRR 2, 023227 (2020)
4. D. Logoteta, PRC 94, 064001 (2016)c

centroid of GMR correlated with derivative of incompressibility

$M = 3\rho K'(\rho)|_{\rho=\rho_c}$ $\rho_c$=0.11 $fm^{-3}$ E.Khan et. al PRL 109, 092501 (2012)

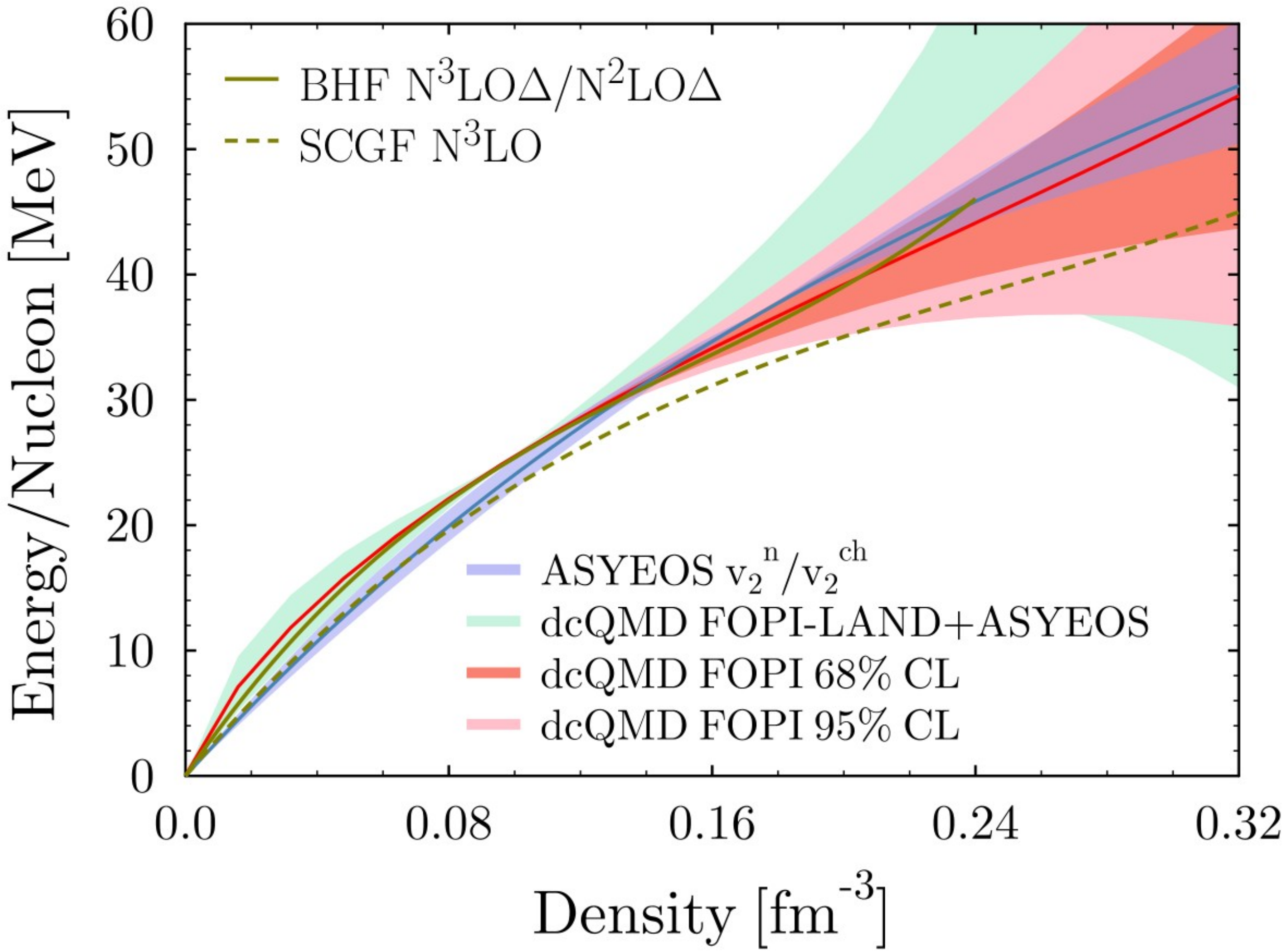
$^{112-124}Sn$, $^{208}Pb$, $^{90}Zr$, $^{144}Sm$ → M=1100±70 MeV

