Nuclear clustering - from the edge of stability to the Fermi energy domain

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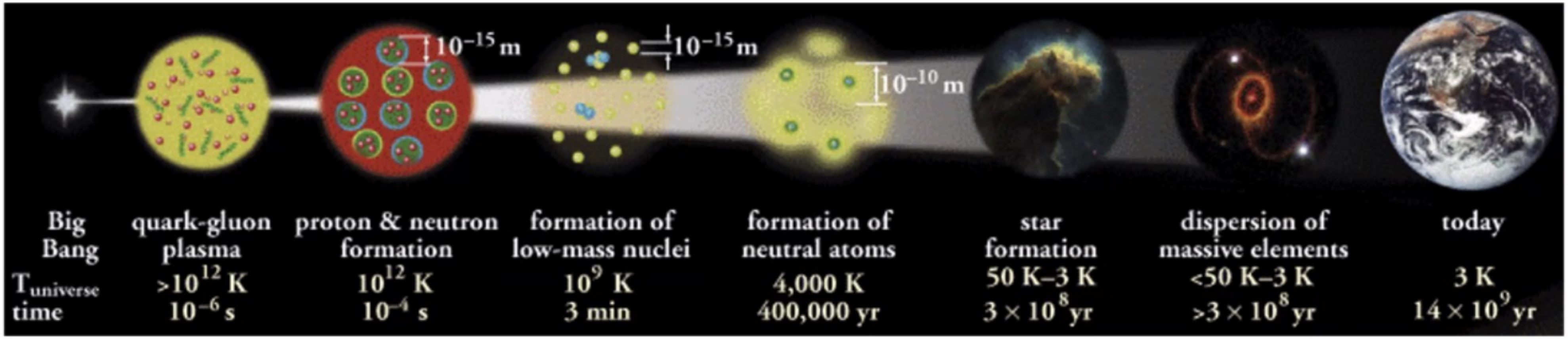
Clustering and fragment production

Clustering is *ubiquitous* in Nature and clearly one of the most *mysterious* processes in Physics. It happens at all scales in time, distances and energies: from the microscopic scales of hadrons and nuclei to the macroscopic scales of living organisms and clusters of galaxies, from the high excitation energies to cold systems

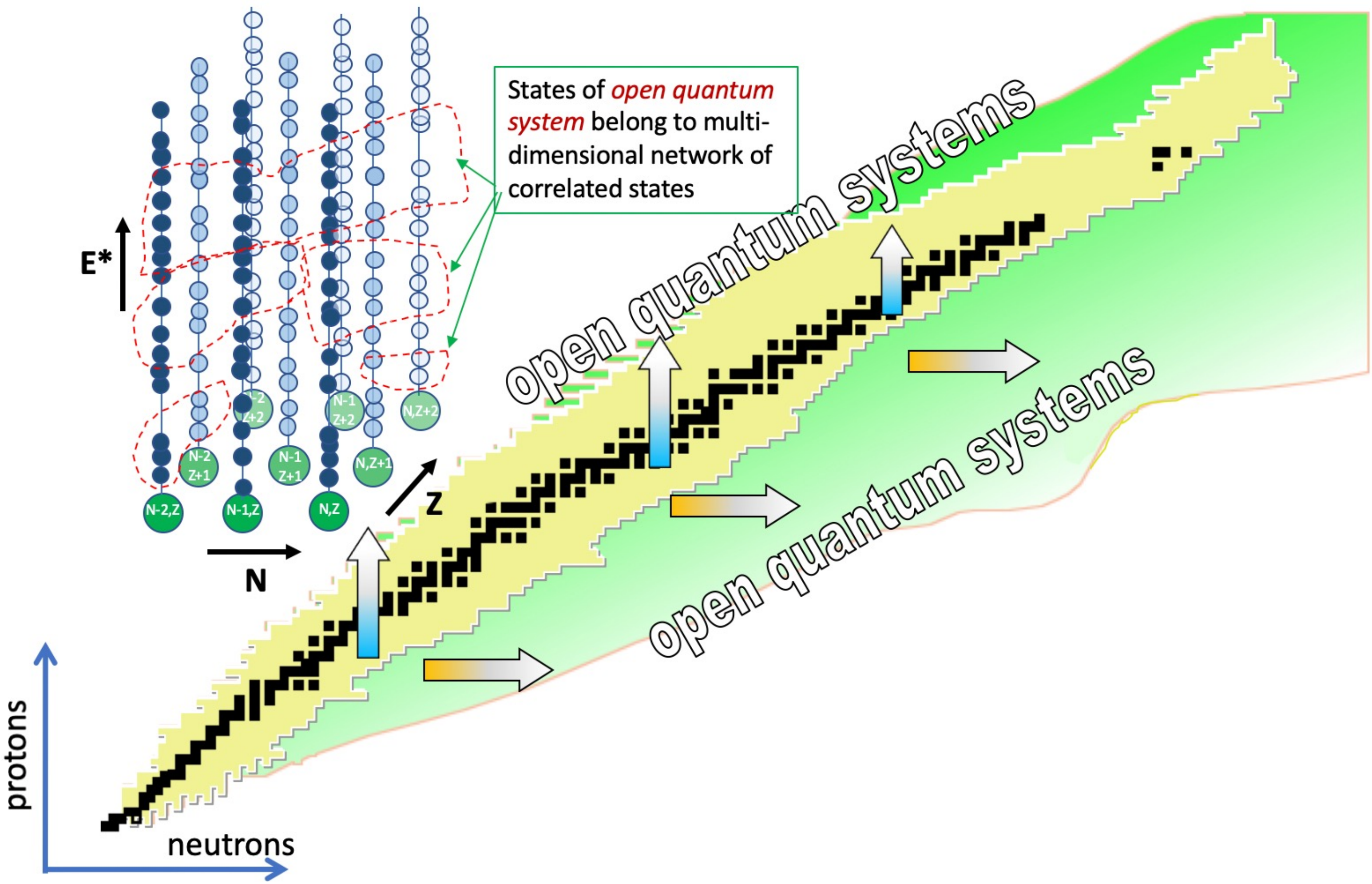


There are many specific reasons for the cluster production but there are also few *generic mechanisms* of the clusterization, independent of individual features of the studied system:

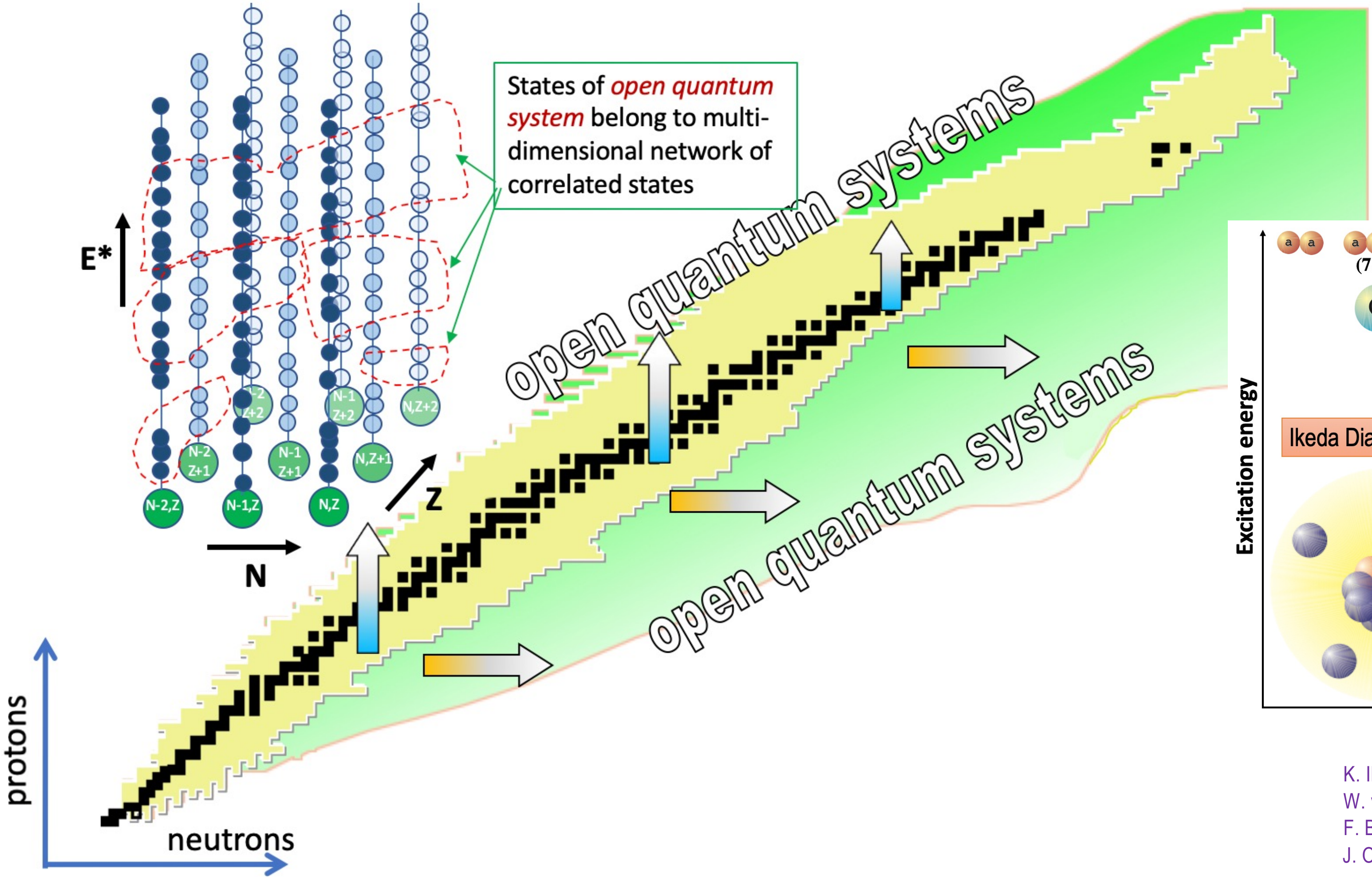
- *statistical mechanism* rooted in the Central Limit Theorem
- mimicry mechanism due to the interaction between the system and its environment
- ...

