Clustering in the (proto)neutron star crust

and the contribution of nuclear experiments

F.Gulminelli, (LPC and Normandie Université, Caen)
NUSYM 2024 GANIL

Transport properties in compact stars

- Most signals from CS involve transport properties
- Some of them mainly concern $\rho \lesssim \rho_0$ matter ≡ clusters
  - NS cooling: B-thermal evolution
  - Relaxation after accretion & deep crust heating
  - CC, PNS cooling & mergers

e-Z, $T \approx 10^8 K$

ν-Z $T > 10^{10} K$

Schmitt&Shternin Springer 2018

- Key micro feature: charge distribution => resistivity

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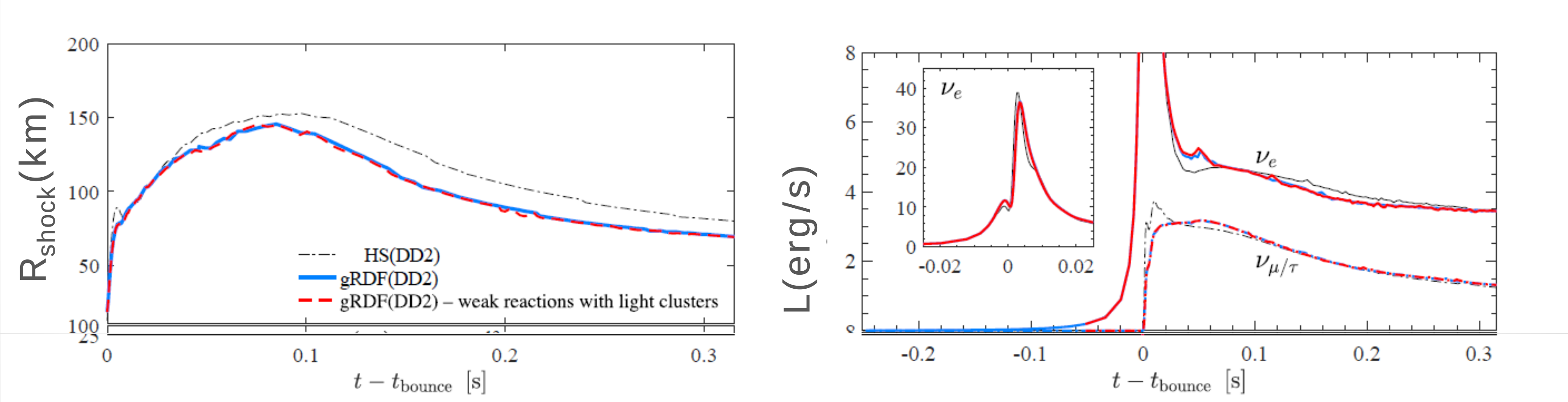
Schmitt&Shternin Springer 2018

- Key micro feature: charge distribution => resistivity
  - T≫0 : distribution of nuclei (or pasta)
  - **light cluster abundancies (indirectly) affect the explosion dynamics and ν–luminosity**



T.Fischer et al Phys. Rev. C **102**(2020) 055807

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$\Big\}$ e-Z, $T \approx 10^8 K$

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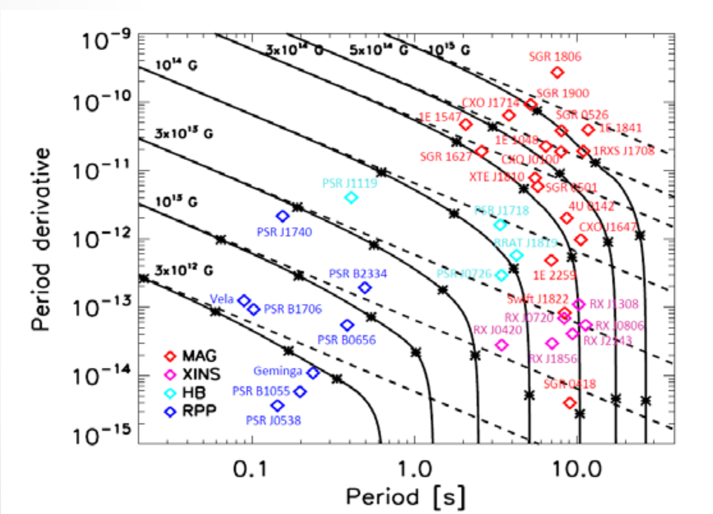
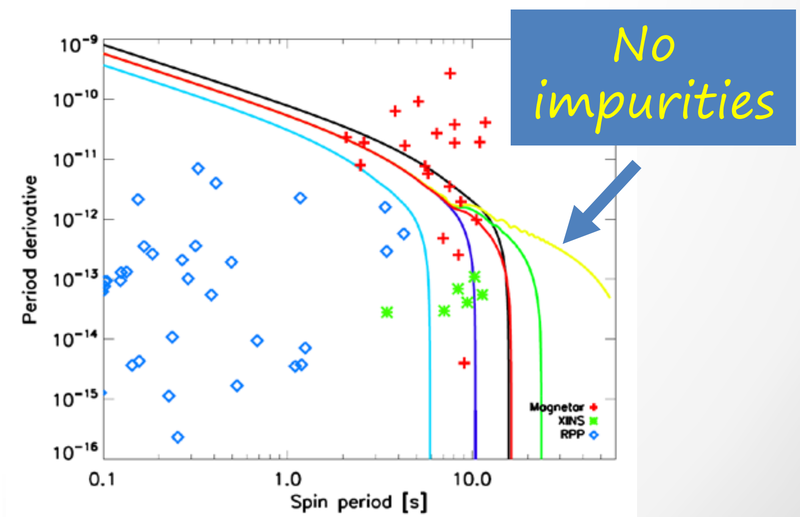
Schmitt&Shternin Springer 2018

- Key micro feature: charge distribution => resistivity
  - T≈0 : impurities in the catalyzed crust
  - **A strong crustal resistivity due to impurities affects the NS magneto-thermal evolution**





J.Pons et al. Nat.Phys. 2013

Transport properties in compact stars

Static structure factor

- **T>T$_m$** $\nu_{e(\nu),ion} \to \sum_j \boldsymbol{n_j} \nu^j_{e(\nu),i}$ $\nu^j_{e(\nu),i} \propto \boldsymbol{S^j(k)}$
- **T<T$_m$** $\nu_{tot} = \nu_{e,ion} + \nu_{e,imp}$ $Z^2 \leftrightarrow \boldsymbol{Q} = \sum_j n_j \left(Z_j - \langle Z \rangle\right)^2$

Impurity factor

Transport properties in compact stars

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Impurity factor

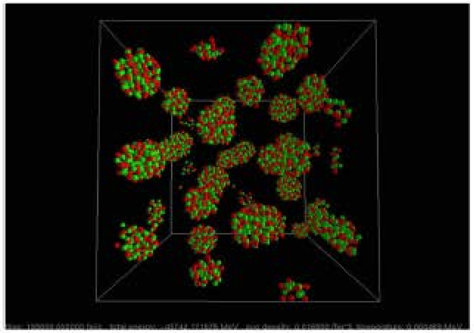
Present situation:

- Q taken as a free parameter in cooling and relaxation simulations A.Deibel et al.ApJ839(2017)
- n_j from Saha equations (Nuclear Statistical Equilibrium)

Z.Lin et al,PRC **102**(2020)045801

Nuclei=quasi-particles
M,B: vacuum values

$$\boxed{n_{AZ} = \frac{g_{AZ}}{2\pi^2\hbar^3}\int_0^\infty dp \left[1 \pm exp\beta\left(\frac{p^2}{2\boldsymbol{M_{AZ}}} - \boldsymbol{B_{AZ}} - A\mu - Z\mu_p\right)\right]^{-1}}$$

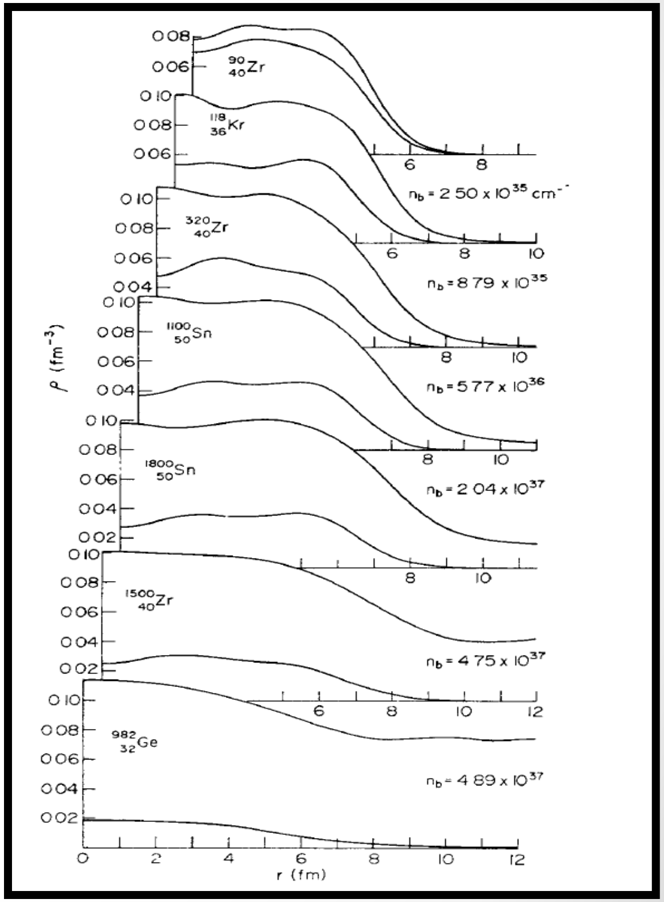


- Spherical approx or S^j from CMD simulations

R.Nandi et al.ApJ852(2018)

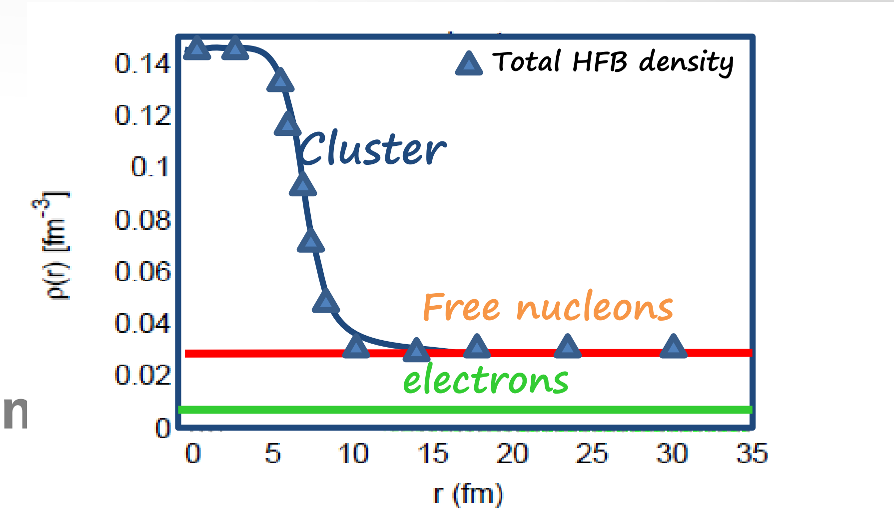
Towards a more controlled theoretical treatment

- Starting point: variational microscopic calculations in the WS cell
- Multi-Component (liquid or solid) plasma: ***clusters as quasi-particles but cluster functional from microscopic WS calculations***



J. W. Negele and D. Vautherin, NPA 207, 298 (1973)

From WS to MCP: mapping in 2 steps



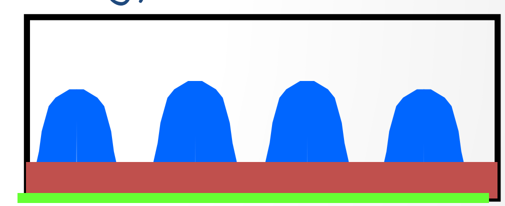
- **WS cell with a microscopic functional** $@(\rho_B, Y_p, T)$:

$$\mathcal{F}_{WS} = \mathcal{E}_{micro} - TS_{micro} = \min(\hat{\rho}_q, \hat{\kappa}_q)$$

=> Optimal particle (and pairing) densities

1. **OCP with cluster DoF**

$$\mathcal{F}_{WS} \equiv \mathcal{F}^{OCP} = \mathcal{F}(\rho_{gq}) + \frac{F_{AZ}}{V_{WS}} = min(A, Z, \rho_{gq})$$

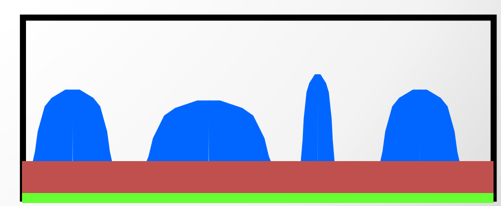


=> Optimal cluster

2. **MCP with cluster DoF**

$$\mathcal{F}^{MCP} = \mathcal{F}(\rho_{gq}) + \sum_{A,Z} n_{AZ} F_{AZ} = min(\{n_{AZ}\}, \rho_{gq})$$

=> Optimal distribution



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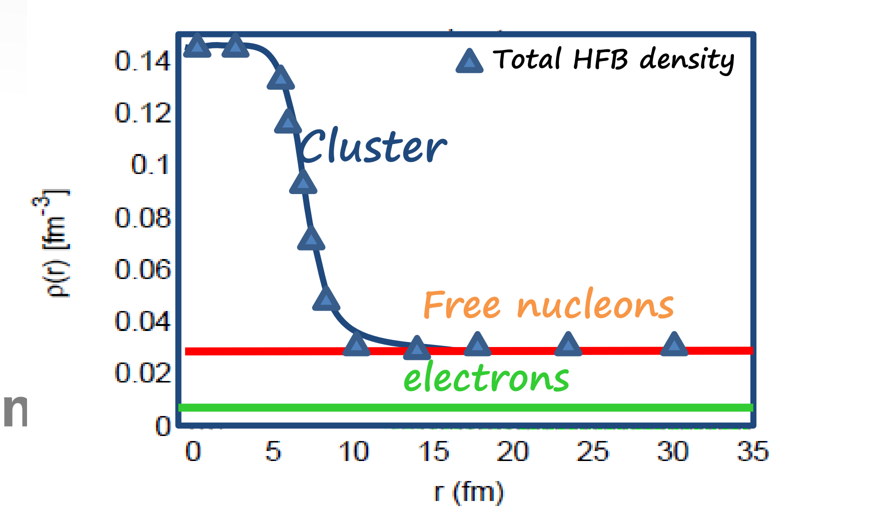

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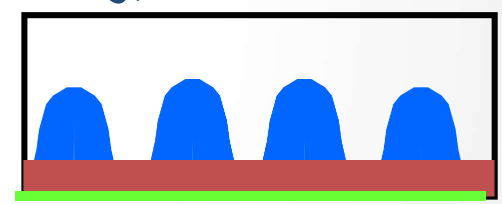

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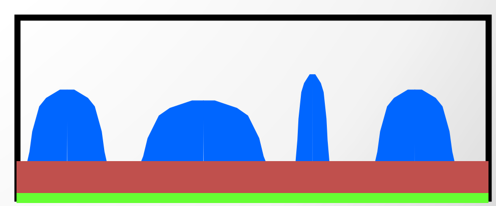
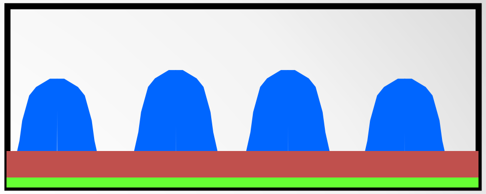


=> *Optimal distribution*

OCP with cluster DoF

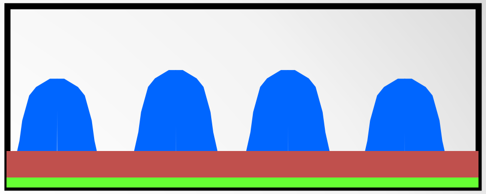


$$\mathcal{F}^{OCP}(A,Z,\rho_{gq}) = \mathcal{F}(\rho_{gq}) + \frac{F_{AZ}}{V_{WS}} \approx \mathcal{F}_{WS}$$

- Below drip: F_{AZ} is just the HFB free energy
- Above drip: $F_{AZ}(A,Z,\rho_{gq})$

$\Rightarrow$ the cluster functional is in-medium modified

OCP with cluster DoF



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- Below drip: F_{AZ} is just the HFB free energy
- Above drip: $F_{AZ}(A,Z,\rho_{gq})$

$\Rightarrow$ the cluster functional is in-medium modified

$\Rightarrow$ Practical implementation: F_{AZ} parametrized as a CLDM with surface parameters fitted from (T)ETF calculation:

$$F_{AZ}^{0} = V_{AZ}\left(\mathcal{F}(\rho_{bulk,q}) - \mathcal{F}(\rho_{gq})\right) + F_{surf} + F_{coul}$$

continuum states subtracted

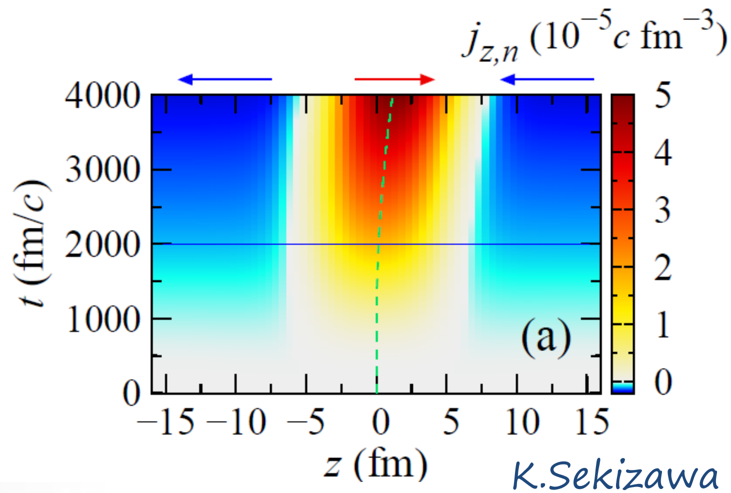
Tubbs&Koonin, ApJ 232 (1979) L59
Bonche,Levit,Vautherin NPA427(1984)278

$\Rightarrow$ Translational degree of freedom: $F_{AZ} = F_{AZ}^{0} + T\left(ln\frac{\lambda_{AZ}^{3}(M^{*})}{gV_f} - 1\right)$

$\Rightarrow$ Effective mass: only bound neutrons are entrained $M^{*} \approx M\left(1 - \frac{\rho_{gn}}{\rho_i}\right)$

Magierski& Bulgac NPA 2004
Martin&Urban PRC 2016

OCP with cluster DoF



Real time TDHF&
inertial mass definition

$$M^* = \frac{eZE_z}{\ddot{z}_{CM}}$$

K.Sekizawa et al PRC 2022

⇒ Translational degree of freedom: $F_{AZ} = F_{AZ}^0 + T\left(ln\frac{\lambda_{AZ}^3(M^*)}{gV_f} - 1\right)$

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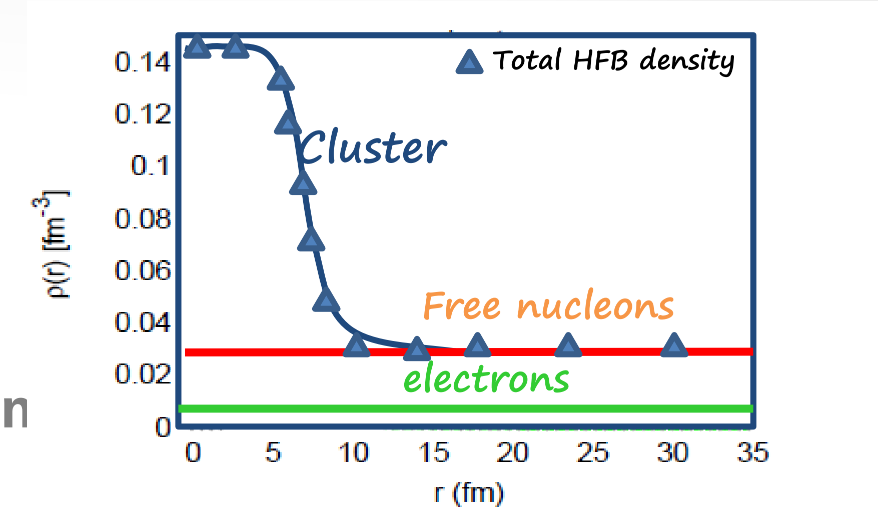

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=> *Optimal particle (and pairing) densities*

1. OCP with cluster DoF

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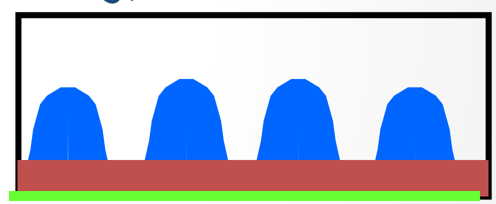
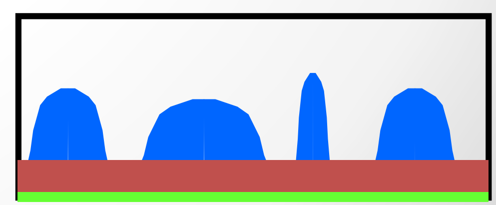


=> *Optimal cluster*

2. MCP with cluster DoF

$$\mathcal{F}^{MCP} = \mathcal{F}(\rho_{gq}) + \sum_{A,Z} n_{AZ} F_{AZ} = min(\{n_{AZ}\}, \rho_{gq})$$

=> *Optimal distribution*



The cluster distribution

Grams 2018, PRC, 97, 035807
Fantina 2020, A&A, 633, A149
Carreau 2020, A&A, 640, A77
Dinh-Thi 2023, A&A, 677, A174

- $d\mathcal{F}_{MCP}(\{n_{AZ}\}) = 0$ leads to:

Continuum subtracted & microscopic energy and level density

$$n_{AZ} = \frac{g_{AZ}}{2\pi^2\hbar^3}\int_0^\infty dp \left[1 \pm \exp\beta\left(\frac{p^2}{2\boldsymbol{M_{AZ}^*}} - \boldsymbol{F_{AZ}^0} - N\mu_n - Z\mu_p + \boldsymbol{R_{AZ}(n_e)}\right)\right]^{-1}$$

Rearrangement $(n_e = \sum_{AZ} Z n_{AZ})$

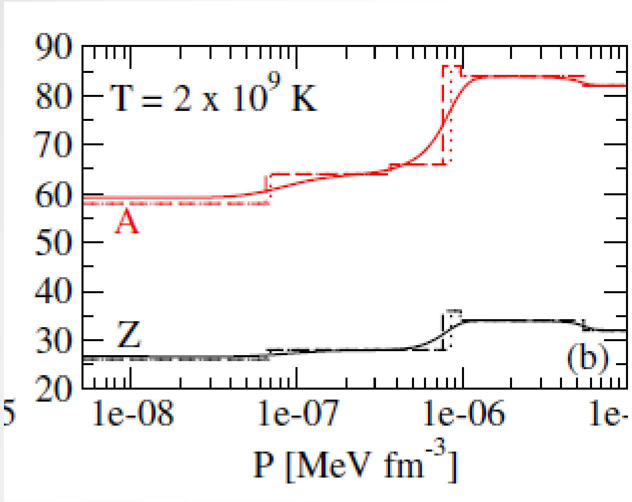
$$\mu_q = \frac{\partial \mathcal{F}_\mu}{\partial \rho_{gq}} + \sum_{AZ} n_{AZ}\frac{\partial F_{AZ}}{\partial \rho_{gq}}\left(1 - \sum_{AZ} n_{AZ}V_{AZ}\right)^{-1} \approx \frac{\partial \mathcal{F}_\mu}{\partial \rho_{gq}} + \frac{1}{V_{WS}^{OCP}}\frac{\partial F_{AZ}^{OCP}}{\partial \rho_{gq}}\left(1 - u_{AZ}^{OCP}\right)^{-1}$$

