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arXiv:2407.19647

Alpha-decay half-lives and symmetry energy in KIDS model

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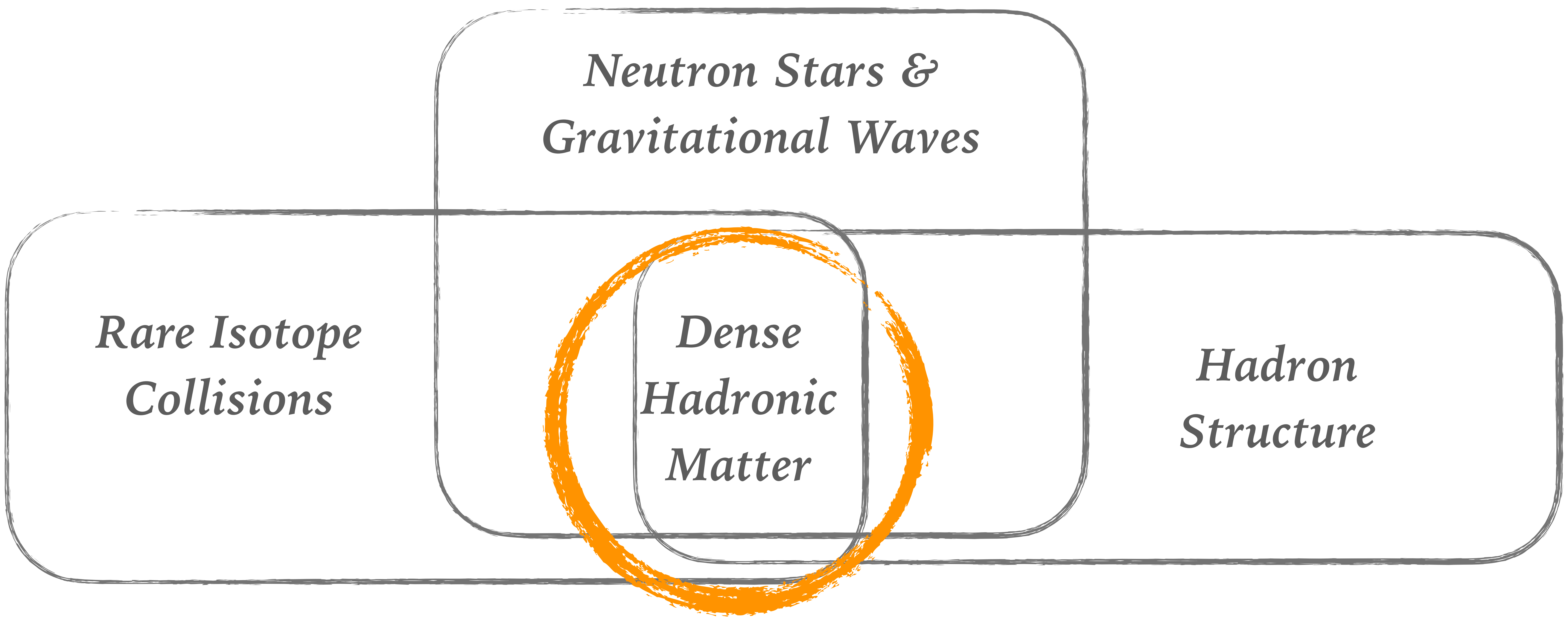
NuSYM2024_GANIL@2024.09.06

Astro-Hadron Physics in the Multi-messenger Era



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Theoretical activities in Korea for RAON

before RAON

Properties of Hadrons

Dense Matter Equation of State

Structure of Neutron Stars

Gravitational Waves

… …

after RAON

Transport Calculations (DJBUU)

Nucleosynthesis

Structure of Finite Nuclei

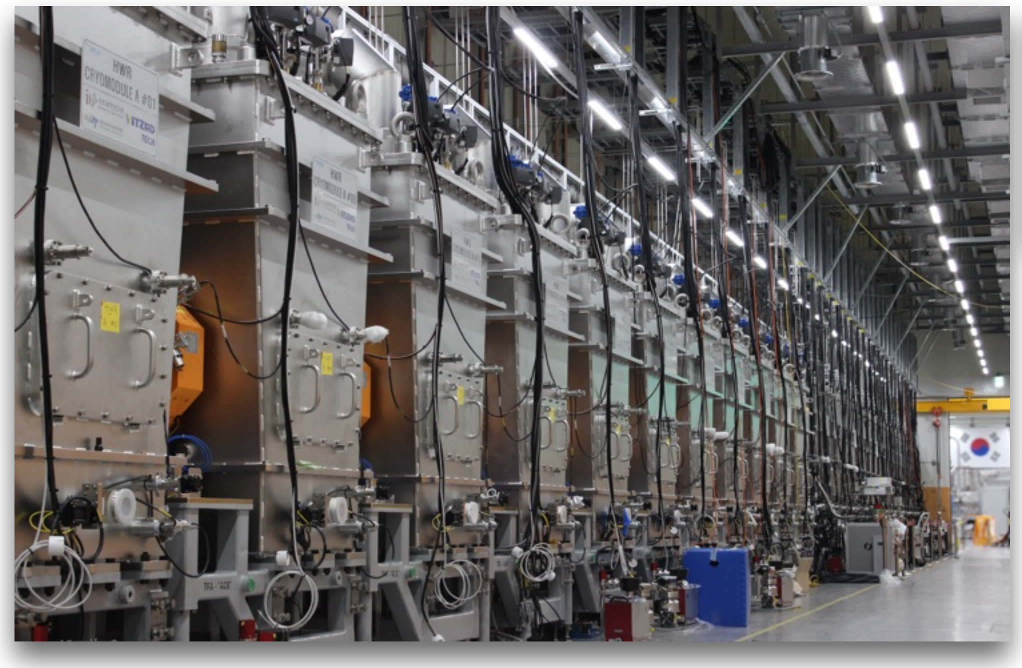
- DRHBc Masss Table Collaboration
- KIDS Model

… …

alpha-decay half-lives

PRC 109, 054310 (2024) with DRHBc

arXiv:2407.19647 with KIDS Model



RAON Rare isotope Accelerator complex for ON-line experiments

KIDS - a versatile framework for the nuclear EoS and EDF

KIDS (Korea-IBS-Daegu-SKKU) formalism

- **Expansion rule**

  Energy density of many-nucleon system expanded in the power of the Fermi momentum

- **Fitting rule**

  Determine the coefficients to reproduce neutron star properties and finite nuclear properties

$$\mathscr{E}(\rho,\delta) = \mathscr{T}(\rho,\delta) + \sum_{i=0}^{N-1} c_i(\delta)\rho^{1+i/3}$$

$$c_i(\delta) = \alpha_i + \beta_i \delta^2$$

$$\delta = (\rho_n - \rho_p)/\rho$$

Expansion parameter: $k_F/m_\rho < 1$ for $\rho < 8\rho_0$.

PRC 98, 065805 (2018), PRC 100, 014312 (2019)
EPJA 56, 157 (2020), PRC 106, 035802 (2022)

Number of parameters & convergence test

PRC 100, 014312 (2019)

4 parameters for pure neutron matter (β_i)

3 parameters for symmetric nuclear matter (α_i)

→ *7 parameters*

TABLE IV. Values of $c_i(1)$ fitted to APR EoS of PNM. The unit of c_i is MeV fm^{3+i} and the units of J, L, $K_{\rm sym}$, $Q_{\rm sym}$, and $R_{\rm sym}$ are MeV.