Quantal regime of clustering

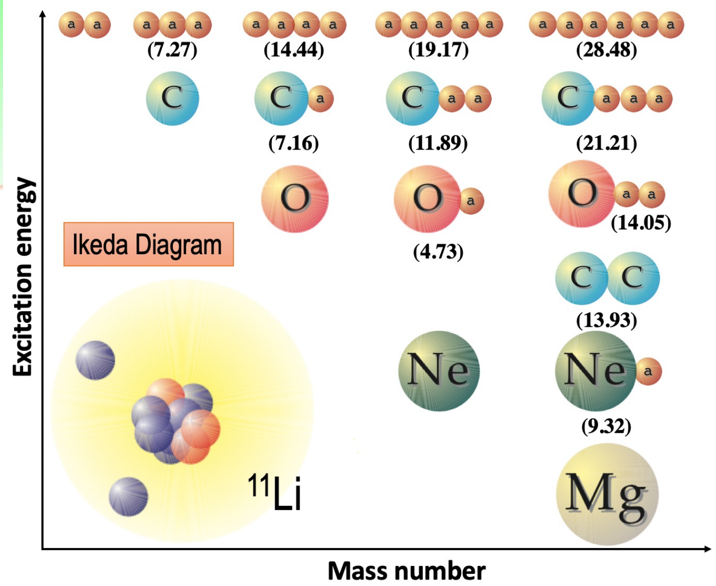
Atomic nucleus: the open quantum system



Atomic nucleus: the open quantum system

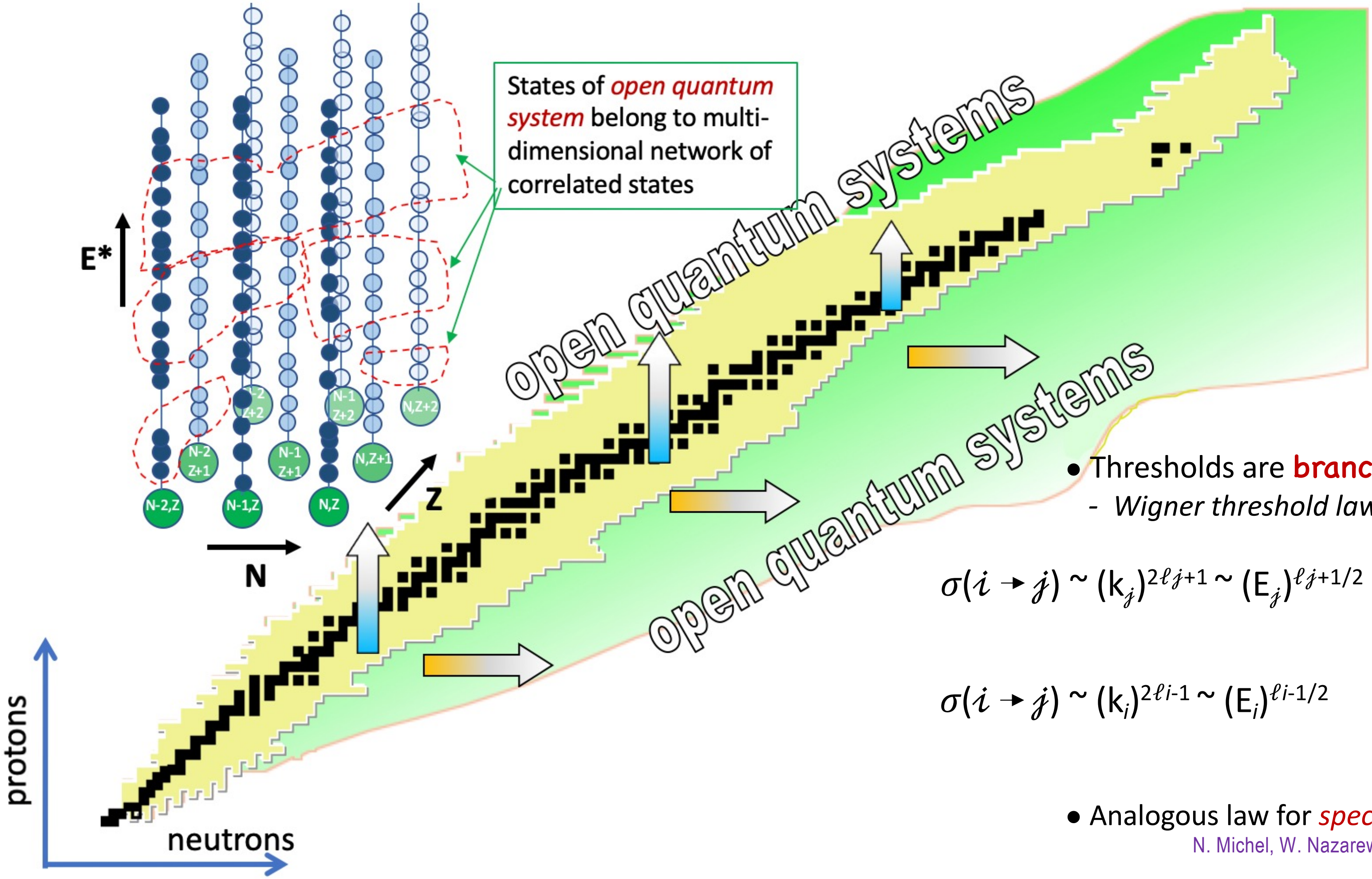


Near-threshold clustering



K. Ikeda, et al., Prog. Theor. Phys. Suppl. E68, 464 (1968)
W. von Oertzen et al., Eur. Phys. J. A 46, 345 (2010)
F. Barker, Proc. Phys. Soc. 84, 681 (1964)
J. Okołowicz et al., Prog. Theor. Phys. Suppl. 196, 230 (2012); Fortschr. Phys. 61, 66 (2013)

Atomic nucleus: the open quantum system



- Thresholds are **branching points** ➡ *nonanalytic behavior*
  - *Wigner threshold law* for elastic and total cross-sections

E.P. Wigner, Phys. Rev. 73, 1002 (1948)

$\sigma(i \to j) \sim (k_j)^{2\ell_j+1} \sim (E_j)^{\ell_j+1/2}$ for endoergic reactions: the production of slow neutral particles

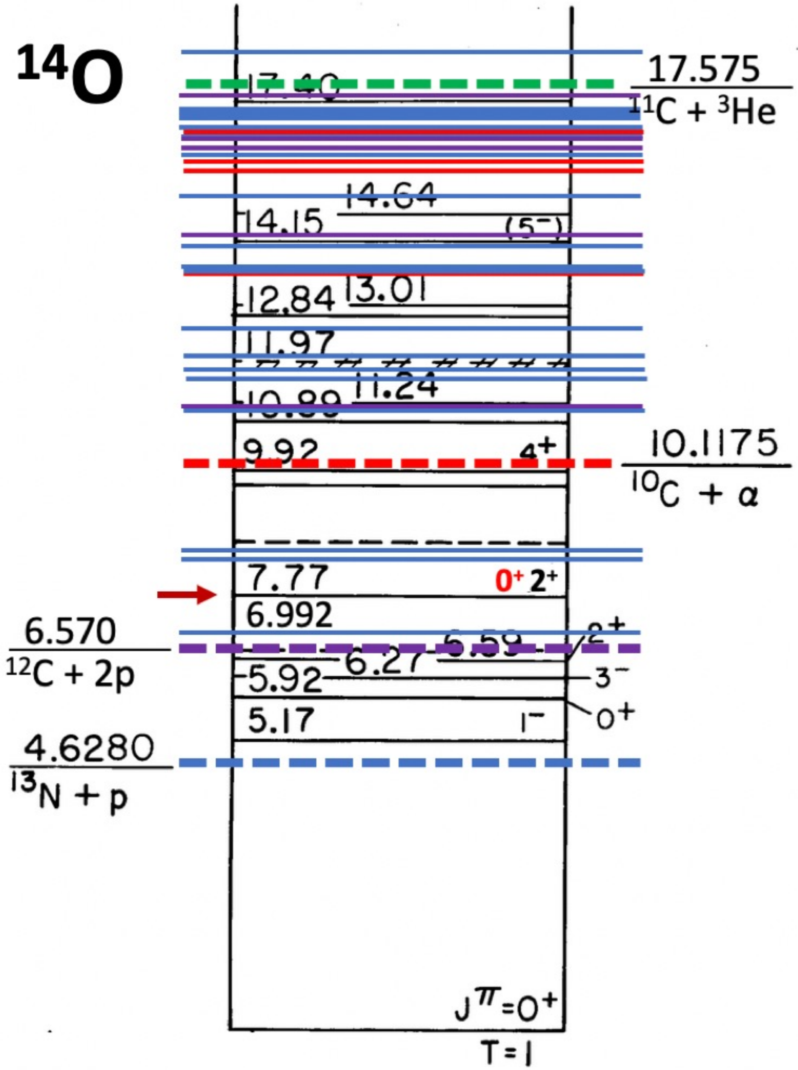
$\sigma(i \to j) \sim (k_i)^{2\ell_i-1} \sim (E_i)^{\ell_i-1/2}$ for exoergic reactions: the absorption of slow neutral particles

