Understanding of expanding and clustering matter: Cluster correlation and momentum fluctuation in AMD

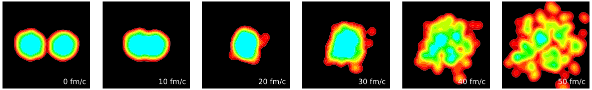
Akira Ono

Tohoku University

NuSYM24: XIIth International Symposium on Nuclear Symmetry Energy,
2024.09.09–13, GANIL, Caen, France

- AMD results compared with SπRIT data (see talks by Mizuki Kurata-Nishimura and Betty Tsang)
- An improvement of AMD: activation of momentum fluctuation (includes Lei Shen's work)
- Collision term and pion production (includes Natsumi Ikeno's work)

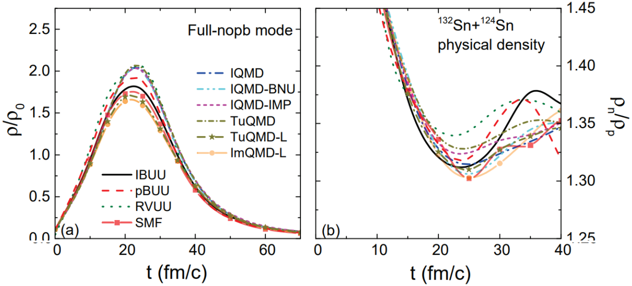
Time evolution of density and neutron-proton asymmetry



Sn+Sn @300 MeV/u (AMD)

$(N/Z)_{sys} = 1.56$

$\rho(t)$ at $r \approx 0$ $\qquad$ $\rho_n(t)/\rho_p(t)$ at $r \approx 0$



^{132}Sn + ^{124}Sn, E/A = 270 MeV, b = 4 fm

Model comparison under the same physical inputs

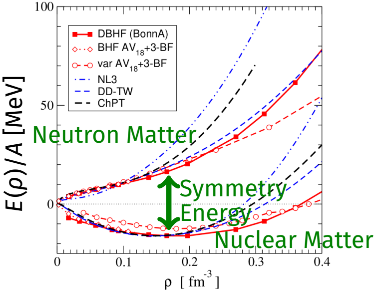
Jun Xu et al. (TMEP), PRC 109, 044609 (2024)

Nuclear matter EOS

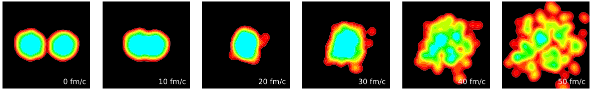
$$\frac{E}{A}(\rho_p, \rho_n) = \left(\frac{E}{A}\right)_0(\rho) + E_{sym}(\rho)\delta^2 + \cdots$$

$$\rho = \rho_p + \rho_n, \quad \delta = (\rho_n - \rho_p)/\rho$$

$$S_0 = E_{sym}(\rho_0), \quad L = 3\rho_0(dE_{sym}/d\rho)_{\rho=\rho_0}$$



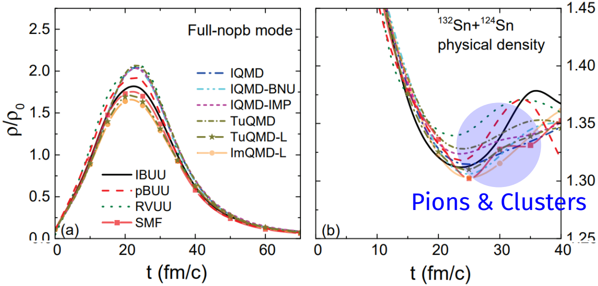
Fuchs and Wolter, EPJA 30, 5 (2006)



Sn+Sn @300 MeV/u (AMD)

$(N/Z)_{sys} = 1.56$

$\rho(t)$ at $r \approx 0$ $\quad\rho_n(t)/\rho_p(t)$ at $r \approx 0$



^{132}Sn + ^{124}Sn, E/A = 270 MeV, b = 4 fm

Model comparison under the same physical inputs

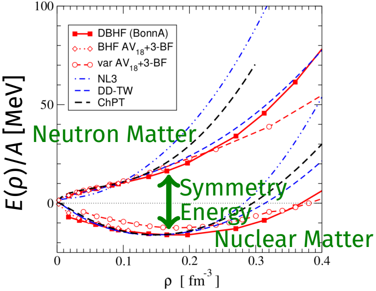
Jun Xu et al. (TMEP), PRC 109, 044609 (2024)

Nuclear matter EOS

$$\frac{E}{A}(\rho_p, \rho_n) = \left(\frac{E}{A}\right)_0(\rho) + E_{sym}(\rho)\delta^2 + \cdots$$

$$\rho = \rho_p + \rho_n, \quad \delta = (\rho_n - \rho_p)/\rho$$

$$S_0 = E_{sym}(\rho_0), \quad L = 3\rho_0(dE_{sym}/d\rho)_{\rho=\rho_0}$$

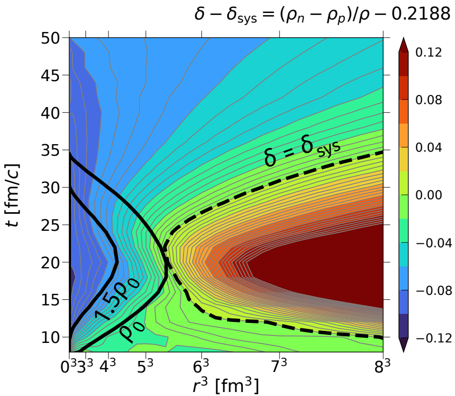


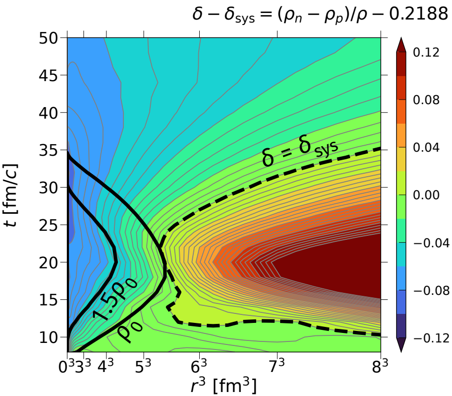
Fuchs and Wolter, EPJA 30, 5 (2006)

Symmetry energy effect on isospin asymmetry $\delta = (\rho_n - \rho_p)/\rho$, in AMD calculation

