湖州师范学院 Huzhou University

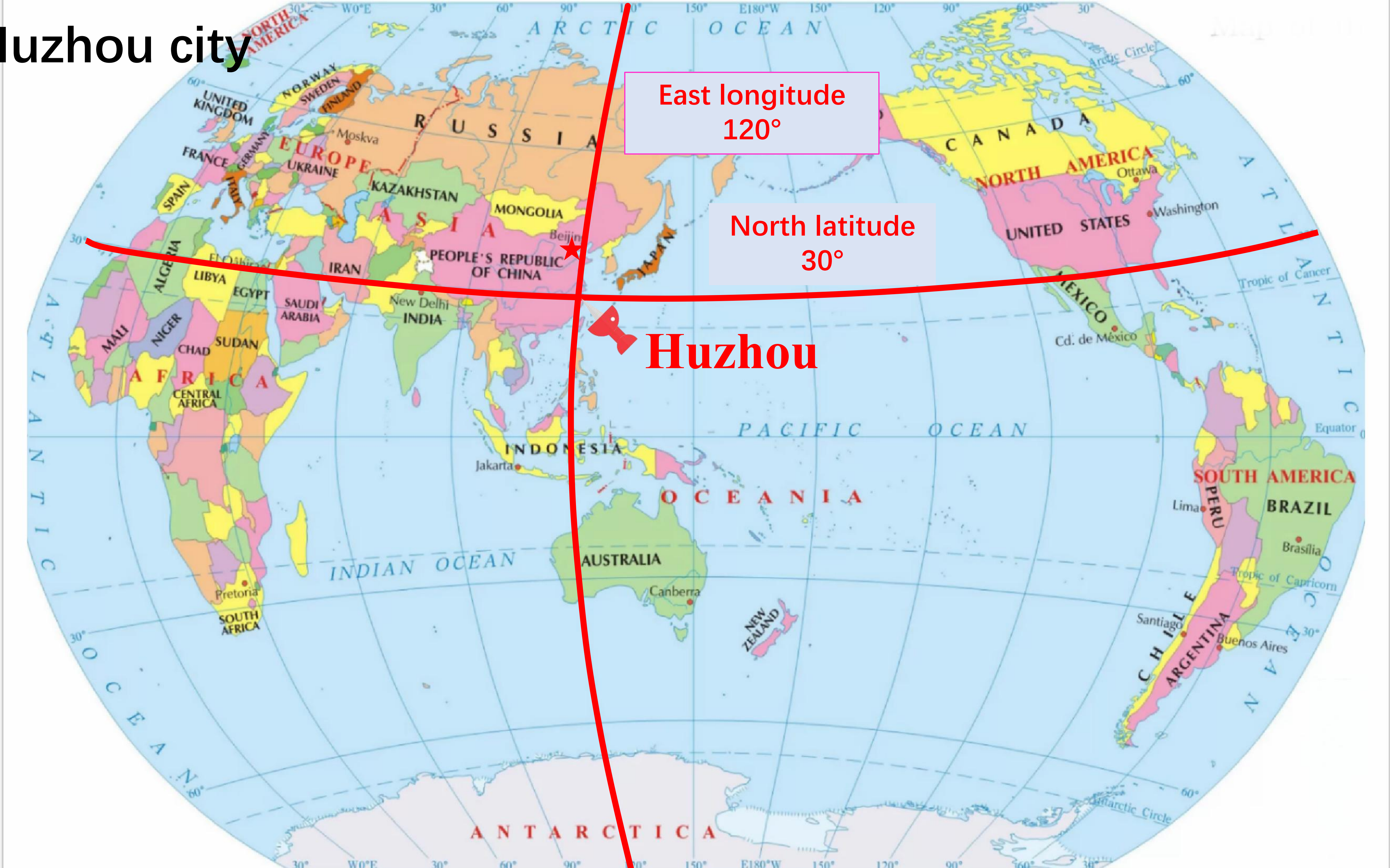
How machine learning affects the inference of the nuclear equation of state

Machine Learning Transforms the Inference of the Nuclear Equation of State

Yongjia Wang (Huzhou University, China)

NUSYM 2024, XIIth International Symposium on Nuclear Symmetry Energy
09.09.2024-15.09.2014, Caen, French

Huzhou city



East longitude
120°
North latitude
30°
Huzhou
ARCTIC OCEAN
RUSSIA
EUROPE
ASIA
AFRICA
OCEANIA
PACIFIC OCEAN
INDIAN OCEAN
ATLANTIC OCEAN
ANTARCTICA
NORTH AMERICA
SOUTH AMERICA
CANADA
UNITED STATES
MEXICO
BRAZIL
PERU
CHILE
ARGENTINA
AUSTRALIA
NEW ZEALAND
INDONESIA
JAPAN
MONGOLIA
PEOPLE'S REPUBLIC OF CHINA
KAZAKHSTAN
INDIA
IRAN
SAUDI ARABIA
EGYPT
LIBYA
ALGERIA
MALI
NIGER
CHAD
SUDAN
CENTRAL AFRICA
SOUTH AFRICA
UNITED KINGDOM
NORWAY
SWEDEN
FINLAND
FRANCE
GERMANY
SPAIN
ITALY
UKRAINE
Moskva
Beijing
New Delhi
Jakarta
Canberra
Pretoria
Ottawa
Washington
Cd. de México
Lima
Brasília
Santiago
Buenos Aires
Arctic Circle
Tropic of Cancer
Equator
Tropic of Capricorn
Antarctic Circle

Outline

Contents

01 Background

02 ML algorithm

- ✓ Bayesian inference
- ✓ Supervised learning

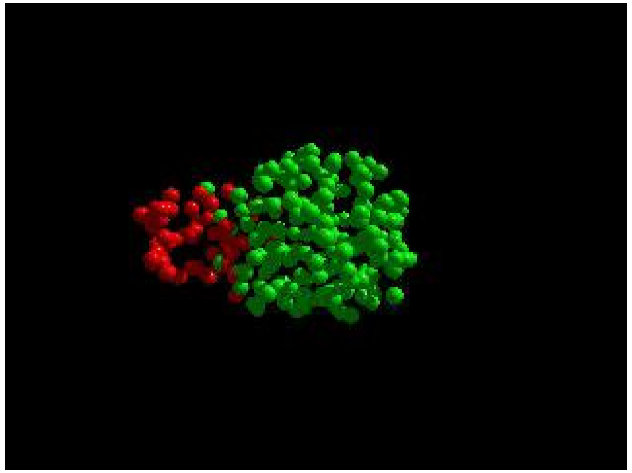
03 Results

- ✓ Bayesian inference on the in-medium nucleon-nucleon cross section
- ✓ Supervised learning on the nuclear symmetry enery

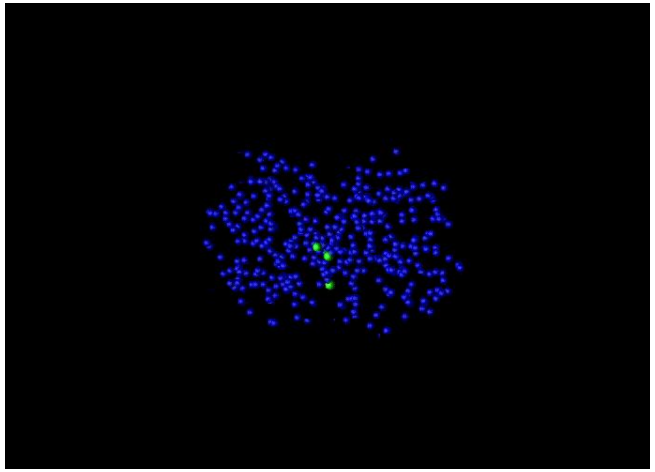
04 Summary and Outlook

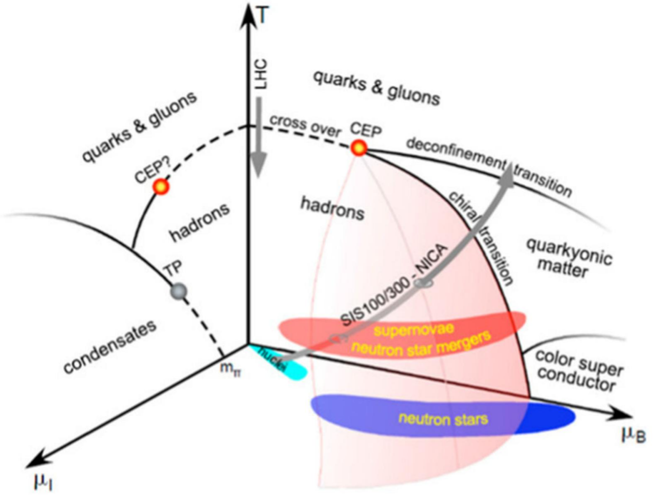
01 Overview of HIC

◆ intermediate energies HIC



Nuclear equation of state





◆ Low energy fusion reaction

~100 MeV- a few GeV/nucleon

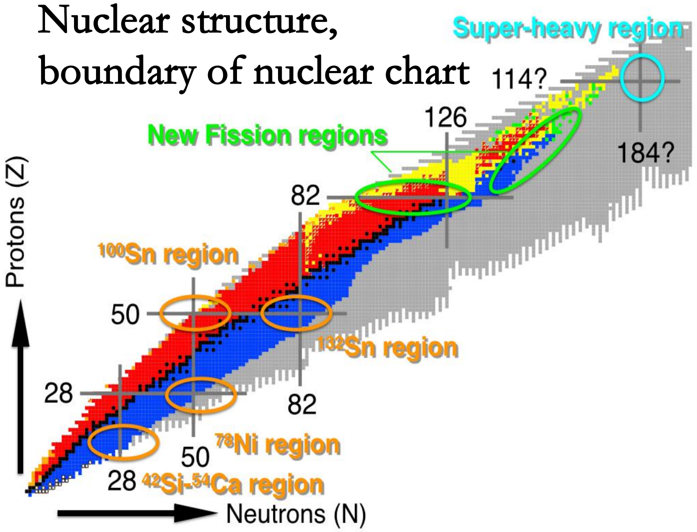
QCD phase diagram, QGP properties

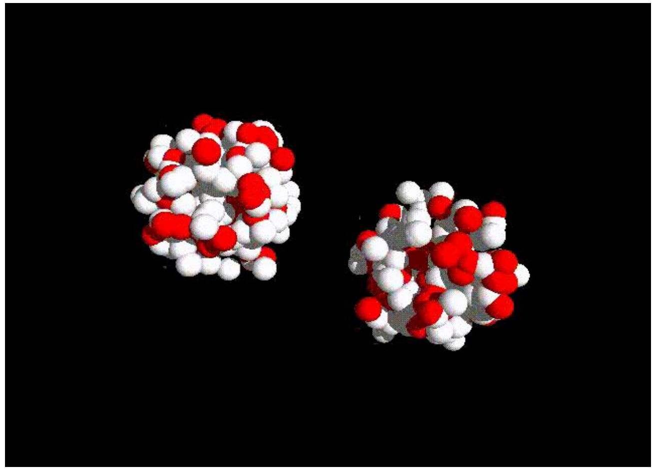
~100 MeV/nucleon

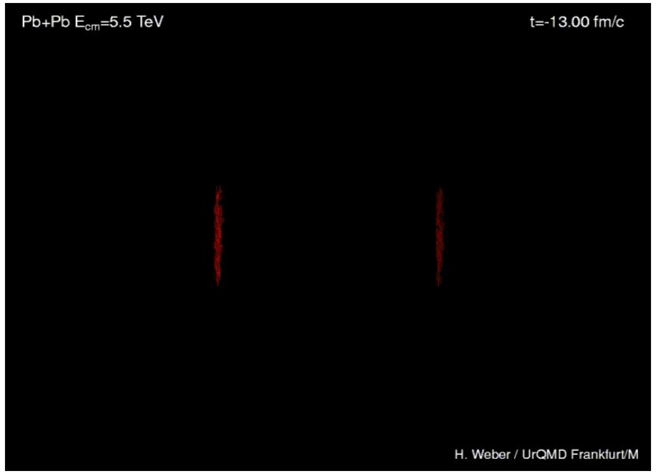
> hundreds of GeV/nucleon

beam energy

Nuclear structure, boundary of nuclear chart







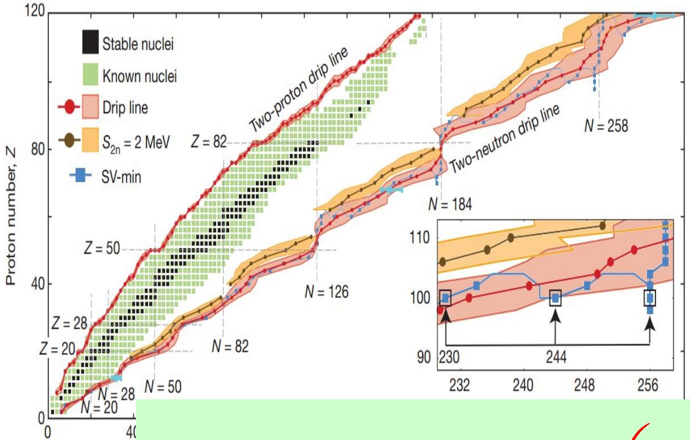
◆ Relativistic HIC

Nuclear equation of state (EOS)

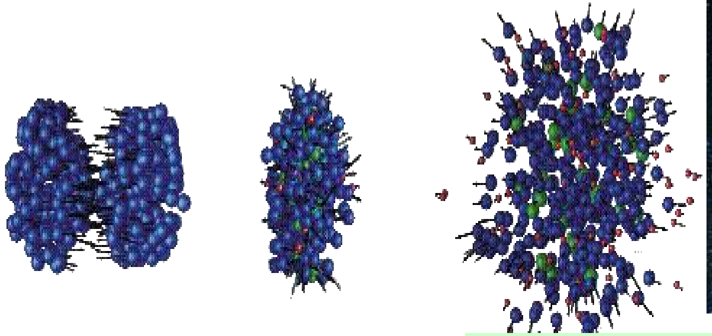
The thermodynamic relationship between the binding energy *E* (or pressure *P*) and density ρ, as well as the isospin asymmetry δ.

$$E(\rho,\delta) = E(\rho,0) + E_{sym}(\rho)\delta^2 + \cdots, \qquad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

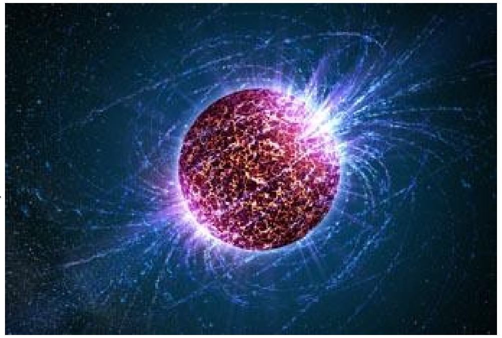
Nuclear landscape



Heavy ion collision



Neutron star



$$E(\rho,0) = E_0 + \frac{K_0}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \ldots,$$

