

Strangeness in equation of state studies at high density

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- Introduction
- π^- / π^+ : S π RIT
- K^0 / K^+ : FOPI
- Σ^- / Σ^+ : ?
- Ξ^- / Ξ^0 : ?
- Threshold effect
- Summary

Supported by US Department of Energy

First experiment on strangeness production in HIC

Production of K^+ Mesons in 2.1-GeV/Nucleon Nuclear Collisions

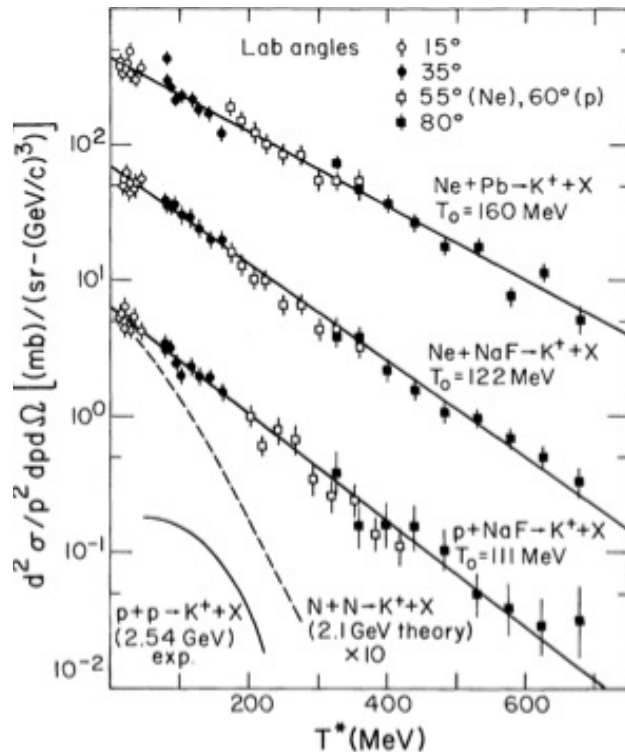
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(Received 9 June 1982)

PRL 49, 989 (1982)

K^+ meson production by 2.1-GeV/nucleon Ne, d , and p projectiles on NaF and Pb targets has been measured. The cross sections depend exponentially upon the kaon energy in the nucleon-nucleon c.m. frame, with an inverse slope T_0 larger than the values obtained from comparable proton and π^- spectra. The angular distribution in this frame is approximately isotropic. We find that $\sigma(\text{Ne} + \text{Pb} \rightarrow K^+ X) / \sigma(\text{Ne} + \text{NaF} \rightarrow K^+ X) > \sigma(d + \text{Pb} \rightarrow K^+ X) / \sigma(d + \text{NaF} \rightarrow K^+ X)$. Data are compared with theoretical predictions.



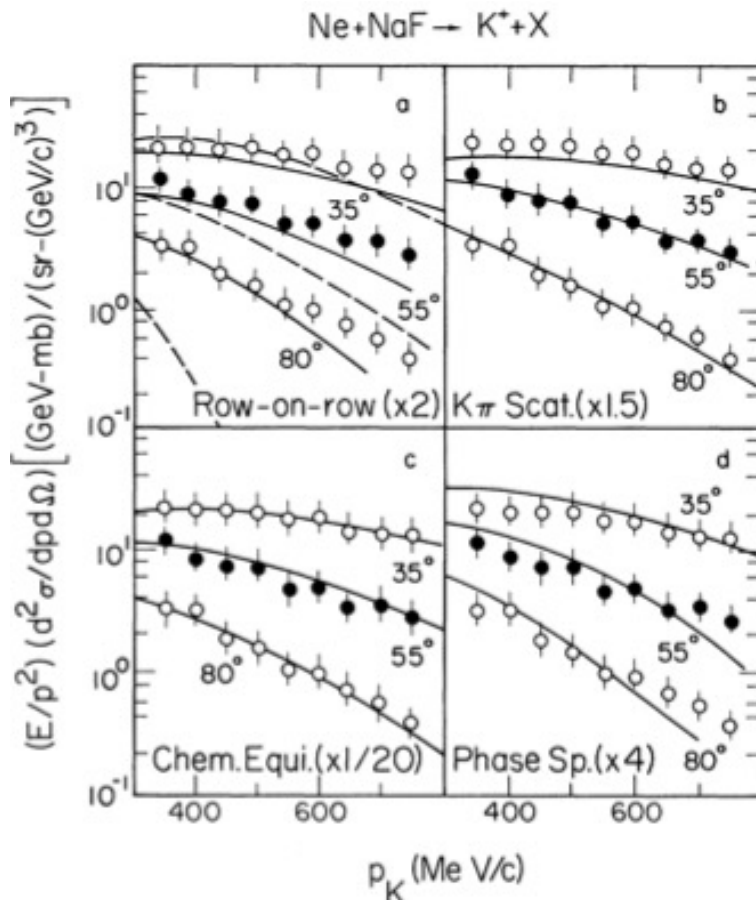
Recently the study of strange-particle (K^+ , Λ , K^-) production¹⁻³ in collisions of relativistic heavy ions has begun. The K^+ mesons are of particular interest since they have an extremely small cross section for absorption and, at low energies, a small cross section, ≈ 13 mb, for scattering on a nucleon. Thus, they may be relatively undistorted by thermalization or multiple scatterings and may, therefore, be more reliable messengers of the early, perhaps very compressed and hot stage of the nuclear collision.

T^* = K^+ kinetic energy in NN center of mass

First theoretical studies on strangeness production

J. Randrup and CMK, NPA 343, 519 (1980)

Abstract: Kaon production in relativistic nuclear collisions is studied on the basis of a conventional multiple-collision model. The input is the differential cross sections for kaon production in elementary baryon-baryon collisions, estimated in a simple model. Inclusive kaon spectra are calculated at 2.1 GeV/nucleon for a number of experimental cases. The calculated kaon yield is approximately isotropic in the mid-rapidity frame and extends considerably beyond the nucleon-nucleon kinematical limit.



- a) Randrup, PLB 99, 9 (1981)
- b) Ko, PRC 23, 2760 (1981)
- c) Asai, Sato & Sano, PLB 98, 19 (1981)
- d) Asai, NPA 365, 519 (1981)

Importance of multiple scattering, strangeness conservation (canonical suppression), and final-state interactions

Nuclear equation of state

Li, Chen & Ko, Phys. Rep. 464, 113 (2008)

EOS of asymmetric nuclear matter

$$E(\rho, \delta) \approx E(\rho, \delta = 0) + E_{\text{sym}}(\rho)\delta^2, \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

Symmetry energy

$$E_{\text{sym}}(\rho) = E_{\text{sym}}(\rho_0) + L \left(\frac{\rho - \rho_0}{3\rho_0} \right) + \frac{K_{\text{sym}}}{2} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 + \frac{J_{\text{sym}}}{3} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^3 \dots$$

Symmetry energy coefficient $E_{\text{sym}}(\rho_0) \approx 30 \text{ MeV}$ from mass formula

Slope $L = 3\rho_0 \left. \frac{\partial E_{\text{sym}}(\rho)}{\partial \rho} \right|_{\rho=\rho_0}$ theoretical values -50 to 200 MeV

Curvature $K_{\text{sym}} = 9\rho_0^2 \left. \frac{\partial^2 E_{\text{sym}}(\rho)}{\partial^2 \rho} \right|_{\rho=\rho_0}$ theoretical values -700 to 466 MeV

Nuclear matter Incompressibility $K(\delta) = K_0 + K_{\text{asy}}\delta^2$, $K_{\text{asy}} = K_{\text{sym}} - 6L$

Empirically, $K_0 \sim 230 \pm 10 \text{ MeV}$, $K_{\text{asy}} \sim -500 \pm 50 \text{ MeV}$, $L \sim 88 \pm 50 \text{ MeV}$

$$E_{\text{sym}}(\rho) \sim 32 (\rho/\rho_0)^\gamma \text{ with } 0.7 < \gamma < 1.1 \text{ for } \rho < 1.2\rho_0$$

▪ Symmetry energy at high densities is practically undetermined !

Microscopic Theory of Pion Production and Sideways Flow in Heavy-Ion Collisions

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(Received 9 July 1984)

Nuclear collisions from 0.3 to 2 GeV/nucleon are studied in a microscopic theory based on Vlasov's self-consistent mean field and Uehling-Uhlenbeck's two-body collision term which respects the Pauli principle. The theory explains simultaneously the observed collective flow and the pion multiplicity and gives their dependence on the nuclear equation of state.

