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中山大学中法核工程与技术学院
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Bayesian model averaging for nuclear symmetry energy from effective proton-neutron chemical potential difference of neutron-rich nuclei

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12 September 2024

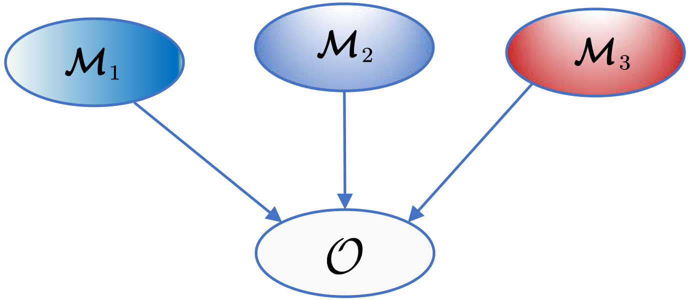
Why Model Averaging?

George E.P. Box

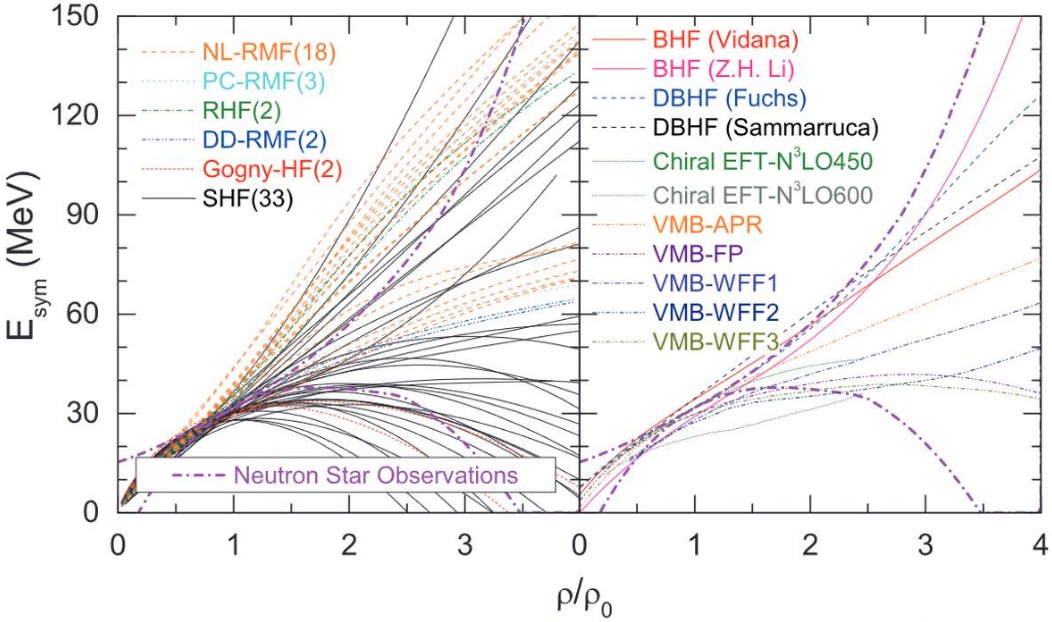
"Essentially,

All models are wrong, but some are useful"

"Which model should we trust?"



Why Model Averaging for symmetry energy?



L.W.Chen, Nucl.Phys.Rev.34,20 (2017).
N.B.Zhang, B.A.Li, Eur.Phys.J.A 55,39(2019)

- ☐ Large uncertainty remains
- Intra-model uncertainty

  Experimental data uncertainty
  Theoretical uncertainty
  Correlation between parameters
  …
- Inter-model uncertainty