SLy4:La100

SLy4

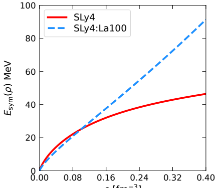




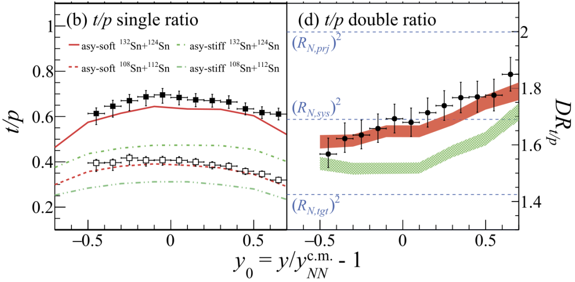
$^{132}\text{Sn} + {}^{124}\text{Sn}$, $b \approx 0$, $E/A = 270$ MeV

$(N, Z)_{\text{sys}} = (156, 100)$

$\delta_{\text{sys}} = [(N - Z)/(N + Z)]_{\text{sys}} = 0.21875$



- $E_{\text{sym}}^{\text{SLy4:La100}}(\rho_1) = E_{\text{sym}}^{\text{SLy4}}(\rho_1)$ at $\rho_1 = 0.10\ \text{fm}^{-3}$
- $L(\rho_0) = \begin{cases} 100\ \text{MeV} & \text{(SLy4:La100)} \\ 46\ \text{MeV} & \text{(SLy4)} \end{cases}$



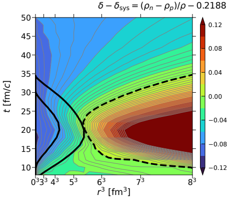
M. Kaneko, Murakami, Isobe, Kurata-Nishimura, Ono, Ikeno et al. (SπRIT), PLB 822 (2021) 136681.

$E_{sym}(\rho)$ dependence of t/p is consistent with:

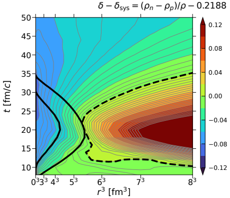
Clusters stems from the central region.

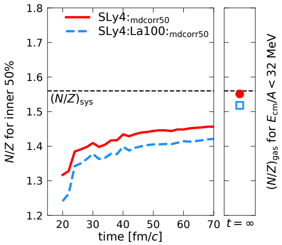
$$\frac{t/p \text{ in } {}^{132}\text{Sn} + {}^{124}\text{Sn}}{t/p \text{ in } {}^{108}\text{Sn} + {}^{112}\text{Sn}} = (t/p \text{ double ratio})$$

Stiff $E_{sym}(\rho)$



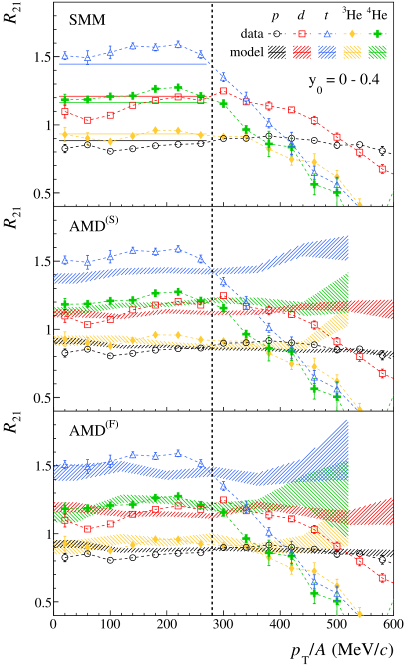
Soft $E_{sym}(\rho)$





Observable:

$$\left.\frac{(N \text{ in } n, d, t, {}^3\text{He}, \alpha)}{(Z \text{ in } p, d, t, {}^3\text{He}, \alpha)}\right|_{E_{cm}/A < 32 \text{ MeV}}$$



J.W. Lee et al. (SπRIT), EPJA (2022) 201.

Isoscaling ratio:

$$R_{21}(N,Z) = \frac{Y(N,Z)\text{ from }^{132}\text{Sn} + {}^{124}\text{Sn}}{Y(N,Z)\text{ from }^{108}\text{Sn} + {}^{112}\text{Sn}}$$

Sn + Sn at 270 MeV/u, central events ($b < 1.5$ fm)

Mid-rapidity $0 < y_0 < 0.4$ ($y_0 = y/y^{c.m}_{NN} - 1$)

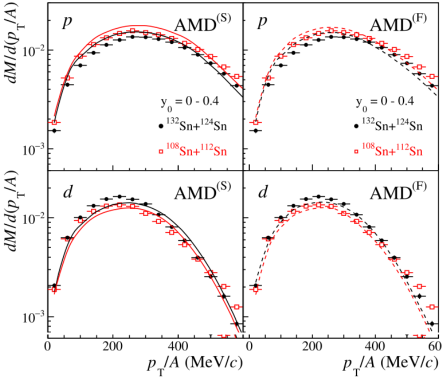
As a function of p_T/A:

- *Isoscaling phenomenon up to $p_T/A < 280$ MeV/c is found*
- *but breaks down for clusters with $p_T/A > 280$ MeV/c.*

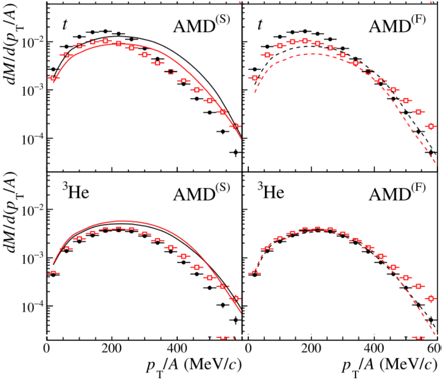
AMD: SLy4 interaction (L = 46 MeV)

SπRIT data: JW Lee et al. (SπRIT), EPJA (2022) 58:201

“$2\sigma_{NN}$” a standard “σ_{NN}”



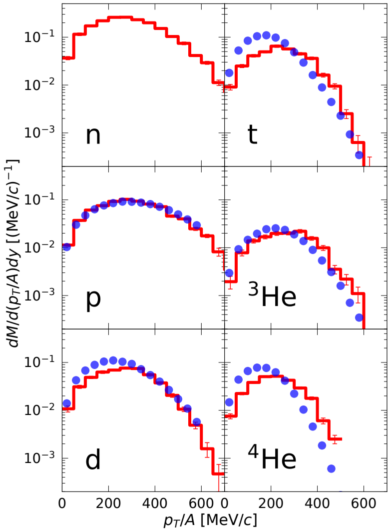
“$2\sigma_{NN}$” a standard “σ_{NN}”



- ● $^{132}Sn + ^{124}Sn$
- □ $^{108}Sn + ^{112}Sn$

Mid-rapidity $0 < y_0 < 0.4$ $(y_0 = y/y_{NN}^{c.m} - 1)$

- Difference between the two systems.
  - See high p_T of t and ^3He.
- Shape of t and ^3He spectra.
  - In AMD, $t \approx {}^3He$.
  - In exp. data, a large t yield at low p_T.



