Determination of the Equation of State from Nuclear Experiments and Neutron Star Observations

Chun Yuen Tsang, Rohit Kumar, Bill Lynch, Betty Tsang, Chuck Horowitz

Based on Nature Astronomy volume 8, 328–336 (2024)

This material is based upon work supported by the U.S. Department of Energy Office of Science under Cooperative Agreement DE-SC0000661, the State of Michigan and Michigan State University. Michigan State University designs and establishes FRIB as a DOE Office of Science National User Facility in support of the mission of the Office of Nuclear Physics.

What is Neutron star (NS)?

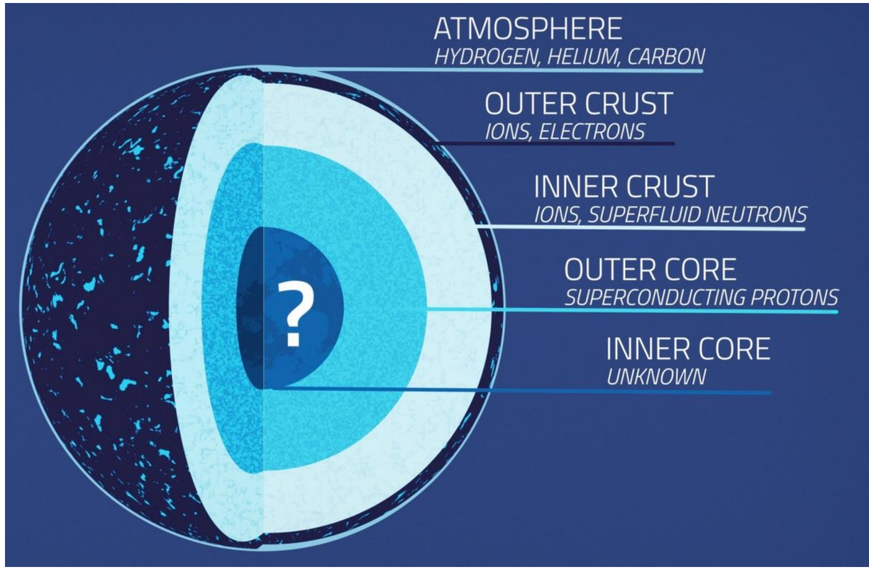
- A dense compact star after a star "die".

- Density increase with depth of NS.
  - NS matter undergoes phase change.

- Material in outer core ~ core of atomic nucleus.
  - Both are (more or less) homogeneous nuclear matter.

- **Goal: Use understanding of nucleus from heavy-ion collision to complement astronomical observation for a comprehensive understanding of NS.**

- Need **nuclear matter EoS** to achieve that goal.

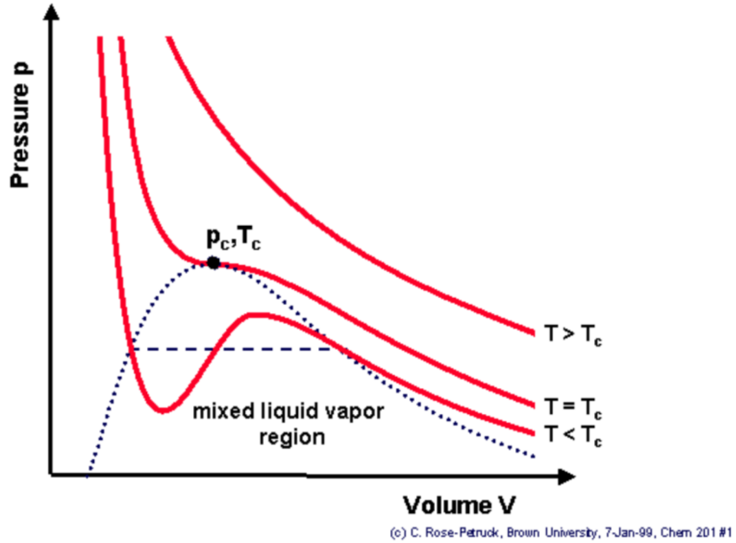


Credit: NASA's Goddard Space Flight Center/Conceptual Image Lab

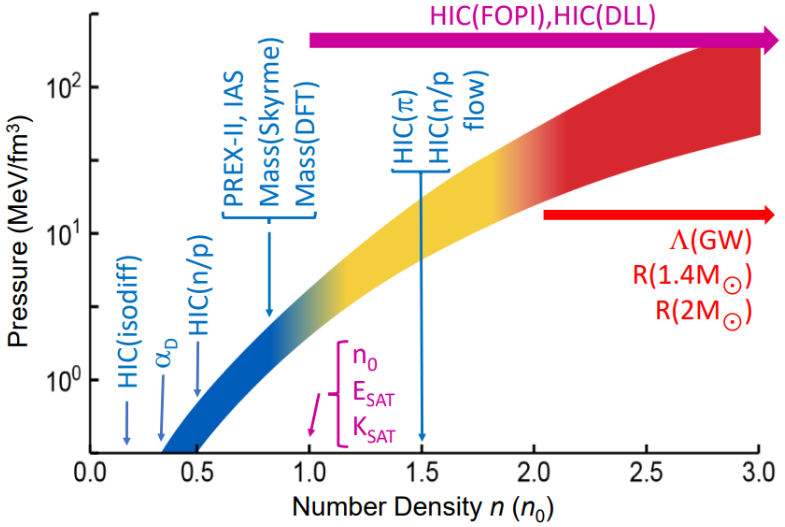
What is Equation of state (EoS)

- Thermodynamic equation that relates state variables. (e.g. Pressure, Volume, etc.)
- EoS tells us properties of the matter. (Incompressibility, heat capacity, etc.)

Recall from High school physics: Van-der waals gas

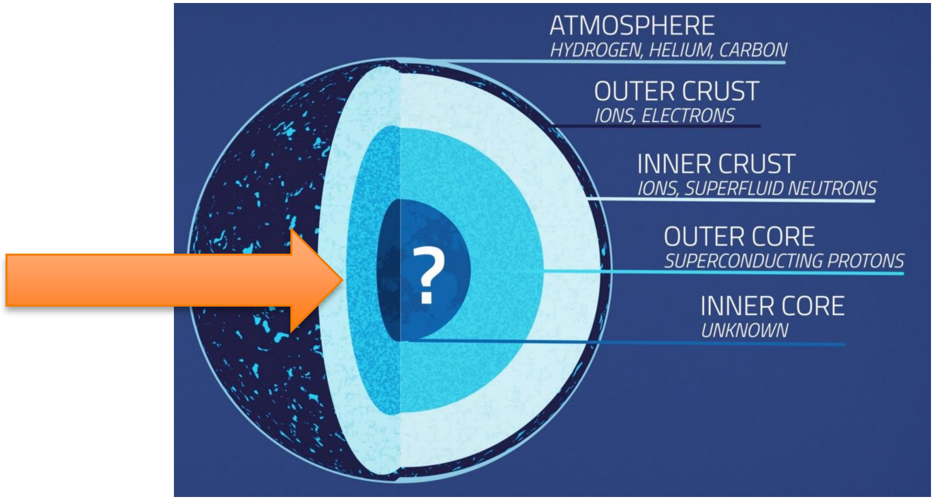


But now we want EoS of ***nuclear matter*** at T=0K



Nature Astronomy volume 8, 328–336 (2024)

Make predictions on mechanical properties of NS.



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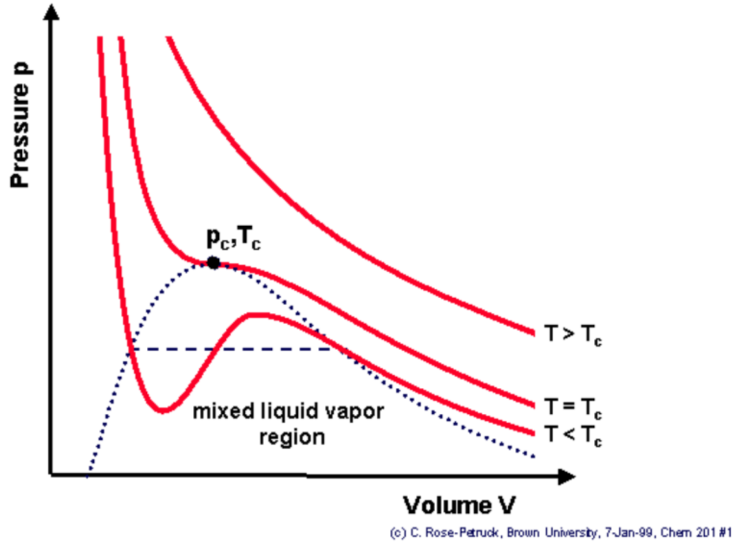

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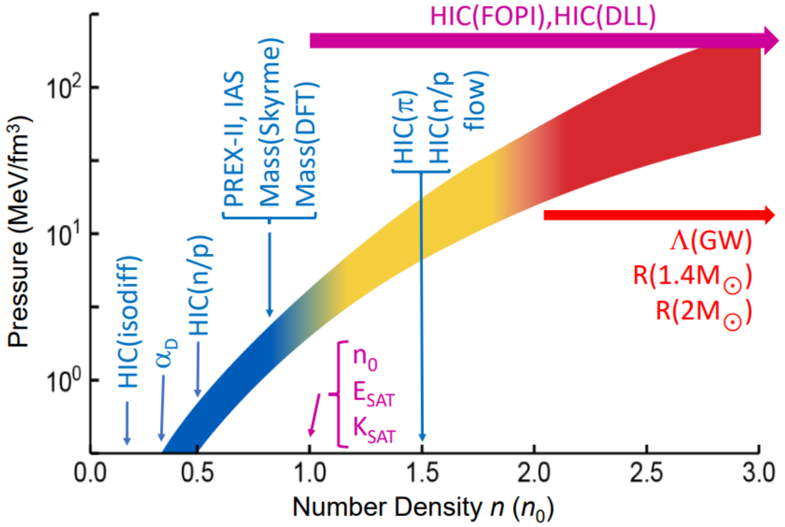

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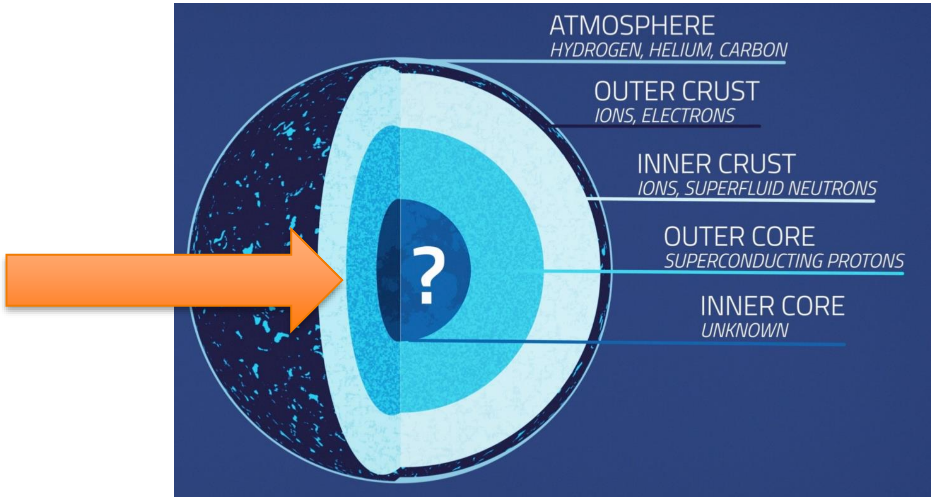
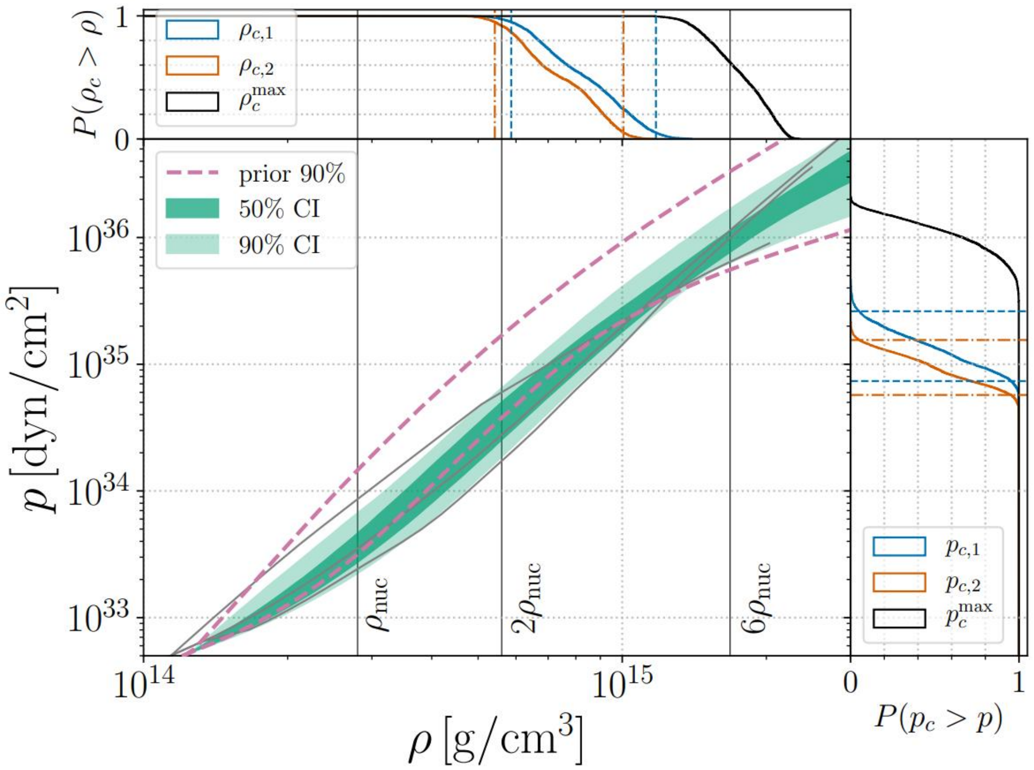


Or from NS observation to constraint on EoS

What was done before

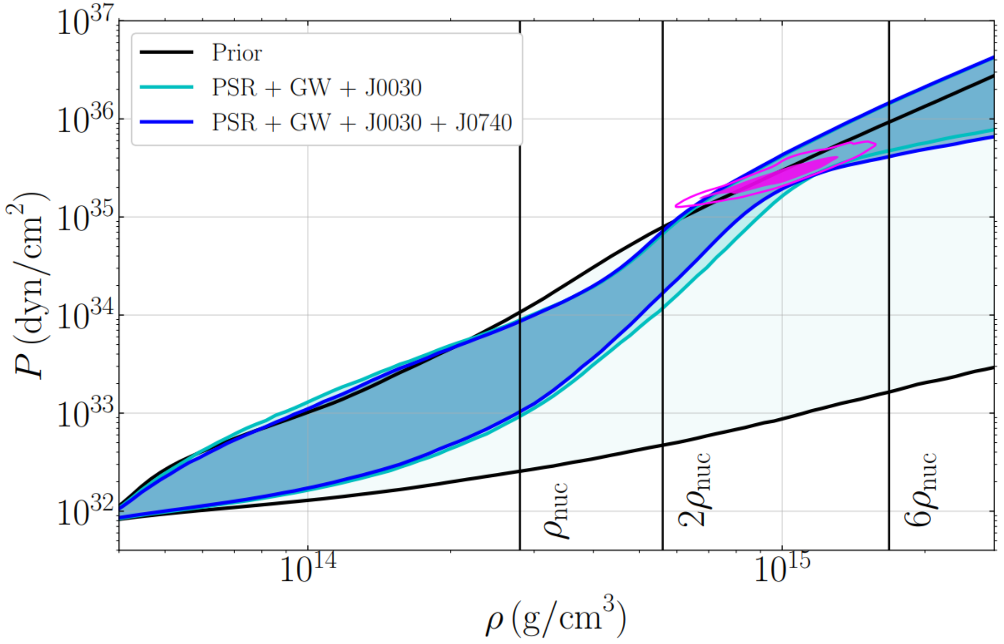
- Phys. Rev. Lett. 121, 161101 (2018)

  Gravitational wave observations of NS merger.



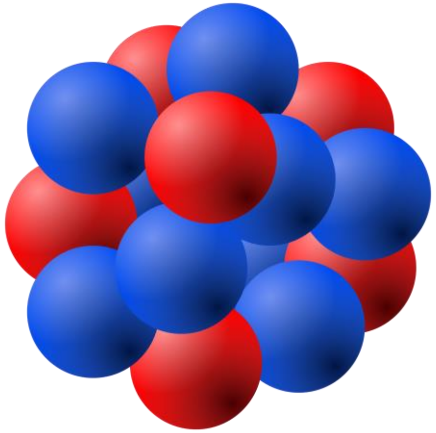
I Legred, et al, Phys. Rev. D 104, 063003

X-ray observations of pulsers.