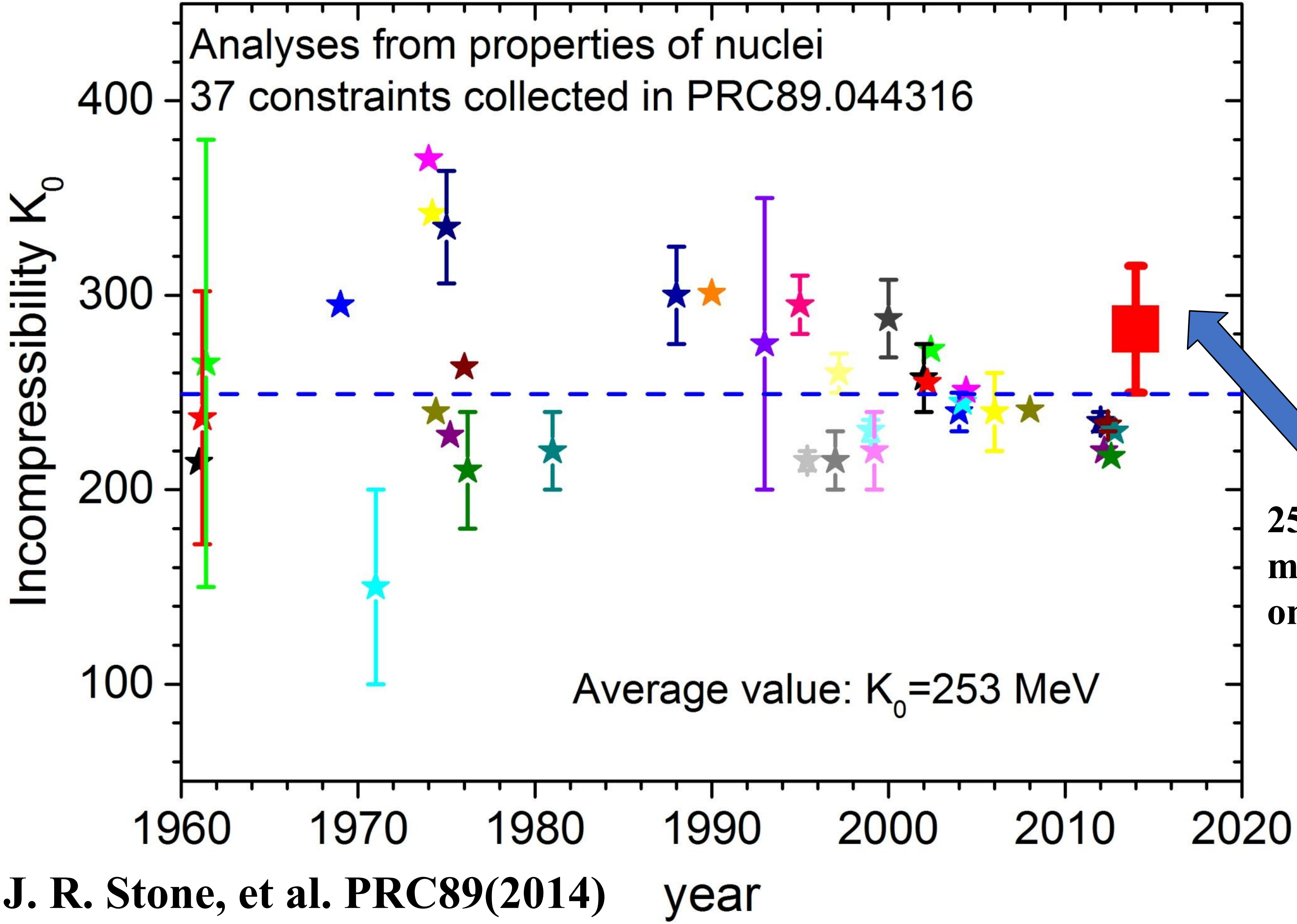
$$K_0 = 9\rho^2\left(\frac{\partial^2 E}{\partial\rho^2}\right)\Bigg|_{\rho=\rho_0}$$

$$E_{sym}(\rho) = S_0 + L\left(\frac{\rho-\rho_0}{3\rho_0}\right) + \frac{K_{sym}}{2}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \ldots$$

$$L = 3\rho\frac{dE_{sym}(\rho)}{d\rho}\Bigg|_{\rho=\rho_0}$$

K_0 and L determine the EOS in the vicinity of the saturation density.

The incompressibility K_0 from properties of nuclei



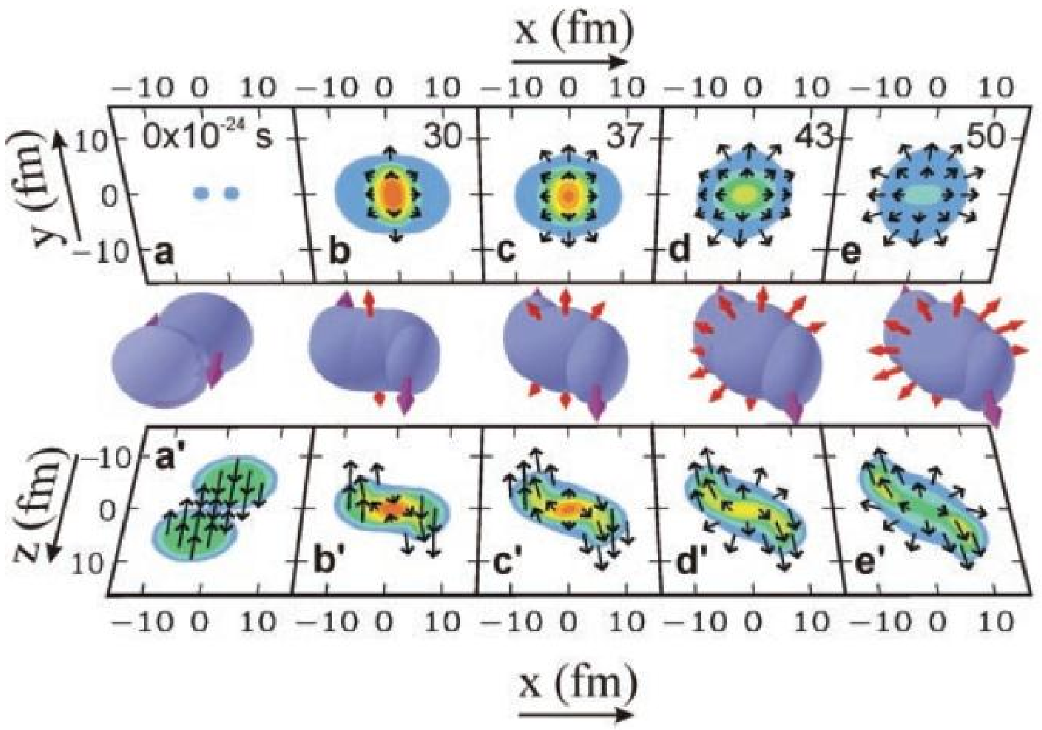
isoscalar giant monopole resonances (GMR), nuclear masses...

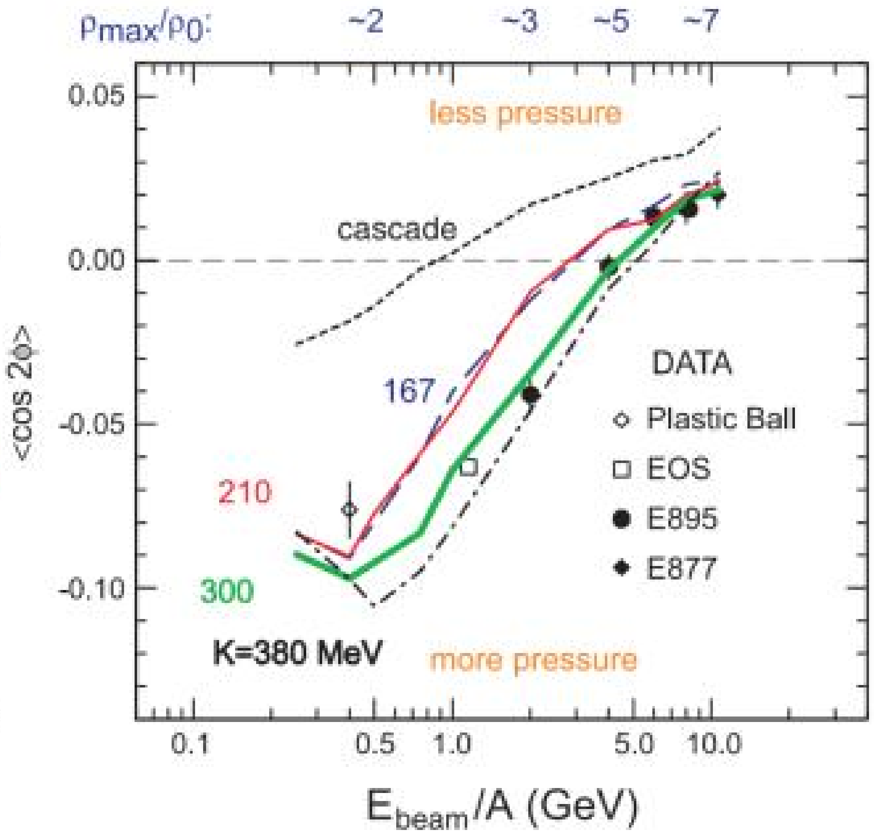
$250<K_0<315$ MeV, based on the most precise and up-to-date data on GMR energies.

J. R. Stone, et al. PRC89(2014)

HIC offers a unique way to create nuclear matter with high density and isospin asymmetry in laboratory.

Determination of the Equation of State of Dense Matter
Pawel Danielewicz, *et al.*
Science **298**, 1592 (2002);
DOI: 10.1126/science.1078070





EOS can be deduced from the comparision bewteen experimental observables and transport model calculations.

01 Nuclear equation of state

Some of highly cited papers

Recent progress and new challenges in isospin physics with heavy-ion reactions

BA Li, LW Chen, CM Ko - Physics Reports, 2008 - Elsevier

… on the reaction aspect of isospin physics, especially heavy-… of isospin physics is to determine the isospin dependence of …) of isospin asymmetric nuclear matter, particularly its isospin-…

☆ 保存 99 引用 被引用次数：1597 相关文章 所有 11 个版本 免费在线GPT »

Equations of state for supernovae and compact stars

M Oertel, M Hempel, T Klähn, S Typel - Reviews of Modern Physics, 2017 - APS

A review is given of various theoretical approaches for the equation of state (EoS) of dense matter, relevant for the description of core-collapse supernovae, compact stars, and compact …

☆ 保存 99 引用 被引用次数：1045 相关文章 所有 11 个版本 免费在线GPT

Constraints on the density dependence of the symmetry energy

MB Tsang, Y Zhang, P Danielewicz, M Famiano, Z Li… - Physical review …, 2009 - APS

… over a range of symmetry energies at saturation density and different representations of the density dependence of the symmetry energy, constraints on the density dependence of the …

☆ 保存 99 引用 被引用次数：946 相关文章 所有 20 个版本 免费在线GPT

01 Nuclear equation of state

Two White papers

Progress in Particle and Nuclear Physics
Available online 19 September 2023, 104080
In Press, Journal Pre-proof What's this?

Review

Dense nuclear matter equation of state from heavy-ion collisions

Agnieszka Sorensen [1], Kshitij Agarwal [2], Kyle W. Brown [3 4], Zbigniew Chajecki [5], Paweł Danielewicz [3 6], Christian Drischler [7], Stefano Gandolfi [8], Jeremy W. Holt [9 10], Matthias Kaminski [11], Che-Ming Ko [9 10], Rohit Kumar [3], Bao-An Li [12], William G. Lynch [3 6], Alan B. McIntosh [10], William G. Newton [12], Scott Pratt [3 6], Oleh Savchuk [3 13], Maria Stefaniak [14 15], Ingo Tews [8], ManYee Betty Tsang [3 6]...Yi Yin [94]

Show more

Add to Mendeley Share Cite

https://doi.org/10.1016/j.ppnp.2023.104080

Get rights and content

Abstract

This White Paper highlights the essential role of hadronic transport simulations of heavy-ion collisions in studies involving the equation of state of nuclear matter. It also elucidates many connections between inferences of the equation of state from heavy-ion collision data and other efforts aiming to understand the properties of nuclear matter.

arXiv > nucl-th > arXiv:2211.02224

Nuclear Theory

[Submitted on 4 Nov 2022 (v1), last revised 8 Nov 2022 (this version, v2)]

Long Range Plan: Dense matter theory for heavy-ion collisions and neutron stars

Alessandro Lovato, Travis Dore, Robert D. Pisarski, Bjoern Schenke, Katerina Chatziioannou, Jocelyn S. Read, Philippe Landry, Paw
Hannah Elfner, Veronica Dexheimer, Rajesh Kumar, Michael Strickland, Johannes Jahan, Claudia Ratti, Volodymyr Vovchenko, Mikh
Hippert, Jacquelyn Noronha-Hostler, Jorge Noronha, Enrico Speranza, Nicolas Yunes, Chuck J. Horowitz, Steven P. Harris, Larry Mc
Stefano Gandolfi, Ingo Tews, M. Coleman Miller, Cecilia Chirenti, Zohreh Davoudi, Jamie M. Karthein, Krishna Rajagopal, Salvatore
Vladimir Skokov, Ulrich Heinz, Christian Drischler, Daniel R. Phillips, Madappa Prakash, Zoltan Fodor, David Radice, Christopher Plu
Fraga, Aleksi Kurkela, James M. Lattimer, Andrew W. Steiner, Jeremy W. Holt, Bao-An Li, Chun Shen, Mark Alford, Alexander Haber,

Since the release of the 2015 Long Range Plan in Nuclear Physics, major events have occurred that reshaped our understanding of quantum chromodyr
of equilibrium. The US nuclear community has an opportunity to capitalize on advances in astrophysical observations and nuclear experiments and enga
matter that connects low- and high-energy nuclear physics, astrophysics, gravitational waves physics, and data science