(without momentum fluctuation $\delta\boldsymbol{p}$)

$$\frac{dM}{d(p_T/A)}:$$

p_T/A distribution in log scale

- mid-rapidity region
- ^{132}Sn + ^{124}Sn, E/A = 270 MeV, $b \approx 0$

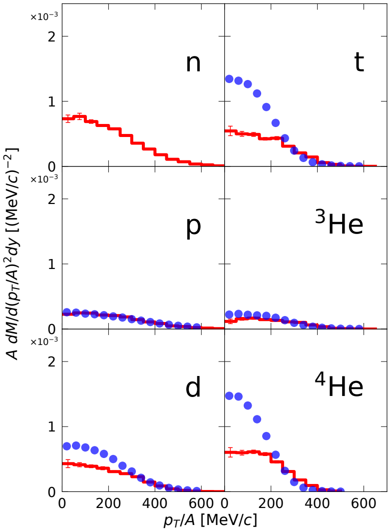
Blue points: SπRIT data, J.W. Lee et al., EPJA (2022) 201.

Red histogram: AMD result with the SLy4 interaction

Deficiency of clusters/matter in the central part of the expanding system, in this AMD calculation.

Need to be solved, also for the determination of $E_{\mathrm{sym}}(\rho)$.

A possible solution: Proper consideration on momentum fluctuation in wave packet molecular dynamics.



(without momentum fluctuation $\delta\boldsymbol{p}$)

$$A \times \frac{1}{(p_T/A)} \frac{dM}{d(p_T/A)}:$$

Mass-weighted p_T/A distribution in linear scale

- mid-rapidity region
- ^{132}Sn + ^{124}Sn, E/A = 270 MeV, $b \approx 0$

Blue points: SπRIT data, J.W. Lee et al., EPJA (2022) 201.

Red histogram: AMD result with the SLy4 interaction

Deficiency of clusters/matter in the central part of the expanding system, in this AMD calculation.

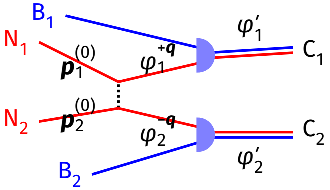
Need to be solved, also for the determination of $E_{sym}(\rho)$.

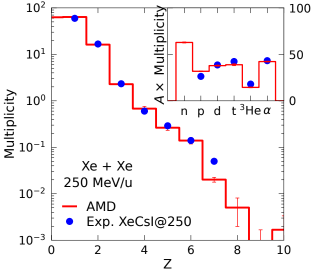
A possible solution: Proper consideration on momentum fluctuation in wave packet molecular dynamics.

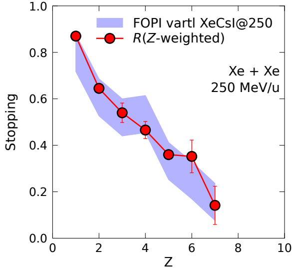
Basic cluster observables and the choice of NN matrix element

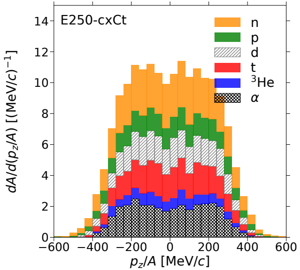
$$v_\mathrm{i} \frac{d\sigma}{d\Omega}(NNNB \to C_1 C_2) = P(\mathrm{C_1 C_2}, p_\mathrm{f,rel}, \Omega) \times \left| M(p_\mathrm{i,rel}, p_\mathrm{f,rel}, \Omega) \right|^2 \times \frac{p^2_\mathrm{f,rel}}{v_\mathrm{f}}$$

$$|M|^2 = \left(\frac{2}{m_N}\right)^2 \frac{d\sigma_{NN}}{d\Omega} \quad \text{with} \quad \sigma_{NN}(\rho', \epsilon) = \sigma_0 \tanh\left(\frac{\sigma_\mathrm{free}(\epsilon)}{\sigma_0}\right), \quad \sigma_0 = 0.8\,(\rho')^{-2/3}$$





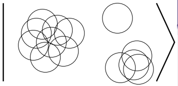




Central Xe + CsI (Xe + Xe) collisions at 250 MeV/nucleon

FOPI Data: Reisdorf et al., NPA 848 (2010) 366.

"Density" $\rho_i'^{(\mathrm{ini/fin})} = \left(\frac{2\nu}{\pi}\right)^{\frac{3}{2}} \sum_{k(\neq i)} \theta\left(p_\mathrm{cut} > |\boldsymbol{P}_i^{(\mathrm{ini/fin})} - \boldsymbol{P}_k|\right) e^{-2\nu(\boldsymbol{R}_i - \boldsymbol{R}_k)^2}$ with a momentum cut $p_\mathrm{cut} = (375\ \mathrm{MeV}/c)\, e^{-\epsilon_\mathrm{cm}/(225\ \mathrm{MeV})}$.



AMD wave function

$$|\Phi(Z)\rangle = \det_{ij}\left[\exp\left\{-\nu\left(\mathbf{r}_j - \frac{\mathbf{Z}_i}{\sqrt{\nu}}\right)^2\right\}\chi_{\alpha_i}(j)\right]$$

$$\mathbf{Z}_i = \sqrt{\nu}\mathbf{D}_i + \frac{i}{2\hbar\sqrt{\nu}}\mathbf{K}_i$$

ν : Width parameter = $(2.5\ \text{fm})^{-2}$

χ_{α_i} : Spin-isospin states = $p\uparrow, p\downarrow, n\uparrow, n\downarrow$

Equation of motion for the wave packet centroids Z

$$\frac{d}{dt}\mathbf{Z}_i = \{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}} + \text{(NN collisions)} + \textit{(some model extensions)}$$

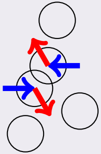
$\{\mathbf{Z}_i, \mathcal{H}\}_{\text{PB}}$: Motion in the mean field

$$\mathcal{H} = \frac{\langle\Phi(Z)|H|\Phi(Z)\rangle}{\langle\Phi(Z)|\Phi(Z)\rangle} + \text{(c.m. correction)}$$

- H: Effective interaction (e.g. Skyrme force)
  — *also in NN collisions?*

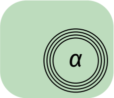
NN collisions

$$W_{i\to f} = \frac{2\pi}{\hbar}|\langle\Psi_f|V|\Psi_i\rangle|^2\delta(E_f - E_i)$$



- $|V|^2$ or σ_{NN} (in medium)
- Pauli blocking

Ono, Horiuchi, Maruyama, Ohnishi, Prog. Theor. Phys. 87 (1992) 1185.