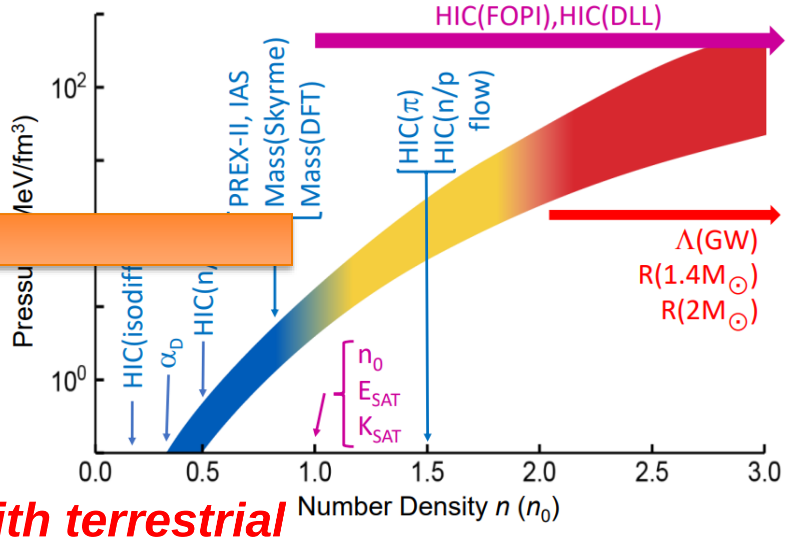
But nuclear EoS also describe nucleus

Make predictions on properties of nucleus.

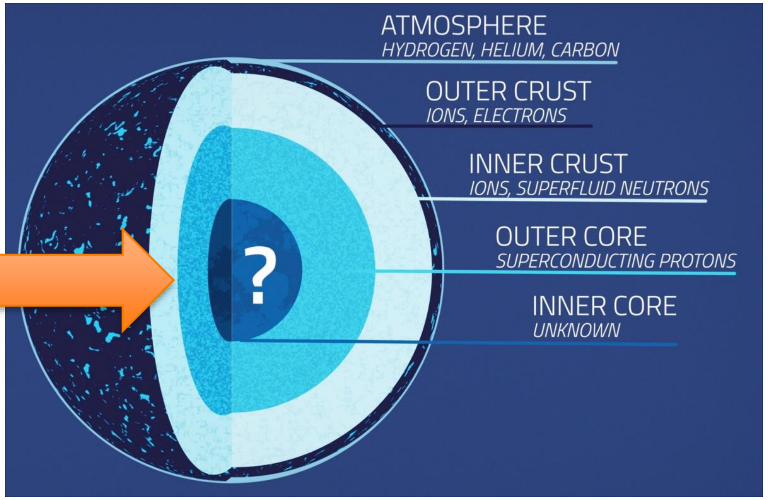


Wikipedia. URL: https://en.wikipedia.org/wiki/Atomic_nucleus#/media/File:Nucleus_drawing.svg

But now we want EoS of ***nuclear matter*** at T=0K



Make predictions on mechanical properties of NS.

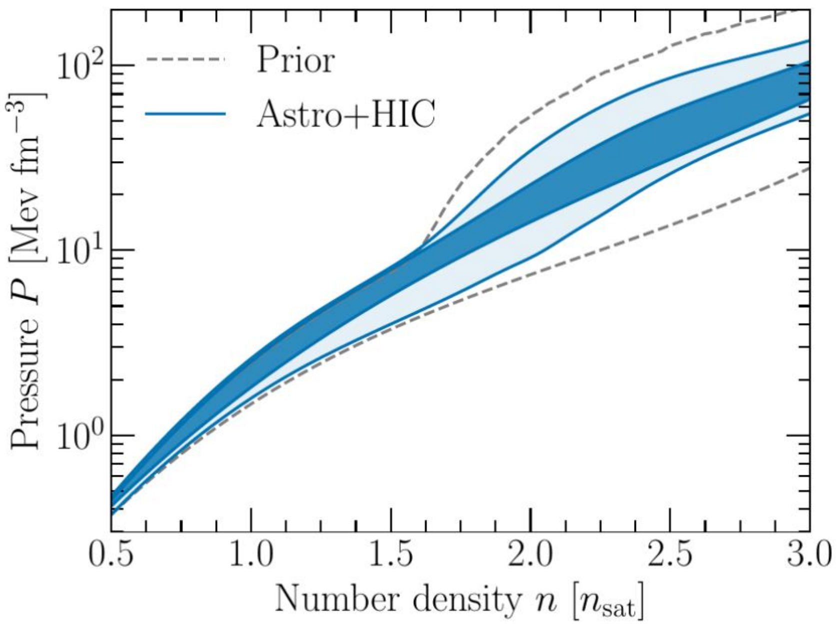


Refine EoS with terrestrial experiments on nucleus

What was done before (cont.)

- Huth, Pang et al., Nature 606, 279 (2022)

(D) HIC and Astro combined:



Chiral effective field up to $1.5n_0$

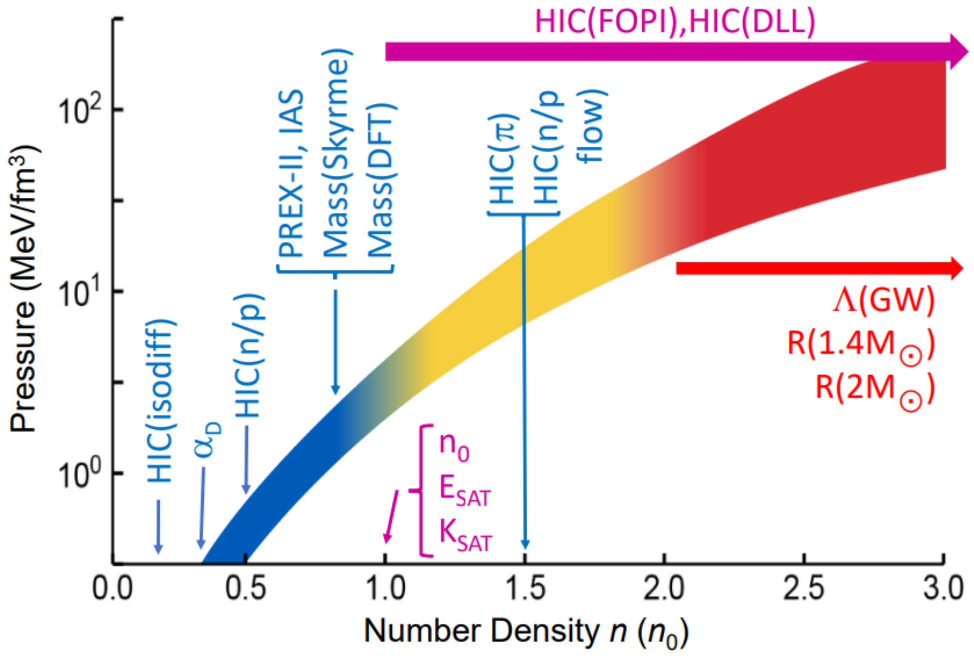
Heavy-ion collisions

Astronomical observables.

- Rely on theory at low density.
- A more data driven method? Constraints < $1.5n_0$ from terrestrial experiments are readily available.

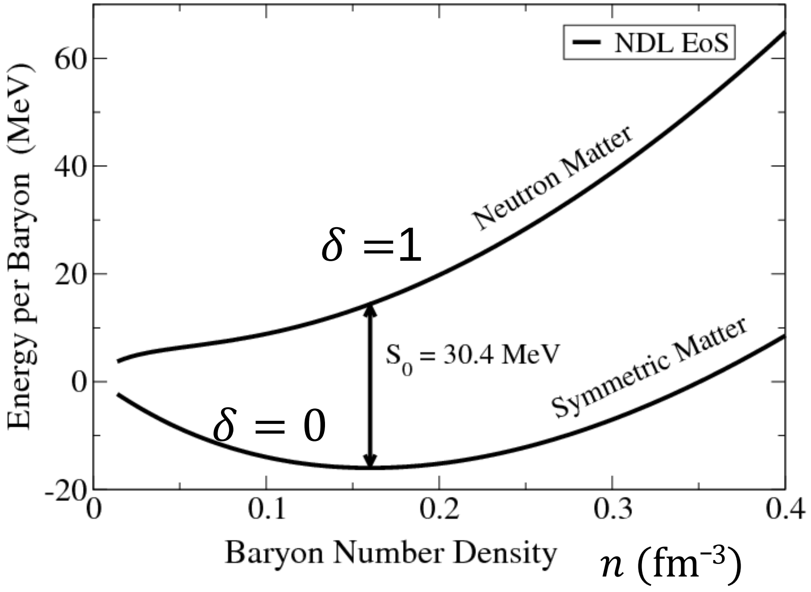
Overview of constraints

- Terrestrial experiments mostly probe low density region. i.e. $n < 2n_0$.
- Astronomical observations are sensitive to higher density region.
- Not shown in 2D plot: Neutron fraction.
  - Atomic nucleus => neutron and proton numbers.
  - NS => mostly neutrons.



Extrapolate with respect to neutron fraction

- Symmetric nuclear matter (SNM): equal density of proton and neutron ($E_{sym}(n)$).
- Neutron matter: Only neutrons.
- Difference between the two: **symmetry energy term $S(n)$.**
- $\Delta E_{neutron\ excess} = S(n)\delta^2$, $\delta = \frac{N-Z}{A}$ called **asymmetry.**
- Terrestrial experiments: low δ.
- Compare results from systems of different δ to constrain S(n).
  - Cutting edge rare isotope beams => isotopes with more neutrons.



Meixner, M. et al. arXiv:1303.0064v1 [astro-ph.HE] (2013)

Short summary

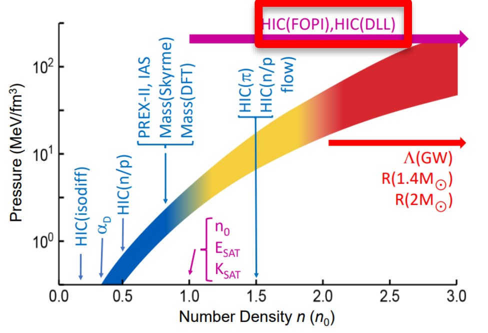
- 3 pieces of information:

1. Terrestrial constraints on symmetric matter.
2. Terrestrial constraints on symmetry energy term.
3. Astronomical observations.

(Some) Terrestrial constraints on symmetric matter.

Note: ρ is used to denote baryon density in nuclear physics instead of n.

Collective flow (HIC(FOPI), HIC(DLL))

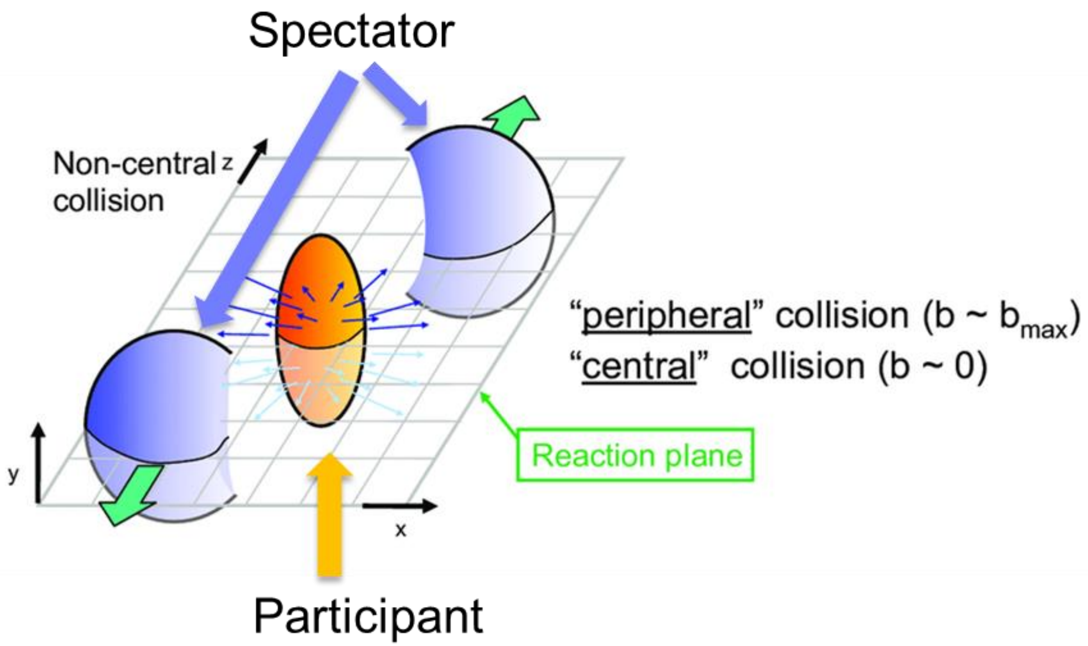


- Flow: Degrees of asymmetries in fragment emission w.r.t. reaction plane.

Nuclear transport model

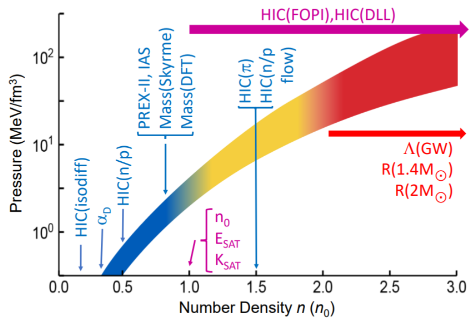
- Fragment azimuthal distribution ⟷ Pressure in participant region.

- Ref. [1] analyzed (elliptical) flow of Au + Au at 0.4 – 1.5 GeV/nucleon from FOPI.
  - $P_{SNM}(2n_0) = 10.3 \pm 2.8$ ***MeV* /fm³ (HIC (FOPI)).**
- Ref. [2] analyzed (transverse and elliptical) flow of Au + Au between 0.15 – 10 GeV/nucleon.
  - $P_{SNM}(2n_0) = 10.1 \pm 3$ **MeV/fm³. (HIC(DLL))**

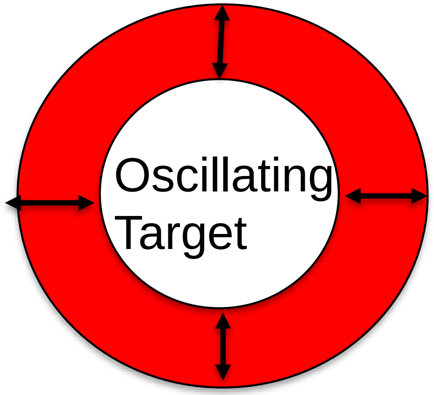


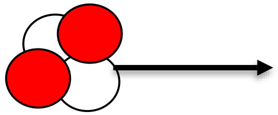
[1]: Nuclear Physics A 945, 112 (2016)
[2]: Science 298, 1592 (2002)

Giant Monopole Resonance (GMR)
*Not shown on the illustration



KE = 240 MeV





List of targets: Zr, Sn, Sm and Pb

KE < 240 MeV, energy transferred to target.

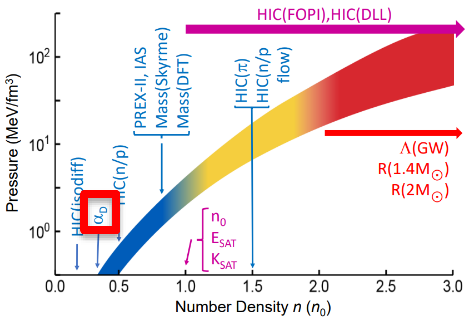
- Energy loss to GMR ⟷ $K_{sat} = 9n_0 \frac{d^2 E_{sat}(n)}{dn^2}$
- Ref. [1] shows $K_{sat} = 231 \pm 5$ MeV

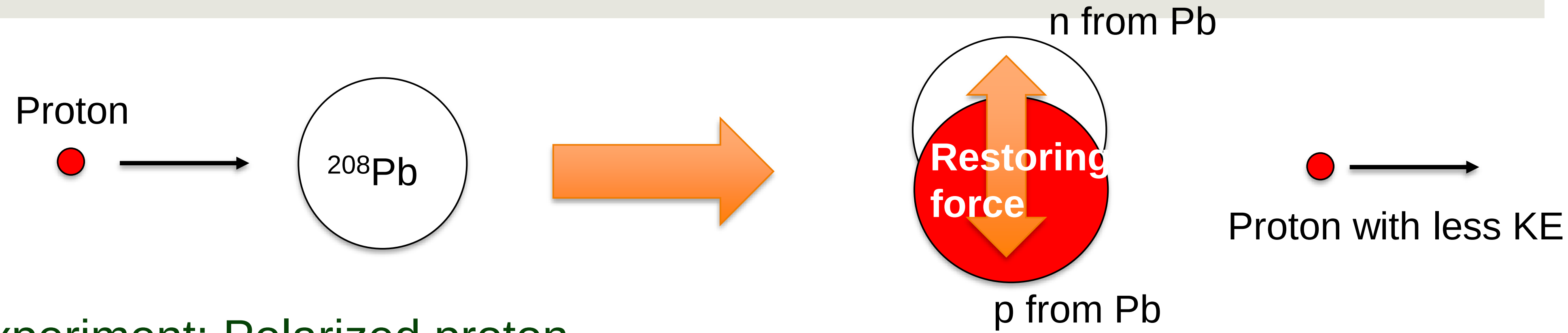
- Updated analysis from Ref. [2] shows $K_{sat} = 230 \pm 40$ MeV

[1]: Youngblood et al., Phys. Rev. Lett. 82, 691 (1999)
[2]: Phys. Rev. C 88, 034319

(Some) Terrestrial constraints on symmetry energy term.

Electric dipole polarizability (α_B)





- Experiment: Polarized proton inelastic scattering at forward angle by RCNP. [1]
- Symmetry force is the restoring force of displaced neutron.
- Constrained $\alpha_B = 20.1 \pm 6\ (fm^3/e^2)$.
- Ref. [2] found correlation $\alpha_B \Rightarrow S(\rho)$.
- $S(n = 0.31 n_0) = 15.91 \pm 0.99$ MeV.