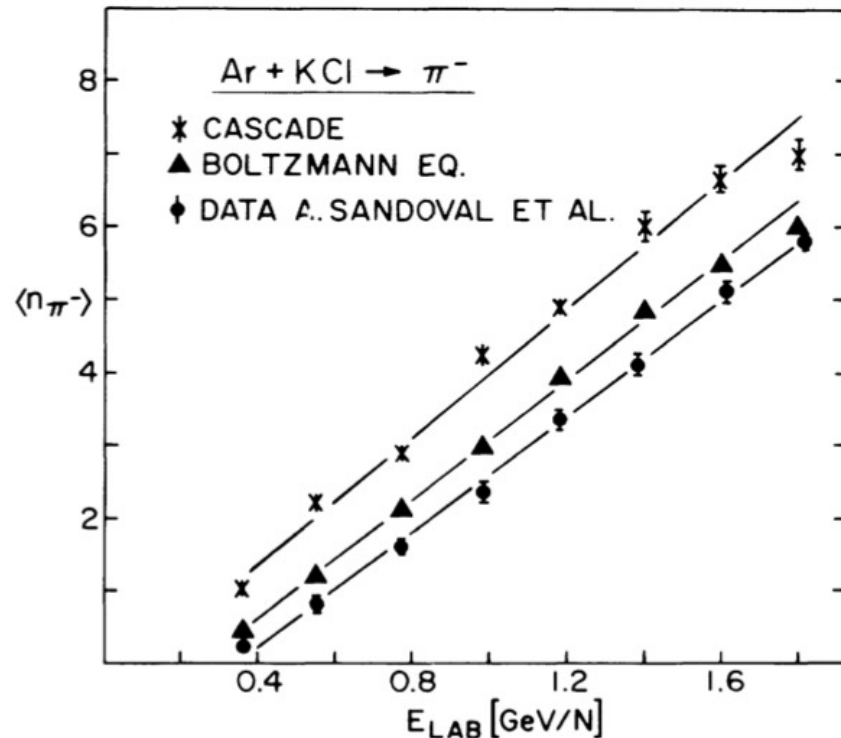
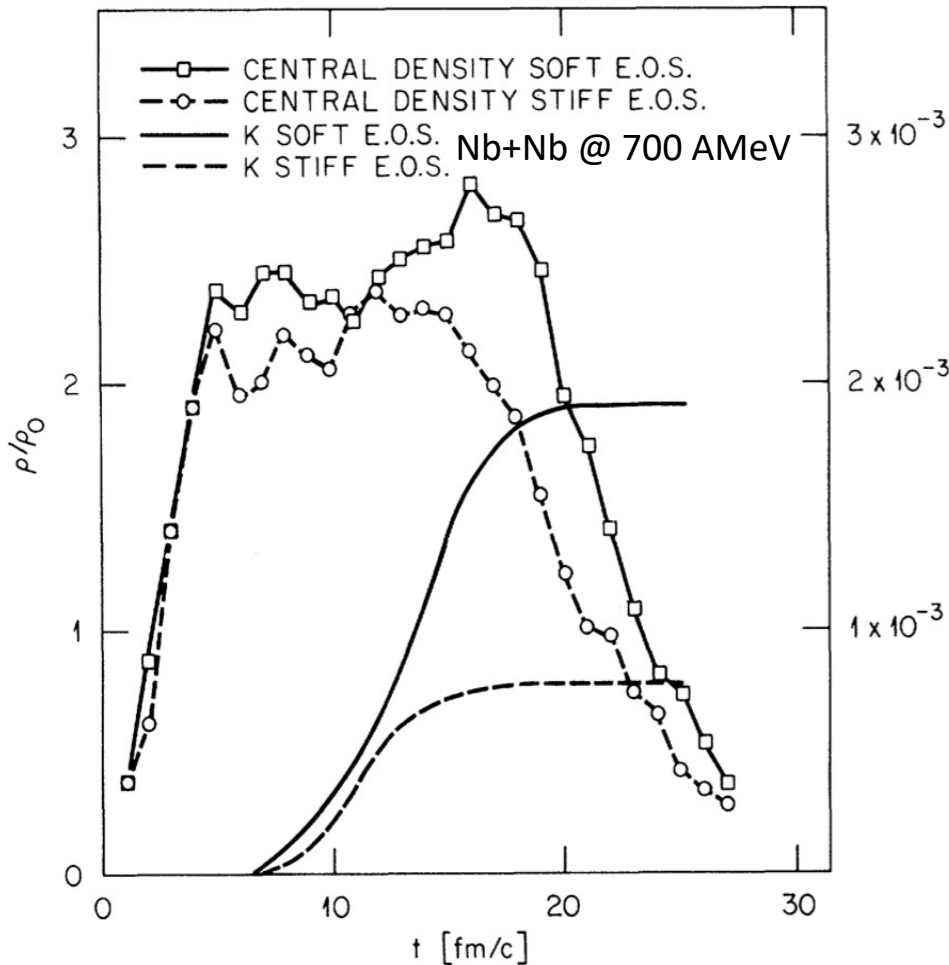


FIG. 2. Pion multiplicity for central collisions ($b < 2.4$ fm) of Ar+KCl. The data (Ref. 6, circles) are compared to the present theory in the "cascade mode" (crosses) and to the same theory with compression energy and phase-space Pauli blocking included (triangles).

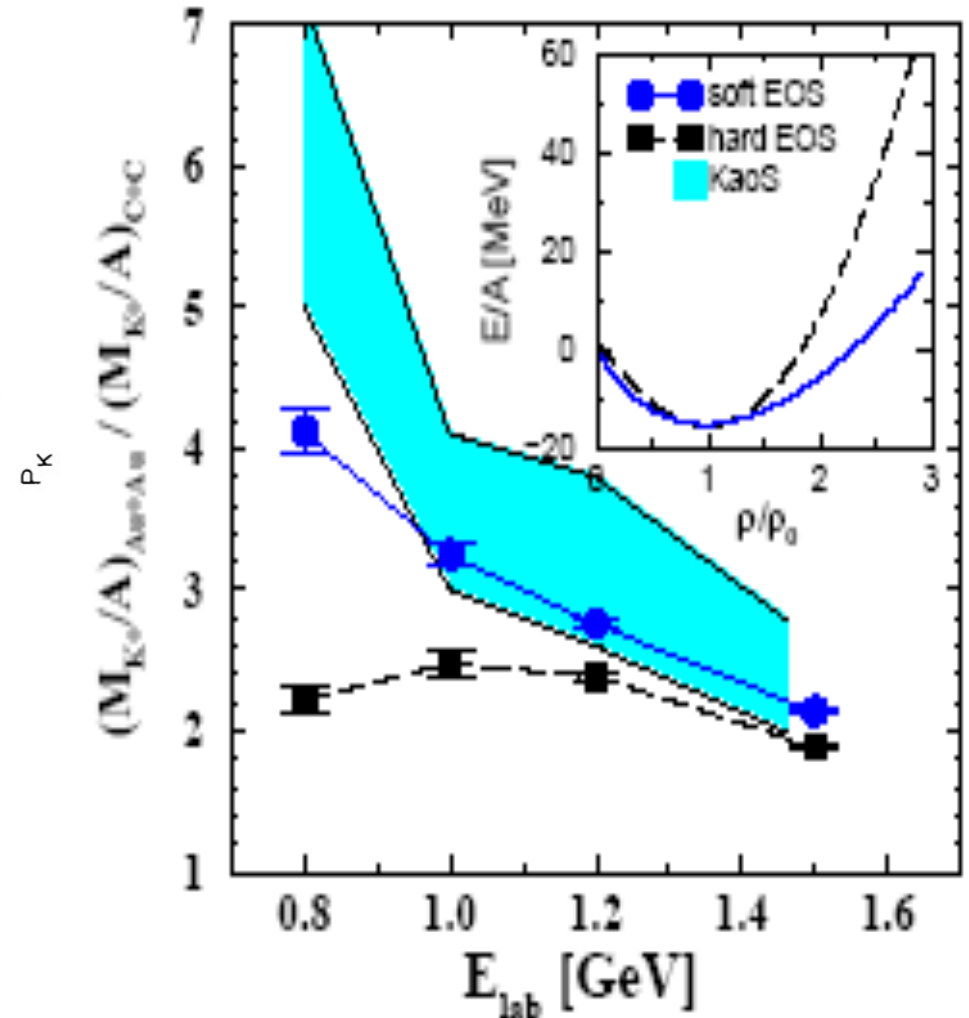
- Although difference between cascade and BUU with mean fields can be 40%, difference between stiff and soft EOS is about 15%.
- No definitive conclusion on stiffness of nuclear EOS from pion yield because the large experimental uncertainty of also 15%.

