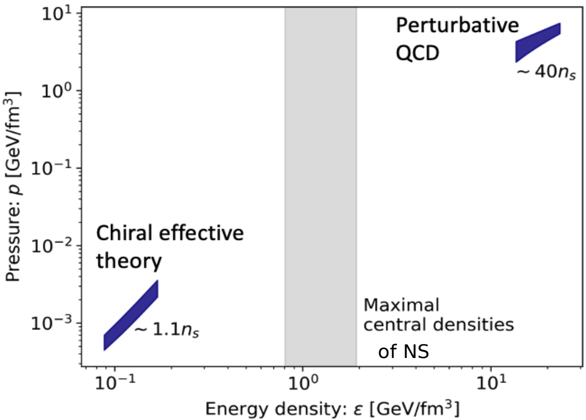
Calibrating the global behaviour of EoS in a machine learning approach

Adil Imam

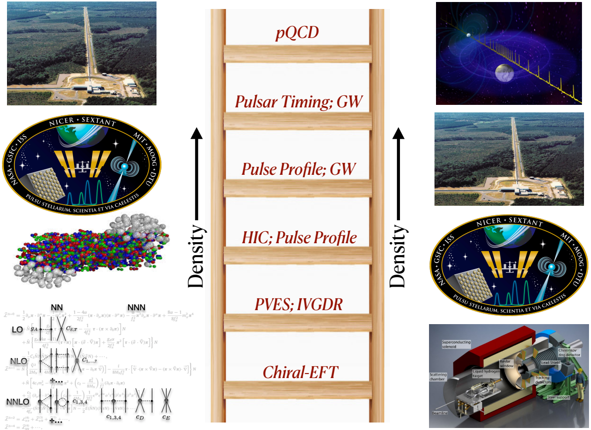
Based on : **arXiv:2407.08553**

INTRODUCTION : SIMPLISTIC VIEW OF PHASE DIAGRAM



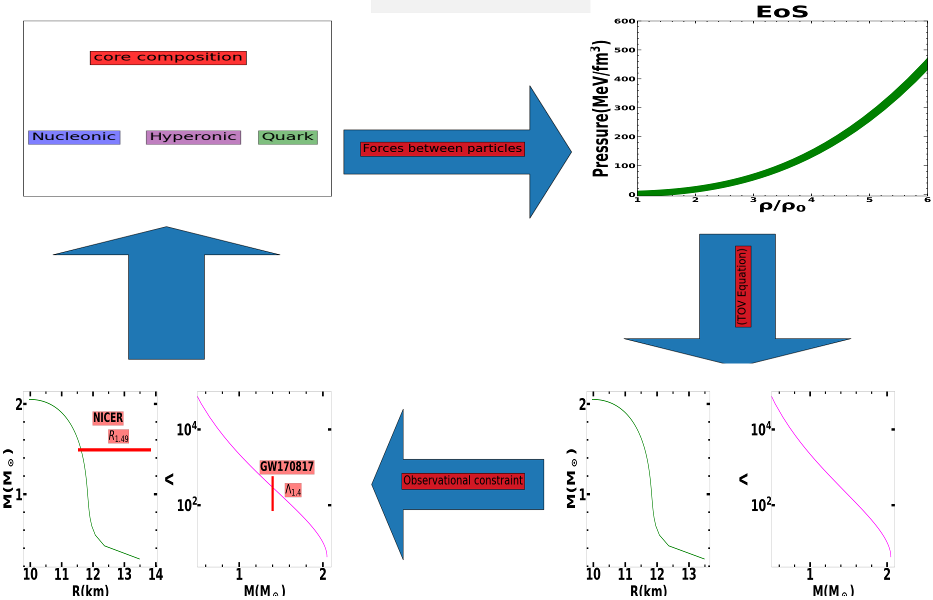
arXiv:2211.11414v1

THE DENSITY LADDER



arXiv:2304.05441v1

Micro-Macro collision



THE EOS FOR NS MATTER :

NUCLEONIC CORE COMPOSITION

- The NS core is composed of neutron (n), proton(p), electron(e) and muon(μ)

The EoS for NS matter :

Nucleonic Core Composition

- The NS core is composed of neutron (n), proton(p), electron(e) and muon(μ)
- The EoS is determined by the features of nucleon-nucleon (N-N) interaction

THE EOS FOR NS MATTER

THE PARABOLIC APPROXIMATION :

