

# Implications of PREX-II and CREX experiments for relativistic nuclear energy density functionals

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Esra Yüksel

School of Mathematics and Physics, University of Surrey,  
Guildford, Surrey, GU2 7XH, UK

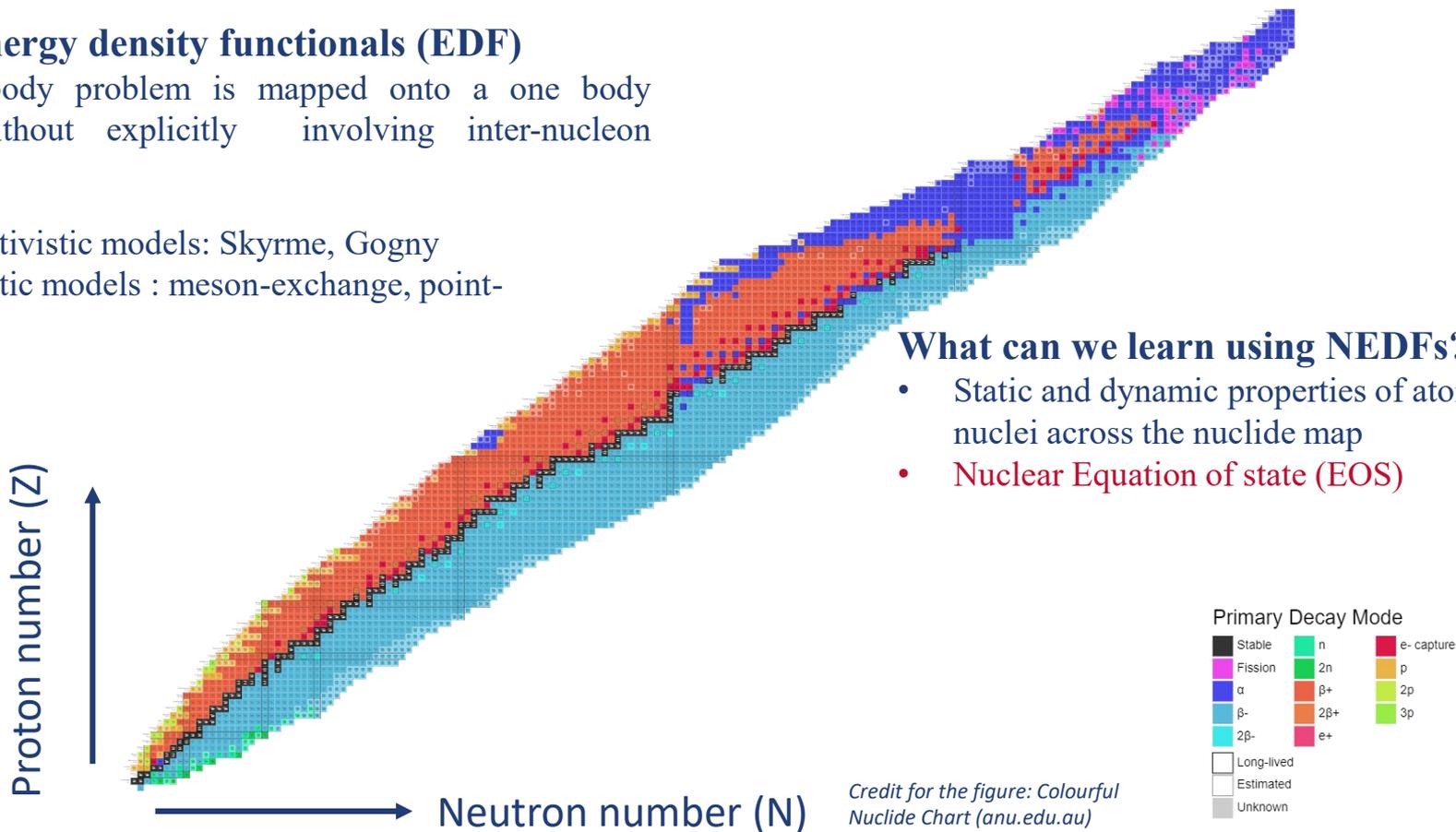
In Collaboration with Nils Paar (Zagreb) , Ante Ravlic (MSU), Tamara Niksic (Zagreb)

XII<sup>th</sup> International Symposium on Nuclear Symmetry Energy (NuSYM24), Caen (France)  
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## Nuclear energy density functionals (EDF)

The many-body problem is mapped onto a one body problem without explicitly involving inter-nucleon interactions!

- Non-relativistic models: Skyrme, Gogny
- Relativistic models : meson-exchange, point-coupling



## What can we learn using NEDFs?

- Static and dynamic properties of atomic nuclei across the nuclide map
- Nuclear Equation of state (EOS)

Credit for the figure: Colourful Nuclide Chart ([anu.edu.au](http://anu.edu.au))

**Point-coupling interaction** → The basis is an effective Lagrangian with four-fermion (contact) interaction terms; isoscalar-scalar (S), isoscalar-vector (V), isovector-vector (TV), derivative term

$$\begin{aligned}
 \mathcal{L} = & \bar{\psi}(i\gamma \cdot \partial - m)\psi \\
 & - \frac{1}{2}\alpha_S(\rho)(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\rho)(\bar{\psi}\gamma^\mu\psi)(\bar{\psi}\gamma_\mu\psi) \\
 & - \frac{1}{2}\alpha_{TV}(\rho)(\bar{\psi}\vec{\tau}\gamma^\mu\psi)(\bar{\psi}\vec{\tau}\gamma_\mu\psi) \\
 & - \frac{1}{2}\delta_S(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) - e\bar{\psi}\gamma \cdot A\frac{1-\tau_3}{2}\psi.
 \end{aligned}$$

- Free nucleon terms
- Point coupling interaction terms
- Derivative term accounting for the leading effects of finite range interactions
- Coupling of protons to the EM field

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- Free nucleon terms
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- The following form of the couplings is employed



$$\alpha_i(\rho) = a_i + (b_i + c_i x)e^{-d_i x} (i \equiv S, V, TV)$$

- Density-dependent coupling functions carry information about the many body correlations!

- The new interactions are optimized by minimizing the  $\chi^2$  function.
- Exp. data on the ground-state properties of nuclei: **mass, radii, s.o splitting, deformation properties, etc.**

$$\chi^2(\mathbf{p}) = \sum_{k=1}^{N_o} \frac{(\mathcal{O}_k^{theo.}(\mathbf{p}) - \mathcal{O}_k^{exp.})^2}{\Delta \mathcal{O}_k^2}$$

- $N_o$  is the number of observables
- The  $\mathcal{O}_k^{theo.}$  ( $\mathcal{O}_k^{exp.}$ ) stands for the calculated (exp.) values
- $\Delta \mathcal{O}_k$  represents the adopted errors.

