# Consistent description of clusters and fragments within upgraded transport models

XII International Symposium on
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# Outline of the presentation

1. **Many-body (MB) correlations and clustering phenomena in nuclear systems**
   - Understanding Equation of State (EOS) for nuclear matter (NM)
   - Phenomenological models based on energy density functionals (EDF)

2. **Extended EDF-based models: recent developments and results**
   - ✏ **Unified (thermodynamic) description of few-body correlations and clusters**
     - Embedding short-range correlations within relativistic mean-field approaches
     - Global mass-shift parameterization for a multi-purposes EOS
   - ✏ **Dynamical approach with light clusters as degrees of freedom (DOF)**
     - Quasi-analytical study of dilute NM with light clusters and in-medium effects
     - Characterization of spinodal instability and growth rate of unstable modes

3. **Further developments and outlooks**
   - Connection between hydrodynamical and linearized Vlasov approach
   - Extensive numerical calculations of the dynamics with light clusters
   - Consistent descriptions of fragment formation mechanisms in heavy-ion collisions

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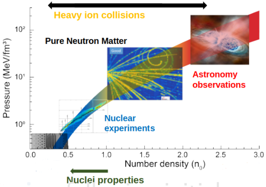
- Heavy-ion collisions (**HIC**) at $E_{\text{beam}} \approx (30 - 300)\,A$MeV $\Rightarrow$ EOS




- **Expansion** following initial **compression**
  $\Rightarrow$ low density ($\rho$) & temperature ($T$)
  - Spinodal instabilities → fragment
  - Few-body correlations → light clusters
- **Phenomenological** EDF with **clusters** DOF
  - NM → (nucleons+nuclei)

**Theoretical challenge**

Consistent dynamical approach for light clusters and heavier fragments

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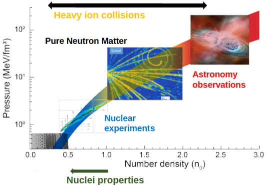


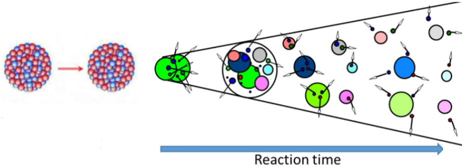


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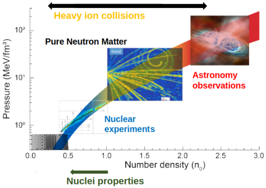


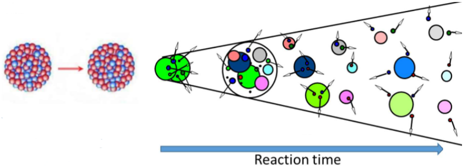


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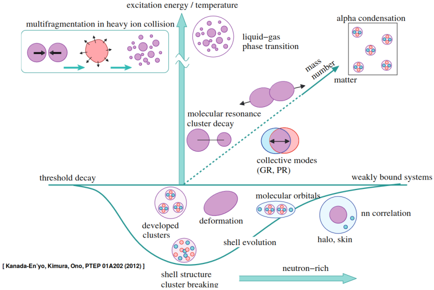

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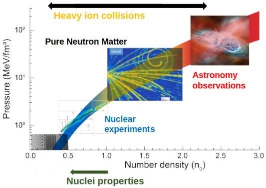


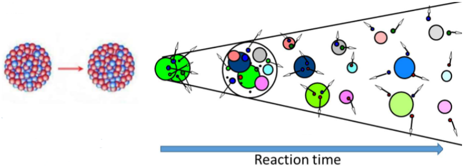


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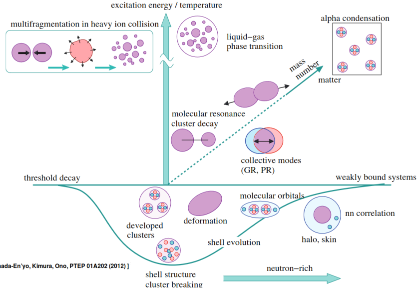