- Energy per nucleon for neutron star matter at a given nucleon density ρ and asymmetry $\delta = (\frac{\rho_n}{\rho} - \frac{\rho_p}{\rho})$ can be written as,

$$\varepsilon(\rho, \delta) = \sum \frac{\partial^n}{\partial \delta^n} \varepsilon(\rho, \delta)|_{\delta=0} = e(\rho, 0) + e_{sym}(\rho)\delta^2 + ...,$$

$e(\rho, 0)$: Energy for SNM and
$e_{sym}(\rho) = \varepsilon(\rho, 1) - e(\rho, 0)$: Density dependent symmetry energy

THE EOS FOR NS MATTER

THE PARABOLIC APPROXIMATION :

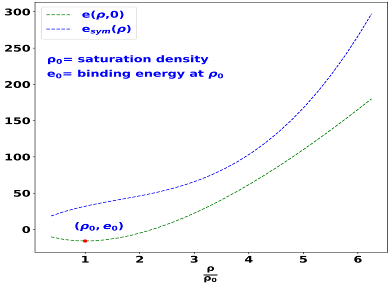


FIGURE: Energy per nucleon for Symmetric Nuclear Matter (SNM)((green curve) and symmetry energy (blue curve)

- Nuclear Matter Parameters(NMPs) $\propto \frac{\partial^n}{\partial\rho^n} e(\rho, 0)$ or $\propto \frac{\partial^n}{\partial\rho^n} e_{sym}(\rho)$

THE EOS OF NS MATTER

NUCLEAR MATTER PARAMETERS (NMPS) :

- Taylor expansion : $\varepsilon(\rho, \delta) = \sum_{n=0}^{4}(a_n + b_n\delta^2)(\frac{\rho-\rho_0}{3\rho_0})^n$

Nuclear Matter Parameters (NMPs) :

- Taylor expansion : $\varepsilon(\rho,\delta) = \sum_{n=0}^{4}(a_n + b_n\delta^2)(\frac{\rho-\rho_0}{3\rho_0})^n$
- **SNM parameters (at ρ_0) :**
- $e_0 \equiv$ Binding energy $= e(\rho,0)|_{\rho=\rho_0} = a_0$
- $K_0 \equiv$ Incompressibility coefficient $\propto \frac{\partial^2 e(\rho,0)}{\partial\rho^2}|_{\rho=\rho_0} = a_2$
- $Q_0(Z_0) \equiv$ Third(Fourth)order derivative $= a_3(a_4)$
- **Symmetry Energy parameters (at ρ_0) :**
- $J_0 \equiv$ Symmetry energy $= e_{sym}(\rho)|_{\rho=\rho_0} = b_0$
- $L_0 \equiv$ Slope of symmetry energy $\propto \frac{\partial e_{sym}(\rho)}{\partial\rho}|_{\rho=\rho_0} = b_1$
- $K_{sym,0} \equiv$ Curvature of symmetry energy $\propto \frac{\partial^2 e_{sym}(\rho)}{\partial\rho^2}|_{\rho=\rho_0} = b_2$
- $Q_{sym,0}(Z_{sym,0}) \equiv$ Third(Fourth)order derivative $= b_3(b_4)$

The EoS of NS matter

The $\frac{n}{3}$ Expansion model :

- $\frac{n}{3}$ Expansion: $\varepsilon(\rho, \delta) = \sum_{n=2}^{6}(c_n + d_n\delta^2)(\frac{\rho}{\rho_0})^{n/3}$

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$$\begin{pmatrix} e_0 \\ 0 \\ K_0 \\ Q_0 \\ Z_0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 & 6 \\ -2 & 0 & 4 & 10 & 18 \\ 8 & 0 & -8 & -10 & 0 \\ -56 & 0 & 40 & 40 & 0 \end{pmatrix} \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}$$

$$\begin{pmatrix} J_0 \\ L_0 \\ K_{\mathrm{sym},0} \\ Q_{\mathrm{sym},0} \\ Z_{\mathrm{sym},0} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & 5 & 6 \\ -2 & 0 & 4 & 10 & 18 \\ 8 & 0 & -8 & -10 & 0 \\ -56 & 0 & 40 & 40 & 0 \end{pmatrix} \begin{pmatrix} d_0 \\ d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix}.$$

Construction of EoS

Physical constraints on the EoS :

Each EoS must satisfies :

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- Symmetry energy : $e_{sym}(\rho) > 0$
- $M_{TOV} \geq$ mass of observed massive NS (non-rotating)
  (M_{TOV} = maximum mass in the stable branch)

