

Microscopic determination of the isospin symmetry breaking energy density functional

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Isospin symmetry breaking of nuclear interaction

- Nuclear interaction: *almost* isospin symmetric

$$v_{pp}^{T=1} \simeq v_{pn}^{T=1} \simeq v_{nn}^{T=1}$$

Miller, Opper, and Stephenson. *Annu. Rev. Nucl. Part. Sci.* **56**, 253 (2006)

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$$v_{pp}^{T=1} \simeq v_{pn}^{T=1} \simeq v_{nn}^{T=1}$$

- Charge symmetry breaking (CSB)**

- Difference between p - p int. and n - n int.

$$v_{\text{CSB}} \equiv v_{nn}^{T=1} - v_{pp}^{T=1} \sim \tau_{zi} + \tau_{zj}$$

- Originates mainly from mass difference of nucleons ($m_p \neq m_n$) and π^0 - η & ρ^0 - ω mixings in meson-exchange process
- Contribute to β term (β^{2n+1} terms) in nuclear EoS

- Charge independence breaking (CIB)**

- Difference between like-particle int. and diff.-particle int.

$$v_{\text{CIB}} \equiv \frac{v_{nn}^{T=1} + v_{pp}^{T=1}}{2} - v_{np}^{T=1} \sim \tau_{zi}\tau_{zj}$$

- Originates mainly from mass difference of pions ($m_{\pi^0} \neq m_{\pi^\pm}$)
- Contribute to SNM and β^2 term (β^{2n} terms) in nuclear EoS

Miller, Opper, and Stephenson. *Annu. Rev. Nucl. Part. Sci.* **56**, 253 (2006)

Isospin symmetry breaking of atomic nuclei

- Isospin symmetry of atomic nuclei is *slightly* broken due to
 - Coulomb interaction
 - Isospin symmetry breaking (ISB) terms of nuclear interaction
- Different properties of mirror nuclei
 - Mass (Okamoto-Nolen-Schiffer anomaly)
 - Ground-state spin, shape, ...
- Isobaric analog energy
- Superallowed β decay
- Finite (negative) neutron-skin thickness $\Delta R_{np} = R_n - R_p$ of $N = Z$ nuclei

Okamoto. *Phys. Lett.* **11**, 150 (1964)

Nolen and Schiffer. *Annu. Rev. Nucl. Sci.* **19**, 471 (1969)

Hoff *et al.* *Nature* **580**, 52 (2020)

Wimmer *et al.* *Phys. Rev. Lett.* **126**, 072501 (2021)

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Most effective interactions do **not** include ISB terms

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Most effective interactions do **not** include ISB terms

→ Nuclear structure calculation with ISB terms is important
to understand these properties quantitatively

Skyrme-like ISB interaction

$$v_{\text{Sky}}^{\text{CSB}}(\mathbf{r}) = \left\{ s_0 (1 + y_0 P_\sigma) \delta(\mathbf{r}) + \frac{s_1}{2} (1 + y_1 P_\sigma) [\mathbf{k}^{\dagger 2} \delta(\mathbf{r}) + \delta(\mathbf{r}) \mathbf{k}^2] + s_2 (1 + y_2 P_\sigma) \mathbf{k}^\dagger \cdot \delta(\mathbf{r}) \mathbf{k} \right\} \frac{\tau_{z1} + \tau_{z2}}{4}$$

$$v_{\text{Sky}}^{\text{CIB}}(\mathbf{r}) = \left\{ u_0 (1 + z_0 P_\sigma) \delta(\mathbf{r}) + \frac{u_1}{2} (1 + z_1 P_\sigma) [\mathbf{k}^{\dagger 2} \delta(\mathbf{r}) + \delta(\mathbf{r}) \mathbf{k}^2] + u_2 (1 + z_2 P_\sigma) \mathbf{k}^\dagger \cdot \delta(\mathbf{r}) \mathbf{k} \right\} \frac{\tau_{z1} \tau_{z2}}{2}$$

$$\begin{aligned} \mathcal{E}_{\text{CSB}}^{\text{H}} &= \frac{s_0}{4} \left(1 + \frac{y_0}{2} \right) (\rho_n^2 - \rho_p^2) + \frac{1}{8} \left[s_1 \left(1 + \frac{y_1}{2} \right) + s_2 \left(1 + \frac{y_2}{2} \right) \right] (\rho_n \tau_n - \rho_p \tau_p) \\ &\quad - \frac{1}{32} \left[3s_1 \left(1 + \frac{y_1}{2} \right) - s_2 \left(1 + \frac{y_2}{2} \right) \right] (\rho_n \Delta \rho_n - \rho_p \Delta \rho_p) - \frac{1}{32} (s_1 y_1 + s_2 y_2) (\mathbf{J}_n^2 - \mathbf{J}_p^2) \end{aligned}$$