Subthreshold kaon production in high-energy HIC

Aichelin & Ko, PRL 55, 2661 (1985)



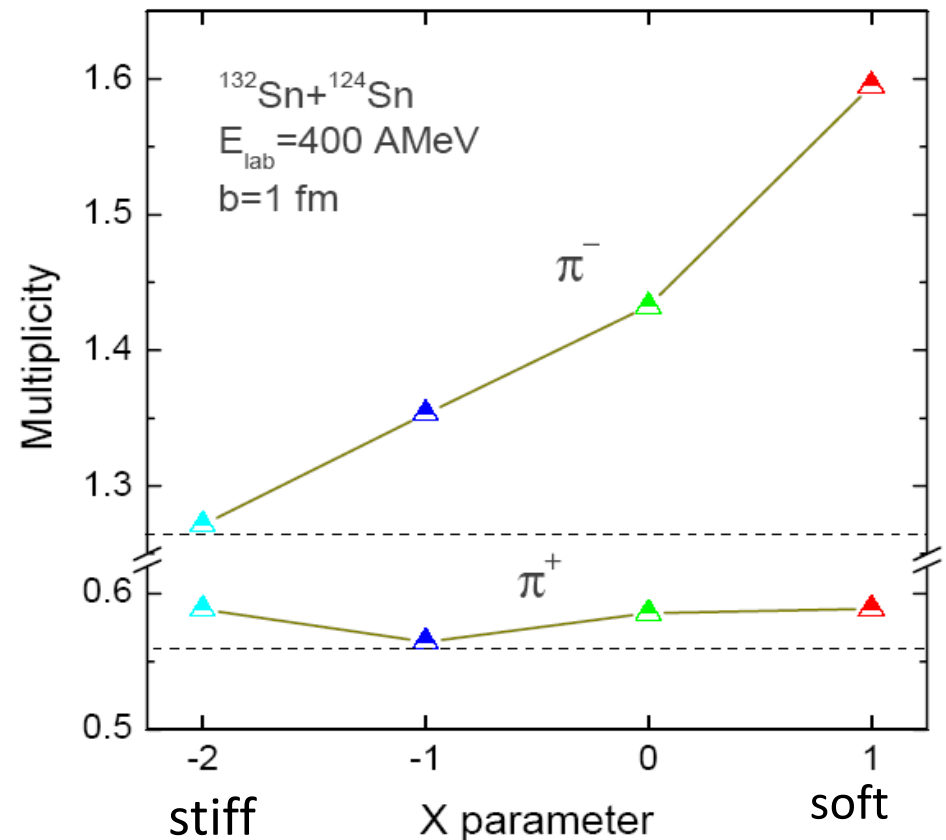
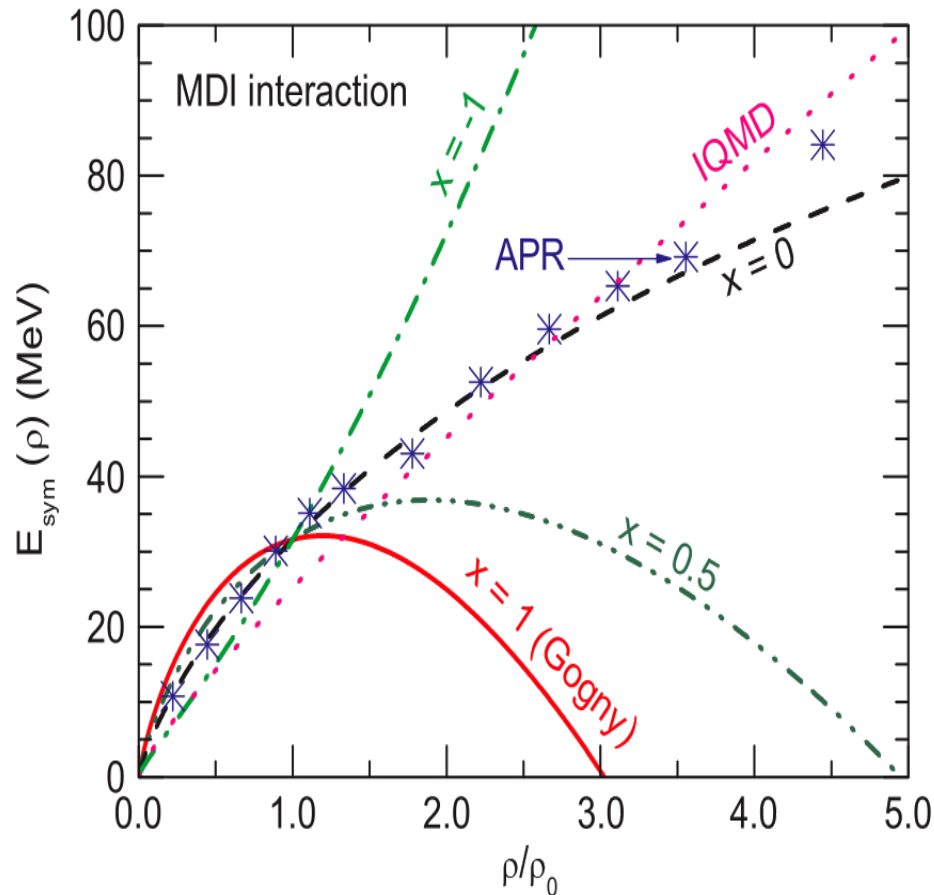
Fuchs, PRL 86, 1974 (2001)



- Kaon production at subthreshold energy in HI collisions, dominated by $NN(\Delta) \rightarrow N\Lambda K$ and $\pi N(\Delta) \rightarrow \Lambda K$, is sensitive to nuclear EOS, and data are consistent with a soft one and that extracted from nucleon flows by Danielewicz, Lacey, and Lynch [Science 298, 1592 (2002)]

Near-threshold pion production with high energy radioactive beams (IBUU)

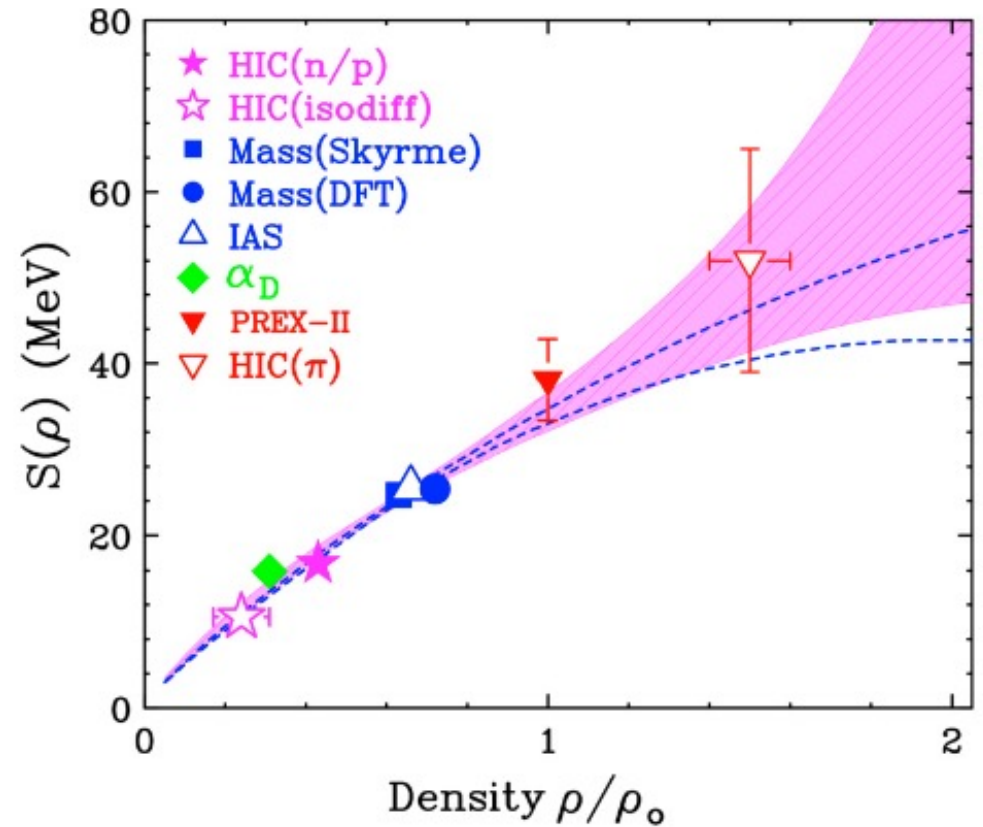
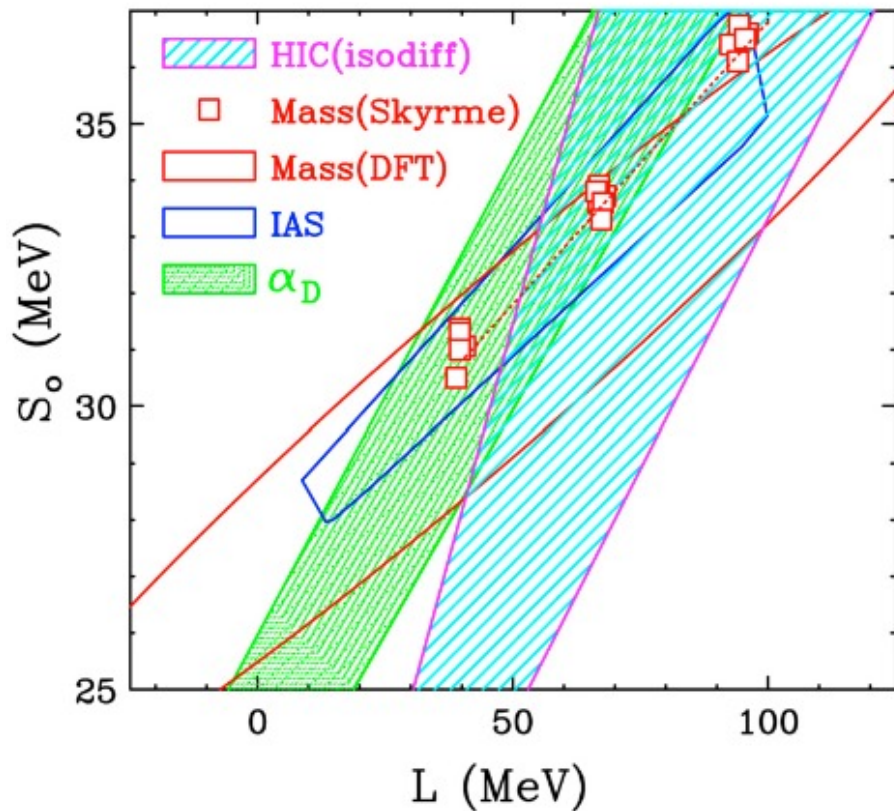
B. A. Li, PRL 88, 192701 (2002)