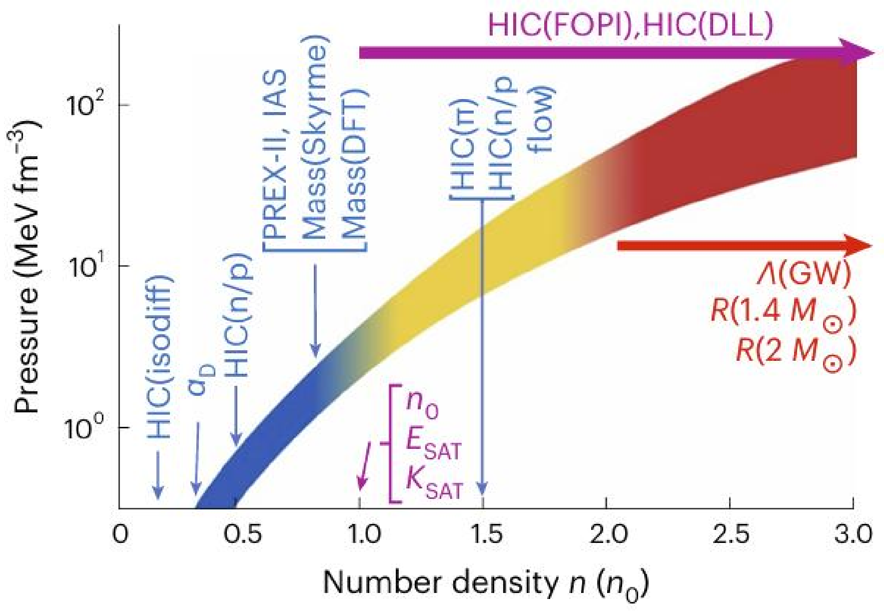
Comments:	70 pages, 3 figures, White Paper for the Long Range Plan for Nuclear Science
Subjects:	**Nuclear Theory (nucl-th)**; High Energy Astrophysical Phenomena (astro-ph.HE); High Energy Physics - Phenomenology (hep-ph)
Report number:	LA-UR-22-31648
Cite as:	arXiv:2211.02224 [nucl-th]
	(or arXiv:2211.02224v2 [nucl-th] for this version)
	https://doi.org/10.48550/arXiv.2211.02224

Submission history

From: Jacquelyn Noronha-Hostler [view email]
[v1] Fri, 4 Nov 2022 02:15:29 UTC (2,372 KB)
[v2] Tue, 8 Nov 2022 01:52:13 UTC (2,373 KB)

Bibliographic Tools | Code, Data, Media | Demos | Related Papers | About arXivLabs

01 Nuclear equation of state



nature

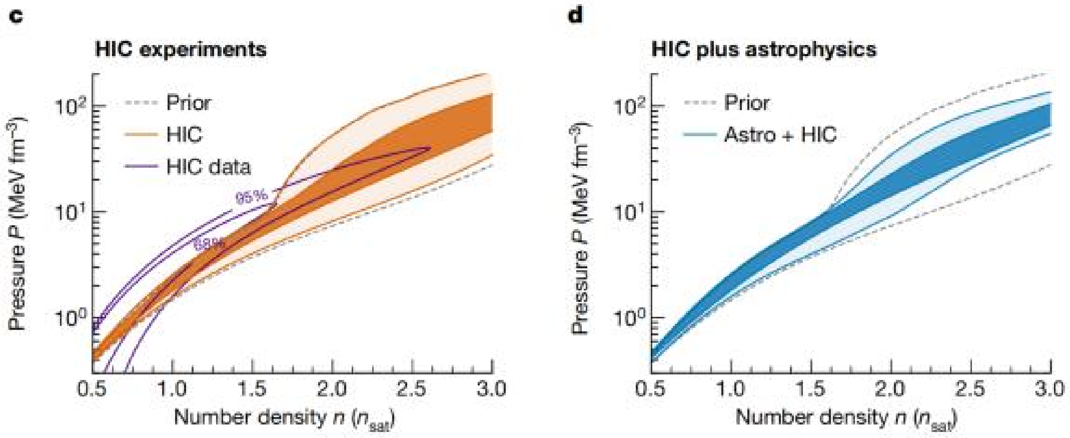
Explore content ⌄ About the journal ⌄ Publish with us ⌄

nature > articles > article

Article | Open Access | Published: 08 June 2022

Constraining neutron-star matter with microscopic and macroscopic collisions

Sabrina Huth, Peter T. H. Pang, Ingo Tews, Tim Dietrich, Arnaud Le Fèvre, Achim Schwenk, Wo… Trautmann, Kshitij Agarwal, Mattia Bulla, Michael W. Coughlin & Chris Van Den Broeck



nature astronomy

Explore content ⌄ About the journal ⌄ Publish with us ⌄ Subscribe

nature > nature astronomy > articles > article

Article | Published: 05 January 2024

Determination of the equation of state from nuclear experiments and neutron star observations

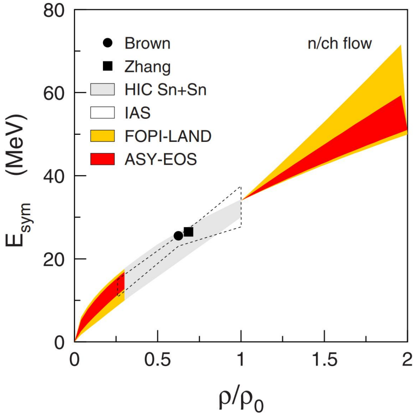
Chun Yuen Tsang, ManYee Betty Tsang, William G. Lynch, Rohit Kumar & Charles J. Horowitz

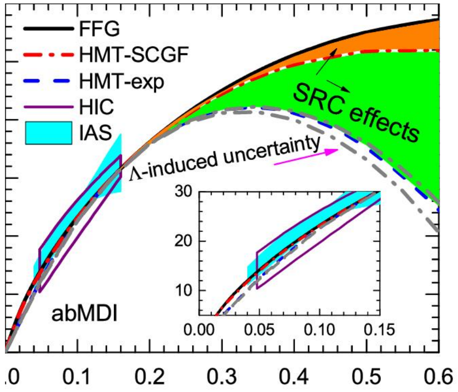
Nature Astronomy **8**, 328–336 (2024) | Cite this article

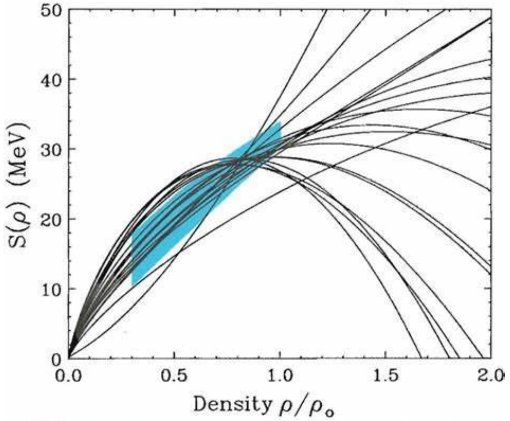
The density-dependent nuclear symmetry energy $E_{sym}(\rho)$

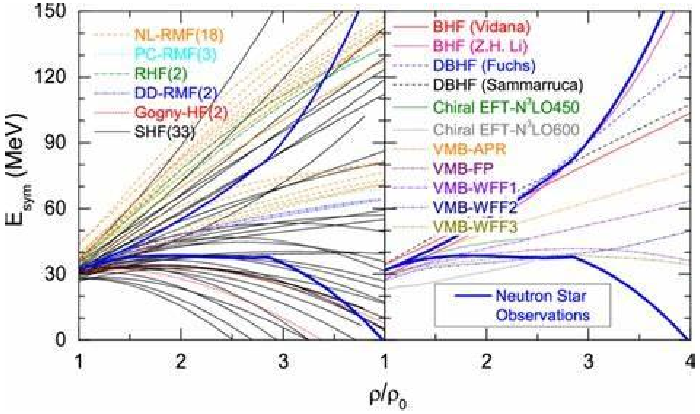
$$E(\rho,\delta)=E(\rho,0)+E_{sym}(\rho)\delta^2+O(\delta^4),$$

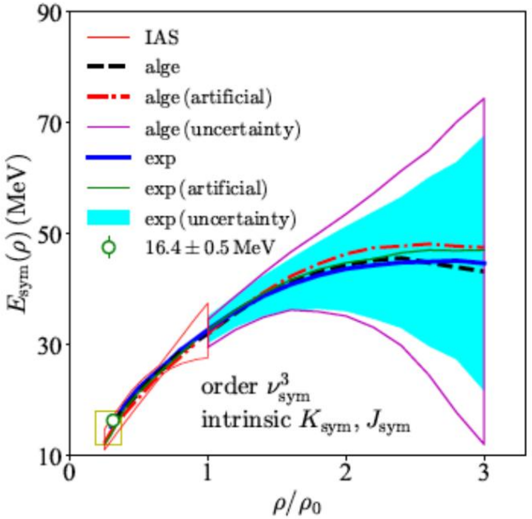
$E_{sym}(\rho)$ is crucial for our understanding of diverse phenomena observed in rare isotopes, nuclear reactions with exotic nuclei, as well as neutron star and its merger.

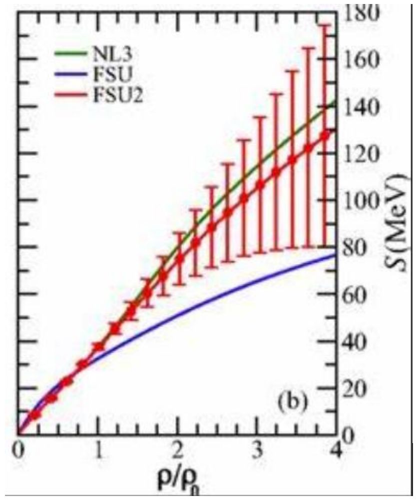


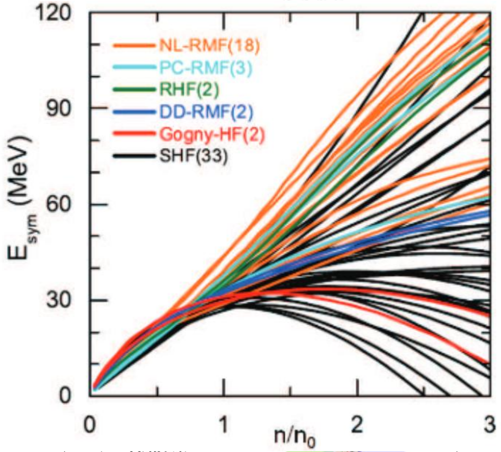


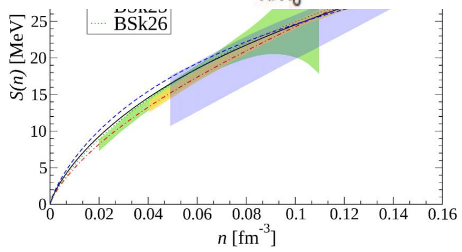


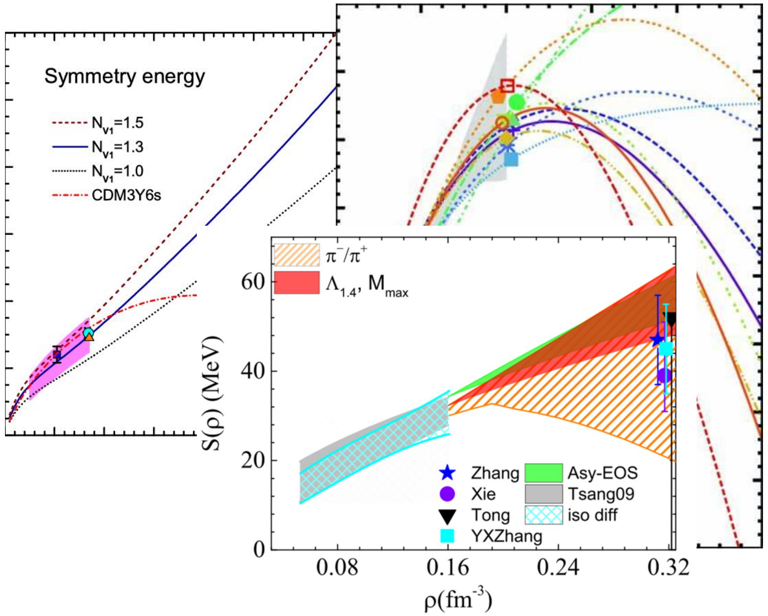












Considerable efforts have been made, however the whole picture of $E_{sym}(\rho)$ is still indistinct.

Nuclear symmetry energy

Nuclear density

The density-dependent nuclear symmetry energy $E_{sym}(\rho)$

$$v_2 = v_{20} + v_{22} \cdot y_0^2.$$

Contents lists available at ScienceDirect

Physics Letters B

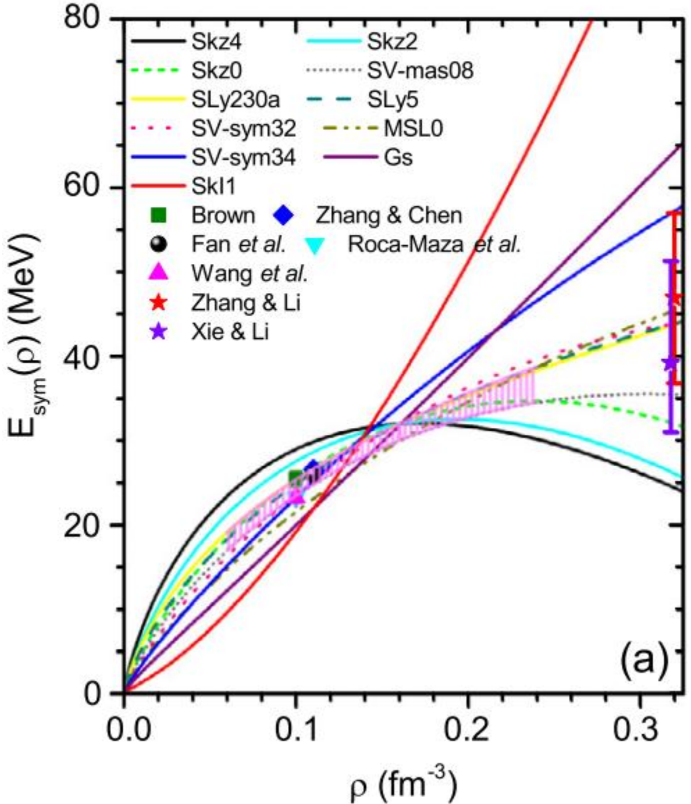
www.elsevier.com/locate/physletb

ELSEVIER

Physics Letters B 802 (2020) 135249

Study of the nuclear symmetry energy from the rapidity-dependent elliptic flow in heavy-ion collisions around 1 GeV/nucleon regime

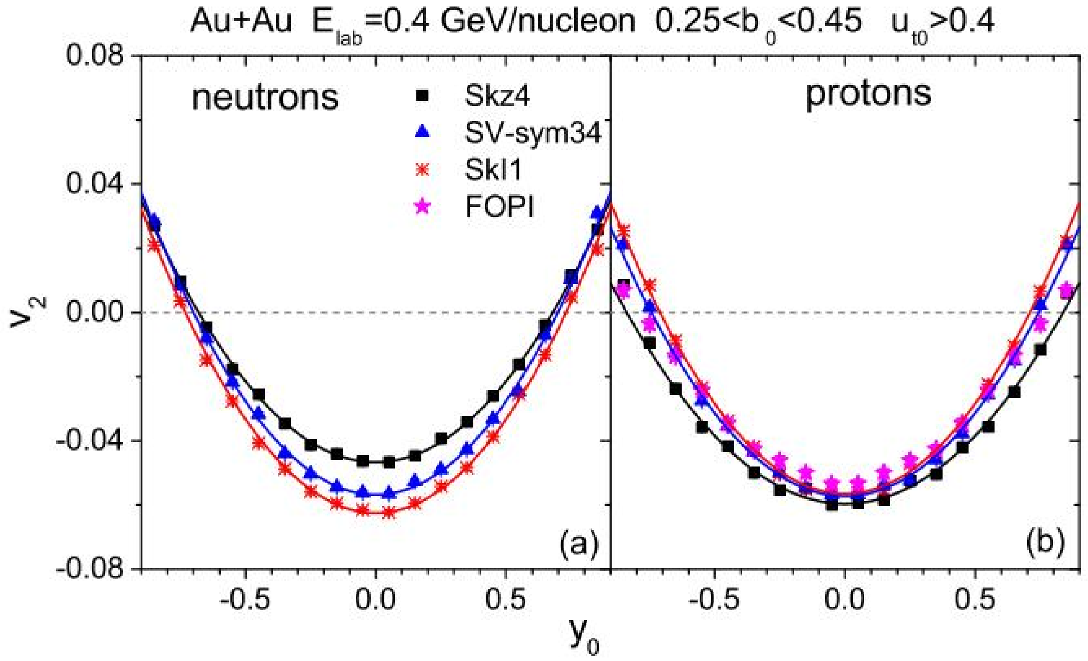
Yongjia Wang[a], Qingfeng Li[a,b,*], Yvonne Leifels[c], Arnaud Le Fèvre[c]