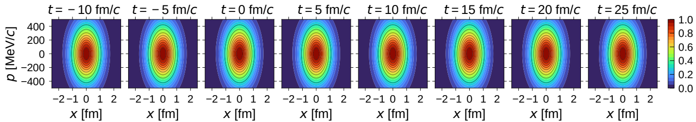


$$= \Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3,\boldsymbol{r}_4) = \prod_{j=1}^{4} e^{-\nu(\boldsymbol{r}_j-\boldsymbol{R})^2} e^{(i/\hbar)\boldsymbol{P}\cdot\boldsymbol{r}_j} = e^{-4\nu(\boldsymbol{r}_\alpha-\boldsymbol{R})^2} e^{(i/\hbar)(4\boldsymbol{P})\cdot\boldsymbol{r}_\alpha} \otimes \Phi_\alpha^{\mathrm{int}} \qquad \boldsymbol{r}_\alpha = \frac{1}{4}\sum_{j=1}^{4}\boldsymbol{r}_j$$

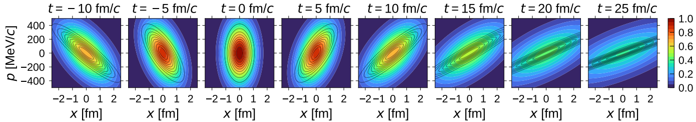
In momentum space, $$\tilde{\Psi}(\boldsymbol{p}_1,\boldsymbol{p}_2,\boldsymbol{p}_3,\boldsymbol{p}_4) = e^{-\frac{1}{16\nu}(\boldsymbol{p}_\alpha-4\boldsymbol{P})^2} e^{-(i/\hbar)\boldsymbol{R}\cdot\boldsymbol{p}_\alpha} \otimes \tilde{\Phi}_\alpha^{\mathrm{int}} \qquad \boldsymbol{p}_\alpha = \sum_{j=1}^{4}\boldsymbol{p}_j$$

- Nucleons in the α cluster are nucleons. (antisymmetrization and Pauli blocking **with other nucleons**)
- Convenient description of **fluctuations/correlations** by compact nucleon wave packets.
- **Wave packet for c.m. motion**: $\Delta x = 1/(2\sqrt{4\nu})$, $\Delta p = \hbar\sqrt{4\nu}$ $\Rightarrow \Delta x \Delta p = \frac{1}{2}\hbar$

Free propagation of a wave packet with a fixed shape:



Correct free propagation:



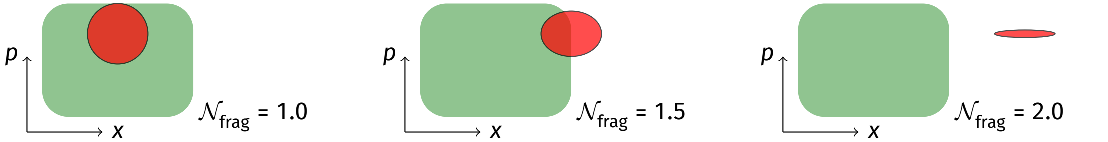
Momentum fluctuation inherent in the wave packet should be activated in some way.

Traditional zero-point energy subtraction plus momentum fluctuation ($\delta \boldsymbol{p}^{\text{one}}$)

Kinetic energy includes zero-point energies: (subtraction of spurious zero-point energies)

$$\mathcal{H} = \left(\sum_{ij} \frac{\boldsymbol{P}_{ij}^2}{2M} B_{ij} B_{ji}^{-1} \right) + \frac{3\hbar^2 \nu}{2M} (A - \mathcal{N}_{\text{frag}}) + \langle V \rangle$$

since AO at al., PTP 87 (1992) 1185.



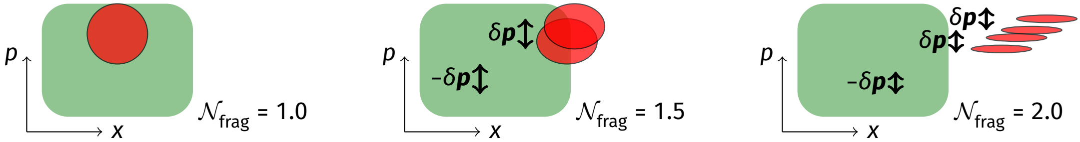
The momentum fluctuation inherent in the original wave packet is lost when the particle is emitted.

Traditional zero-point energy subtraction plus momentum fluctuation ($\delta\boldsymbol{p}^{\text{one}}$)

Kinetic energy includes zero-point energies: (subtraction of spurious zero-point energies)

$$\mathcal{H} = \left(\sum_{ij} \frac{\boldsymbol{P}_{ij}^2}{2M} B_{ij} B_{ji}^{-1}\right) + \frac{3\hbar^2 \nu}{2M}(A - \mathcal{N}_{\text{frag}}) + \langle V \rangle$$

since AO at al., PTP 87 (1992) 1185.



Recover the momentum distribution inherent in the wave packet by giving fluctuation $\delta\boldsymbol{p}$, according to the change of the isolation.

Correlation appears between the wave packet and the environment.

- For momentum conservation, the recoil momentum $-\delta\boldsymbol{p}$ is given to the environment.
- Energy must be conserved by adjusting some internal degrees of freedom of the environment.

New: Consistency between the fragment number $\mathcal{N}_{\text{frag}}$ and the momentum width.

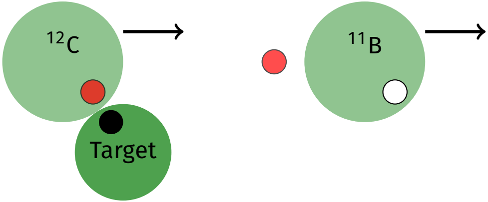
$$\text{w.p. width: } \sigma_p^2 = 3\hbar^2\nu(2 - \mathcal{N}_{\text{frag}}) \quad \Leftrightarrow \quad \text{fluctuation: } \langle \delta\boldsymbol{p}^2 \rangle = 3\hbar^2\nu(\mathcal{N}_{\text{frag}} - \mathcal{N}_{\text{frag}}^0)$$

Note: The actual formula is more general when many wave packets are considered simultaneously.