[Kanada-En'yo, Kimura, Ono, PTEP 01A202 (2012)]

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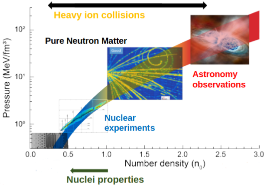


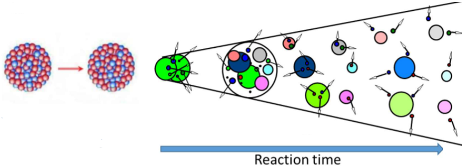


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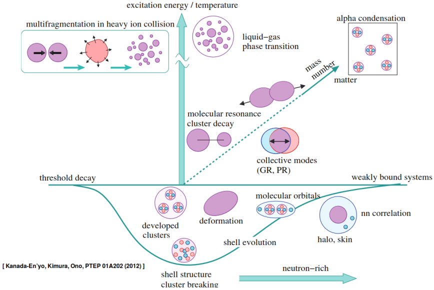


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# In-medium (Mott) effects and cluster dissolution

- Cluster **dissolution** approaching saturation from below
  $\Rightarrow$ Mott effect ruled by Pauli-blocking
- Generalized relativistic density functional (**GRDF**)
  [S. Typel et al., PRC 81, 015803 (2010)]
  - Microscopic in-medium effects
  - (Effective) binding energy $\rightarrow B^{\text{eff}} = B - \Delta m$
- $\Delta m^{(\text{low})}$ from in-medium MB **Schrödinger equation** [G. Röpke, NPA 867 (2011) 66–80]
- Parameterization $\Delta m(\rho, \beta, T, \mathbf{P}_{\text{c.m.}}) \Rightarrow$ **heuristic** $\Delta m^{(\text{high})}$ beyond Mott density
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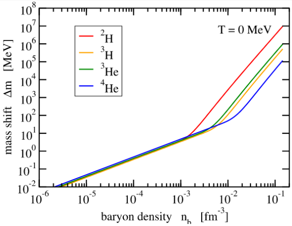


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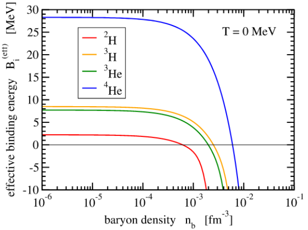

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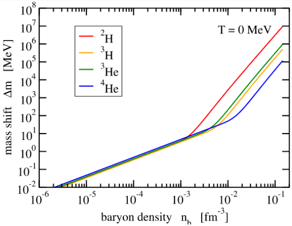


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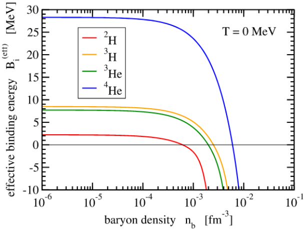

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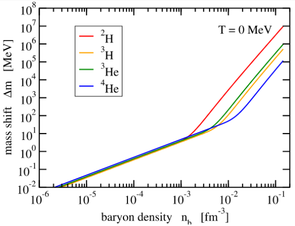


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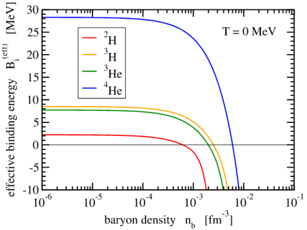

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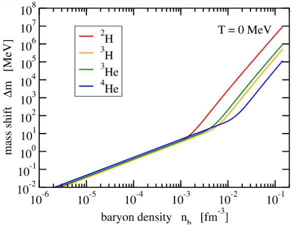


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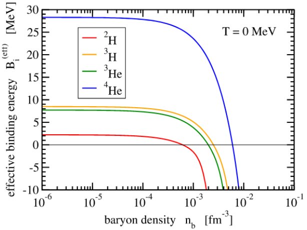