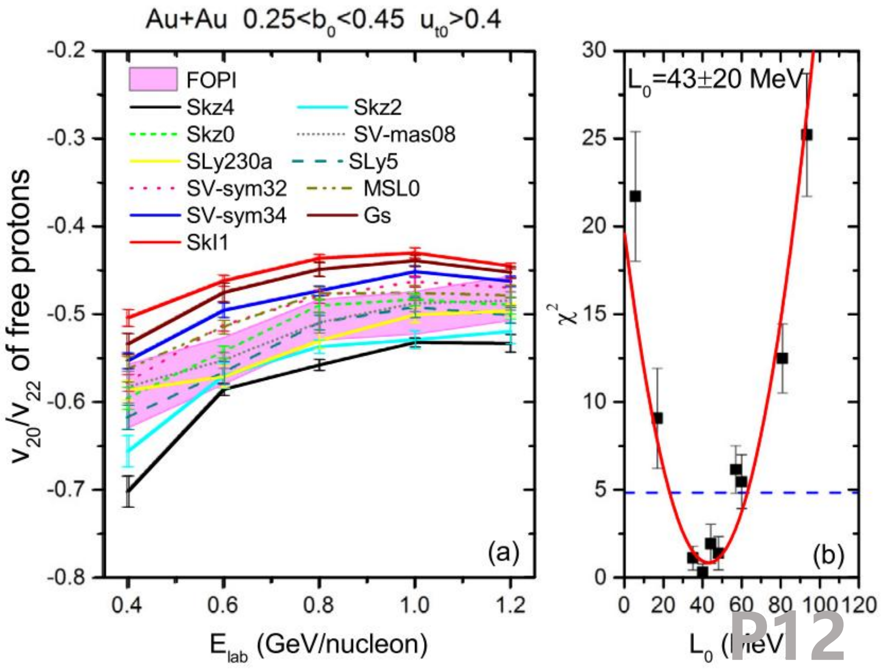


Model calculations: considering different interactions that exhibit different types of $E_{sym}(\rho)$.

Experimental data: the rapidity-dependent elliptic flow.

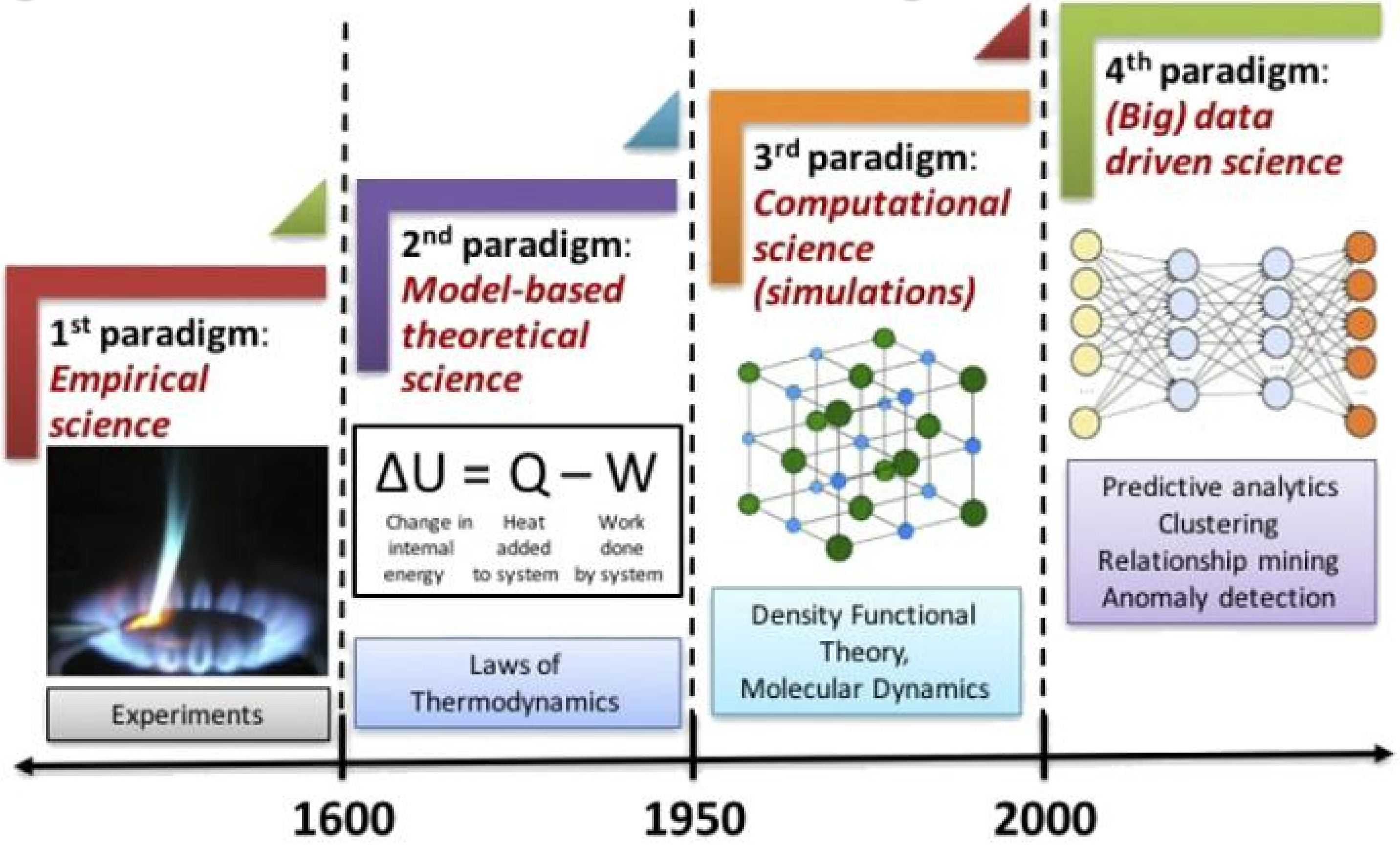
By using UrQMD model, together with the FOPI data on elliptic flow, the slope parameter of Esym(ρ) can be constrained.





01 Background

Because of the update and iteration of computer techniques, the paradigm of scientific research has changed.



2017

FIG. 1. The four paradigms of science: empirical, theoretical, computational, and data-driven.

APL Mater. 4, 053208 (2016); https://doi.org/10.1063/1.4946894

01 Background

Use of computer — Personal compute — computer cluster — GPU

Appearance of transport model
V(B)UU、QMD，AMD

1985

1995

Vast amount of Exp. data

Isospin physics

2005

Rapid development of transport model

2015

More precise Exp. data

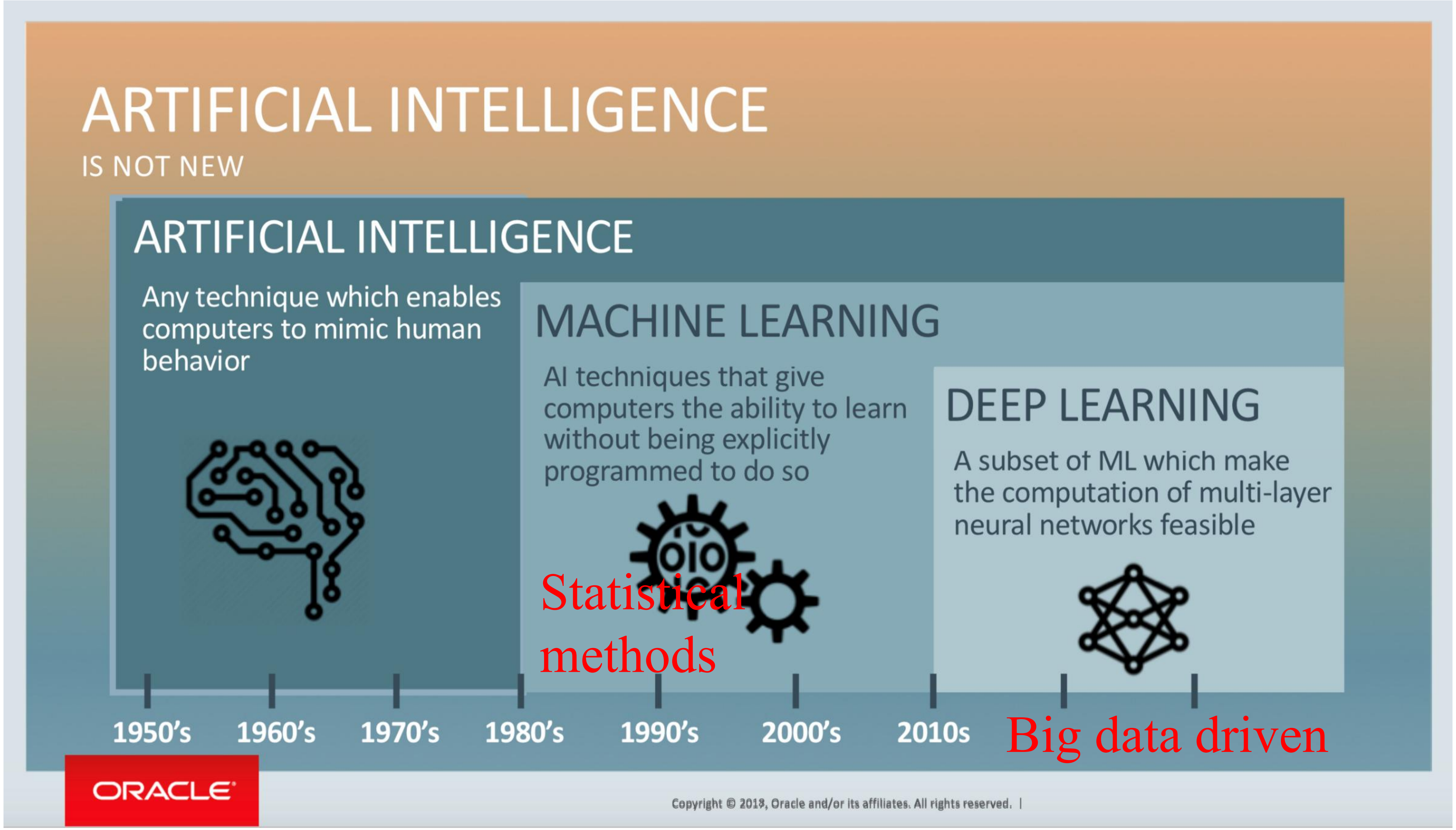
Nuclear structure+HIC + Neutron star

2025

More Exp. FAIR, FRIB, HIAF, NICA

Higher density, higher orders, higher accuracy, higher dimension

02 Machine learning



02 Machine learning

Convolutional Neural Network (CNN)

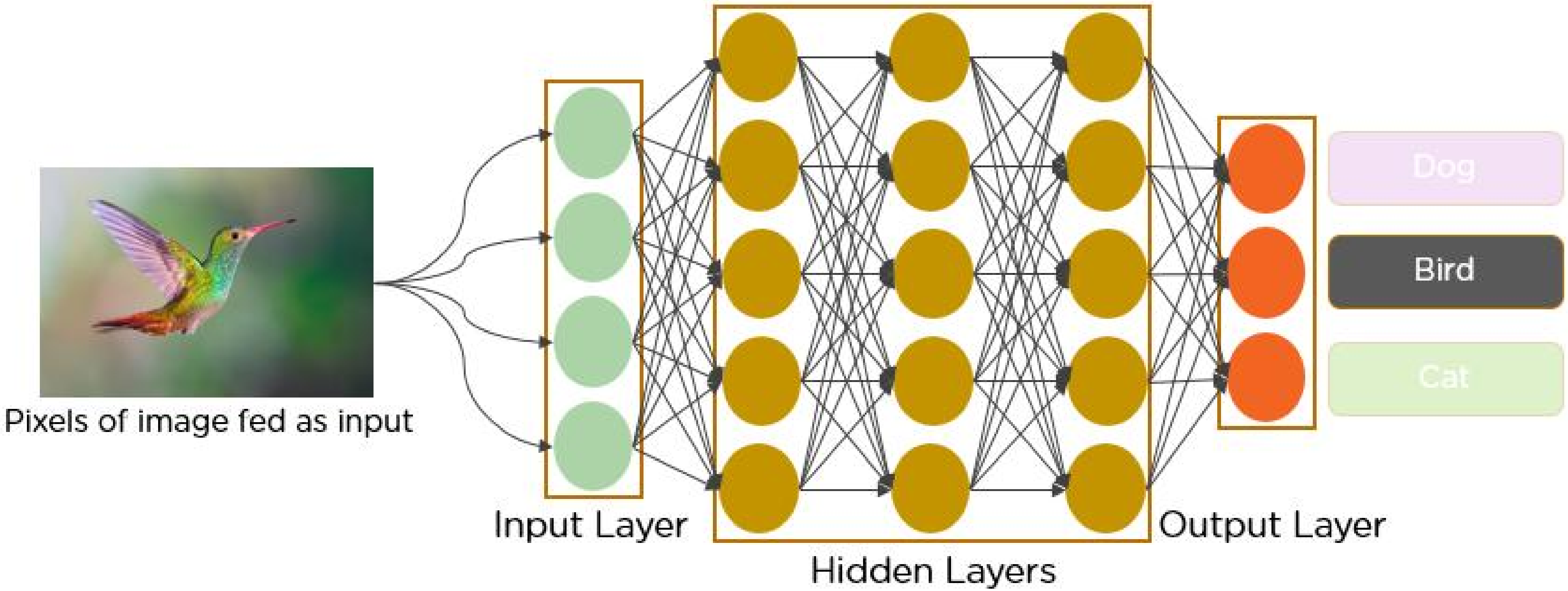
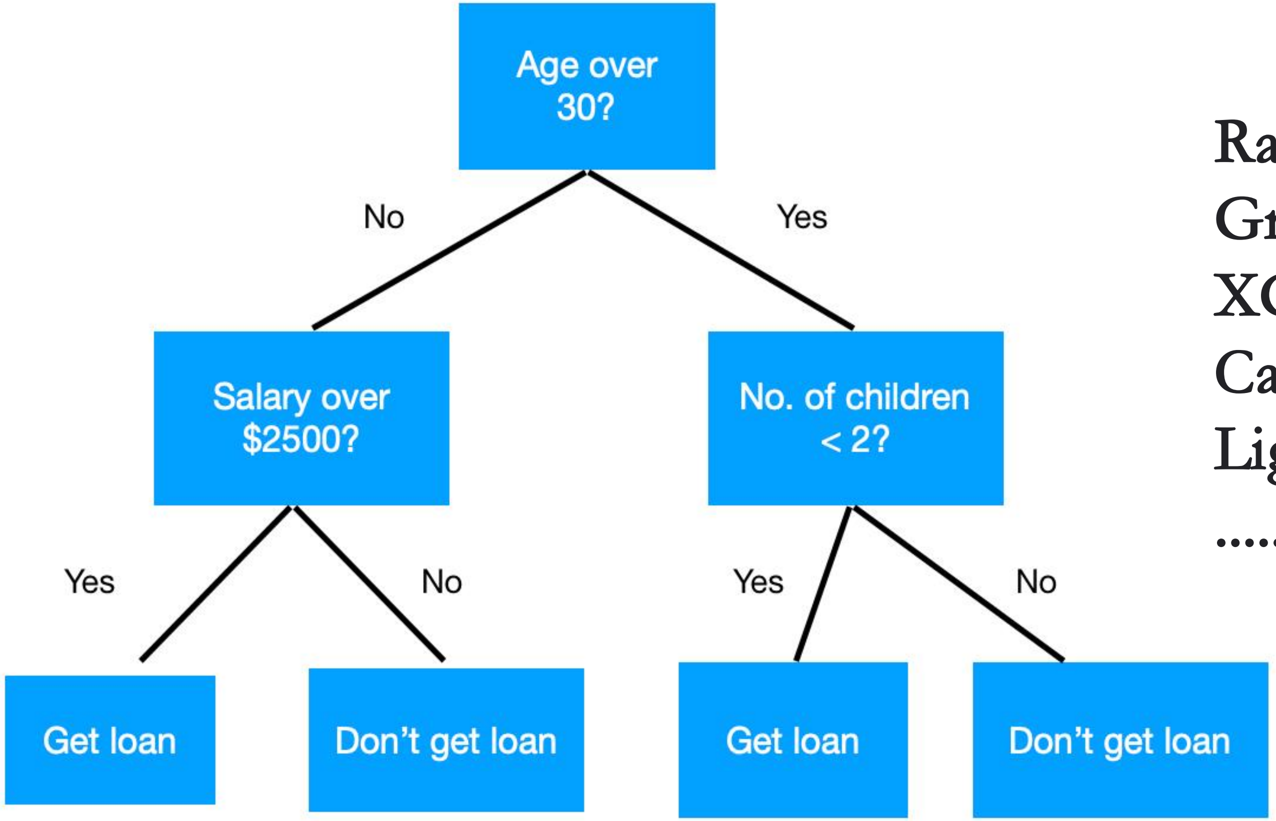