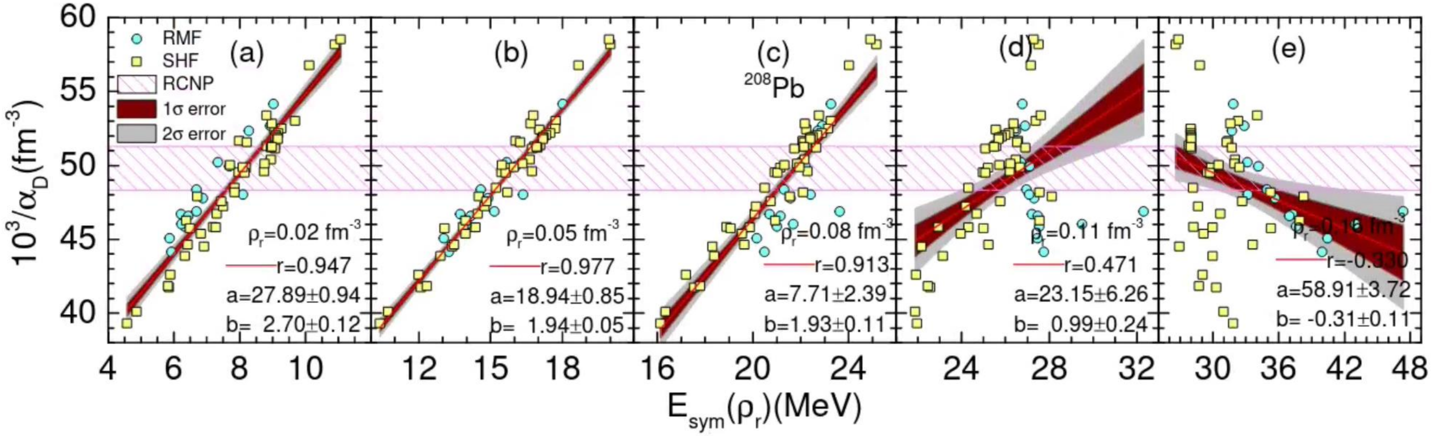
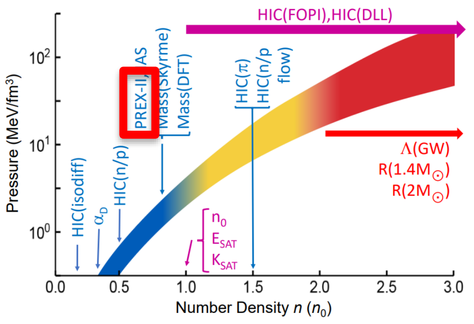


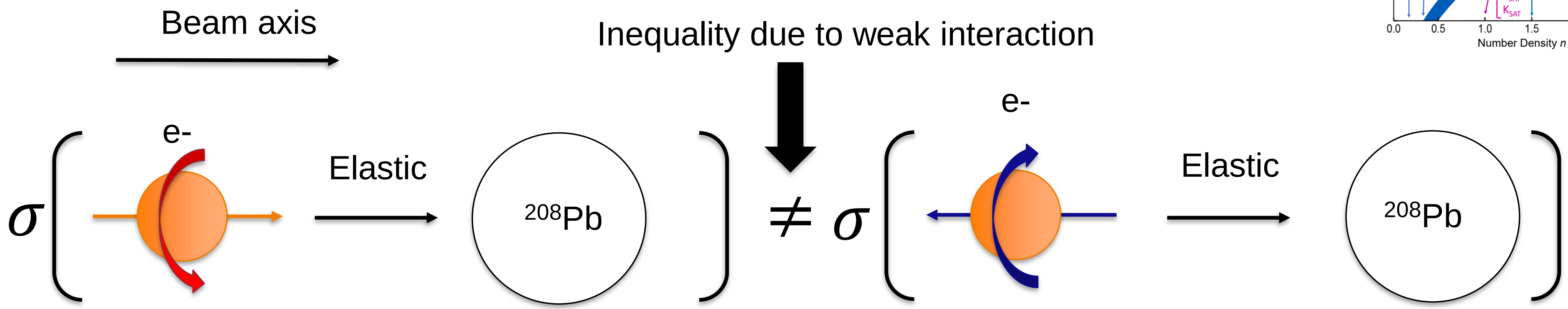
Fig. from Ref. [2]

[1]: Phys. Rev. Lett. 107, 062502 (2011)
[2]: Phys. Rev. C 92, 031301(R) (2015)

U.S. Department of Energy Office of Science
National Science Foundation
Michigan State University

Lead Radius Experiment (PREX-II)

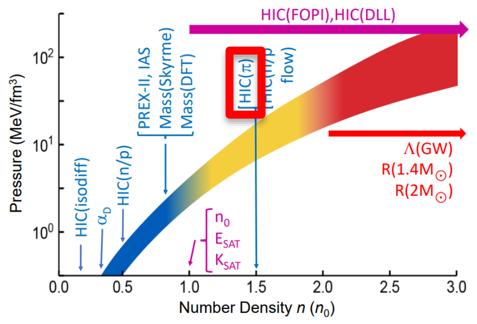




- Neutron weak charge >> proton weak charge => Neutron radius is measured.
- Compare to proton radius with EM probe => Neutron skin thickness $\mathrm{R_{skin}}$
- Ref. [1] shows $\mathrm{R_{skin}} = 0.283 \pm 0.071$ fm.
- Ref. [2] shows correlation $\mathrm{R_{skin}} \Rightarrow \mathrm{P_{sym}}(n) = \frac{dS(n)}{dn}$
- $P_{sym}(2n_0/3) = 2.38 \pm 0.75$ MeV /fm^3

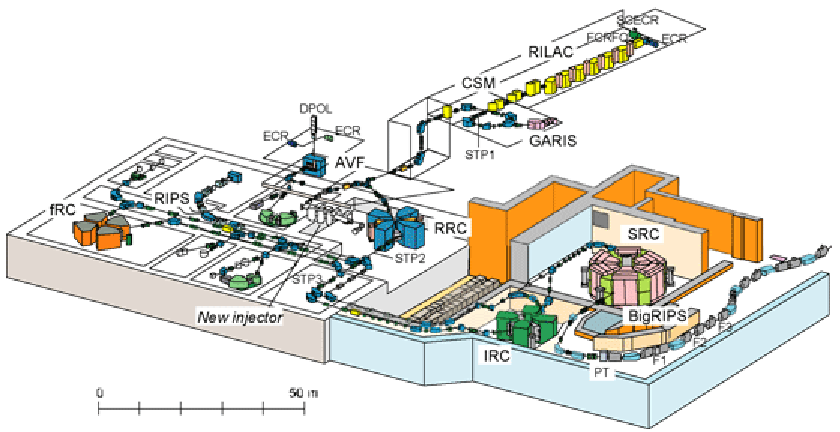
[1]: Phys. Rev. Lett. 126, 172502 (2021)
[2]: Phys. Rev. Lett. 126, 172503 (2021)

Pion ratio (HIC(π))



RIBF produced and accelerated Sn at 270 MeV/nucleons

RIBF Accelerators



URL: https://www.nishina.riken.jp/ribf/accelerator/overview.html

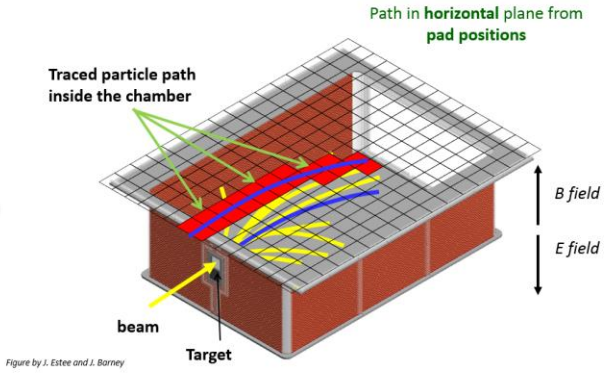
^{108}Sn*

^{112}Sn

^{124}Sn

^{132}Sn*

Fixed target ^{112}Sn or ^{124}Sn

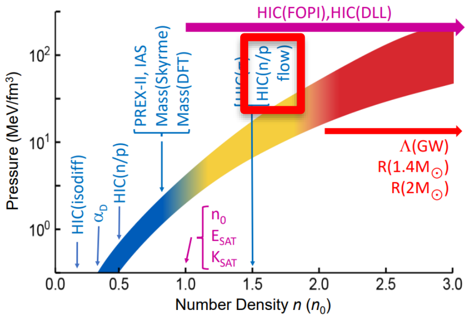


Pions detected by SπRIT TPC particle detector

- RIKEN RIBF produce short-lived* neutron rich-(poor-)Tin isotopes.
- $\pi^{+(-)}$ are created indirectly from p-p(n-n) collisions.
- π^+/π^- from heavy-ion collision ⟷ dcQMD model ⟷ EoS parameters.
- Ref. [1] shows: $\boldsymbol{S(1.45n_0) = 52 \pm 13}$ **MeV and** $\boldsymbol{P_{sym}(1.45n_0) = 10.9 \pm 8.7}$ **MeV/fm³**.

[1]: Phys. Rev. Lett. 126, 162701 (2021)

Neutron to proton flow ratios (HIC(n/p flow))

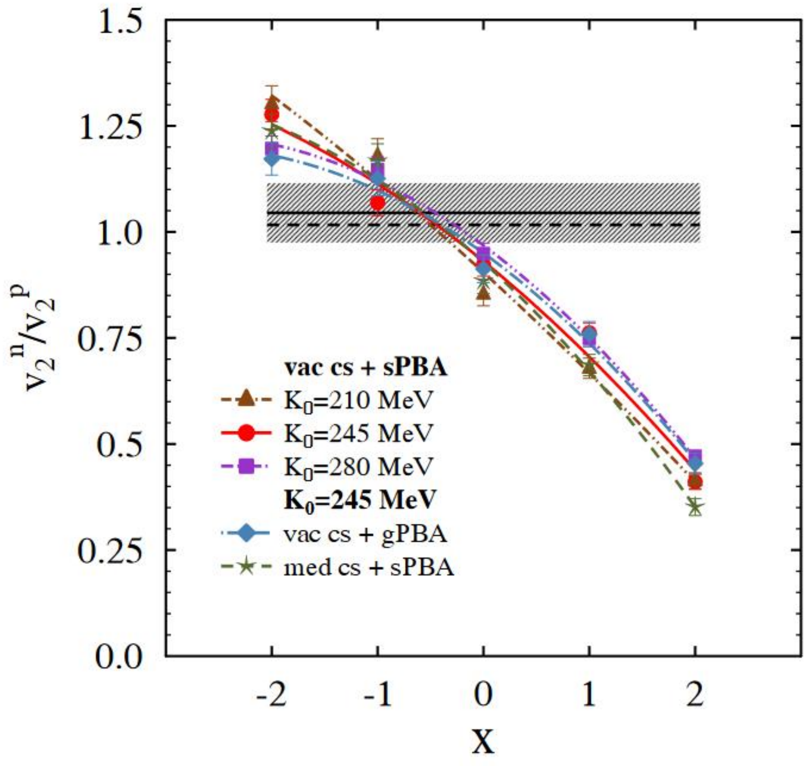


- Reference: Phys. Lett. B 697, 471 (2011), Phys. Rev. C 94, 034608 (2016), Eur. Phys. J. A 54, 40 (2018)

- Experiment: Au + Au at 400 MeV/nucleon.

- Elliptical flow ratios measured by FOPI-LAND and ASYEOS is compared to predictions from a QMD model with MDI2 potential.

- $P_{sym}(1.5n_0) = 12.1 \pm 8.4$ MeV/fm^3.

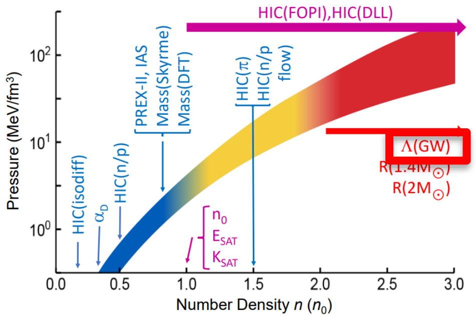


Nucleus have been probed in various ways by nuclear physicists.

Should make use of them more.

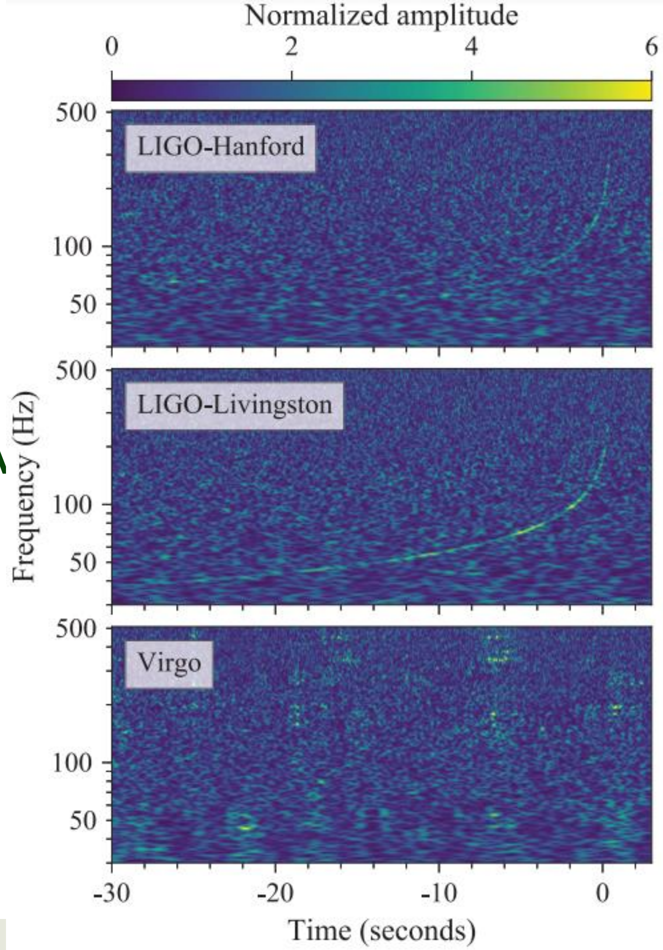
Astronomical observations

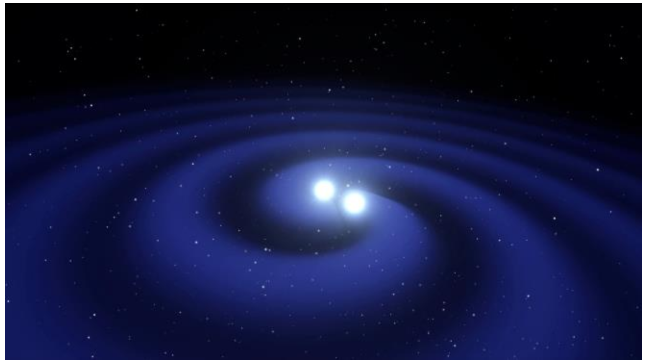
Gravitational wave from neutron star merger.



- Reference: Phys. Rev. Lett. 119, 161101 (2017), Phys. Rev. Lett. 121, 161101 (2018)

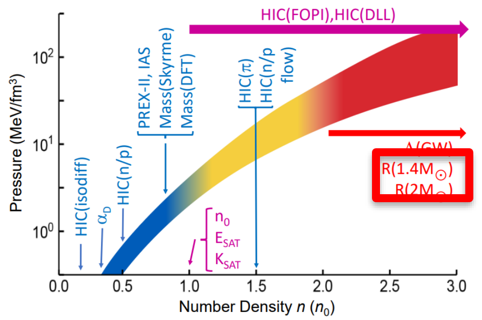
- GW170817:
  - LIGO-VIRGO gravitational wave interferometer detects gravitational-wave signal from merger of two neutron stars.
- Mechanical properties of NS can be inferred.
  1. NS deformed due to tidal force.
  2. Deformation energy is “stolen” from orbital energy.
  3. Orbital period differs from point mass calculation.
- “Toughness” of NS is quantified as dimensionless Tidal deformability Λ
- $\Lambda(1.4 M_\odot) = 190^{+390}_{-120}$



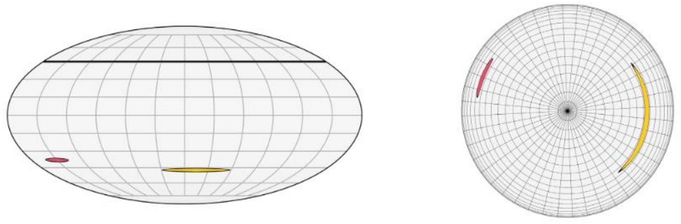


Credit: ESO/L. Calçada.
https://www.esa.int/ESA_Multimedia/Videos/2017/10/Neutron_star_merger

Pulsars observed with NICER and XMM-Newton



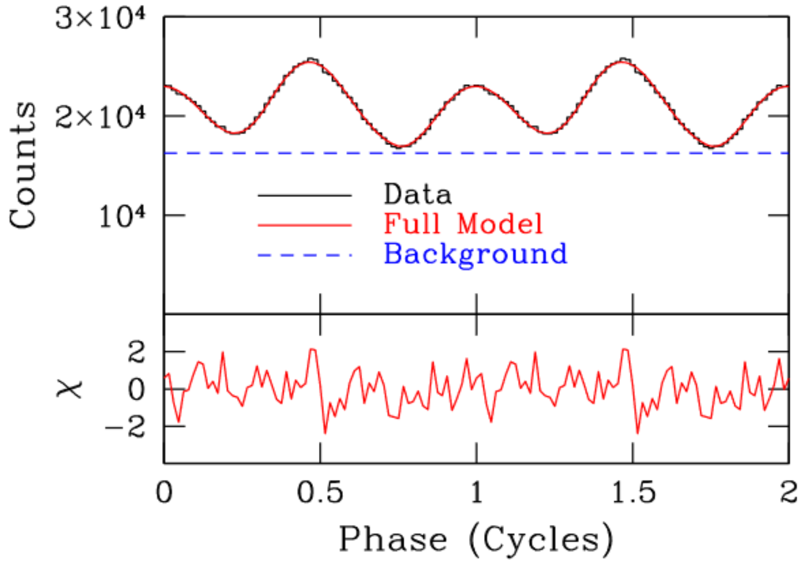
- Reference: Riley et al. (2019), AJL 887(1), L21. Riley et al. (2021), AJL 918(2), L27. Miller et al. (2019), AJL 887(1), L24. Miller et al. (2021), AJL 918(2), L28.
- Observation: X-ray waveform of pulsar PSR J0030+0451 and PSR J07040+6620 observed using the NICER (and in conjunction with XMM-Newton for the latter).
- X-ray from hot-spots of pulsers inform us of their spacetime parameters such as mass and radius.