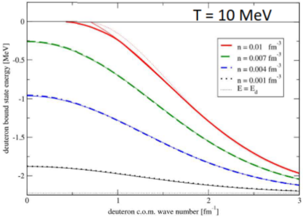

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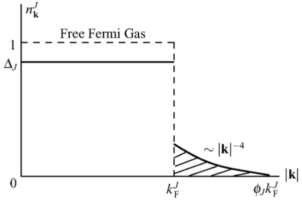
# Short-range correlations within GRDF model

- NM beyond Mott density: free Fermi gas
  $\Rightarrow$ **step** function in **momentum distribution** at zero T
- Nucleon knock-out in inelastic electron scattering
  [O. Hen et al. (CLAS Coll.), Science 346, 614 (2014)]
  - **Smearing** of Fermi surface in cold nucleonic matter
  - High momentum tail (**HMT**) decreasing with $\sim |\mathbf{k}|^{-4}$
- Nucleon-nucleon short-range correlations (**SRCs**)
  - Tensor components or repulsive core of nuclear forces
- Embedding (effectively) SRCs in **GRDF** model using quasi-clusters as surrogate
  [S. Burrello, S. Typel, EPJA 58, 120 (2022)]
  - **Two-body** correlations in $np\ ^3S_1$ channel $\Rightarrow$ quasi-deuteron
  - $\approx 20\%$ of nucleons in pairs $\Rightarrow$ Quasi-deuteron mass fraction $X_d(\rho_0) = 0.2$
  - $T = 0 \Rightarrow$ **condensate** of quasi-deuterons under chemical **equilibrium**
  - **Interpolation** between $\Delta m_d^{(\mathrm{low})}$, $\Delta m_d^{(\mathrm{high})}$, $\Delta m_d(\rho_0)$



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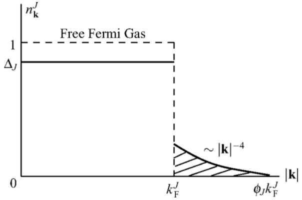


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$$\mu_d = \mu_n + \mu_p \Rightarrow \boxed{m_d^* + \Delta m_d^{(\mathrm{high})} + V_d' = \sqrt{k_n^2 + (m_n^*)^2} + V_n' + \sqrt{k_p^2 + (m_p^*)^2} + V_p'}$$

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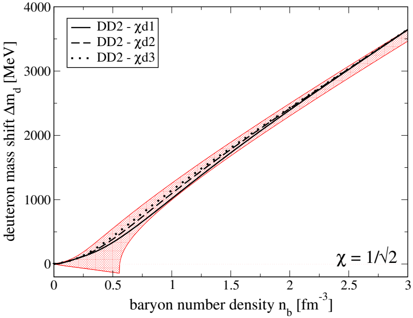
# Mass-shift parametrization and impact on EOS

- **Unified** mass-shift **parameterization** $(\gamma = 1)$ [S. Burrello, S. Typel, EPJA 58, 120 (2022)]

$$X_d(\rho_b) \Leftarrow \Delta m_d(x) = \frac{ax}{1+bx} + cx^{\eta+1}\left[1-\tanh(x)\right] + fx^{\gamma}\tanh(gx), \qquad x = \frac{\rho_b}{\rho_0}$$




| | $a$ | $b$ | $c$ | $\eta$ | $f$ | $g$ |
|---|---|---|---|---|---|---|
| DD2 - $\chi$d1 | 541.726060 | 243.472387 | 99.677247 | 1.656159 | 181.113975 | 0.18 |
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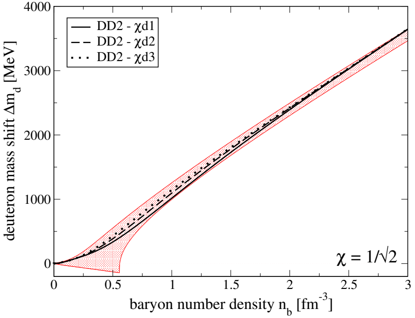