$$\begin{aligned} \mathcal{E}_{\text{CSB}}^{\text{X}} &= -\frac{s_0}{4} \left(\frac{1}{2} + y_0 \right) (\rho_n^2 - \rho_p^2) - \frac{1}{8} \left[s_1 \left(\frac{1}{2} + y_1 \right) - s_2 \left(\frac{1}{2} + y_2 \right) \right] (\rho_n \tau_n - \rho_p \tau_p) \\ &\quad + \frac{1}{32} \left[3s_1 \left(\frac{1}{2} + y_1 \right) + s_2 \left(\frac{1}{2} + y_2 \right) \right] (\rho_n \Delta \rho_n - \rho_p \Delta \rho_p) + \frac{1}{32} (s_1 - s_2) (\mathbf{J}_n^2 - \mathbf{J}_p^2) \end{aligned}$$

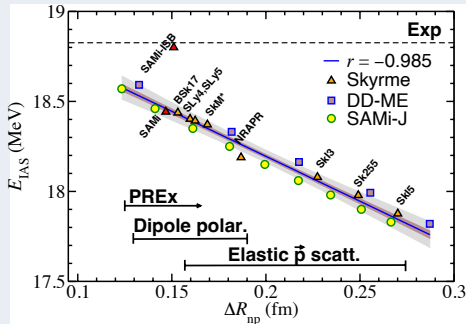
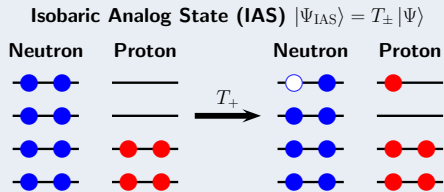
$$\begin{aligned} \mathcal{E}_{\text{CIB}}^{\text{H}} &= \frac{u_0}{4} \left(1 + \frac{z_0}{2} \right) (\rho_n - \rho_p)^2 + \frac{1}{8} \left[u_1 \left(1 + \frac{z_1}{2} \right) + u_2 \left(1 + \frac{z_2}{2} \right) \right] (\rho_n - \rho_p) (\tau_n - \tau_p) \\ &\quad - \frac{1}{32} \left[3u_1 \left(1 + \frac{z_1}{2} \right) - u_2 \left(1 + \frac{z_2}{2} \right) \right] (\rho_n - \rho_p) (\Delta \rho_n - \Delta \rho_p) - \frac{1}{32} (u_1 z_1 + u_2 z_2) (\mathbf{J}_n - \mathbf{J}_p)^2 \end{aligned}$$

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Sagawa, Colò, Roca-Maza, and Niu. *Eur. Phys. J. A* **55**, 227 (2019)

Naito, Colò, Roca-Maza, and Sagawa. *Phys. Rev. C* **107**, 064302 (2023)

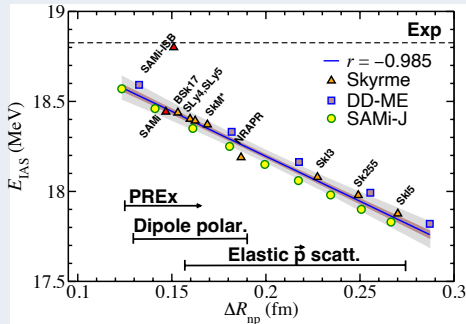
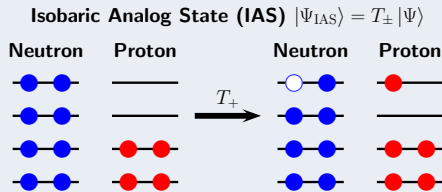
ISB effects on isobaric analog energy and neutron-skin thickness



- There is a correlation between E_{IAS} and ΔR_{np} of ^{208}Pb
- Without ISB terms, exp. values of E_{IAS} and ΔR_{np} cannot be described at the same time

Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

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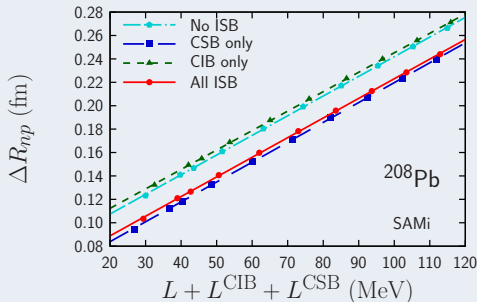
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(before PREX-II data)