Image-like data, deep and complex structure, huge number of parameter, time-consuming, low explainability and high generalizability

Bayesian Neural Network (BNN), PointNet, Recurrent Neural Network (RNN) ...

02 Machine learning

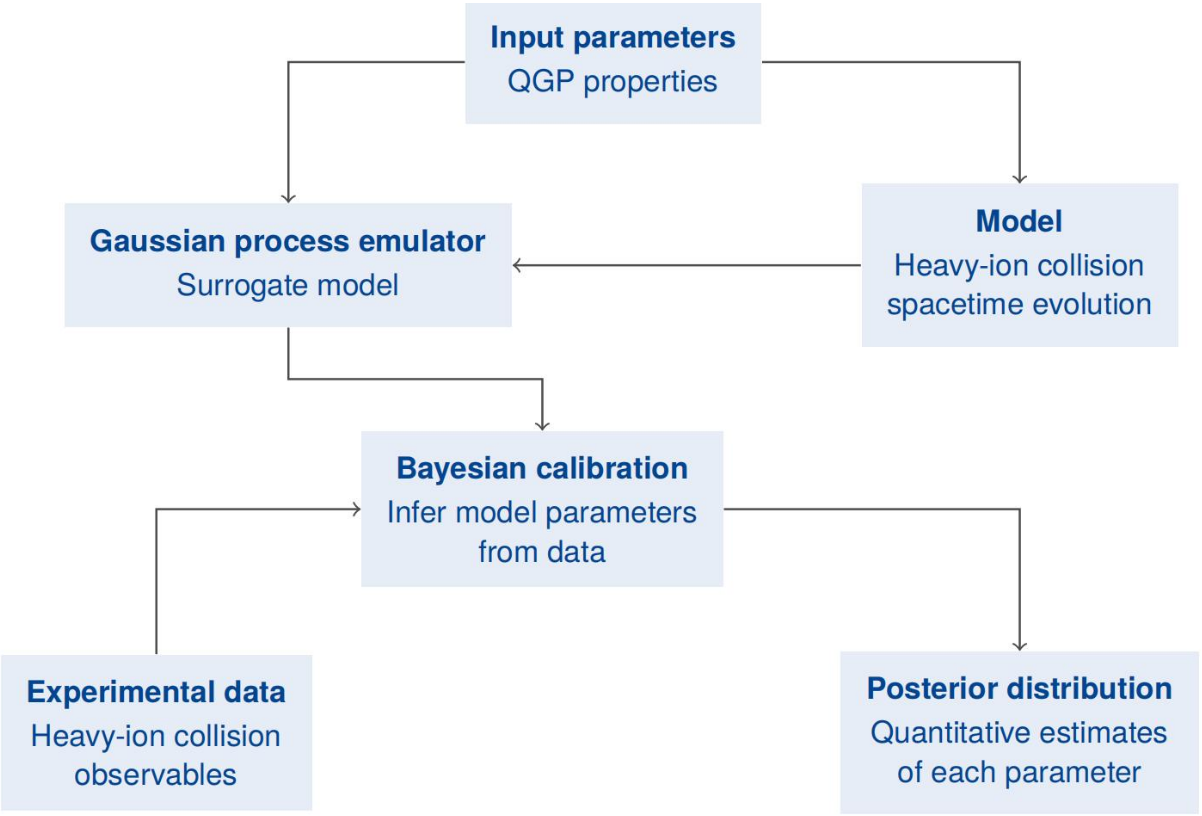
Decision-tree based algorithm



Random Forest
Gradient Boosting Decision Trees（GBDT）
XGBoost（eXtreme Gradient Boosting）
CatBoost
LightGBM (Light Gradient Boosting Machine)
……

Feature data, white-box algorithm, faster training speed and higher efficiency, lower memory usage, capable of handling large-scale data.

02 Machine learning



Bernhard J E, Moreland J S, Bass S A. Bayesian estimation of the specific shear and bulk viscosity of quark–gluon plasma[J]. Nature Physics, 2019, 15(11): 1113-1117.

Bayesian inference

$$\mathcal{P}(\boldsymbol{\theta}|\mathbf{y}_{\rm exp}) \propto \mathcal{P}(\mathbf{y}_{\rm exp}|\boldsymbol{\theta})\mathcal{P}(\boldsymbol{\theta}).$$

Prior distribution

Posterior distribution

likelihood function

$$\Delta\mathbf{y}(\boldsymbol{\theta}) = \mathbf{y}(\boldsymbol{\theta}) - \mathbf{y}_{\rm exp}$$

$$\ln[\mathcal{P}(\mathbf{y}_{\rm exp}|\boldsymbol{\theta})] = -\frac{1}{2}\Delta\mathbf{y}(\boldsymbol{\theta})^T\Sigma^{-1}\Delta\mathbf{y}(\boldsymbol{\theta}) - \frac{1}{2}\ln[(2\pi)^n \det\Sigma].$$

Physics Letters B 833 (2022) 137348

Constraining parameters from mutli observables.

02 Machine learning

Bayesian inference

Physics Letters B 799 (2019) 135045

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

ELSEVIER

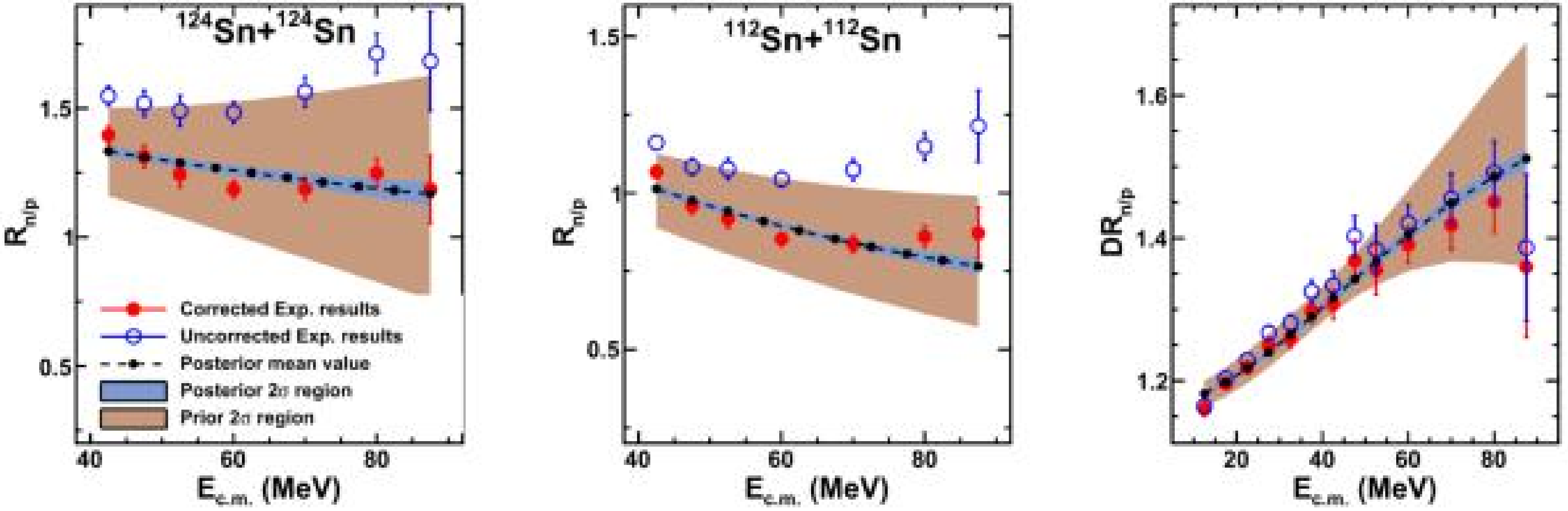
SπRIT+ImQMD

Constraining the symmetry energy with heavy-ion collisions and Bayesian analyses

P. Morfouace [a,*], C.Y. Tsang [a], Y. Zhang [b], W.G. Lynch [a], M.B. Tsang [a], D.D.S. Coupland [a], M. Youngs [a], Z. Chajecki [c], M.A. Famiano [c], T.K. Ghosh [e], G. Jhang [a], Jenny Lee [d], H. Liu [f], A. Sanetullaev [a], R. Showalter [a], J. Winkelbauer [a]

Table 1
Model parameter values for prior distribution. 49 sets of calculation have been performed within this 4D model space using a Latin hyper-cube sampling.

Parameter range
$25.7 \leq S_0 \leq 36$ (MeV)
$32 \leq L \leq 120$ (MeV)
$0.6 \leq m_s^*/m_N \leq 1.0$
$0.6 \leq m_v^*/m_N \leq 1.2$



02 Machine learning

PHYSICAL REVIEW LETTERS

Highlights Recent Accepted Collections Authors Referees Search Press About Editori

Editors' Suggestion Open Access

Access by Huzho

QCD Equation of State of Dense Nuclear Matter from a Bayesian Analysis of Heavy-Ion Collision Data

Manjunath Omana Kuttan, Jan Steinheimer, Kai Zhou, and Horst Stoecker
Phys. Rev. Lett. **131**, 202303 – Published 16 November 2023

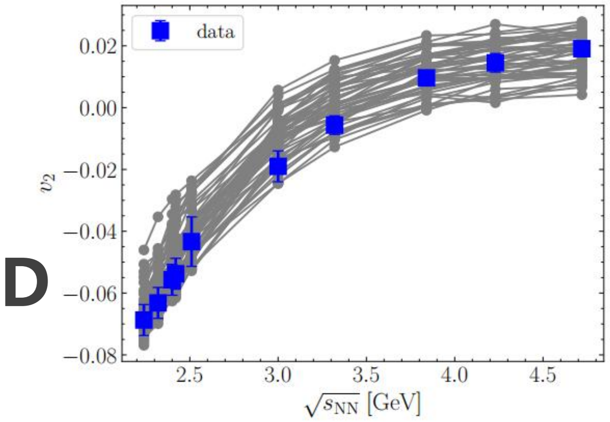
Article | References | Citing Articles (1) | Supplemental Material | PDF | HTML | Export Citation

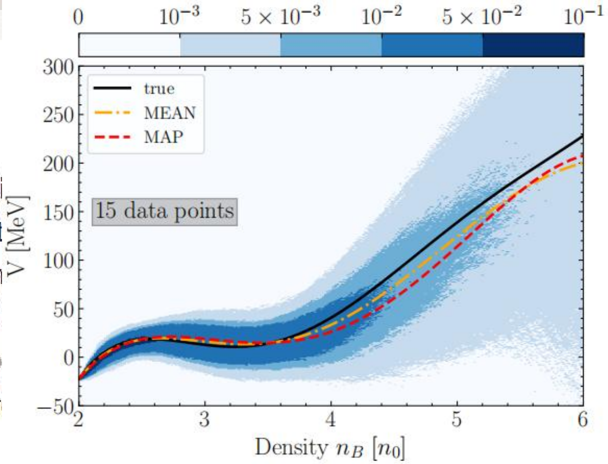
Bayesian inference

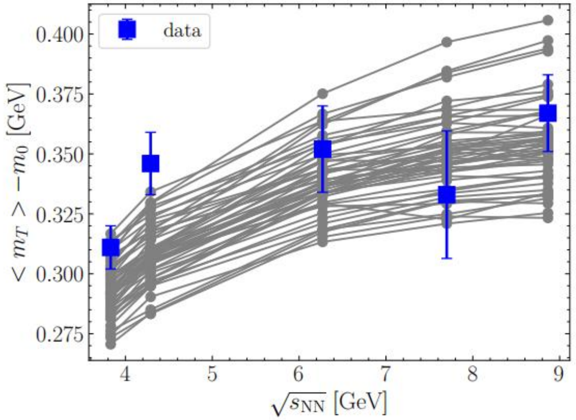
E895、NA49、STAR+UrQMD

ABSTRACT

Bayesian methods are used to constrain the density dependence of the QCD equati
for dense nuclear matter using the data of mean transverse kinetic energy and ellipt
from heavy ion collisions (HICs), in the beam energy range $\sqrt{s_{NN}} = 2\text{–}10$ GeV. Th
tight constraints on the density dependent EOS up to 4 times the nuclear saturation
extracted EOS yields good agreement with other observables measured in HIC exp
constraints from astrophysical observations both of which were not used in the infer