  Variations across different models

→ **Possible bias**

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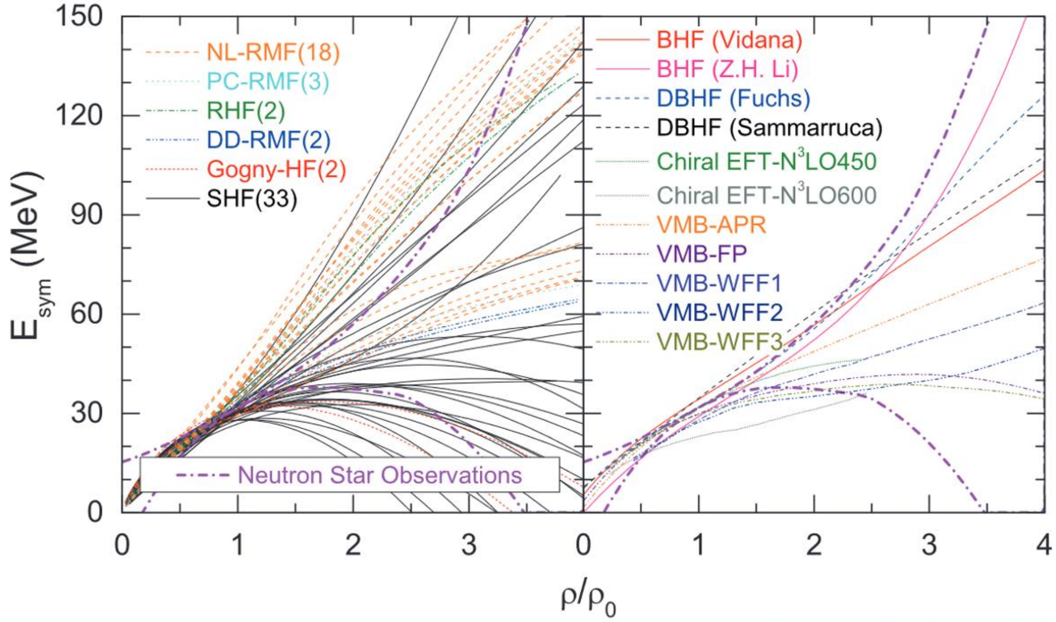

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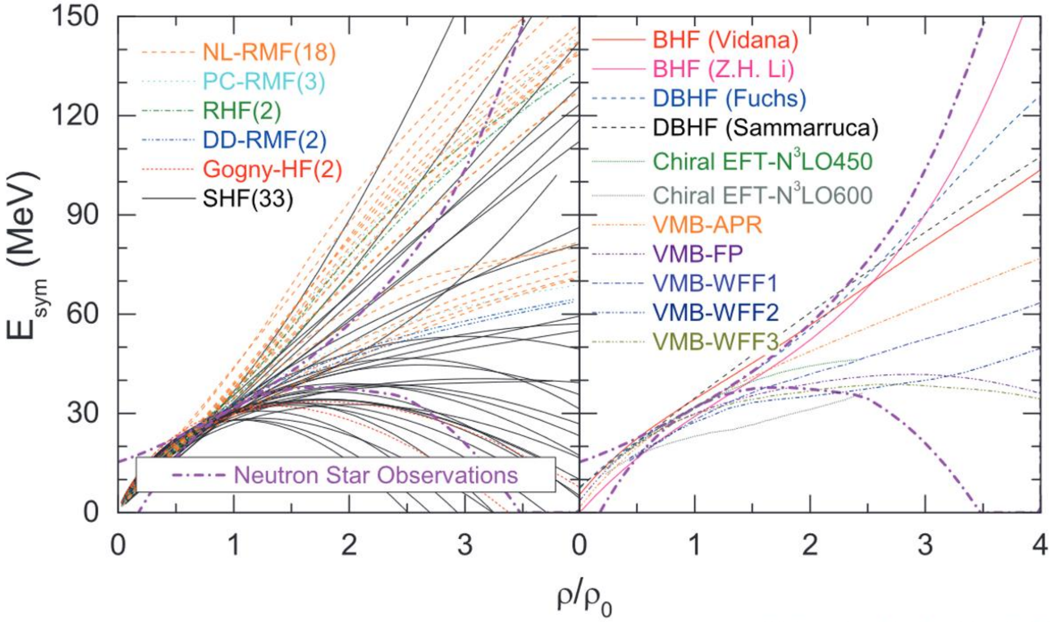