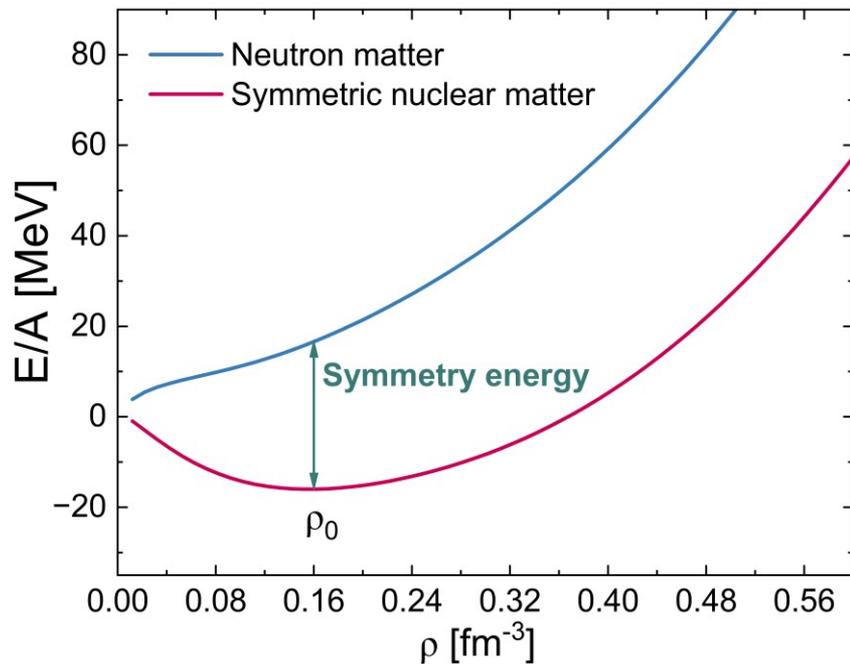
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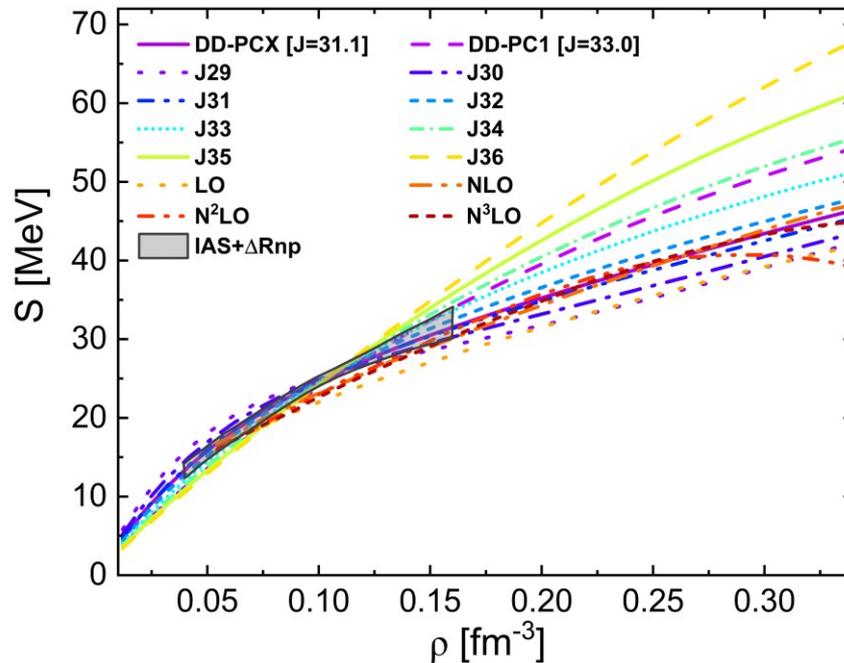
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- These observables are **not sufficient to constrain the isovector channel of the effective interaction**, which is particularly significant for neutron-rich nuclei, neutron skins, symmetry energy, and neutron stars.

**Symmetry energy is not well constrained!**  
We need relevant observables to constrain the isovector channel of the effective interactions!

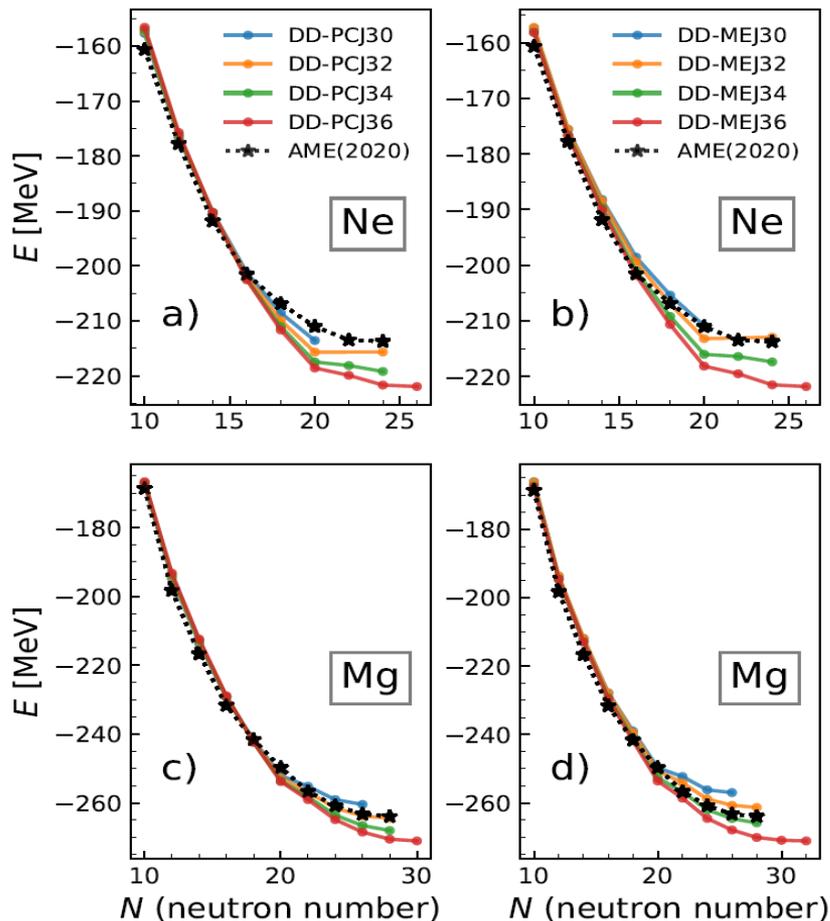


$$\frac{E}{A}(\rho, \delta) = \frac{E}{A}(\rho, 0) + S(\rho)\delta^2 + \mathcal{O}[\delta^4]$$