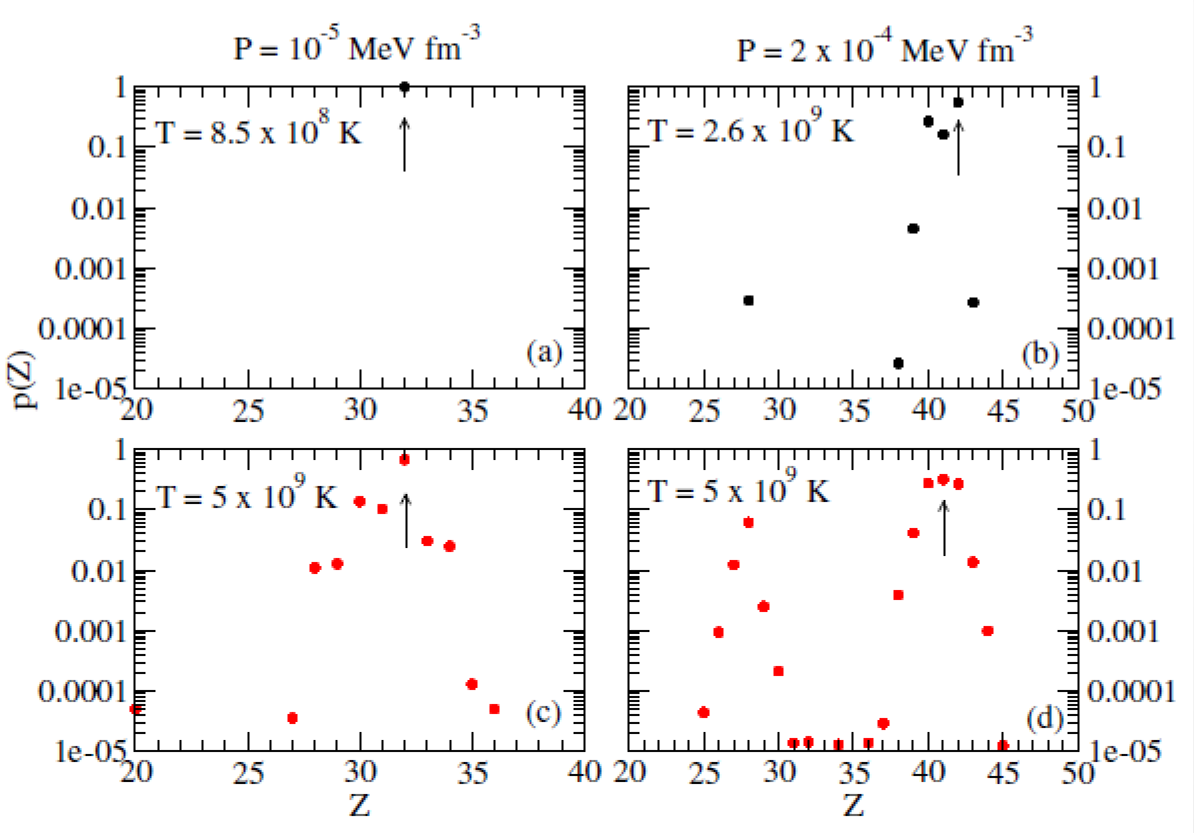
$$= \mu_q^{OCP}$$

Self-consistent $\mu(\rho)$

Perturbation 1st order in $n_g - n_g^{OCP}$

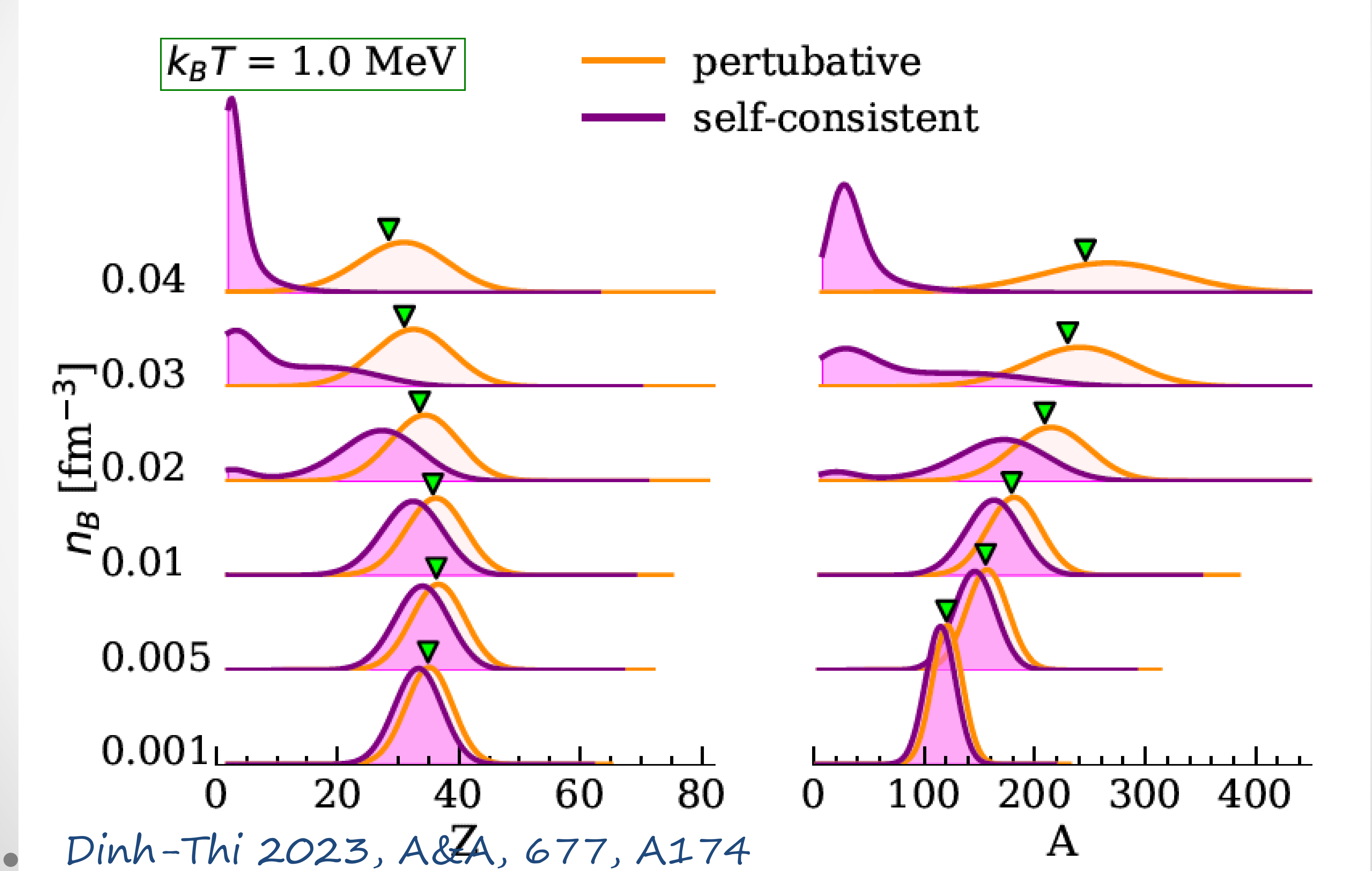
Nuclear distribution in the outer crust



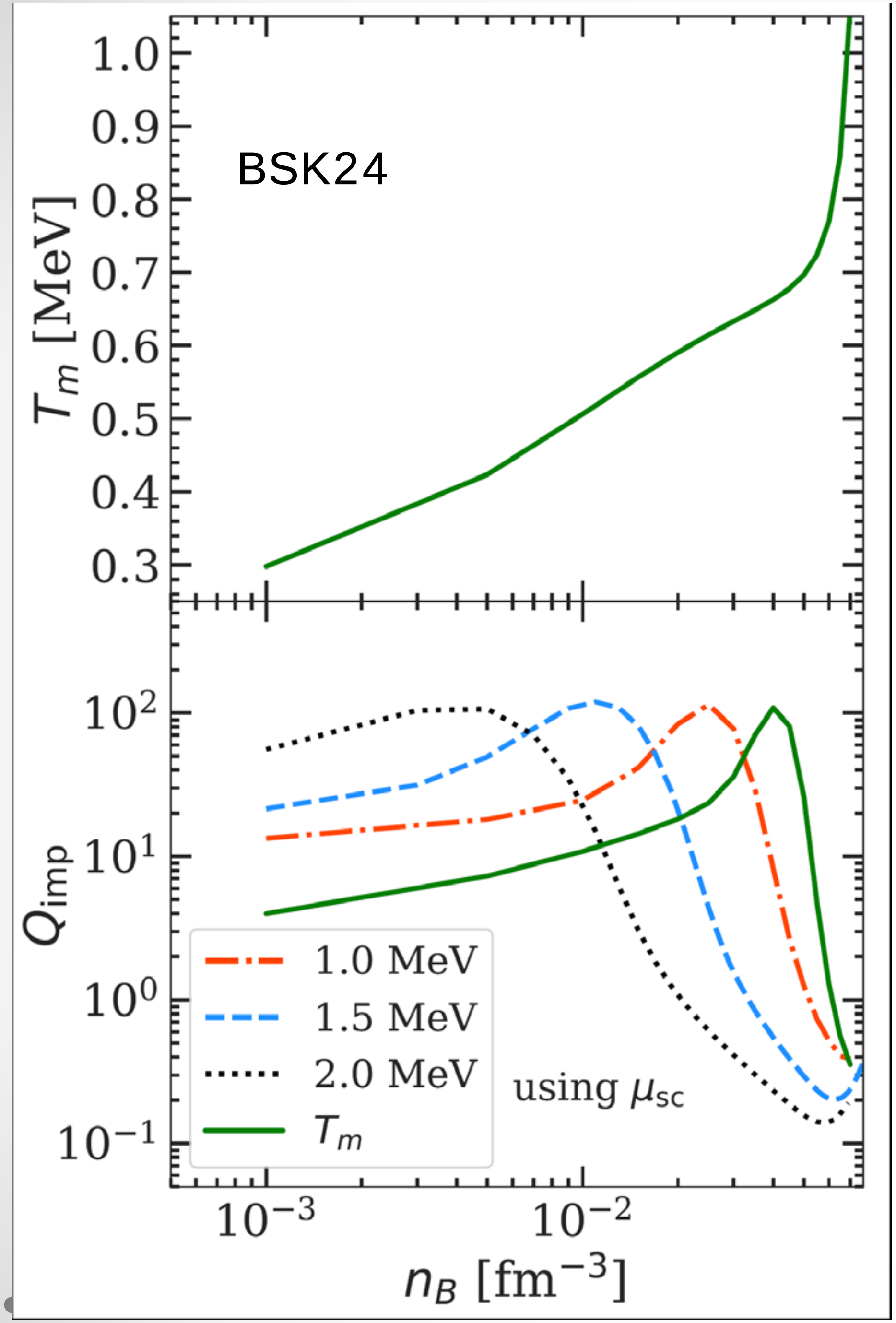


Fantina 2020, A&A, 633, A149

Nuclear distribution in the inner crust



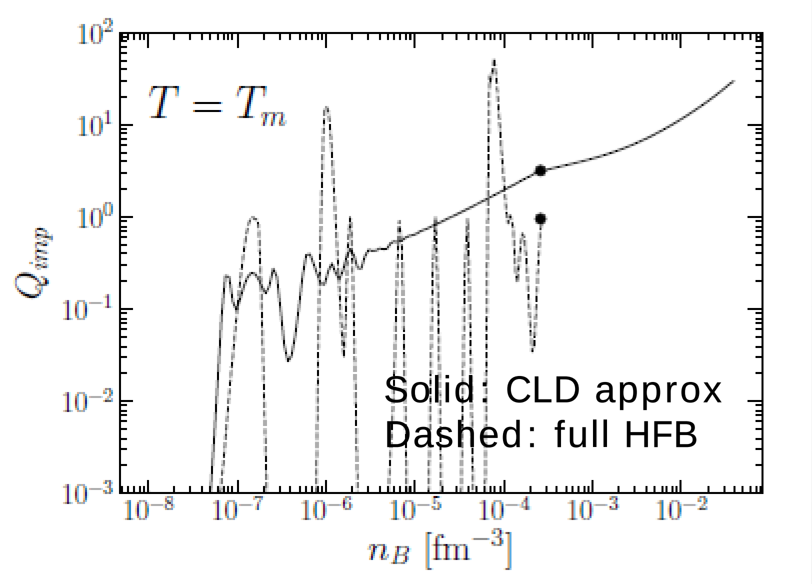
- Dinh-Thi 2023, A&A, 677, A174



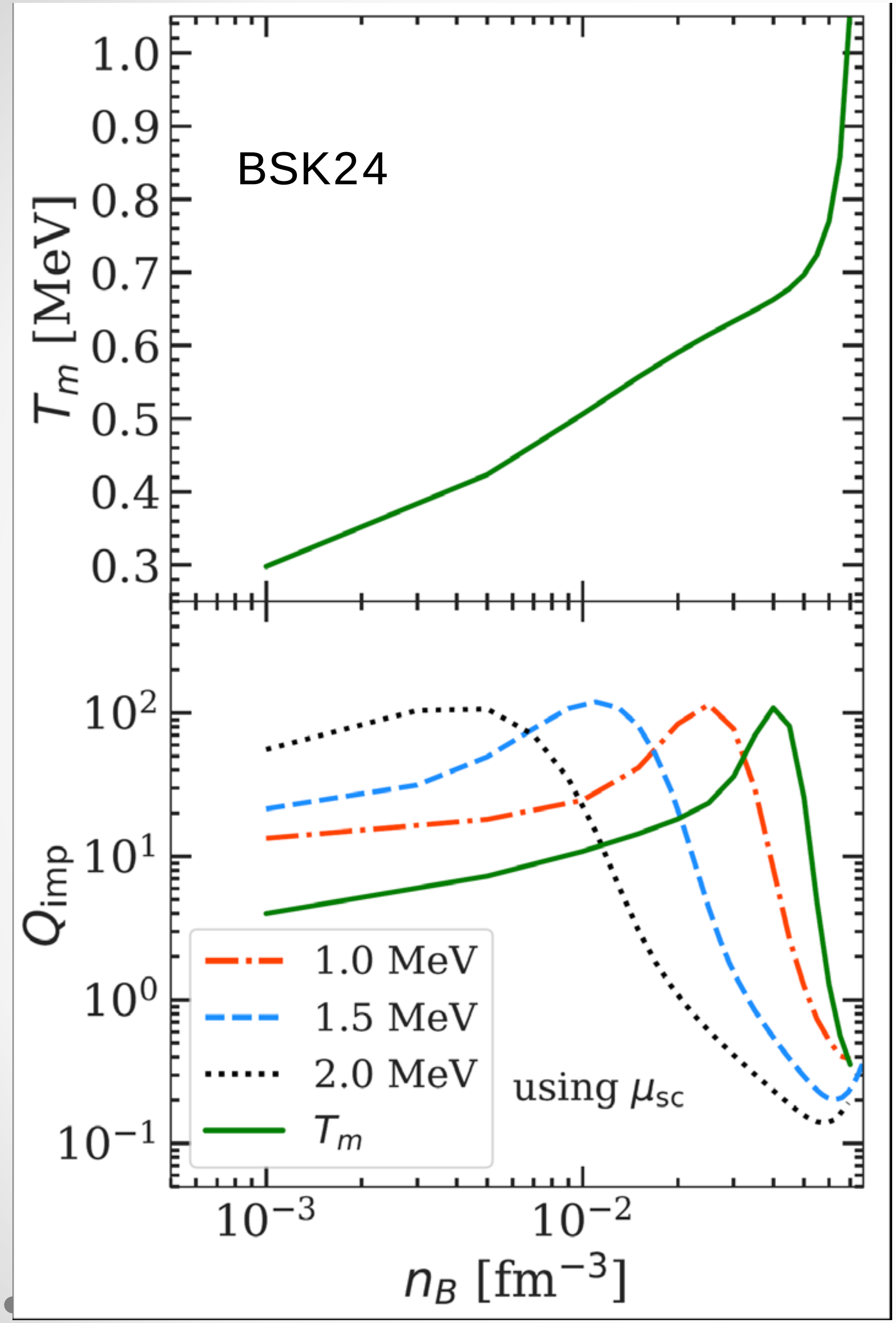
Impurity factor

$$Q = \sum_j n_j \left(Z_j - \langle Z \rangle\right)^2$$

Carreau 2020, A&A, 640, A77



Dinh-Thi 2023, A&A, 677, A174

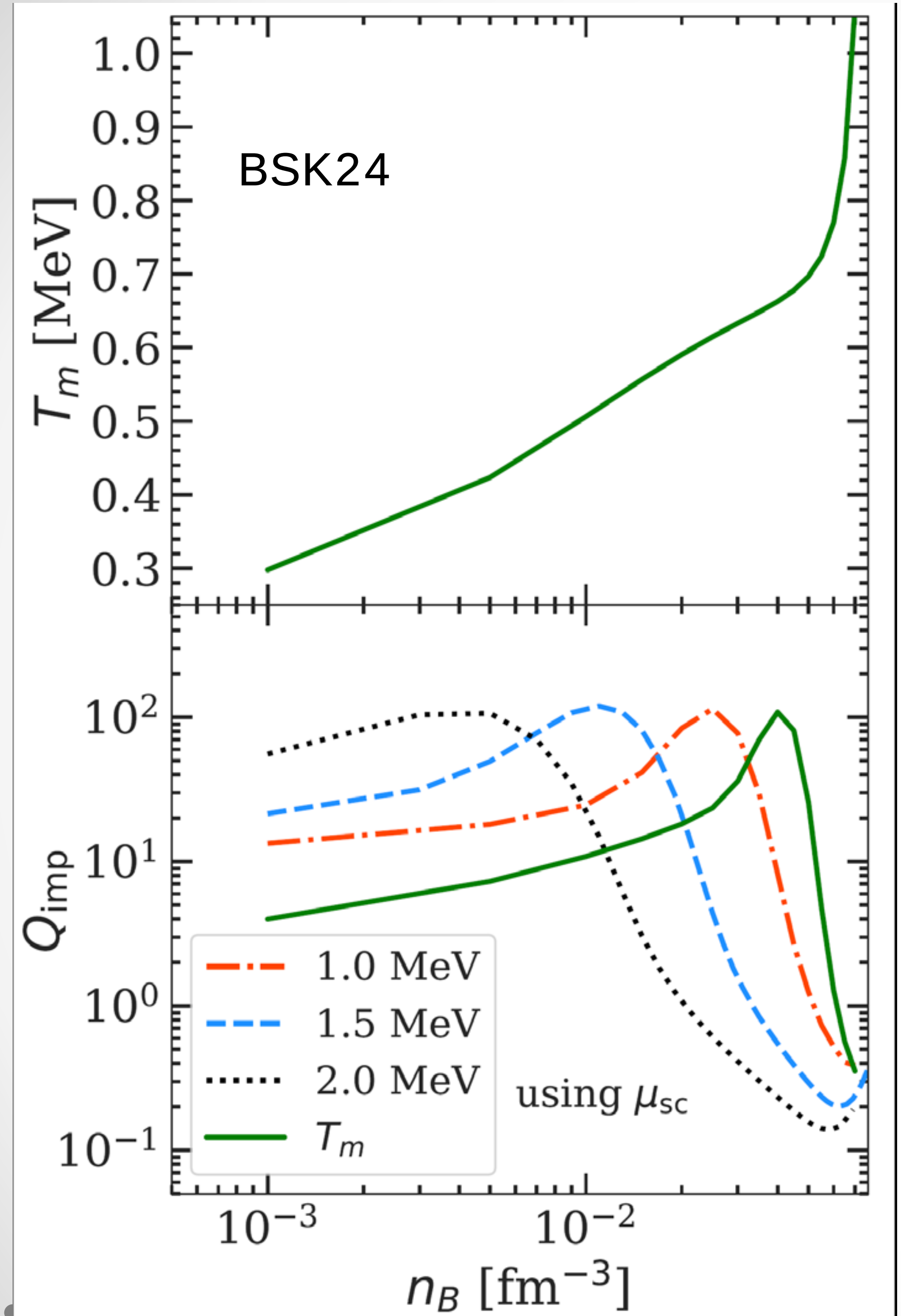


Impurity factor

$$Q = \sum_j n_j \left(Z_j - \langle Z \rangle\right)^2$$

- First microscopic calculation of $Q_{\rm imp}$
- If the cluster distribution is frozen at the crystallization temperatures, the crust is highly resistive even without pasta!

Dinh-Thi 2023, A&A, 677, A174



Impurity factor

$$Q = \sum_j n_j \left(Z_j - \langle Z \rangle\right)^2$$

- First microscopic calculation of Q_{imp}
- If the cluster distribution is frozen at the crystallization temperatures, the crust is highly resistive even without pasta!
- However: poor description of light clusters with the leptodermous expansion
  $F^0_{AZ} = F_{bulk} + F_{surf} + F_{coul}$

Dinh-Thi 2023, A&A, 677, A174

Light clusters: modeling the in-medium effects

- NSE with a momentum cut-off   S.Burrello, PRC 2024

$$n_{AZ} = \frac{g_{AZ}}{2\pi^2\hbar^3}\int_{\boldsymbol{\Lambda_{AZ}(n,T)}}^{\infty} dp\left[1 \pm exp\beta\left(\frac{p^2}{2M_{AZ}} - B_{AZ}^{exp} - A\mu - Z\mu_p\right)\right]^{-1}$$

=> See Stefano's talk!