π^- yield is sensitive to the symmetry energy $E_{\text{sym}}(\rho)$ since they are mostly produced in the neutron-rich region, with softer one giving more π^- than stiffer one. Difference between π^-/π^+ from super soft ($x = 1$) and super stiff ($x = -1$) $E_{\text{sym}}(\rho)$ is, however, only about 30%, which makes it very challenging to determine $E_{\text{sym}}(\rho)$ from data using transport models.

Density dependence of nuclear symmetry energy

W. G. Lynch and M. B Tsang, PLB 830, 137098 (2022)

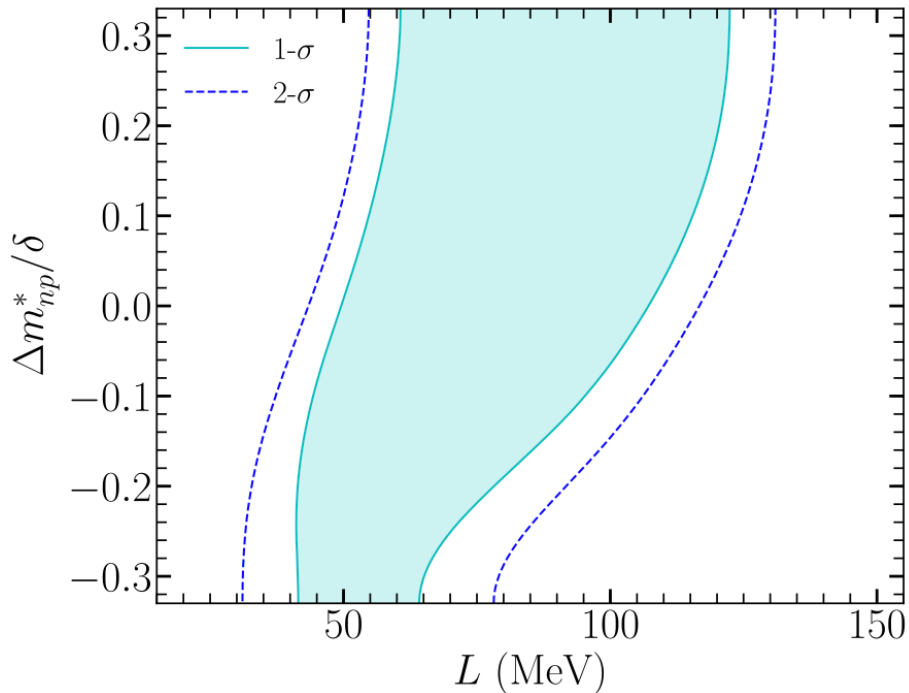


Instead of constraining the symmetry energy S_0 and its slope parameter L at nuclear matter saturation density (left figure), each measured observable should be used to determine the symmetry energy at the density it is most sensitive (right figure).

Probing the Symmetry Energy with the Spectral Pion Ratio

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(S π RIT Collaboration)



- $NN \leftrightarrow N\Delta$ and $\Delta \leftrightarrow N\pi$ are dominant reactions for pion production in Sn+Sn at 270A MeV.
- $\Delta m_{np}^*/\delta$ and L correlations extracted from the single spectral ratio of $^{132}\text{Sn}+^{124}\text{Sn}$ and $^{108}\text{Sn}+^{112}\text{Sn}$ reactions using dcQMD (Dan Cozma). The green shaded region lies 68% confidence level for data with $p_T > 200$ MeV/c. The dotted blue lines denote contours corresponding to the 95% confidence level.

$$\rightarrow S_0 = 35.3 \pm 2.8 \text{ MeV}, L = 79.9 \pm 37.6 \text{ MeV.}$$

Pion production in $^{132}\text{Sn} + ^{124}\text{Sn}$ and $^{112}\text{Sn} + ^{108}\text{Sn}$ at 270A MeV and $b = 3$ fm