(a) (b)

PSR J0030+0451	Riley et al	Miller et al
M	$1.34^{+0.15}_{-0.16} M_\odot$	$1.44^{+0.15}_{-0.14} M_\odot$
R	$12.71^{+1.14}_{-1.19}$ km	$13.02^{+1.24}_{-1.09}$ km

PSR J07040+6620	Riley et al	Miller et al
M	$2.072^{+0.07}_{-0.07} M_\odot$	$2.08^{+0.07}_{-0.07} M_\odot$
R	$12.39^{+1.30}_{-0.98}$ km	$13.7^{+2.6}_{-1.5}$ km



Miller et al. (2019), AJL 887(1), L24

Put everything together.

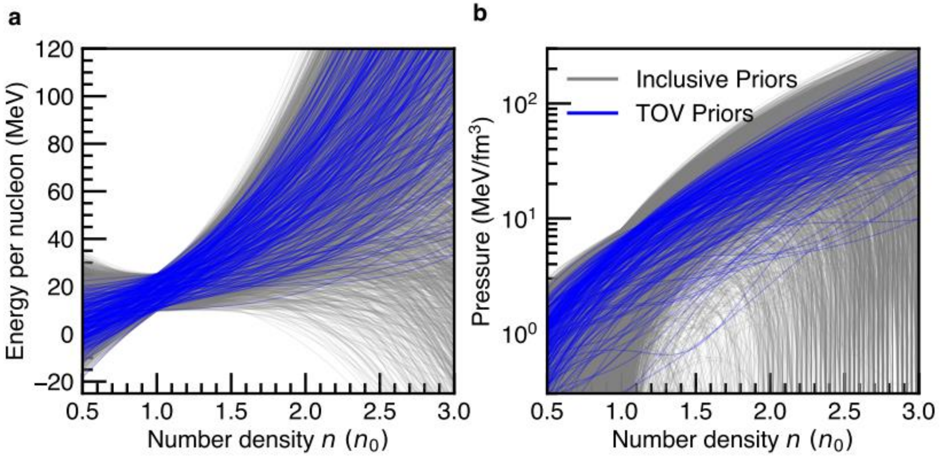
Impose all constrains on EoS

- Flexible family of nuclear EoS models: Meta-modelling EoSs.
- Straightforward comparison to terrestrial constraints.
- Expected astronomical observables (Λ and R) of each EoS are calculated with TOV equation.

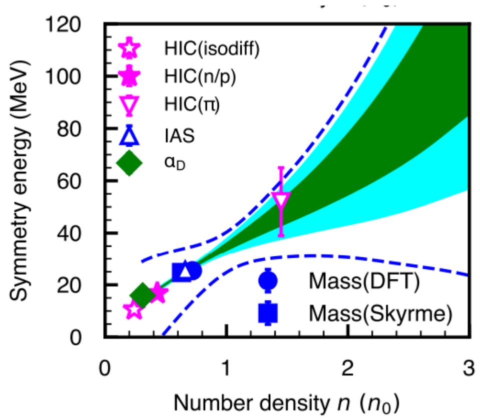
Generate random EoSs with uniform prior

Parameters	Priors
$K_{\rm SAT}$ (MeV)	[0, 648]
$Q_{\rm SAT}$ (MeV)	[-1100, 2100]
S_0 (MeV)	[24.7, 40.3]
L (MeV)	[-11.4, 149.4]
$K_{\rm sym}$ (MeV)	[-328.5, 237.9]
$Q_{\rm sym}$ (MeV)	[-489, 1223]
$Z_{\rm sym}$ (MeV)	[-10110, 2130]
$P_{\rm SNM}(4n_0)$ (MeV/fm^3)	[0, 300]

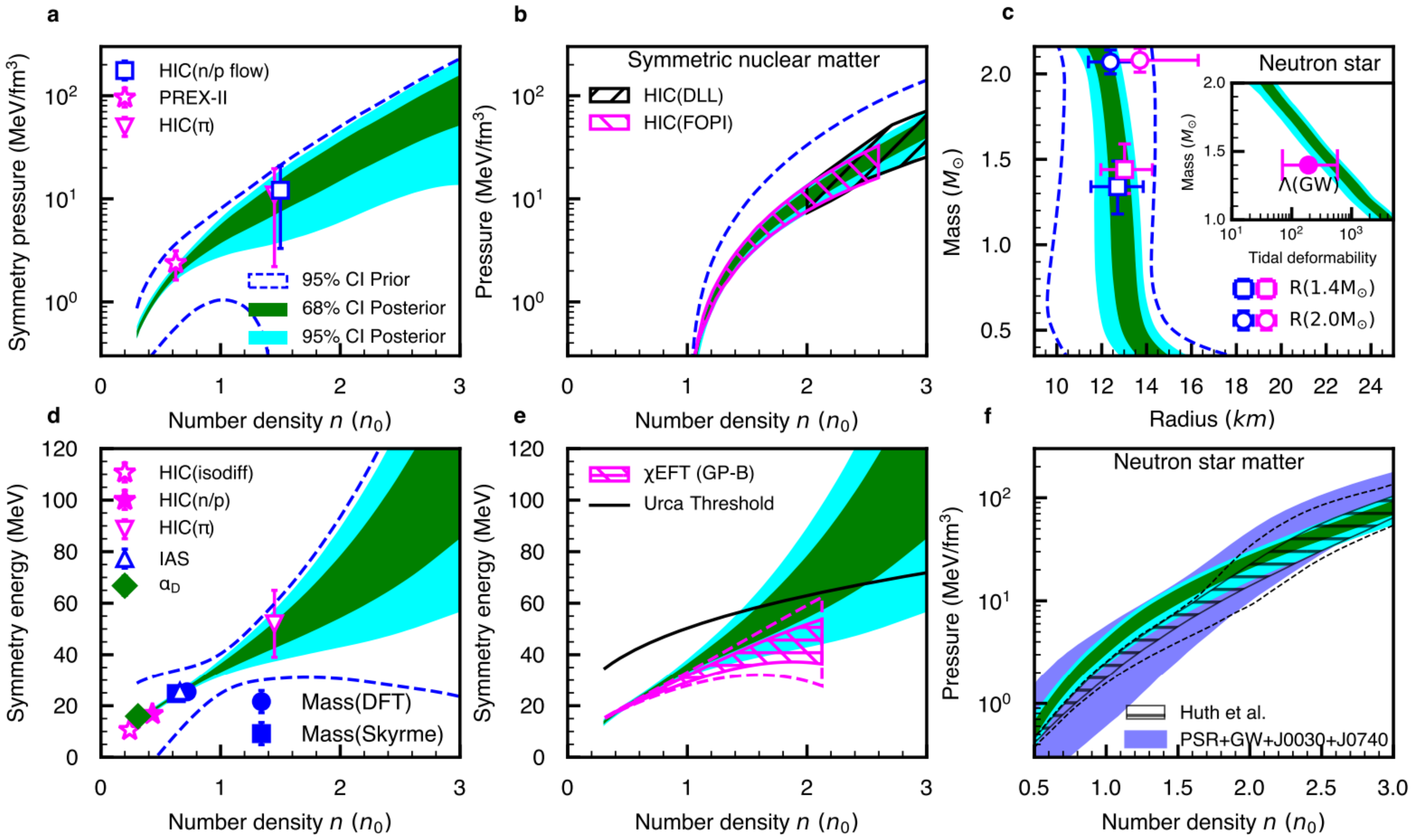
Select EoSs that forms NS



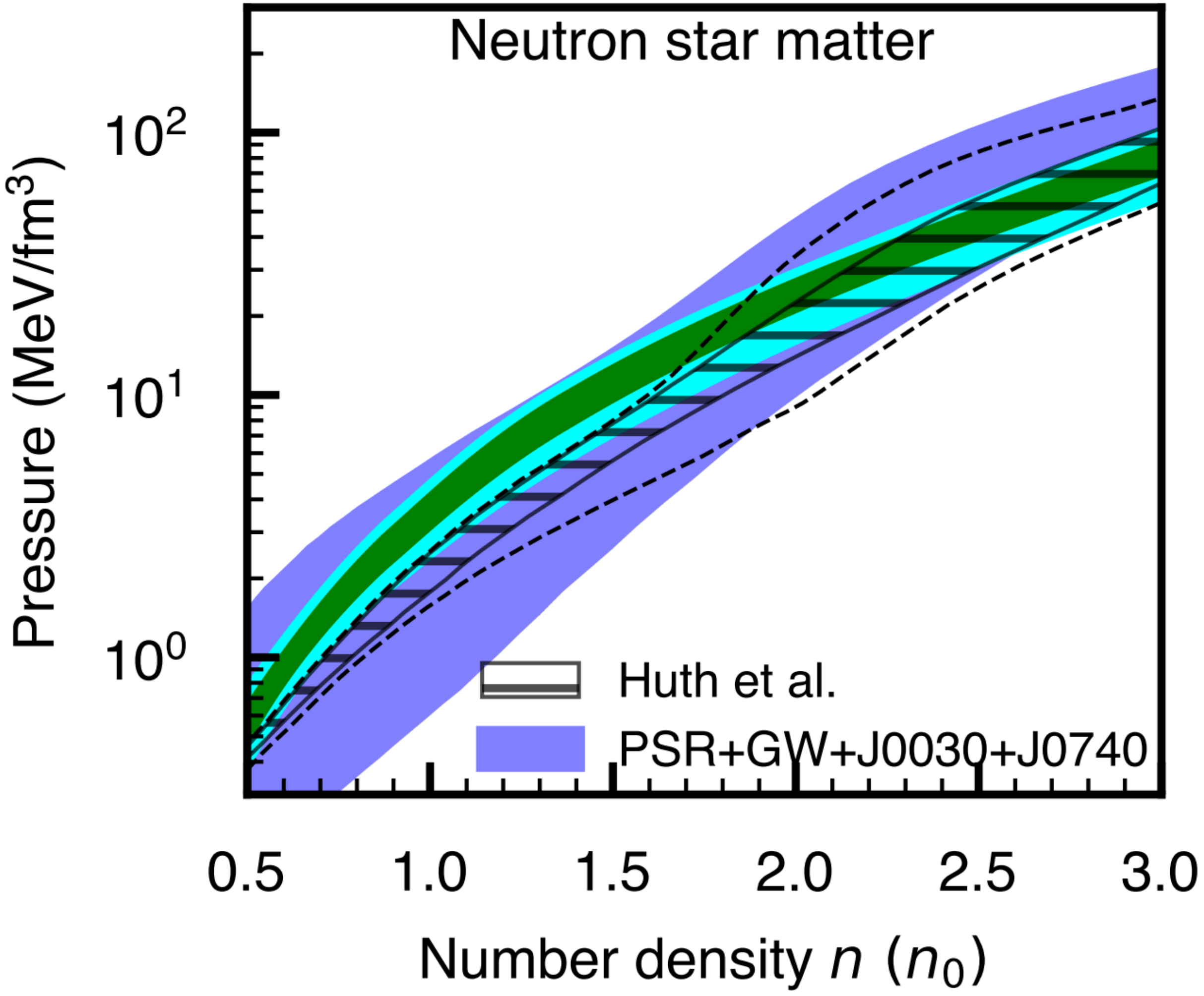
Final selection of EoSs that agrees with all constraints



Impose all constrains on EoS (cont.)



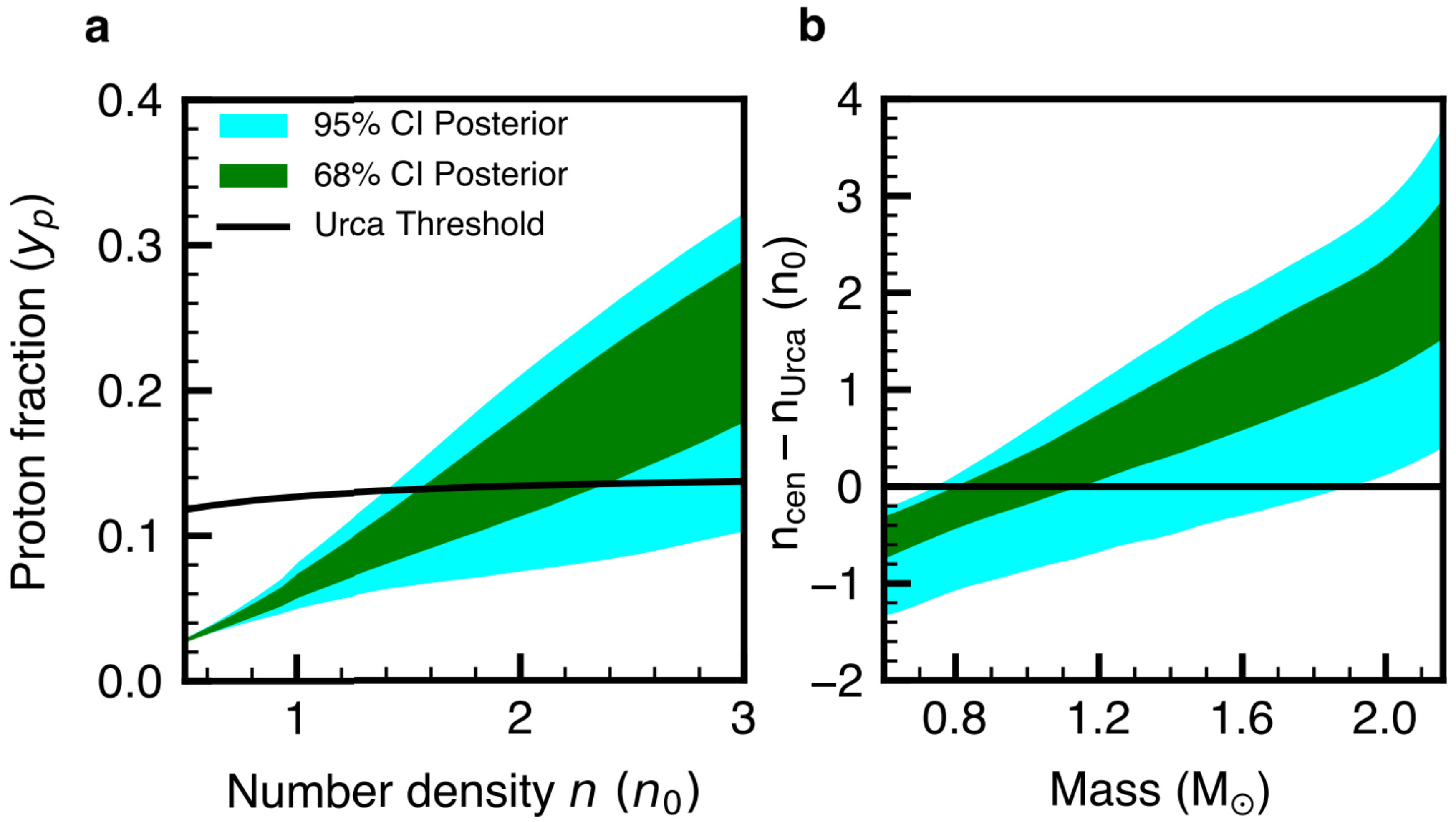
Comparison to other studies



Huth, Pang et al., Nature 606, 279 (2022)

- Chiral effective field up to $1.5n_0$
- Heavy-ion collisions
- Astronomical observables.