Light clusters: modeling the in-medium effects

- Effective masses⇔self-energies in the RMF approximation

$$\mathcal{L} = -\sum_{i\in F}\bar{\psi}_i(\gamma_\mu D_{\mu i} + m_i^*)\,\psi_i - \frac{1}{2}\sum_{i\in S}\left[(D_{\mu i}\psi_i)^2 + m_i^{*2}\psi_i^2\right] + \sum_{i\in V}\left[(D_{\mu i}\psi_{\nu i} - D_{\nu i}\psi_{\mu i})^2 - \frac{1}{2}m_i^{*2}\psi_{\mu i}^2\right]$$

$$+\mathcal{L}_{\sigma\omega\rho}$$

n,p,t,^{3}He,... | ^{4}He, ^{6}He,... | d,...

$$m_{AZ}^* = m_{AZ}^{vac} - g_s^{AZ}(n)\langle\sigma\rangle$$

S.Typel et al,PRC81(2010)015803

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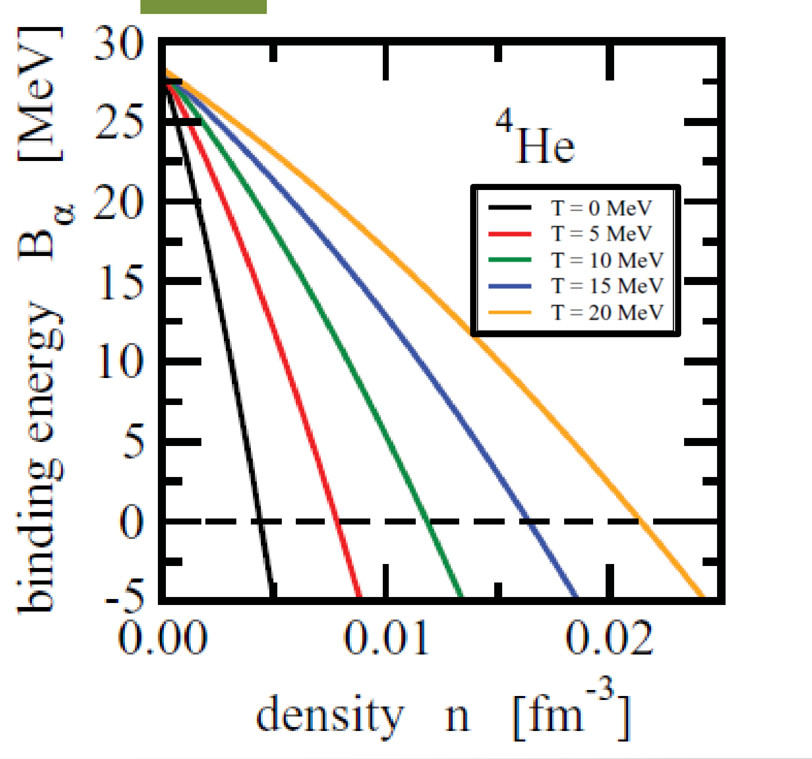
n,p,t,^{3}He,... | ^{4}He, ^{6}He,... | d,...

$$m_{AZ}^* = m_{AZ}^{vac} - g_s^{AZ}(n)\langle\sigma\rangle$$
$$\approx m_{AZ}^{vac} - Ag_s(n)\langle\sigma\rangle - \Delta B_{AZ}(n,T)$$

ad-hoc prescription



- S.Typel et al,PRC81(2010)015803

Light clusters: modeling the in-medium effects

- Effective masses⇔self-energies in the RMF approximation

$$m^*_{AZ} = Am + E^0_{AZ}$$

$$\boldsymbol{\mathcal{L}} = -\sum_{i\in F} \bar{\psi}_i(\gamma_\mu D_{\mu i} + m_i^*)\,\psi_i - \frac{1}{2}\sum_{i\in S}\left[\left(D_{\mu i}\psi_i\right)^2 + m_i^{*2}\psi_i^2\right] + \sum_{i\in V}\left[\left(D_{\mu i}\psi_{\nu i} - D_{\nu i}\psi_{\mu i}\right)^2 - \frac{1}{2}m_i^{*2}\psi_{\mu i}^2\right]$$

$$+\boldsymbol{\mathcal{L}}_{\sigma\omega\rho}$$

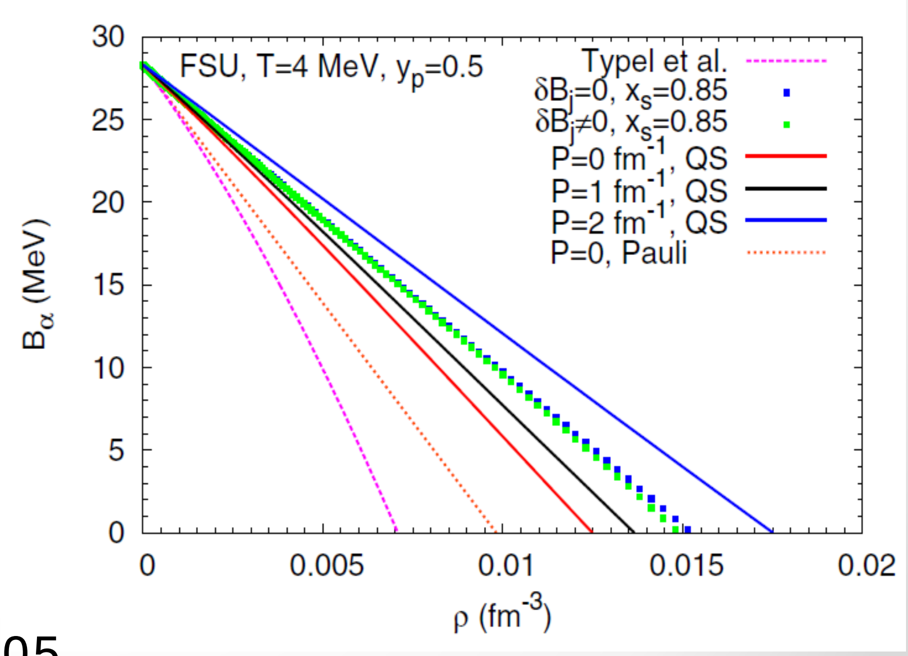
n,p,t,^{3}He,...

^{4}He, ^{6}He,...

d,...

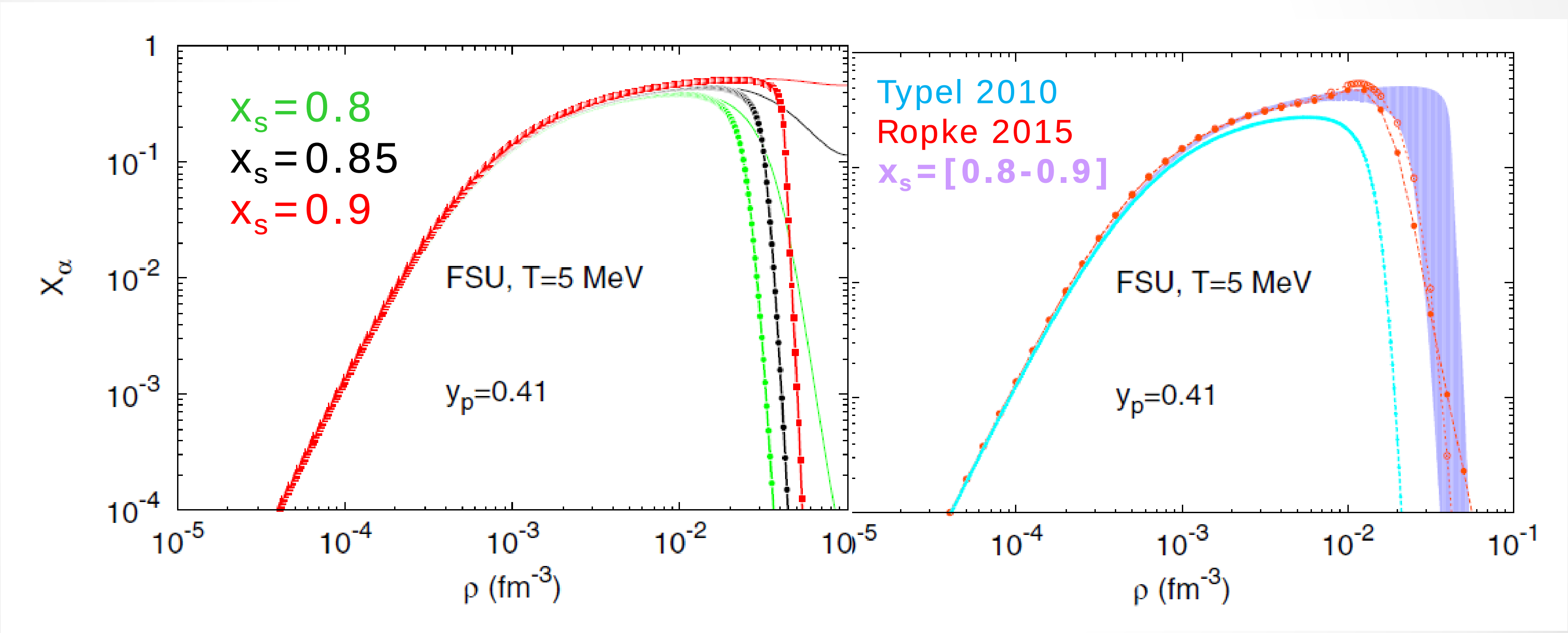
$$m^*_{AZ} = m^{vac}_{AZ} - g_s^{AZ}(n)\langle\sigma\rangle$$
$$\approx m^{vac}_{AZ} - A\boldsymbol{x_s}(n,T)g_s(n)\langle\sigma\rangle$$

Free parameter $0<x_s<1$



H.Pais et al,PRC97(2018)045805

Calibrating the in-medium effects



H.Pais et al,PRC97(2018)045805

Calibration on INDRA data

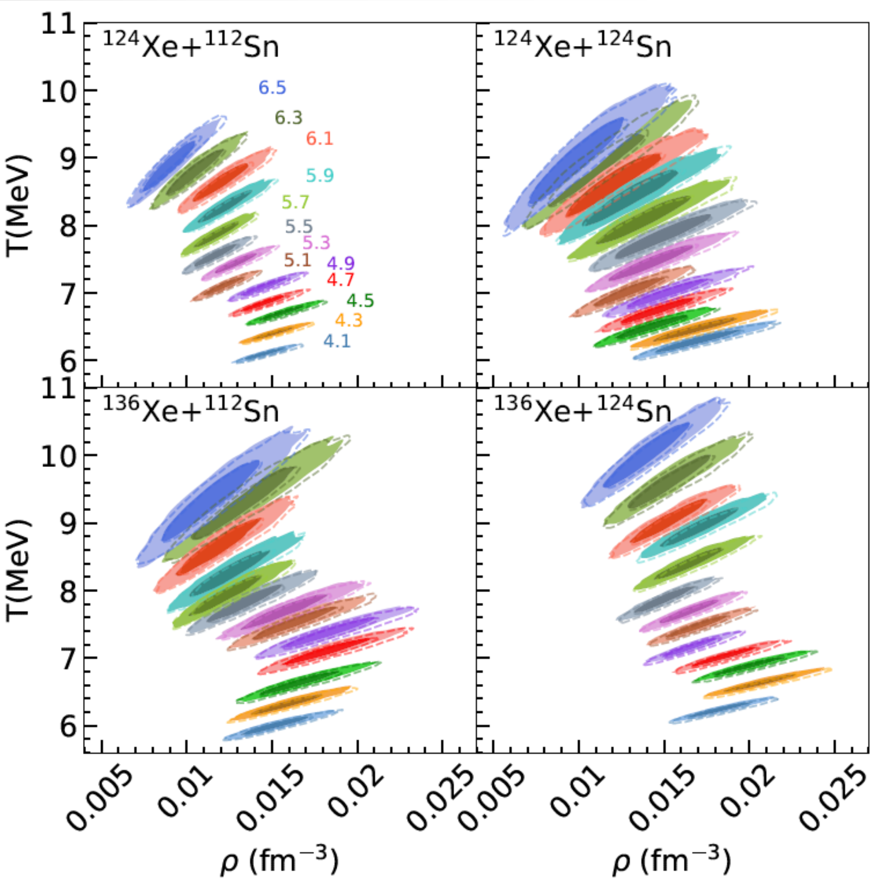
=> See Rémi's talk!

- 4 different data sets $X_i = 1, \cdots, 4$ from INDRA ^{136,124}Xe+ ^{124,112}Sn @ 32 A.MeV*
- EoS Model: FSU-RMF
- Bayesian inference (NSMC**) of the unknown parameters $\boldsymbol{\theta_i = (T, n_B, x_s(T, n_B))_i}$ directly from the mass fractions for each sample $\{\omega(A,Z)\}_i$ labelled by the radial velocity bin and data set $i = (v_i, X_i)$

$$\boldsymbol{p_i(\theta|\{\omega_{AZ}\}) = \frac{p_\theta}{Z} \prod_{AZ} \mathcal{L}\left(\omega^i_{AZ}|\theta\right) \quad \mathcal{L}\left(\omega^i_{AZ}|\theta\right) = e^{-\frac{\left(\omega^{exp}_{AZ}(i) - \omega^{th}_{AZ}(\theta)\right)^2}{2\sigma^2_{AZ}(i)}}}$$

* A.Rebillard-Soulié et al, JPhysG 51(2023)015104
** PyMultiNest: J. Buchner 10.1214/23-ss144 (2021)

Calibration on INDRA data

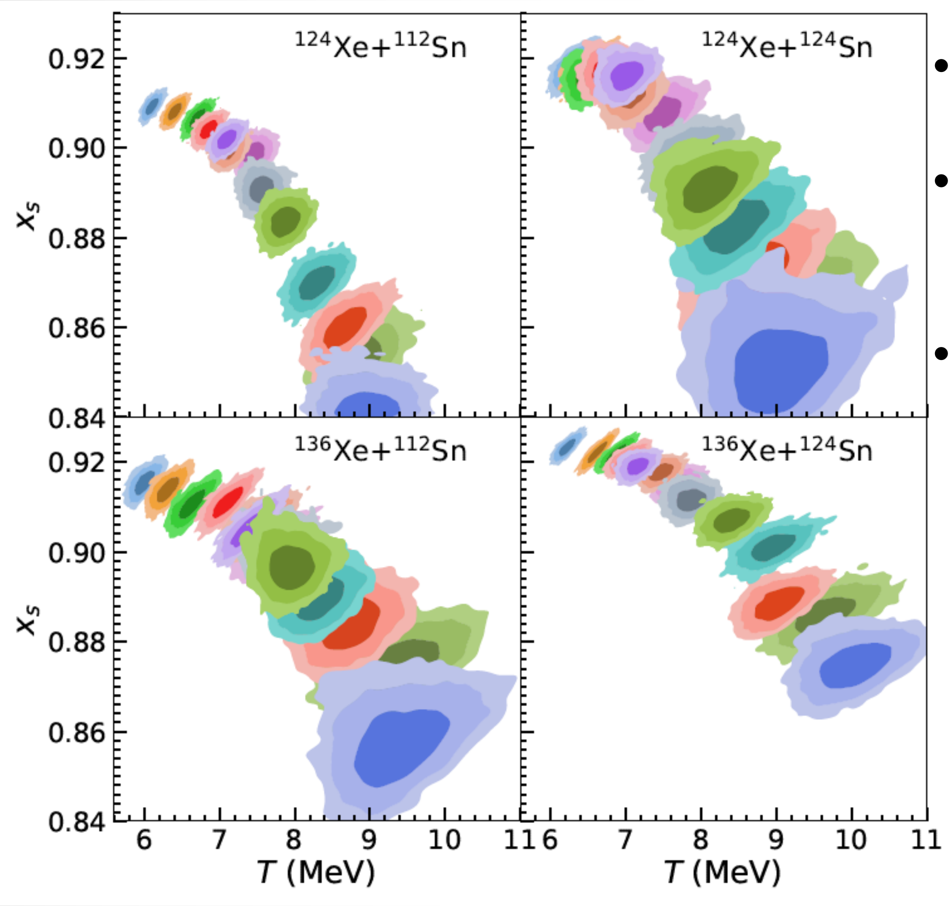


Solid: FSU Dashed: DD2

- The temperature and density estimation is model independent

T.Custodio et al, arXiv:2407.02307

Calibration on INDRA data



- The temperature and density estimation is model independent
- The independent analyses of the 4 systems lead to ~ compatible results for x(n,T)
- This confirms the validity of the statistical approach

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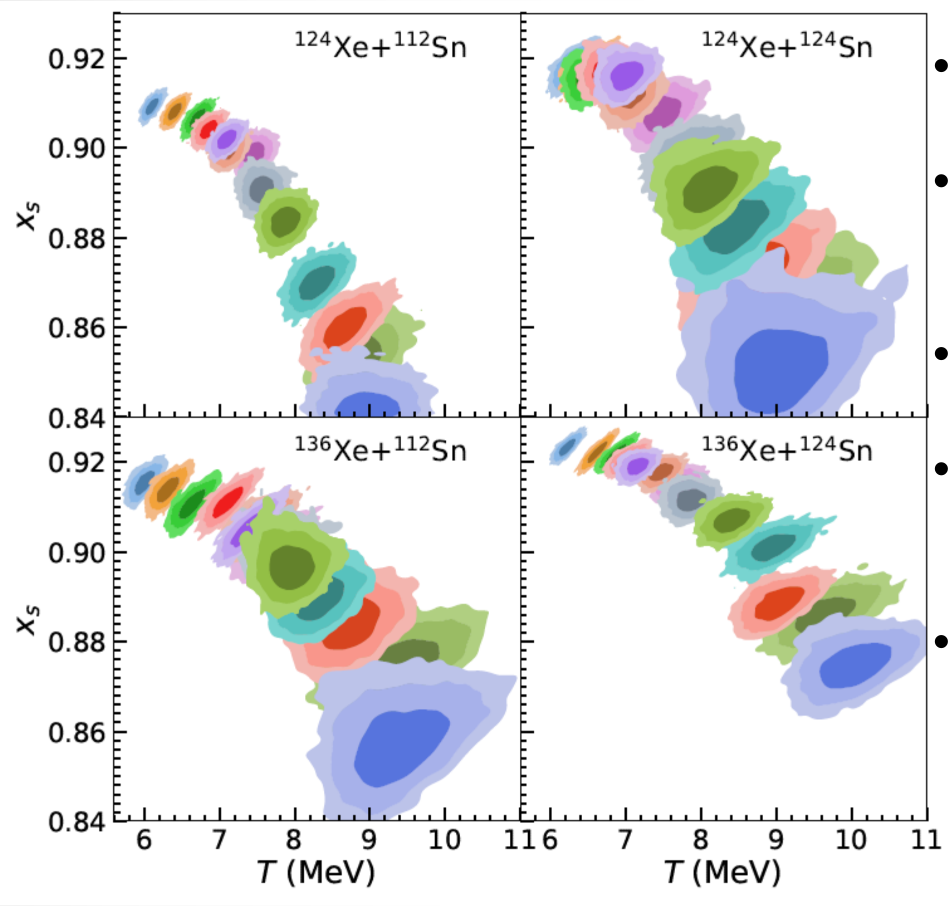


- The temperature and density estimation is model independent
- The independent analyses of the 4 systems lead to ~ compatible results for x(n,T)
- This confirms the validity of the statistical approach
- The in-medium binding energy shift decreases with increasing temperature as expected
- Still, important effect for T as high as T=10 ! => increased cluster suppression wrt NSE

T.Custodio et al, arXiv:2407.02307

Conclusions

- A thermodynamically consistent formalism to calculate matter composition from a given microscopic energy functional
  - Leptodermous approximation for the cluster free energy
  - Effective mass with entrainment effects
  - Subtraction of continuum states and in-medium modified surface energies

=> First microscopic evaluation of the impurity factor

- Improved treatment of the in-medium energy functional of light clusters
  - Only the binding energy shift is extracted from the microscopic energy functional
  - Calibration of the unknown coupling on INDRA data

=> Temperature dependent light clusters binding energy shifts

Collaboration

Multi-Component plasma

- **H.Dinh-Thi** (Rice Univ., TX USA)
- F.Gulminelli, A.F.Fantina (LPC&Ganil, Caen)

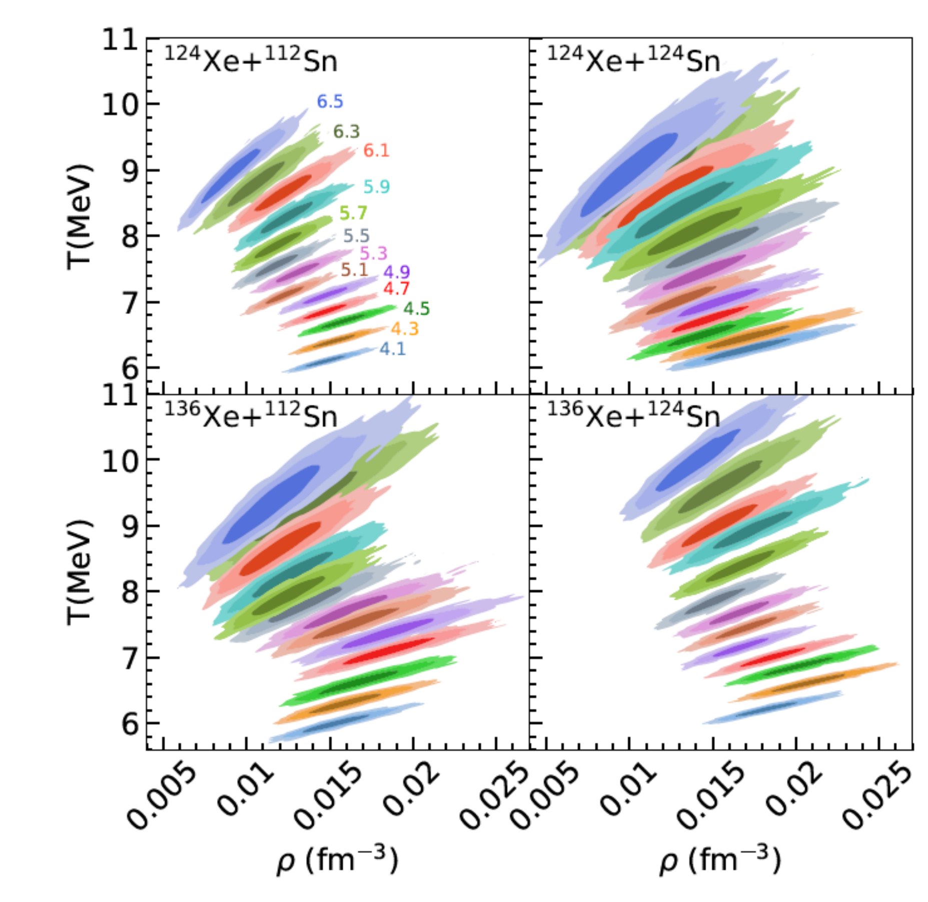
Master Projet MAC, In2p3 and IRP ACNu, CNRS

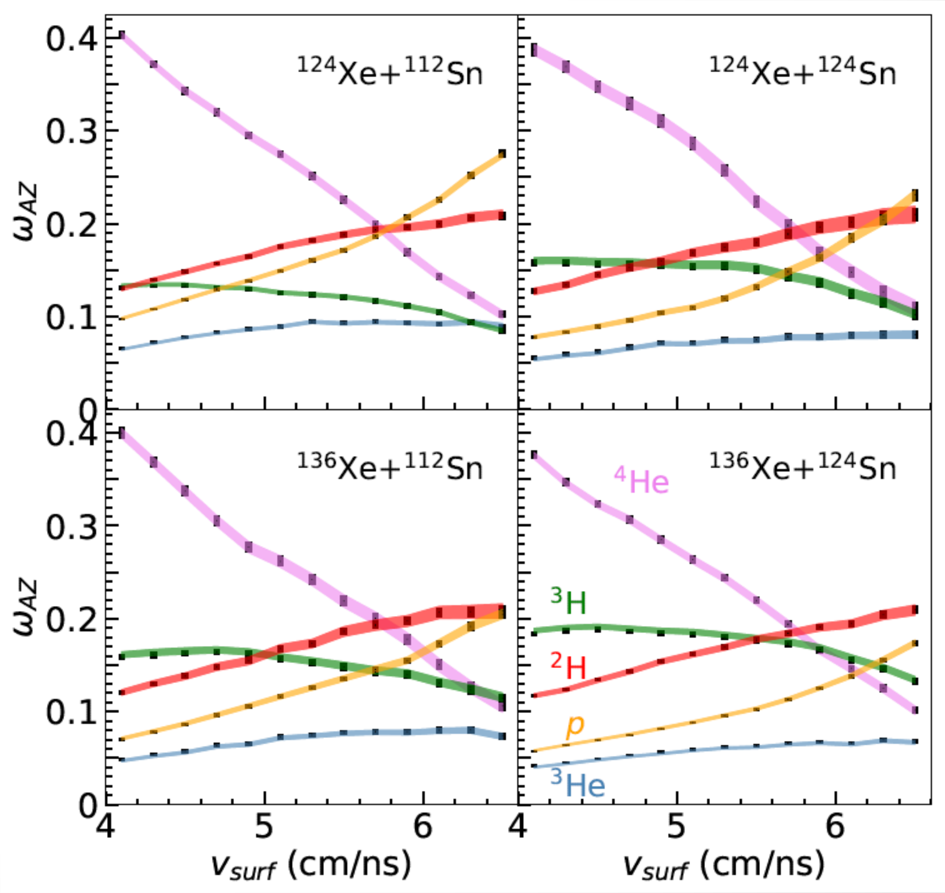
Cluster couplings calibration

- **T.Custodio** (Coimbra&LPC)
- F.Gulminelli, *A.Rebillard-Soulié, R.Bougault, D.Gruyer (LPC&Ganil, Caen)*
- H.Pais, C.Providencia, T.Mallik (Coimbra Univ.)