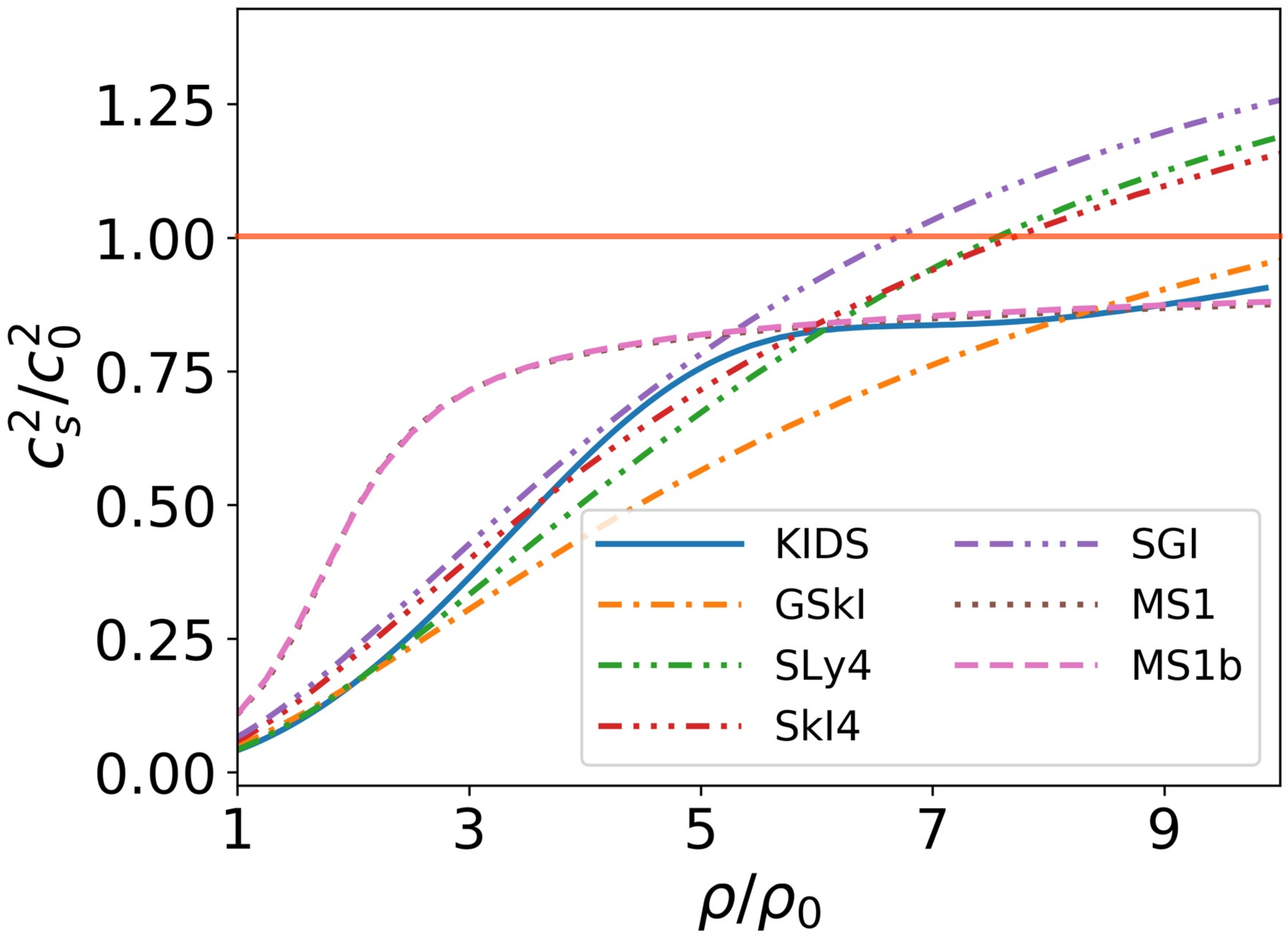
Model	N	$c_0(1)$	$c_1(1)$	$c_2(1)$	$c_3(1)$	$c_4(1)$	$c_5(1)$	χ_n^2	J	L	$K_{\rm sym}$	$Q_{\rm sym}$	$R_{\rm sym}$
P3	3	−266.72	133.50	281.38	–	–	–	5.3×10^{-4}	32.6	53.5	−129.7	422.3	−2421.8
P4	4	−407.94	990.09	−1321.86	937.14	–	–	1.4×10^{-4}	32.[illegible]	49.2	−156.3	583.1	−2469.7
P5	5	−224.16	−479.28	2814.48	−3963.71	2075.79	–	6.3×10^{-5}	33.0	51.4	−166.8	461.4	−1388.4
P6a	6	−224.81	−473.46	2795.50	−3935.18	2056.11	4.94	6.3×10^{-5}	33.0	51.4	−166.8	461.6	−1391.7
P6b	6	−28[illegible].99	110.63	604.05	−10.59	−1312.44	1117.70	6.4×10^{-5}	33.0	51.5	−163.8	450.0	−1545.9
P6c	6	−313.98	400.88	−463.41	1864.00	−2891.61	1630.37	6.5×10^{-5}	33.0	51.5	−162.3	446.6	−1631.2

J, L, K_{sym}, … can be chosen independently, so one can check the correlation more systematically.

$$S(\rho) = J + L\,x + \tfrac{1}{2}K_{\rm sym}\,x^2 + \mathcal{O}(x^3) \qquad K_\tau \equiv K_{\rm sym} - 6L - \frac{Q_0}{K_0}L. \qquad x \equiv (\rho - \rho_0)/3\rho_0$$

Speed of sound

EPJA 56, 157 (2020)



KIDS-A, B, C, D

$$E(\rho,\,\delta) = E(\rho) + S(\rho)\delta^2 + O(\delta^4),$$
$$E(\rho) = E_B + \frac{1}{2}K_0 x^2 + O(x^3),$$
$$S(\rho) = J + Lx + \frac{1}{2}K_{\rm sym}x^2 + \frac{1}{6}Q_{\rm sym}x^3 + O(x^4).$$

- **Nuclear matter: determine 7 model constants**
  - $\rho_0, E_B, K_0, J, L, K_{\rm sym}, Q_{\rm sym}$
  - vary K_0 (220-260:10), J (30-34:1), L (40-70:1), Kτ (-360,-420,-480)
- **Nuclear properties: determine additional 2 model constants (9 in total)**
  - Binding energy and charge radius of ^{40}Ca, ^{48}Ca and ^{208}Pb

$$\chi_6^2 \equiv \sum_{n=1}^{6}\left(\frac{O_n^{\rm exp} - O_n^{\rm calc}}{O_n^{\rm exp}}\right)^2$$