NS cooling by direct URCA process



Beta decay:

$n \rightarrow p + e^- + \bar{\nu}_e$

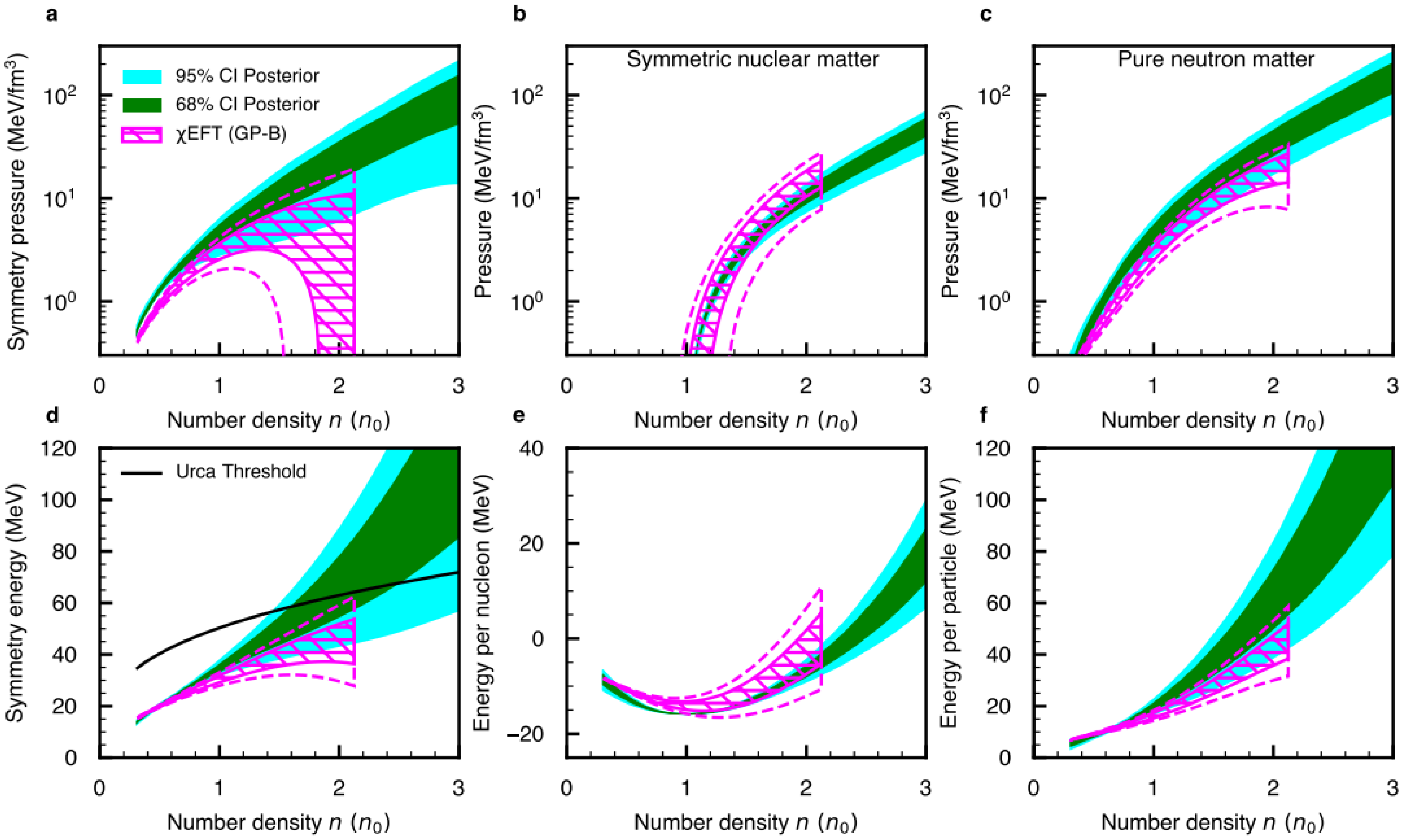
Cycle

Cool by emitting neutrinos

Electron capture:

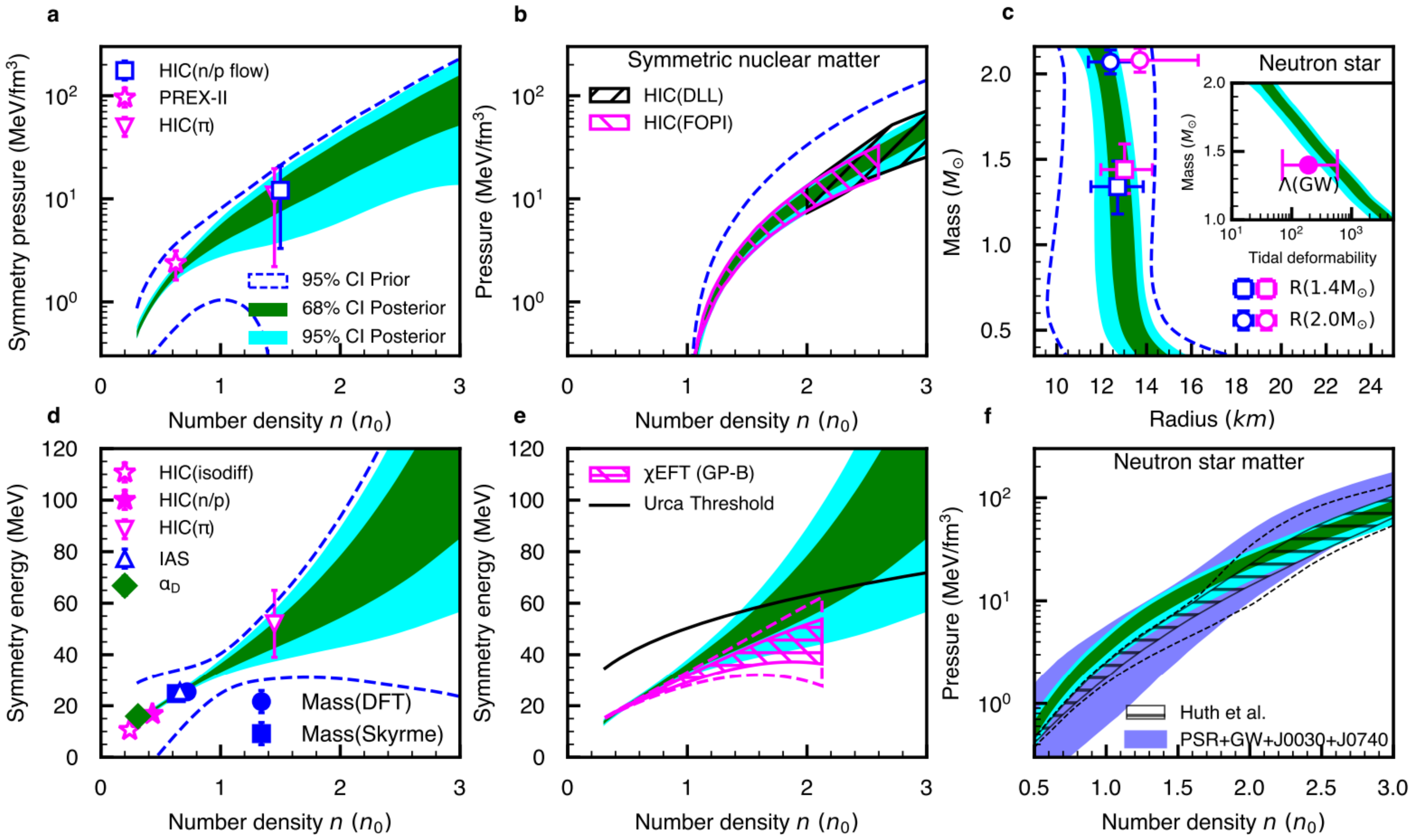
$p + e^- \rightarrow n + \nu_e$

Compare to χEFT



References:
Phys. Rev. Lett. 122, 042501 (2019). URL https://link.aps.org/doi/10. 1103/PhysRevLett.122.042501.
Phys. Rev. Lett. 125, 202702 (2020). URL https://link.aps.org/doi/10.1103/PhysRevLett.125.202702.
Phys. Rev. C 102, 054315 (2020). URL https://link.aps.org/ doi/10.1103/PhysRevC.102.054315

Summary

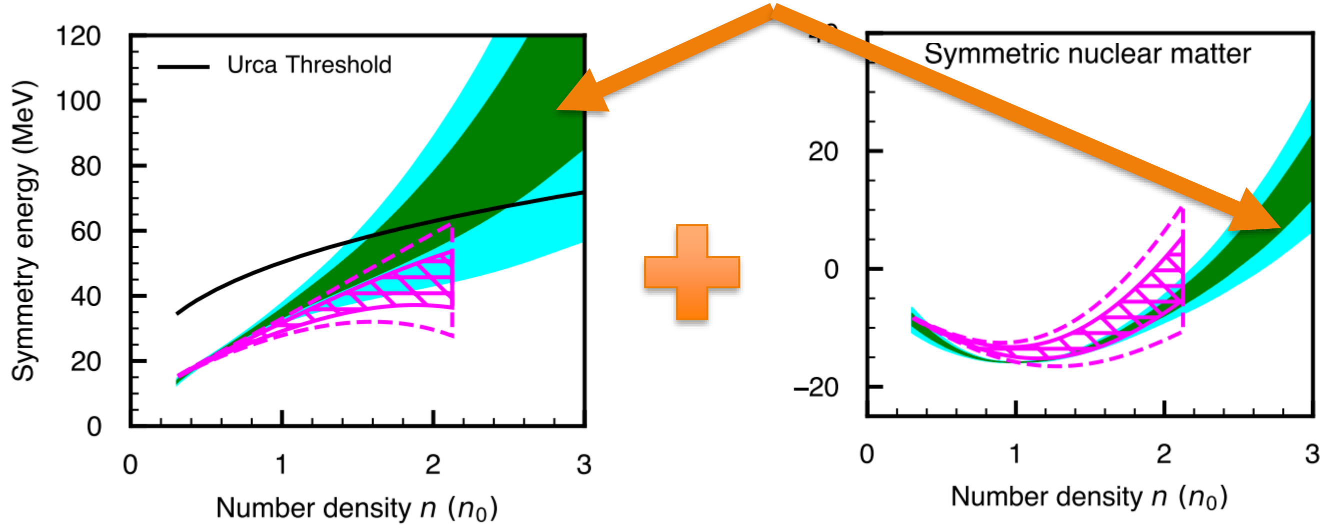


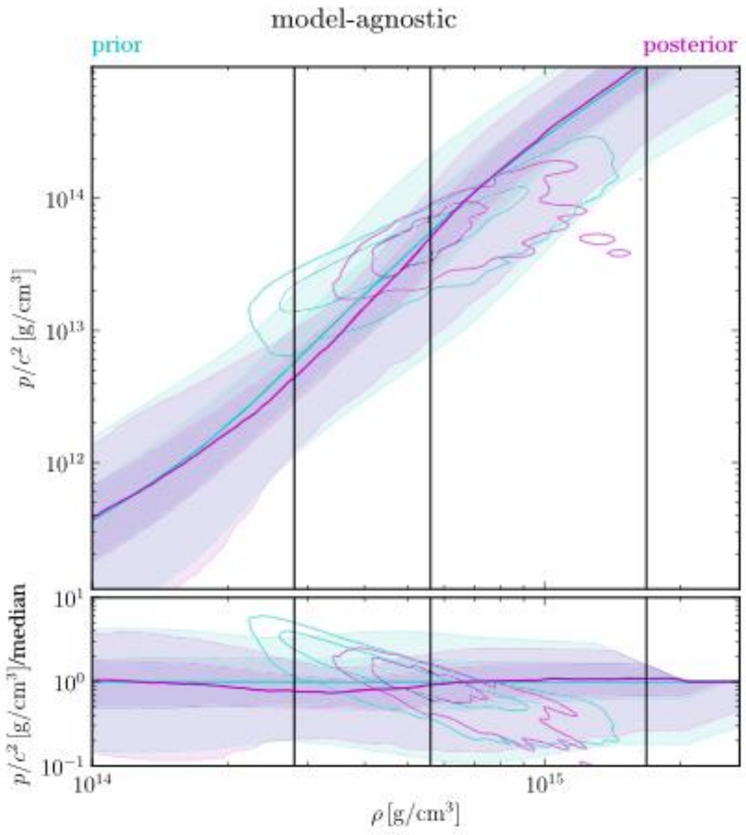
a
HIC(n/p flow)
PREX-II
HIC(π)
95% CI Prior
68% CI Posterior
95% CI Posterior
Symmetry pressure (MeV/fm³)
Number density n (n₀)
b
Symmetric nuclear matter
HIC(DLL)
HIC(FOPI)
Pressure (MeV/fm³)
Number density n (n₀)
c
Neutron star
Λ(GW)
Tidal deformability
R(1.4M⊙)
R(2.0M⊙)
Mass (M⊙)
Radius (km)
d
HIC(isodiff)
HIC(n/p)
HIC(π)
IAS
αD
Mass(DFT)
Mass(Skyrme)
Symmetry energy (MeV)
Number density n (n₀)
e
χEFT (GP-B)
Urca Threshold
Symmetry energy (MeV)
Number density n (n₀)
f
Neutron star matter
Huth et al.
PSR+GW+J0030+J0740
Pressure (MeV/fm³)
Number density n (n₀)

Future

- Need constraint on high density part of symmetry energy term.
- More astronomical observation.
- Even more flexible EoS models.
  - Non-parametrize curves (e.g. Gaussian process)?
  - Astrophysicists are already doing it for neutron star EoS (see right).
  - Separate into symmetry energy term and symmetric term?

Replace with two independent GP?

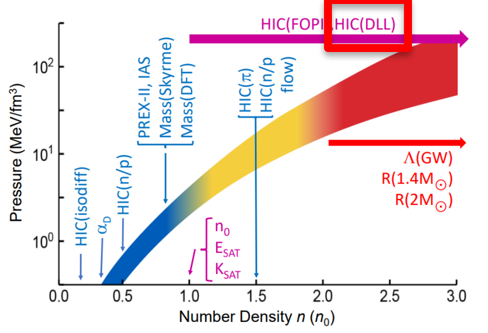




P. Landry and R. Essick, Phys. Rev. D 99, 084049 (2019)

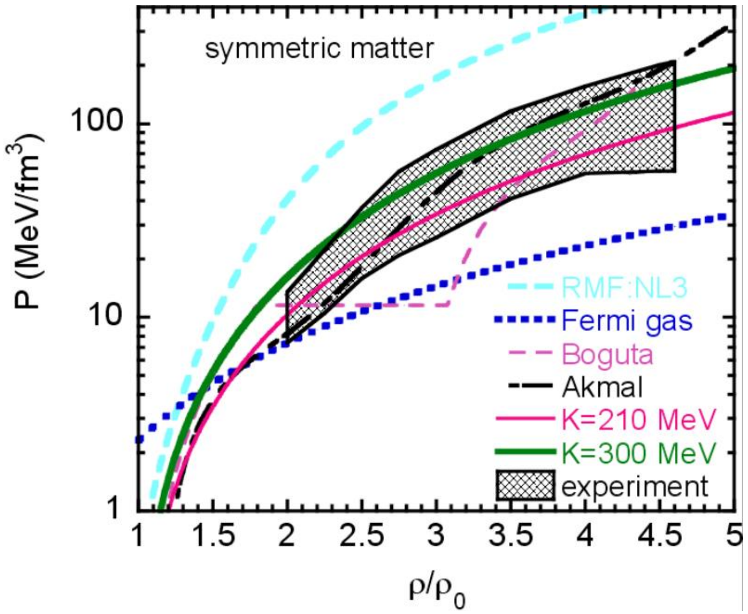
Backup slides

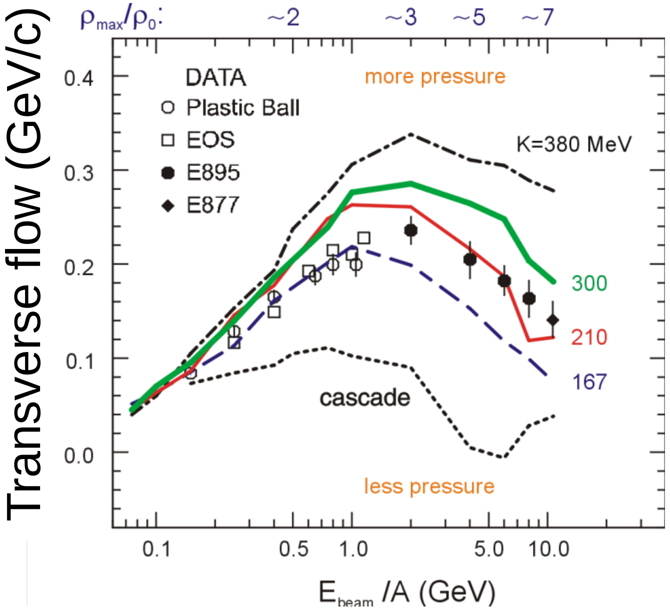
Other constraints

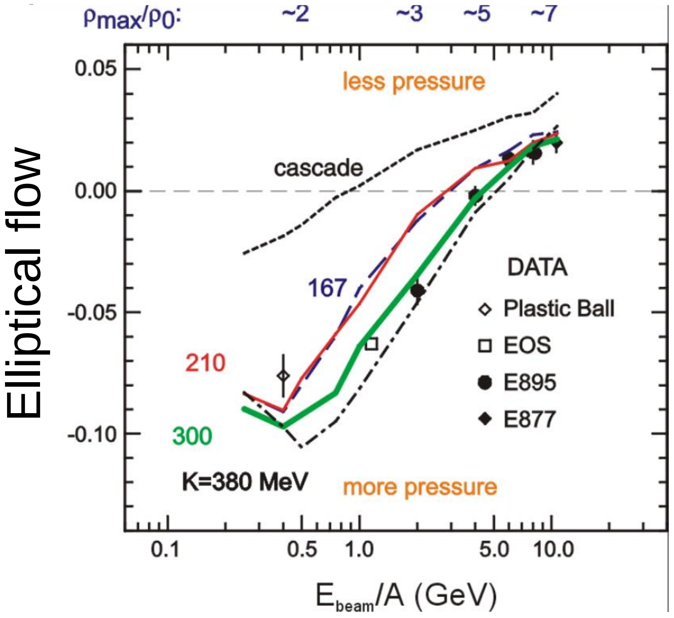


Heavy-ion collisions (HIC (DLL))

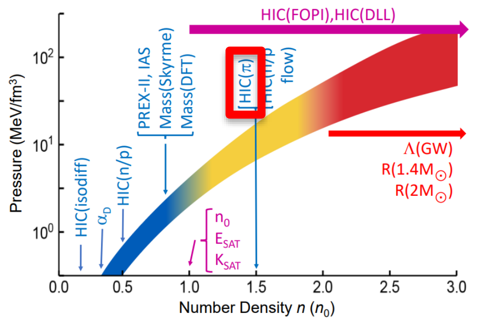
- Reference: Danielewicz, Lacey, Lynch, Science 298, 1592 (2002).
- Experiment: transverse and elliptical flow from Au + Au between 0.15 – 10 GeV/nucleon.
- Compare to predictions from transport models for the most probable nuclear EoS.
- $P_{SNM}(2n_0) = 10.1 \pm 3$ MeV/fm³.