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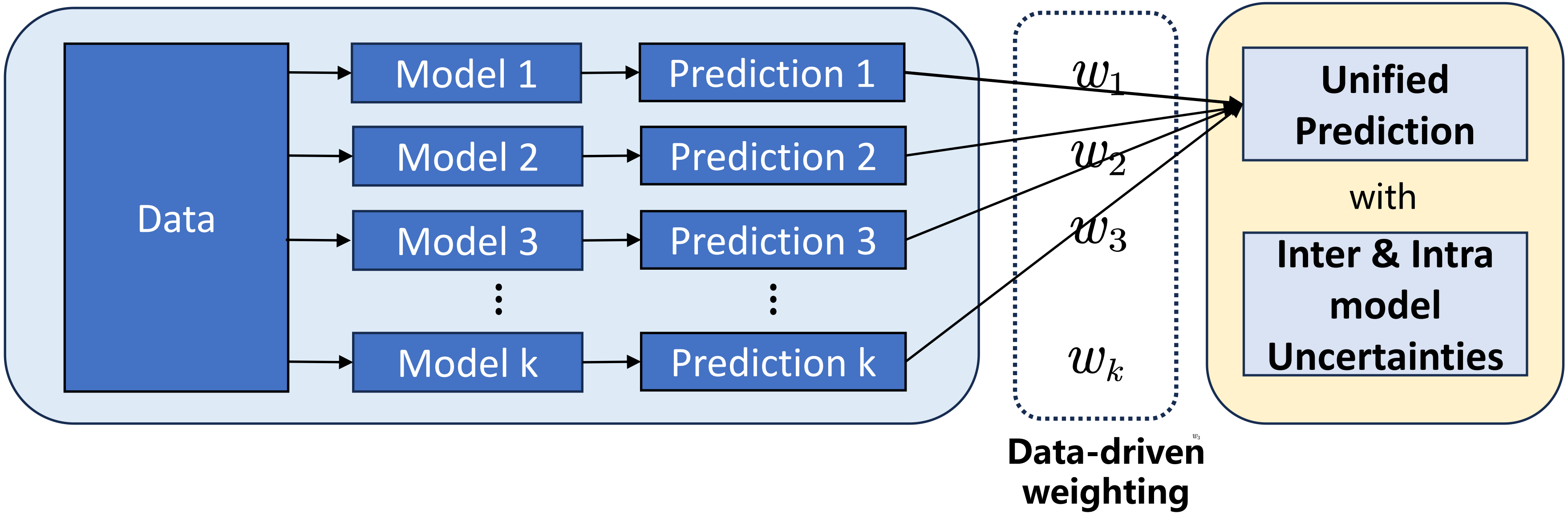
How to deal with it ?

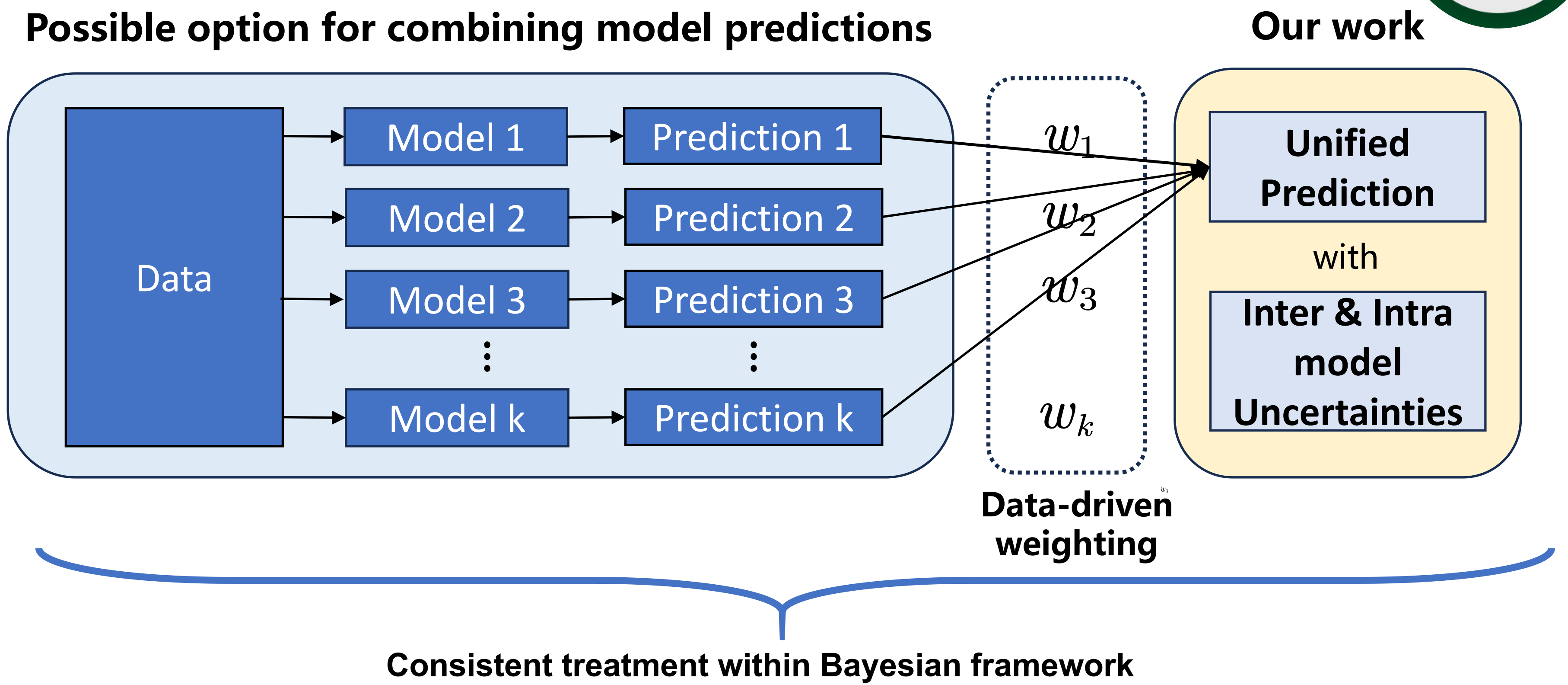
Model Averaging

Possible option for combining model predictions

Our work



Model Averaging



Bayesian Analysis

Under model $\mathcal{M}$' s assumption

The likelihood function of observing y given the model $\mathcal{M}$ predictions at Θ