03 Results

3.1 Nuclear symmetry energy

Contents lists available at ScienceDirect

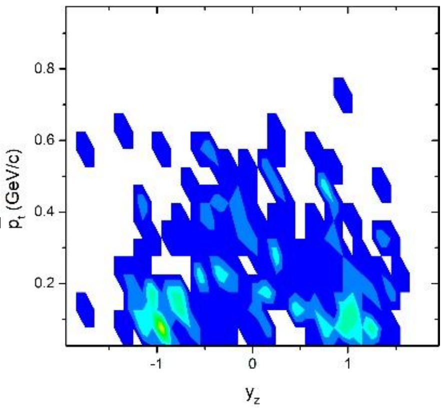
Physics Letters B

www.elsevier.com/locate/physletb

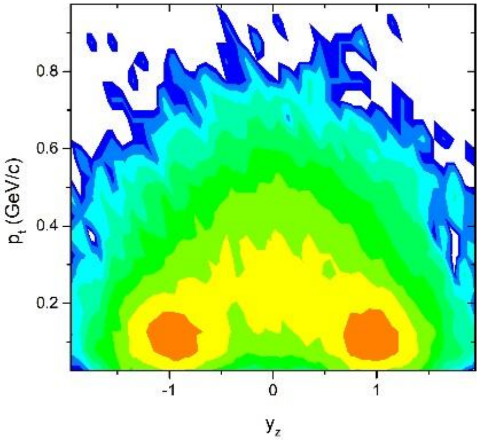
Physics Letters B 822 (2021) 136669

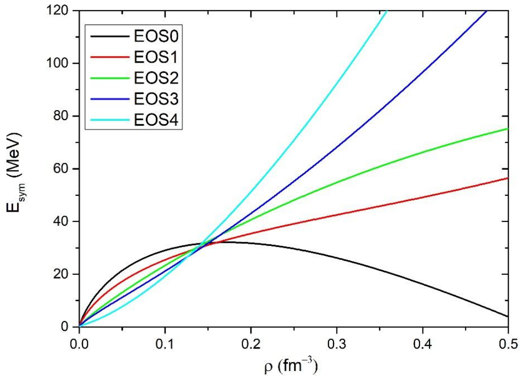
Finding signatures of the nuclear symmetry energy in heavy-ion collisions with deep learning

Yongjia Wang[a,*], Fupeng Li[a,b], Qingfeng Li[a,c,**], Hongliang Lü[d], Kai Zhou[e]

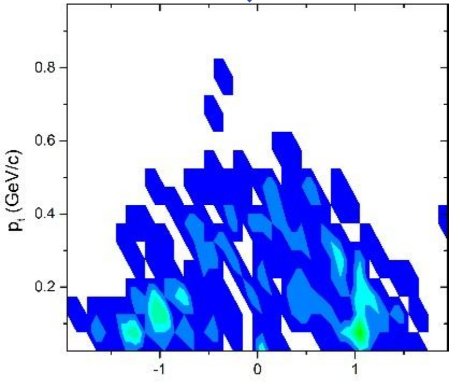


100 events

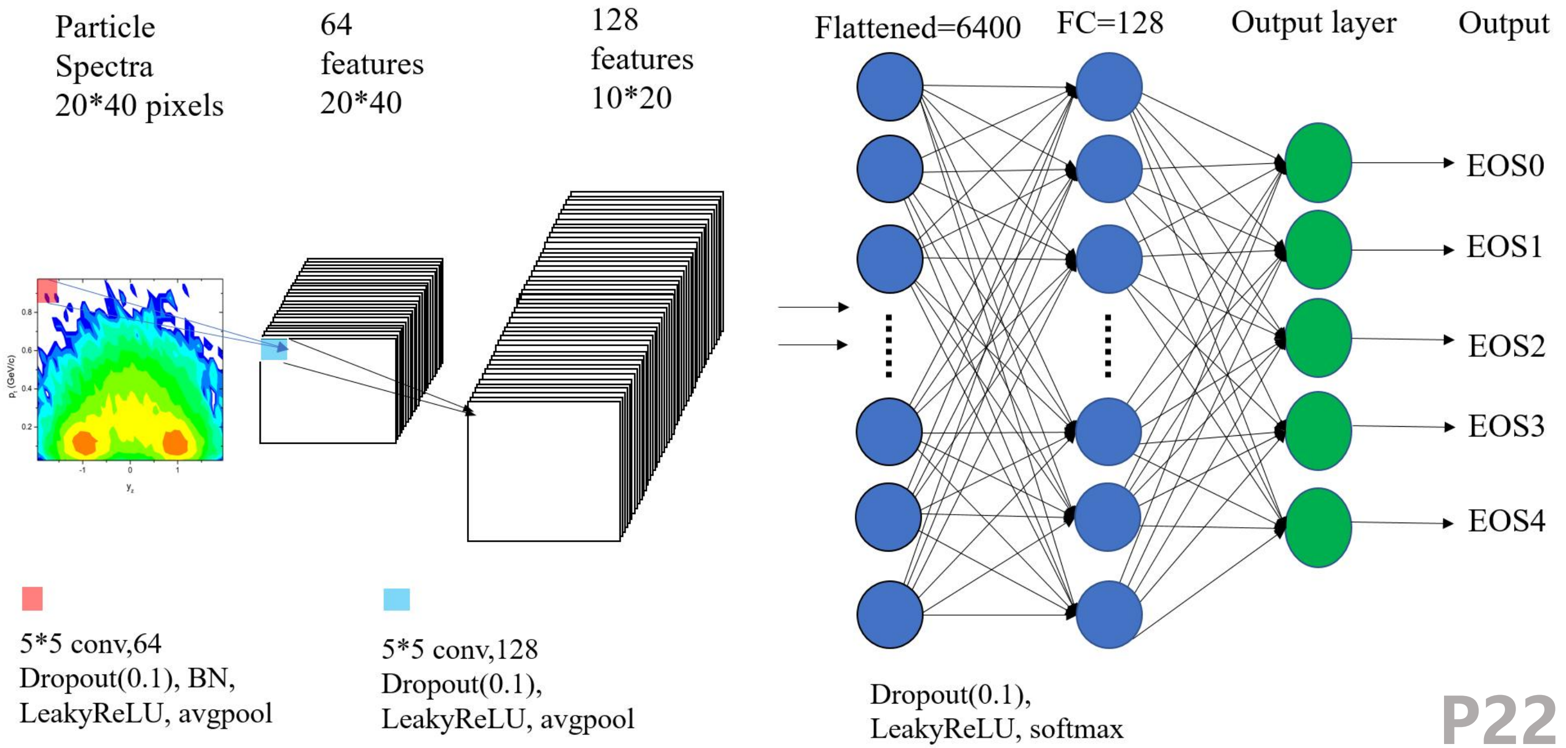




UrQMD



y_z: -2.0 to 2.0 with 40 bins,
p_t: 0 to 1 GeV/c with 20 bins



03 Results

3.1 Nuclear symmetry energy

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

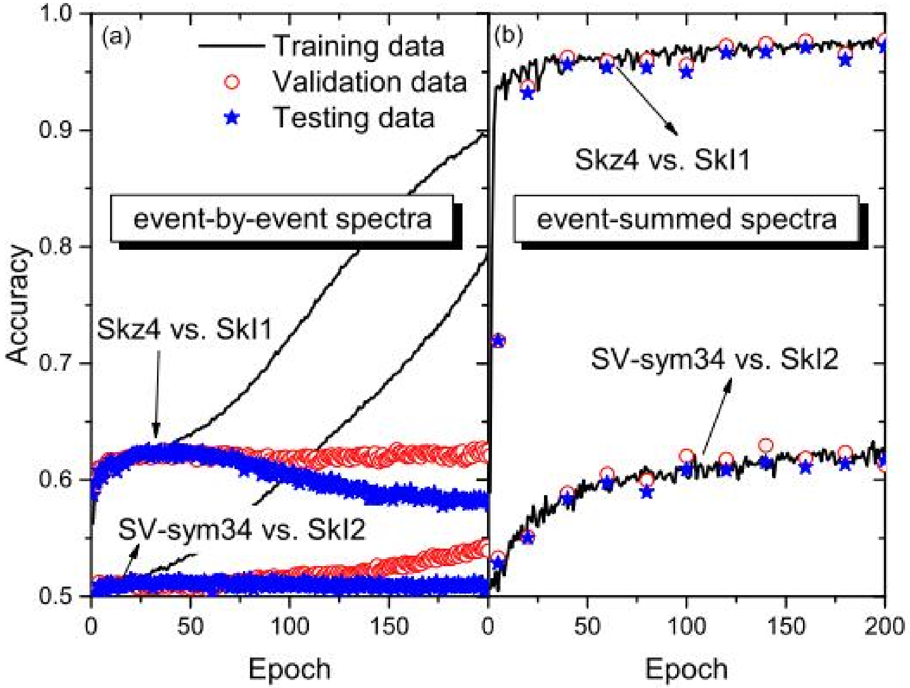
Physics Letters B 822 (2021) 136669

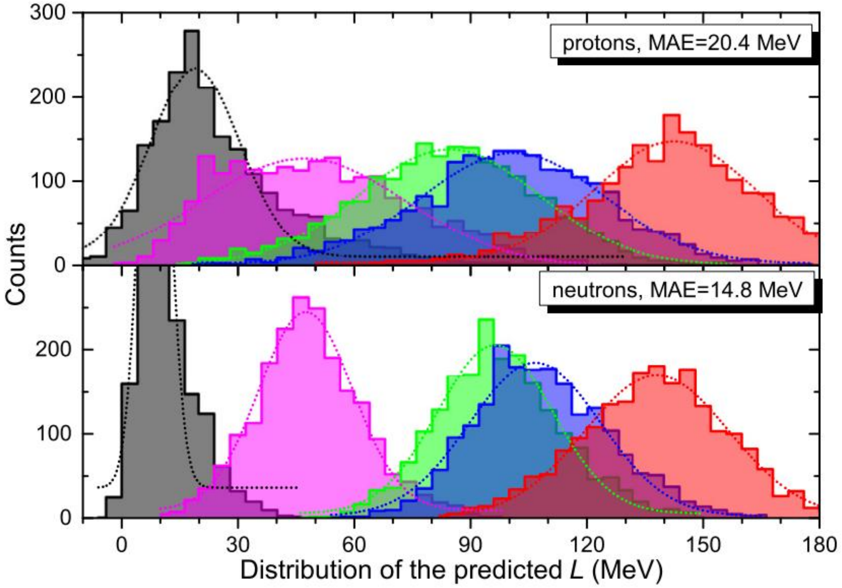
Finding signatures of the nuclear symmetry energy in heavy-ion collisions with deep learning

Yongjia Wang[a,*], Fupeng Li[a,b], Qingfeng Li[a,c,**], Hongliang Lü[d], Kai Zhou[e]

The mean values of predicted L and its standard deviation σ obtained with Gaussian fit.

	L_{true}	Proton spectra		Neutron spectra	
		L_{pred}	σ	L_{pred}	σ
Skz4	5.8	18.9	11.5	8.5	3.7
SLy230a	44.3	46.9	26.5	47.5	12.1
SV-sym36	81.2	84.5	23.3	96.4	14.8
SkI2	106.4	100.9	23.6	106.7	17.0
SkI1	159.0	142.6	20.7	138.1	18.5



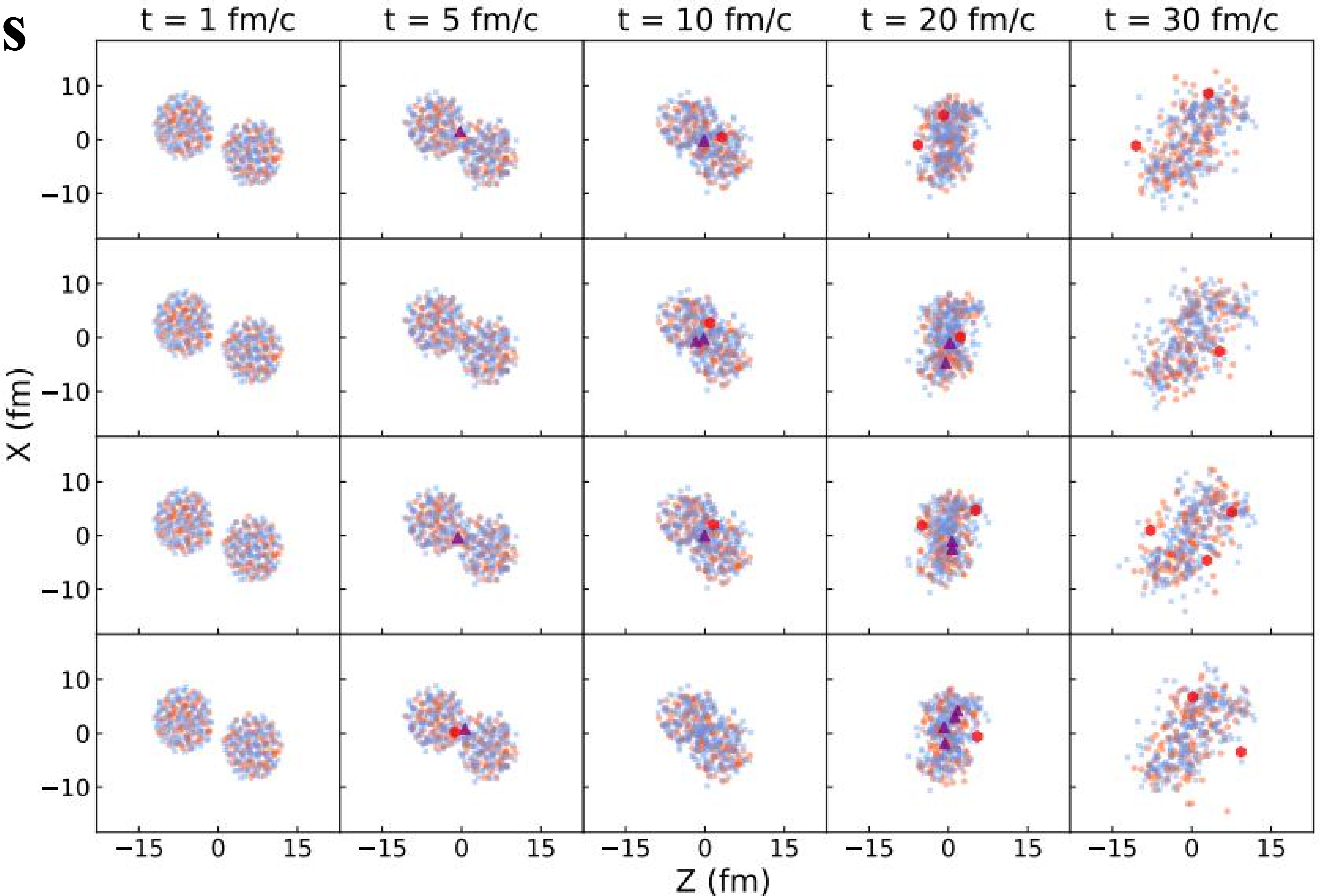


Fingerprints of $E_{sym}(\rho)$ on the transverse momentum and rapidity distributions of protons and neutrons can be identified by convolutional neural network algorithm.