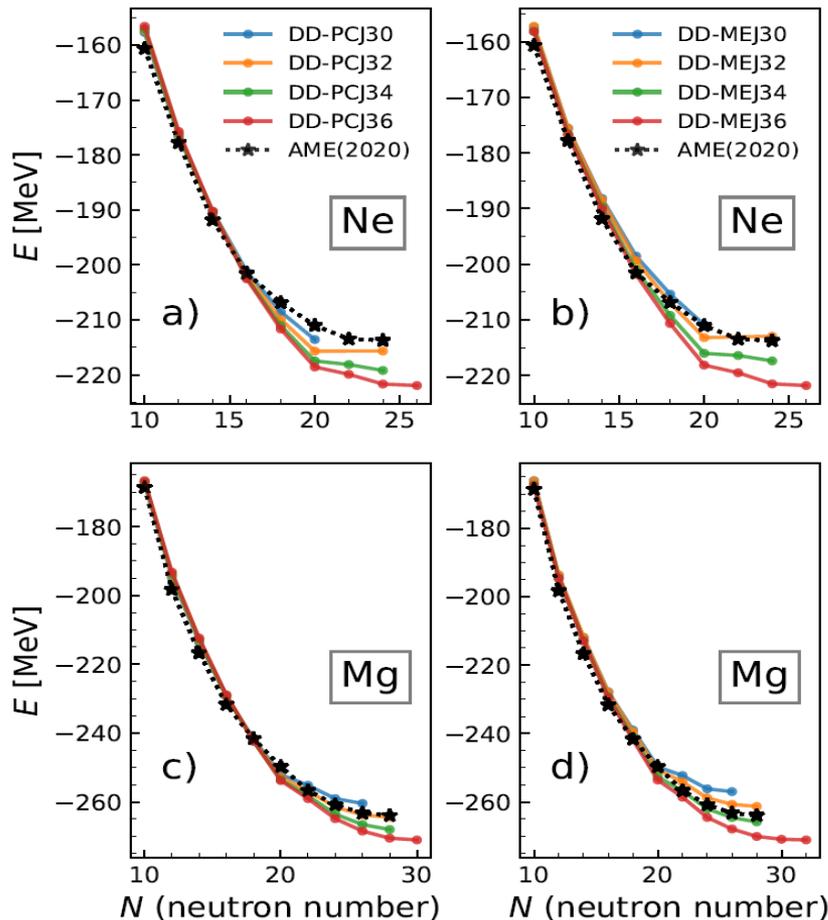


$$S(\rho) = J + L\left(\frac{\rho - \rho_0}{3\rho_0}\right) + \frac{1}{2}K_{sym}\left(\frac{\rho - \rho_0}{3\rho_0}\right)^2 + \mathcal{O}[(\rho - \rho_0)^3],$$

- E. Yüksel, T. Oishi, N. Paar, Universe, 7, 71 (2021).
- Chiral effective-field theory ( $\chi$ EFT): Drischler, C.; Furnstahl, R.J.; Melendez, J.A.; Phillips, D.R. Phys. Rev. Lett. 2020, 125, 202702.
- IAS +  $\Delta$ Rnp constraints: Danielewicz, P.; Lee, J. Symmetry energy II: Isobaric analog states. Nucl. Phys. A 2014, 922, 1–70.



- ✓ Axially-deformed RHB with the DD-PCJ and DD-MEJ effective interactions.
- ✓ The effect of increasing  $J$  on the binding energy is negligible for nuclei closer to the valley of stability.
- ✓ In general, **increasing  $J$  (L) makes nuclei more bound**, with a more pronounced effect towards the neutron drip line. Thus, the position of the two-neutron drip lines is also affected.



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### Drip line nuclei (exp.):

$^{34}\text{Ne}$  – favors  $J = 32, 34$  MeV

$^{40}\text{Mg}$  (?) – favors  $J = 32, 34$  MeV

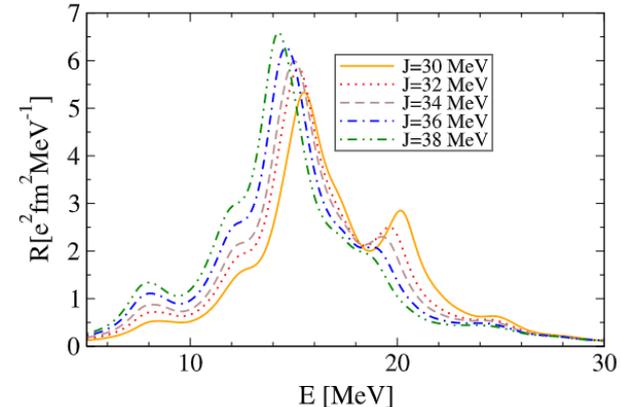
- Excitations in nuclei and EOS are related to each other!

**Nuclear collective excitations can provide important constraints for the EDFs!**

Isoscalar Excitations, **isovector excitations (dipole polarizability)**, charge-exchange modes