Momentum distribution of projectile-like fragment ($\delta \boldsymbol{p}^{\mathrm{one}}$)

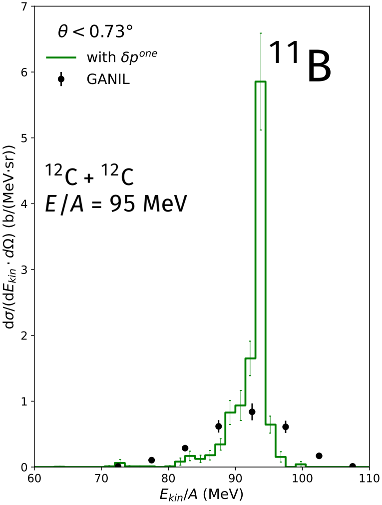
Lei Shen, A. Ono, Y.G. Ma, arXiv:2408.17029

Production of ^{11}B by one-proton removal from ^{12}C projectile.



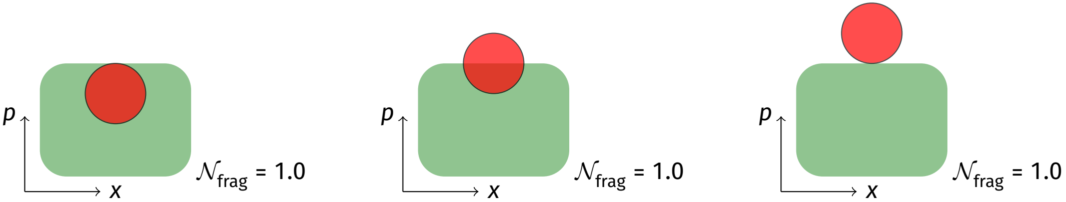
With only one-body momentum fluctuation $\delta \boldsymbol{p}^{\mathrm{one}}$, the ^{11}B momentum distribution is too sharp.

- ⇐ Fluctuation $\delta \boldsymbol{p}$ is canceled because sufficient energy cannot be supplied from the internal d.o.f. of the ^{11}B residue.
- ⇒ Fluctuation should be considered at the *NN* collision.

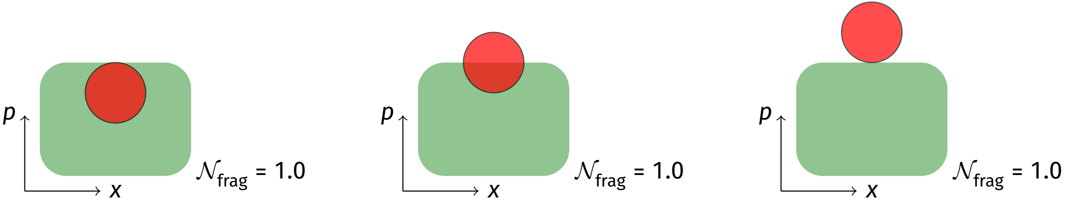


GANIL data: J. Dudouet et al., PRC 89, 064615 (2014). http://hadrontherapy-data.in2p3.fr/

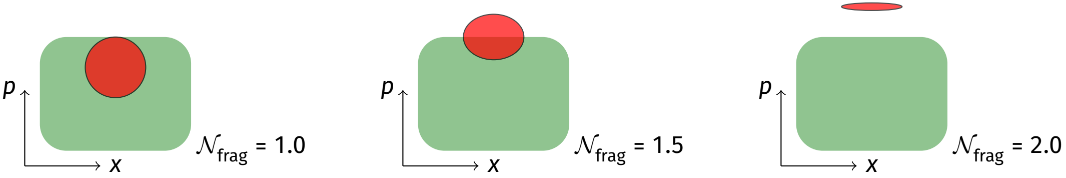
Traditional:



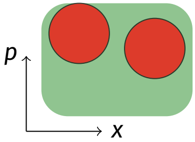
Traditional:

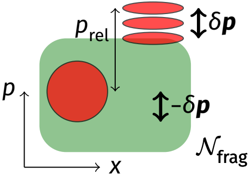


New: Measure the isolation in phase space.



Momentum fluctuation at each two-nucleon collision ($\delta\boldsymbol{p}^{\text{coll}}$)





Scattering may change the isolation of the nucleon(s).

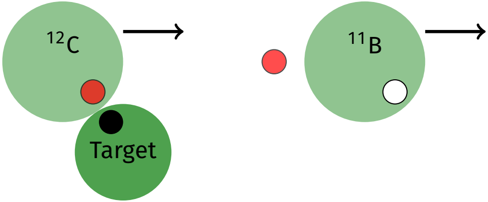
- When the isolation of a nucleon increases by the scattering, the momentum fluctuation $\delta\boldsymbol{p}$ is given to compensate the decrease of the wave packet momentum width.
- For momentum conservation, the recoil $-\delta\boldsymbol{p}$ is given to the environment.
- Energy conservation is achieved by adjusting the final relative momentum p_{rel} between the scattered nucleons. **(Important source of energy)**

The scattered nucleon(s) may form a cluster after the fluctuation $\delta\boldsymbol{p}$ is applied, and then the energy conservation is achieved by adjusting p_{rel}.

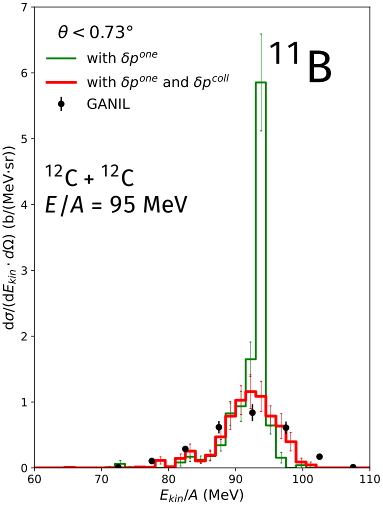
c.f. **"Fermi boost" in AMD** by W. Lin, X. Liu, R. Wada et al., PRC 94, 06409 (2016).

Lei Shen, A. Ono, Y.G. Ma, arXiv:2408.17029

Production of ^{11}B by one-proton removal from ^{12}C projectile.



