Nuclear equation of state from nuclear experiments and neutron stars observations

NuSYM 2024, Caen

Pietro Klausner

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Collaborators

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Marco **Antonelli** (L.P.C. Caen)

Structure of the presentation

Nuclear equation of state from nuclear experiments and neutron stars observations

- First Part: constraints on EoS from nuclear experiments

- Bayesian inference
- Skyrme Interaction

- Second Part: constraints on EoS from Neutron Stars observations

- Second Bayesian inference

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Parameters of the model

Parameters

ρ_0, E_0, K_0, J, L	Nuclear matter parameters
G_0, G_1	Surface term parameters
W_0	Spin-orbit parameter
$m_0^*/m, m_1^*/m$	Effective masses

0 = isoscalar; 1 = isovector

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1-to-1 correspondence with usual Skyrme parameters[1]!

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1-to-1 correspondence with usual Skyrme parameters[1]!

Prior distribution

Par.	Units	Lower limit	Upper limit
ρ_0	$[\mathrm{fm}^{-3}]$	0.150	0.175
E_0	[MeV]	-16.50	-15.50
K_0	[MeV]	180.00	260.00
J	[MeV]	24.00	40.00
L	[MeV]	-20.00	120.00
G_0	$[\mathrm{MeV\ fm}^5]$	90.00	170.00
G_1	$[\mathrm{MeV\ fm}^5]$	-90.00	70.00
W_0	$[\mathrm{MeV\ fm}^5]$	60.00	190.00
m_0^*/m		0.70	1.10
m_1^*/m		0.60	0.90

[1] L.-W. Chen et al. Phys. Rev. C 80, 014322 (2009)

Observable chosen for the fit

“hfbcs-qrpa[1]” code to compute observables from parameters

[1]G. Colò, X. Roca-Maza, arXiv:2102.06562v1 [nucl-th]

Observable chosen for the fit

Ground-state properties			
	$B.E.$ [MeV]	$R_{\rm ch}$ [fm]	$\Delta E_{\rm SO}$ [MeV]
^{208}Pb	$1636.4 \pm 2.0^*$	$5.50 \pm 0.05^*$	$2.02 \pm 0.50^*$
^{48}Ca	$416.0 \pm 2.0^*$	$3.48 \pm 0.05^*$	$1.72 \pm 0.50^*$
^{40}Ca	$342.1 \pm 2.0^*$	$3.48 \pm 0.05^*$	-
^{56}Ni	$484.0 \pm 2.0^*$	-	-
^{68}Ni	$590.4 \pm 2.0^*$	-	-
^{100}Sn	$825.2 \pm 2.0^*$	-	-
^{132}Sn	$1102.8 \pm 2.0^*$	$4.71 \pm 0.05^*$	-
^{90}Zr	$783.9 \pm 2.0^*$	$4.27 \pm 0.05^*$	-

"hfbcs-qrpa[1]" code to compute observables from parameters

$B.E.$: Binding Energy

R_{ch} : Charge radius

ΔE_{SO} : Spin-orbit splitting

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Isoscalar resonances		
	E^{IS}_{GMR} [MeV]	E^{IS}_{GQR} [MeV]
^{208}Pb	13.5 ± 0.5*	10.9 ± 0.5*
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E^{IS}_{GMR} : IsoScalar Giant monopole resonance excitation energy (constrained)

E^{IS}_{GQR} : IsoScalar Giant quadrupole resonance excitation energy (centroid)

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	α_D [fm^3]	$m(1)$ [MeV fm^2]	A_{PV} (ppb)
^{208}Pb	19.60 ± 0.60	961 ± 22	550 ± 18
^{48}Ca	2.07 ± 0.22	-	2668 ± 113

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$B.E.$: Binding Energy

R_{ch} : Charge radius

ΔE_{SO} : Spin-orbit splitting

E^{IS}_{GMR} : IsoScalar Giant monopole resonance excitation energy (constrained)

E^{IS}_{GQR} : IsoScalar Giant quadrupole resonance excitation energy (centroid)

α_D: Nuclear polarizability

$m(1)$: EWSR of IVGDR

A_{PV}: Parity violating asymmetry

* Theoretical error

[1]G. Colò, X. Roca-Maza, arXiv:2102.06562v1 [nucl-th]

The need for emulation

→ Computing all the observables → ~ 2 hours!