The prior probability of quantities of interest Θ before being confronted with the experimental measurements y

$$p(\mathcal{O}|\mathbf{y},\mathcal{M}) = \frac{\mathcal{L}(\mathbf{y}|\mathcal{O},\mathcal{M})\pi(\mathcal{O}|\mathcal{M})}{p(\mathbf{y}|\mathcal{M})}$$

The posterior probability distribution of quantities of interest Θ given experimental measurements y

The marginal likelihood/Evidence The probability of model $\mathcal{M}$ giving experimental measurements y

Bayesian Model Averaging (BMA)

- Each model's contribution is weighted by its model posterior probability

$$p(\mathcal{O}|\boldsymbol{y}) = \sum_i p(\mathcal{O}|\boldsymbol{y}, \mathcal{M}_i)\, p(\mathcal{M}_i|\boldsymbol{y})$$

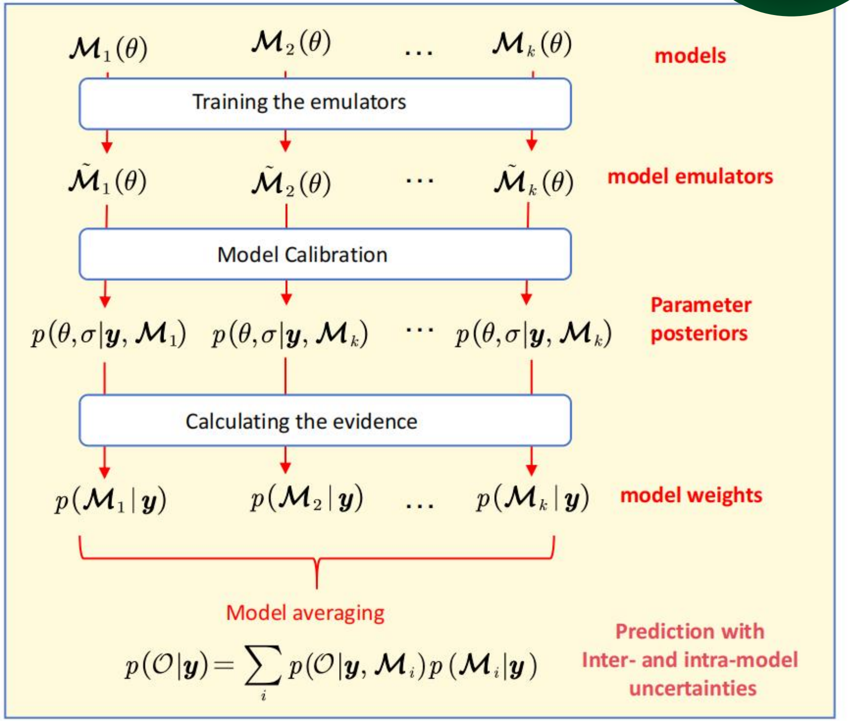
- Model posterior probability: a weighting factor

$$p(\mathcal{M}_i|\boldsymbol{y}) = \frac{p(\boldsymbol{y}|\mathcal{M}_i)\,\pi(\mathcal{M}_i)}{\sum_\ell p(\boldsymbol{y}|\mathcal{M}_\ell)\,\pi(\mathcal{M}_\ell)}$$

- The model prior $\pi(M_i)$ is our preference on M_i before seeing the data

- Bayesian evidence/marginal likelihood: measures the probability that the model reproduces the experimental data

$$p(\boldsymbol{y}|\mathcal{M}_i) = \int p(\boldsymbol{y}|\theta_i, \sigma_i, \mathcal{M}_i)\,\pi(\theta_i, \sigma_i|\mathcal{M}_i)\, d\theta_i d\sigma_i$$



V. Cirigliano *et al*, J. Phys. G 49, 120502 (2022)

Effective Proton-neutron chemical potential difference

- Effective chemical potential

$$\mu_{\rm n} = \frac{\partial B(N,Z)}{\partial N} \approx \frac{B(N+2,Z)-B(N-2,Z)}{4}, \quad (1)$$

$$\mu_{\rm p} = \frac{\partial B(N,Z)}{\partial Z} \approx \frac{B(N,Z+2)-B(N,Z-2)}{4}, \quad (2)$$