Campus France PESSOA program (project PHC 47833UB)

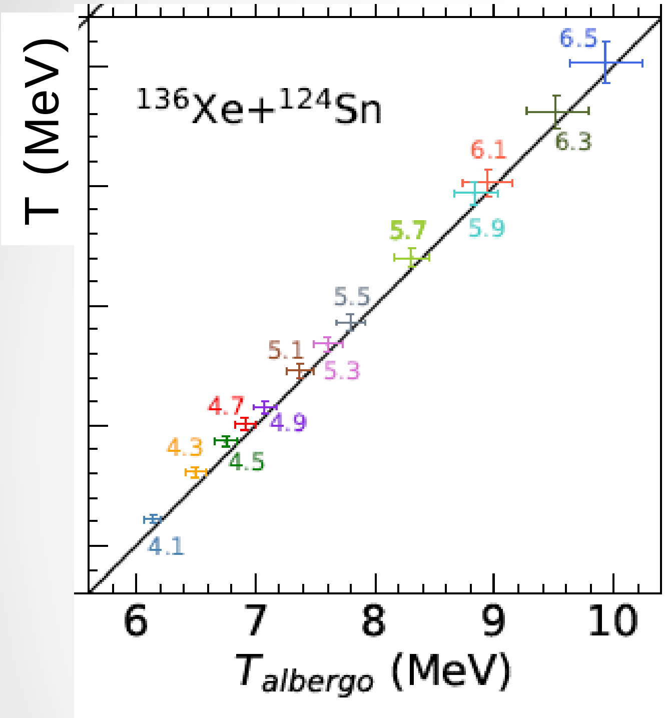
Calibration on INDRA data

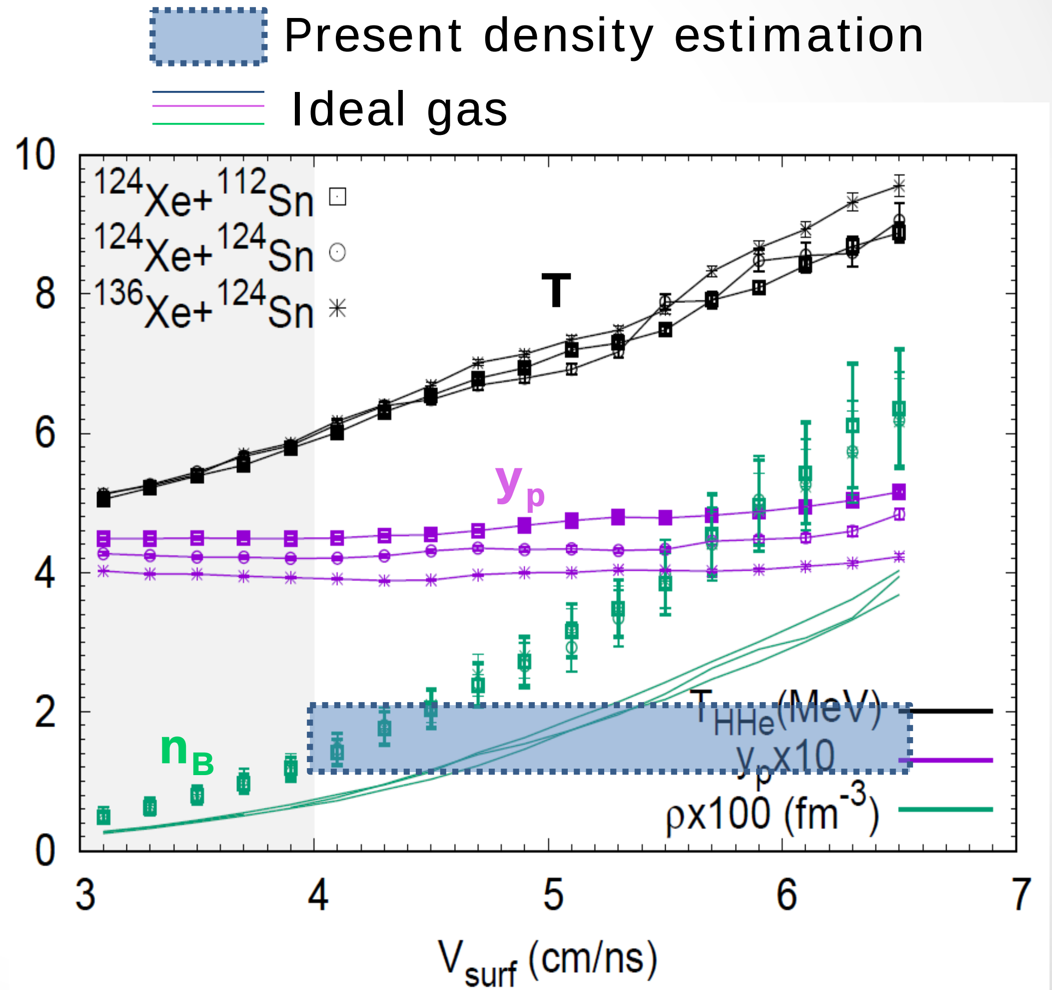




Comparison with the parameter estimation

from the classical ideal gas

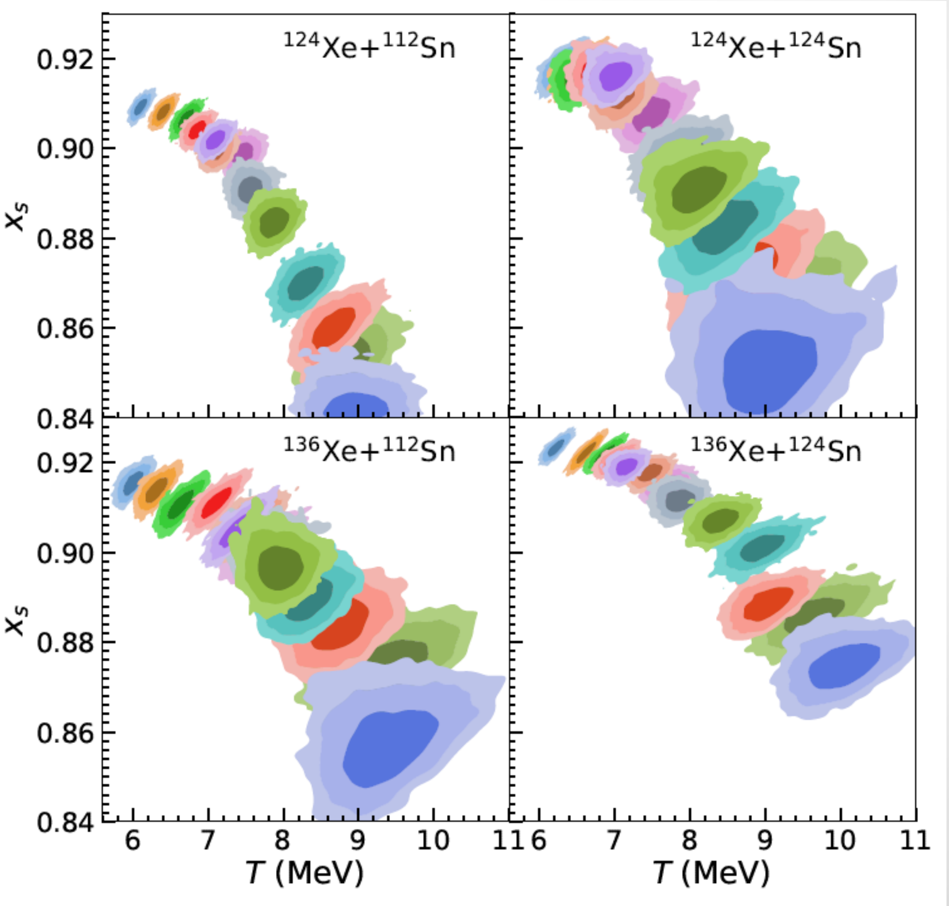




H.Pais et al Jphys.G 47(2020)105204

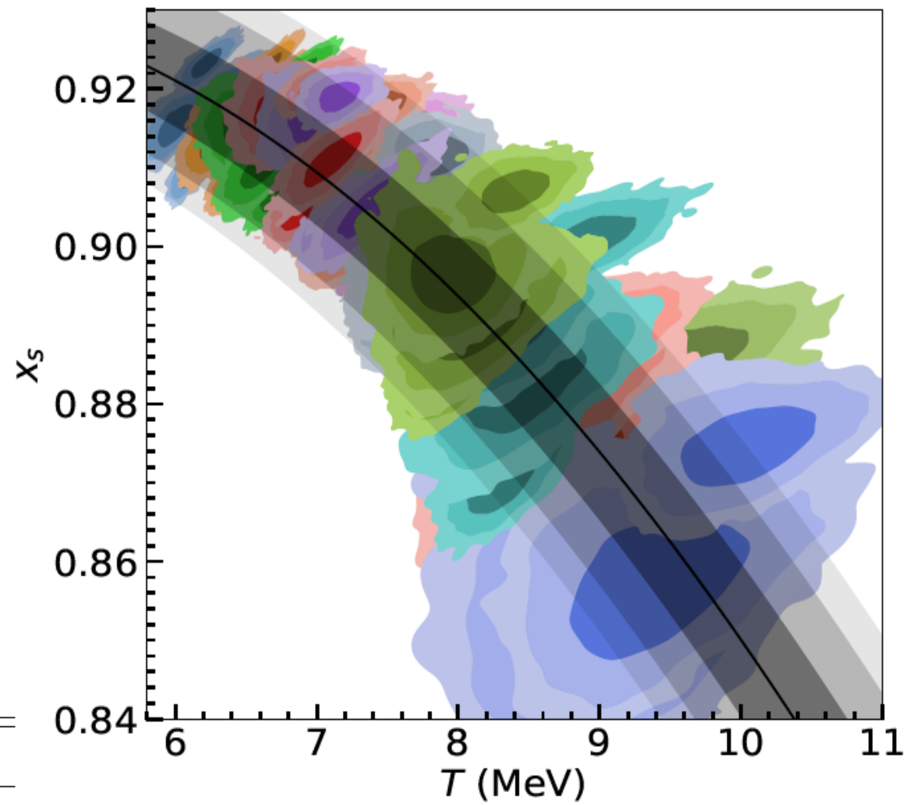
Posteriors for the coupling modification $x_s(n_B,T)$

- The independent analyses of the 4 systems lead to compatible results as a function of T
- This confirms the validity of the statistical approach



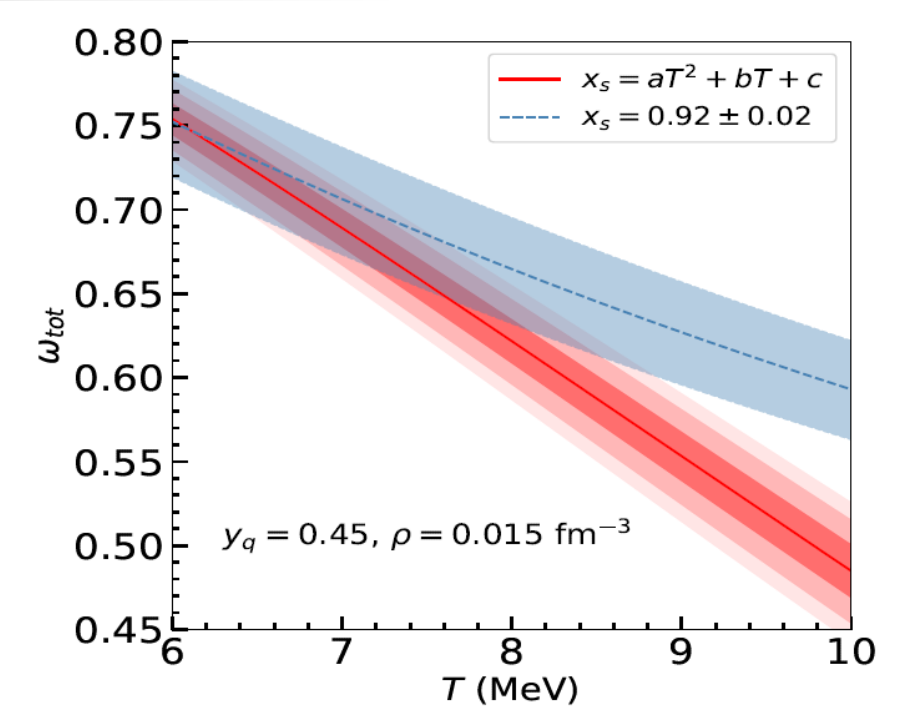
Posteriors for the coupling modification $x_s(n_B,T)$

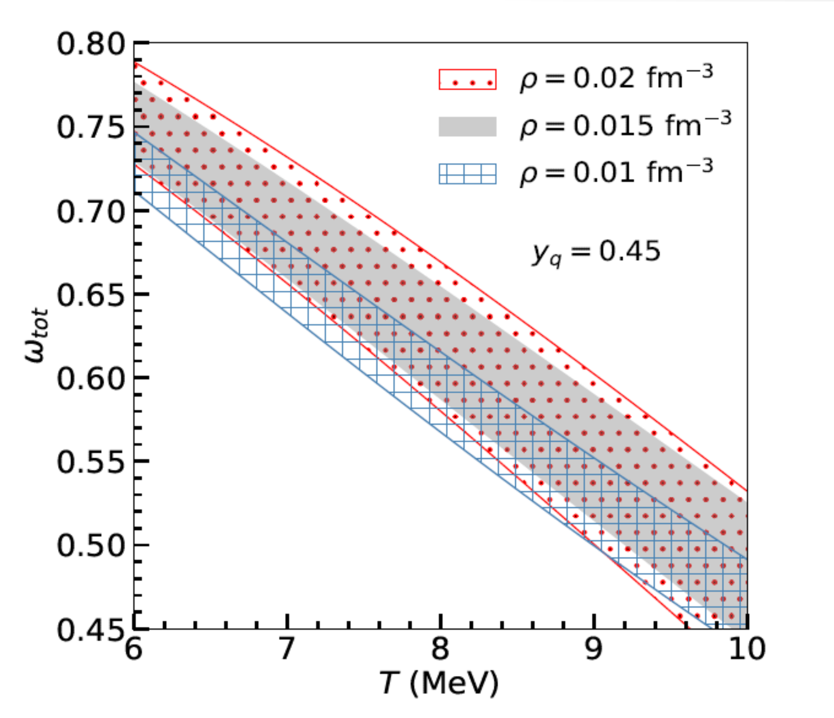
- The independent analyses of the 4 systems lead to compatible results as a function of T
- This confirms the validity of the statistical approach
- Quadratic fit (*Python lmfit*) $x_s = aT^2 + bT + c$
- Hyp: the n_B dependence can be neglected in the probed n_B range



Parameter	Unit	Median	1σ	2σ
a	MeV^{-2}	-0.00203	± 0.00003	± 0.00006
b	MeV^{-1}	0.01477	± 0.00047	± 0.00093
c		0.90560	± 0.0018	± 0.00355

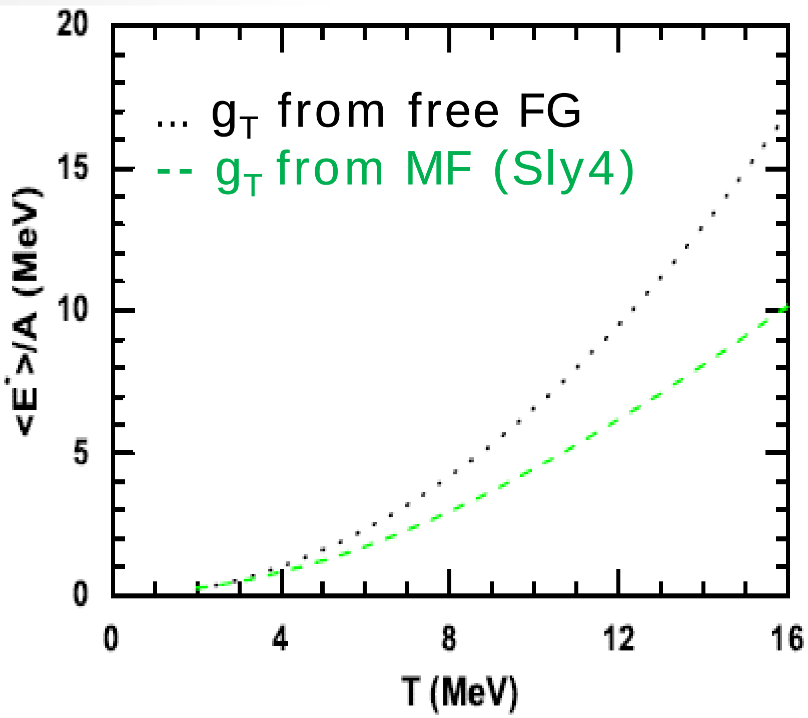
Predictions for general purpose EoS

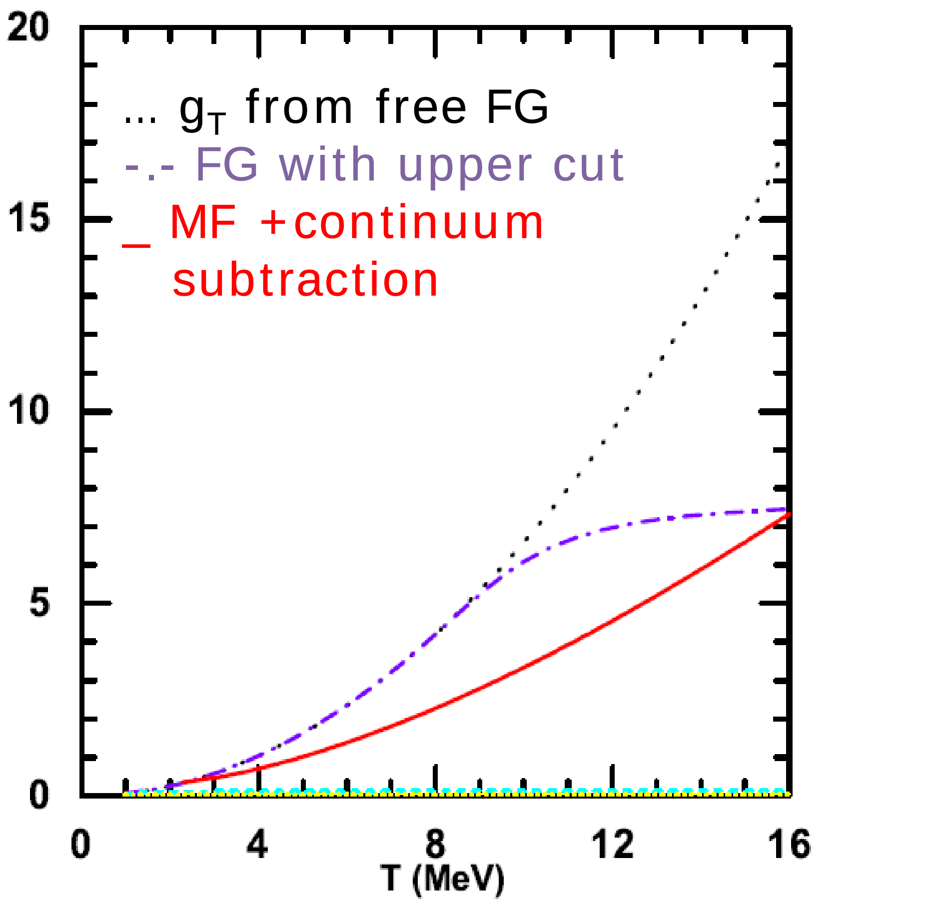




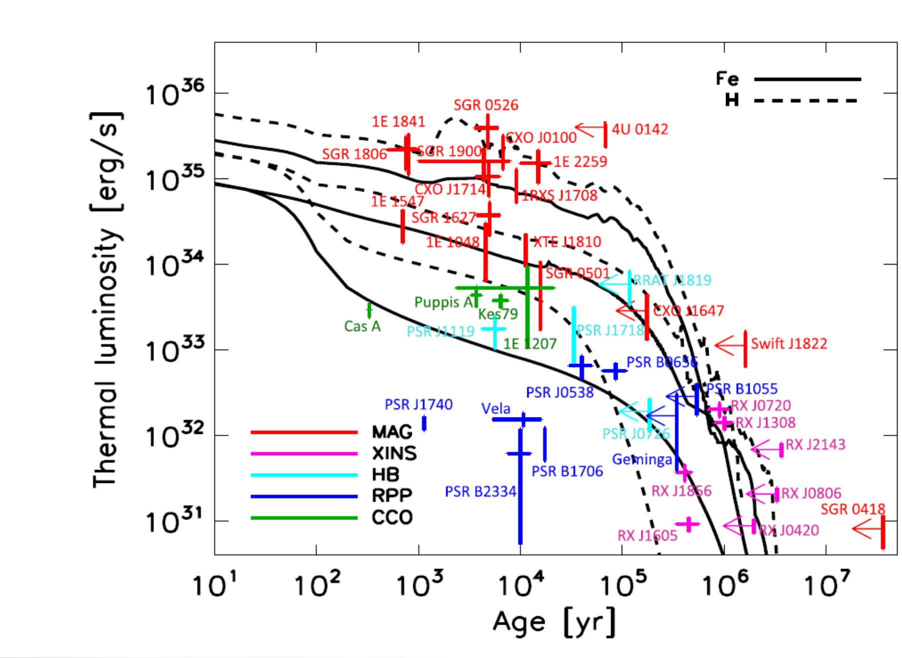
….. To be continued

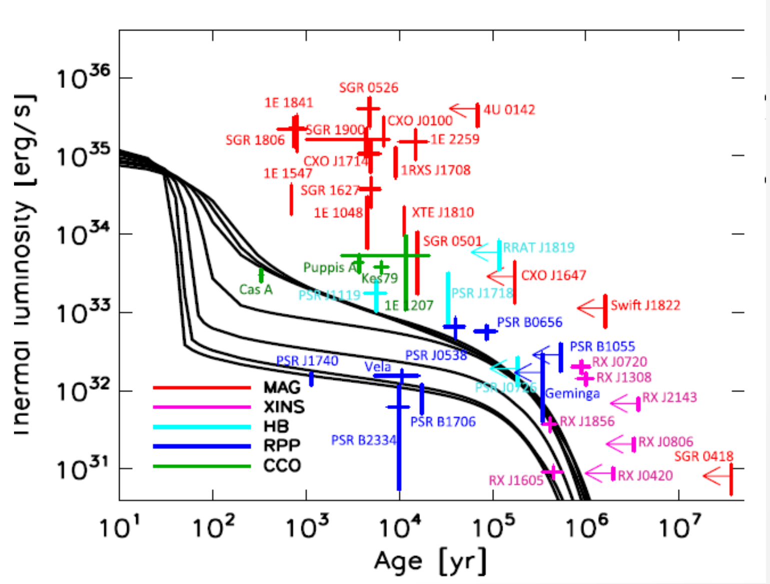
Effect of the microscopic entropy and continuum subtraction