- Analogous law for *spectroscopic factors*

N. Michel, W. Nazarewicz., M. Płoszajczak, Phys. Rev. C(R) 75, 031301 (2007)

➡ Shell model for *open* quantum systems

Atomic nucleus: the open quantum system



- Nuclear states are *embedded in the scattering continuum*

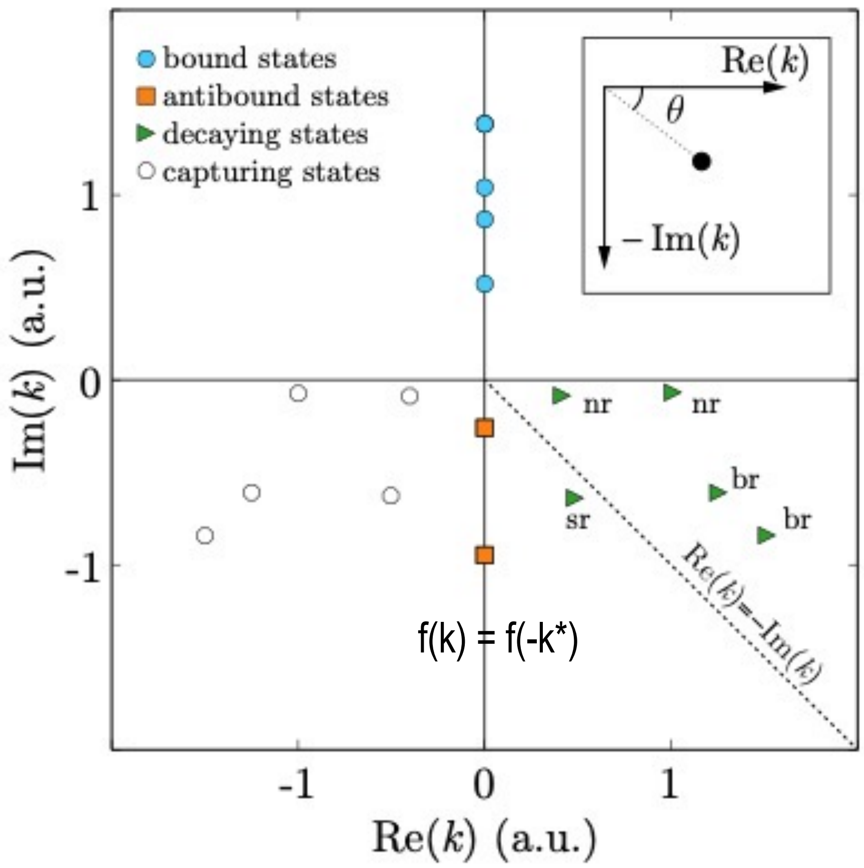
- Couplings to various particle emission channels are crucial for the properties of near-threshold states

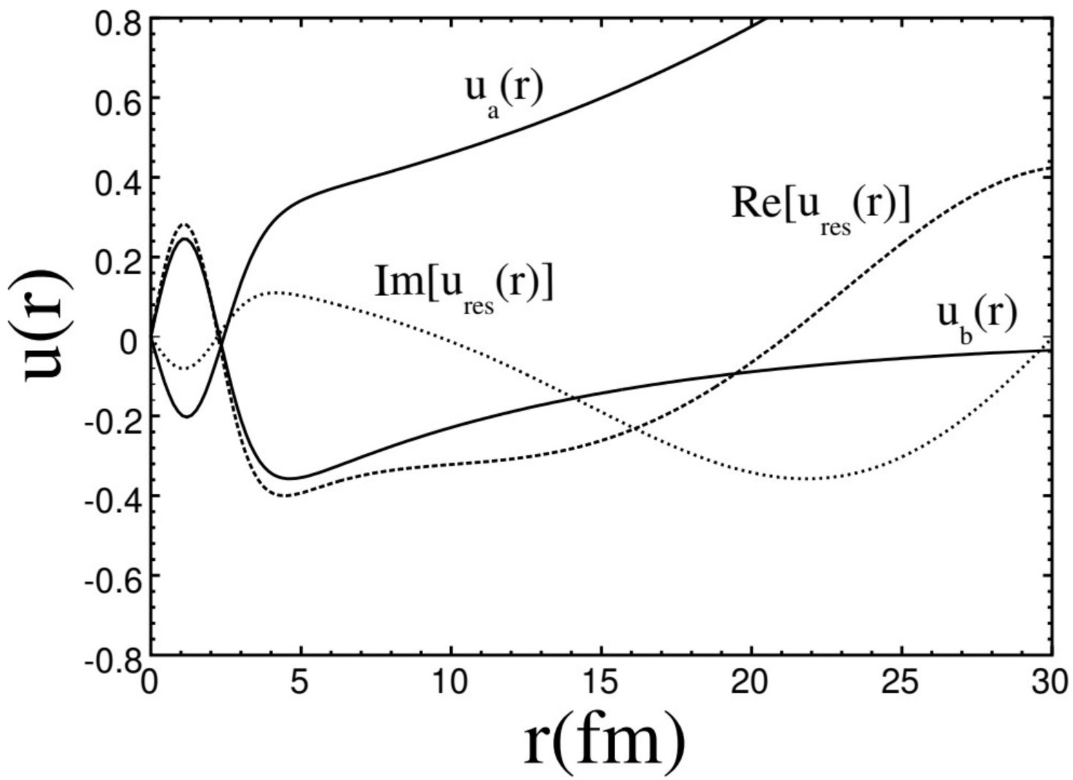
- **Unitarity** is the fundamental property of QM yet 'mainstream' nuclear theory describes nucleus in *unitarity violating schemes*

⟹ *'Unitarity crisis'* in nuclear theory

Shell model for open quantum systems

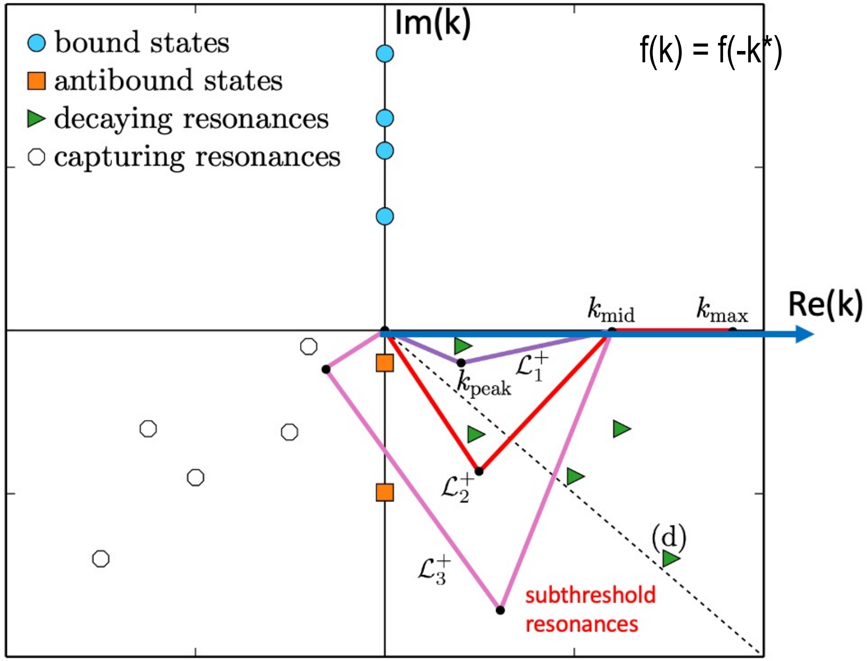
Gamow poles: Quasi-stationary extension in the complex k-plane





- Bound and resonance states: same object
- Same behavior inside the nucleus
- Asymptote is different for bound, virtual, and resonance states

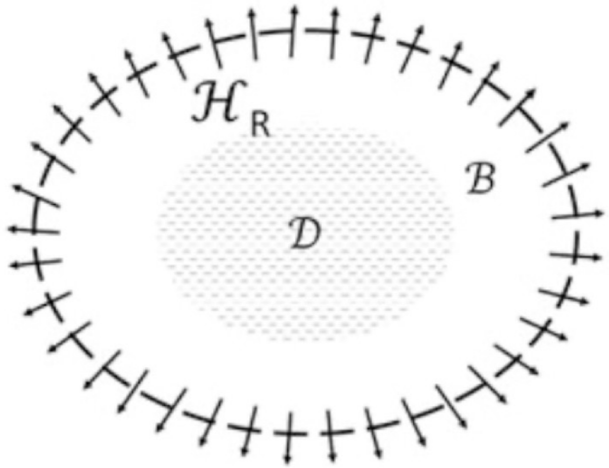
Hermitian QM in rigged Hilbert space



$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k|dk = 1 \; ; \; \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