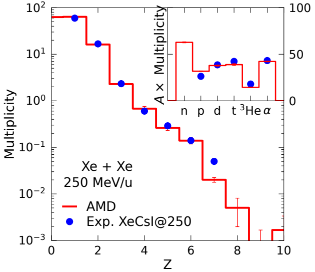
The momentum distribution of the ^{11}B is improved significantly, by considering δp^{coll} at each NN collision in addition to the one-body fluctuation δp^{one}.

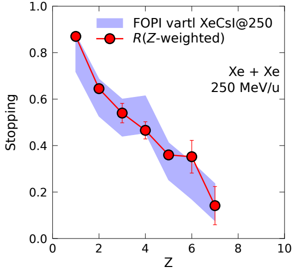


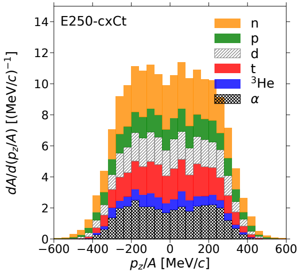
GANIL data: J. Dudouet et al., PRC 89, 064615 (2014).
http://hadrontherapy-data.in2p3.fr/

Basic observables without and with momentum fluctuations

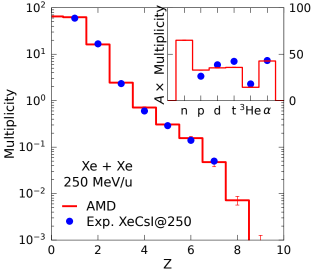
Without $\delta\boldsymbol{p}$

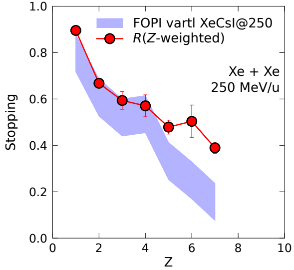


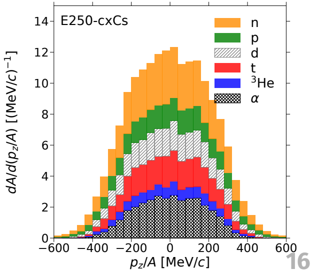




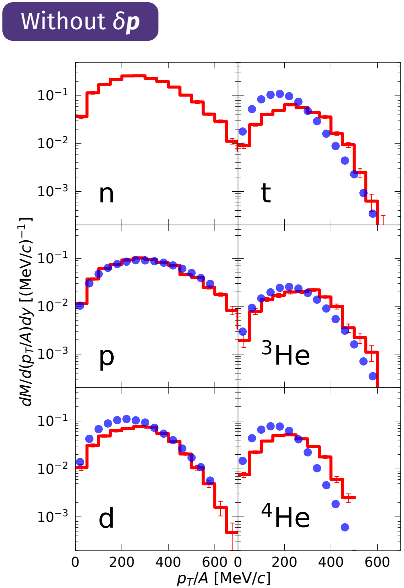
With $\delta\boldsymbol{p}^{\text{one}}$ & $\delta\boldsymbol{p}^{\text{coll}}$

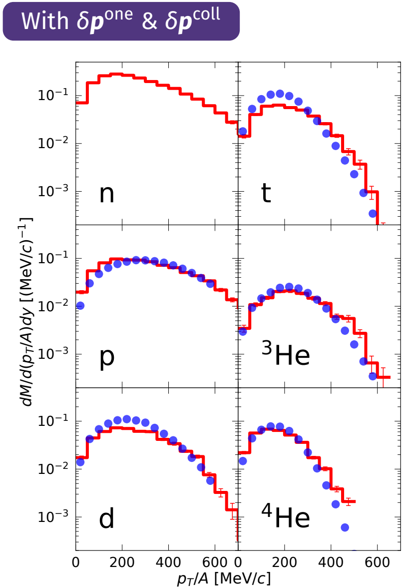






Transverse momentum spectra of charged particles without and with δp



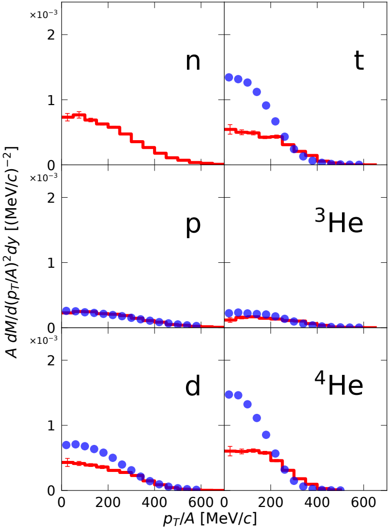


^{132}Sn + ^{124}Sn, E/A = 270 MeV, $b \approx 0$

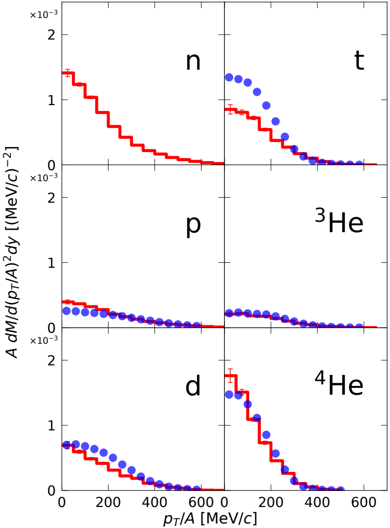
Blue points: SπRIT data, J.W. Lee et al. (SπRIT), EPJA (2022) 201.

Transverse momentum spectra of charged particles without and with δp

Without $\delta \boldsymbol{p}$



With $\delta \boldsymbol{p}^{\mathrm{one}}$ & $\delta \boldsymbol{p}^{\mathrm{coll}}$

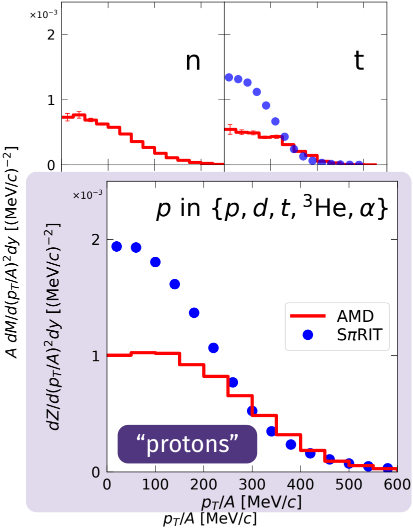


^{132}Sn + ^{124}Sn, E/A = 270 MeV, $b \approx 0$

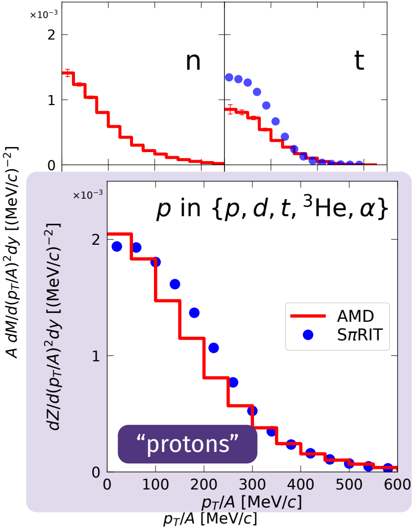
Blue points: SπRIT data, J.W. Lee et al. (SπRIT), EPJA (2022) 201.