this work: M=1110±40 MeV



# Symmetry Energy

Input: $S(0.62\rho_0)$=25.5 MeV




L=63 ± 10 MeV 68%CL
$S(\rho_0)$=34.3 ± 1.1 MeV
$S(2\rho_0)$=54.2 ± 12.4 MeV

ASYEOS result:

L=72 ± 13 MeV 68%CL $S_0$=34 MeV
L=63 ± 11 MeV 68%CL $S_0$=31 MeV

P. Russotto et al., PRC 94, 034608 (2016)

Microscopic calculations:

SCGF: A. Carbone, PRR 2, 023227 (2020)

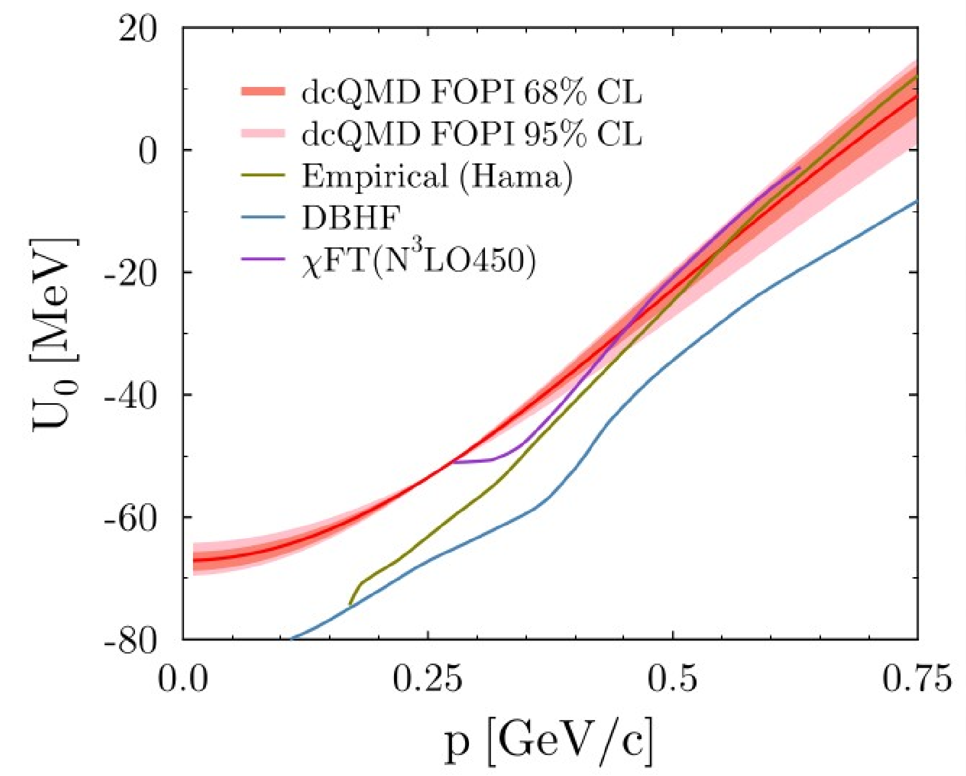
BHF: D. Logoteta, PRC 94, 064001 (2016)c