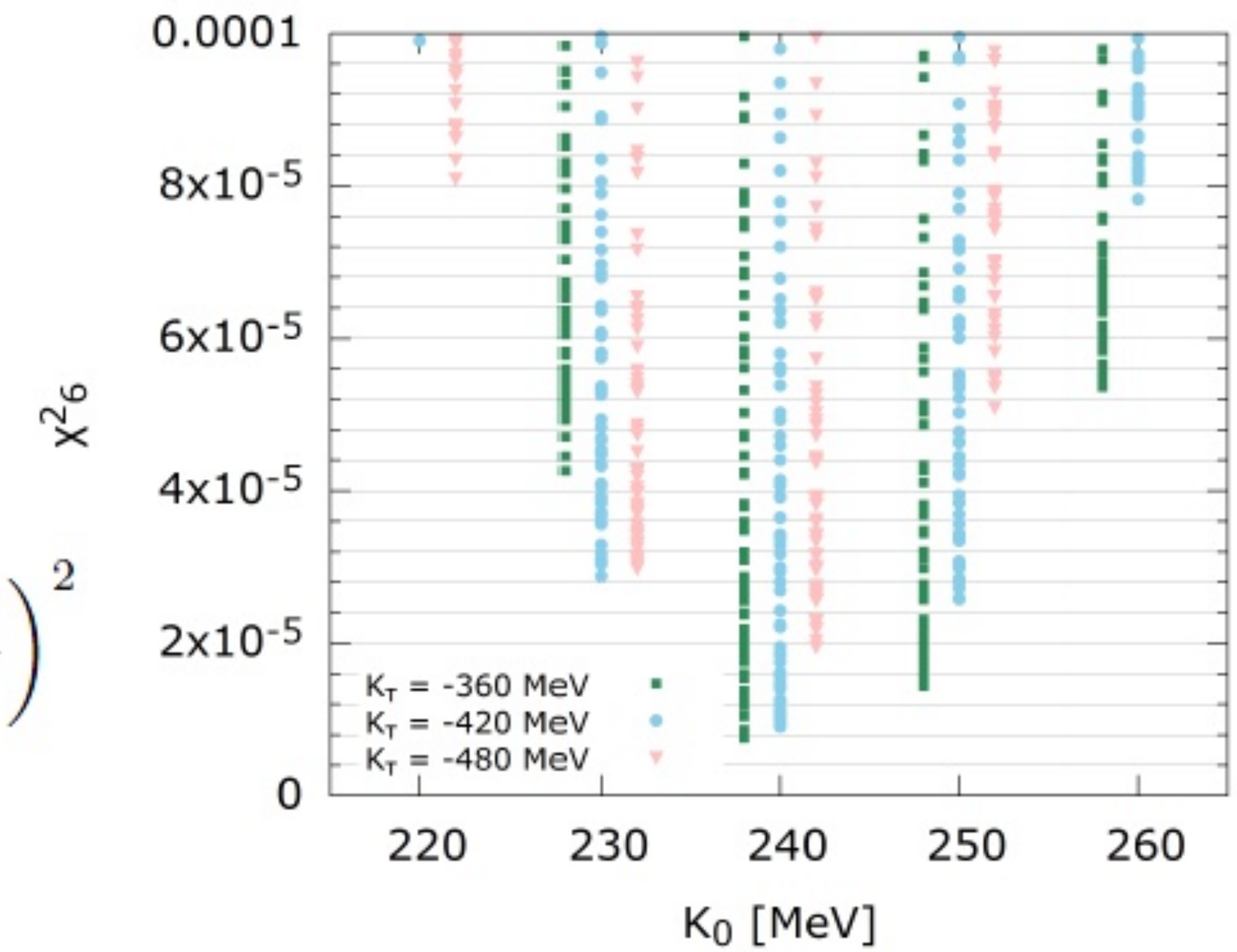
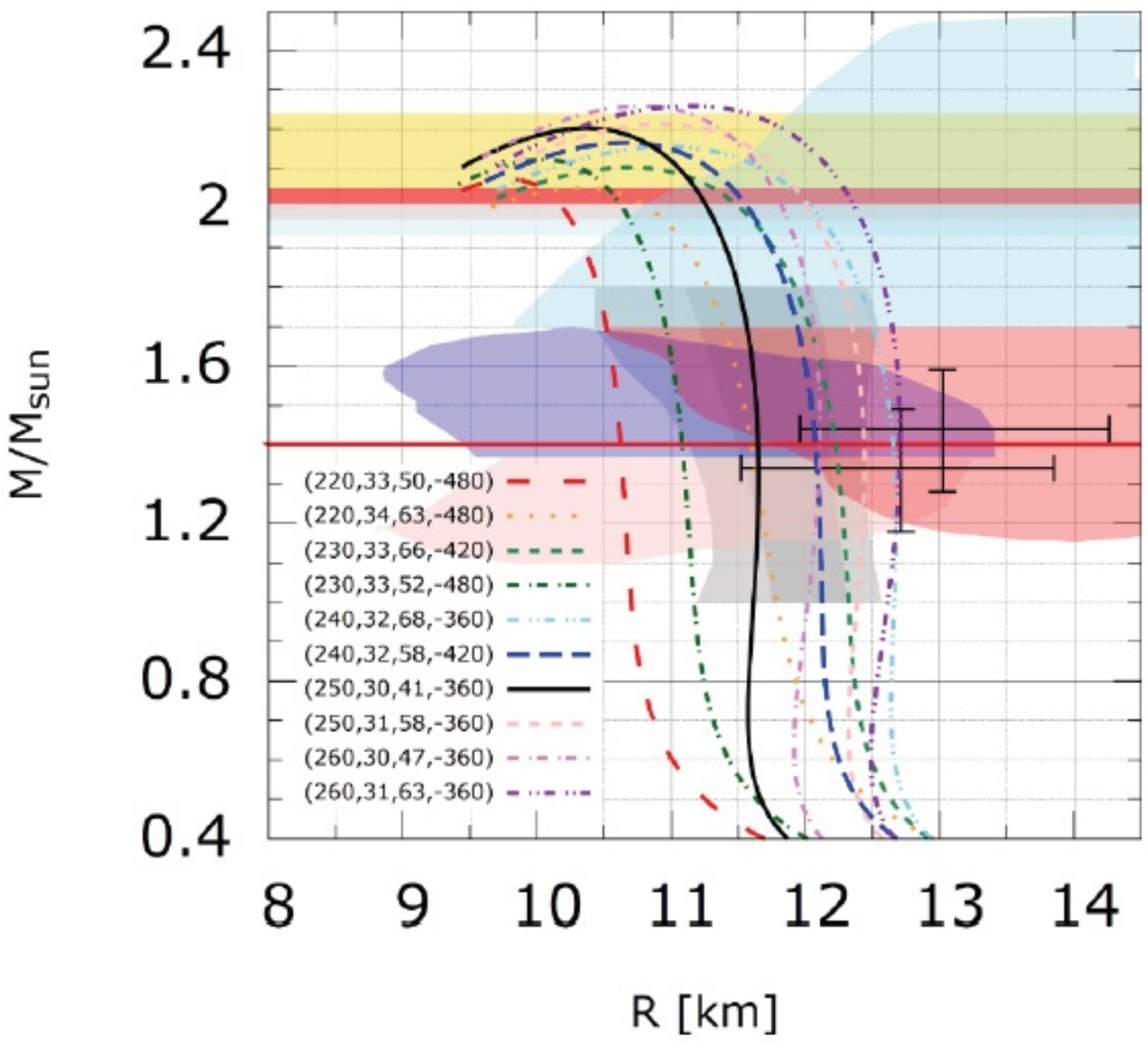


Table 1. EoS parameters $(J,\,L,\,K_\tau)$ giving the two smallest χ_6^2 values for each K_0 value.