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(Emulator for Bayesian inference)

[1]https://madai.phy.duke.edu/

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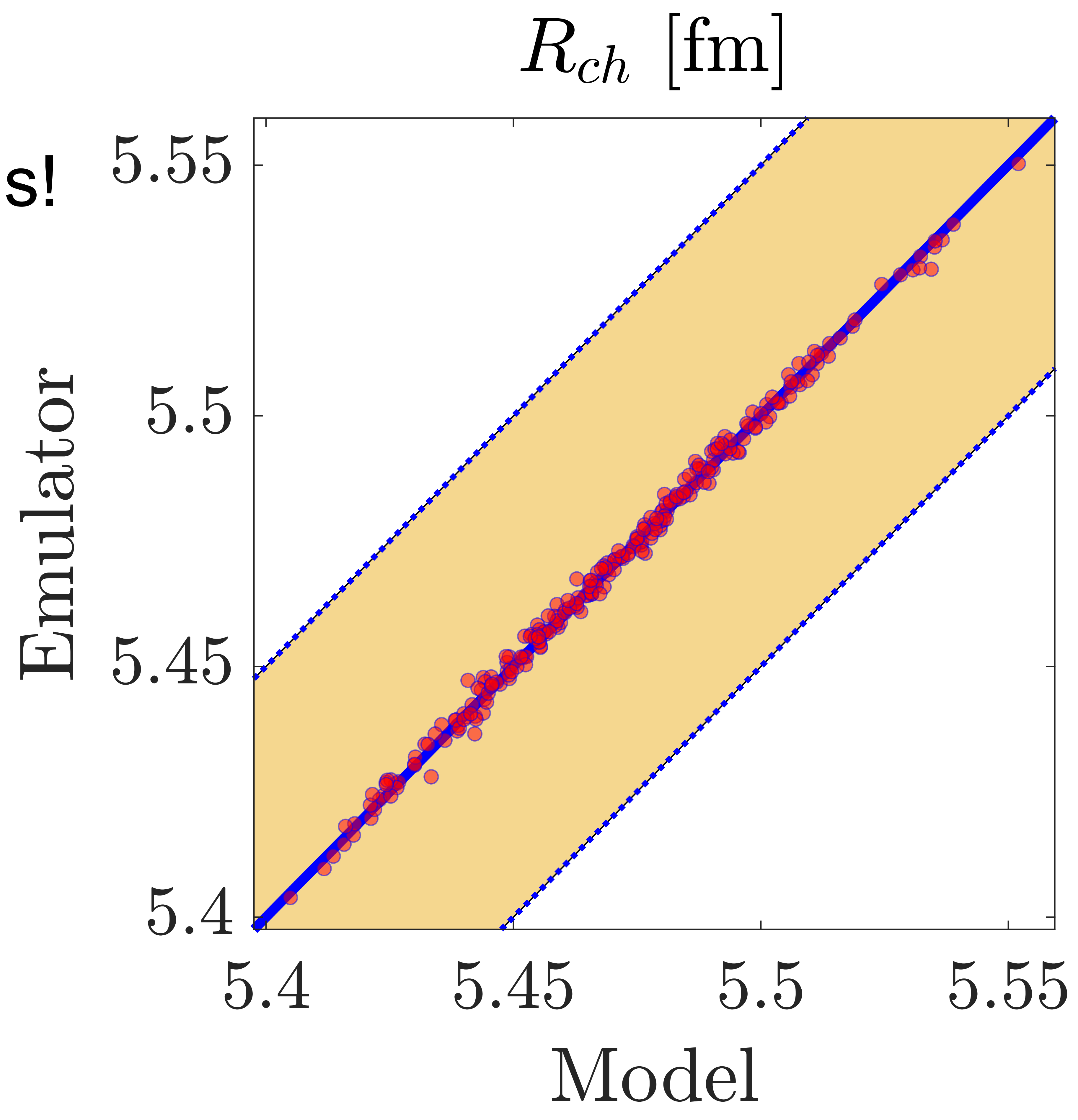

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Progressive marginalized posteriors