Transverse momentum spectra of charged particles without and with δp

Without $\delta \boldsymbol{p}$



With $\delta \boldsymbol{p}^{\mathrm{one}}$ & $\delta \boldsymbol{p}^{\mathrm{coll}}$

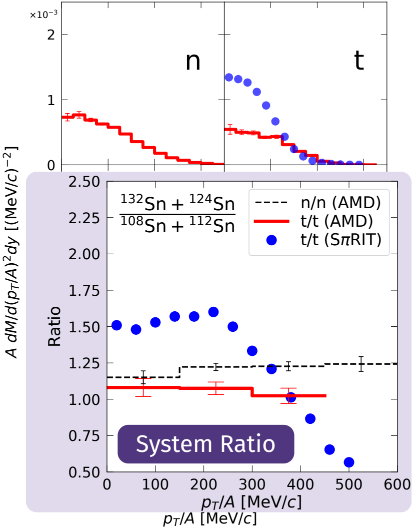


^{132}Sn + ^{124}Sn, E/A = 270 MeV, $b \approx 0$

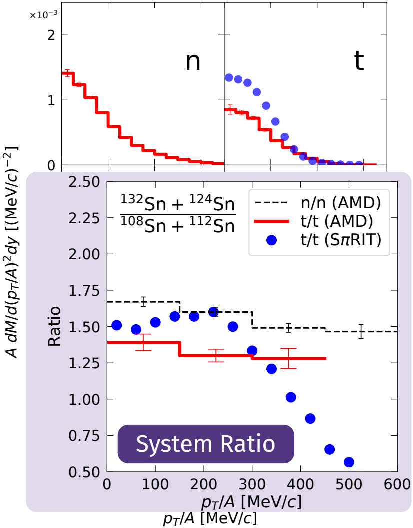
Blue points: SπRIT data, J.W. Lee et al. (SπRIT), EPJA (2022) 201.

Transverse momentum spectra of charged particles without and with δp

Without $\delta \boldsymbol{p}$



With $\delta \boldsymbol{p}^{\mathrm{one}}$ & $\delta \boldsymbol{p}^{\mathrm{coll}}$



^{132}Sn + ^{124}Sn, E/A = 270 MeV, $b \approx 0$

Blue points: SπRIT data, J.W. Lee et al. (SπRIT), EPJA (2022) 201.

Collision term under potential

Boltzmann/BUU equation for heavy-ion collisions:

$$\frac{\partial f}{\partial t} = \underbrace{\frac{\partial h}{\partial \boldsymbol{r}}\cdot\frac{\partial f}{\partial \boldsymbol{p}} - \frac{\partial h}{\partial \boldsymbol{p}}\cdot\frac{\partial f}{\partial \boldsymbol{r}}}_{\text{mean-field propagation}} + \underbrace{\int |\boldsymbol{v}|\frac{d\sigma}{d\Omega}\{f_3 f_4(1-f)(1-f_2) - f f_2(1-f_3)(1-f_4)\}\frac{d\boldsymbol{p}_2 d\Omega}{(2\pi\hbar)^3}}_{\text{collision term, e.g., } nn \to nn,\, p\Delta^-,\, n\Delta^0}$$

Initial state:

$+\boldsymbol{p}_i \rightarrow$ 1 $\quad$ $\leftarrow -\boldsymbol{p}_i$ 2

Final state:

$+\boldsymbol{p}_f \nearrow 3$

$-\boldsymbol{p}_f \swarrow 4$

Collision rate/cross-section: $\sigma(\boldsymbol{p}_1, \boldsymbol{p}_2; \text{environment})$ for $1 + 2 \to 3 + 4$

$$v_i \frac{d\sigma}{d\Omega} \propto \int |M|^2 \delta(E_f - E_i) p_f^2 dp_f = |M|^2 \frac{p_f^2}{dE_f/dp_f} \qquad \therefore \quad \frac{d\sigma}{d\Omega} = \left(\frac{p_i}{v_i}\frac{p_f}{v_f}\right)\cdot |M|^2 \cdot \frac{p_f}{p_i}\cdot A(m)$$

Potentials at the space-time point of the collision enter in the energies:

$$E_i = \sqrt{m_1^2 + p_1^2} + U_1 + \sqrt{m_2^2 + p_2^2} + U_2, \quad E_f = \sqrt{m_3^2 + p_3^2} + U_3 + \sqrt{m_4^2 + p_4^2} + U_4$$

We must solve

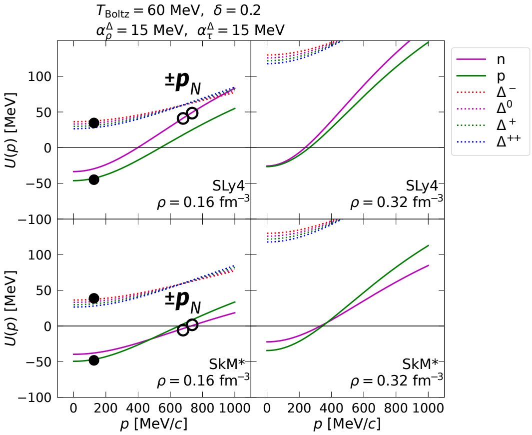
$$E_f(\boldsymbol{p}_f) = E_i$$

sJAM code: Ikeno and Ono, PRC 108, 044601 (2023)

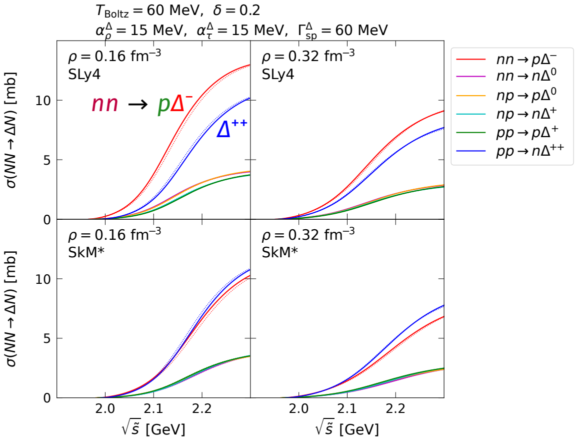
- $\mu_i^* = p_i/v_i$ and $\mu_f^* = p_f/v_f$ are the effective reduced masses in the initial and final states. They are *reduced* by the momentum dependence of the mean field.
- We assume the matrix element $|M|^2$ is not strongly affected by the potential.
- The final momentum factor p_f is determined by the energy conservation, which can be strongly affected by the potential. E.g, the threshold of $NN \to N\Delta$ (endothermic reaction) is determined by $p_f = 0$.