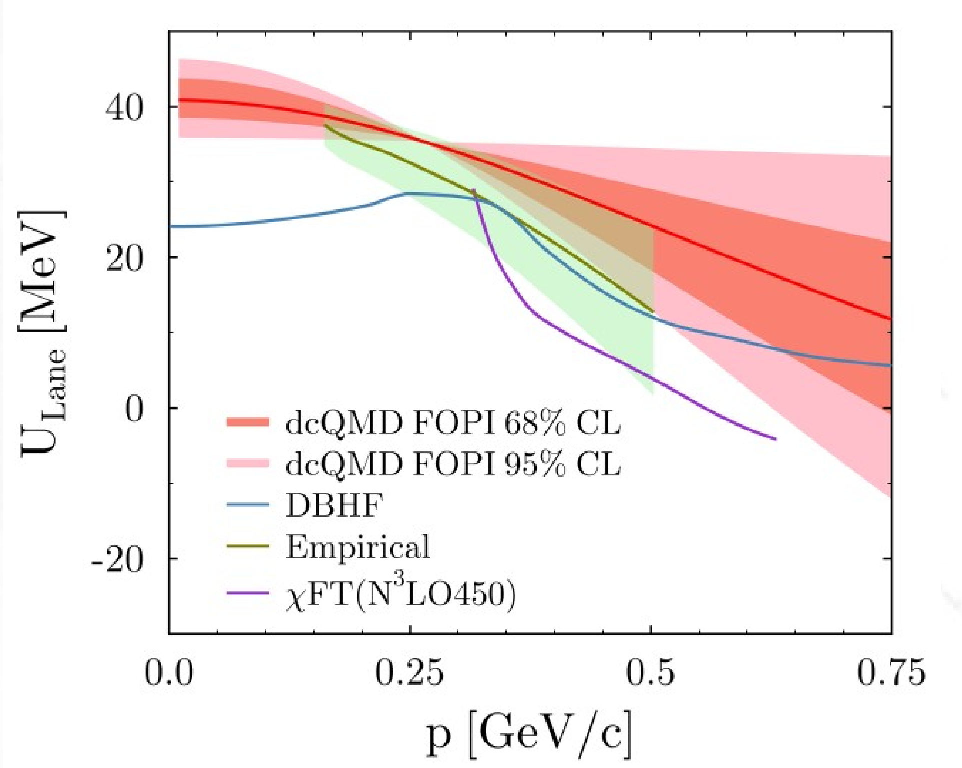
# Momentum dependent optical potential

**Isoscalar Potential**

**Lane Potential**

at saturation density




DBHF: van Dalen et al., PRC 72, 065803 (2005)

Empirical Lane: B.A. Li, PRC 69, 064602 (2004)

χFT:J.W. Holt et al., PRC 93, 064603 (2016)

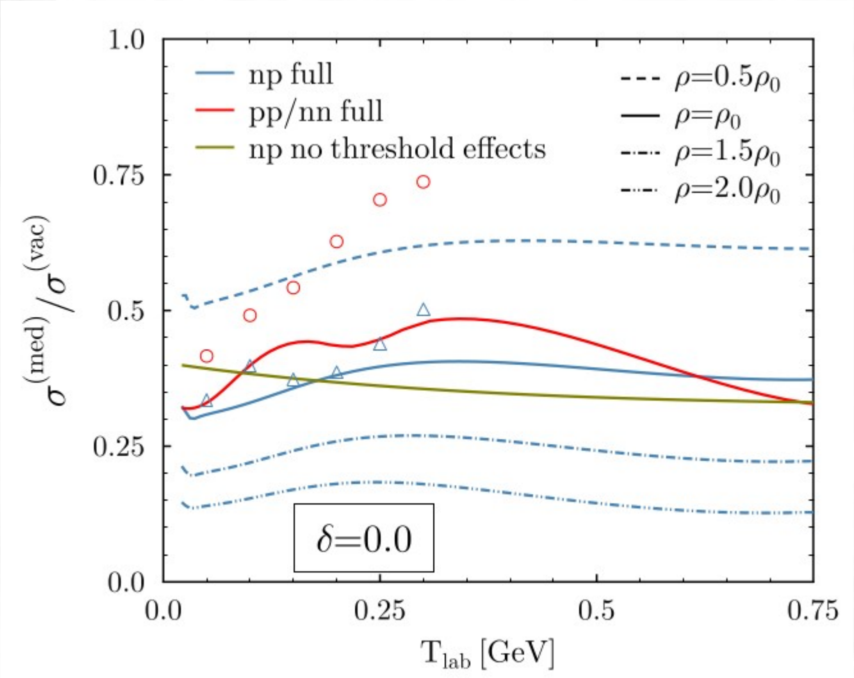
# In-medium $\sigma_{NN}$ (T=0 MeV Fermi)