G. Jhang et al. [$S\pi$ RIT & TMEP], PLB 813, 136016 (2021); H. Wolter et al. [TMEP], PPNP 125, 103962 (2022)

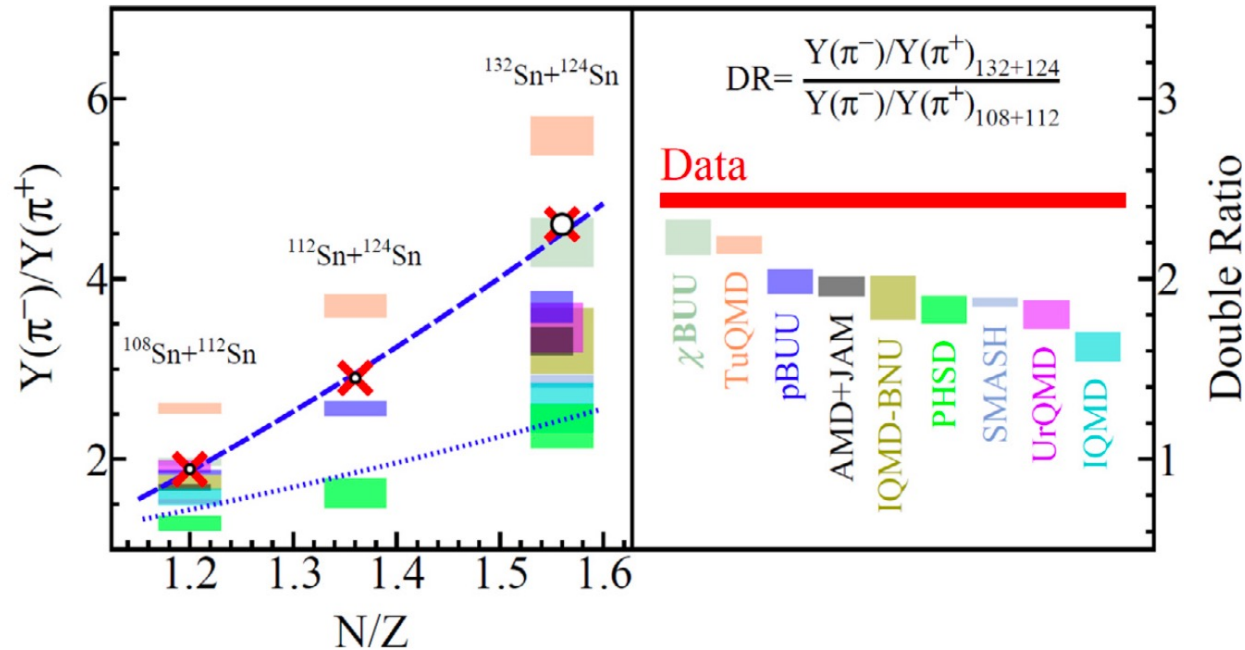


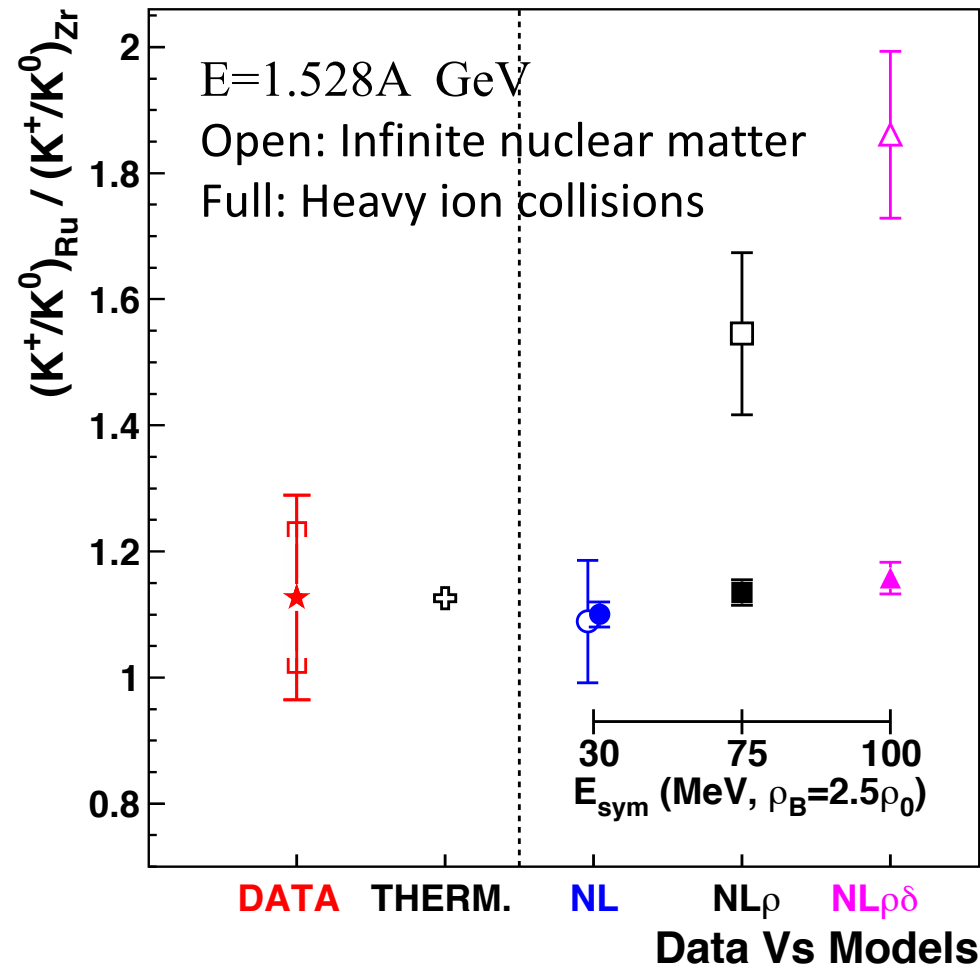
Fig. 13. (Left panel) Charged pion yield ratios as a function of N/Z . The experimental data are shown as crosses with the circles representing the experimental errors. The results of the calculations are represented by colored boxes for the different codes identified by their color in the right panel. The upper and lower boundaries of the boxes give the result for the soft and stiff symmetry energy choices for each code, i.e., the height of the boxes is representative for the sensitivity to the stiffness of the symmetry energy. The dashed blue line is a power-law fit with the function $(N/Z)^{3.6}$, while the dotted blue line represents $(N/Z)^2$ of the system. (Right panel) Double pion yield ratios for $^{132}\text{Sn} + ^{124}\text{Sn}$ and $^{108}\text{Sn} + ^{112}\text{Sn}$. The data and the uncertainty are given by the red horizontal bar, while the results of the transport models are shown by the colored boxes, in a similar way as in the left panel.

Although no transport models can perfectly describe the data yet, the symmetry energy effect in some models is larger than the experimental errors.

Symmetry energy effect on K^+/K^0 ratio

Lopez et al. (FOPI Collaboration) , Phys. Rev. C75, 011901(R) (2007)

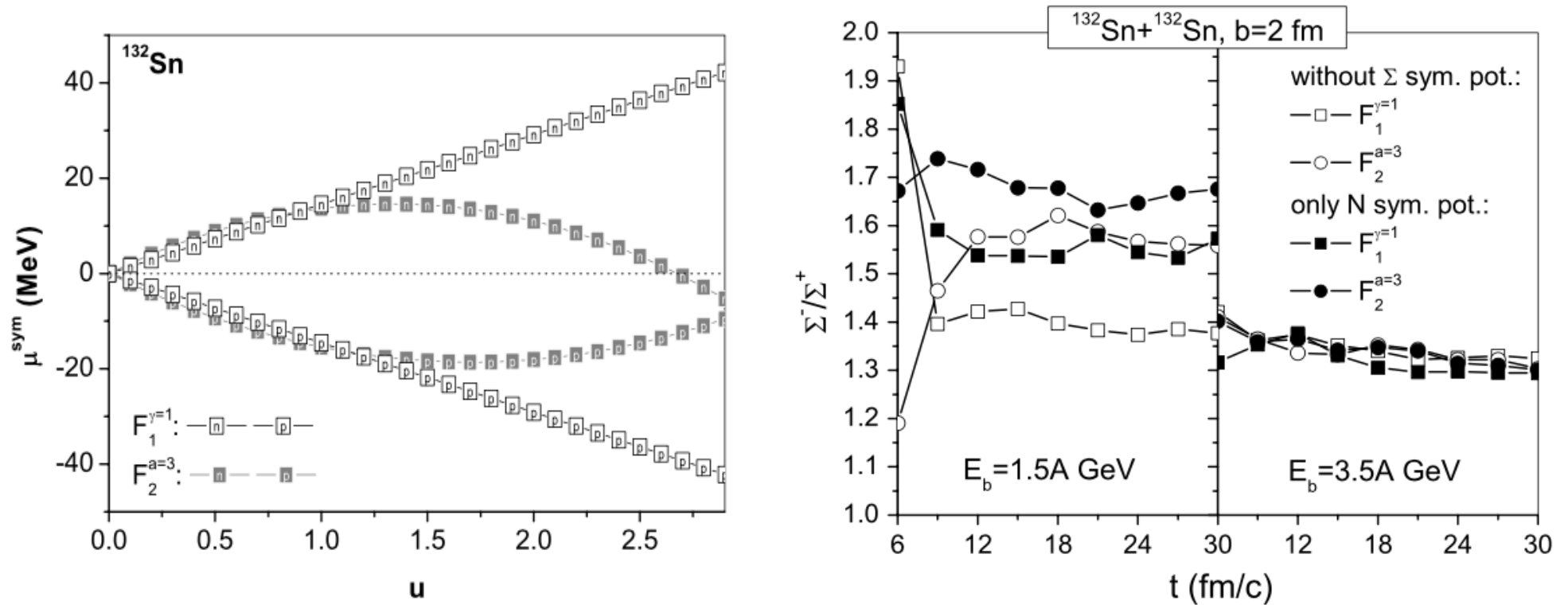
G. Ferini, M. Colonna, T. Gaitanos, & M. Di Toro, NPA 762, 147 (2005)



- Symmetry energy effect negligible in collisions at 1.528A GeV, which is only slightly below the threshold energy of ~ 1.60 GeV, and is expected to be larger at deep subthreshold energy.