T. Berggren, Nucl. Phys. A109, 265 (1968)
T. Lind, Phys. Rev. C47, 1903 (1993)

N. Michel et al, PRL 89 (2002) 042502
R. Id Betan et al, PRL 89 (2002) 042501
N. Michel, et al, J. Phys. G37 (2010) 064042
N. Michel, M. Płoszajczak,
«*Gamow Shell Model: The Unified Theory of Nuclear Structure and Reactions* »
Lecture Notes in Physics, Vol. 983 (2021)



Gamow shell model (GSM)

$$|SD_i\rangle = |u_{i_1} \ldots u_{i_A}\rangle \Rightarrow \sum_k |SD_k\rangle\langle SD_k| \cong 1$$

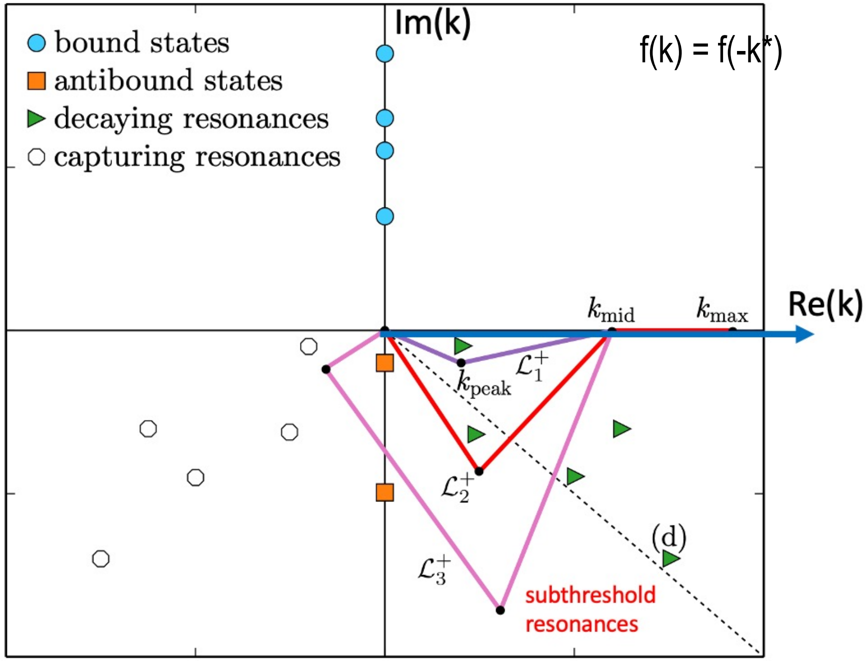
$$H \rightarrow [H]_{ij} = [H]_{ji}$$

Rigged Hilbert Space inner product

$$\langle\tilde{u}_n|u_n\rangle = \int_0^\infty dr \tilde{u}_n^*(r) u_n(r)$$

- Unitary formulation of the nuclear Shell Model
- Complex-symmetric eigenvalue problem
- No identification of reaction channels

Hermitian QM in rigged Hilbert space



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$$|SD_i\rangle = |u_{i_1}\ldots u_{i_A}\rangle \Rightarrow \sum_k |SD_k\rangle\langle SD_k| \cong 1$$

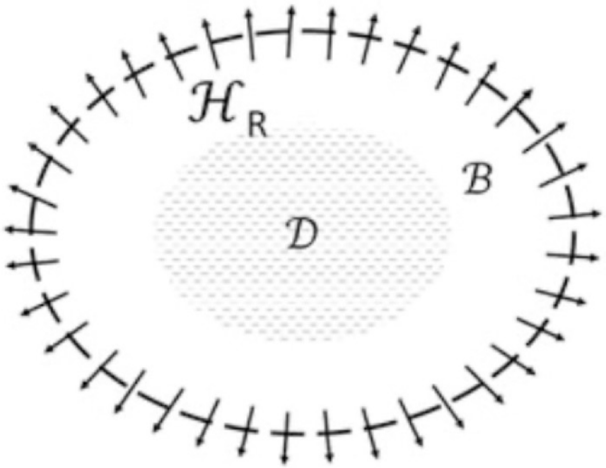
T=0

- **np** bound state (deuteron): k=+i0.2315 fm^{-1}

T=1

- **np** virtual state (deuteron): k=-i0.044 fm^{-1}
- **nn** virtual state: k=-i0.0559(33) fm^{-1}
  V.A. Babenko, N.M. Petrov, Phys. At. Nucl. 76, 684 (2013)
- **pp** threshold resonant state: k=(0.0647-i0.0870) fm^{-1}
  L.P. Kok, Phys. Rev. Lett. 45, 427 (1980)

Hermitian QM in rigged Hilbert space



Y. Jaganathen et al, Phys. Rev. C 88, 044318 (2014)
K. Fossez et al., Phys. Rev. C 91, 034609 (2015)
A. Mercenne et al., Phys. Rev. C 99, 044606 (2019)

N. Michel, M.Płoszajczak,
«*Gamow Shell Model: The Unified Theory of Nuclear Structure and Reactions* »
Lecture Notes in Physics, Vol. 983, (Springer Verlag, 2021)

GSM – Coupled-channel representation

$$|\Psi^J_M\rangle = \sum_{\mathrm{c}} \int_0^{+\infty} |(\mathrm{c},r)^J_M\rangle \, \frac{u_{\mathrm{c}}^{JM}(r)}{r} r^2 \, dr$$

$$|(\mathrm{c},r)\rangle = \hat{\mathcal{A}}[|\Psi_{\mathrm{T}}^{J_{\mathrm{T}}}; N_T, Z_T\rangle \otimes |r \; L_{\mathrm{CM}} \; J_{\mathrm{int}} \; J_{\mathrm{P}}; n, z\rangle]^J_M$$

$$H\,|\Psi^J_M\rangle = E\,|\Psi^J_M\rangle \;\rightarrow\; \sum_{\mathrm{c}} \int_0^{\infty} r^2 \left(H_{\mathrm{cc'}}(r,r') - E N_{\mathrm{cc'}}(r,r')\right) \frac{u_{\mathrm{c}}(r)}{r} = 0$$

$$H_{\mathrm{cc'}}(r,r') = \langle(\mathrm{c},r)|\,\hat{H}\,|(\mathrm{c}',r')\rangle$$

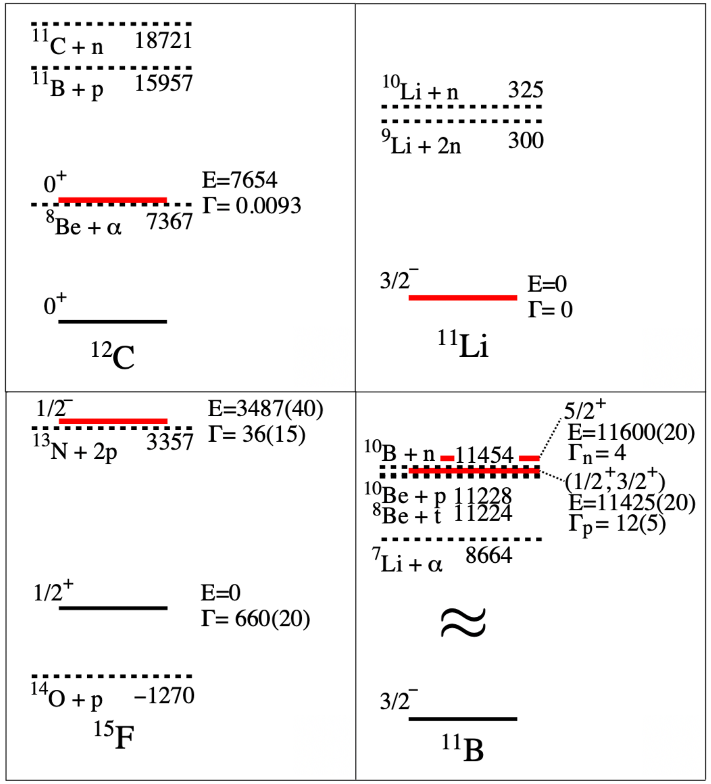
$$N_{\mathrm{cc'}}(r,r') = \langle(\mathrm{c},r)|(\mathrm{c}',r')\rangle$$

- Entrance and exit reaction channels defined
  - → Unification of nuclear structure and reactions
- Calculation in relative coordinates of core cluster orbital shell model coordinates
- Center-of-mass handled by recoil term in the Hamiltonian
- Scattering wave functions are the many-body states
- Antisymmetry handled
- Reaction channels with different (binary) mass partitions
- Core is arbitrary

Near-threshold states and origin of clustering

α-clustering "…α-cluster states can be found in the proximity of α-particle decay threshold…" K. Ikeda, N. Takigawa, H. Horiuchi (1968)

But this is only the tip of the iceberg!