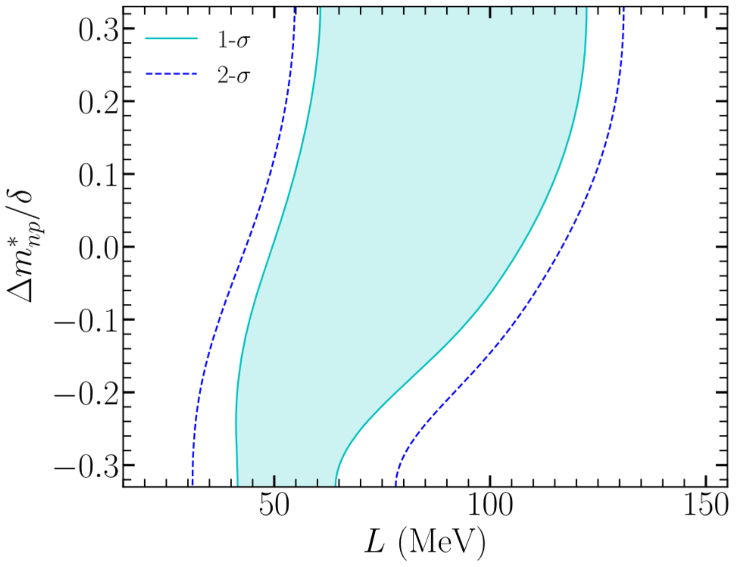


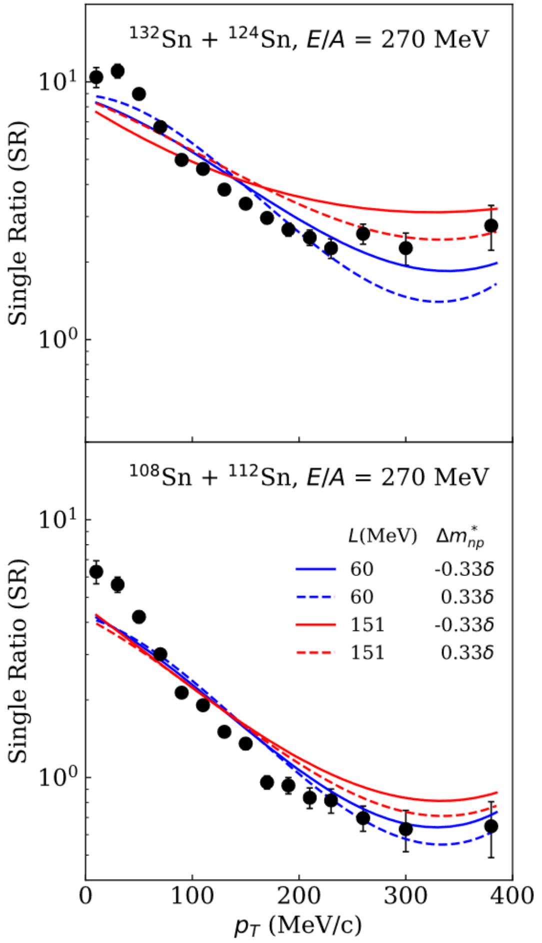
Pion ratio (HIC(π))



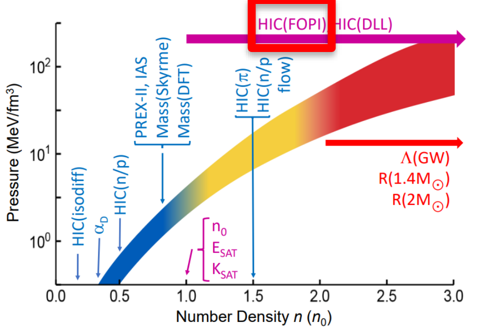
- Reference: Phys. Rev. Lett. 126, 162701 (2021)
- Experiment: Neutron-rich rare isotopes of Sn on Sn at 270 MeV/nucleons.
- Compare π^+/π^- predictions from dcQMD to data.
- $\pi^{+(-)}$ are created indirectly with p-p(n-n) collisions. Ratios singles out neutron contribution.

- $S(1.45n_0) = 52 \pm 13$ MeV.
- $P_{sym}(1.45n_0) = 10.9 \pm 8.7$ MeV/fm³.

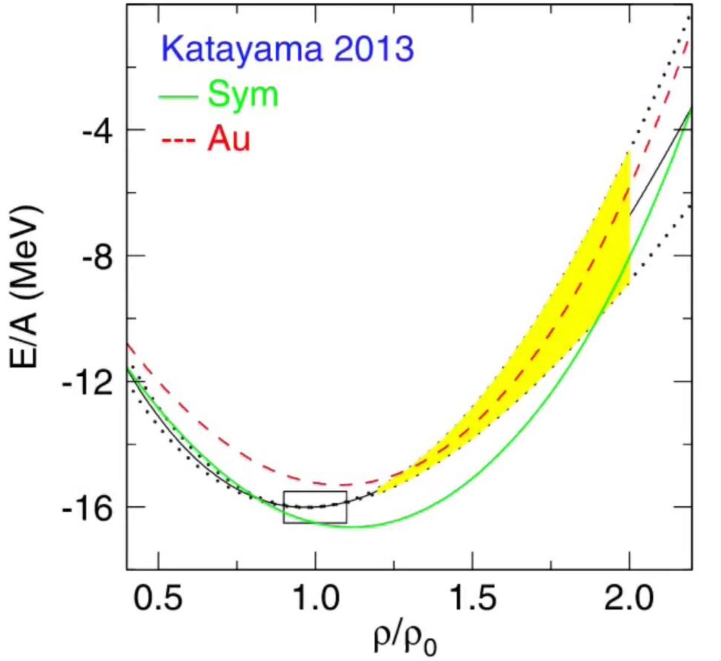


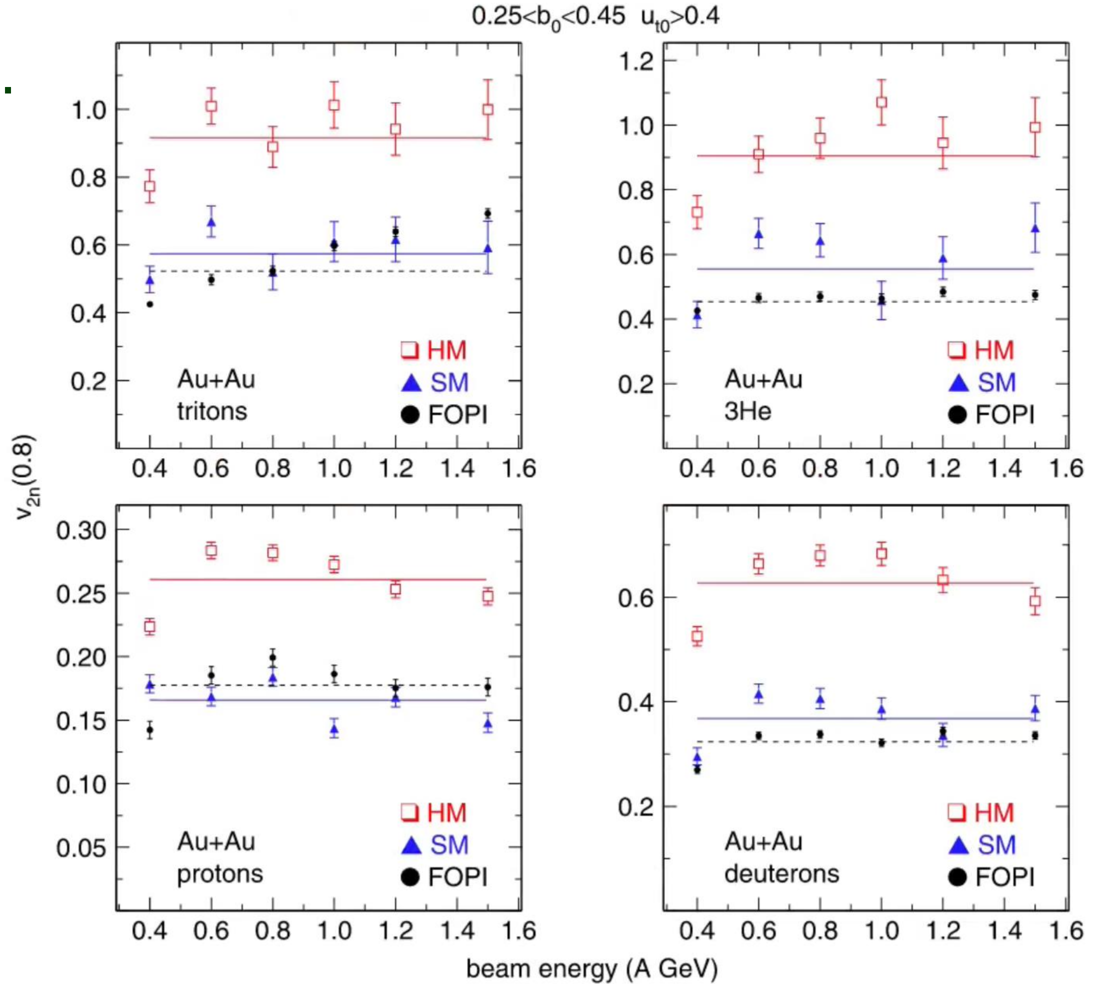


Heavy-ion collisions (HIC (FOPI))

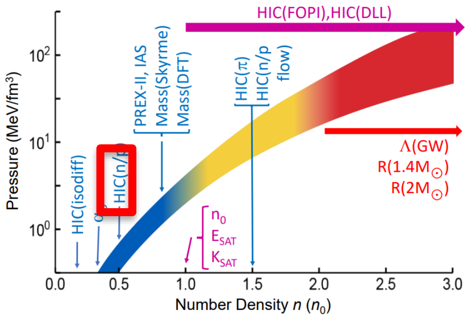


- Reference: Le Fevre, Leifels, Reisdorf, Aichelin, Hartnack, Nuclear Physics A 945, 112 (2016)
- Experiment: Au + Au at 0.4 – 1.5 GeV/nucleon from FOPI.
- Compare observed elliptical flow to IQMD predictions.
- $P_{SNM}(2n_0) = 10.3 \pm 2.8$ MeV.

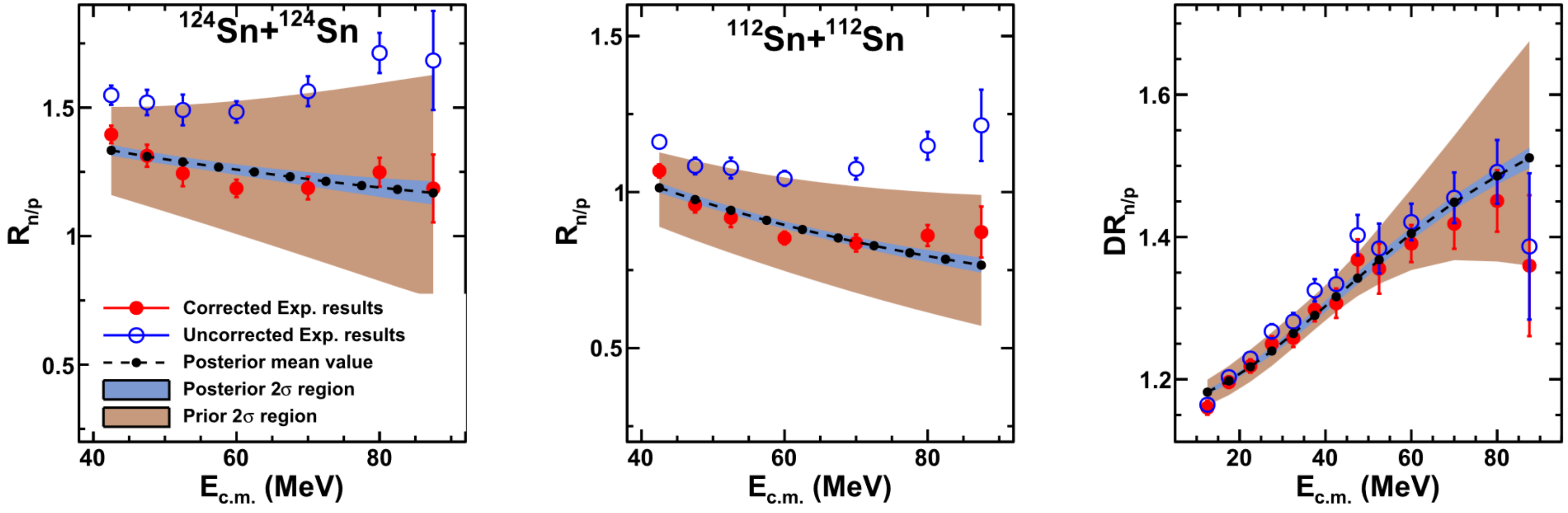


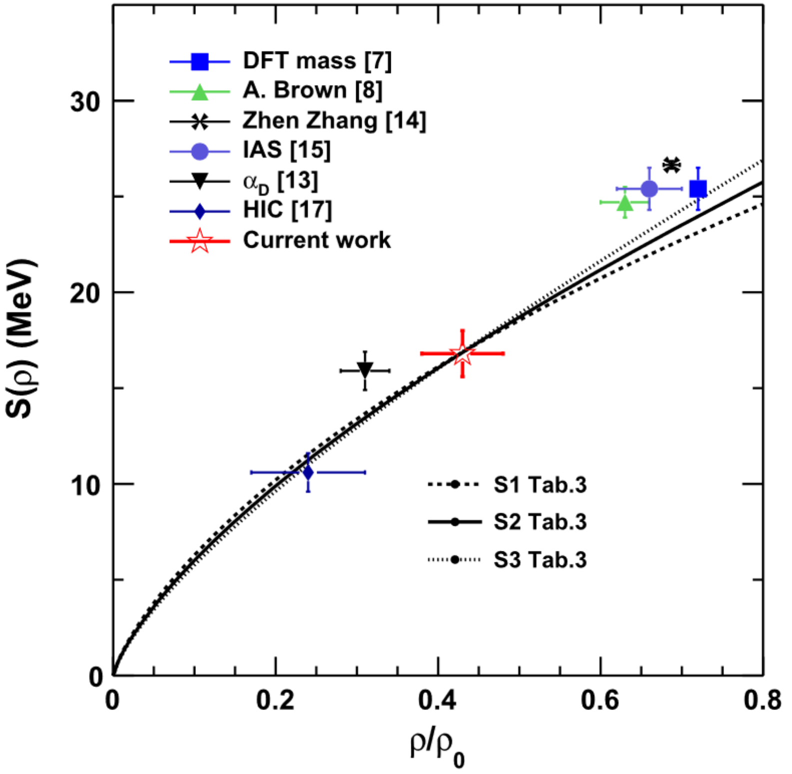


Neutron to proton ratios (HIC(n/p))

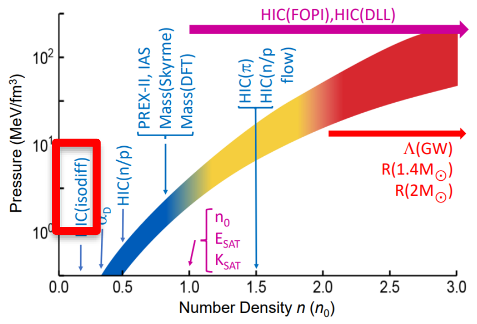


- Reference: Phys. Lett. B 799, 135045 (2019)
- Experiment: Neutron to proton yield from $^{112}Sn+^{112}Sn$ and $^{124}Sn + ^{124}Sn$ collisions at 120 MeV/nucleon.
- Compared to predictions from ImQMD-Sky with Bayesian analysis.
- $S((0.43 \pm 0.05)n_0) = 16.8 \pm 1.2$ MeV/fm³.