- Proton-neutron chemical potential differences

$$\Delta\mu^*_{\rm pn} = \frac{1}{4}\left[B(N,Z+2)-B(N,Z-2) - B(N+2,Z)+B(N-2,Z)\right]$$

- Semi empirical mass formula

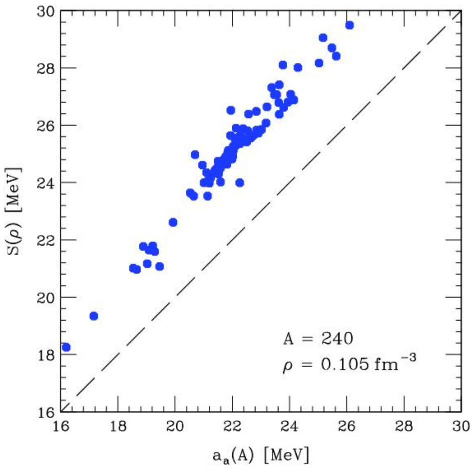
$$B(N,Z) = a_{\rm v}A - a_{\rm s}A^{2/3} - a_{\rm c}\frac{Z^2}{A^{1/3}} - a_{\rm sym}I^2A + E_{\rm mic},$$

pairing and shell effects

- Expected sensitivity

$$\Delta\mu^*_{\rm pn} \simeq a_{\rm c}\left[\frac{1-Z}{(A-2)^{1/3}} - \frac{1+Z}{(A+2)^{1/3}}\right] + a_{\rm sym}\frac{4A^2I}{A^2-4} \simeq -2a_{\rm c}\frac{Z}{A^{1/3}} + 4a_{\rm sym}I$$

$$\Delta\mu^*_{\rm pn} \propto a_{\rm sym} \approx E_{\rm sym}(\rho_r)$$



Pawel Danielewicz, Jenny Lee, Nuclear Physics A 922 (2014)
M. Centelles, Phys. Rev. Lett. 102, 122502 (2009)
L.-W. Chen, Phys. Rev. C 83, 044308 (2011)
N. Wang, L. Ou, and M. Liu, Phys. Rev. C 87, 034327(2013)

Non relativistic & covariant EDFs

$G_s, G_v, W_0 \quad m^*_{s,0}/m, m^*_{v,0}/m$

$\rho_0, E_0(\rho_0), K_0 \quad E_{\rm sym}(\rho_0), L$

$m^*_{\rm Dirac}/m, m_\sigma, c_\omega$

Analytical transformation with
pseudo-observables in nuclear matter

Standard Skyrme Hartree Fock (SHF) Model

$$\begin{aligned} v(\boldsymbol{r}_1, \boldsymbol{r}_2) &= t_0(1 + x_0 P_\sigma)\delta(r) \\ &+ \frac{1}{2} t_1 (1 + x_1 P_\sigma)[\mathbf{k}'^2 \delta(\mathbf{r}) + \delta(\mathbf{r})\mathbf{k}^2] \\ &+ t_2(1 + x_2 P_\sigma)\mathbf{k}' \cdot \delta(r) k \\ &+ \frac{1}{6} t_3 (1 + x_3 P_\sigma)[\rho(R)]^\alpha \delta(r) \\ &+ i W_0 \sigma \cdot [\mathbf{k}' \times \delta(\mathbf{r})\mathbf{k}] \end{aligned}$$