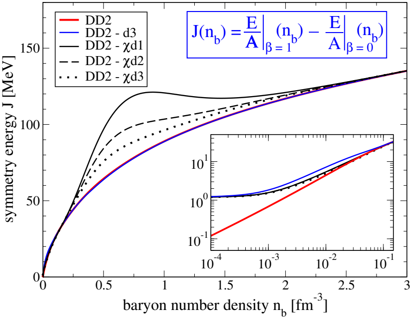


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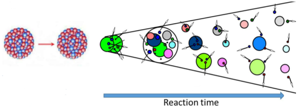
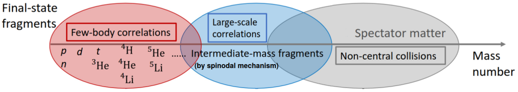
- **Dynamical** processes modelizations ⇒ **Transport** theories
  - Lack of consistent description of light and heavier **fragments**




- **Kinetic** approach of light-nuclei production in HIC at intermediate energies
  - Boltzmann–Uehling–Uhlenbeck model + collision integral (Mott effect)

    [R. Wang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, & Z. Zhang, PRC 108, L031601 (2023)]

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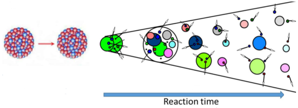


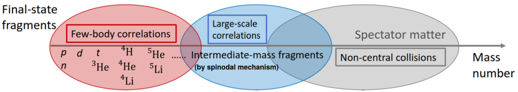


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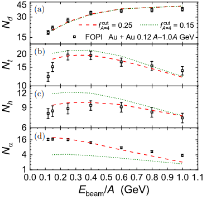
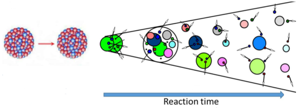
$$\left(\partial_t + \nabla_{\mathbf{p}}\varepsilon_\tau \cdot \nabla_{\mathbf{r}} - \nabla_{\mathbf{r}}\varepsilon_\tau \cdot \nabla_{\mathbf{p}}\right) f_\tau = I_\tau^{\mathrm{coll}}[f_n, f_p, \ldots], \qquad \tau = n, p, d, t, h, \alpha$$

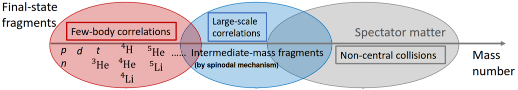
$$\langle f_N \rangle_A \equiv \int d\mathbf{p} f_N\left(\frac{\mathbf{P}}{A} + \mathbf{p}\right) \rho_A(\mathbf{p}) \leq f_A^{\mathrm{cut}}$$



# Kinetic approach for HIC with light-clusters DOF

- **Dynamical** processes modelizations ⇒ **Transport** theories
  - Lack of **consistent** description of **light** and heavier **fragments**




- **Kinetic** approach of light-nuclei **production** in HIC at intermediate energies
  - **Boltzmann**–Uehling–Uhlenbeck model + collision integral **cut-off** (Mott effect)

[R. Wang, Y.-G. Ma, L.-W. Chen, C. M. Ko, K.-J. Sun, & Z. Zhang, PRC 108, L031601 (2023)]

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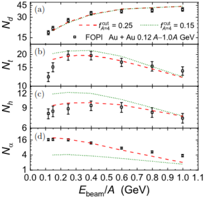


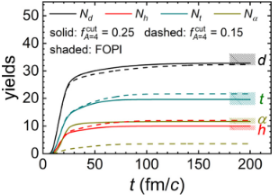


## Our goal

Assess if light **clusters** (from **compression** phase) affect **spinodal** instability (**expansion** stage)