K_0	$(J,\,L\,K_\tau)$	$\chi^2(\times10^{-5})$
220	(33, 50, −480)	9.45
	(34, 63, −480)	8.61
230	(33, 66, −420)	3.04
	(33, 52, −480)	3.01
240	(32, 68, −360)	0.75
	(32, 58, −420)	0.89
250	(30, 41, −360)	1.50
	(31, 58, −360)	1.43
260	(30, 47, −360)	5.55
	(31, 63, −360)	6.03

KIDS-A, B, C, D

- **Neutron star**



- **Final selection**

$11.8 \leq R_{1.4} \leq 12.5$ km

Model	K_0	J	L	K_{sym}
KIDS-A	230	33	66	−139.5
KIDS-B	240	32	58	−162.1
KIDS-C	250	31	58	−91.5
KIDS-D	260	30	47	−134.5

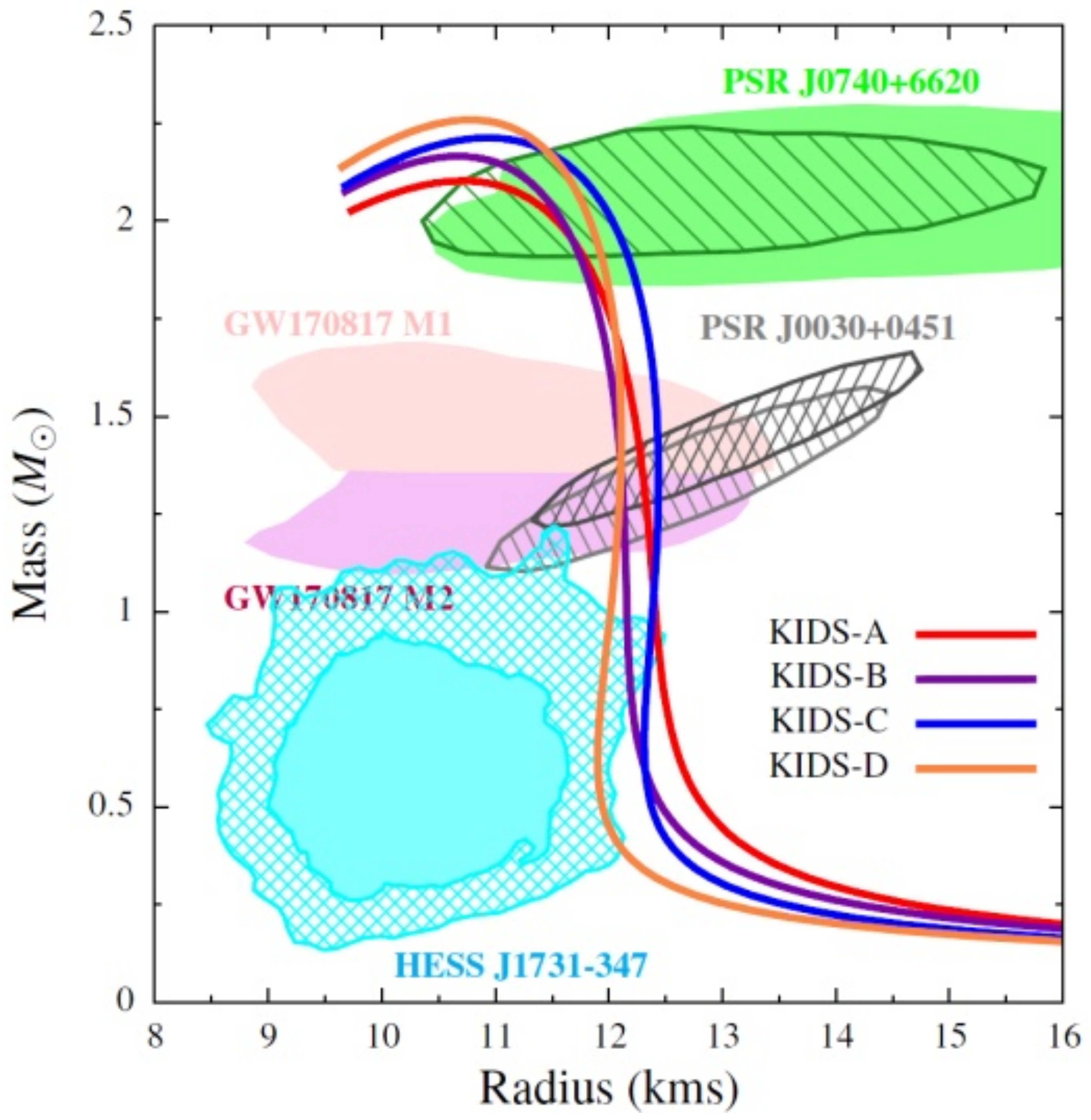
Hadron-quark phase transition with Vector MIT Bag & KIDS

Frot. Astron. Space Sci. 11, 1421839 (2024)

Model	Gv/Gs	ρ^H_t/ρ_0	ρ^Q_t/ρ_0	M_{max}/M_{sun}
KIDS-A	0.35	1.44	2.47	2.08
	0.4	6.5	7.26	2.1
KIDS-D	0.35	1.76	2.9	2.08
	0.4	4.56	5.14	2.16

pure hadronic matter for $\rho < \rho_t^H$

pure quark matter for $\rho > \rho_t^Q$



Examples of applications - quadruple deformation

PRC 108, 044316 (2023)