$t_0 \sim t_3, x_0 \sim x_3, \alpha, W_0$

Non-linear Relativistic Mean-Field (RMF) Model

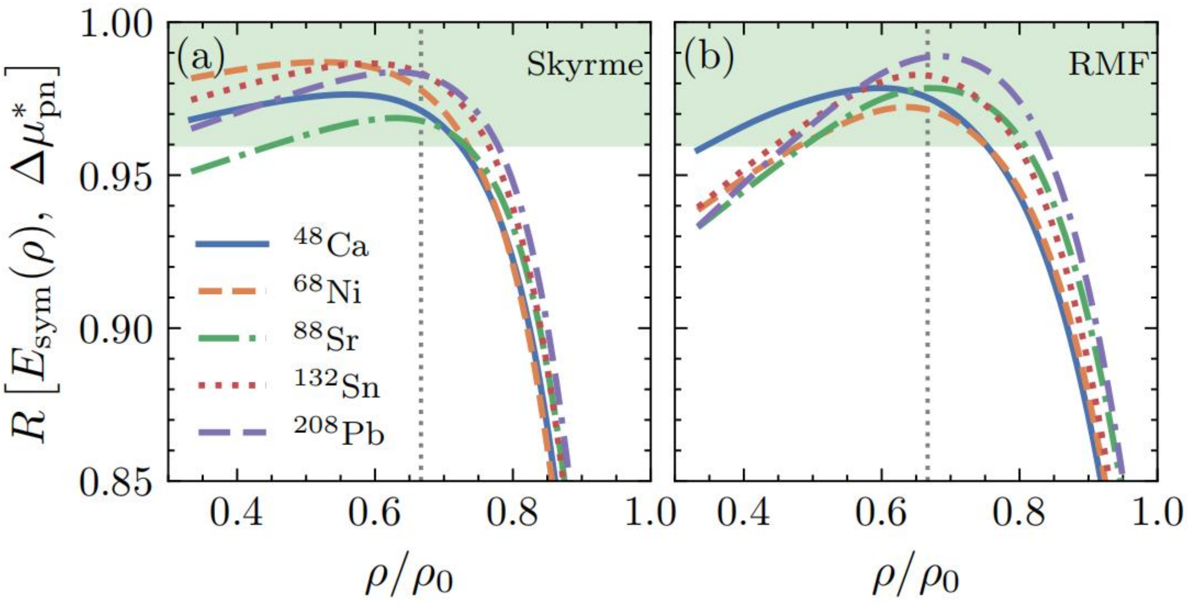
$$\begin{aligned} \mathcal{L} =& \bar{\psi}(i\partial_\mu \gamma^\mu - m)\psi - e\bar{\psi}\gamma_\mu \frac{1+\tau_3}{2} A^\mu \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \\ &+ g_\sigma \sigma \bar{\psi}\psi - g_\omega \omega_\mu \bar{\psi}\gamma^\mu \psi - g_\rho \vec{\rho}_\mu \bar{\psi}\gamma^\mu \vec{\tau}\psi \\ &+ \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} b_\sigma M (g_\sigma \sigma)^3 - \frac{1}{4} c_\sigma (g_\sigma \sigma)^4 \\ &- \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4} c_\omega (g_\omega^2 \omega_\mu \omega^\mu)^2 \\ &- \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu + \frac{1}{2}\Lambda_V (g_\rho^2 \vec{\rho}_\mu \vec{\rho}^\mu)(g_\omega^2 \omega_\mu \omega^\mu), \end{aligned}$$

$g_\sigma, g_\omega, g_\rho, b_\sigma, c_\sigma, c_\omega, \Lambda_V, m_\sigma$

$\Delta\mu^*_{\rm pn}$ for 5 doubly magic nuclei

$E_{\rm sym}(\rho)$ at different densities

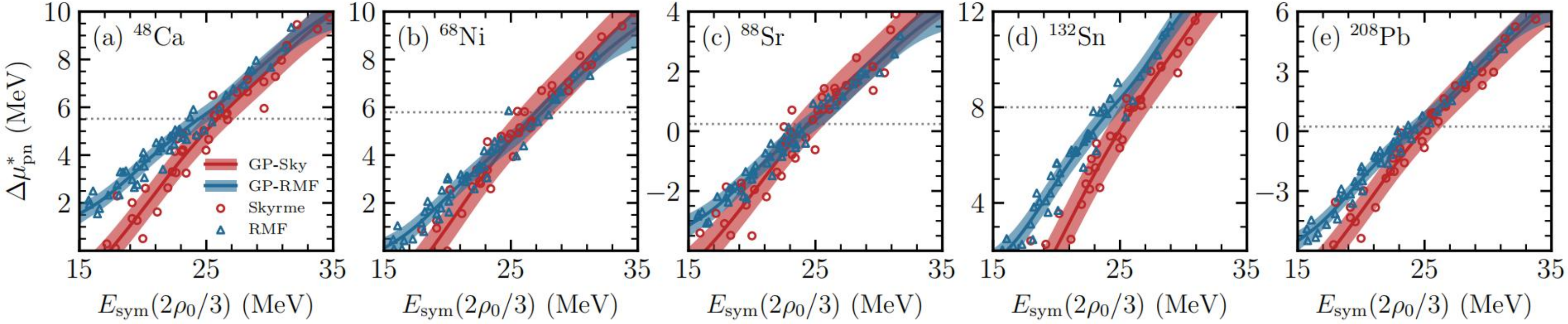
Pearson correlation coefficient



- ✦ $R[A,B] = \frac{\mathrm{Cov(A,B)}}{\sqrt{\mathrm{Var(A)Var(B)}}}$
- ✦ A strong linear correlation between the $\Delta\mu^*_{pn}$ and the symmetry energy at subsaturation densities
- ✦ High sensitivity around $2\rho_0/3$