An example: $NN \to N\Delta$ cross sections in neutron-rich nuclear matter

Two different momentum dependences of neutrons $U_n(p)$ and protons $U_p(p)$.



$\sigma(NN \to N\Delta)$ with the initial nucleon momenta $\pm \boldsymbol{p}_N$ in nuclear matter, as a function of $\sqrt{\tilde{s}} = 2\sqrt{m_N^2 + \boldsymbol{p}_N^2}$.

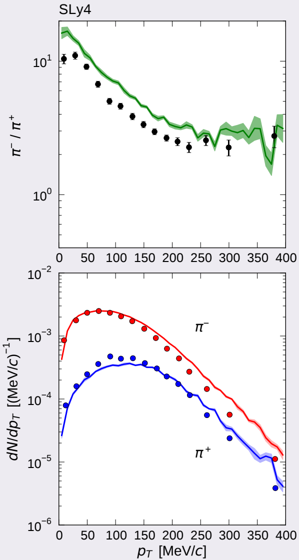


$$\sigma(\pm \boldsymbol{p}_N; \text{environment}) \sim p_f \sim \sqrt{\epsilon^*} \quad \text{with} \quad \epsilon^* = \underbrace{2\sqrt{m_N^2 + (\pm \boldsymbol{p}_N)^2} - m_N - m_\Delta}_{\text{same as in vaccum}} + \underbrace{U_1(+\boldsymbol{p}_N) + U_2(-\boldsymbol{p}_N) - U_3(0) - U_\Delta(0)}_{\text{effect of potentials}}$$

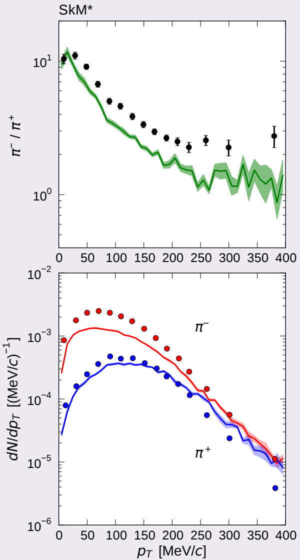
Strong impact on the isospin dependence of Δ production by e.g. $2U_n(p_N) - U_p(0)$. $p_N \gtrsim 400$–500 MeV/c.

Pion production from ^{132}Sn + ^{124}Sn at 270 MeV/u

SLy4 ($m_n^* < m_p^*$)



SkM* ($m_n^* > m_p^*$)



Spectra of π^- and π^+ (lower) and the π^-/π^+ ratio (upper)

Lines AMD+sJAM calculation with potentials in the collision term.

Points SπRIT experimental data. J. Estee et al., PRL 126, 162701 (2021)

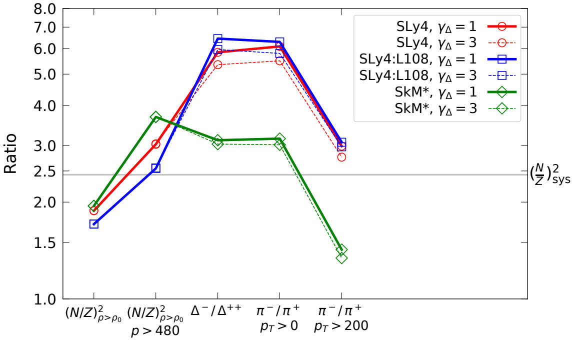
The charged pion ratio π^-/π^+ is strongly affected by the momentum dependence of

$$U_n(p) - U_p(p) = 2\delta\, U_{\mathrm{sym}}(p)$$

Ikeno and Ono, PRC 108, 044601 (2023)

Summary of N/Z, Δ^-/Δ^{++} and π^-/π^+ ratios

^{132}Sn + ^{124}Sn at 270 MeV/u



Ikeno and Ono, PRC 108, 044601 (2023)

Effect of isospin splitting of Δ potential

[Small splitting] >≈ [Large splitting]

$(N/Z)^2$ in the high density region ($\rho > \rho_0$)

[Soft $E_{\rm sym}$: SkM*, SLy4] > [Stiff $E_{\rm sym}$: SLy4:L108]

$(N/Z)^2$ at high density ($\rho > \rho_0$) and high momentum ($|\boldsymbol{p} - \boldsymbol{p}_{\rm rad}| > 480$ MeV/c)

[$m_n^* > m_p^*$: SkM*] > [$m_n^* < m_p^*$: SLy4]

$(NN \to N\Delta^-)/(NN \to N\Delta^{++})$ production ratio

[SkM*] ≪ [SLy4] < [SLy4:L108]

Final π^-/π^+ ratio

[SkM*] ≪ [SLy4], [SLy4:L108]

High-momentum π^-/π^+ ratio ($p_T > 200$ MeV/c)

Simple Coulomb effect

Activation of momentum fluctuation $\delta\boldsymbol{p}^{\mathrm{one}}$ and $\delta\boldsymbol{p}^{\mathrm{coll}}$ — an improvement of AMD

- This improved the momentum distribution of the ^{11}B fragments from ^{12}C + ^{12}C reactions. [Lei Shen et al., arXiv:2408.17029]
- In Sn + Sn at 270 MeV/nucleon, this increased the clusters and nucleons near the center of mass of the expanding system.
- ⇒ Seems to affect symmetry energy observables,
  such as the isoscaling ratio Y(^{3}H from ^{132}Sn + ^{124}Sn) / Y(^{3}H from ^{108}Sn + ^{112}Sn).

SπRIT data compared with AMD results indicate:

- Clusters (in particular ^{3}H) are enhanced near the center in neutron-rich systems.
  - Mott effect in AMD with Skyrme force is too strong? Something else?

Fluctuations and correlations in transport models

- use of wave packets
- BUU + fluctuation
- clusters as new d.o.f.

- One-body mean field propagation
- Two-nucleon collisions