Roca-Maza, Colò, and Sagawa. *Phys. Rev. Lett.* **120**, 202501 (2018)

Neutron-skin thickness of ^{208}Pb and nuclear equation of state

$$\frac{E}{A} = \varepsilon_0 + \frac{K_\infty}{2} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 + \dots + \left[J + L \left(\frac{\rho - \rho_0}{3\rho_0} \right) + \frac{K_{\text{sym}}}{2} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 \right] \beta^2$$



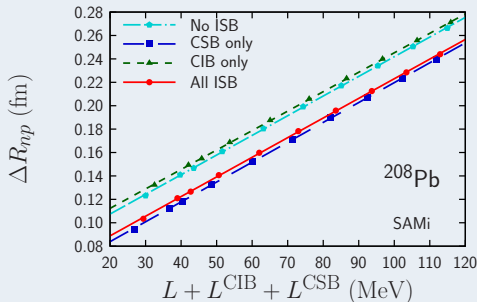
- L vs ΔR_{np} correlation estimated by SAMi-J family
- On top of SAMi-J family, ISB terms are considered
- SAMi-ISB strengths is used
 - CSB $s_0 = -26.3 \text{ MeV fm}^3$
 - CIB $u_0 = 25.8 \text{ MeV fm}^3$
 - $y_0 = z_0 = -1$ (the others are zero)

- Difference between estimated L_{full} without & that with ISB is 11.1 MeV (CSB contrib. 13.9 MeV, CIB contrib. -2.7 MeV) → Change of L is 12 MeV
 - Note: These values depend on the strengths of ISB terms
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- Difference between estimated L_{full} without & that with ISB is 11.1 MeV (CSB contrib. 13.9 MeV, CIB contrib. -2.7 MeV) → Change of L is 12 MeV
- Note: These values depend on the strengths of ISB terms → Determination of s_j and u_j are crucial!

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Phys. Rev. C* **107**, 064302 (2023)

Determination of Skyrme-like CSB parameters

Here, we assume $y_0 = y_1 = -1$ and $y_2 = +1$

- Phenomenological determination $O(10) \text{ MeV fm}^3$
 - $s_0 = -26.3 \text{ MeV fm}^3$ (IAE of ^{208}Pb)
 - $s_0 \simeq -10 \text{ MeV fm}^3$ (MDE and TDE)
 - $s_0 \simeq 22 \text{ MeV fm}^3$, $s_1 \simeq -28 \text{ MeV fm}^5$, $s_2 \simeq -16 \text{ MeV fm}^5$ (MDE and TDE)
- Extract from *ab initio* data $O(1) \text{ MeV fm}^3$
 - $s_0 \simeq -2 \text{ MeV fm}^3$ (ΔE_{tot} of ^{48}Ca - ^{48}Ni , CC & χEFT)
 - $s_0 \simeq -3 \text{ MeV fm}^3$ (ΔE_{tot} of ^{10}Be - ^{10}C , VMC & AV18)

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Nuovo Cim. C* **47**, 52 (2024)

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→ To understand this deviation,

we attempted to determine CSB parameters using fundamental theory

QCD sum rule

Naito, Colò, Liang, Roca-Maza, and Sagawa. *Nuovo Cim. C* **47**, 52 (2024)

Neutron-Skin Thickness and Charge Radii Difference

Determination from QCD sum rule

- Chiral condensation $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0$ is related to p - n mass difference

$$\Delta_{np}(\rho) = C_1 \left(\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} \right)^{1/3} - C_2 \quad C_1 = -a\gamma \quad \gamma = \frac{\langle \bar{d}d \rangle_0}{\langle \bar{u}u \rangle_0} - 1$$

obtained by QCD sum rule ($\gamma = -(7.8^{+3.7}_{-1.8}) \times 10^{-3}$, $C_1 = 5.24^{+2.48}_{-1.21}$ MeV)

- $\langle \bar{q}q \rangle / \langle \bar{q}q \rangle_0$ can be calculated by chiral perturbation theory

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} \simeq 1 + k_1 \frac{\rho}{\rho_0} + k_2 \left(\frac{\rho}{\rho_0} \right)^{5/3} + \dots \quad k_1 = -\frac{\sigma_{\pi N} \rho_0}{f_\pi^2 m_\pi^2} \quad k_2 = -k_1 \frac{3k_{F0}^2}{10m_N^2}$$

$\sigma_{\pi N}$: π - N sigma term, f_π : pion decay constant

Hatsuda, Høgaasen, and Prakash. *Phys. Rev. Lett.* **66**, 2851 (1991)