M. Qiu, B. J. Cai, L.-W. Chen et al. Phys. Lett. B 849 (2024) 138435

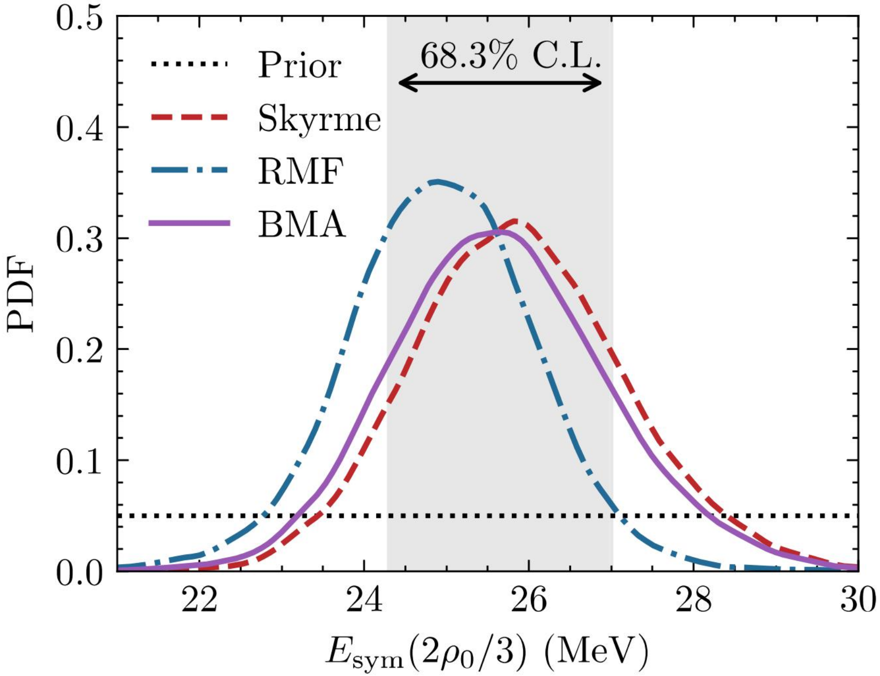
Gaussian Process(GP)-Sky & GP-RMF



- Tune Gaussian processes using the results of 50 Skyrme EDFs and 50 covariant EDFs.
  - *Surmise* python package by BAND collaboration
- GP predictions with uncertainties.

M. Plumlee, O. Surer, S. M. Wild, and M. Y.-H.Chan, surmise 0.2.0, https://surmise.readthedocs.io/en/latest/

Symmetry energy at $2\rho_0/3$



- Posterior by Sequential Monte Carlo algorithm from PyMCv4.0

O. Abril-Pla, et al., PeerJ Computer Science 9,e1516 (2023)

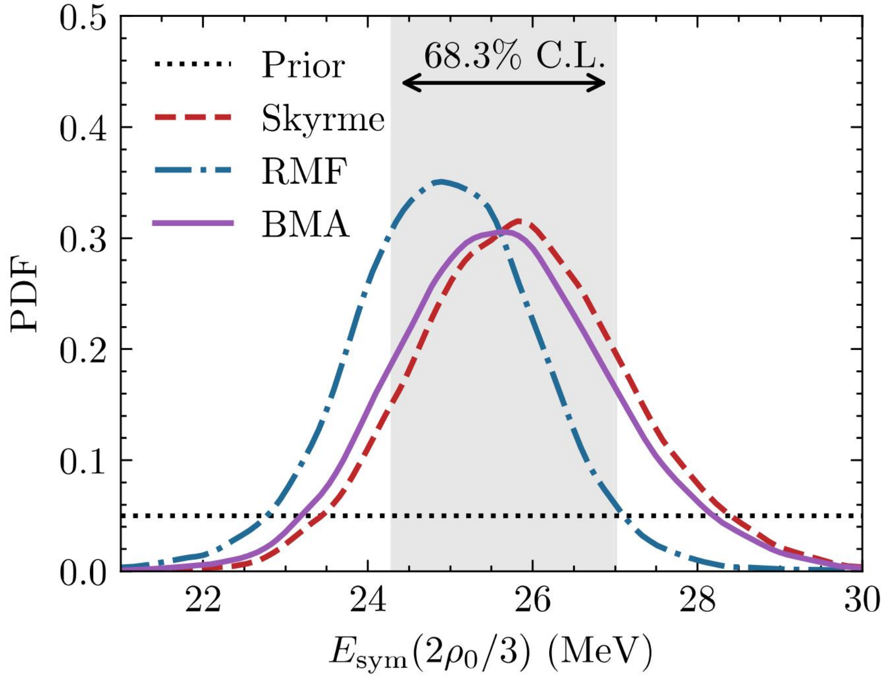
Skyrme EDFs $E_{\text{sym}}(2/3\rho_0) = 25.8^{+1.3}_{-1.2}$ MeV