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- The parameters which are almost unknown:
  $Q_0, Z_0, K_{sym,0}, Q_{sym,0}, Z_{sym,0}$

- Constraining the EoS means constraining these parameters

Reverse Engineering : Bayesian Inference

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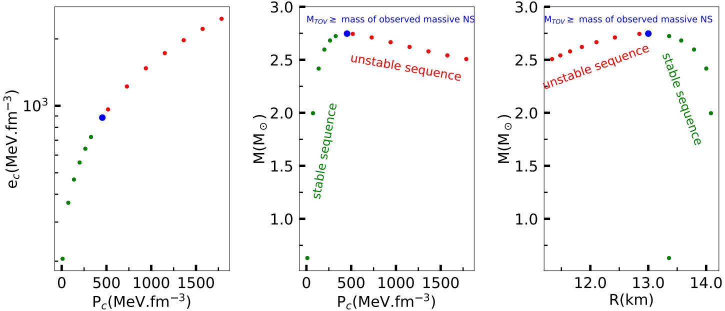
Calibrating the global behaviour of EoS

- NMPs $\longrightarrow$ EoS $\longrightarrow$ TOV $\longrightarrow$ NS properties

- NMPs ⟶ EoS ⟶ TOV ⟶ NS properties



- For one single EoS with 100 stars in the stable sequence takes 1 second then for 10^6 EoS it takes ∼ 11 days with a single CPU core

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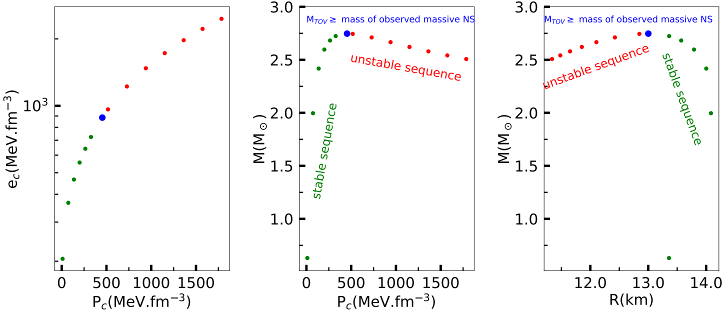

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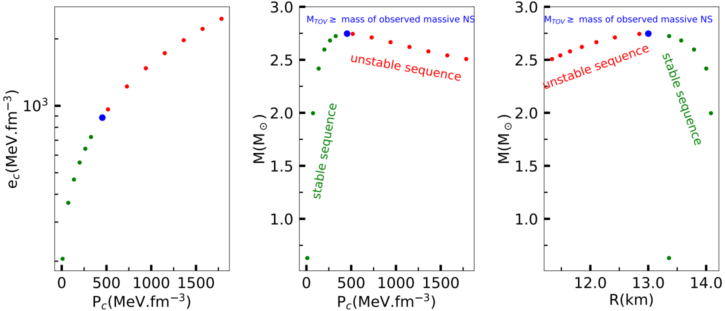

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- We use a machine learning approach : Symbolic Regression

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Symbolic Regression :

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- The symbolic regression process involves searching through a space of mathematical expressions
- A K-fold validation procedure will give multiple equations in each of the k fold
- To choose the best equation one has to perform a goodness of fit

SYMBOLIC REGRESSION :

- $$R^2 = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}$$

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- $$\text{RMSE} = \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{n}}$$

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-

$$
\begin{aligned}
M_{\max} &= 0.14\hat{K}_0 - 0.02\hat{L}_0 + 0.27\hat{Q}_0 + 0.16\hat{Z}_0 + 2.33 \\
R_{1.4} &= -0.31(\hat{J}_0 - \hat{Q}_0 - \hat{Q}_{\mathrm{sym}0}) - 0.62\hat{K}_{\mathrm{sym}0} + 1.75\hat{L}_0 + 14.22 \\
\Lambda_{1.4} &= -54.02(\hat{J}_0 - \hat{K}_0 - \hat{Q}_0 - \hat{Q}_{\mathrm{sym}0}) + 76.09\hat{K}_{\mathrm{sym}0} \\
&+ 178.12\hat{L}_0 + 737.93
\end{aligned}
$$

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CALIBRATING THE GLOBAL BEHAVIOUR OF EOS

DATA FOR BAYESIAN INFERENCE :

- 12 experimental data on symmetry energy, pressure due to symmetry energy and symmetric nuclear matter
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- GW170817,mass-radius posterior data from NICER