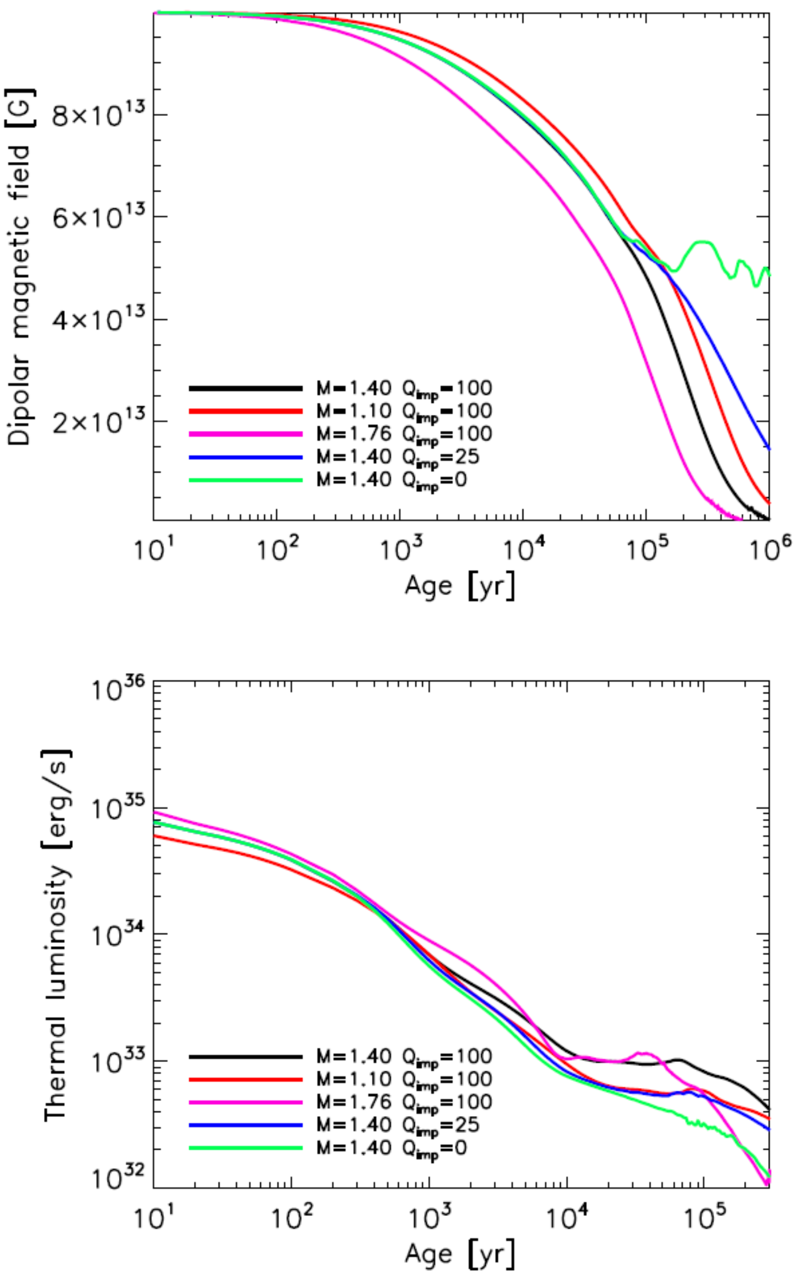


S.Mallik, FG, PRC 2021



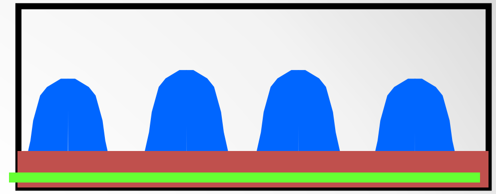


D.Vigano et al, MNRAS 434(2013)123



D.Vigano et al, MNRAS 434(2013)123

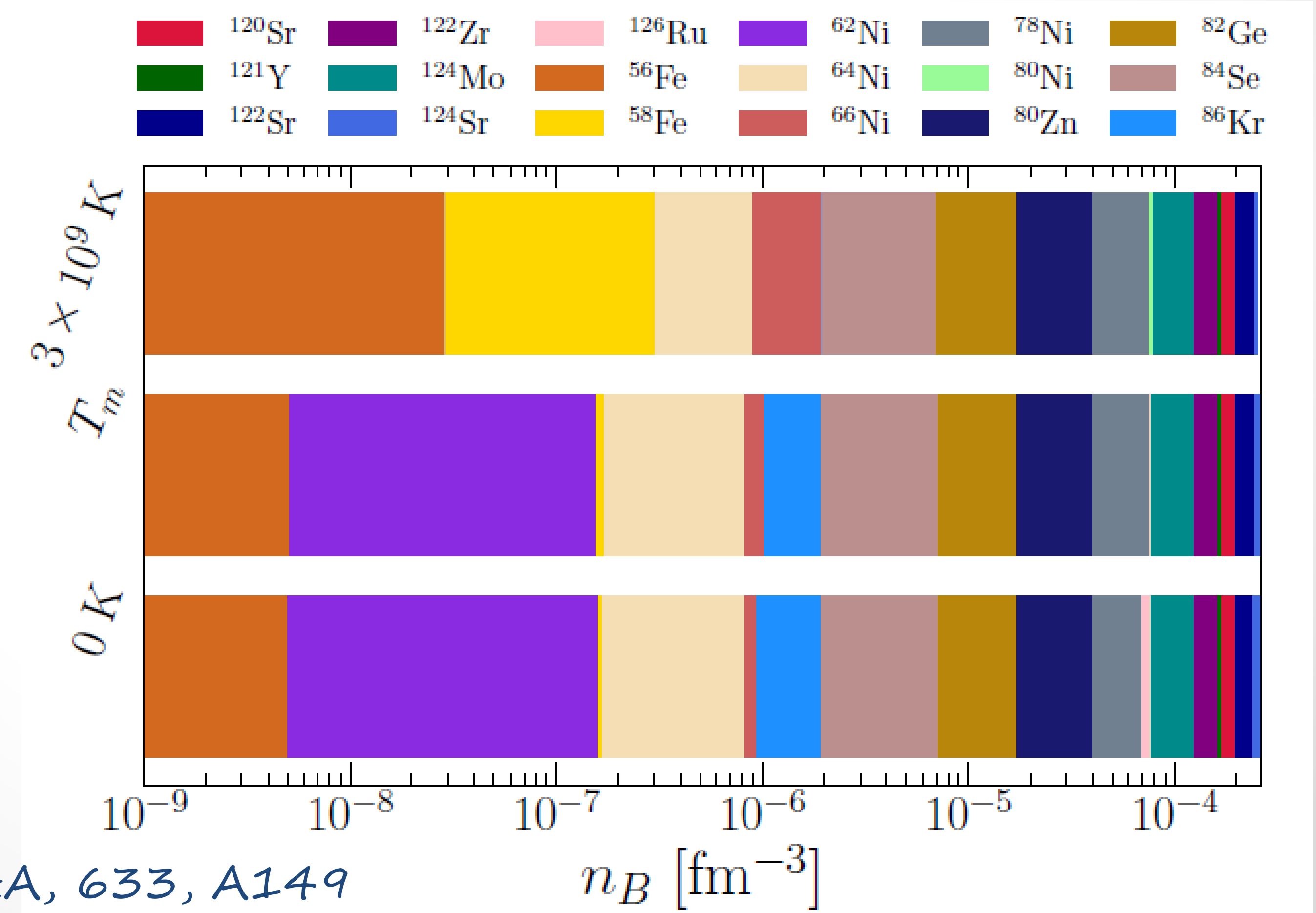
OCP with cluster DoF



$$\mathcal{F}^{OCP}(A,Z,\rho_{gq}) = \cancel{\mathcal{F}(\rho_{gq})} + \frac{F_{AZ}}{V_{WS}} = \mathcal{F}_{WS}$$

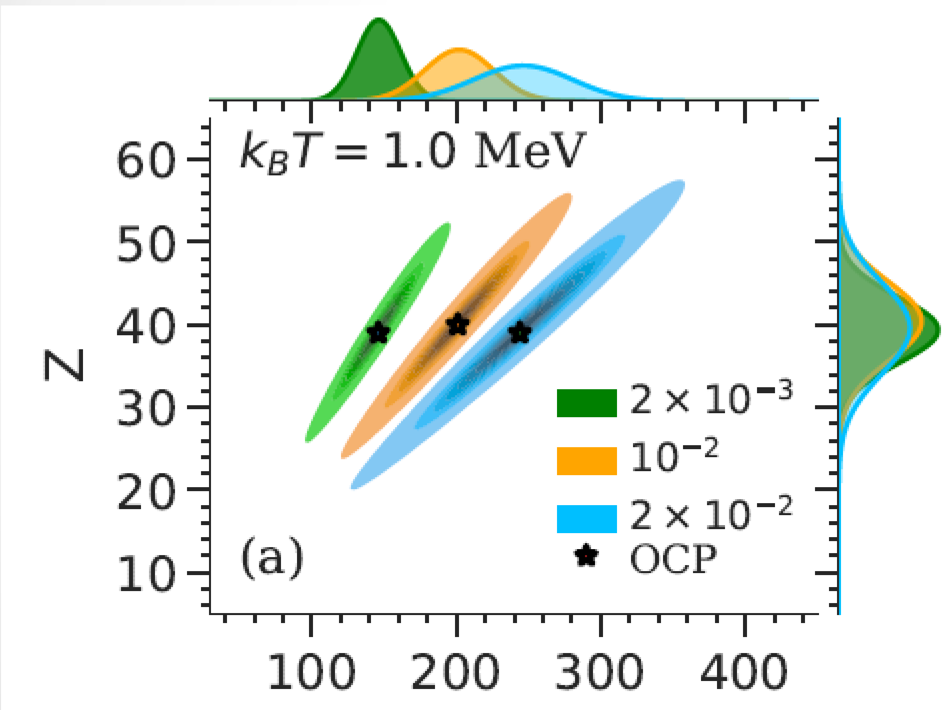
- Below drip: F_{AZ} is just the HFB free energy

BSK-24

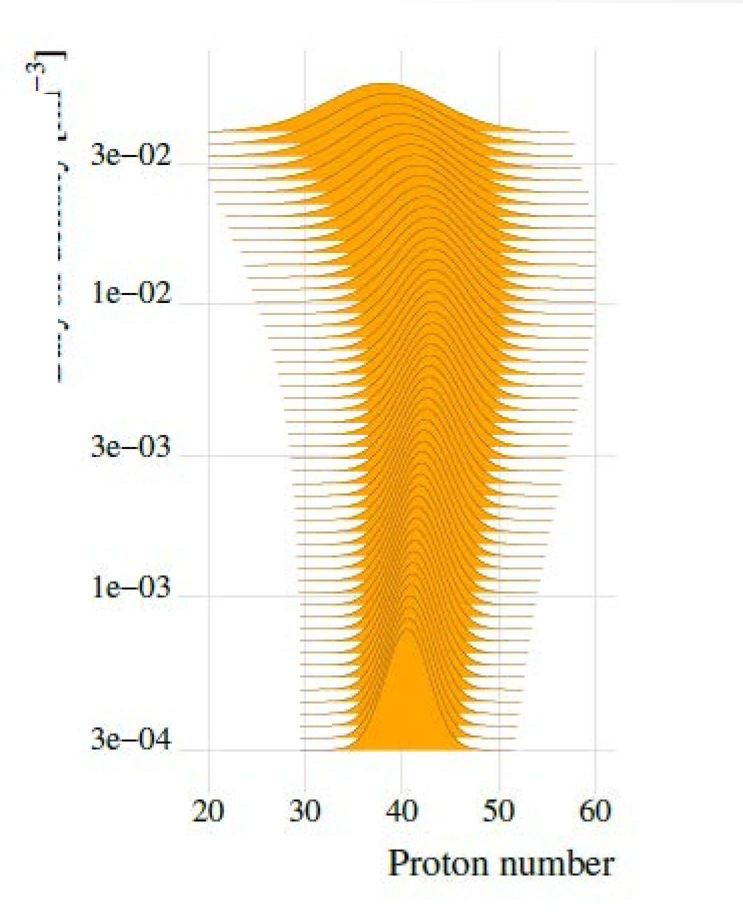


Fantina 2020, A&A, 633, A149

Nuclear distribution in the inner crust



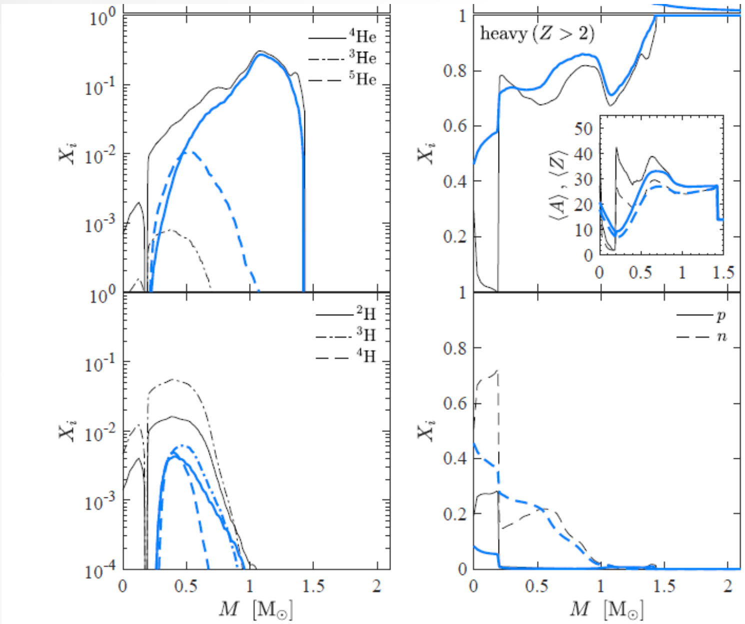
Dinh-Thi 2022, to be submitted



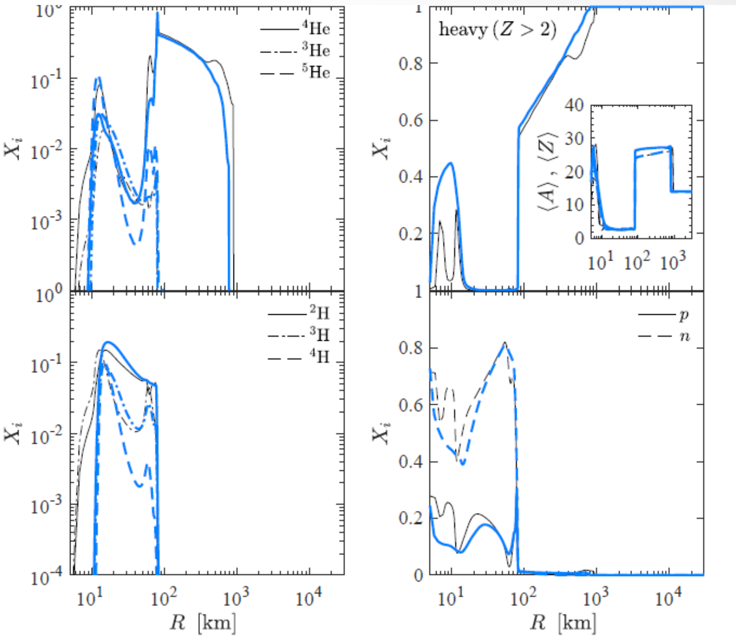
Carreau 2020, A&A, 640, A77

T.Fischer et al Phys. Rev. C **102**(2020) 055807

SN dynamics and cluster in-medium effects

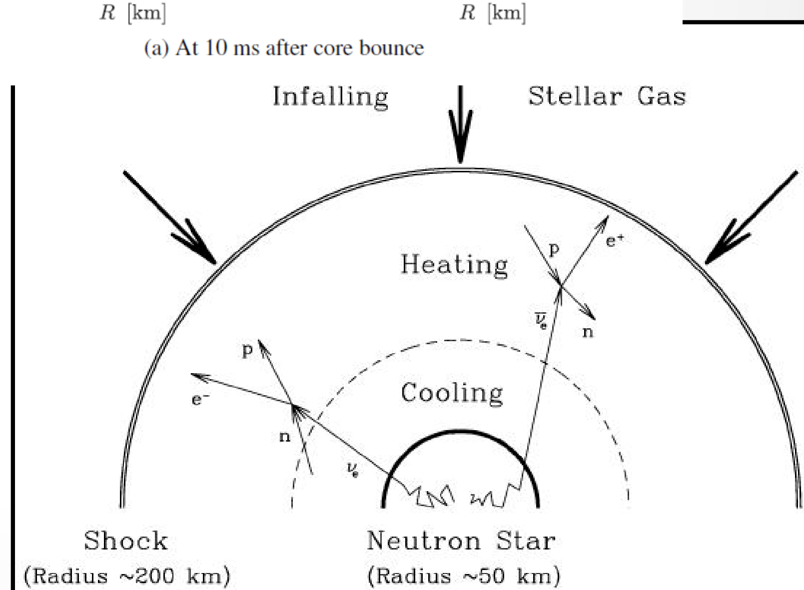


(a) At about 0.5 ms before core bounce



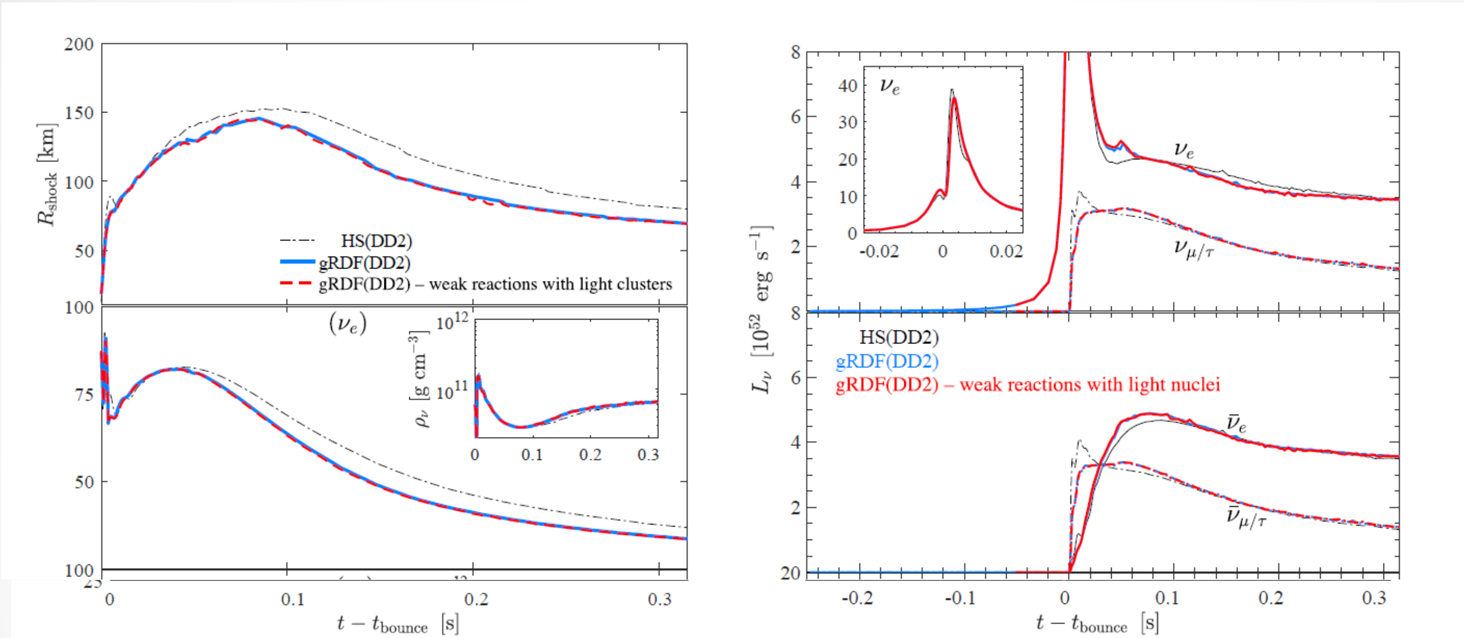
(a) At 10 ms after core bounce

- The energy deposition in the gain region depends on the position of the ν–sphere
- Coherent scattering off nuclei is a crucial source of opacity
- Composition depends on the in-medium modifications to the binding energy



T.Fischer et al Phys. Rev. C **102**(2020) 055807

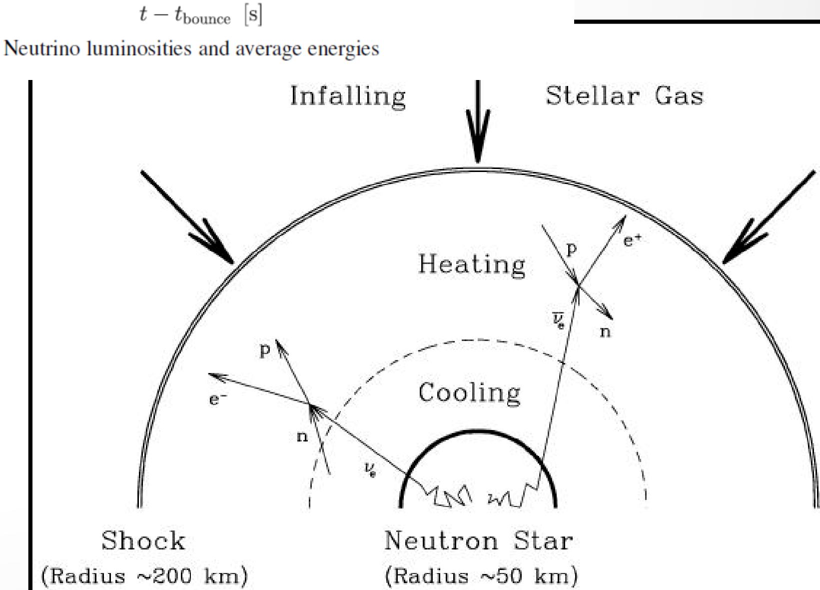
SN dynamics and cluster in-medium effects



(a) Shock radii and neutrinospheres

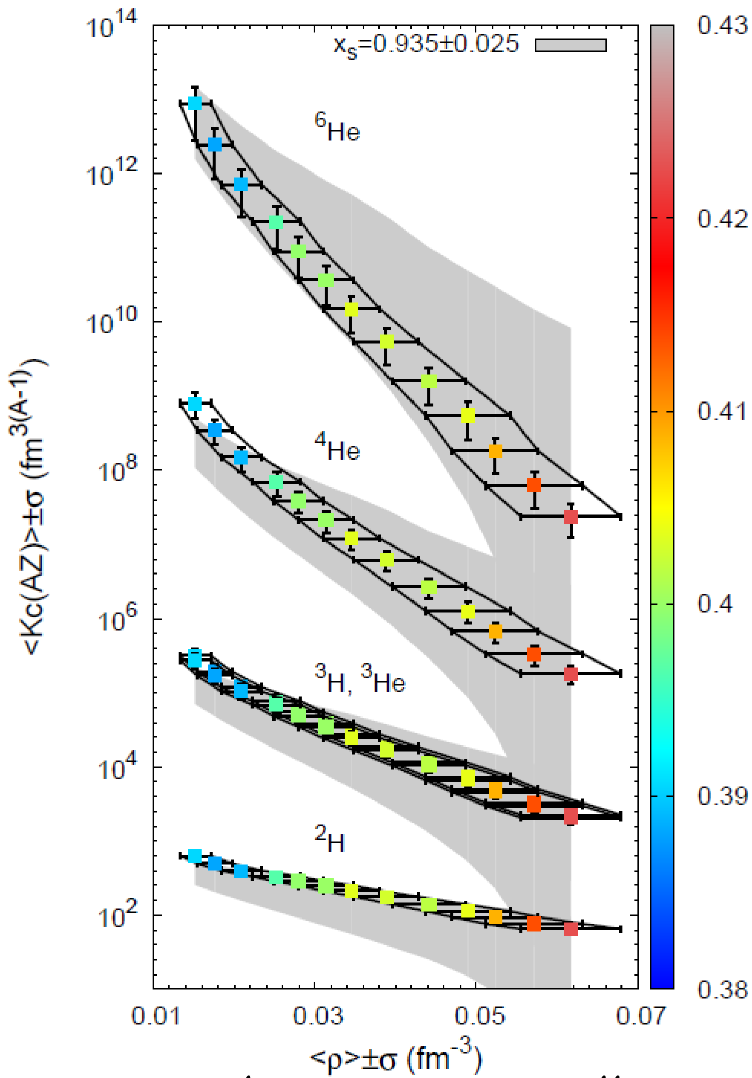
(b) Neutrino luminosities and average energies

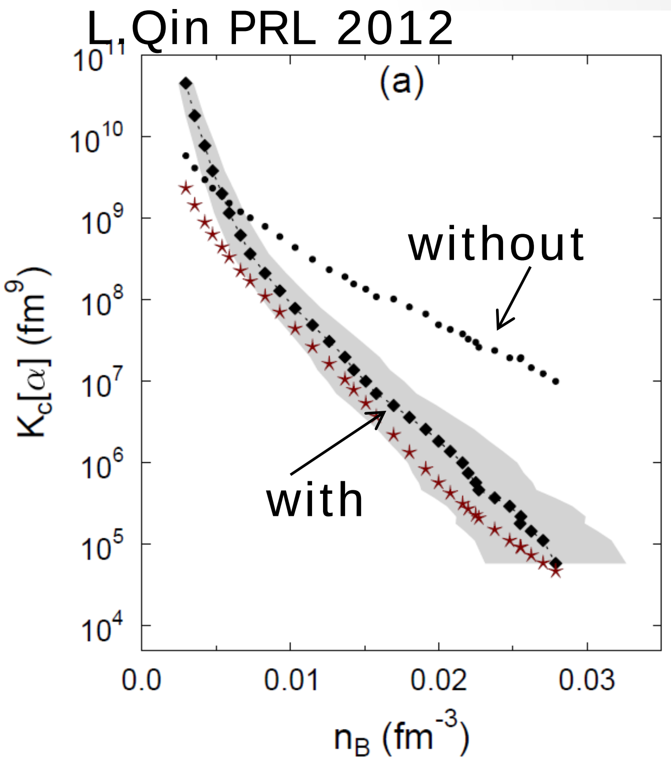
- The energy deposition in the gain region depends on the position of the ν–sphere
- Coherent scattering off nuclei is a crucial source of opacity
- Composition depends on the in-medium modifications to the binding energy
-