Symmetry energy effect on Σ^-/Σ^+ ratio

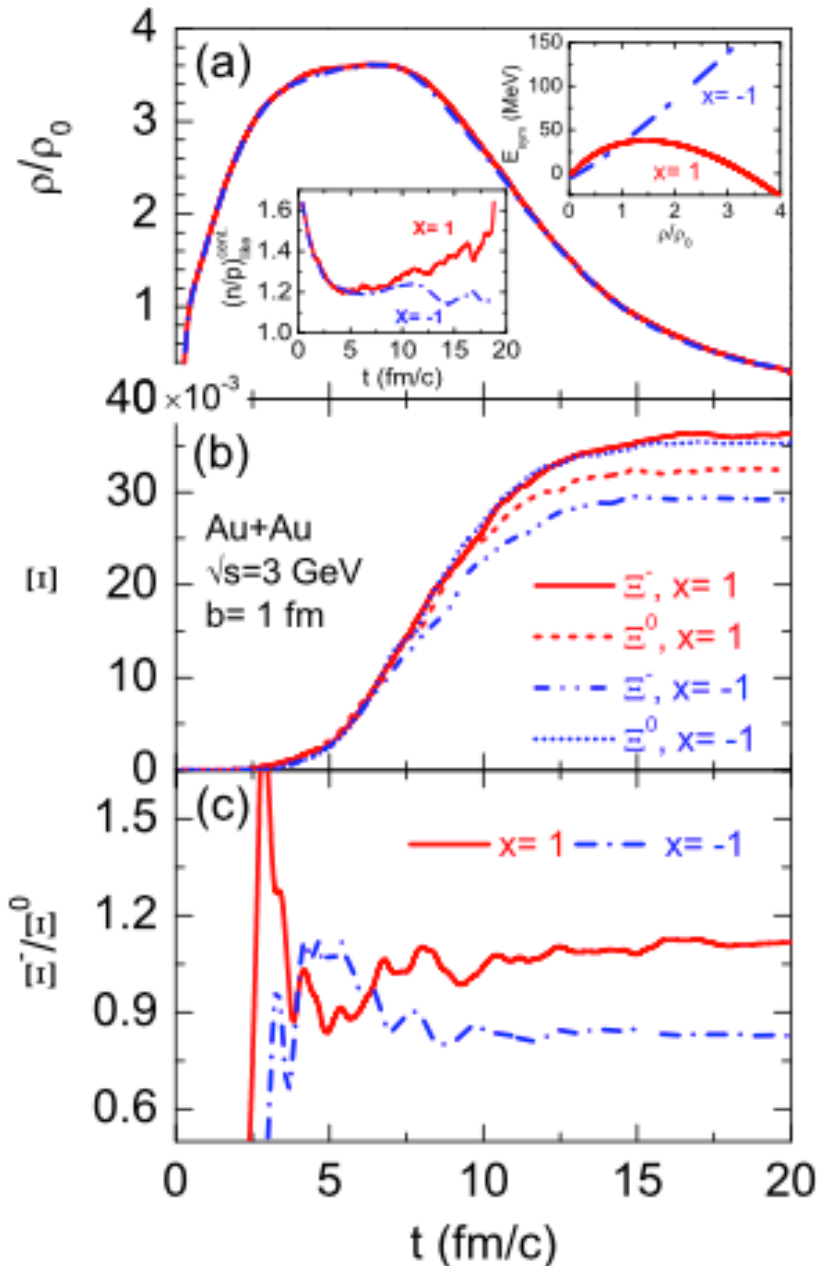
Li, Li, Zhao & Gupta, PRC 71, 054907 (2005)



- Based on extended UrQMD with symmetry potentials and $BB \rightarrow \Sigma X, MB \rightarrow \Sigma X, B^* \rightarrow \Sigma X, \Sigma M \rightarrow B^*$ reactions.
- Symmetry energy effect is seen at 1.5A GeV, which is slightly below the threshold energy of ~ 1.79 GeV, with the soft one giving a Σ^-/Σ^+ about 5% and 14% larger than those from the stiff symmetry energy with and without Σ mean-field potentials, respectively.
- A much smaller symmetry energy effect on Σ^-/Σ^+ is, however, seen in Au+Au at 1.5A GeV based on LQMD [Ding-Chang Zhang et al., CPL 38, 092501 (2021)].

Symmetry energy effect on Ξ^-/Ξ^0 ratio

Gao-Chan Yong, Bao-An Li, Zhi-Gang Xiao, and Zi-Wei Lin, PRC 106, 024902 (2022)



- Based on AMPT with the inclusion of mean-field potentials for baryons according to the quark number counting rule.
- A supersoft symmetry energy ($x=1$) gives about 24% larger Ξ^-/Ξ^0 than a superhard symmetry energy ($x=-1$) in Au+Au collisions at $\sqrt{s} = 3$ GeV, which is below the threshold energy of 3.25 GeV.
- The symmetry energy effect is expected to be enhanced in collisions of more neutron-rich nuclei.
- However, it is not known if the present model can describe the HAHES data on Ξ^- from Ar+KCl at 1.76A GeV or $\sqrt{s} = 2.61$ GeV because hadron yields at the initial stage of hadronic evolution in AMPT are determined from fragmentation of strings produced in initial nucleon-nucleon collisions and the important $YY \leftrightarrow N\Xi$ reactions [Li, Chen, Ko & Lee, PRC 85, 064902 (2012)] are not included in the present model.

In-medium threshold effects on pion production

$$U_{asy}^{\Delta^{++}} = U_{asy}^p, \quad U_{asy}^{\Delta^+} = \frac{2}{3}U_{asy}^p + \frac{1}{3}U_{asy}^n, \quad U_{asy}^{\Delta^0} = \frac{1}{3}U_{asy}^p + \frac{2}{3}U_{asy}^n, \quad U_{asy}^{\Delta^-} = U_{asy}^n$$

- $pn \rightarrow p\Delta^0$

Initial-state potential: $U_p + U_n$

Final-state potential: $U_p + U_{\Delta^0} = U_p + U_p/3 + 2/3U_n$

→ difference in initial and final potentials:

$(U_n - U_p)/3 > 0$ in neutron-rich matter

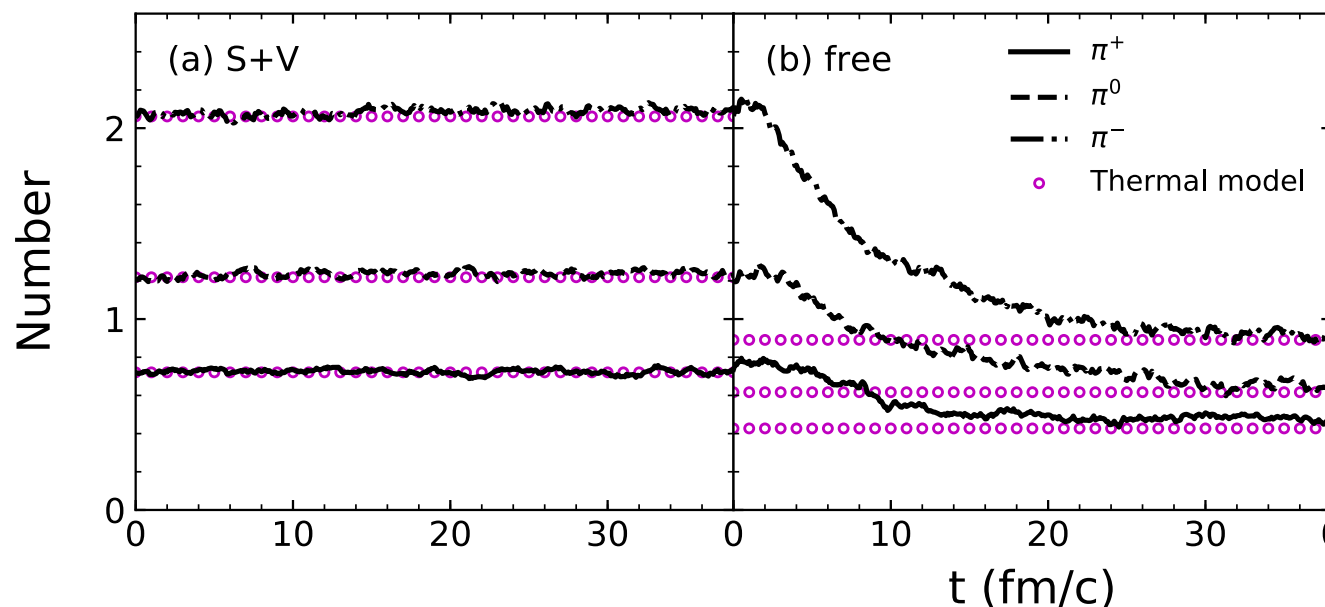
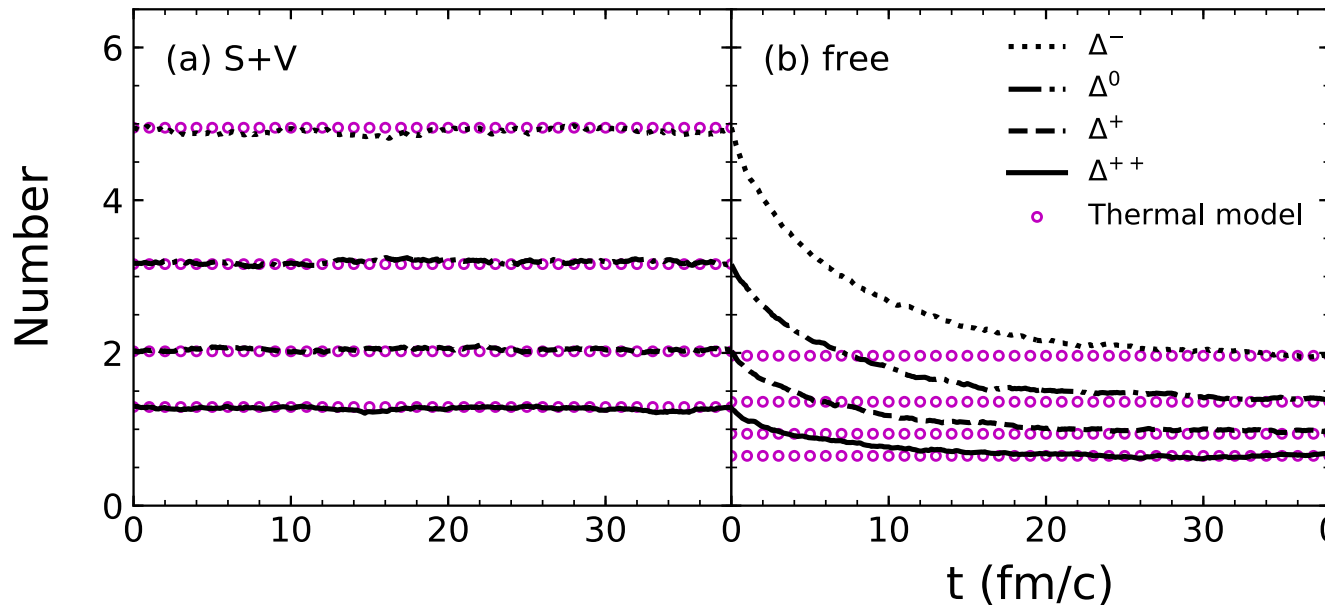
→ reduced production threshold