- Other cases: ^{6}He, ^{6}Li, ^{7}Be, ^{7}Li, ^{11}O, ^{11}C, ^{17}O, ^{20}Ne, ^{26}O, ^{24}Mg,…
- *Various clusterings*: ^{2}H, ^{3}He, ^{3}H, 2p, 2n
- *Astrophysical relevance* of near-threshold resonances for α- and proton-capture reactions of nucleosynthesis

- *'Fortuitous'* appearance of correlated states close to open channels?
  ➡ They cannot result from any particular feature of the NN interaction or any dynamical symmetry of the nuclear many-body problem

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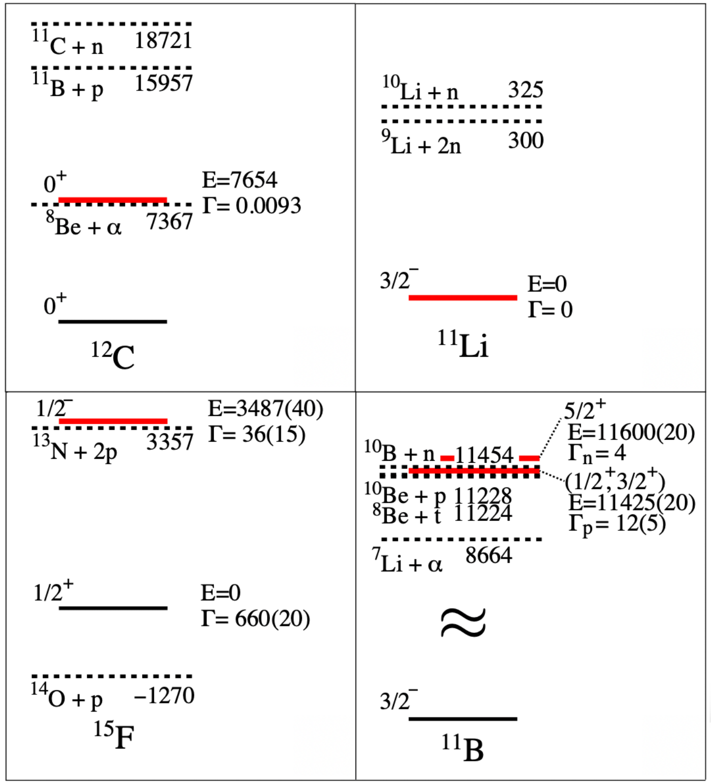

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neutral particles

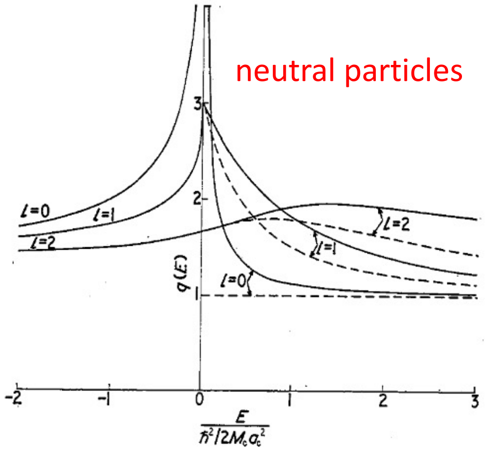

Figure 1. Enhancement factors for neutron channels with orbital angular momenta l = 0, 1 and 2 and reduced widths $\gamma_{\lambda c}{}^2 = \hbar^2/M_c a_c{}^2$ as functions of channel energy E (in units of $\hbar^2/2M_c a_c{}^2 \simeq 1$ MeV). Full curves give values of $q(E)$, broken curves values of $q_1(E)$.

charged particles

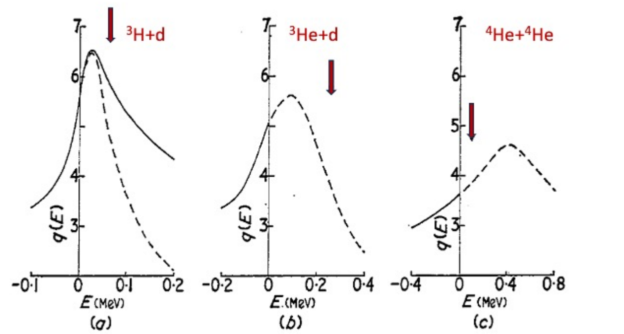

Figure 2. Enhancement factors for channels (a) ^{3}H + d, (b) ^{3}He + d, (c) ^{4}He + ^{4}He, all with l = 0 and with values of a_c and $\gamma_{\lambda c}{}^2$ given in the text. Full curves give values of $q(E)$, broken curves values of $q_1(E)$. Arrows indicate energies of observed levels of ^{5}He, ^{5}Li and ^{8}Be.

F. Barker, Proc. Phys. Soc. 84, 681 (1964)

- The appearance of near-threshold resonances can be explained in terms of the increased density of levels that have large reduced width
- The enhancement of the level density is largest for low-barrier potentials, i.e., for low - ℓ partial waves

Near-threshold states and origin of clustering

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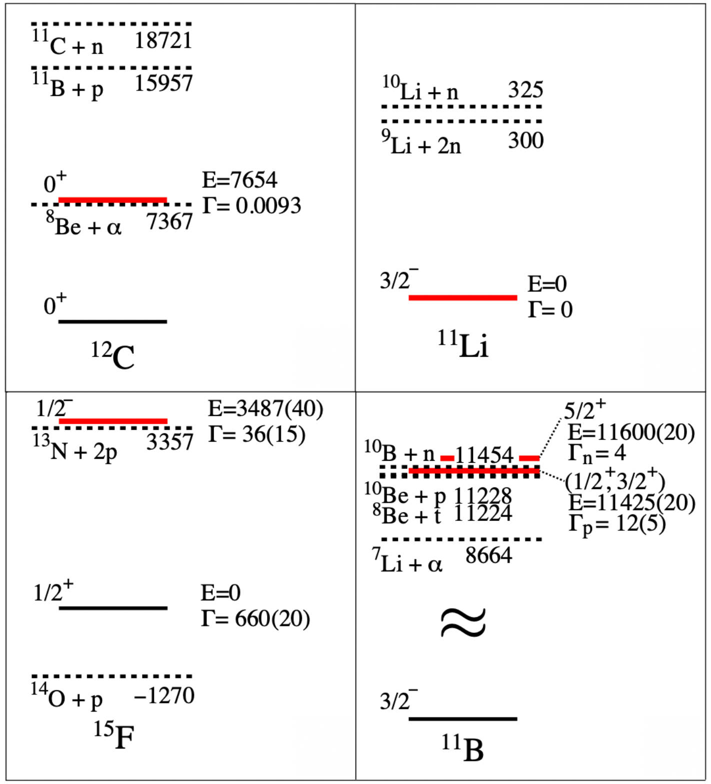

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Continuum shell model perspective

J. Okołowicz, M. P., W. Nazarewicz, Prog. Theor. Phys. Suppl. 196, 230 (2012); Fortschr. Phys. 61, 66 (2013)

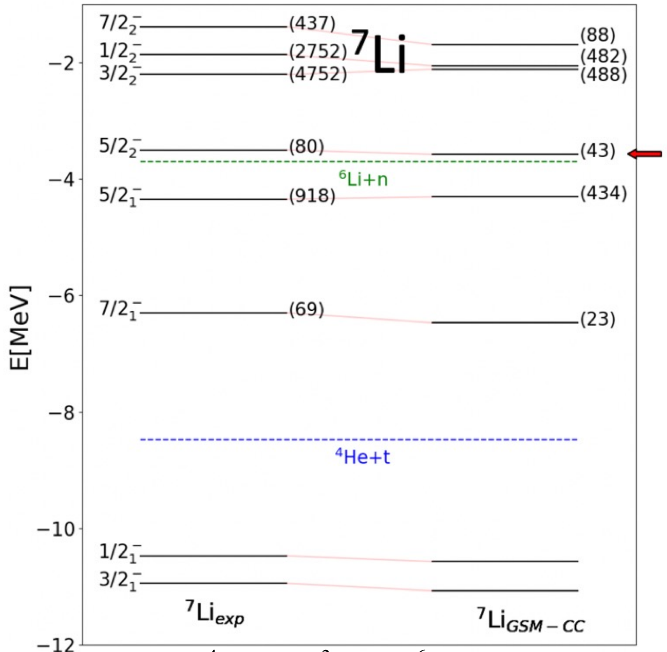
- The appearance of correlated (cluster) states close to open channels is the generic *open quantum system phenomenon* related to the collective rearrangement of SM wave functions due to the coupling via the continuum

- Specific aspects:
  - Energetic order of particle emission thresholds depends on (nuclear) Hamiltonian
  - Absence of stable cluster entirely composed of like nucleons