**DD-PCX ( $J = 31.12$  MeV and  $L = 46.32$  MeV)**

E. Yüksel, T. Marketin, and N. Paar Phys. Rev. C 99, 034318 (2019).



D. Vretenar, et.al, Phys. Rev. C, 68, 024310, 2003.

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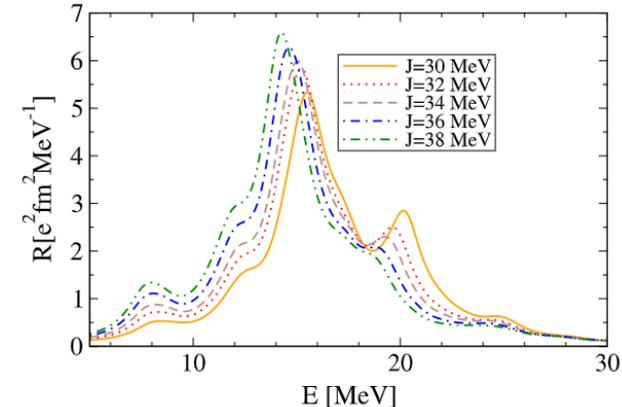
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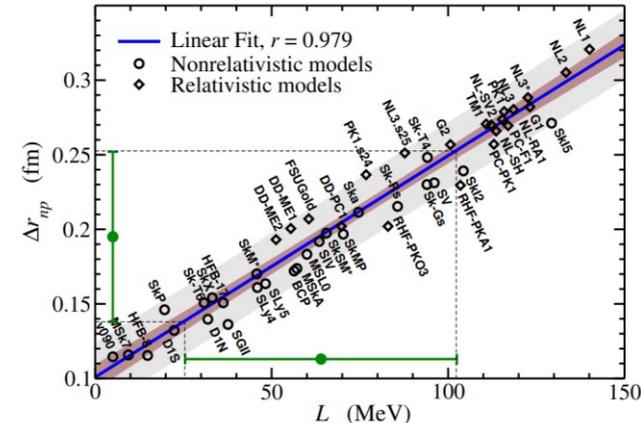
E. Yüksel, T. Marketin, and N. Paar Phys. Rev. C 99, 034318 (2019).

- The **neutron skin thickness**  $\Delta r_{np} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$  is one of the possible observables that could be used to probe the isovector channel of the EDFs.

**The neutron skin thickness  $\Delta r_{np} \rightarrow$  Experimental data has large uncertainties and model dependent!**



D. Vretenar, et al, Phys. Rev. C, 68, 024310, 2003.



X. Roca-Maza, et al, Phys. Rev. Lett. 106, 252501, 2011.

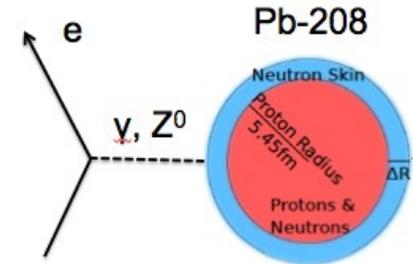
The recent precise parity-violating electron scattering experiments on  $^{48}\text{Ca}$  (CREX) and  $^{208}\text{Pb}$  (PREX-II) provide new insight into the neutron skin thickness in nuclei.

## $^{48}\text{Ca}$ (CREX)

- Neutron skin thickness is related to the details of the nuclear force
- Microscopic calculations are possible.

## $^{208}\text{Pb}$ (PREX)

- Provides a link to the EOS of neutron rich matter
- Laboratory to test neutron star structure



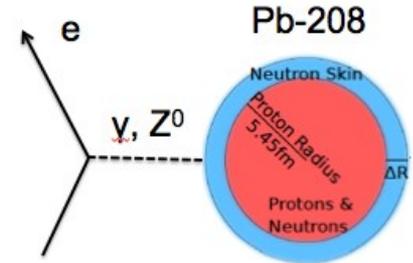
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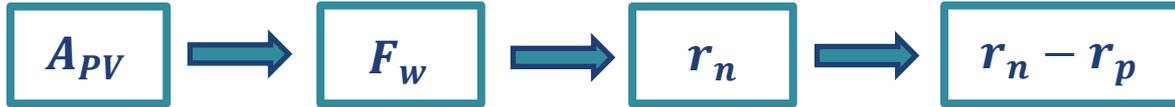
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## Parity-violating asymmetry



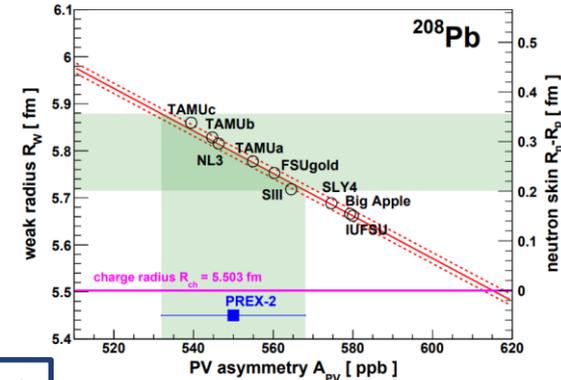
Directly accessible by experiment!

**PREX-II:  $\Delta R_{np}$  ( $^{208}\text{Pb}$ ) =  $0.283 \pm 0.071$  fm**

D. Adhikari et al., Phys. Rev. Lett., 126 (2021), 172502.

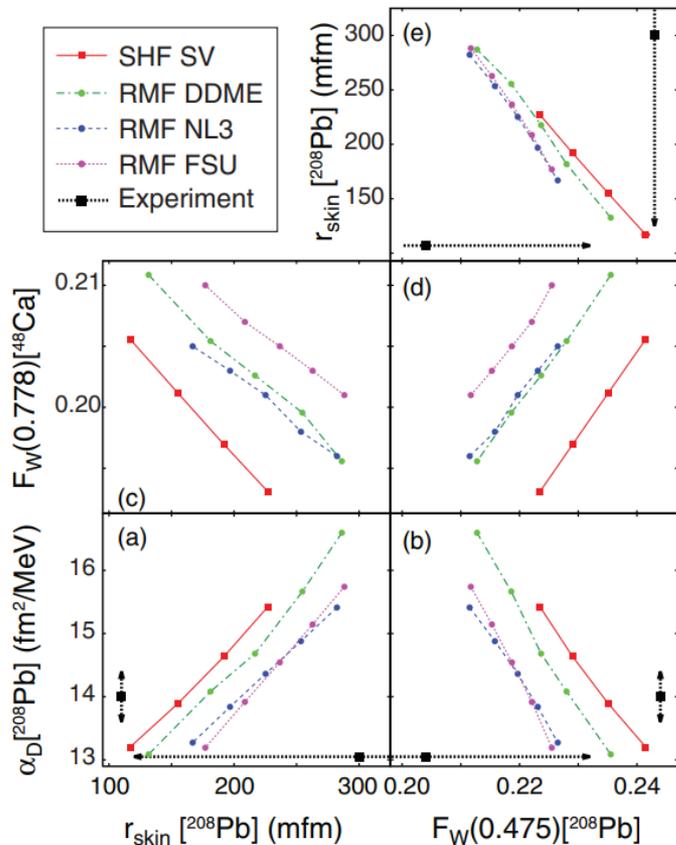
**CREX:  $\Delta R_{np}$  ( $^{48}\text{Ca}$ ) =  $0.121 \pm 0.026(\text{exp}) \pm 0.024(\text{model})$**

D. Adhikari et al. Phys. Rev. Lett. 129, 042501 (2022).



**Uncertainty is still large!**

# IS THERE ANY CORRELATION BETWEEN $F_W$ AND $\Delta R_{np}$ ?



Systematic trends for the following isovector observables: the neutron skin, electric-dipole polarizability, weak-charge form factors.