- First studied by Ferini, Colonna, Gaitanos and Di Toro (NPA 762, 147 (2005)) in a relativistic transport model

Effects of energy conservation on chemical equilibrium in hot dense symmetric nuclear matter (RVUU with $NL\rho$)

Zhang & Ko, PRC 97,
014910 (2018)

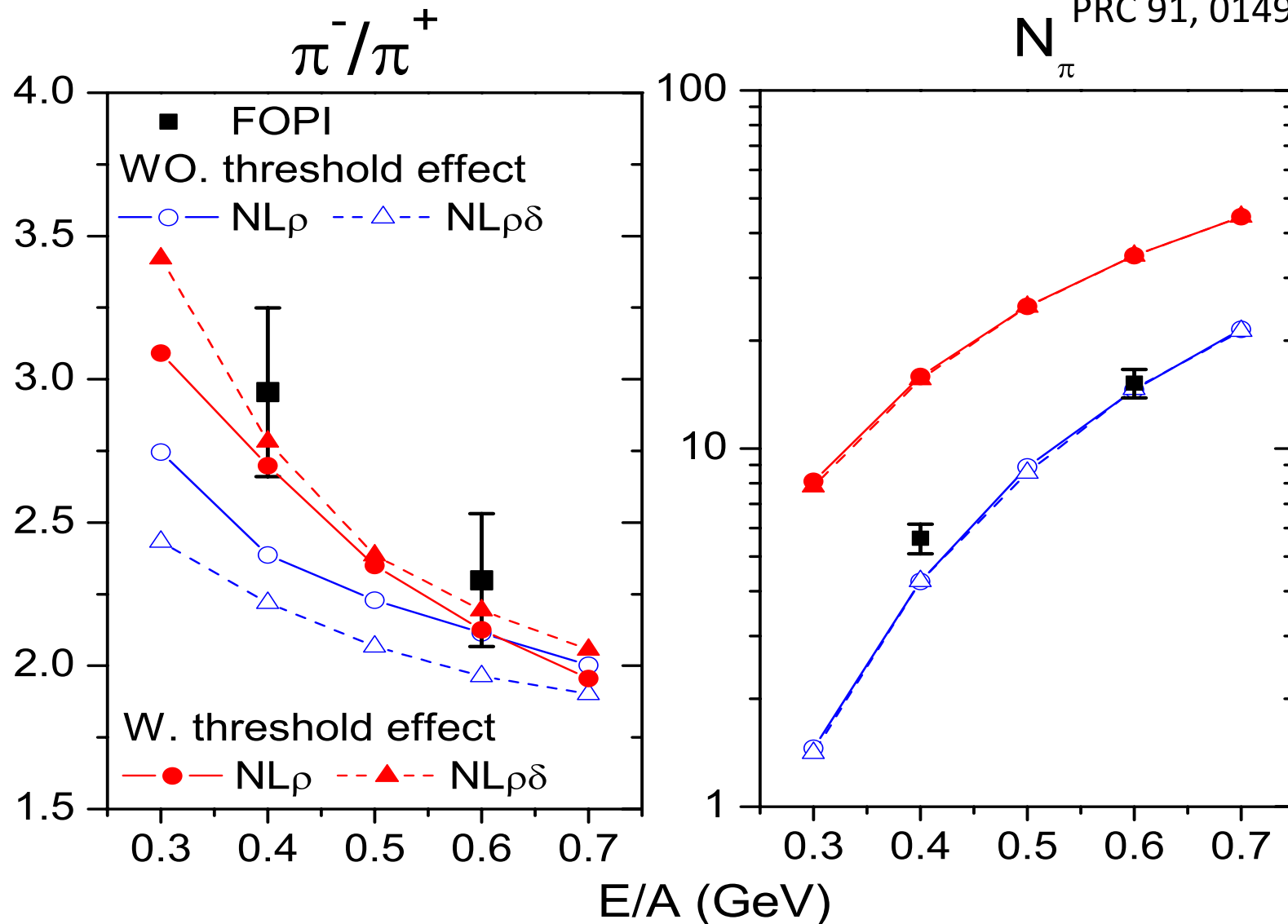
Nucleons, Deltas and pions
in a box at $T= 60$ MeV,
 $\rho = 0.24 \text{ fm}^{-3}$,
 $\rho_I = 0.096 \text{ fm}^{-3}$



- Including potentials in the energy conservation during collisions keeps correct equilibrium distributions.
- Treating collisions as in free space, as done in essentially all transport models, leads to equilibrium distributions without potential effects.

In-medium threshold effects on π^-/π^+ ratio

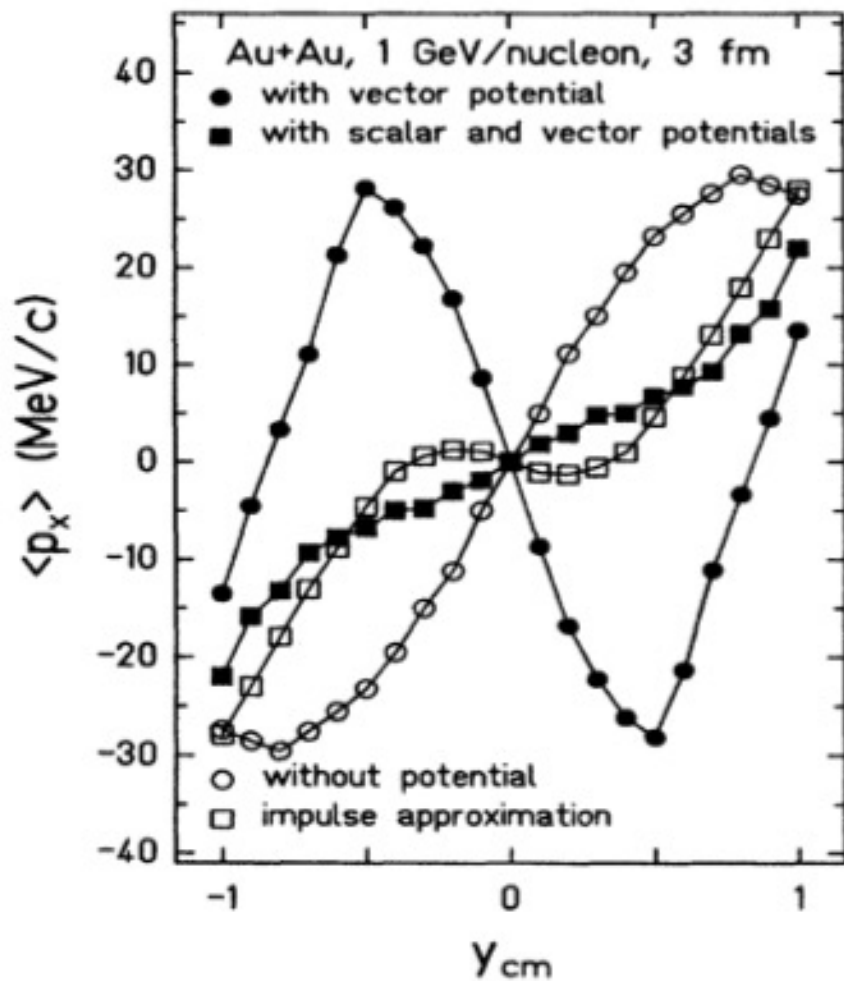
T. Song and CMK,
PRC 91, 014901 (2015)