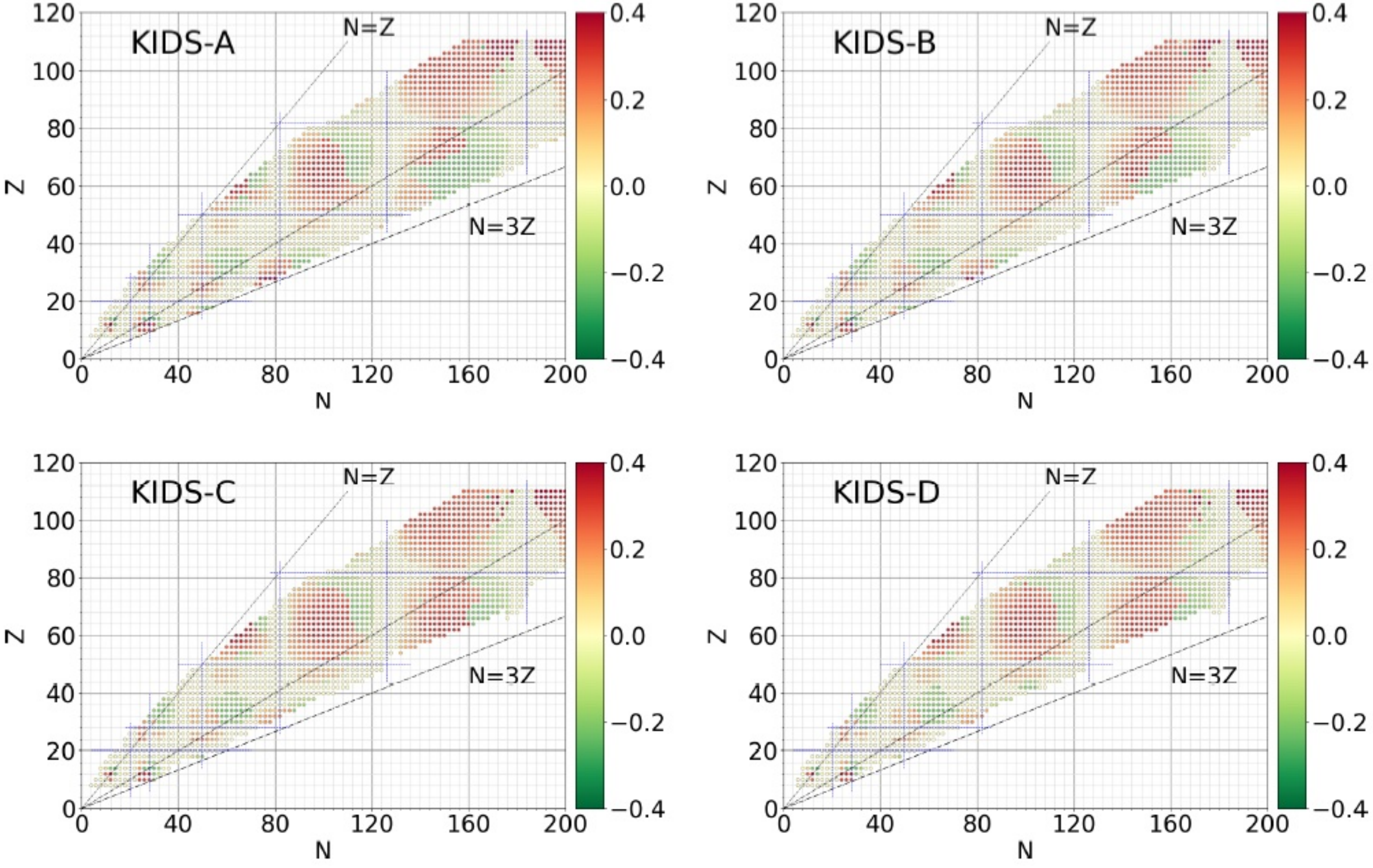


FIG. 3. Calculated quadrupole deformation $\beta_{2,p}$ for bound nuclei obtained by employing the KIDS-A–D models.

Formulas for alpha-decay half-lives

PRC 109, 054310 (2024)

Basic ingredients

- WKB approximation
- Cluster-formation model
- Folding potential

$$T_{1/2} = \frac{\hbar \ln 2}{\Gamma},$$

$$\Gamma = P_\alpha N_f \frac{\hbar^2}{4\mu} P_{\rm tot}.$$

$$P_\alpha = \frac{2S_p + 2S_n - S_\alpha}{S_\alpha},$$

P_α and Q_α from AME2020

$$N_f = \frac{1}{2}\int_0^\pi N_f(\beta)\sin\beta d\beta,$$

$$P_{\rm tot} = \frac{1}{2}\int_0^\pi \exp\left[-2\int_{r_2(\beta)}^{r_3(\beta)} k(r',\beta)dr'\right]\sin\beta d\beta,$$

with

$$N_f(\beta) \simeq \left[\int_{r_1(\beta)}^{r_2(\beta)} \frac{dr'}{2k(r',\beta)}\right]^{-1},$$

$$k(r,\beta) = \sqrt{\frac{2\mu}{\hbar^2}|Q_\alpha - V(r,\beta)|}.$$

$$V(r,\beta) = V_l(r) + V_C(r,\beta) + V_N(r,\beta),$$

$$V_l(r) = \frac{\hbar^2}{2\mu}\frac{(l+1/2)^2}{r^2},$$

$$V_C(r,\beta) = \int d\mathbf{r}_d d\mathbf{r}_\alpha \rho_d^p(\mathbf{r}_d)\rho_\alpha^p(\mathbf{r}_\alpha)\frac{e^2}{s},$$

$$V_N(r,\beta) = \lambda\int d\mathbf{r}_d d\mathbf{r}_\alpha \rho_d(\mathbf{r}_d)\rho_\alpha(\mathbf{r}_\alpha)v(\mathbf{s}).$$

$$v(s) = 7999\frac{e^{-4s}}{4s} - 2134\frac{e^{-2.5s}}{2.5s} - 276\left(1 - 0.005\frac{Q_\alpha}{A_\alpha}\right)\delta(s),$$

$$\rho_\alpha(r) = 0.4229\exp(-0.7024r^2)$$

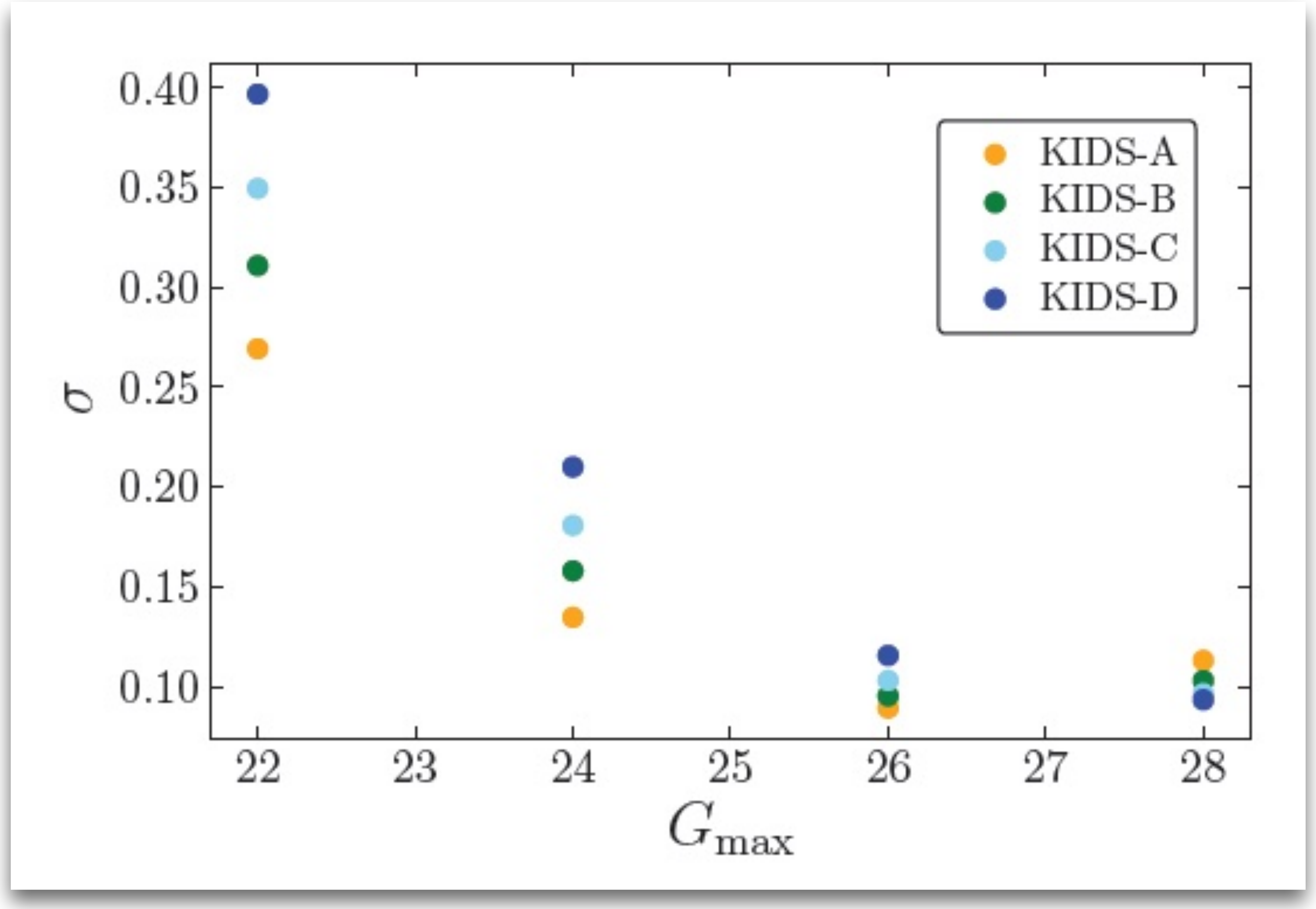
Results

nuclei considered

Z_{par}	84 (Po)	86 (Rn)	88 (Ra)	90 (Th)	92 (U)
A_{par}	186-224	194-230	202-234	208-238	216-242