Chemical constants from multi-fragmentation

$$K_c(A,Z) = \frac{\rho_{pa}(A,Z)}{\rho_{pa}(1,1)^Z\ \rho_{pa}(1,0)^N}$$





R.Bougault & INDRA coll. JPG (2019)
H.Pais & INDRA coll. PRL (2020)

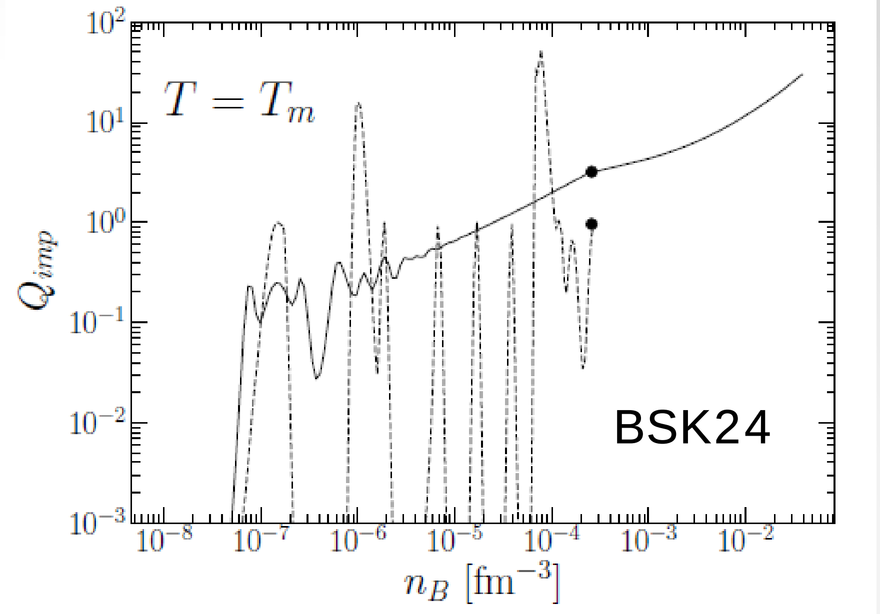
Applications

- Impurity factor

=> extension to pasta to be done

- e-transport coefficients

=> To be calculated



T.Carreau, A.Fantina, FG submitted to A&A

$$\kappa = \frac{\pi k_F^3 T}{12 e^4 m_e^{*2} \Lambda_{ep}^{\kappa}},$$

$$\sigma = \frac{k_F^3}{4\pi e^2 m_e^{*2} \Lambda_{ep}^{\sigma}},$$

$$\eta = \frac{k_F^5}{60\pi e^4 m_e^{*2} \Lambda_{ep}^{\eta}},$$

$$\Lambda_{ep}^{\kappa,\sigma}(A,Z,d) = \int_{q_0}^{2k_F} dq\, q^3 u^2(q) S_q(A,Z,d) \left(1 - \frac{q^2}{4m_e^{*2}}\right)$$

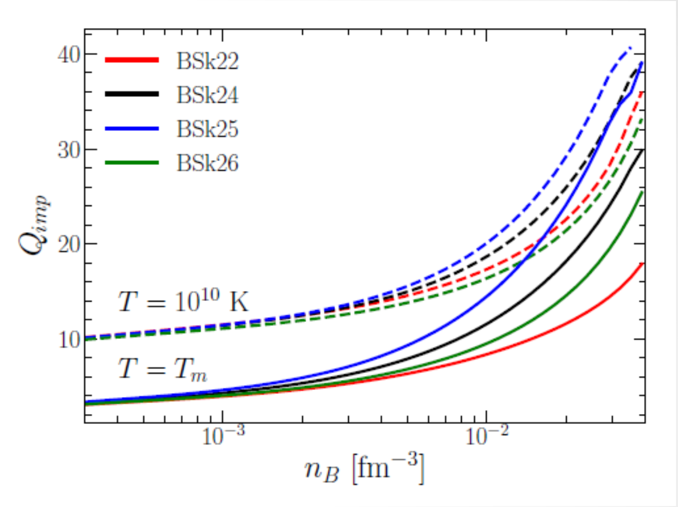
$$\Lambda_{ep}^{\eta}(A,Z,d) = \int_{q_0}^{2k_F} dq\, q^3 u^2(q) S_q(A,Z,d) \left(1 - \frac{q^2}{4m_e^{*2}}\right)\left(1 - \frac{q^2}{4k_F^2}\right)$$

$$\Lambda_{ep}^{\kappa,\sigma,\eta} = \sum_{A,Z,d} p_{AZ,d}\, \Lambda_{ep}^{\kappa,\sigma,\eta}(A,Z,d)$$

Potekhin, et al. (1999), A&A, 346, 34
Chugunov & Yakovlev (2005) ARep, 49, 724

Phenomena

- NS oscillations
- PNS cooling (SN1987a ??)
- Mergers ???

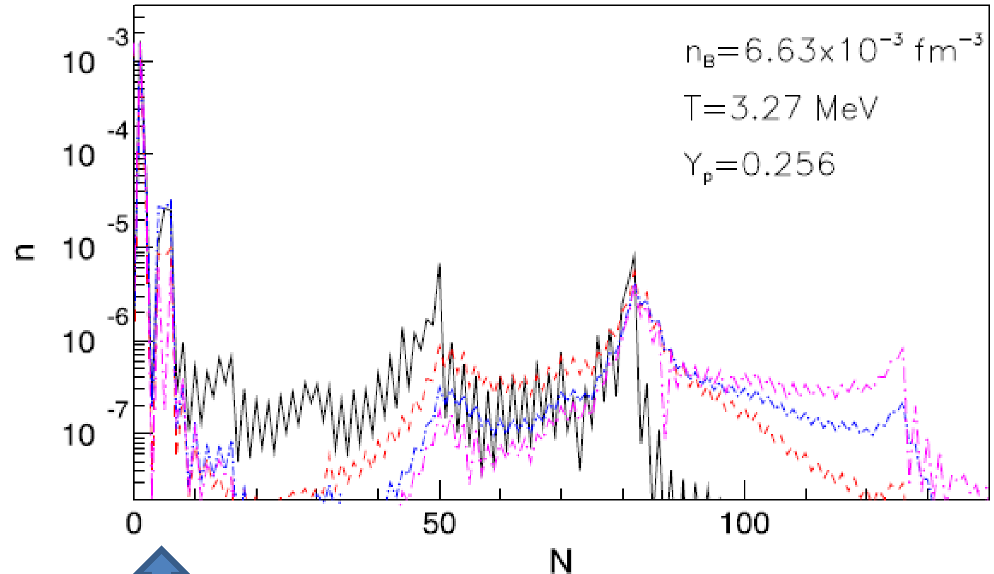


Strategy

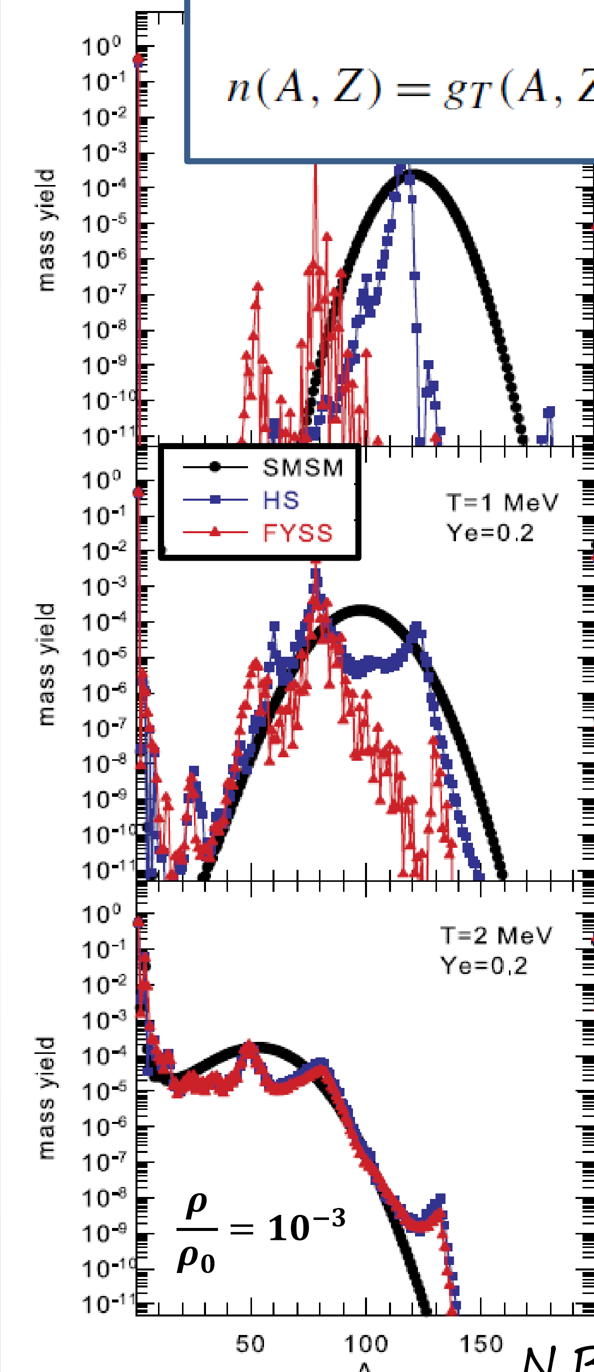
- What exactly do we need to calculate?
- Which format (table, code..) ?
- Microscopic functional: calculate for BSK22-26 ?
- Meta-modelling: Tews functionals? Most probable posterior EoS only?

$$n(A,Z) = g_T(A,Z)\left(\frac{M_{A,Z}T}{2\pi\hbar^2}\right)^{3/2}\exp\left[\frac{(A-Z)\mu_n + Z\mu_p - M(A,Z) + \delta E}{T}\right]$$

A.Raduta, FG NPA 983(2019)252



Different level densities (g_T)



Different models (M, $\mu(\rho)$, δE, g_T)

N.Buyukcizmeci et al NPA 907(2013)13

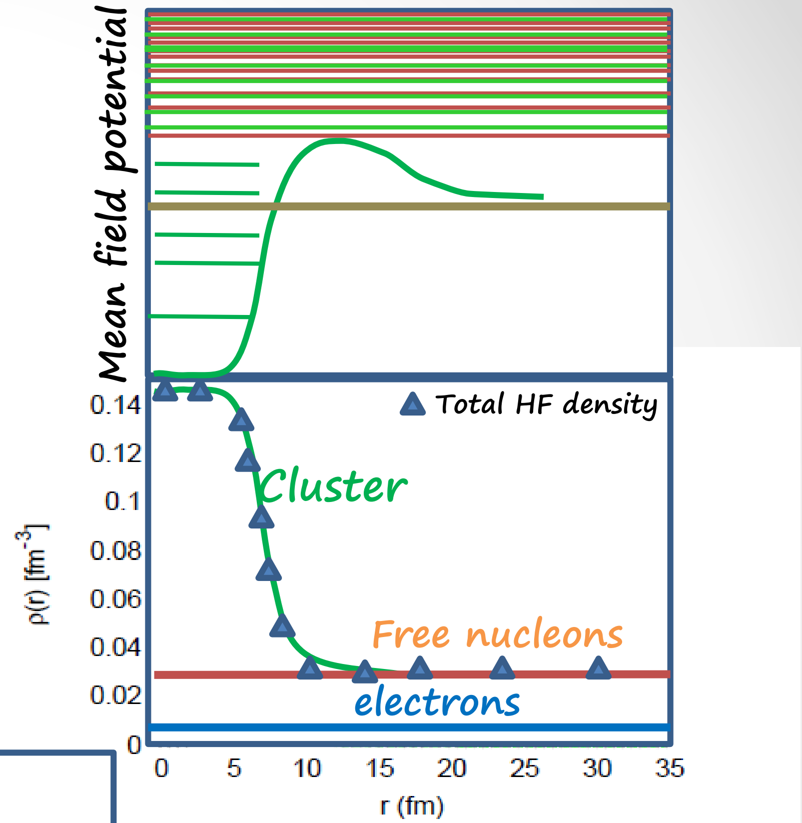
T>0: continuum states

- Double counting of continuum states if we switch to cluster DoF
- Easy subtraction in the GC ensemble

$$Z_{\beta\mu_n\mu_p} = \prod_{i,q}\left(1 + \exp\left(\alpha_q - \beta e_i^q\right)\right)$$

$$\Omega_N = \Omega_{Ng} - \Omega_g \qquad \Omega_N = -T \ln \mathscr{Z}^{(N)}_{\beta\mu_n\mu_p}$$

Tubbs&Koonin, ApJ 232 (1979) L59
Bonche,Levit,Vautherin NPA427(1984)278



$$F_N(A, I, \rho_g, y_g) =$$

$$= -TV_N \ln \mathfrak{z}^{mf,N}_{\beta,\mu_n,\mu_p} + \mu_n N_n + \mu_p N_p + E_{coul} + E_{surf}(A,I)$$

$$= V_N\left[v(n_c,\delta_c) - v(n_g,\delta_g) - \sum_q \left(U_{c,q} n_{c,q} - U_{g,q} n_{g,q}\right)\right]$$

$$- \sum_{q=n,p}\left[\frac{2V_N}{3}\left\{\xi_{c,q} - \xi_{g,q}\right\} - \mu_q N_q\right]; + E_{coul} + E_{surf}(A,I)$$

S.Mallik, FG, to be submitted

Surface tension and correlations with NM properties

$I_{crust,1.4} / I_{1.4}$

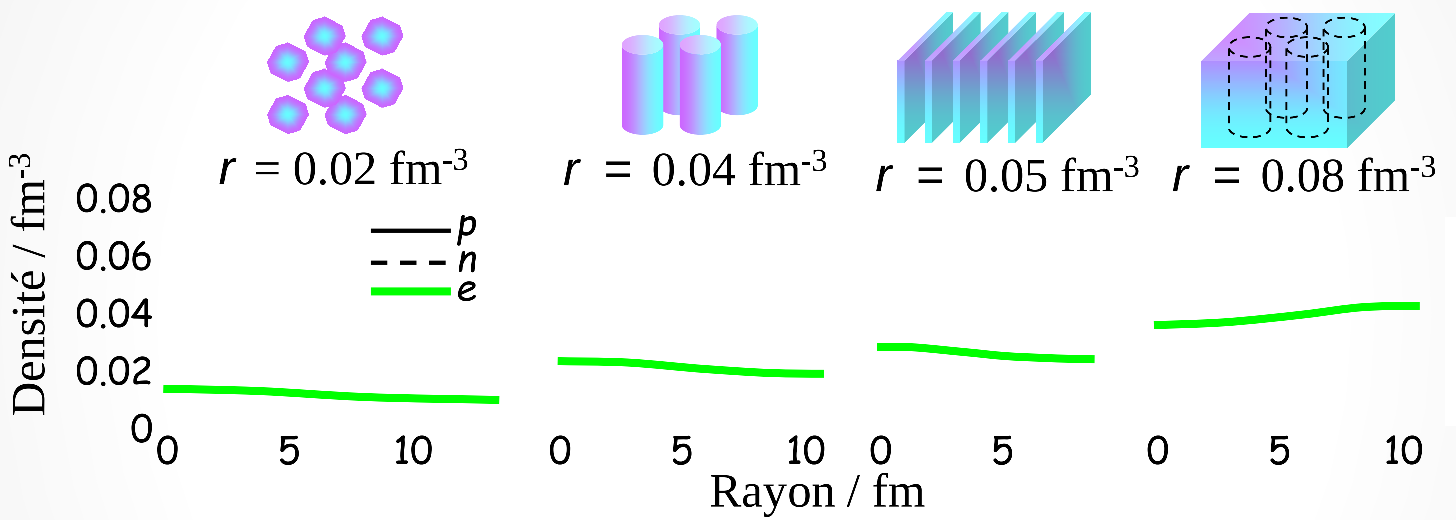
	n_{sat}	E_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	m^*/m	$\Delta m^*/m$	b	p	σ_0	b_s	n_t	P_t	$I_{crust,1.4}$
LD p=3	0.01	-0.05	0.20	-0.06	-0.06	-0.24	-0.38	0.42	0.77	-0.32	0.00	0.07	-0.04	0.00	0.04	0.19	0.90	0.93	0.92
HD p=3	-0.03	0.24	0.26	-0.22	0.07	0.26	-0.08	-0.33	0.46	-0.10	-0.03	0.04	-0.10	0.00	-0.23	-0.02	0.71	0.94	0.79
LD	0.04	-0.21	0.16	-0.05	-0.04	-0.09	-0.23	0.26	0.52	-0.20	-0.01	0.04	-0.02	0.56	0.46	0.58	0.92	0.96	0.94
HD	-0.02	0.14	0.24	-0.21	0.08	0.25	-0.08	-0.31	0.42	-0.09	-0.02	0.04	-0.09	0.25	-0.09	0.18	0.73	0.95	0.79
Prior	0.02	0.12	0.21	-0.11	0.03	0.25	-0.03	-0.46	0.48	-0.11	-0.04	-0.14	-0.15	0.11	-0.08	0.10	0.40	0.96	0.72

	E_{sat}	n_{sat}	K_{sat}	Q_{sat}	Z_{sat}	E_{sym}	L_{sym}	K_{sym}	Q_{sym}	Z_{sym}	p	σ_0	b_s	σ_{0c}	β
LD+HD	-0.06	-0.05	0.33	-0.35	0.18	0.05	-0.19	-0.28	0.52	-0.14	0.13	0.06	0.12	-0.03	-0.06
LD+HD $p = 3$	-0.05	-0.06	0.34	-0.35	0.18	0.03	-0.22	-0.30	0.54	-0.16	0.00	0.05	0.03	-0.02	-0.08
LD (w/ P) $p = 3$	-0.00	-0.03	0.23	-0.24	0.10	-0.04	-0.33	-0.02	0.73	-0.24	0.00	0.00	0.05	0.00	-0.04
LD $p = 3$	-0.07	-0.03	0.30	-0.27	0.12	0.16	-0.13	-0.35	0.50	-0.08	0.00	0.07	-0.05	0.00	-0.11
HD $p = 3$	0.24	-0.03	0.29	-0.44	0.20	0.11	-0.03	-0.16	0.44	-0.15	0.00	-0.25	-0.21	0.27	-0.00
Prior $p = 3$	0.19	-0.03	0.25	-0.40	0.14	0.13	-0.01	-0.25	0.42	-0.09	0.00	-0.19	-0.21	0.24	-0.05

The ambiguity is under control if we make a UNIFIED modelling

T.Carreau, PRC 2020 +
Thomas Carreau, PhD thesis

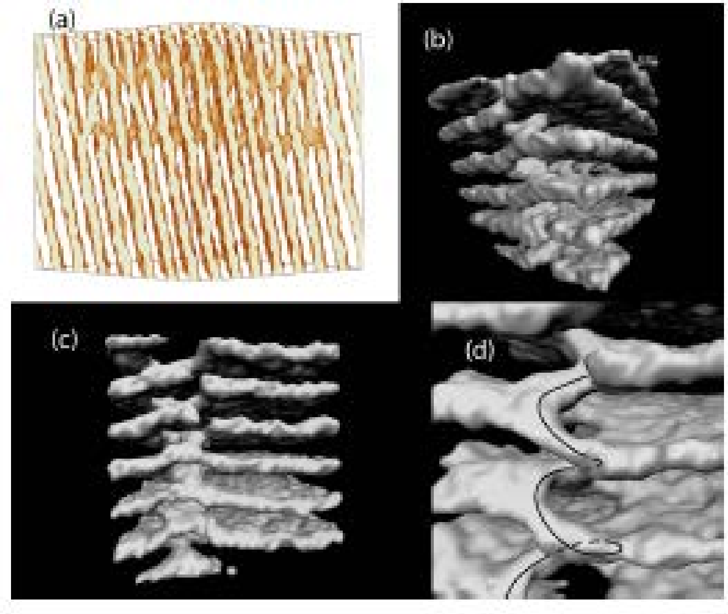
Geometry fluctuations



- $d\mathcal{F}_{MCP}(\{n_{AZ,d}\}) = 0$ leads to:

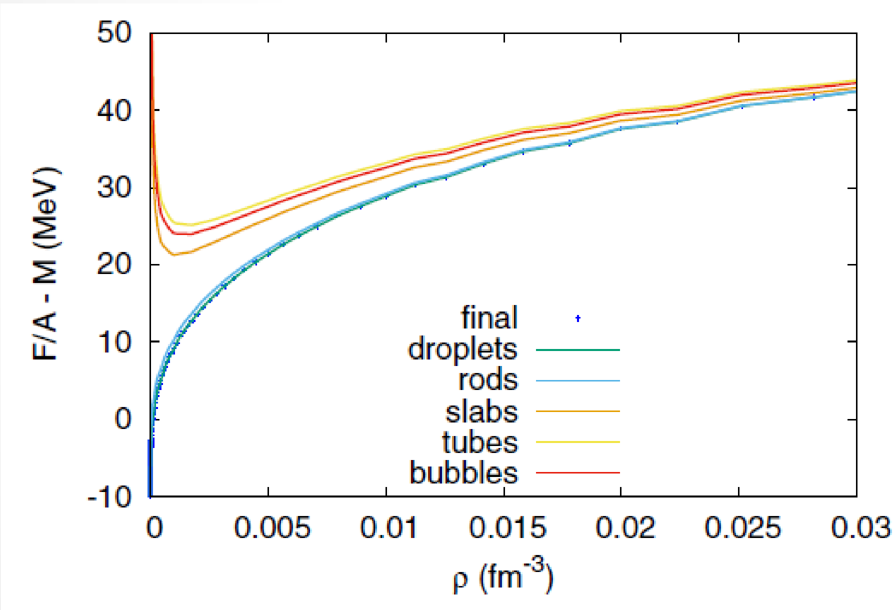
$$p_{AZ,d} \propto \exp\beta\left[N\mu_n + Z\mu_p - F_{i,d} + R_{AZ,d}(n_e)\right]$$

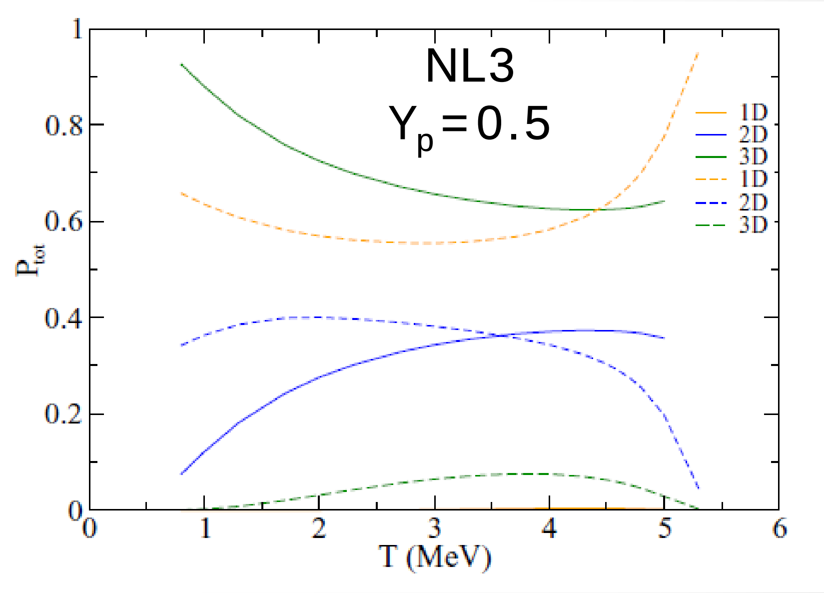
Schneider, Horowitz PRL 2015



Geometry fluctuations

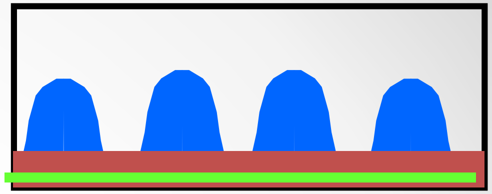
Solid: $\rho_B = 0.01$ fm^{-3}
Dashed: $\rho_B = 0.03$ fm^{-3}