$$\frac{d\sigma^{(med)}}{d\Omega} = (2\pi)^4 \frac{m_1^* \, m_2^*}{k_i^* \sqrt{s_i^*}} |M_{fi}^{(med)}(\rho, \delta, \{\tau\})|^2 \frac{k_f^* \, m_{1'}^*, m_{2'}^*}{\sqrt{s_f^*}}$$

$$|M_{fi}^{(med)}(\rho, \delta, \{\tau\})|^2 = \frac{1}{2}(|M_{fi}^{(vac)}(\tilde{s}_i)|^2 + |M_{fi}^{(vac)}(\tilde{s}_f)|^2)$$
$$\times \exp\left[(\alpha + \beta_1 \, \delta + \beta_2 \, (\tau_1 + \tau_2)\delta) \frac{\rho}{\rho_0}\right]$$

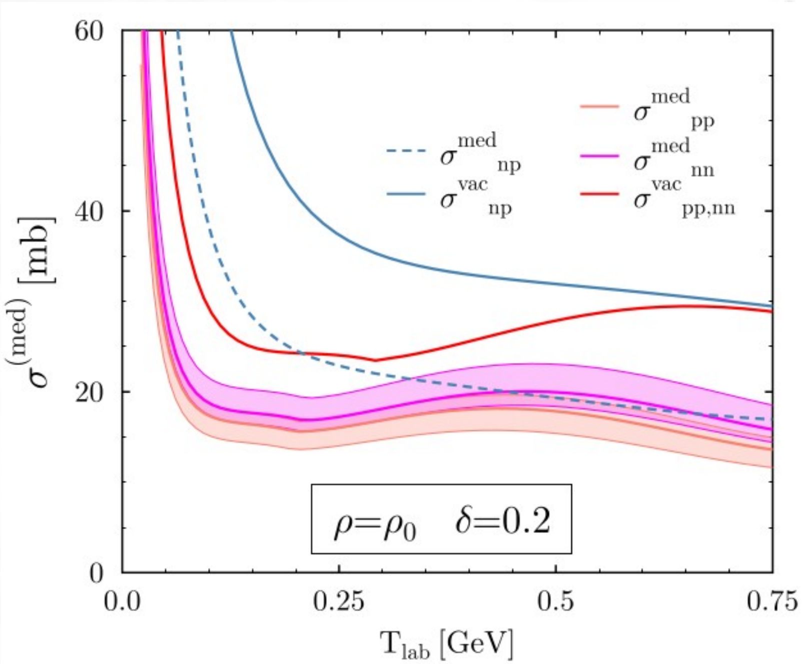
$$\sqrt{\tilde{s}_{i,f}} - 2\, m_N = \sqrt{s_{i,f}^*} - \sqrt{s_{th}^*} + U_{i,f} - U_{th}$$

reduction factor in symmetric matter




asymmetric nuclear matter

P=0.0




Li, Machleidt, PRC 48, 1702 (1993)
Li, Machleidt, PRC 49, 566 (1994)

qualitative agreement
with microscopical models

H. Zhang et al., IJMPE 19, 1788 (2010)
F. Sammarruca, EPJA 50, 22 (2014)