Mapping NS properties to NMPs

Importance of E_{pnm} and M :

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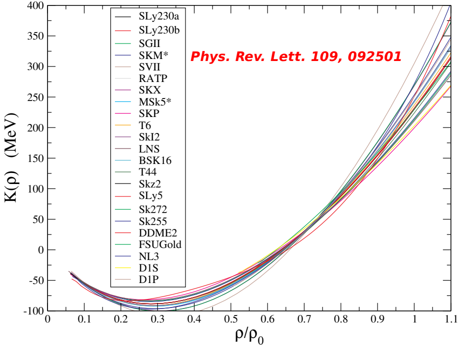

FIGURE: Variation of incompressibility coefficient, K with density.

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TABLE: S_0 : All the data except E_{pnm} and M_c; S_{ALL} : All the data.

	Scenario	K_0	Q_0	J_0	L_0	K_{sym0}
ref	-	266	-90	33.17	67	-47
$\mathcal{T}_{SRM}$	S_0	256^{+46}_{-56}	-148^{+255}_{-266}	$32.92^{+1.36}_{-1.33}$	65^{+14}_{-14}	-27^{+81}_{-79}
	S_{ALL}	262^{+21}_{-23}	-167^{+175}_{-194}	$32.97^{+1.29}_{-1.26}$	65^{+14}_{-14}	-26^{+79}_{-80}
$\mathcal{T}_{TOV}$	S_{ALL}	262^{+20}_{-20}	-203^{+182}_{-176}	$32.74^{+1.24}_{-1.24}$	63^{+14}_{-13}	-34^{+76}_{-74}

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- Bayesian inferences are performed with realistic data including E_{pnm} and M_c

Calibrating the global behaviour of EoS

TOV vs Symbolic Regression models :

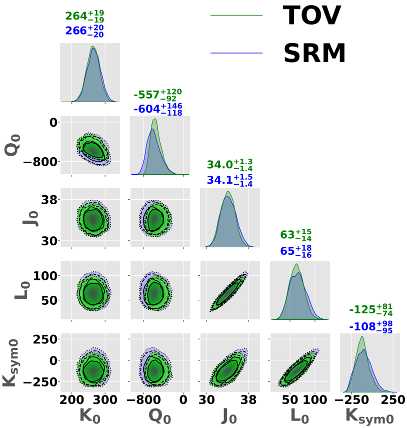

Figure: Posterior obtained using symbolic regression models and TOV solutions

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- Performing Bayesian inference to calibrate the EoS using NS observation is computationally very expensive
- Symbolic regression based equations are constructed to replace EoS computation and solutions of TOV equations
- Using TOV and SRMs in Bayesian inference give comparable results while SRMs provides results within 20 minutes which is 100 times faster than the traditional TOV approach

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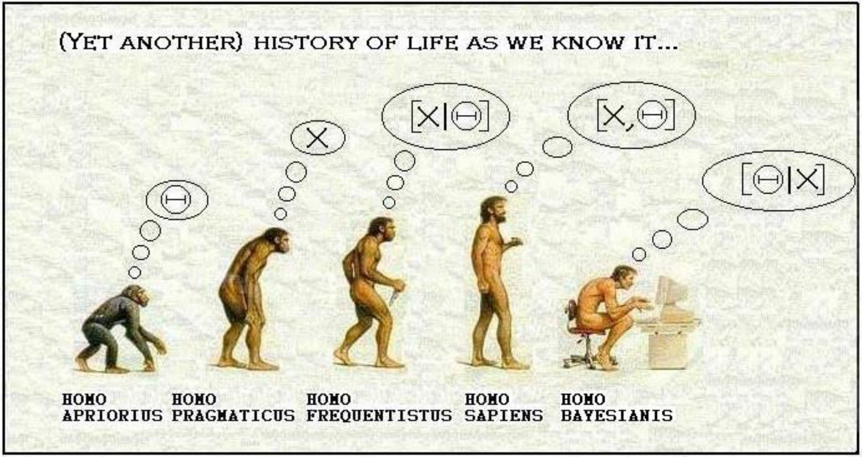
2. Dr. Tuhin Malik
3. Dr. Naresh K. Patra

Thank You



(YET ANOTHER) HISTORY OF LIFE AS WE KNOW IT...
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X
Θ
HOMO APRIORIUS
HOMO PRAGMATICUS
HOMO FREQUENTISTUS
HOMO SAPIENS
HOMO BAYESIANIS