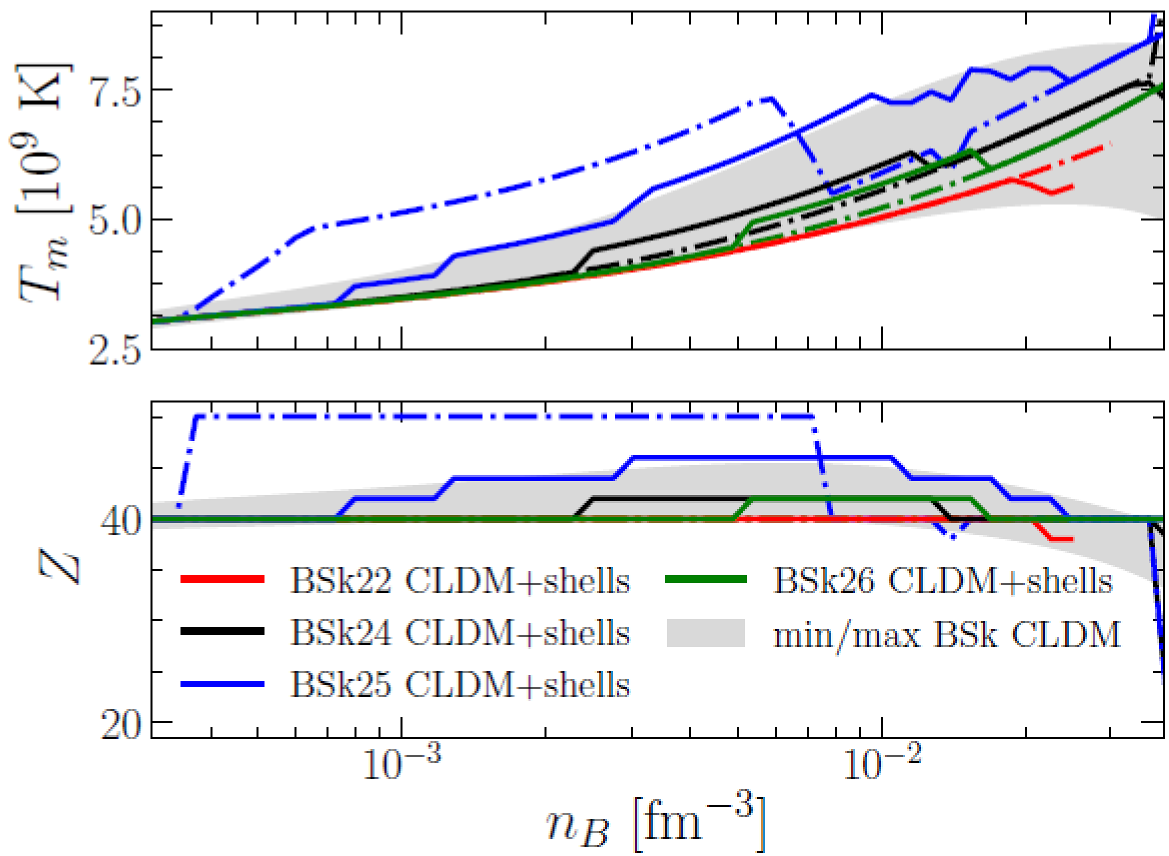
C.Barros, D.Menezes, FG, PRC 101 (2020) 035211

OCP with cluster DoF



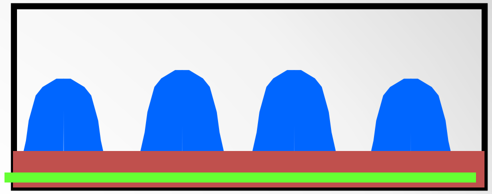
$$\mathcal{F}^{OCP}(A, Z, \rho_{gq}) = \mathcal{F}(\rho_{gq}) + \frac{F_{AZ}}{V_{WS}} = min$$

Carreau 2020, A&A, 635, A84



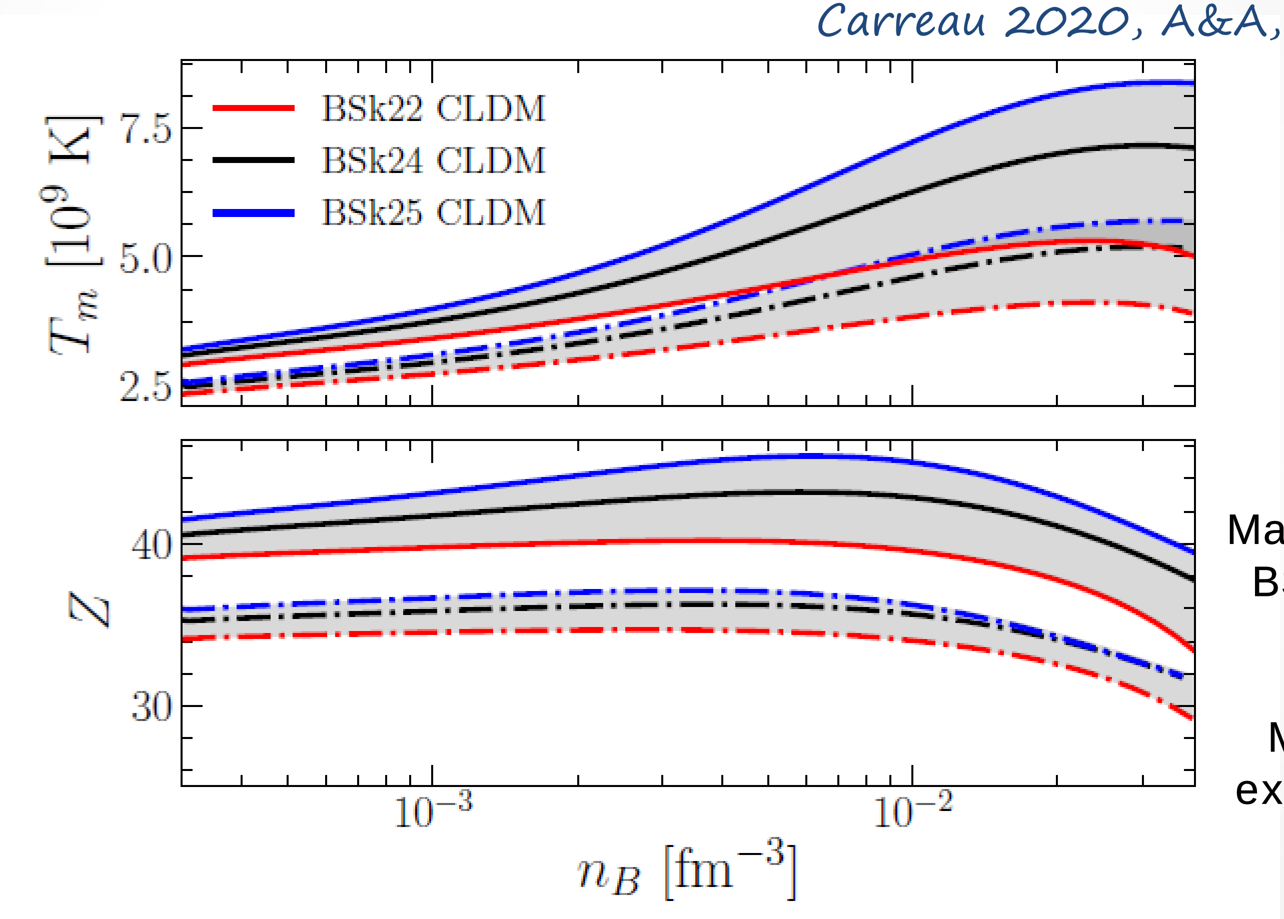
=> The uncertainty in the shell treatment is within the fonctional uncertainty.

OCP with cluster DoF



$$\mathcal{F}_{AZ}(A,Z,\rho_{gq}) = \mathcal{F}_{\mu}(\rho_{gq}) + \frac{F_i}{V_{AZ}} = min$$

Carreau 2020, A&A, 635, A84



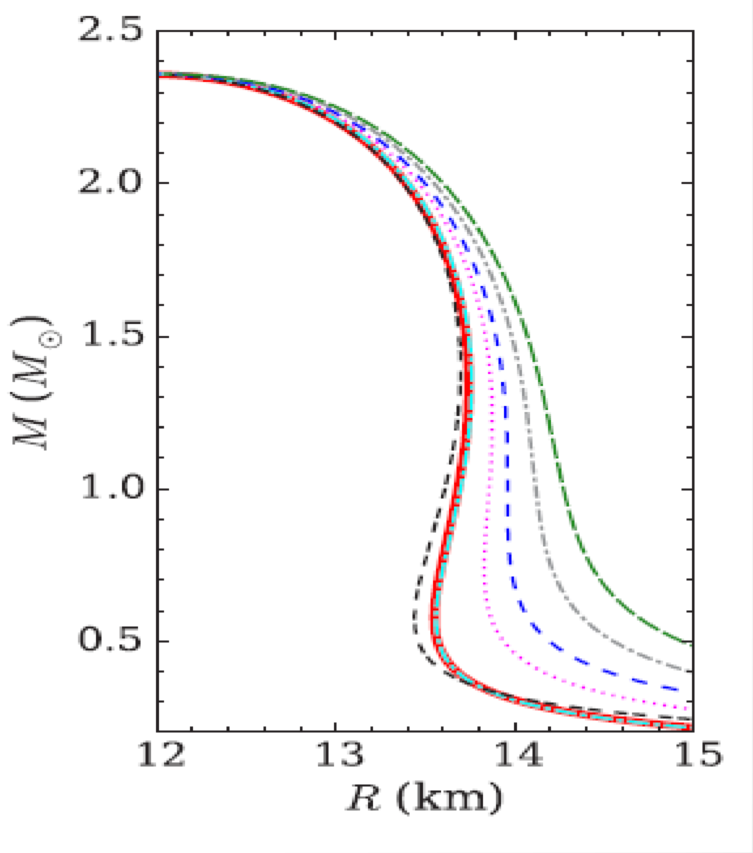
Mass fit on BSK ETF

Mass fit on exp.spherical nuclei

=> Importance of a unified treatment

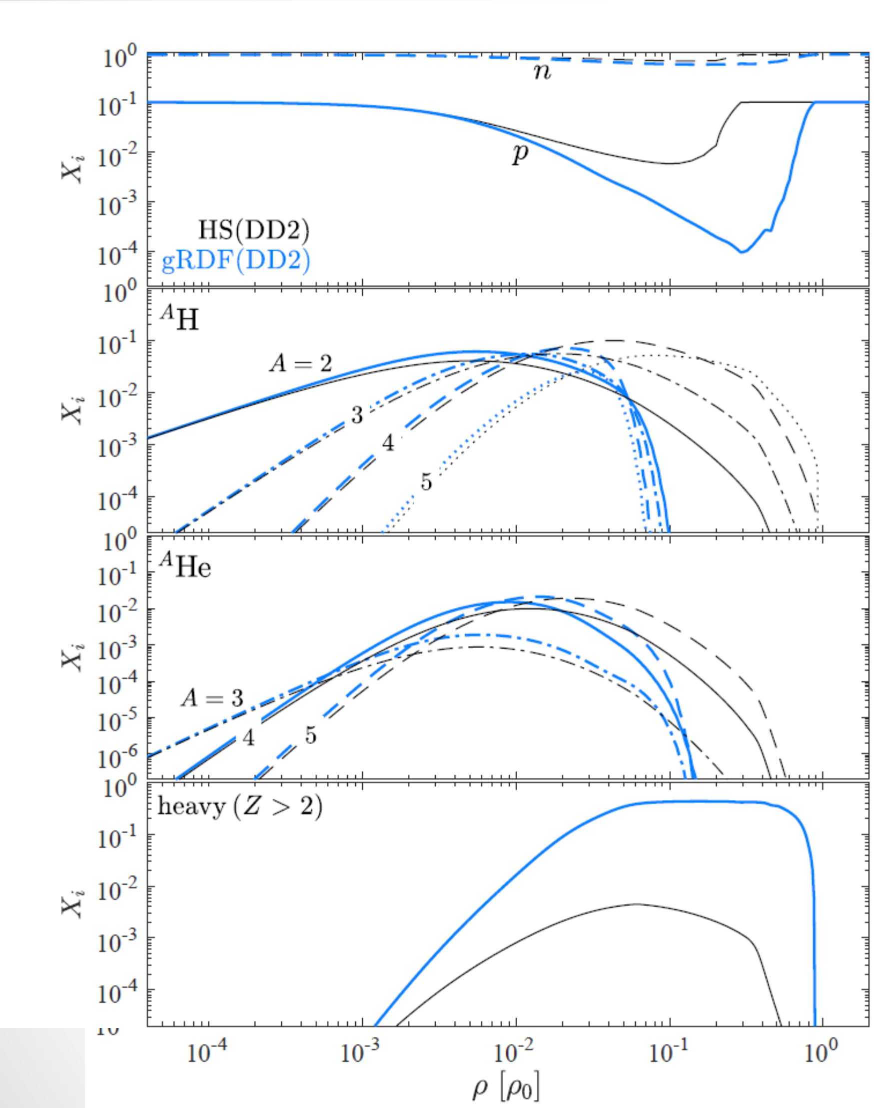
Plan

1. ***Motivation: transport properties in compact stars***
2. ***From a microscopic EDF to the finite temperature nuclear distribution: unified T>0 EoS for astrophysics***
3. ***Results: melting temperature and impurity factor: the unexpected role of light nuclear clusters***



M.Fortin et al. PRC 94, 035804

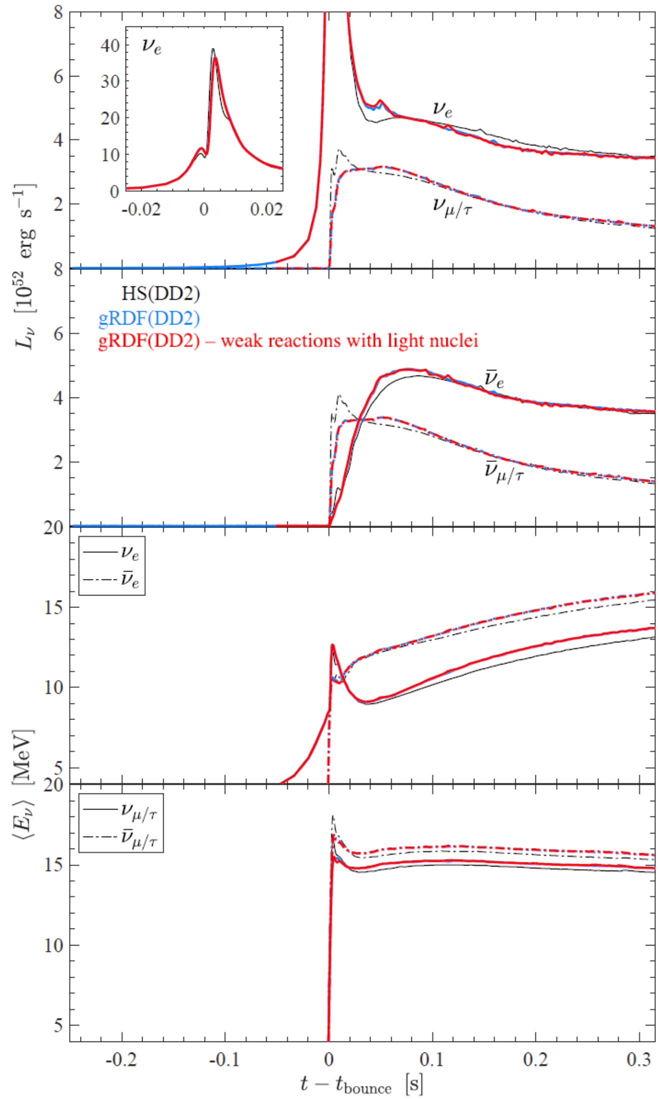
Clusters in-medium effects



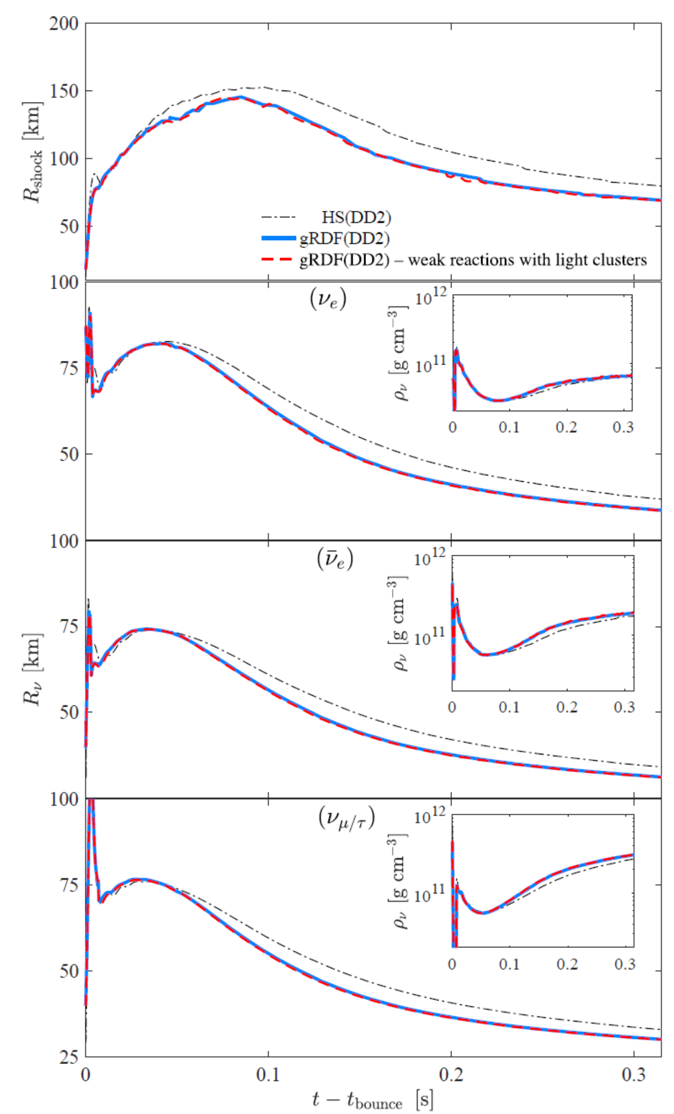
T.Fischer et al PRC102(2020)

- the ETF approach is not very realistic for light clusters!
- Alternative approaches: in-medium modified meson couplings H.Pais, FG PRC 97(2018)045805; quasi-particle virial expansion G.Roepke, PRC101 (2020) 064310

Effect on the CCSN dynamics



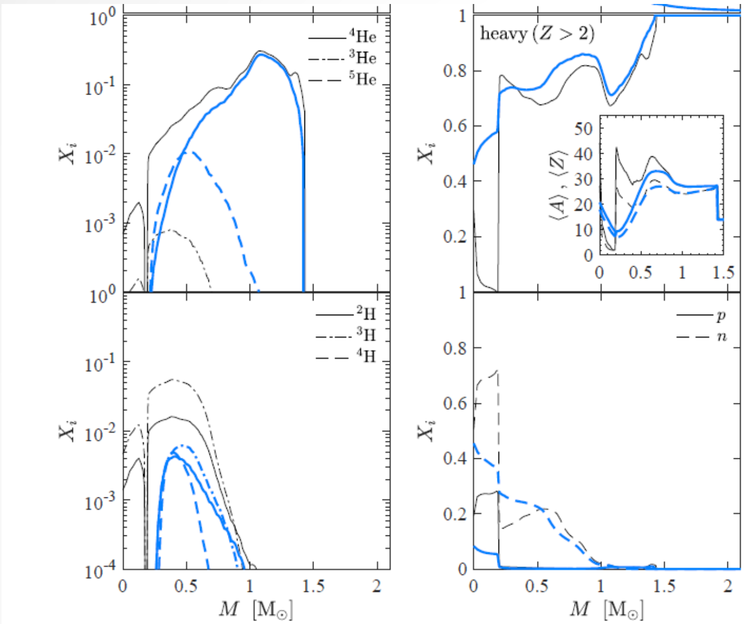
(b) Neutrino luminosities and average energies



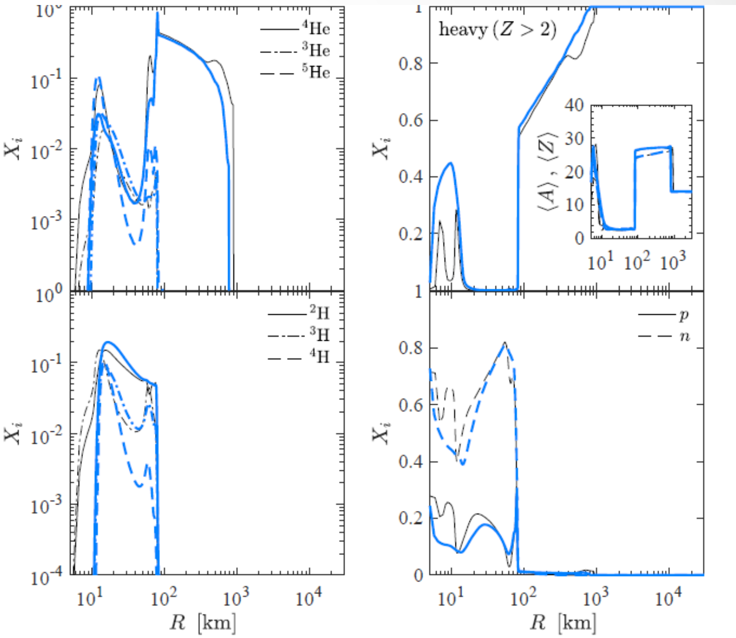
T.Fischer et al PRC102(2020)