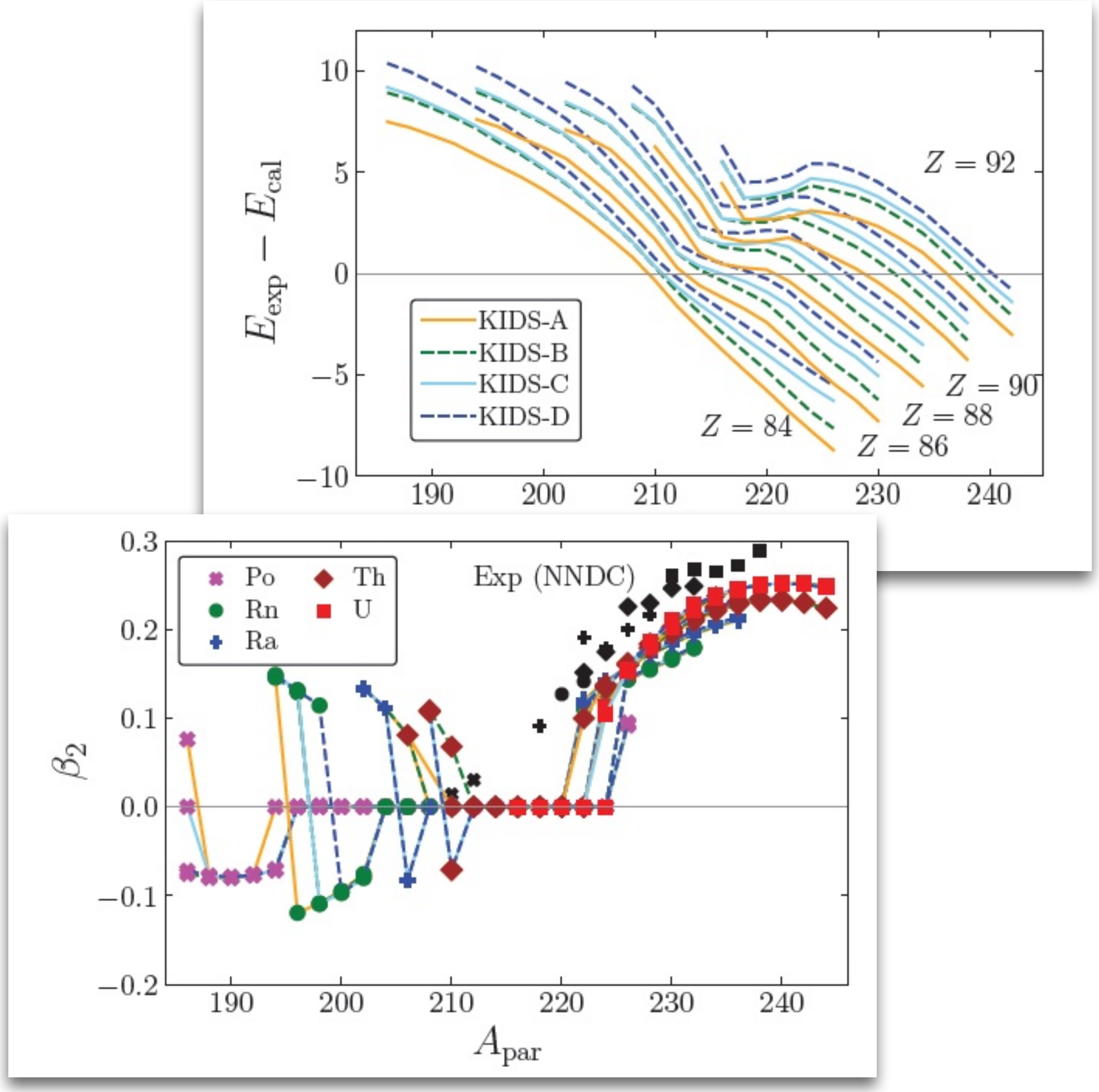
- **Adjusting G value**

$$\int_0^{\pi}\int_{r_1(\beta)}^{r_2(\beta)}\sqrt{\frac{2\mu}{\hbar^2}|Q_\alpha - V(r,\beta)|}\sin\beta dr d\beta = (2n+1)\frac{\pi}{2}$$
$$= (G - l + 1)\frac{\pi}{2}.$$



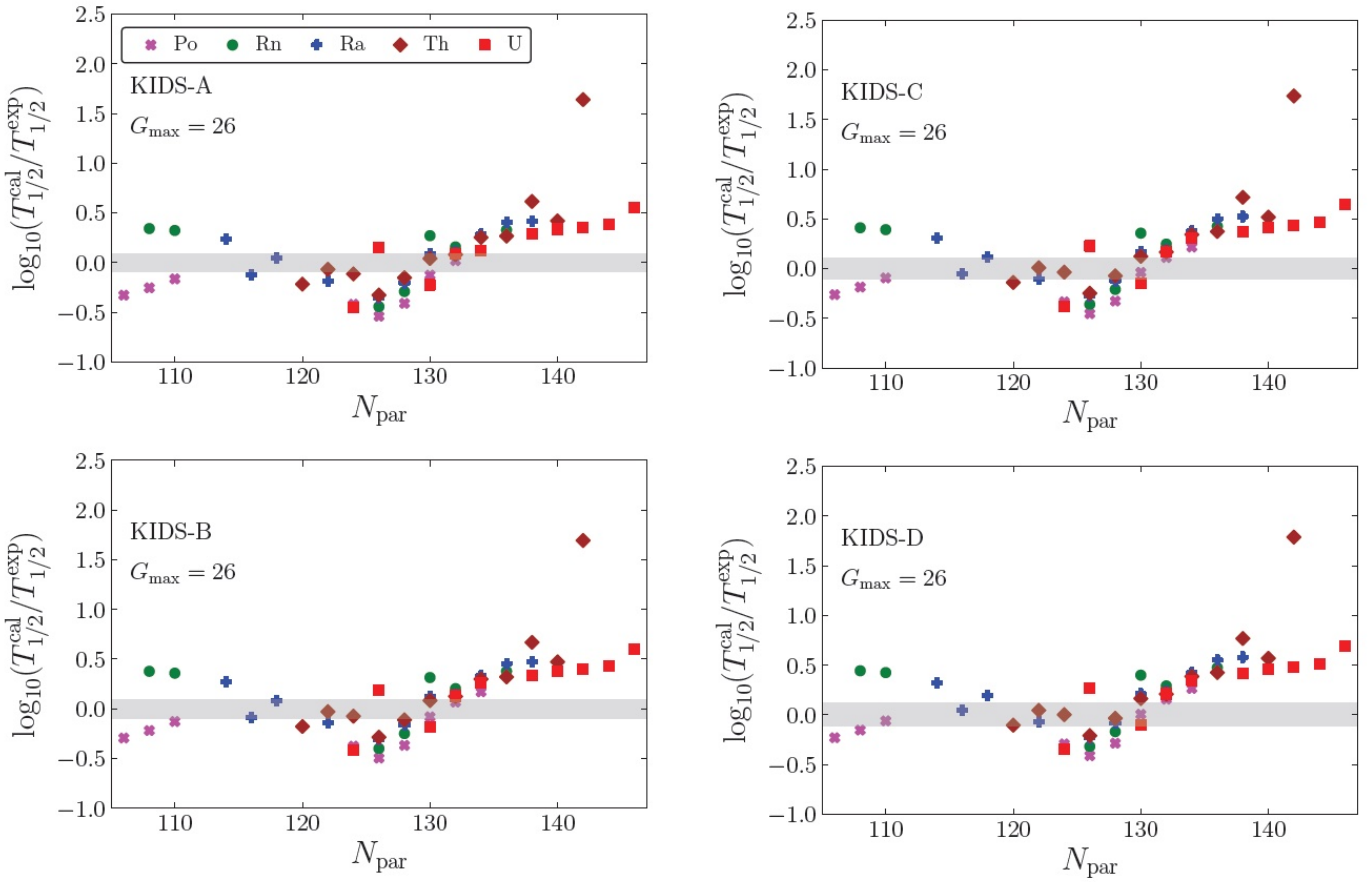
$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\log_{10} T_{1/2,i}^{cal} - \log_{10} T_{1/2,i}^{exp}\right)^2}$$