- A strong correlation has been found between the weak-form factor, dipole polarizability and the neutron skin.
- $F_W \rightarrow$  directly accessible by the experiment and has low uncertainties!

## $^{48}\text{Ca}$ (CREX)

$$F_W(0.8733 \text{ fm}^{-1}) = 0.1304 \pm 0.0052(\text{stat}) \pm 0.0020(\text{sys})$$

D. Adhikari et al., PRL 129, 042501 (2022).

## $^{208}\text{Pb}$ (PREX-II)

$$F_W(0.3978 \text{ fm}^{-1}) = 0.368 \pm 0.013$$

D. Adhikari et al., PRL 126, 172502 (2021).

S. Abrahamyan et al., PRL 108, 112502 (2012).

- Within the energy density functional (EDF) framework, we investigate the implications of CREX and PREX-II data on **nuclear matter symmetry energy and isovector properties of finite nuclei: neutron skin thickness and dipole polarizability.**

- **Ground-state properties of selected nuclei**

- Binding energies (34 nuclei)
- Charge radii (26 nuclei)
- Mean pairing gaps (15 nuclei) of the selected open-shell nuclei

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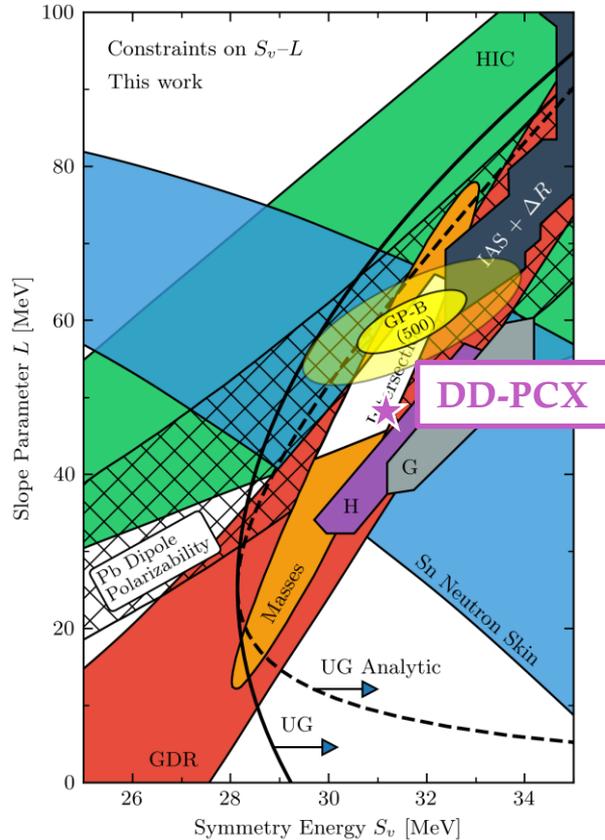
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## Three new relativistic point coupling interactions

**DDPC-CREX** – constrained by  $F_W$  ( $^{48}\text{Ca}$ )

**DDPC-PREX** – constrained by  $F_W$  ( $^{208}\text{Pb}$ )

**DDPC-REX** – constrained by  $F_W$  ( $^{48}\text{Ca}$ ) &  $F_W$  ( $^{208}\text{Pb}$ )

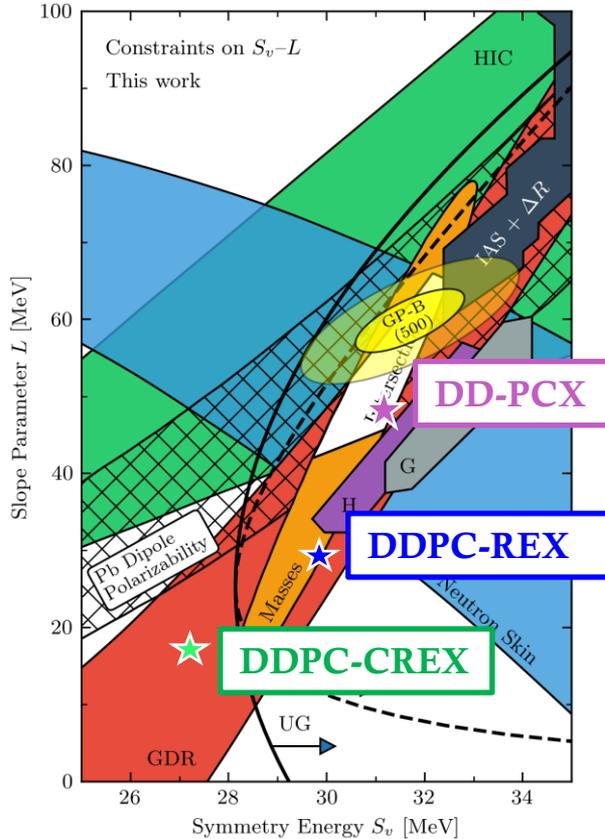


The nuclear matter properties at saturation density for the DDPC-CREX, DDPC-PREX, and DDPC-REX interactions. The properties for the DD-PC1 [20] and DD-PCX [21] interactions are also given for comparison. The uncertainties of the obtained values are provided within the parenthesis.

	$E/A$ (MeV)	$m_D^*/m$	$K_0$ (MeV)	$J$ (MeV)	$L$ (MeV)
DDPC-CREX	-15.989(16)	0.5672(5)	225.48(1.55)	27.01(23)	19.60(1.01)
DDPC-PREX	-16.108(19)	0.5680(7)	235.41(2.42)	36.18(0.80)	101.78(9.34)
DDPC-REX	-16.019(16)	0.5696(5)	242.95(76)	28.86(0.33)	30.03(2.06)
DD-PC1	-16.061	0.580	230.0	33.0	70.1
DD-PCX	-16.026(18)	0.5598(8)	213.03(3.54)	31.12(32)	46.32(1.68)

\* DD-PCX: Constrained using nuclear collective excitations!