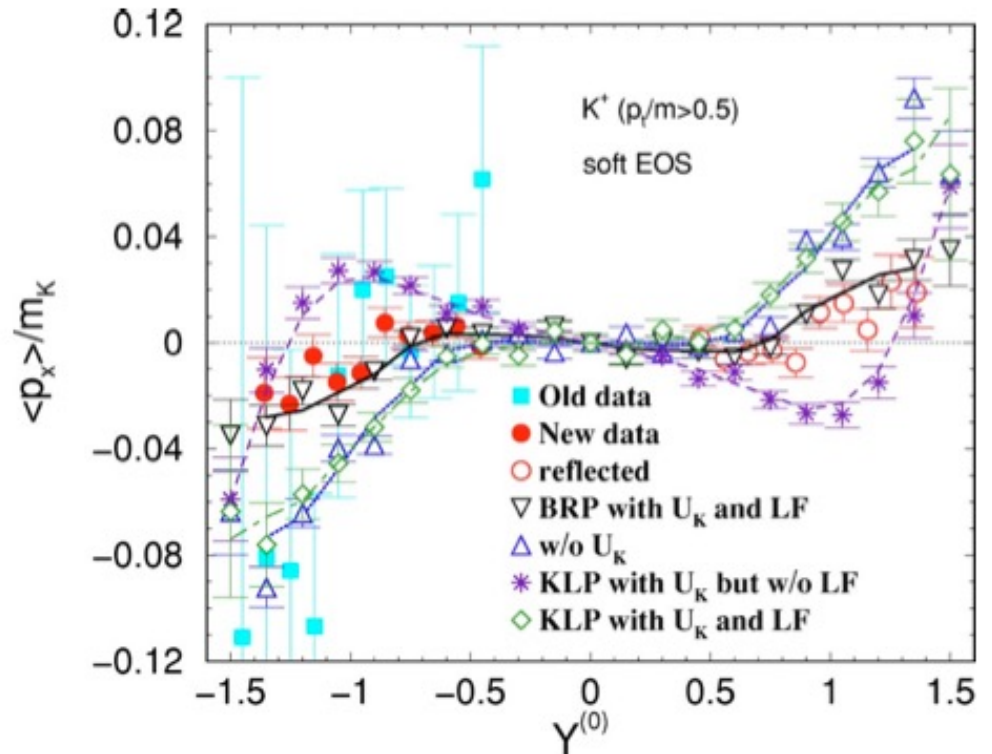
- In-medium threshold effects increase the total pion yield, the π^-/π^+ ratio and reverse the effect of symmetry energy.

Kaon flow as a probe of kaon potential

Li & Ko, PRL 74, 235 (1994)



Zheng, Fuchs et al., PRC 69, 034907 (2004)



- Kaon flow is sensitive to its potential, and a repulsive potential is needed to describe data.

Kaon potentials in nuclear medium

Ko & Li, JPG 22, 1673 (1996);
Ko, Koch & Li, ARNPS 47, 505 (1997)

Chiral effective Lagrangian → repulsive potential for kaons and attractive potential for antikaons

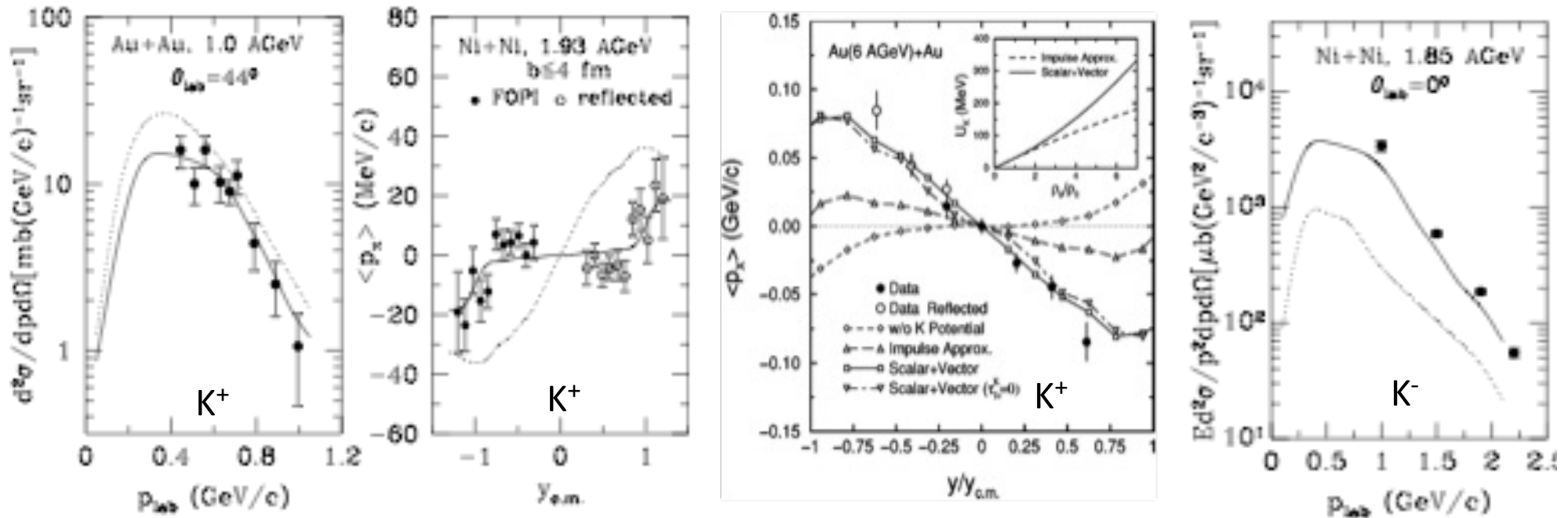
$$U_{K,\bar{K}} = \omega_{K,\bar{K}} - \omega_0, \quad \omega_0 = \sqrt{m_K^2 + p^2}$$

$$\omega_{K,\bar{K}} = \sqrt{m_K^2 + p^2 - a_{K,\bar{K}}\rho_s + (b_K\rho_B)^2} \pm b_K\rho_B$$

$$a_K = 0.22 \text{ GeV}^2 \text{ fm}^3, \quad a_{\bar{K}} = 0.45 \text{ GeV}^2 \text{ fm}^3$$

$$b_K = 0.33 \text{ GeV}^2 \text{ fm}^3$$

$$\Rightarrow U_K = 20 \text{ MeV}, U_{\bar{K}} = -120 \text{ MeV at } \rho_0 = 0.16 \text{ fm}^{-3}$$



- Experimental data on spectrum and directed flow are consistent with repulsive kaon and attractive antikaon potentials → threshold effect on their production.

Summary

- Subthreshold kaon production has been shown to be a good probe to the stiffness of the EOS of normal nuclear matter at $2-3 \rho_0$.
- Heavy ion collisions at various energies allows the possibility to determine the nuclear symmetry energy at various densities, with the π^-/π^+ ratio measured by S π RIT already providing information on the value of $S(\rho)$ at $\rho \approx 1.5 \rho_0$, although with large uncertainty.
- To extend the density reached in HIC, it has been proposed to study the ratio of isospin partners of strange hadrons, such as the K^0/K^+ , Σ^-/Σ^+ and Ξ^-/Σ^0 ratios, in HIC at energy below their respective production threshold, as these particles are produced at densities higher than $1.5 \rho_0$.
- Because of the lack of anti quarks in the considered strange hadrons, they cannot be destroyed in the expanding nuclear matter after their production and are thus cleaner probes of $S(\rho)$ than the π^-/π^+ ratio.
- Threshold effects due to hadron potentials can, however, reverse the effect of nuclear symmetry, which makes it a challenge to extract information on $S(\rho)$.
- Both experimental and theoretical studies are needed to achieve this goal.