- **Accuracy of the model**



Results: Half-life as a function of N_{par}

arXiv:2407.19647

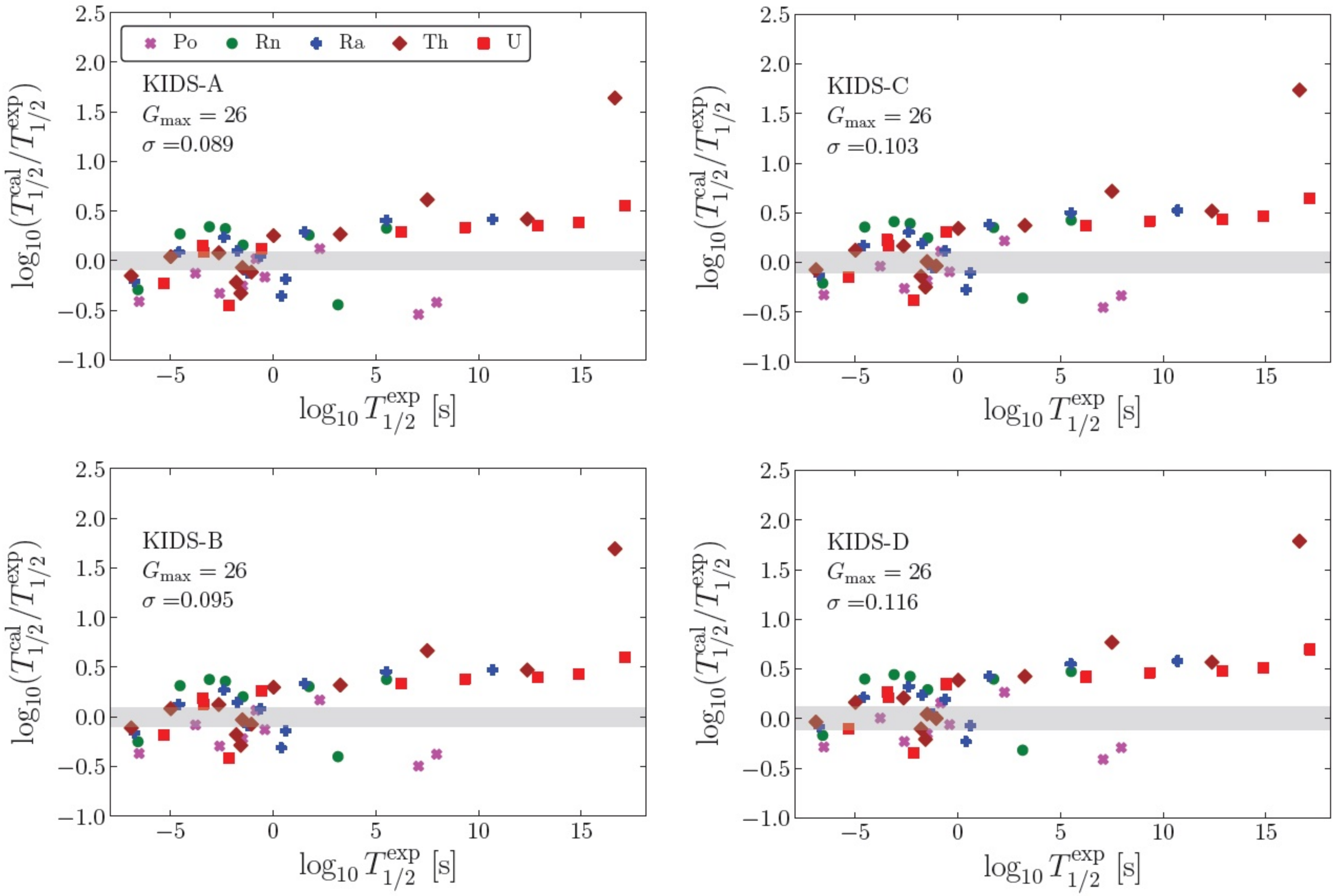


- Results are mostly in [-0.5:0.5]
- Minima at N=126

- Tend to increase with large N
- Shortest in KIDS-A
- Longest in KIDS-D

Results: Half-life as a function of $T^{exp}_{1/2}$

arXiv:2407.19647



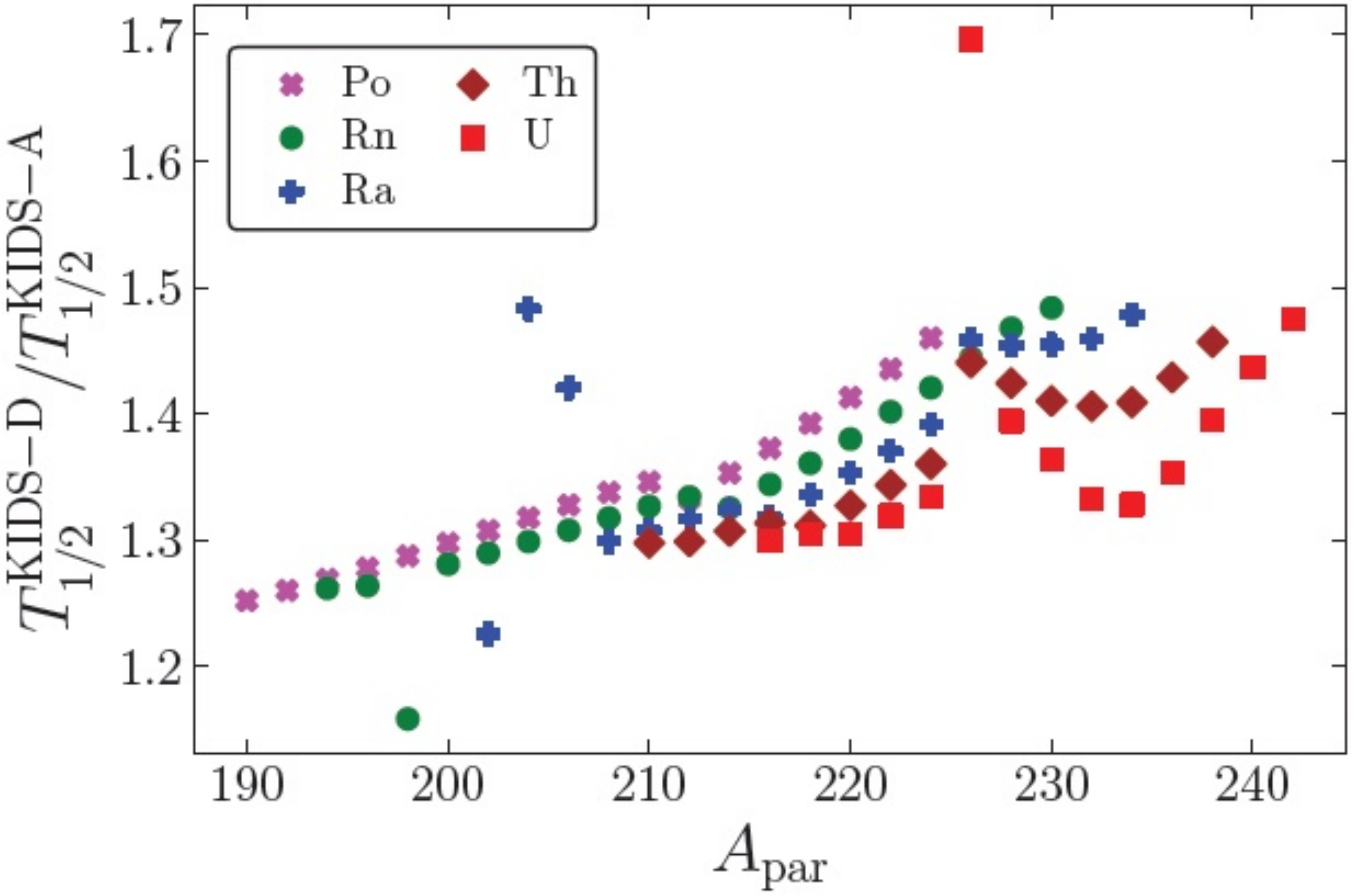
- 1 day = 86,400 seconds (~10^5)
- T < 1 day: random distribution