$\sigma = 0.4^{+0.4}_{-0.2}$ MeV

Nonlinear RMF $E_{\text{sym}}(2/3\rho_0) = 24.9 \pm 1.1$ MeV

$\sigma = 0.8^{+0.6}_{-0.3}$ MeV

Symmetry energy at 2ρ_0/3



- Model posterior probability

$$p(\mathcal{M}_i\,|\,\boldsymbol{y}) = \frac{p(\boldsymbol{y}\,|\,\mathcal{M}_i)\pi(\mathcal{M}_i)}{\sum_{\ell} p(\boldsymbol{y}\,|\,\mathcal{M}_\ell)\pi(\mathcal{M}_\ell)}$$

- Equal prior preference
- Evidence ratio Sky/RMF ≈ 3.3

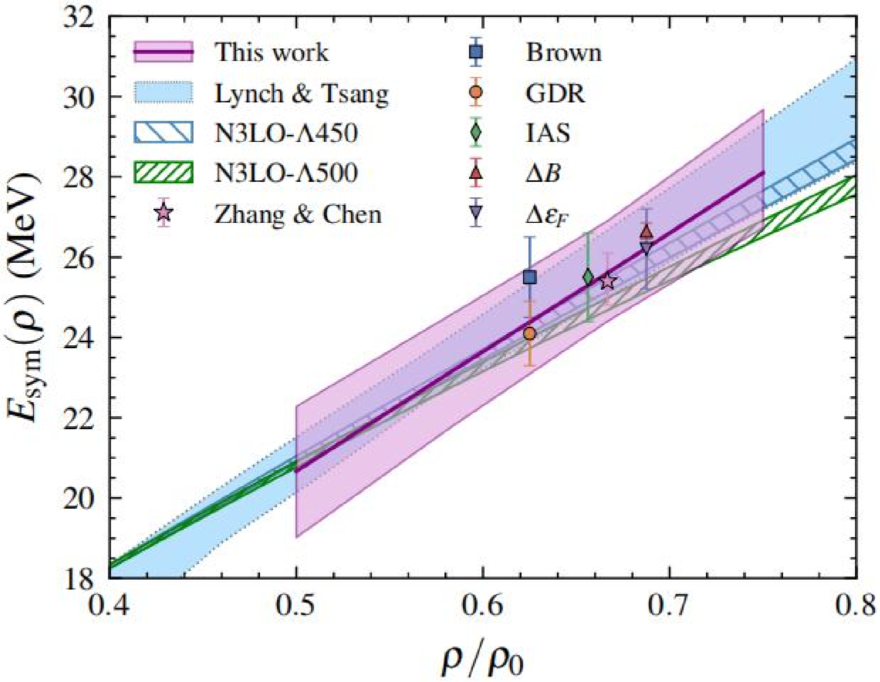
Skyrme * 3.3/4.3

RMF * 1/4.3

→ BMA

- $E_{\rm sym}(2/3\rho_0)$ is inferred to be $25.6^{+1.4}_{-1.3}$ MeV

Symmetry energy at subsaturation densities



- Brown: Doubly magic nuclei
  B.A.Brown, Phys. Rev. Lett. 111, 232502 (2013)
- Lynch & Tsang: various terrestrial and astrophysical constraints
  W.G.Lynch and M.B.Tsang, Phys. Lett. B. 830, 137098 (2022)
- Zhang & Chen: Doubly magic nuclei+PREX+CREX
  Z. Zhang and L.-W. Chen, Phys. Rev. C. 108, 024317 (2023)
- GDR: Giant dipole resonance
  L. Trippa *et al*, Phys. Rev. C 77, 061304 (2008)
- IAS: Isobaric analog states
  Pawel Danielewicz, Jenny Lee, Nuclear Physics A 922 (2014)
- ΔB: Isotope binding energy difference
  Z. Zhang and L.-W. Chen, Phys. Lett. B 726, 234 (2013)
- $\Delta\varepsilon_F$: Fermi energy difference
  N. Wang, L. Ou, and M. Liu, Phys. Rev. C 87, 034327(2013)

Summary

- Within both the non-relativistic Skyrme EDFs and the nonlinear RMF model, the effective proton-neutron chemical potential difference $\Delta\mu_{pn}^{*}$ of neutron-rich nuclei is found to be strongly sensitive to the symmetry energy $E_{sym}(\rho)$ around $2\rho_0/3$,
- We carried out a Bayesian model averaging analysis based on Gaussian process emulators to extract the symmetry energy around $2\rho_0/3$ $25.6^{+1.4}_{-1.3}$ MeV
- Since both the intra- and inter-model uncertainties are taken into account in our BMA analyses, the present results are statistically more reliable.

- Inclusion of more experimental observables and more theoretical models

Thank you for your attention!