## ★ DDPC-PREX



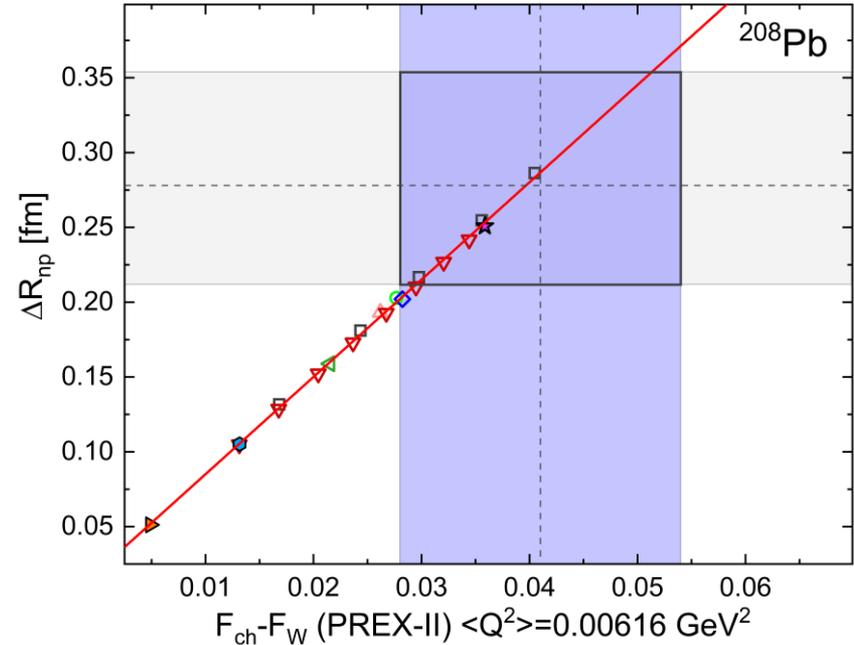
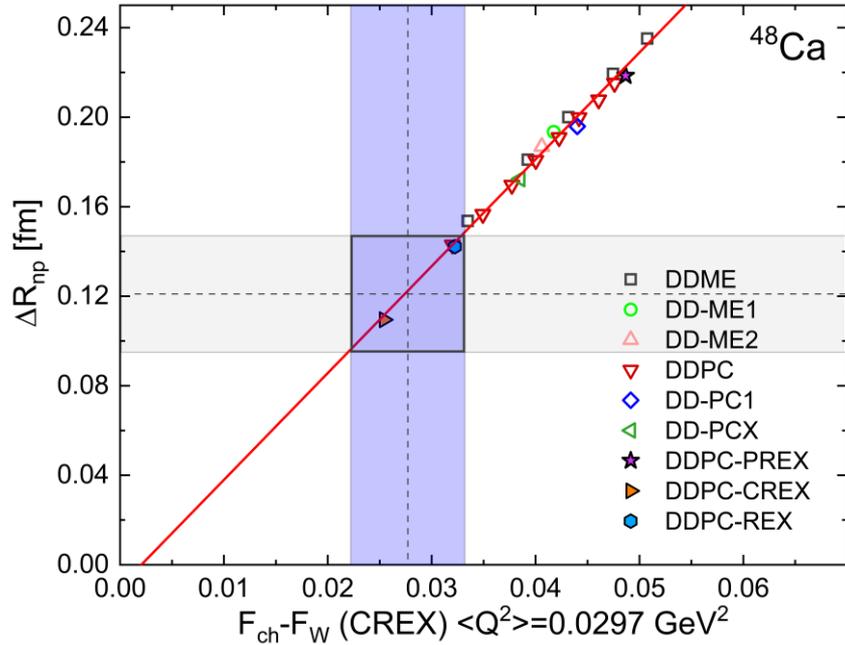
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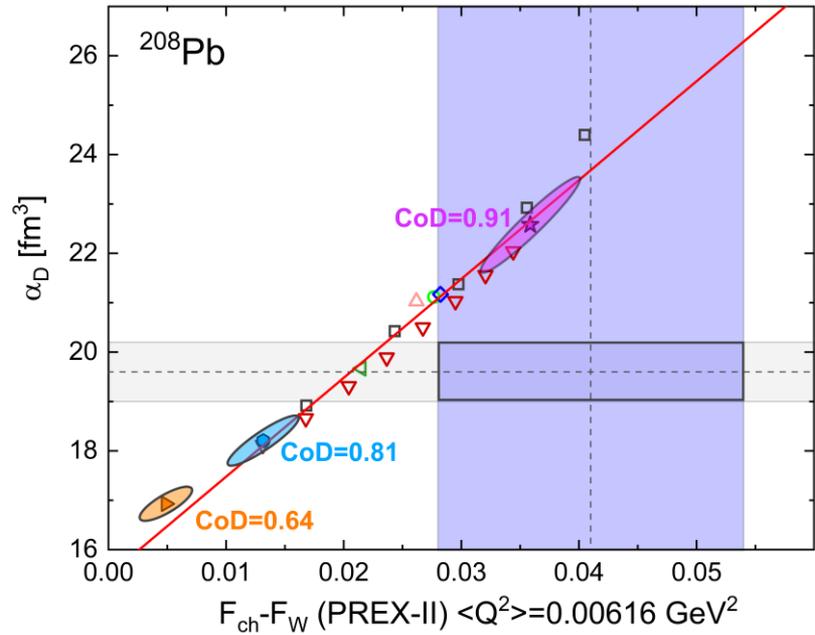
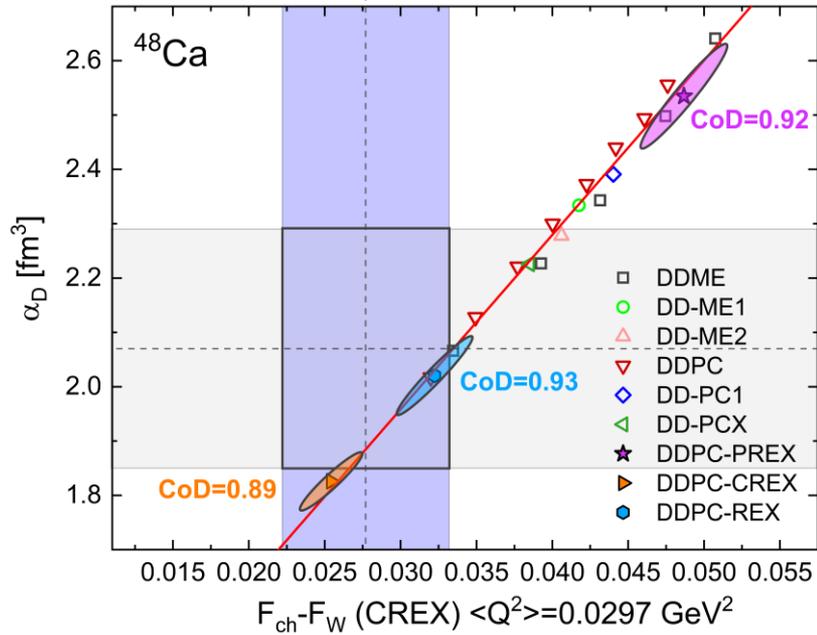
\* DD-PCX: Constrained using nuclear collective excitations!