# Density-dependent (Mott) momentum cut-off

- Non-relativistic framework ⇒ **dynamical** treatment more easily carried out
- Cut-off (Mott) momentum $\Lambda_j$ for Pauli-blocking

$$\rho_j = g_j \int_{|\mathbf{p}|>\Lambda_j} \frac{d\mathbf{p}}{(2\pi\hbar)^3} f_j \qquad j = n, p, d \qquad (\Lambda_q = 0, \text{ for } q = n, p)$$

- Chemical equilibrium ⇒ $X_d = \frac{\rho_d}{\rho_0}$ consistent with **benchmark** calculations [cf. Röpke]

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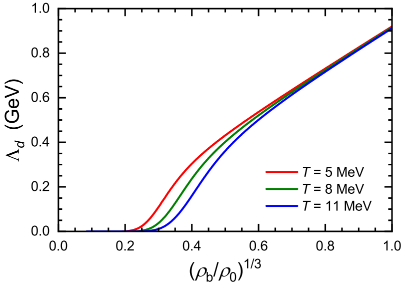


[R. Wang, S. Burrello, M. Colonna, F. Matera, arXiv:2405.02157, accepter for PRC-Letter]

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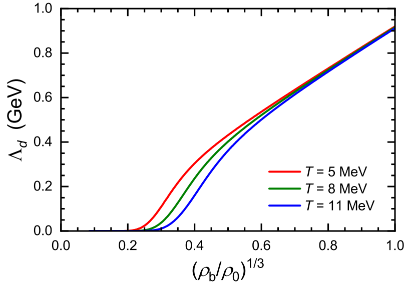
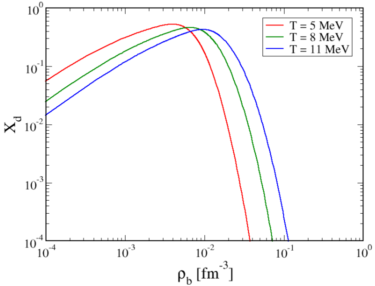



[R. Wang, S. Burrello, M. Colonna, F. Matera, arXiv:2405.02157, accepter for PRC-Letter]

# Linearized Vlasov equations for NM+deuterons

- **Linear** response to **collision-less** Boltzmann ⇒ linearized **Vlasov** equations for NMd

$$\partial_t\left(\delta f_j\right)+\nabla_{\mathbf{r}}(\delta f_j)\cdot\nabla_{\mathbf{p}}\varepsilon_j-\nabla_{\mathbf{p}}f_j\cdot\nabla_{\mathbf{r}}(\delta\varepsilon_j)=0 \quad\Rightarrow\quad \delta\rho_j=-\chi_j\sum_l\left(F_0^{jl}+\tilde{F}_\lambda^{jl}\right)\delta\rho_l-\delta_{jd}\sum_l\Phi_\lambda^{dl}\delta\rho_l$$

- **Single-particle** energy $\varepsilon_j\equiv\frac{\delta\mathcal{E}}{\delta f_j(\mathbf{p})}$ (from **EDF** $\mathcal{E}=\mathcal{K}+\mathcal{U}$)

$$\varepsilon_j=\frac{p^2}{2m_j}+U_j+\tilde{\varepsilon}_j^\lambda \qquad \left(\tilde{\varepsilon}_j^\lambda\propto\Phi_\lambda^{dj}\sim\frac{\partial\Lambda_d}{\partial\rho_j}\right)$$

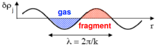
- Momentum-independent **Skyrme**-like interaction (= for bound and free nucleons)

$$\mathcal{U}=\frac{A}{2}\frac{\rho_b^2}{\rho_0}+\frac{B}{\alpha+2}\frac{\rho_b^{\alpha+2}}{\rho_0^{\alpha+1}}+\frac{C(\rho)}{2}\frac{\rho_3^2}{\rho_0}+\frac{D}{2}(\nabla_{\mathbf{r}}\rho_b)^2-\frac{D_3}{2}(\nabla_{\mathbf{r}}\rho_3)^2$$