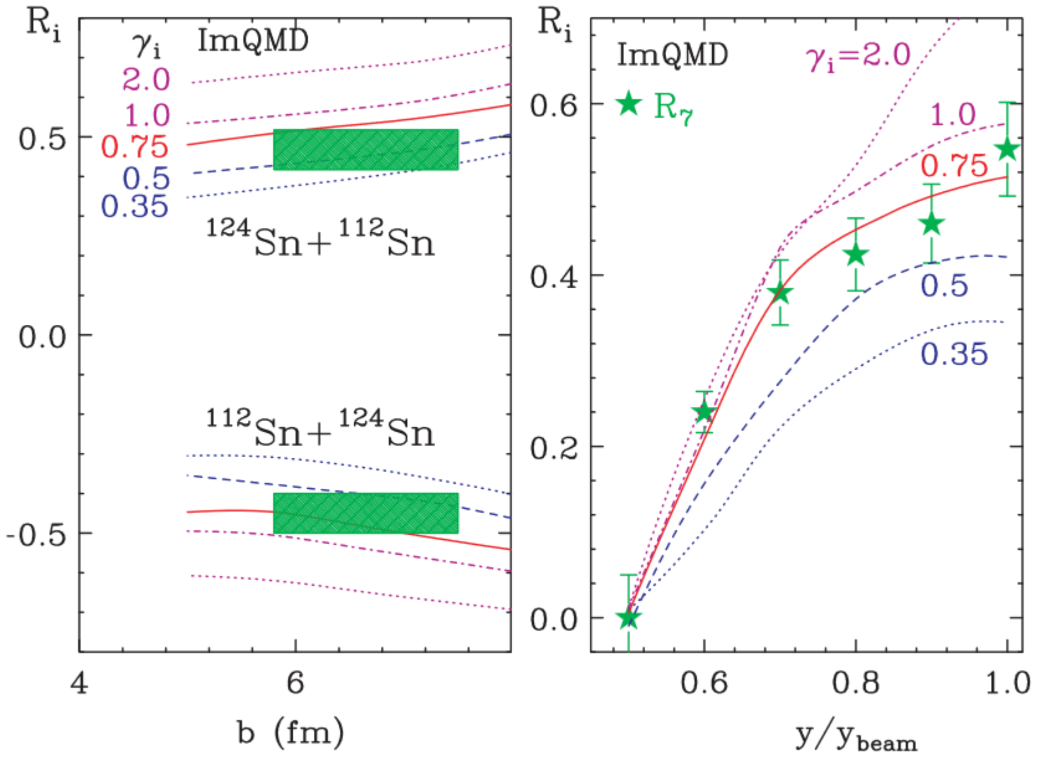
Isospin diffusion (HIC(isospin))



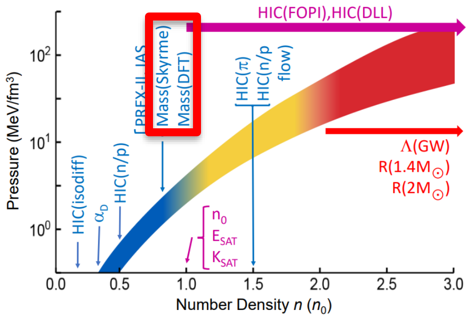
- Reference: Phys. Rev. C 102, 122701 (2009), Phys. Lett. B 830,137098 (2022)
- Experiment: Neutron to proton yield from $^{112}Sn+^{112}Sn$ and $^{124}Sn + ^{124}Sn$ and mixed collisions at 50 MeV/nucleon.
- Isospin diffusion:

$$R_i(X) = 2\frac{X-(X_{A+A}+X_{B+B})/2}{X_{A+A}-X_{B+B}}, \qquad (3)$$

- i.e. How close mixed collision is to symmetric collision.
- X is $\ln(Y(^7Li)\,/Y(^7Be))$ in this case.
- $S((0.22 \pm 0.07)n_0) = 10.3 \pm 1.0$ MeV/fm^3.



Mass(Skyrme), Mass(DFT)



Mass(Skyrme)

- Reference: Phys. Rev. Lett. 111, 232502 (2013), Phys. Rev. C 89, 011307(R) (2014)
- Data: Binding energies, charge radii, single-particle energies of doubly magic nuclei.
- Compare data with predictions of 18 Skyrme energy density functionals.
- $S\big((0.63 \pm 0.03)n_0\big) = 24.7 \pm 0.8$ MeV.

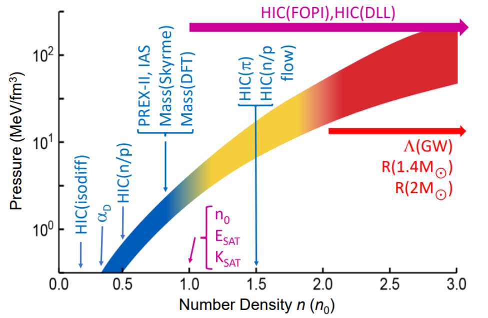
Mass(DFT)

- Reference: Phys. Rev. C 82, 024313 (2010), Phys. Rev. C 85, 024304 (2012)
- Data: Masses of nuclei at $40 \leq A \leq 264$.
- Predictions from density functional theory (DFT) are fitted to data.
- $S\big((0.72 \pm 0.01)n_0\big) = 25.4 \pm 1.1$ MeV.

TABLE II. Optimized parameter set UNEDF1. Listed are bounds used in the optimization, final optimized parameter values, standard deviations, and 95% confidence intervals.

$\mathbf{x}$	Bounds	$\hat{\mathbf{x}}^{(\mathrm{fin.})}$	σ	95% CI
ρ_c	[0.15,0.17]	0.15871	0.00042	[0.158, 0.159]
E^{NM}/A	[−16.2,−15.8]	−15.800	—	—
K^{NM}	[220, 260]	220.000	—	—
$a_{\mathrm{sym}}^{\mathrm{NM}}$	[28, 36]	28.987	0.604	[28.152, 29.822]
$L_{\mathrm{sym}}^{\mathrm{NM}}$	[40, 100]	40.005	13.136	[21.841, 58.168]
$1/M_s^*$	[0.9, 1.5]	0.992	0.123	[0.823, 1.162]
$C_0^{\rho\Delta\rho}$	[−∞, +∞]	−45.135	5.361	[−52.548, −37.722]
$C_1^{\rho\Delta\rho}$	[−∞, +∞]	−145.382	52.169	[−217.515, −73.250]
V_0^n	[−∞, +∞]	−186.065	18.516	[−211.666, −160.464]
V_0^p	[−∞, +∞]	−206.580	13.049	[−224.622, −188.538]
$C_0^{\rho\nabla J}$	[−∞, +∞]	−74.026	5.048	[−81.006, −67.046]
$C_1^{\rho\nabla J}$	[−∞, +∞]	−35.658	23.147	[−67.663, −3.654]

Isobaric analogue state (ISA), *Not shown on the illustration



Isobaric analogue state

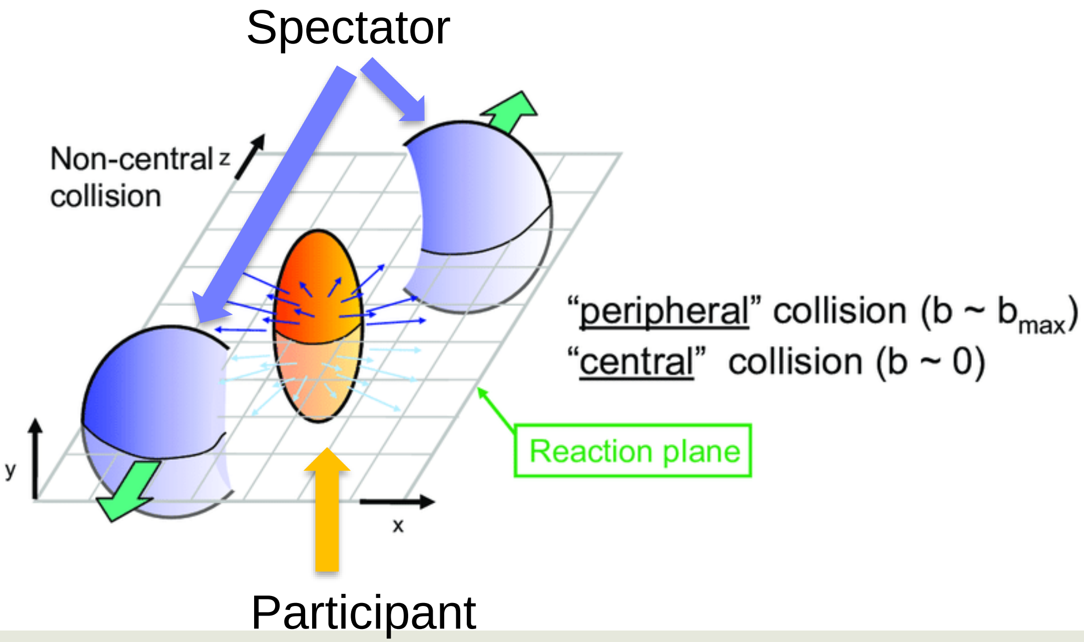
- Reference: Nucl. Phys. A 958, 147 (2017)

- Experiment: Quasielastic change exchange (p,n) reactions, proton and neutron elastic scattering on Ca, Zr, Sn and Pb at 10 – 50 MeV.

- $E_{sym}\left((0.66 \pm 0.04)n_0\right) = 25.4 \pm 1.1$ MeV.

Collective flow

- Collective flow is the degree of asymmetry in the azimuth distribution of reaction fragments with respect to reaction plane, the plane that the perpendicular displacement vector between target and projectile and beam axis span.
- Collisions are rarely head-on, so spectator nucleons often blocks the emission of participant nucleons along reaction plane.
- If mean field is very repulsive, particles are promptly ejected. Spectators don't have time to clear the path of emission, resulting in strong asymmetry, vice versa.
- Flow is quantified with v_1 (directed flow) and v_2 (elliptical flow) defined as,
  - $\frac{dN}{d\Phi} \propto 1 + 2v_1 cos\phi + 2v_2 cos2\phi + \cdots$
- Higher order terms are not constructed due to statistics



Basics of transport models

- Consider phase space of a single nucleon f(t, r, p). Without collisions, phase space volume won't change. This give rise to the Vlasov equation,

$$\frac{\partial f}{\partial t} + \frac{dr}{dt} \cdot \frac{\partial f}{\partial r} + \frac{dp}{dt} \cdot \frac{\partial f}{\partial p} = 0$$

- Heuristically, Boltzmann transport equation can be thought of as Vlasov equation with Hamiltonian equation of motion (dr/dt = dH/dp and dp/dt = -dH/dr) and collision terms,

$$\frac{\partial f}{\partial t} + \frac{dH}{dp} \cdot \frac{\partial f}{\partial r} - \frac{dH}{dr} \cdot \frac{\partial f}{\partial p} = I_{coll}(f, \frac{d\sigma}{d\Omega})$$

- EoS is embedded in the potential energy part of the Hamiltonian.
- $d\sigma/d\Omega$ is the in-medium nucleon-nucleon cross-section.
- Solve this equation with Monte Carlo simulation of test particles. Pauli blocking is considered at each time-step.
- An approach to solving it is through Quantum Molecular Dynamics (QMD) simulation.

Two implementations of QMD models

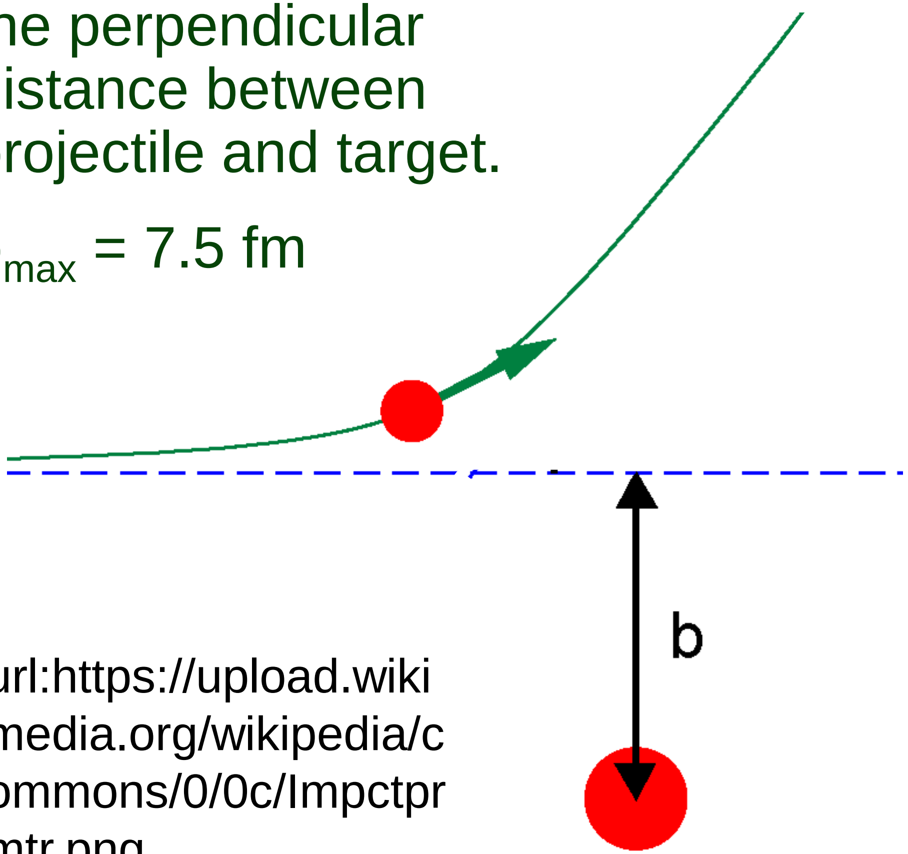
- ImQMD-Sky model by Yingxun Zhang in 2003 (Y. Zhang, M. Tsang, Z. Li, and H. Liu, Physics Letters B **732**, 186 (2014)).
  - The mean-field takes the form of Skyrme interaction, a family of EoS that is commonly used in nuclear physics.
  - The in-medium cross section is formulated as $\sigma^{med} = \left(1 - \frac{\eta\rho}{\rho_0}\right)\sigma^{free}$, where η is a free parameter.
  - This code does not produce pions.
- dcQMD model by Dan Cozma in 2019 (M. Cozma, Physics Letters B **700**, 139 (2011)).
  - Allows the isospin-dependent potential of nucleons to be different from that of Δ(1231) resonance. This is important for pion calculation as it was shown that total pion yield is only accurately reproduced if the potential of Δ(1231) resonance is varied.
  - Also considers total energy balance due to in-medium potential in a collision, which has a pronounced effect in its sensitivity to pion observables.

Coalescence

- Transport models can only propagate nucleons. Most do not simulate how nucleons merge to form isotopes before final emission.
- ***Coalescence***, or the after burner, takes the final nucleon distribution from QMD models and group nucleons into isotopes. Usually nucleons that are within some threshold distances in phase space from each other are grouped into isotopes.
- This is ***unreliable*** as physical coalescence involves multi-body correlations that are not well understood. ***Difficult to calculate it right.***
- Therefore the predicted spectra for light fragments are not reliable. To circumvent this difficulty, we construct observables that are less sensitive to coalescence process,
  1. Examine particles that do not form isotopes (e.g. pions).
  2. Take ratios of similar observables or same observable from different reactions such that effects of coalescence cancels out.
  3. Sum up all constituent protons from all the emitted fragments to recover the final nucleon distribution before coalescence.

Terminology in this presentation

- Impact parameter b is the perpendicular distance between projectile and target.
- b_{max} = 7.5 fm



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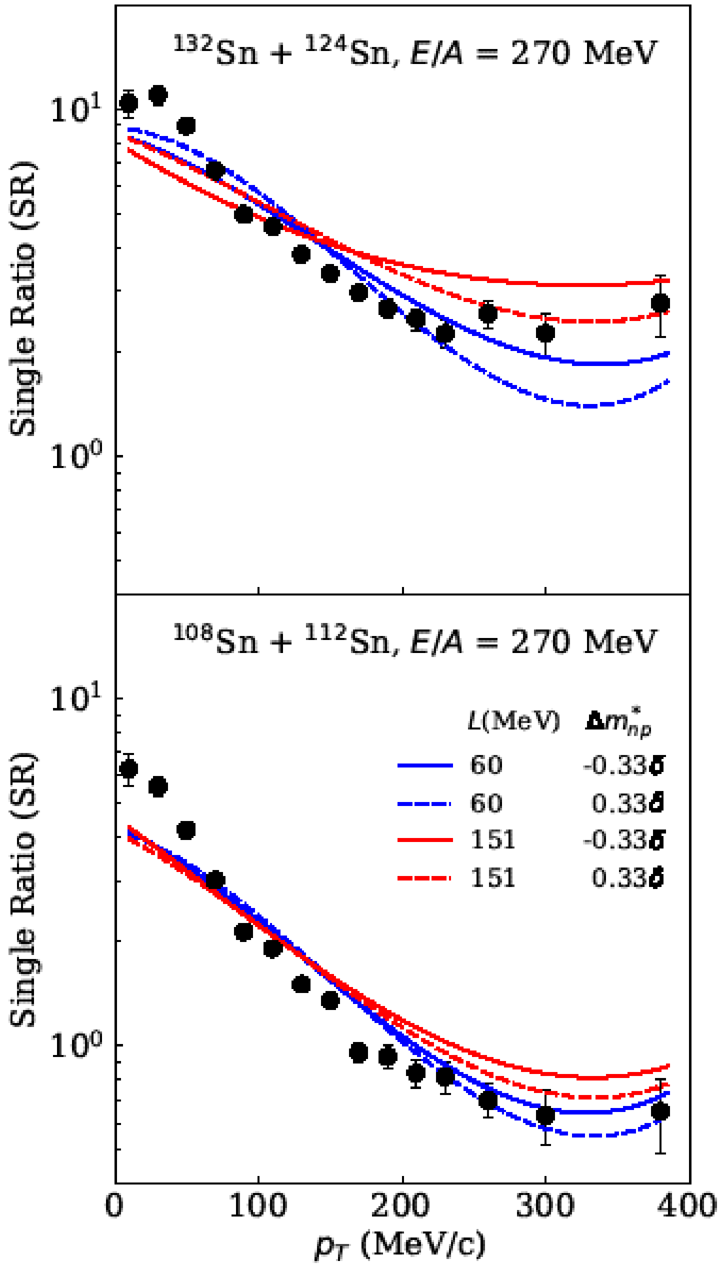
- EoSs are expanded around $x = (\rho - \rho_0)/3\rho_0$ in a Taylor series, (ρ_0=saturation density)
  - $E(symmetric\ matter) = E_{sat} + K_{sat}x^2 + Q_{sat}x^3 + Z_{zat}x^4 + \cdots$
  - $S(\rho) = S_0 + Lx + K_{sym}x^2 + Q_{sym}x^3 + Z_{sym}x^4 +$ ...
  - Subscript of “sat” refers to E(symmetric matter).
  - Subscript of “sym” refers to symmetry energy term.
- Effective-mass splitting,