Goda and Jido. *Phys. Rev. C* **88**, 065204 (2013)

Sagawa, Naito, Roca-Maza, and Hatsuda. *Phys. Rev. C* **109**, L011302 (2024)

Neutron-Skin Thickness and Charge Radii Difference

Determination from QCD sum rule

Mirror nuclei mass difference in local density approximation obtained by

- Skyrme HF calculation with s_0 , s_1 , and s_2
- QCD sum rule and the local density approximation



Sagawa, Naito, Roca-Maza, and Hatsuda. *Phys. Rev. C* **109**, L011302 (2024)

Neutron-Skin Thickness and Charge Radii Difference

Determination from QCD sum rule

The mass difference originating from ISB reads

$$\Delta_{\text{Skyrme}} \simeq -\frac{s_0(1-y_0)}{4}\rho - \frac{1}{10}\left(\frac{3\pi^2}{2}\right)^{2/3} [s_1(1-y_1) + 3s_2(1+y_2)]\rho^{5/3}$$

$$\Delta_{\text{QCDSR}} = C_1 \left[1 - \left(\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} \right)^{1/3} \right] \simeq C_1 \left\{ 1 - \left[1 + k_1 \frac{\rho}{\rho_0} + k_2 \left(\frac{\rho}{\rho_0} \right)^{5/3} \right]^{1/3} \right\}$$

$$\simeq C_1 \left[\frac{1}{3} \frac{\sigma_{\pi N}}{f_\pi^2 m_\pi^2} \rho - \frac{1}{10} \left(\frac{3\pi^2}{2} \right)^{2/3} \frac{1}{m_N^2} \frac{\sigma_{\pi N}}{f_\pi^2 m_\pi^2} \rho^{5/3} \right]$$

We obtain

$$s_0(1-y_0) \simeq -\frac{4}{3} \frac{C_1 \sigma_{\pi N}}{f_\pi^2 m_\pi^2} = -15.5_{-12.5}^{+8.8} \text{ MeV fm}^3$$

$$s_1(1-y_1) + 3s_2(1+y_2) \simeq \frac{1}{m_N^2} \frac{C_1 \sigma_{\pi N}}{f_\pi^2 m_\pi^2} = 0.52_{-0.29}^{+0.42} \text{ MeV fm}^5$$

(Error bar is due to $\sigma_{\pi N}$)

Sagawa, Naito, Roca-Maza, and Hatsuda. *Phys. Rev. C* **109**, L011302 (2024)

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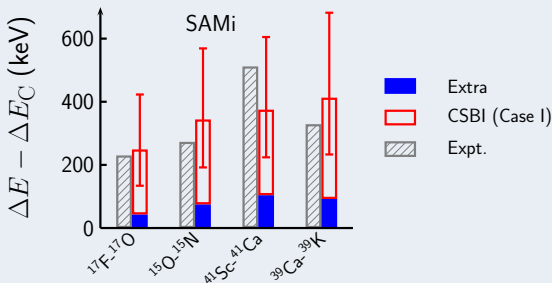
s_1 & s_2 terms may be small

(Error bar is due to $\sigma_{\pi N}$)

Sagawa, Naito, Roca-Maza, and Hatsuda. *Phys. Rev. C* **109**, L011302 (2024)

Neutron-Skin Thickness and Charge Radii Difference

ONS anomaly and QCD-CSB interaction



- “Extra” contribution is not enough to describe ΔE
 - Higher-order correction for the Coulomb interaction
 - Change of kinetic energy due to $m_p \neq m_n$
- QCD-CSB interaction describe ΔE quite nicely
→ ONS anomaly may be solved?

Sagawa, Naito, Roca-Maza, and Hatsuda. *Phys. Rev. C* **109**, L011302 (2024)

Conclusion

- $O(\beta)$ term originating from CSB in EoS is related to
 - the restoration of the chiral symmetry breaking
 - effective mass (self-energy) of nucleons in medium→ QCD sum rule approach works to understand it
- QCD-CSB interaction can describe ONS anomaly

Perspectives

- How about CIB interaction, originating from $m_{\pi^0} \neq m_{\pi^\pm}$?
Pion is NG boson → in-medium effect is important
- Ongoing
 - QCD-based CIB interaction (w/ Colò, Hatsuda, Roca-Maza, Sagawa)
 - Relativistic CSB EDF (w/ Cheoun, Sagawa, Tanimura)
- Ultimate goal
 - **“Complete & accurate” nuclear EDF**
 - **Can we understand “medium effect” from QCD?**

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Thank you for attention!!