- **Density-dependent** (Mott) momentum **cut-off** ⇒ extra-terms in both $\delta\rho_j$ and $\varepsilon_j$

$$\rho_j=g_j\int_{|\mathbf{p}|>\Lambda_j}\frac{d\mathbf{p}}{(2\pi\hbar)^3}f_j \quad j=n,p,d \quad\rightarrow\quad \delta\rho_j(\mathbf{r},t)=g_j\int_{|\mathbf{p}|>\Lambda_j}\frac{d\mathbf{p}}{(2\pi\hbar)^3}\delta f_j-\delta_{jd}\sum_{l=n,p,d}\Phi_\lambda^{dl}\delta\rho_l$$

- $\Phi_\lambda^{dl}\neq 0$ ⇒ adding **in-medium** effects for cluster appearance/dissolution in **dynamics**
- **Landau** procedure $\left(F_0^{jl}\sim\frac{\partial U_j}{\partial\rho_l},\tilde{F}_\lambda^{jl}\sim\frac{\partial\tilde{\varepsilon}_j^\lambda}{\partial\rho_l}\right)$ for $\delta f_j\sim\sum_{\mathbf{k}}\delta f_j^{\mathbf{k}}\,e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$



# Dispersion relation and spinodal instability region

- **Solving** linearized Vlasov equations ⇒ dispersion relation $\omega = \omega(k)$

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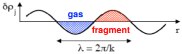
- $\omega = \text{Im}(\omega) \Leftrightarrow$ unstable mode (spinodal region)

[R. Wang, S. Burrello, M. Colonna, F. Matera, arXiv:2405.02157]



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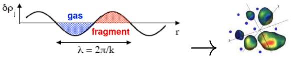

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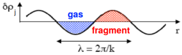


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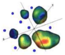
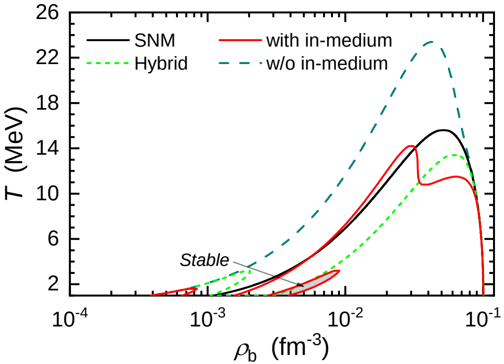


- $\omega = 0$ (**Lindhard** functions $\chi_j = 1$) $\Rightarrow$ **border**




**Legend**

| | |
|---|---|
| red solid line | full |
| teal dashed line | $\Phi_\lambda^{dl} = \tilde{F}_\lambda^{jl} = 0$ |

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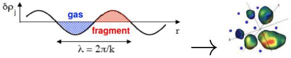


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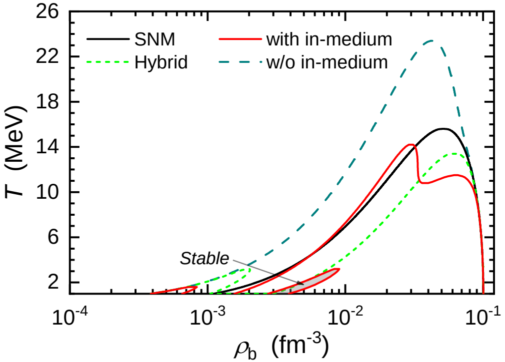


**Legend**

- full (solid red)
- $\Phi_\lambda^{dl} = \tilde{F}_\lambda^{jl} = 0$ (dashed)

**In-medium effects in dynamics**

- Dawn of meta-stable region

[G. Röpke et al, NPA 970, 224 (2018)]