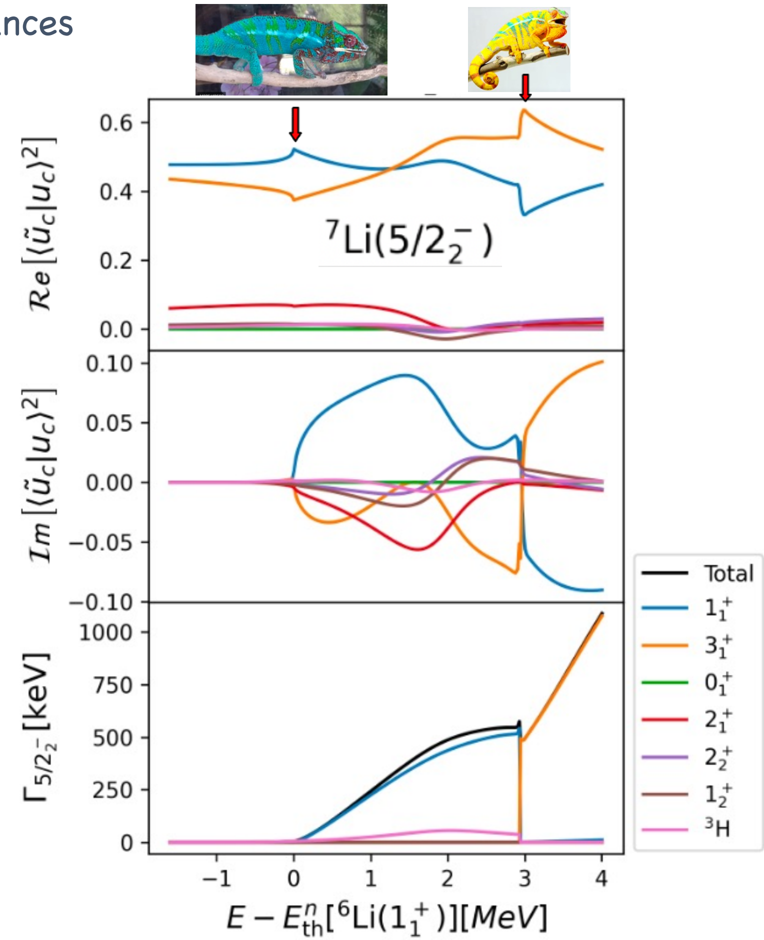
- With increasing strength of Coulomb potential, the near-threshold clustering becomes weaker and moves to higher energies

Mimicry mechanism of clusterization

Chameleon nature of resonances



$[|^4\mathrm{He}\rangle \otimes |^3\mathrm{H}\rangle], [|^6\mathrm{Li}\rangle \otimes |n\rangle]$



- Hamiltonian: 1-body potential, 2-body FHT interaction

  H. Furutani et al, Prog. Theor. Phys. 62, 981 (1979)

  ^{3}H wave functions calculated using N^{3}LO$_{(2\text{-body})}$ interaction

- Channels: ^{6}Li(K^π): $K^\pi = 1_1^+, 1_2^+, 3_1^+, 0_1^+, 2_1^+, 2_2^+$

  n: $\ell_j = s_{1/2}, p_{1/2}, p_{3/2}, d_{3/2}, d_{5/2}, f_{5/2}, f_{7/2}$

  ^{3}H(L): $L \equiv {}^{2J_{int}+1}[L_{CM}]_{JP} = {}^2S_{1/2}, {}^2P_{1/2}, {}^2P_{3/2}, {}^2D_{3/2}, {}^2D_{5/2}, {}^2F_{5/2}, {}^2F_{7/2}$

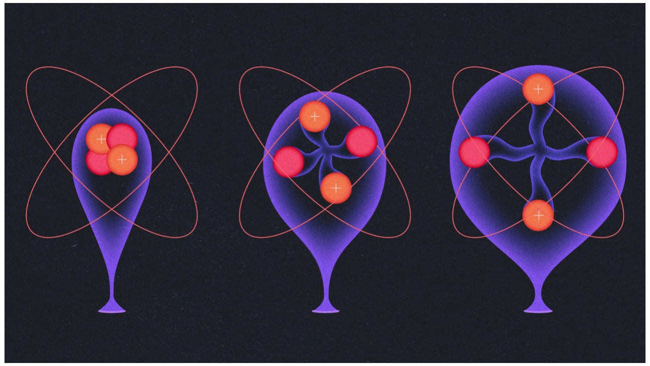
- The resonance (*chameleon*) changes its structure (*skin color*) as a result of the alignment (*mimicry*) with the nearby new reaction channel (*changing environment*)

  J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

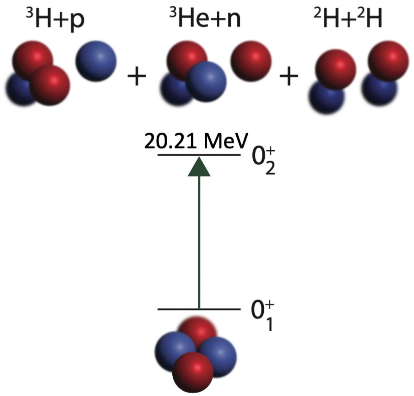
Mimicry mechanism of clusterization

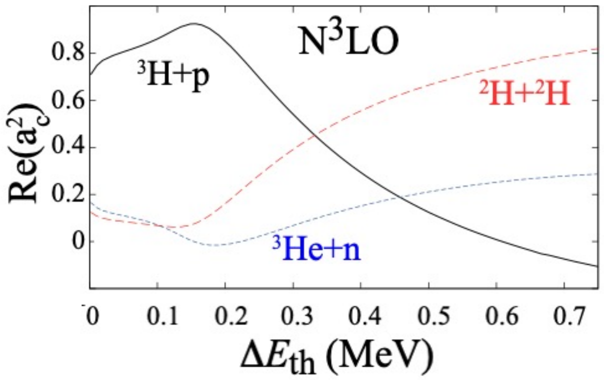
Structure of 0^+ resonance of the α particle

Is the first excited state of ^{4}He inflating like a balloon?



or it is the cluster state?



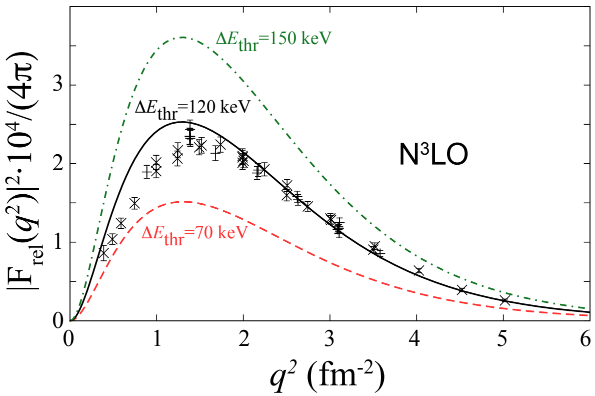


N. Michel, W. Nazarewicz, M. Ploszajczak, PRL 131, 242502 (2023)

- Strong continuum coupling between [t + p], [^{3}He + n], [d + d] reaction channels

- First excited state of ^{4}He is an aligned state dominated by the [t + p] channel

- Monopole form factor is fairly sensitive to interactions, threshold positions and resonance energy



Exp.: S. Kegel et al., PRL 130, 152502 (2023)

$$F_{\rm rel}(q^2) = \left(\frac{4\pi\sqrt{f_p(q^2)}}{Z}\right) \int dr_{\rm rel}\, r_{\rm rel}^2\, j_0(qr_{\rm rel})\, \rho^{\rm (if)}(r_{\rm rel})$$

$$f_p(q^2) = 1/(1+0.0548q^2)^4 \qquad \rho^{\rm (if)}(r_{\rm rel}) = \langle\Psi_f|\sum_{i=1}^{Z}\frac{\delta(r_{\rm rel}-r_i)}{r_{\rm rel}\, r_i}|\Psi_i\rangle$$