T.Fischer et al Phys. Rev. C **102**(2020) 055807

SN dynamics and cluster in-medium effects

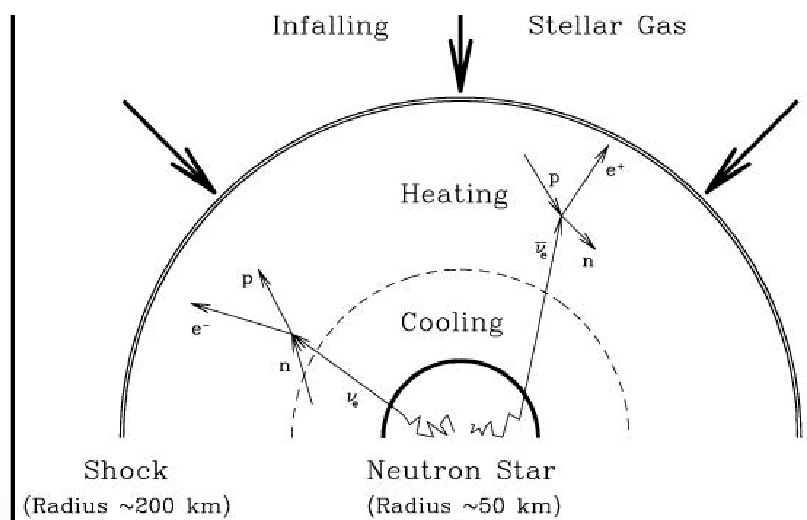


(a) At about 0.5 ms before core bounce



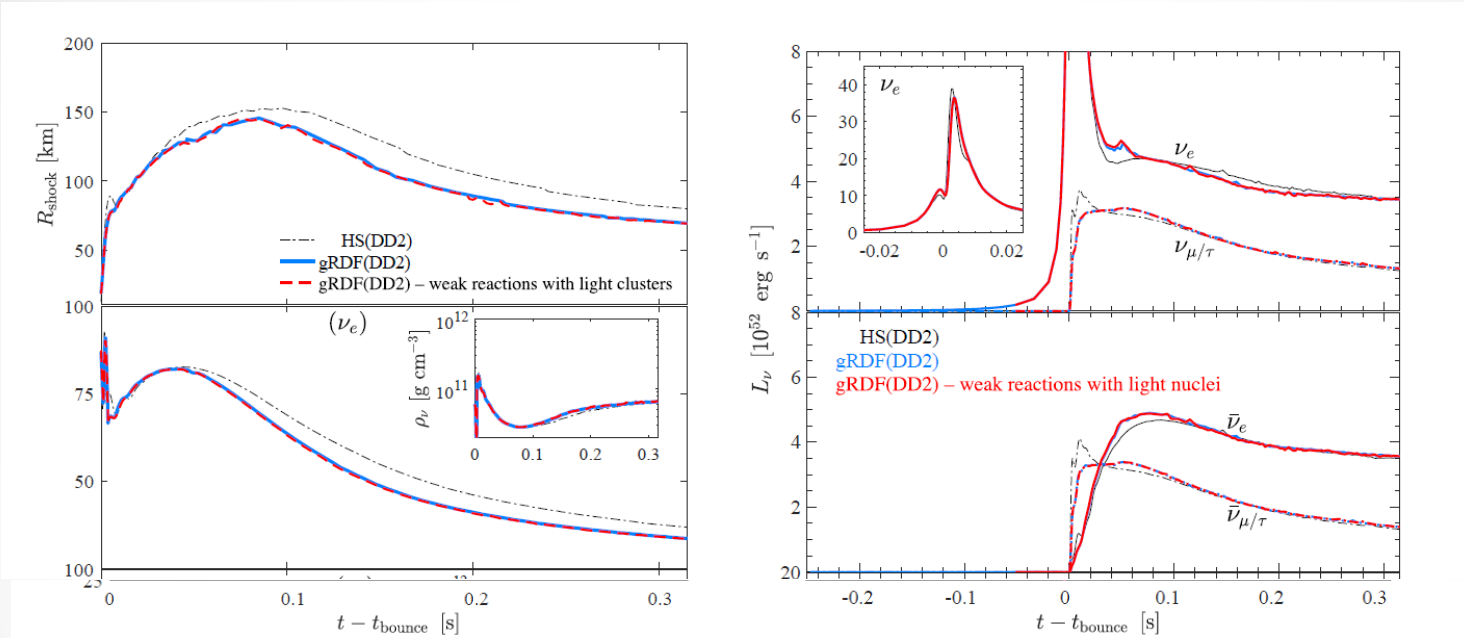
(a) At 10 ms after core bounce

- The energy deposition in the gain region depends on the position of the ν–sphere
- Neutrino opacity is due to scattering off nucleons and nuclei
- **Composition depends on the in-medium modifications to the binding energy**



T.Fischer et al Phys. Rev. C **102**(2020) 055807

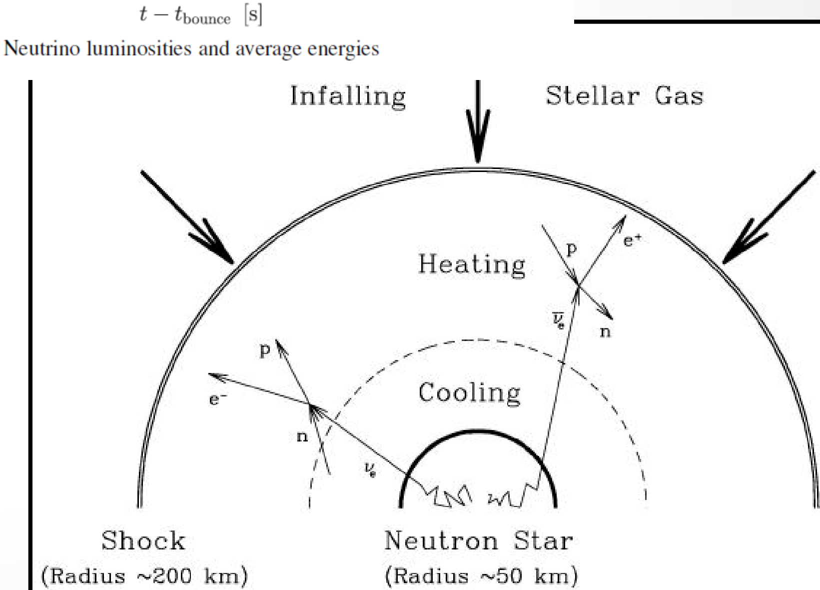
SN dynamics and cluster in-medium effects



(a) Shock radii and neutrinospheres

(b) Neutrino luminosities and average energies

- The energy deposition in the gain region depends on the position of the ν–sphere
- Neutrino opacity is due to scattering off nucleons and nuclei
- **Binding energy shifts might affect the ν–sphere position**
- **and ν luminosity**



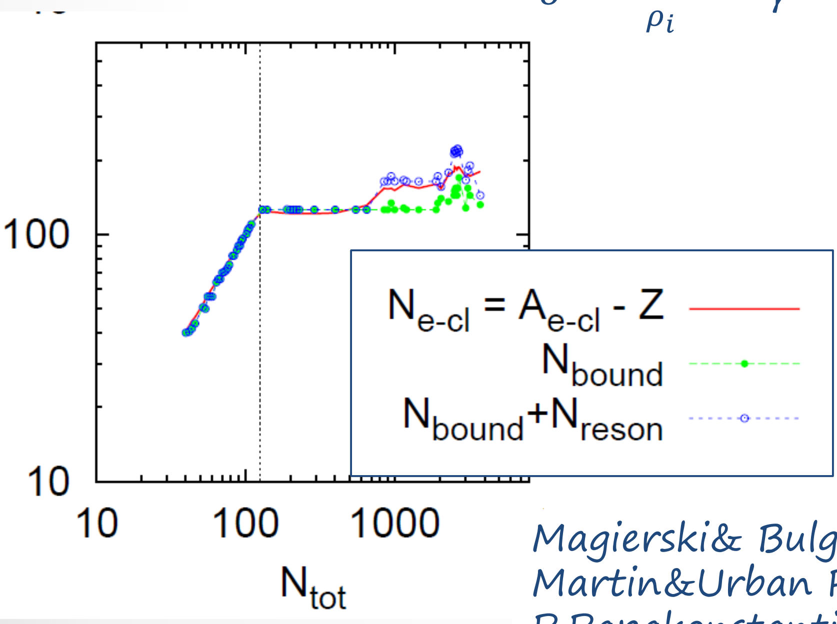
OCP with cluster DoF

$$F_{AZ} = F_{AZ}^0 + T\left(ln\frac{\lambda_{AZ}^3(M^*)}{gV_f} - 1\right)$$

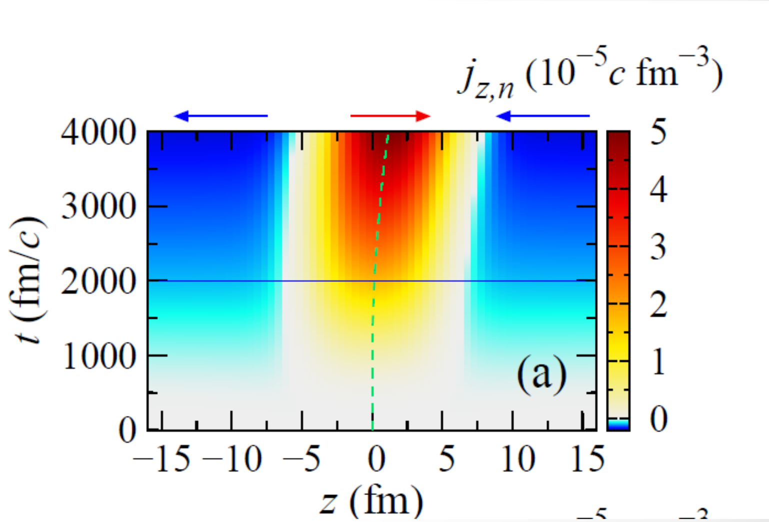
- CM degree of freedom: translation ($T>T_m$)
- n-p interaction: (only) bound neutrons are entrained by the ion

$$M^* = M\left(1 - \delta^f + \frac{\left(\delta^f - \gamma\right)^2}{\delta^f + 2\gamma}\right) \approx M\left(1 - \frac{\rho_{gn}}{\rho_i}\right)$$

$$\delta^f = \frac{\rho_n^f}{\rho_i} \qquad \gamma = \frac{\rho_{gn}}{\rho_i}$$



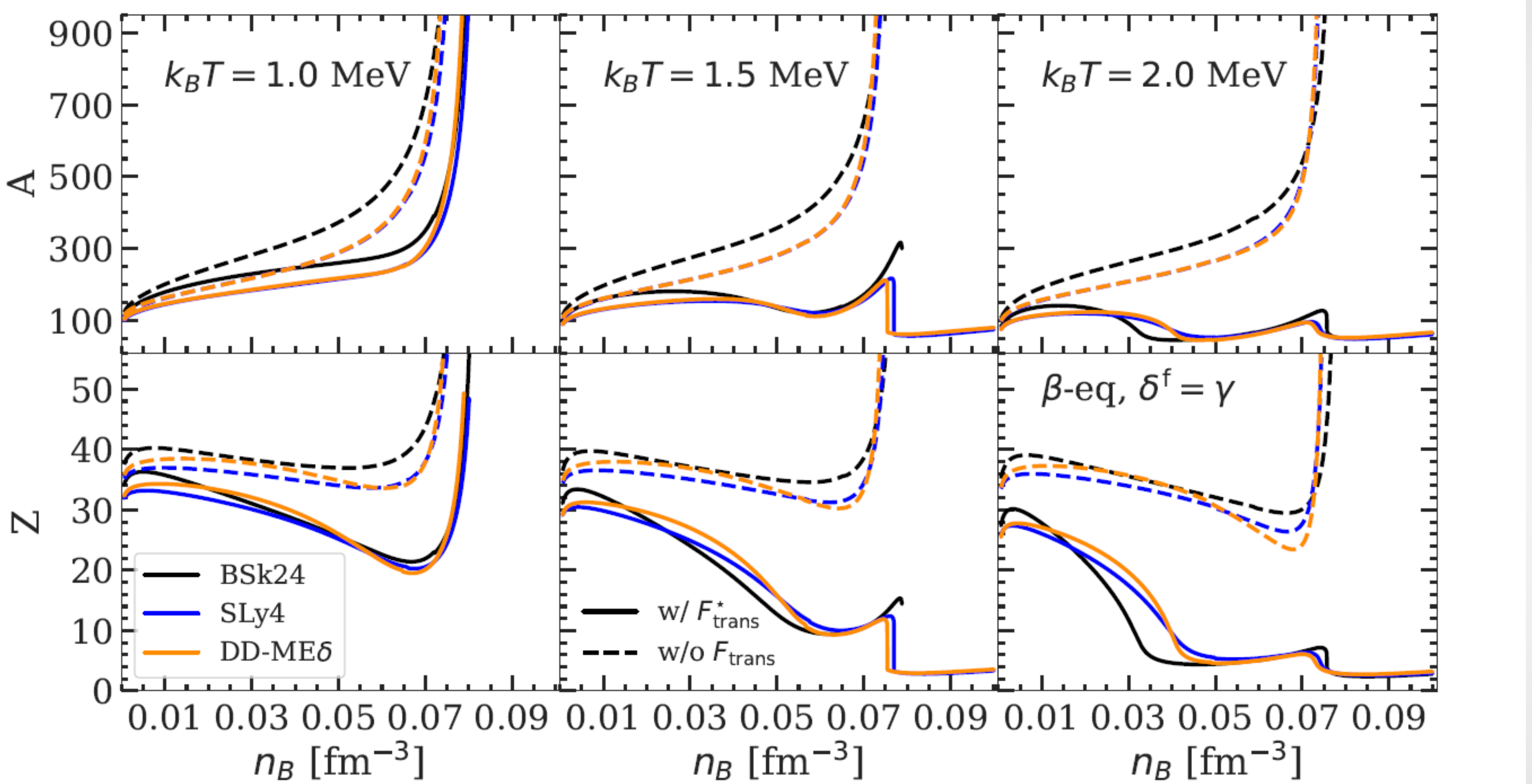
Magierski& Bulgac NPA 2004
Martin&Urban PRC 2016
P.Papakonstantinou et al, PRC 2013



K.Sekizawa et al PRC 2022

OCP with cluster DoF

$$F_{AZ} = F^0_{AZ} + T\left(ln\frac{\lambda^3_{AZ}(M^*)}{gV_f} - 1\right)$$

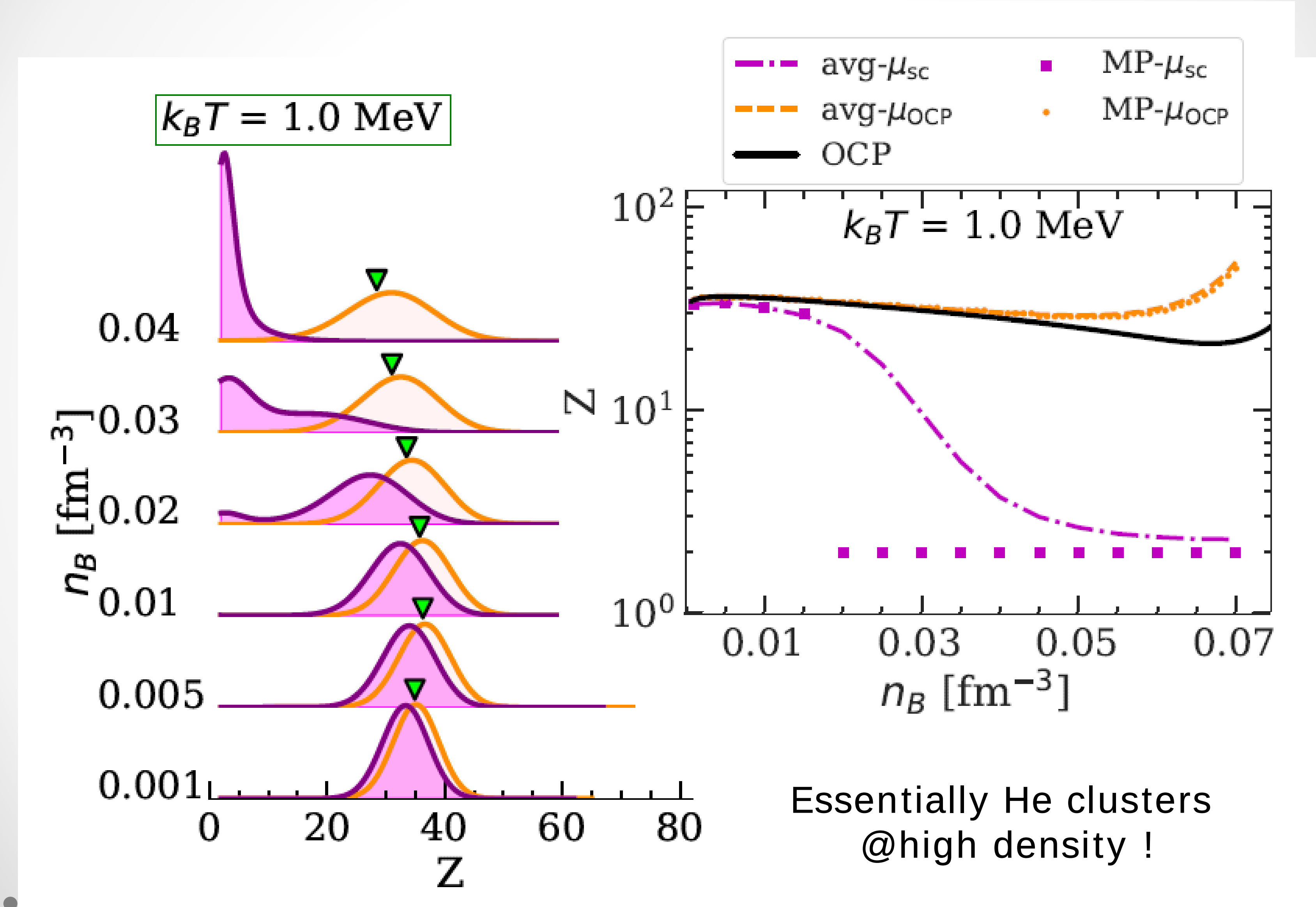


Essentially He clusters
@high density !

H.DinhThi et al, A&A 2023

The cluster distribution

Dinh-Thi 2023, to be submitted



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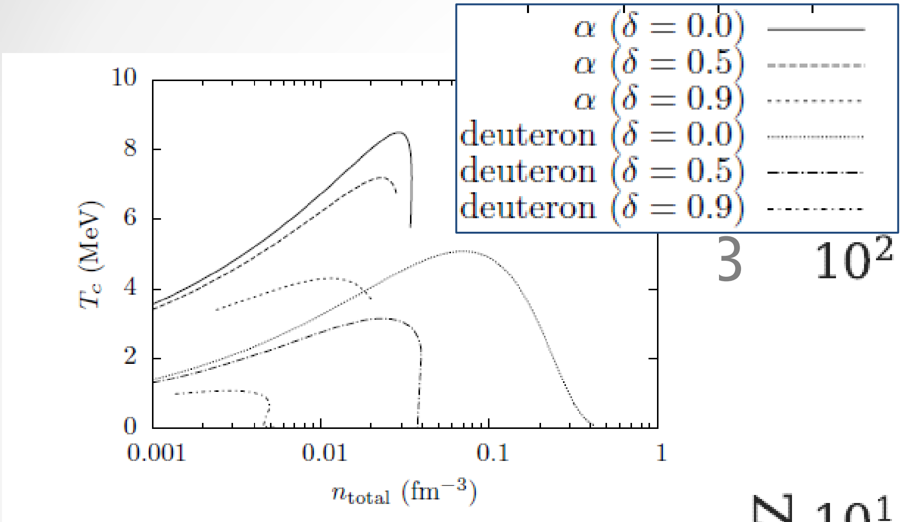
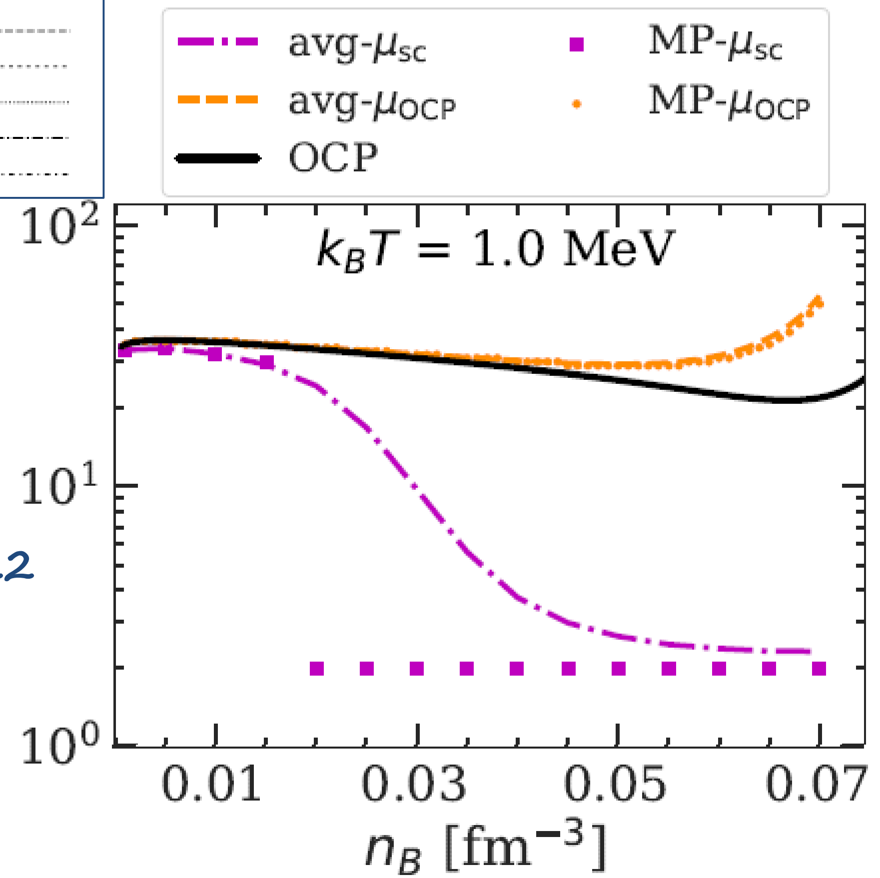
The cluster distribution

Dinh-Thi 2022, to be submitted

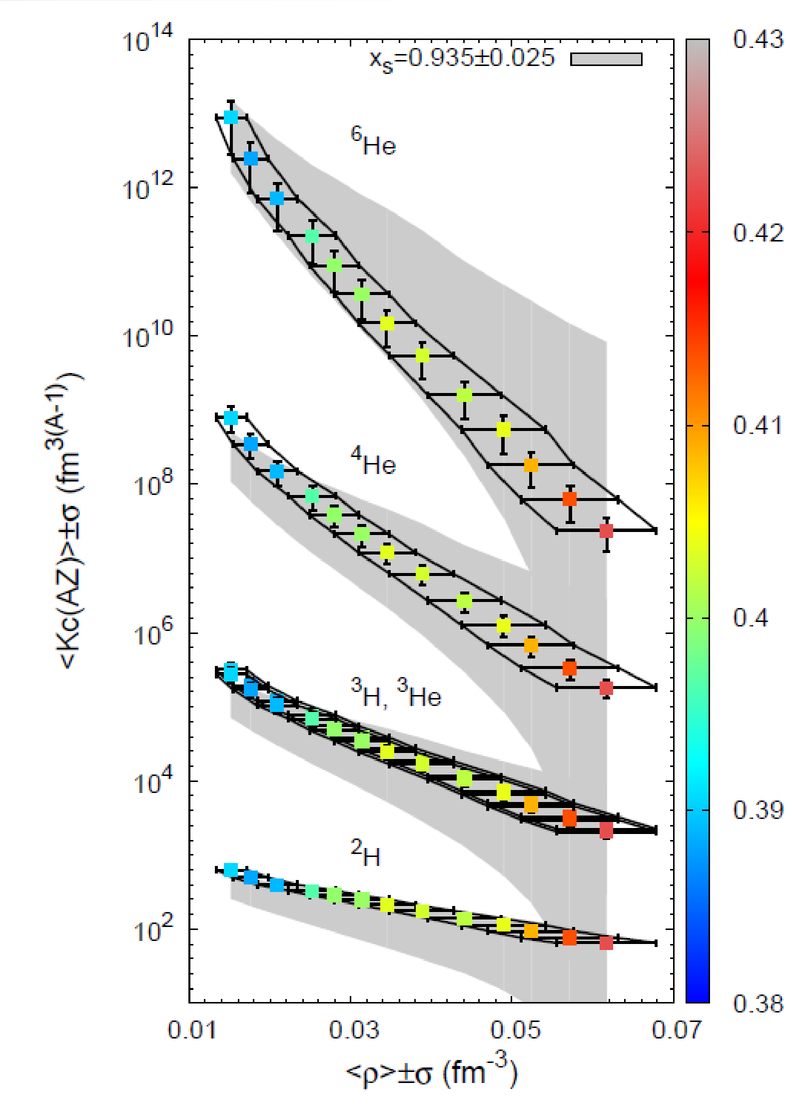


Takaaki Sogo, Gerd Röpke, and Peter Schuck, Phys. Rev. C 82, 034322



Essentially He clusters @high density !

Calibrating the in-medium effects



$$K_C(N,Z) = \frac{n_{NZ}}{n_{01}^Z n_{10}^N} = \frac{\omega_{NZ}}{A\omega_{01}^Z \omega_{10}^N} n_B^{-(A-1)}$$

n=density
ω =mass fraction

- INDRA Xe+Sn central collision data sampled in bins of radial velocity(≡emission time?)
- Mapping: v_i *EXP* ⇔ $(n_B, T, y_P)_i$ *RMF*
- y_P deduced from data
- T,n_B tentatively deduced from a (modified) ideal gas formula
- Hyp: x_s=cst.

=> $x_s = 0.935 \pm 0.025$

H.Pais & INDRA coll. PRL 125(2020)012701