- Slowdown of instability rate

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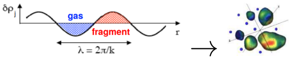
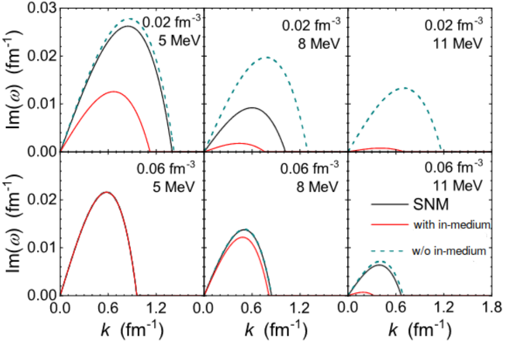



- $\mathrm{Im}(\omega) \Rightarrow$ **growth rate** of density fluctuations

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- **Direction of instability** in space of density fluctuations: $\frac{\delta\rho_S}{\delta\rho_d}$ $(\rho_S = \rho_n + \rho_p)$
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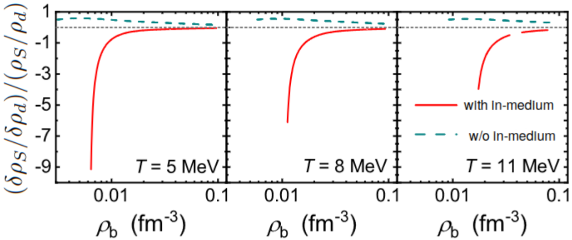


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- **NMd with no in-medium effects:**
  - **Favored growth** of instabilities
  - **Cooperation** to form fragments
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  - Deuterons move to low densities
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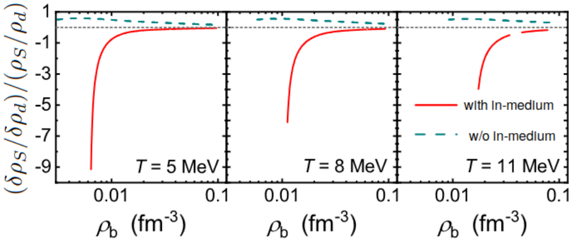


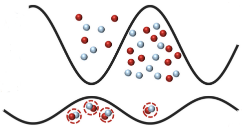

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- **Scaling** factor for **deuteron** coupling strenght in $\mathcal{U}(\rho)$ (with $\rho = \sum_j A_j \eta_j \rho_j$)
  - $\eta_d = 1 \Rightarrow$ nucleons **bound** in deuterons feel the same potential as free nucleons
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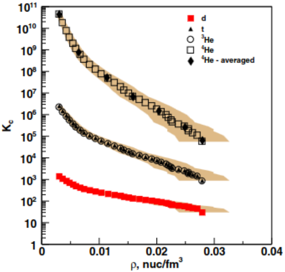
  [L. Qin et al., PRL 108, 172701 (2012); R. Bougault et al., J. Phys. G 47, 025103 (2020)]

- **Alternative** framework for spinodal instability $\Rightarrow$ **Hydrodynamical** approach
  $\Rightarrow$ hydrodynamics vs linearized Vlasov with **density-dependent cut-off**

  [S. Burrello, M. Colonna, R. Wang, in preparation]



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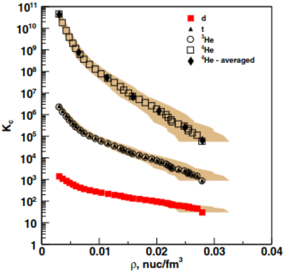