! The symmetry energy parameters for DDPC-CREX and DDPC-PREX interactions are outside rather broad ranges of their values obtained in different studies.

Esra Yüksel and Nils Paar, Physics Letters B, 836, 137622, 2023.



- All interactions that have previously been established as very successful in describing nuclear properties remain outside the experimental ranges!
- **The new RNEDFs cannot provide a description for  $^{48}\text{Ca}$  and  $^{208}\text{Pb}$  simultaneously!**



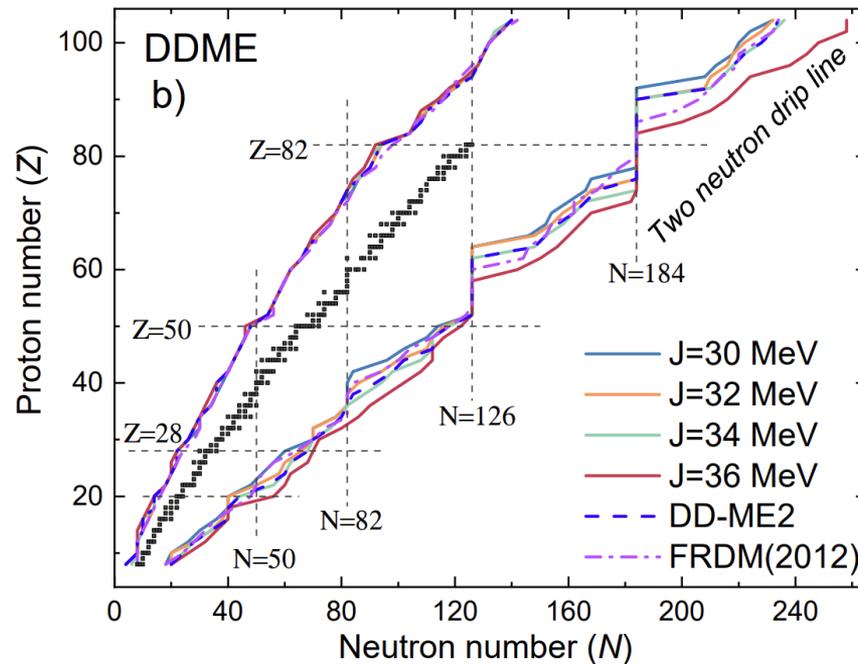
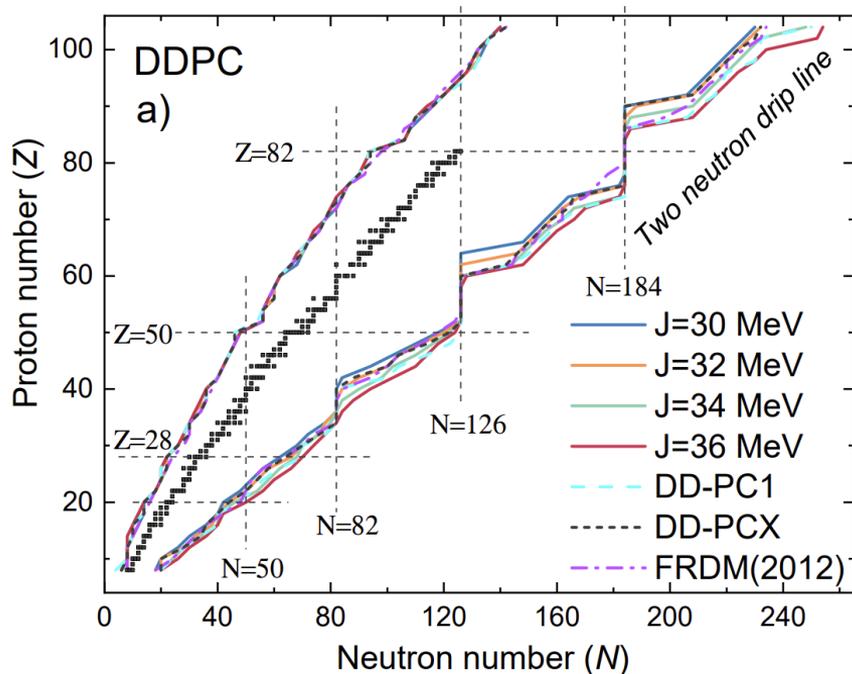
The dipole polarizability  $\alpha_D$  of  $^{48}\text{Ca}$  and  $^{208}\text{Pb}$  as a function of the form factor difference  $F_{ch} - F_W$  using relativistic EDFs.