E. Hiyama, B.F. Gibson, M. Kamimura, PRC 70, 031001(4) (2004)

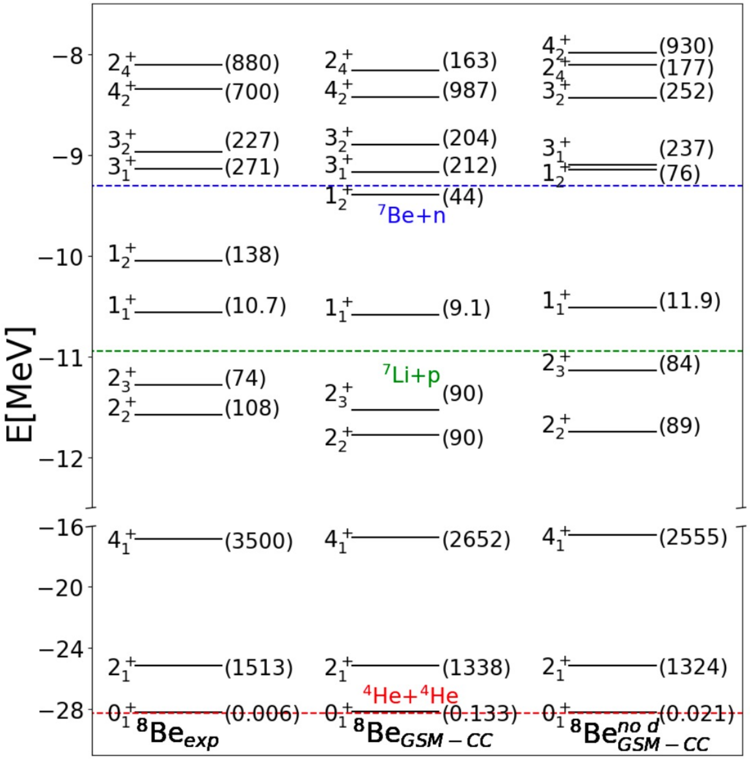
Mimicry mechanism of clusterization

Near-threshold clustering in ^{8}Be

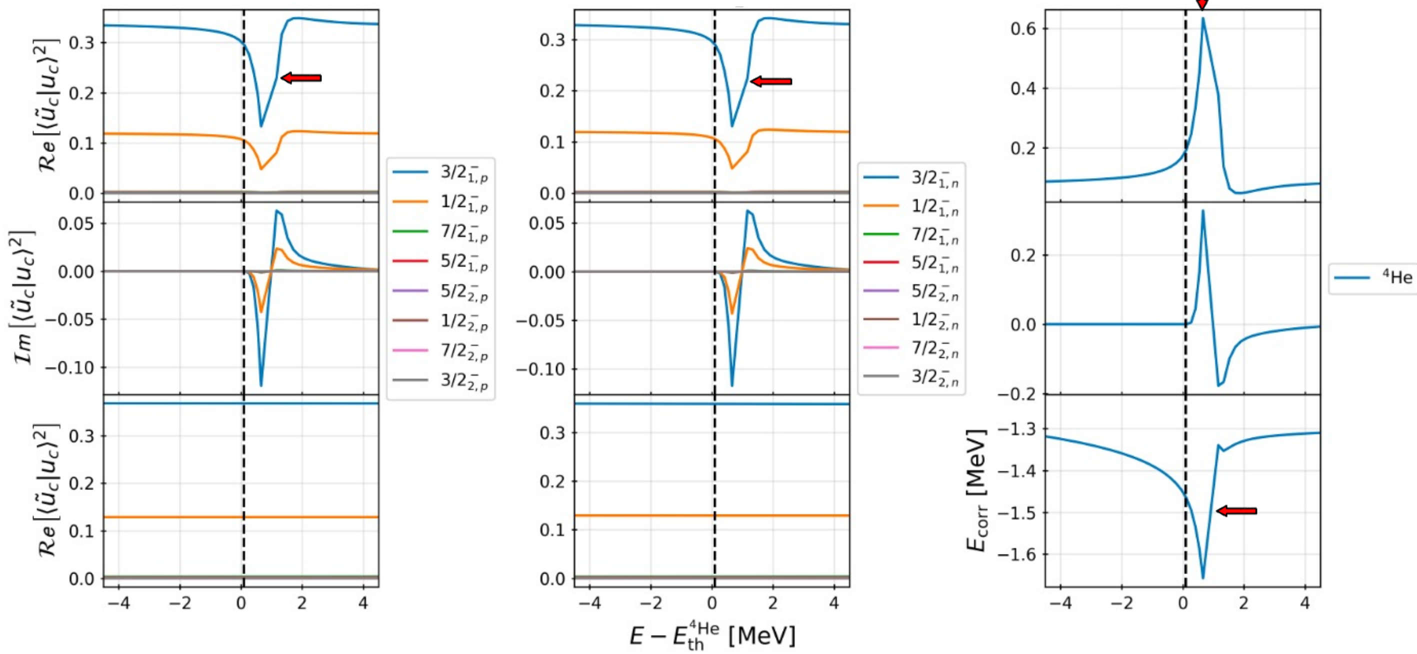
Continuum coupling correlation energy ➡ $E^{(\mathrm{corr})}_{J^\pi,M} = \langle \tilde{\Psi}^J_M | H | \Psi^J_M \rangle - \langle \tilde{\Phi}^{J;(\alpha)}_M | H | \Phi^{J;(\alpha)}_M \rangle \equiv \mathcal{E}_{J^\pi,M} - \mathcal{E}^{(\alpha)}_{J^\pi,M}$

$$|\Phi^{J;(\alpha)}_M\rangle = \sum_{c;c\neq\alpha} \int_0^{+\infty} |(c,r)^J_M\rangle \frac{\bar{u}^{JM}_c(r)}{r} r^2\, dr$$

^{8}Be



Near-threshold *alignment* of ^{8}Be(0_1^+)



Mass partitions:

$[|^4\mathrm{He}\rangle \otimes |^4\mathrm{He}\rangle]$, $[|^7\mathrm{Li}\rangle \otimes |p\rangle]$, $[|^7\mathrm{Be}\rangle \otimes |n\rangle]$, $[|^6\mathrm{Li}\rangle \otimes |d\rangle]$

Near-threshold clustering is the *emergent phenomenon* in SM for open quantum systems

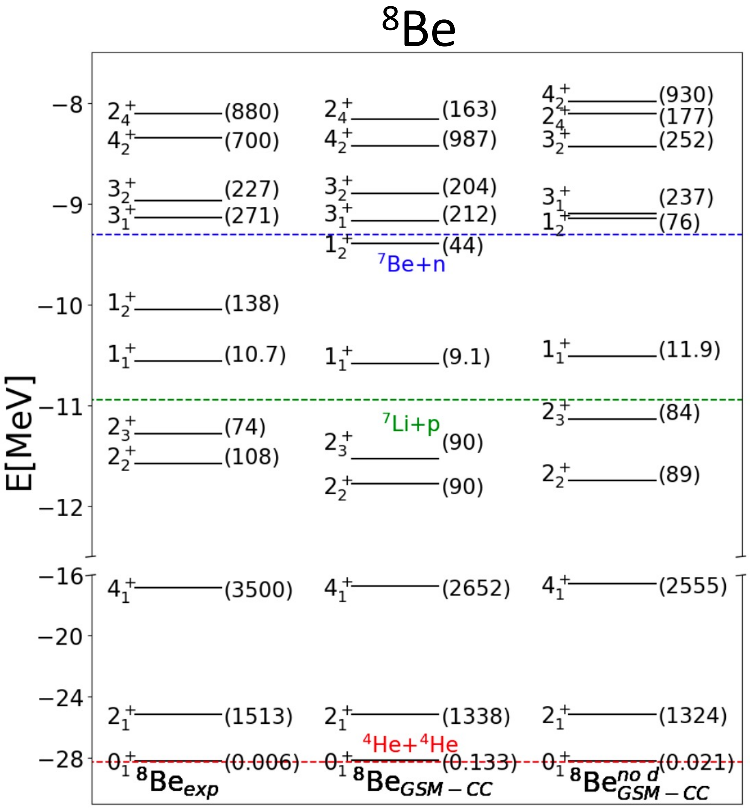
J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

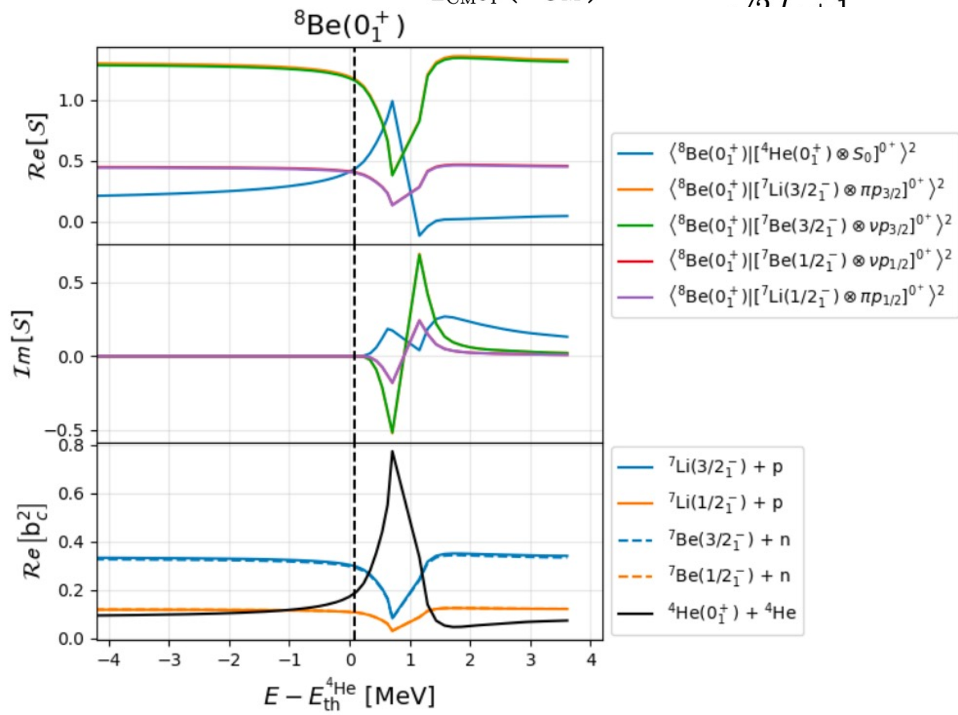
Mimicry mechanism of clusterization

Near-threshold clustering in ^{8}Be

Spectroscopic factor (GSM-CC) ➡ $\mathcal{S}^2_{L_{\rm CM}J_{\rm P}} = \int_0^{+\infty} u_c(r)^2\,dr + \left[\sum_{N_{CM}} \mathcal{A}^2_{L_{\rm CM}J_{\rm P}}(N_{CM}) - \int_0^{+\infty} u_c(r)^2\,dr\right]^{\rm (HO)}$

$$\mathcal{A}_{L_{\rm CM}J_{\rm P}}(N_{CM}) = \frac{\langle \Psi_A || A^\dagger_{N_{CM}L_{CM}J_{\rm P}} || \Psi_{A-k}\rangle}{\sqrt{2J_A+1}}$$





Near-threshold clustering is the *emergent phenomenon* in SM for open quantum systems

J.P. Linares Fernandez, et al, Phys. Rev. C 108, 044616 (2023)

Mass partitions:

$[|^4{\rm He}\rangle \otimes |^4{\rm He}\rangle]$, $[|^7{\rm Li}\rangle \otimes |{\rm p}\rangle]$, $[|^7{\rm Be}\rangle \otimes |{\rm n}\rangle]$, $[|^6{\rm Li}\rangle \otimes |{\rm d}\rangle]$