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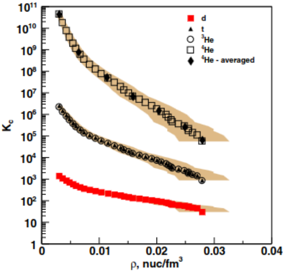
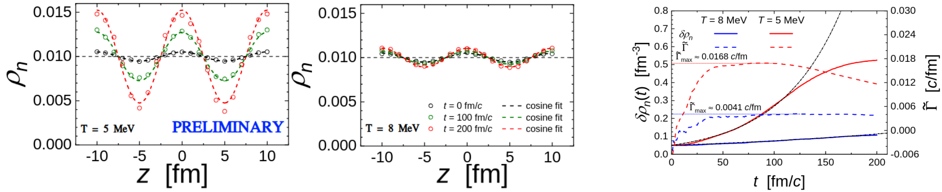



**Work in progress**

- **Extensive** calculations (other light clusters, ANM)
  - Different parameterizations for interaction & **cut-off**
- **Consistent** description of HIC **fragmentation** mechanisms
  - Beyond quasi-analytical $\Rightarrow$ numerical calculations

# Numerical Vlasov solution: preliminary results




- Density **fluctuations** $\delta\rho_n(t)$ (**exponentially**) increase $\Rightarrow$ spinodal instability
- No constant growth factor $\tilde{\Gamma}(t) = \ln\left[\frac{\delta\rho_n(t)}{\delta\rho_n(t=0)}\right]/t \Rightarrow$ not **uniform** system
- Test-particle method $\Rightarrow$ triangular profiles
  - S-finite size reduces growth rate

$$S(\mathbf{r}_i - \mathbf{r}) = \frac{1}{(nl/2)^6} g(\Delta x)\, g(\Delta y)\, g(\Delta z)$$

$$g(q) = \left(\frac{nl}{2} - |q|\right)\theta\left(\frac{nl}{2} - |q|\right)$$

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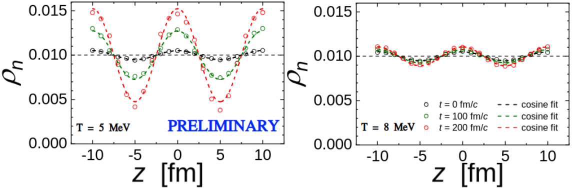


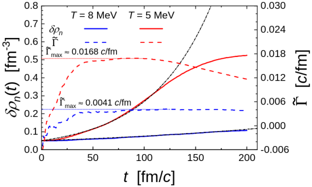


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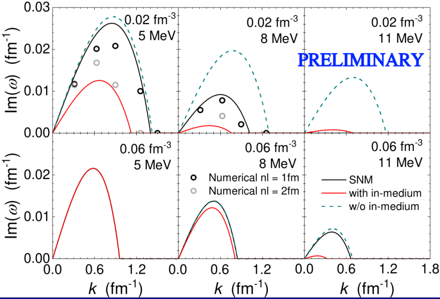

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# Final remarks and conclusions

## Main topic

- Description of **correlations** & clustering with phenomenological **EDF** models
- **Dynamics** of dilute NM with light **clusters** DOF and local **in-medium** effects

## Main results

- Unified **mass-shift** parametrization for deuterons & **SRCs** and impact on EOS
- Role of clusters on SNM **spinodal** instability and fragmentation dynamics
- Impact of in-medium effects on **growth rates** and distillation mechanism

## Further developments and outlooks

- Screening effects for **bound** nucleons and connection with **hydrodynamics**
- Extension to ANM with other light clusters and **effective interaction**
- Numerical calculations & **consistent** description of **HIC** fragment formation

# Final remarks and conclusions

## Main topic

- Description of **correlations** & clustering with phenomenological **EDF** models
- **Dynamics** of dilute NM with light **clusters** DOF and local **in-medium** effects

## Main results

- Unified **mass-shift** parametrization for deuterons & **SRCs** and impact on EOS
- Role of clusters on SNM **spinodal** instability and fragmentation dynamics
- Impact of in-medium effects on **growth rates** and distillation mechanism

## Further developments and outlooks

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THANK YOU FOR YOUR ATTENTION!