$$\Delta m^*_{np} = \frac{m^*_n - m^*_p}{m_N}$$

J. Estee, et al., Phys. Rev. Lett. 126, 162701 (2021).

Pion spectra ratio

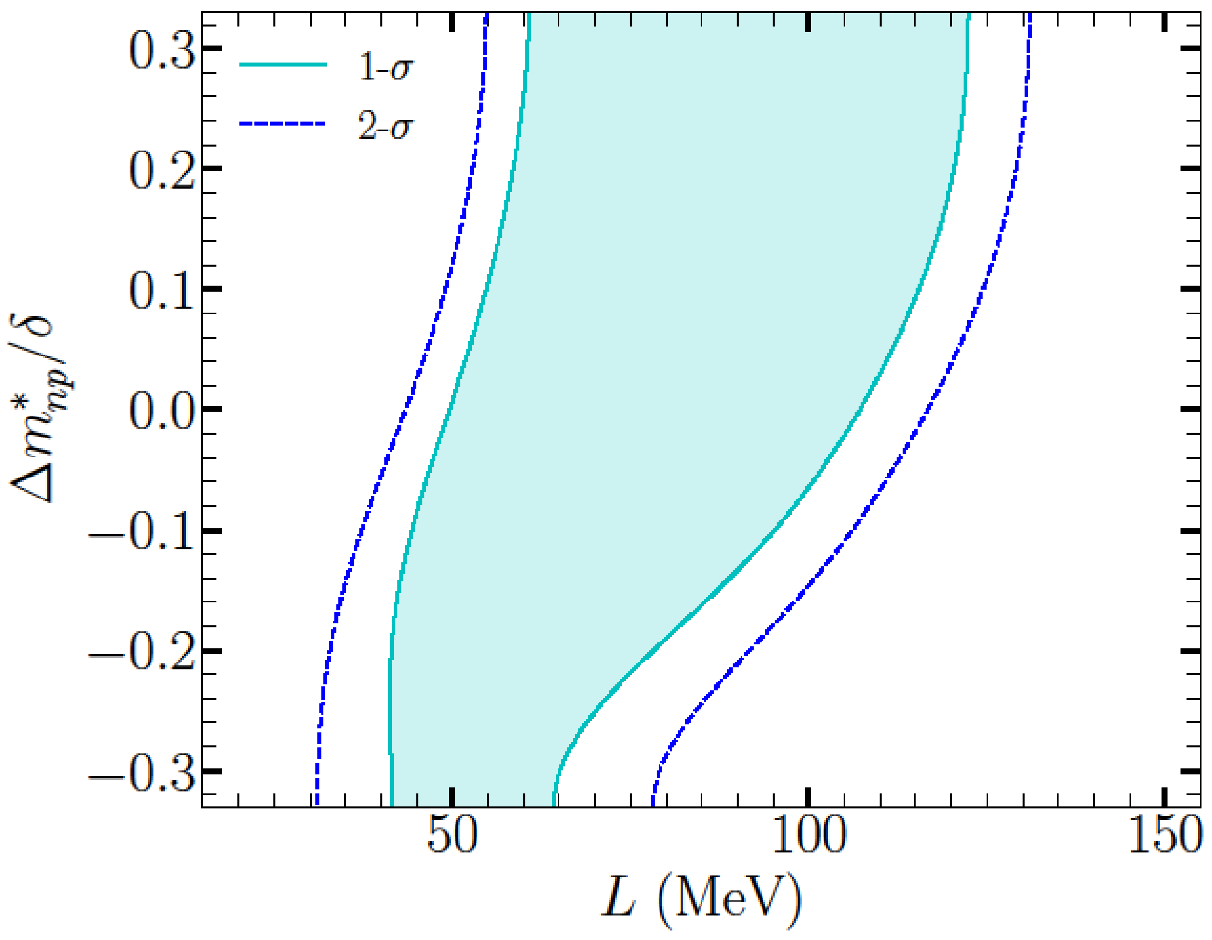
- Ratios of π^- to π^+ spectra are taken for a few reasons:
  - Effects that acts similarly on different pions are cancelled out to eliminate sensitivity on effects that are not being considered here.
  - Symmetry energy effects are magnified due to symmetry forces acting on π^- and π^+ with opposite sign.
  - Systematic uncertainties due to detector errors (if any) are cancelled out in the division.
  - <b> = 2.1 fm, pions are generated mostly from central collisions.
- Configuration of dcQMD:
  - Best fitted pion and delta potential are used.
  - Only L and $\Delta m_{np}/\delta$ are allowed to vary. Other parameters are fixed to values from previous studies.
  - K_{sat} = 250 MeV, Q_{sat} = -350 MeV, K_{sym} = -488 + 6.728L and $S(0.67\rho_0) = 25.5\ MeV$
  - L is 1st derivative, K is 2nd derivative, Q is 3rd derivative. "Sat" for symmetric matter and "sym" for symmetry energy term.



Results of the comparison

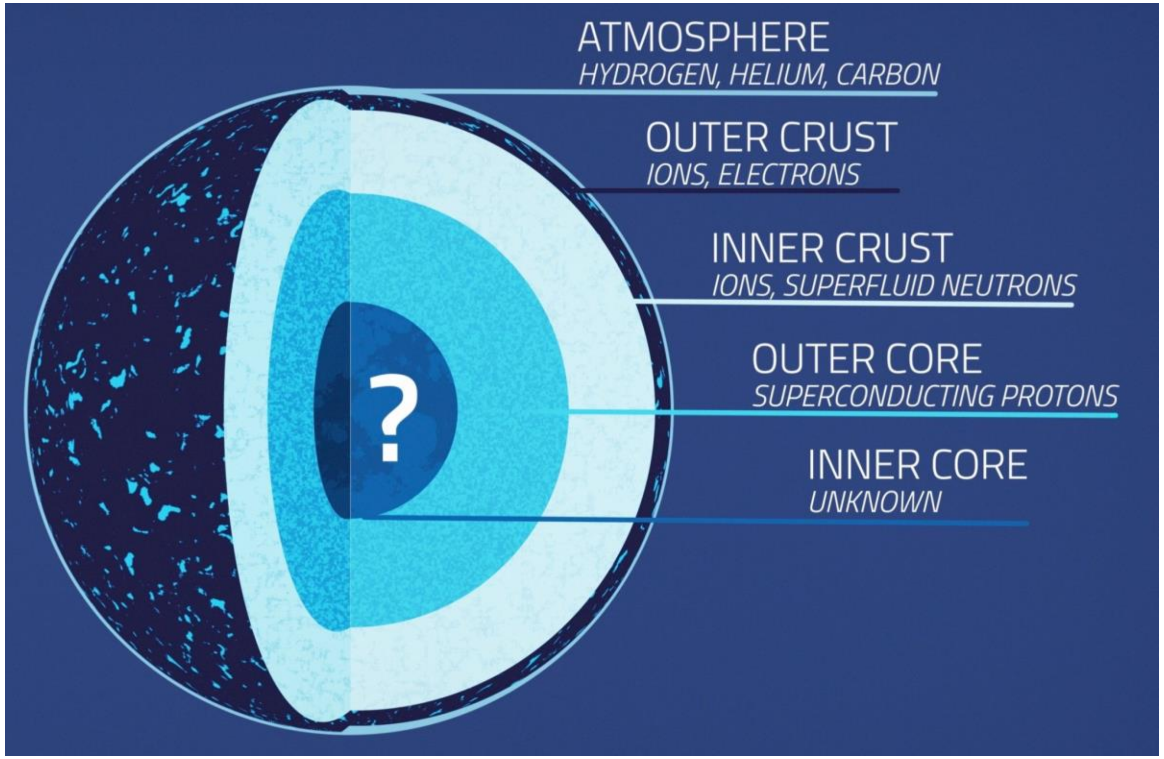
- Only spectra at $p_T > 200\ MeV/c$ are compared as low energy effects, such as Coulomb interaction, diminish at high momentum. The cut isolates the effect of symmetry energy.
- Chi-square analysis is performed to constraint nuclear EoS parameters.
- Without constraint on $\Delta m^*_{np}/\delta$, we have L = 79.9 +/- 37.6 MeV and S_0= 35.5+/-2.9 MeV.
- The result is a correlation between L and $\Delta m^*_{np}/\delta$. If constraint on $\Delta m^*_{np}/\delta$ is improved, L will be better constrained.
- Pion results are the focus of Justin Estee's thesis. My work tighten his constraint by incorporating light fragment observables.

J. Estee, et al., Phys. Rev. Lett. 126, 162701 (2021).



Construction of EoSs

- Outer crust: Crustal EoS from (G. Baym, 1971)
- Inner crust: Spline connection between outer and inner crust.
- Outer core: Beta-equilibrated meta-modelling EoS.
- Inner core: Exist only if speed of sound of outer core = c. At that density, switch to EoS with constant speed of sound = c.



Credit: NASA's Goddard Space Flight Center/Conceptual Image Lab

All experimental/observational constraints

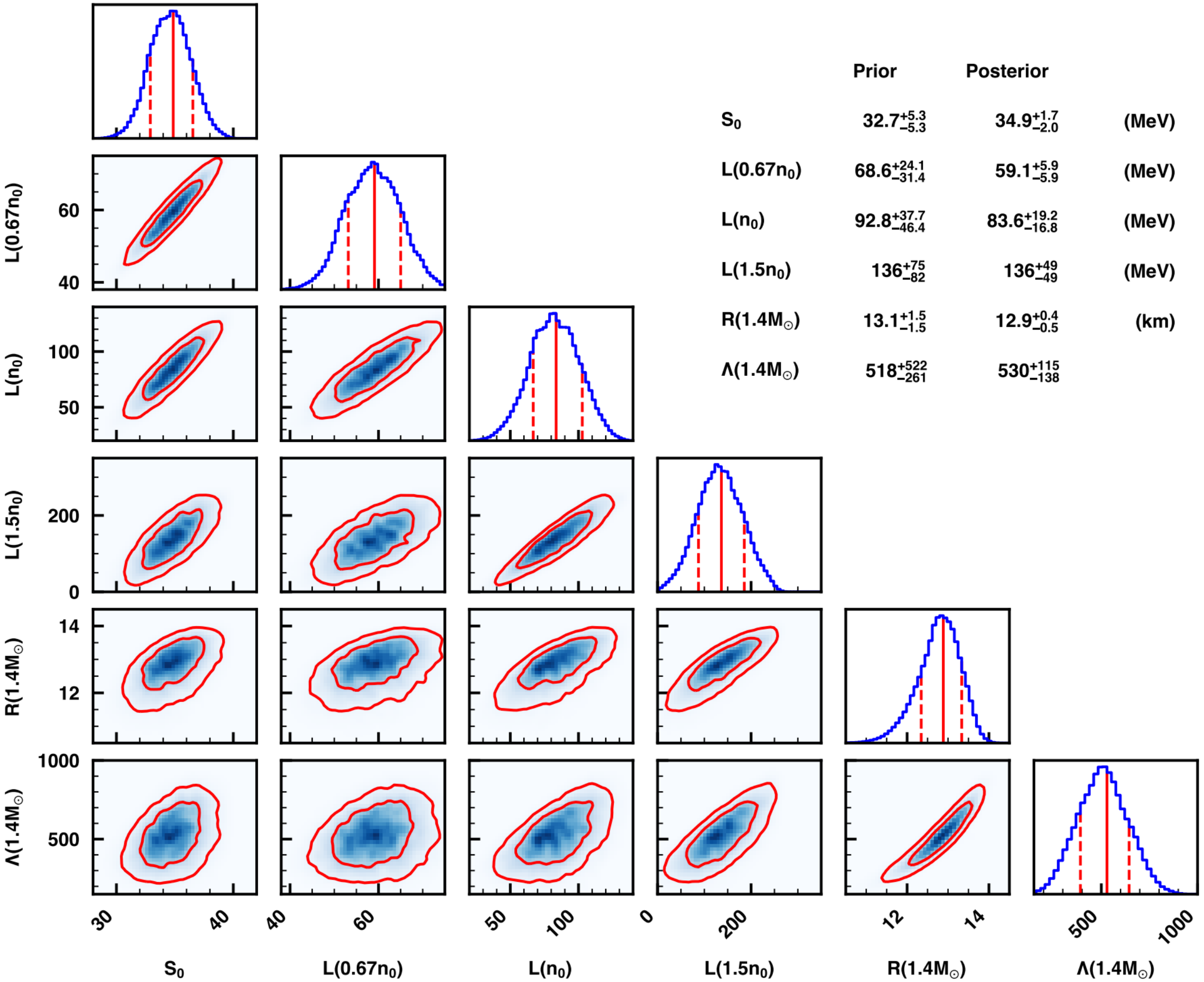
TABLE I. **List of Constraints used in Bayesian Analysis**. The NICER constraints marked with * are given half weight in the Bayesian analysis as the 4 constraints listed here come from two measurements analyzed by two different groups.

Symmetric matter Constraints	n (fm^{-3})	P_{SNM} (MeV/fm^3)	K_{SAT} (MeV)	Ref.
HIC(DLL)	0.32	10.1 ± 3.0		[12]
HIC(FOPI)	0.32	10.3 ± 2.8		[13]
GMR	0.16		230 ± 30	[14]

Asymmetric matter Constraints	n (fm^{-3})	$S(n)$ (MeV)	P_{sym} (MeV/fm^3)	Ref.
Nuclear structure				
α_D	0.05	15.9 ± 1.0		[15]
PREX-II	0.11		2.38 ± 0.75	[16, 17, 28]
Nuclear masses				
Mass(Skyrme)	0.101±0.005	24.7 ± 0.8		[18, 28]
Mass(DFT)	0.115±0.002	25.4 ± 1.1		[19, 28]
IAS	0.106±0.006	25.5 ± 1.1		[20, 28]
Heavy-ion collisions				
HIC(Isodiff)	0.035±0.011	10.3 ± 1.0		[21, 28]
HIC(n/p ratio)	0.069 ± 0.008	16.8 ± 1.2		[22, 28]
HIC(π)	0.232±0.032	52 ± 13	10.9 ± 8.7	[23, 28]
HIC(n/p flow)	0.240		12.1 ± 8.4	[24–26, 28]

Astronomical Constraints	$M(\odot)$	R (km)	Λ	Ref.
LIGO	1.4		190^{+390}_{-120}	[11]
*Riley PSR J0030+0451	$1.34^{+0.15}_{-0.16}$	$12.71^{+1.14}_{-1.19}$		[6]
*Miller PSR J0030+0451	$1.44^{+0.15}_{-0.14}$	$13.02^{+1.24}_{-1.06}$		[7]
*Riley PSR J0740+6620	$2.07^{+0.07}_{-0.07}$	$12.39^{+1.30}_{-0.98}$		[8]
*Miller PSR J0740+6620	$2.08^{+0.07}_{-0.07}$	$13.7^{+2.6}_{-1.5}$		[9]

Posterior



	Prior	Posterior	
S_0	$32.7^{+5.3}_{-5.3}$	$34.9^{+1.7}_{-2.0}$	(MeV)
$L(0.67n_0)$	$68.6^{+24.1}_{-31.4}$	$59.1^{+5.9}_{-5.9}$	(MeV)
$L(n_0)$	$92.8^{+37.7}_{-46.4}$	$83.6^{+19.2}_{-16.8}$	(MeV)
$L(1.5n_0)$	136^{+75}_{-82}	136^{+49}_{-49}	(MeV)
$R(1.4M_\odot)$	$13.1^{+1.5}_{-1.5}$	$12.9^{+0.4}_{-0.5}$	(km)
$\Lambda(1.4M_\odot)$	518^{+522}_{-261}	530^{+115}_{-138}	

Parameters	Priors	Posteriors (Median ± 68% CI)
K_{SAT} (MeV)	[0, 648]	221^{+23}_{-27}
Q_{SAT} (MeV)	[-1100, 2100]	-640^{+273}_{-249}
S_0 (MeV)	[24.7, 40.3]	$34.9^{+1.7}_{-2.0}$
L (MeV)	[-11.4, 149.4]	$83.6^{+19.2}_{-16.8}$
K_{sym} (MeV)	[-328.5, 237.9]	-6^{+102}_{-102}
Q_{sym} (MeV)	[-489, 1223]	692^{+377}_{-548}
Z_{sym} (MeV)	[-10110, 2130]	-149^{+1585}_{-1982}
$P_{SNM}(4n_0)$ (MeV/fm^3)	[0, 300]	213^{+60}_{-66}
Predictions		**Posteriors (Median ± 68% CI)**
$R(1.4M_\odot)$ (km)		$12.9^{+0.4}_{-0.5}$
$\Lambda(1.4M_\odot)$		530^{+115}_{-138}
S_{01} (MeV)		$24.0^{+0.5}_{-0.5}$
L_{01} (MeV)		57^{+4}_{-6}
K_{01} (MeV)		-35^{+42}_{-42}
Q_{01} (MeV)		200^{+115}_{-148}
Z_{01} (MeV)		-528^{+335}_{-335}

Implications on neutron stars