Statistical regime of clustering

Statistical mechanism of clusterization

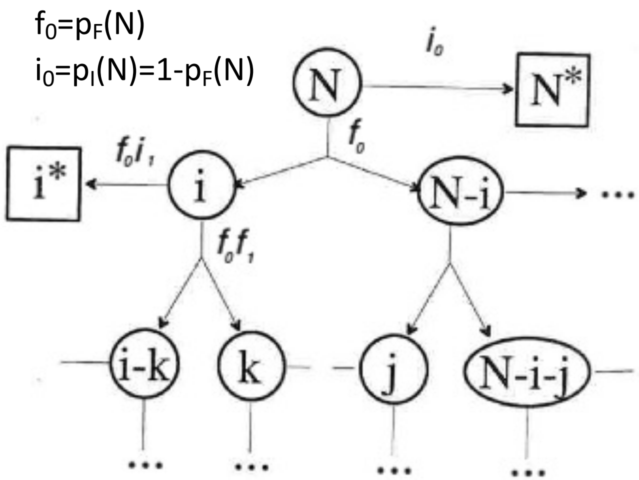
Quantal (mimicry) regime of clustering
Individual reaction thresholds are crucial

→

Classical (statistical) regime of clustering
Quantum features in the cluster production are unimportant

Fragmentation scenario

Various hybrids of the Fragmentation–Inactivation model



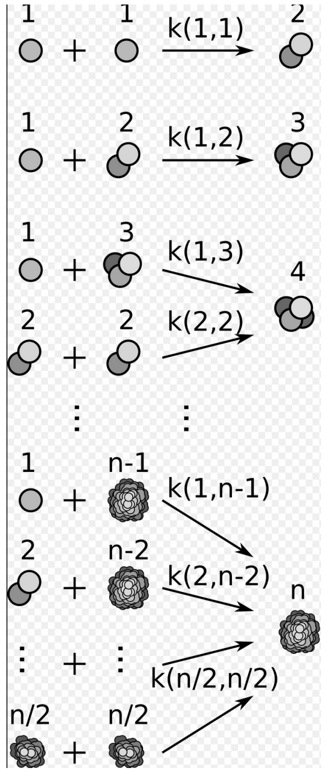
$F_{j,k-j}$: fragmentation kernel

I_k : inactivation kernel

R. Botet, M. Ploszajczak
Universal Fluctuations – The Phenomenology of Hadronic Matter
World Scientific Lecture Notes in Physics, Vol. 65 (2002)

Aggregation scenario

Equilibrium models: Fisher droplet model, Ising model, percolation model
Off-equilibrium models: Smoluchowski model of gelation



Smoluchowski equation

$$\frac{dc_s}{dt} = \frac{1}{2}\sum_{i+j=s} \boldsymbol{A}_{i,j} c_i\, c_j - \sum_{j} \boldsymbol{A}_{s,j} c_s\, c_j$$

$\boldsymbol{A}_{i,j}$: aggregation kernel

Exp.: $\boldsymbol{A}_{\omega I,\omega j} = \omega^{\alpha} \boldsymbol{A}_{i,j}$

S. Simons (1986)

Clustering in heavy ion collisions

Observables: cluster size and multiplicity of clusters

Δ - scaling of the normalized probability distribution $P_{<m>}[m]$ of the variable m for different 'system sizes' <m>

$$<m>^{\Delta} P_{<m>}[m] = \Phi(z_{(\Delta)}) \qquad 0 < \Delta \leq 1$$

R. Botet, M. Ploszajczak, Phys. Rev. E62, 1825 (2000)

$$z_{(\Delta)} = (m - m^{*}) / <m>^{\Delta}$$

most probable value   average value

If the scaling holds then the scaling relation holds independently of any phenomenological reasons to change <m>

The finite system exhibits the '*second scaling law*' (Δ=1/2) in the *ordered phase* and the '*first scaling law*' (Δ=1) in the *disordered phase*. The crossover close to the *critical point* happens with the continuous Δ - scaling.

Fragmentation scenario

Order parameter : average cluster multiplicity <n>

Cluster-size distribution : $n(s) \sim s^{-\omega}$, $\omega < 2$

Anomalous dimension : $g = \omega - 1$

Aggregation scenario

Order parameter : average size of the largest cluster $<s_{max}>$

Cluster-size distribution : $n(s) \sim s^{-\omega}$, $\omega > 2$

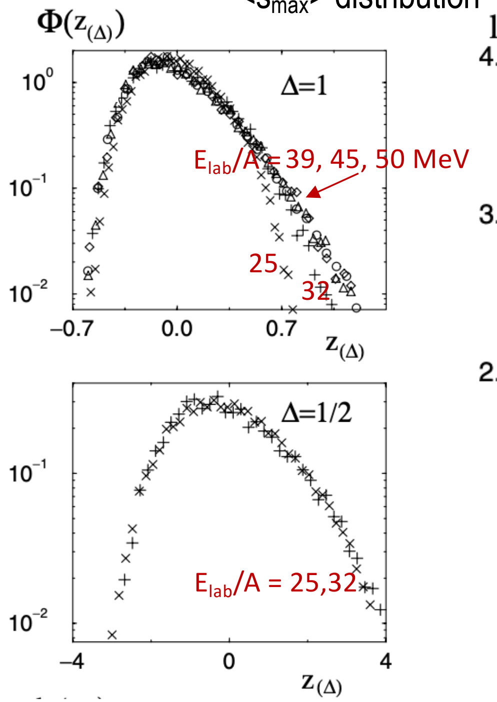
Anomalous dimension : $g = 1/(\omega - 1)$

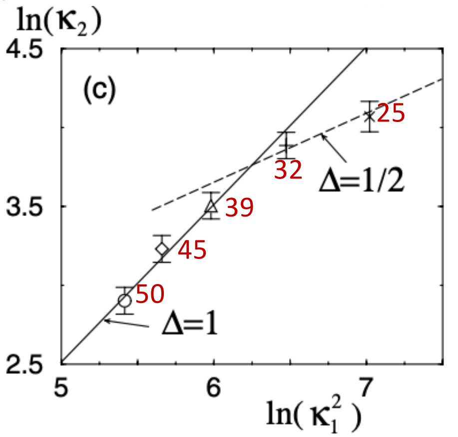
Clustering in heavy ion collisions

Order parameter fluctuations

Xe + Sn central collisions

<s_{max}> distribution

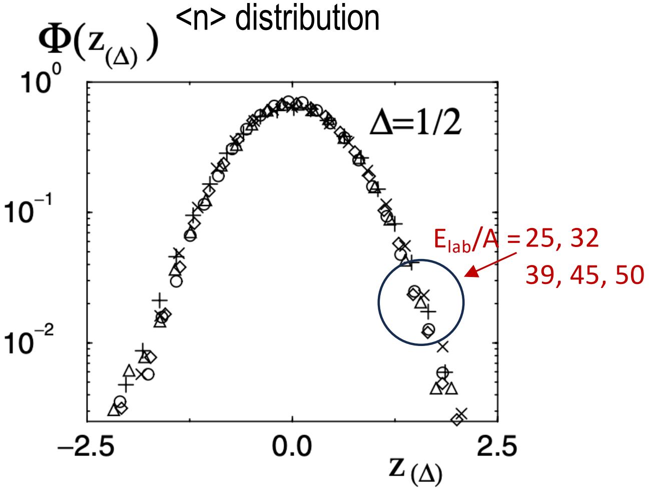




$\kappa_1 = \langle m \rangle,\ \kappa_2 = \langle m^2 \rangle - \langle m \rangle^2$

κ_1 =<s_{max}>

<n> distribution



Exp.: INDRA Coll., R. Bougault et al., in Proc. of the XXXIV Int. Winter Meeting on Nuclear Physics, Bormio, Italy, (1997)
INDRA Coll., N. Marie et al., Phys. Lett. B 391, 15 (1997).

Clustering in central heavy-ion collisions is governed by the aggregation scenario

R. Botet, M. Ploszajczak and INDRA Coll., Phys. Rev. Lett. 86, 3514 (2001)

Message to take

- Two generic clusterization mechanisms have been identified in atomic nucleus:
  - the statistical mechanism of clusterization (*aggregation scenario*), rooted in the CLT
  - the quantum mechanism of clusterization (*mimicry scenario*) in low energy near-threshold states

- Quantum states in the vicinity of a particle emission threshold belong to the category of *open quantum systems* having unique properties which distinguish them from *closed quantum systems*

- Proximity of the threshold (branching point) induces the collective mixing of eigenstates resulting in a single *aligned* eigenstate of the open quantum system Hamiltonian (→ *chameleon resonance*)

- Chameleon resonances are important astrophysically

- The correlated (cluster) states in a vicinity of reaction channel thresholds are the generic manifestations of *quantum openness* of a many-body system related to the *collective rearrangement* of wave functions due to their mutual coupling via the continuum

- Clustering in the mimicry scenario is the *emergent phenomenon* associated with the branch point singularity at the particle emission threshold.
  - ➡ Essential role of the *unitarity*!

- With increasing excitation energy, number of states and reaction channels grows rapidly and quantum aspects of the clusterization are gradually gone. The clusterization process randomizes and simplifies, i.e. the fragment production is governed by few kernel functions, generic statistical mechanism and the CLT

- The richness of nuclear interaction and the existence of nucleons in four distinct states (proton/neutron, spin-up/spin-down) make studies on the near-threshold phenomena in atomic nucleus unique

Thanks to my collaborators:

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Thank You