**We implemented the recent PREX and CREX experimental data in the optimization of the RNEDFs.**

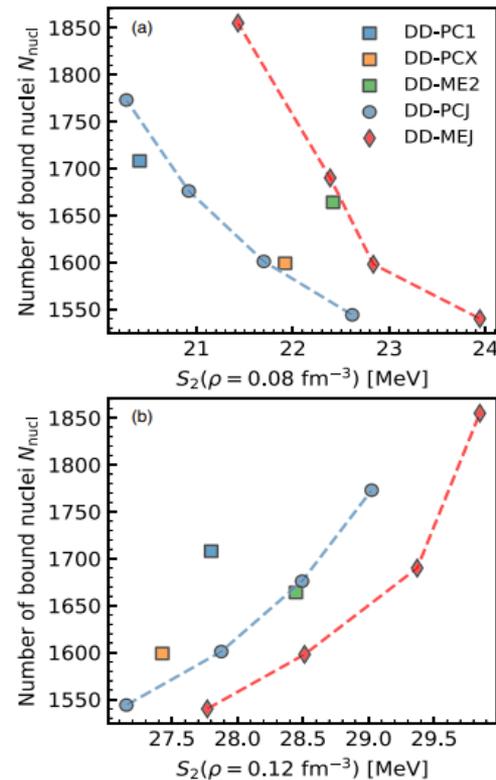
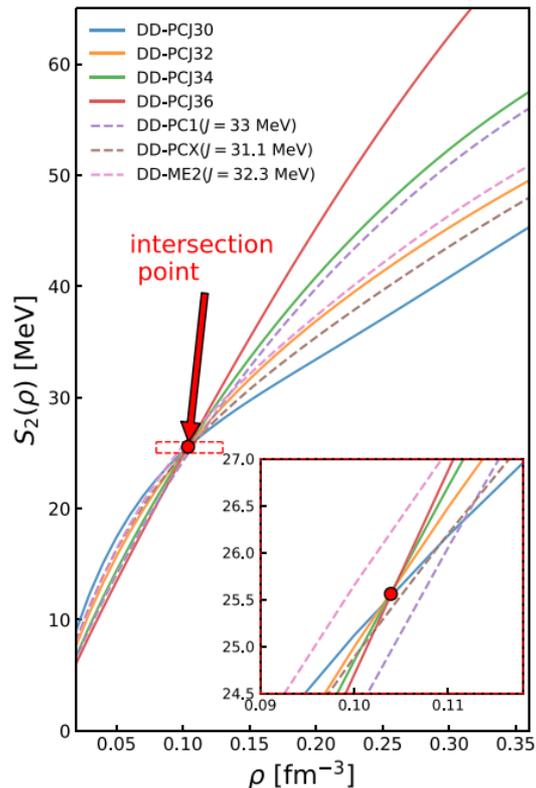
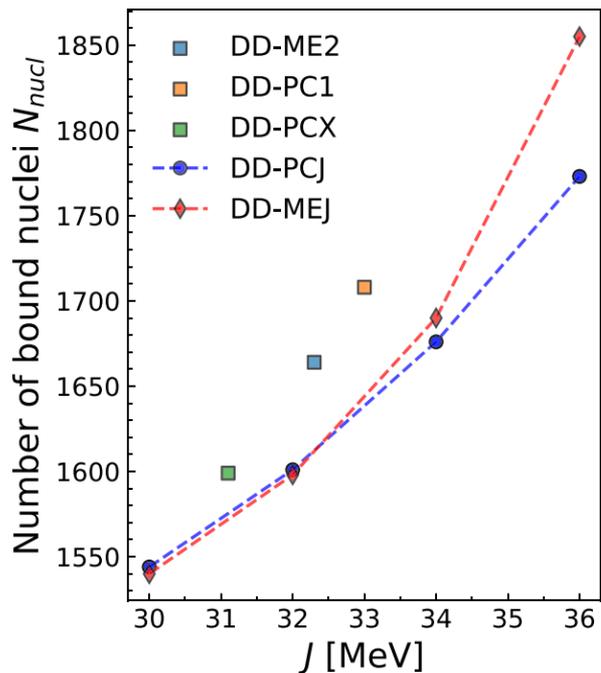
- ✓ The optimization of the isovector channel of the REDFs is of utmost importance to better describe the nuclear properties, especially away from the stability line.
- ✓ DDPC-CREX, DDPC-PREX, DDPC-REX functionals established using the nuclear ground state properties + weak form factors from parity violating electron scattering experiments on  $^{48}\text{Ca}$  (CREX) and  $^{208}\text{Pb}$  (PREX II).
- ✓ Presented analysis shows that CREX and PREX-II experiments could not provide consistent constraints for the isovector sector of the EDFs, and further theoretical and experimental studies are required.
- ✓ Further EDF and ab-initio studies, alongside with novel experimental investigations (Parity violation at MESA Mainz?) are needed to resolve the current puzzling implications of the parity violating electron scattering data...

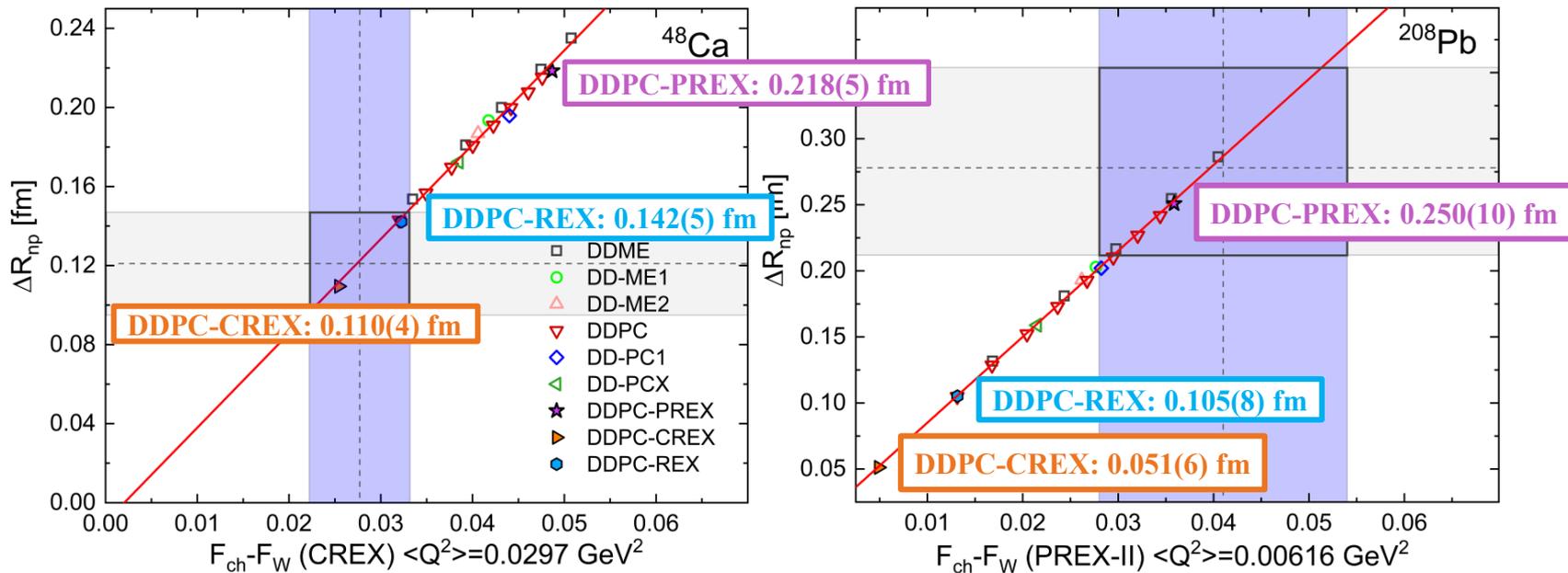
THANKS FOR LISTENING  
Any questions?

In Collaboration with  
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The impact of increasing  $J$  on the proton drip line is negligible. However, the two-neutron drip line shifts systematically towards a higher neutron number as  $J$  increases.





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