- T > 1 day: overshoots the experiment
- Small uncertainty in Γ gives large difference

alpha-decay half-lives vs symmetry energy

Model	J	L
KIDS-A	33	66
KIDS-D	30	47

$T^D_{1/2}/T^A_{1/2}$ as a function of A_{par}



- The ratios are in the range 1.25-1.5
- For $A_{par} \leq 224$, ratio increases rather monotonically
- Above 224, there are minima at A_{par}=230 (Ra), 232 (Th), 234 (U) for which N_{par}=142

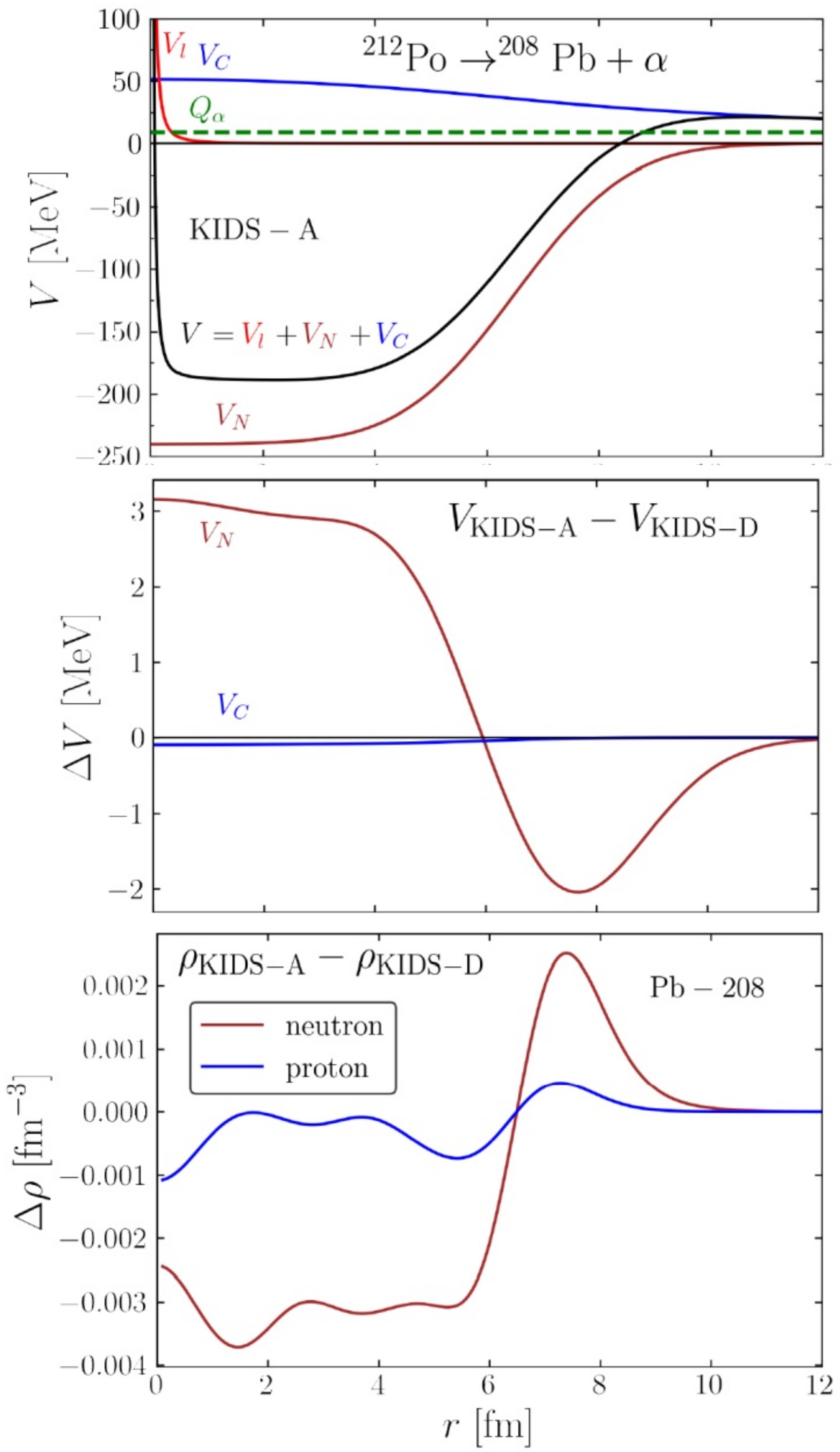
  ** Is there any special meaning for N_{par}=142 ?
- Is the ratio 1.25-1.5 big enough?

  ** Neutron skin thickness of 208Pb:

  ΔR_{np}(KIDS-D)/ΔR_{np}(KIDS-A) = 1.40

Diagnose the origin of the correlation

Model	J	L
KIDS-A	33	66
KIDS-D	30	47



- Tunneling barrier is determined by V_C and V_N
- Barrier builds up at r~8.5 fm
- Density in the surface region is critical

- V_C is identical in the two models
- In r≤6 fm, nuclear potential of KIDS-A is larger
- In 6-10 fm, KIDS-A is smaller: more cancellation with V_C
- Stronger cancellation makes barrier lower → shorter half-life

- Lower density in the core with KIDS-A
- Depletion in the core is compensated by the distribution in 6-10 fm
- Symmetry energy → different density & potential → different half-life

Summary

- KIDS formalism provides a unified description of finite nuclei and infinite nuclear matter.
- Models are constrained by nuclear properties and neutron star data.
- Alpha-decay half-lives are reproduced with factor 1/3 – 3
- **Soft symmetry energy gives longer half-lives.**
- Interesting behavior happens at N=142.