03 Results

Event-by-event
Au+Au
b=5 fm
Elab=0.4A GeV

Can we decode information event-by-event?



03 # Results

3.1 Nuclea

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

ELSEVIER

Physics Letters B 835 (2022) 137508

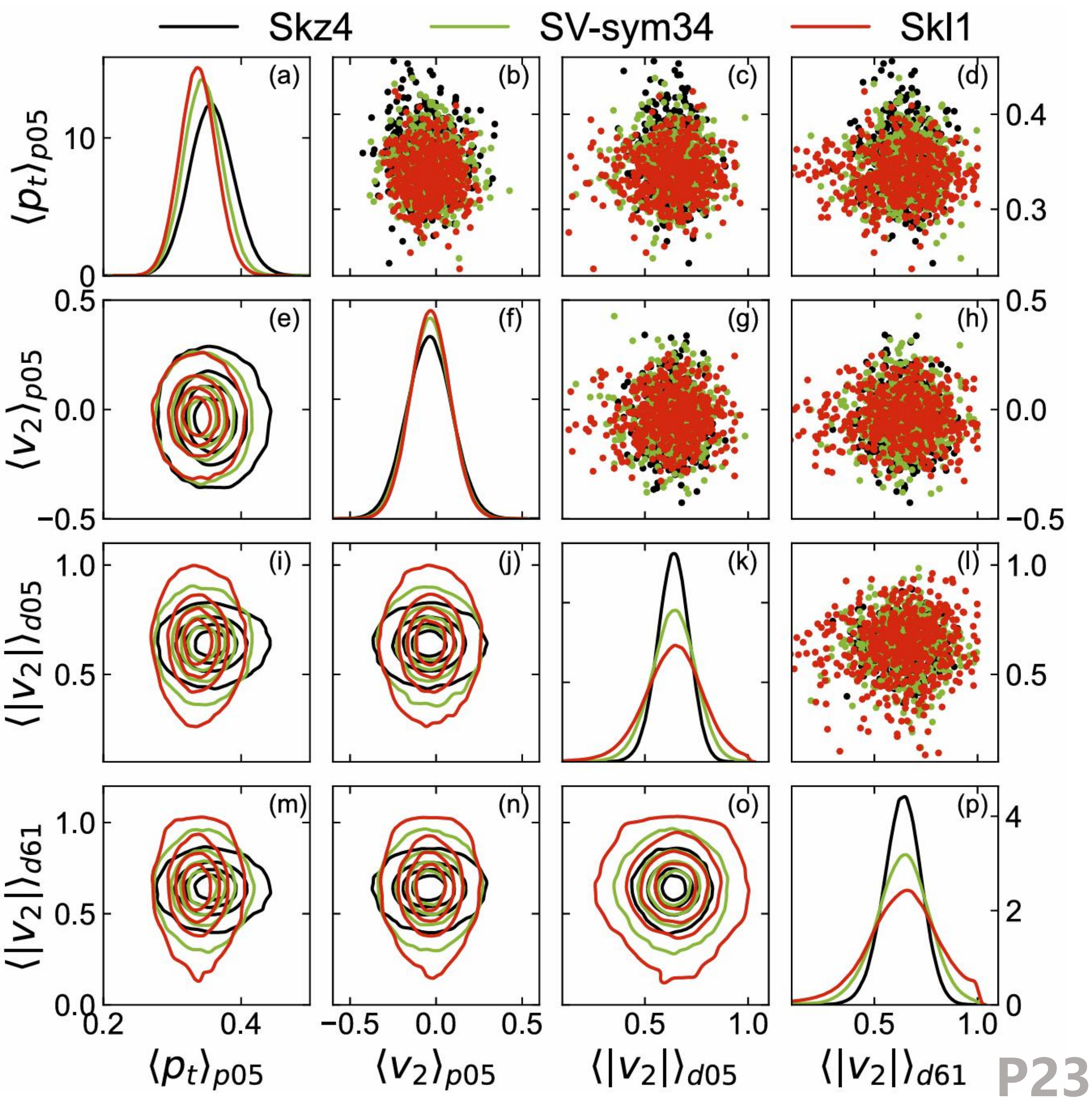
Decoding the nuclear symmetry energy event-by-event in heavy-ion collisions with machine learning

Yongjia Wang [a], Zepeng Gao [a,b], Hongliang Lü [c], Qingfeng Li [a,d,*]

Particles	Rapidity window	Feature	Description
Free protons	$\|y_0\| < 0.5$	$\langle p_t \rangle_{p05}$	Mean value of p_t
		$\langle \|p_x\| \rangle_{p05}$	Mean value of $\|p_x\|$
		$\langle \|p_y\| \rangle_{p05}$	Mean value of $\|p_y\|$
		$\langle v_2 \rangle_{p05}$	Mean value of v_2
		$\langle \|v_1\| \rangle_{p05}$	Mean value of $\|v_1\|$
		$\langle \|v_2\| \rangle_{p05}$	Mean value of $\|v_2\|$
		$\langle \|v_3\| \rangle_{p05}$	Mean value of $\|v_3\|$
	$0.6 < \|y_0\| < 1.0$	$\langle p_t \rangle_{p61}$	Mean value of p_t
		$\langle \|p_x\| \rangle_{p61}$	Mean value of $\|p_x\|$
		$\langle \|p_y\| \rangle_{p61}$	Mean value of $\|p_y\|$
		$\langle v_2 \rangle_{p61}$	Mean value of v_2
		$\langle \|v_1\| \rangle_{p61}$	Mean value of $\|v_1\|$
		$\langle \|v_2\| \rangle_{p61}$	Mean value of $\|v_2\|$
		$\langle \|v_3\| \rangle_{p61}$	Mean value of $\|v_3\|$

Particles	Rapidity window	Feature
Deuterons	$\|y_0\| < 0.5$	$\langle p_t \rangle_{d05}$
		$\langle \|p_x\| \rangle_{d05}$
		$\langle \|p_y\| \rangle_{d05}$
		$\langle \|p_z\| \rangle_{d05}$
		$\langle v_2 \rangle_{d05}$
		$\langle \|v_1\| \rangle_{d05}$
		$\langle \|v_2\| \rangle_{d05}$
		$\langle \|v_3\| \rangle_{d05}$
	$0.6 < \|y_0\| < 1.0$	$\langle p_t \rangle_{d61}$
		$\langle \|p_x\| \rangle_{d61}$
		$\langle \|p_y\| \rangle_{d61}$
		$\langle \|p_z\| \rangle_{d61}$
		$\langle v_2 \rangle_{d61}$
		$\langle \|v_1\| \rangle_{d61}$
		$\langle \|v_2\| \rangle_{d61}$
		$\langle \|v_3\| \rangle_{d61}$

30 event-by-event obseravbles related to momenta of protons and deuterons.



03 Results

3.1 Nuclear symmetry energy

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

ELSEVIER

Physics Letters B 835 (2022) 137508

Decoding the nuclear symmetry energy event-by-event in heavy-ion collisions with machine learning

Yongjia Wang [a], Zepeng Gao [a,b], Hongliang Lü [c], Qingfeng Li [a,d,*]

Feature importance

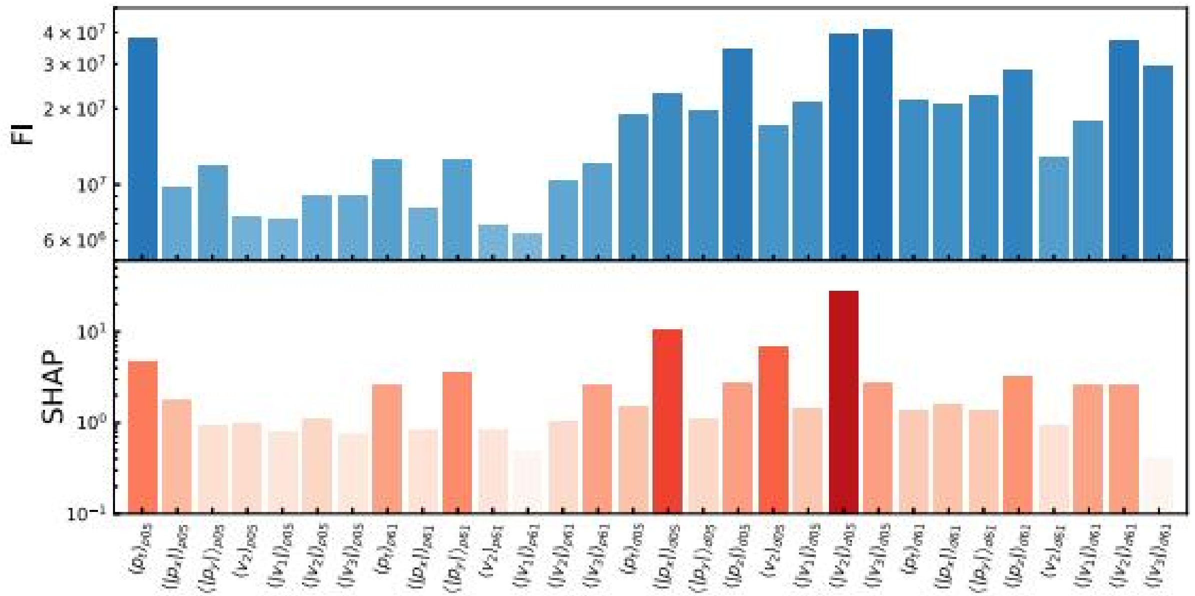


Table 1
The mean values of predicted $L(\rho_0)$ and their standard deviation σ obtained with Gaussian fit. All units are in MeV.

	$L^{true}(\rho_0)$	*Testdata*1 (MAE=29.6) $\langle L^{pred}(\rho_0)\rangle$	σ	*Testdata*2 (MAE=29.4) $\langle L^{pred}(\rho_0)\rangle$	σ	*Testdata*3 (MAE=29.4) $\langle L^{pred}(\rho_0)\rangle$	σ	*Testdata*4 (MAE=27.8) $\langle L^{pred}(\rho_0)\rangle$	σ
Skz4	5.8	44.1	16.8	43.3	16.1	38.4	16.4	48.0	17.0
SLy230a	44.3	52.3	19.4	51.3	17.5	47.3	19.0	58.7	20.2
SV-sym32	57.0	71.3	25.1	69.1	23.2	66.6	25.3	82.9	25.8
SV-sym34	81.2	78.8	27.2	76.6	24.8	73.9	27.2	93.0	27.6
SkI2	106.4	82.8	27.9	79.6	25.7	77.7	28.1	98.6	28.2
SkI1	159.0	114.9	29.7	110.3	29.8	109.7	31.5	140.8	22.6

Good generalizability.

- **Fingerprints of $E_{sym}(\rho)$ can be decoded from a large set of observables in HICs on an event-by-event basis by the trained machine learning algorithm.**
- **With feature attribution methods, the most important features that drive predictions can be identitied.**

3.2 Bayesian Inference of the in-medium nucleon-nucleon cross section

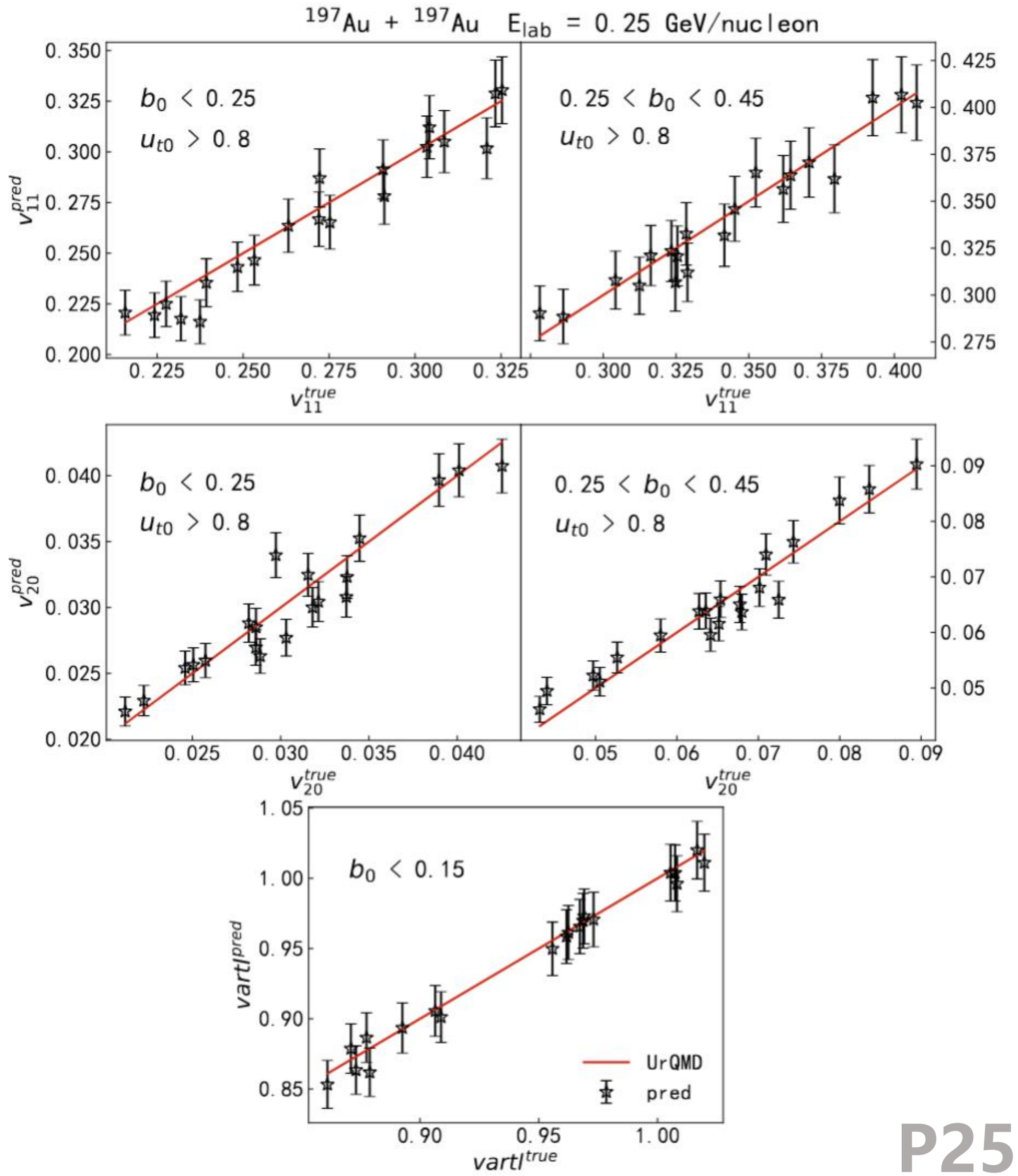
Table I. List of observables used in the analysis in ^{197}Au + ^{197}Au collisions with a beam energy of 0.25 GeV/nucleon

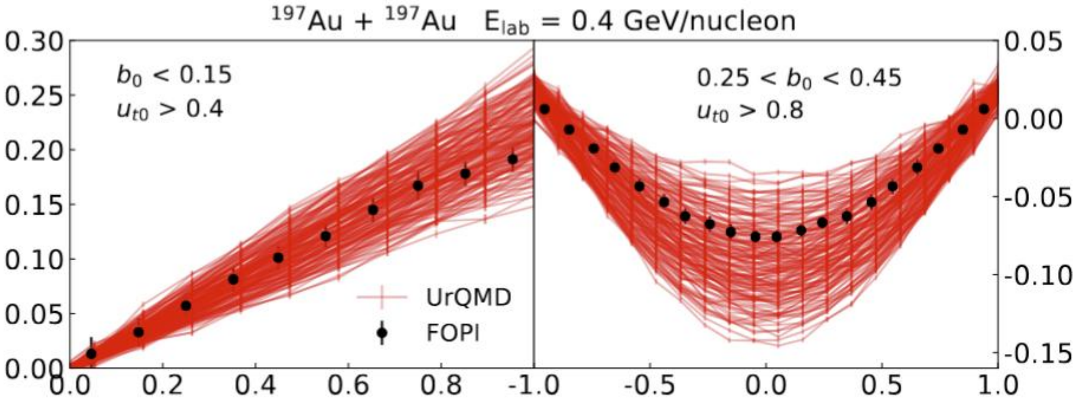
Observable	b_0	u_{t0}	value
v_{11}	$b_0 < 0.25$	$u_{t0} > 0.8$	0.23±0.01
	$0.25 < b_0 < 0.45$	$u_{t0} > 0.8$	0.37±0.01
$-v_{20}$	$b_0 < 0.25$	$u_{t0} > 0.8$	0.026±0.001
	$0.25 < b_0 < 0.45$	$u_{t0} > 0.8$	0.046±0.005
$vartl$	$b_0 < 0.15$	None	0.891±0.041

Table III. Parameters used in the present work

Para. Name	Description	Prior ranges
K_0	Incompressibility	[180, 380]
m^*	Isoscalar effective mass	[0.6, 0.95]
F	In-medium correction factor	[0.5, 1.0]

Gaussian process (GP) model is trained as an emulator of UrQMD model to interpolate the simulation results in the parameter space.

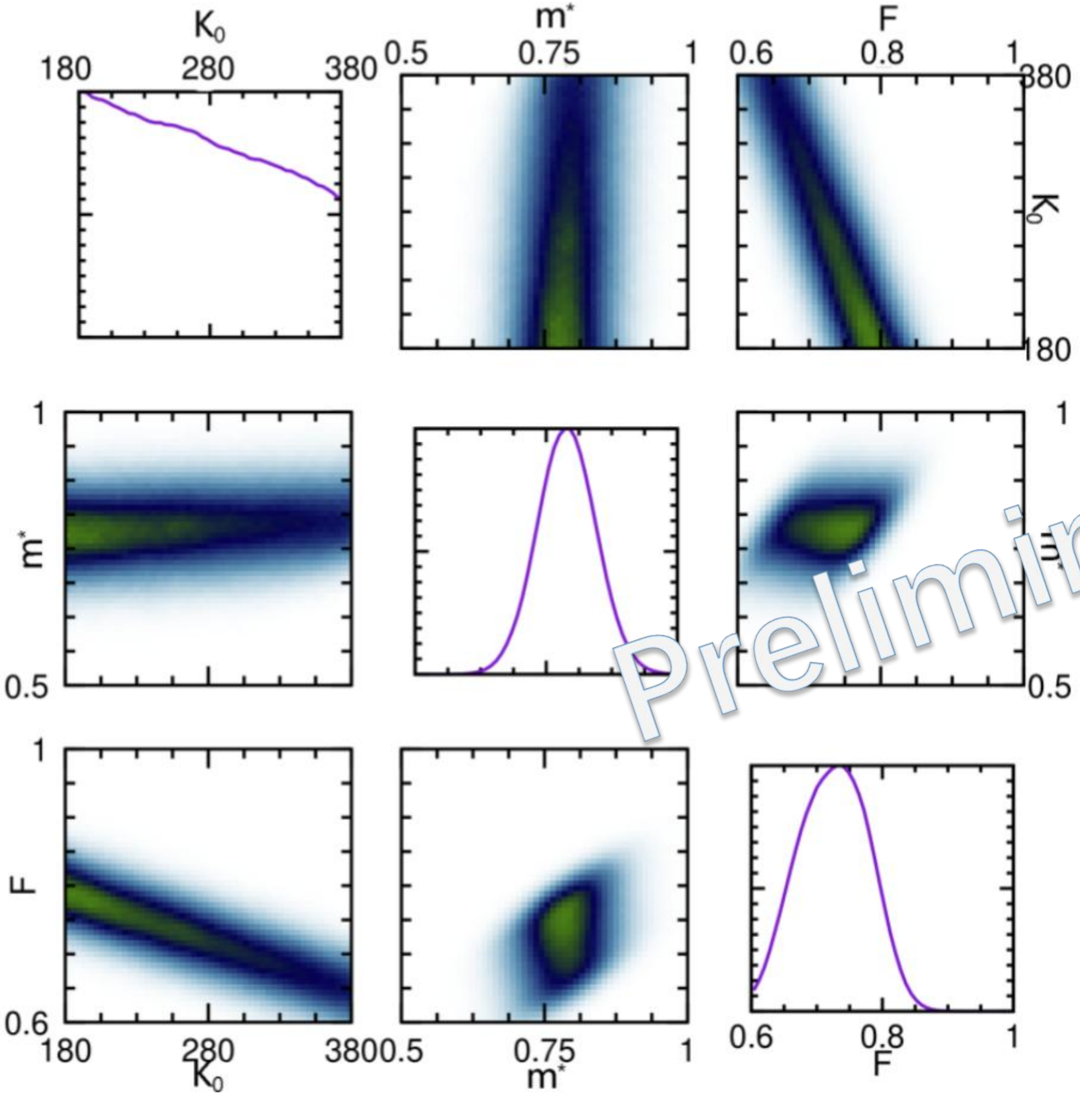


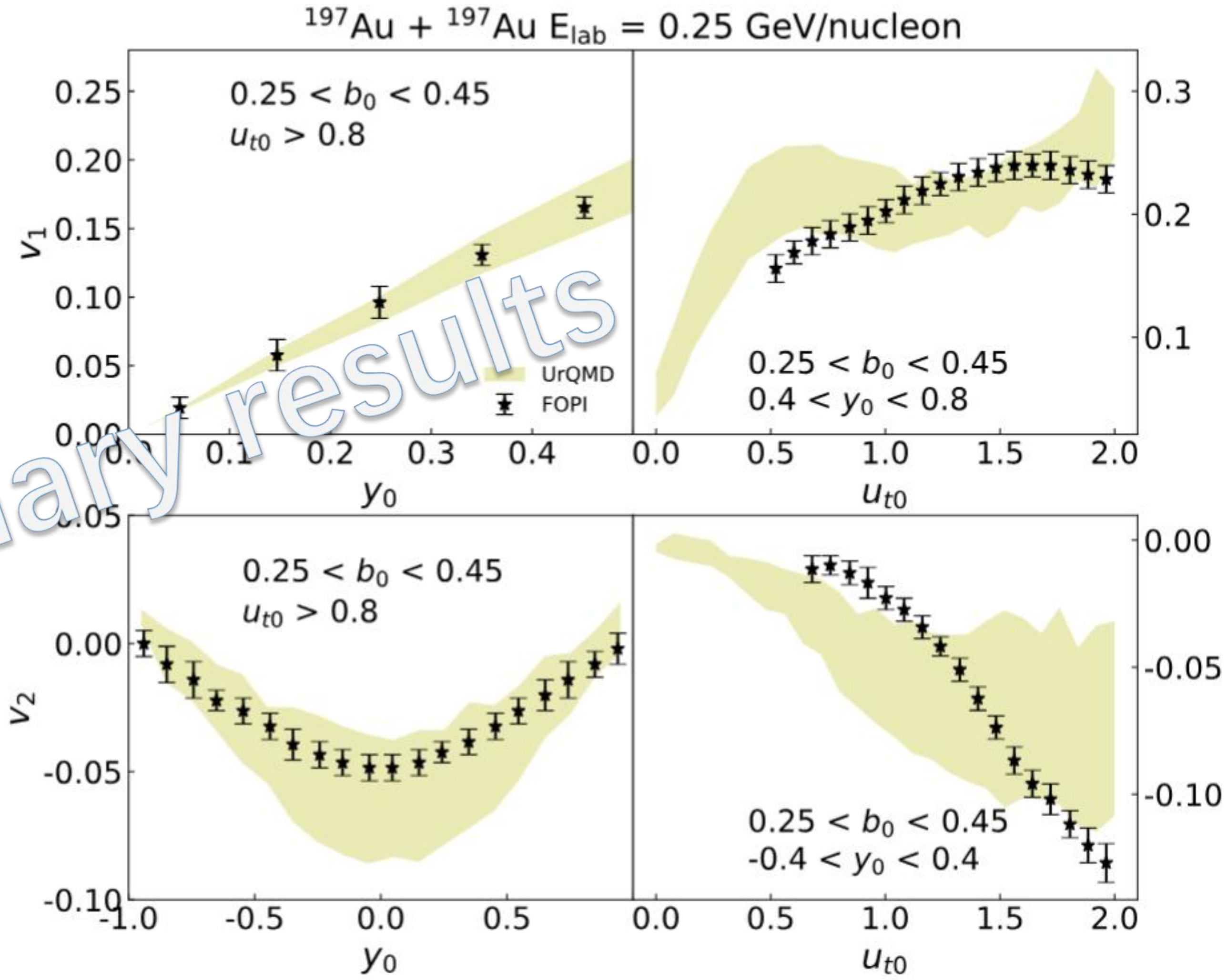


03

Results

3.2 Bayesian Inference of the in-medium nucleon-nucleon cross section





Summary and outlook

FRONTIERS OF PHYSICS

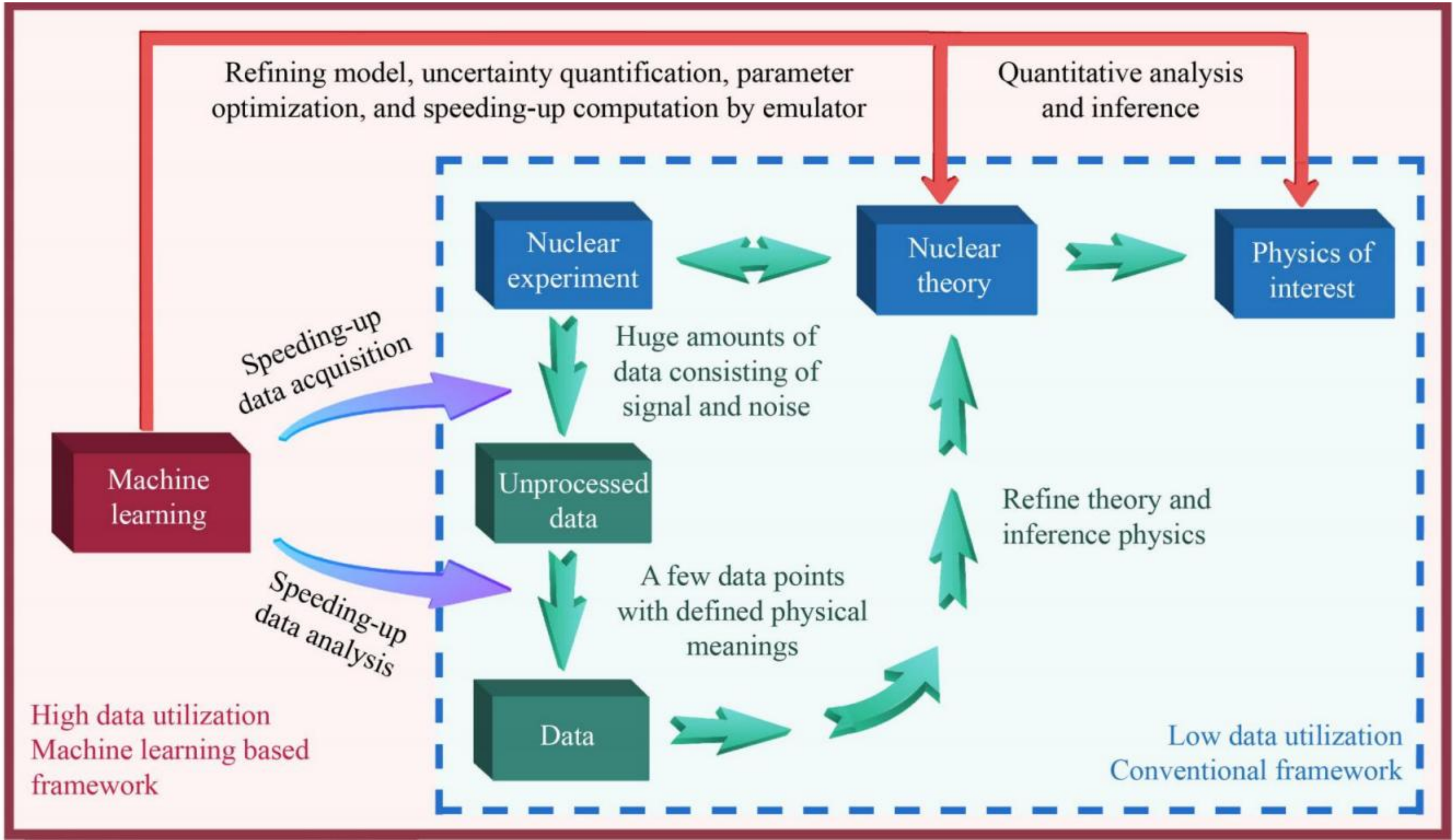
VIEW & PERSPECTIVE
Volume 18 / Issue 6 / 64402 / 2023

Machine learning transforms the inference of the nuclear equation of state

Yongjia Wang[1], Qingfeng Li[1,2,3,†]

Advantages:

- ✓ constraining multi- parameters from multi- observables
- ✓ estimating uncertainties



04 Summary and Outlook

Mergence of ML and heavy-ion physics

01 Improving the quality of data

Developing more sophisticated models, or using different models to generate data

02 Introducing physical information into ML algorithms

Using input features with defined physical meanings or by considering physical symmetries and laws when constructing architectures of ML algorithms

03 Using experiences of ML applications in other fields

Condensed matter physics and particle physics.

04 Introducing the latest developments of ML into tools for studying nuclear physics

A diverse array of ML algorithm has been developed and continue to be refined to cover a wide variety of data types and tasks, this is a sufficiently large and diverse pool of tools feasible to study heavy-ion physics

Thanks!