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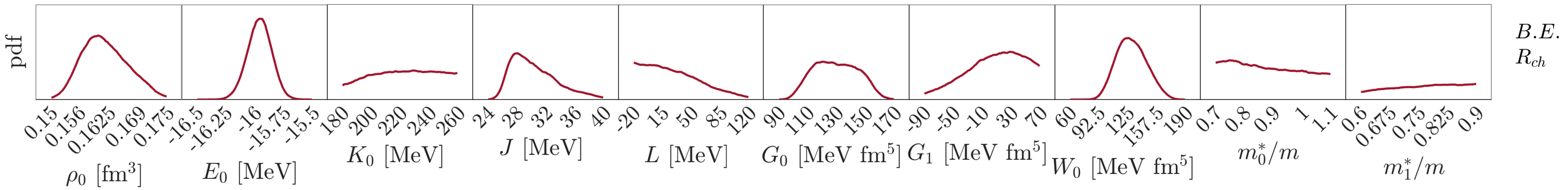
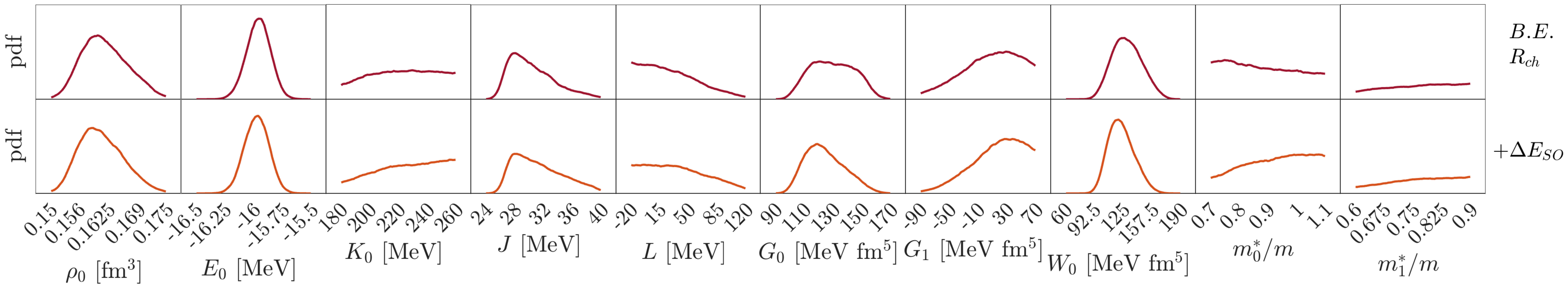


Progressive marginalized posteriors



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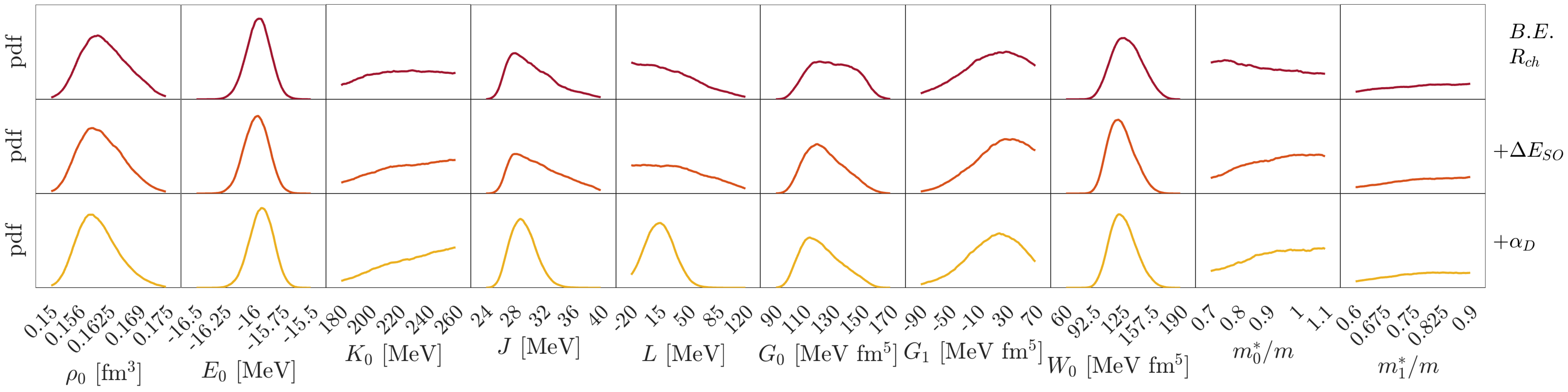

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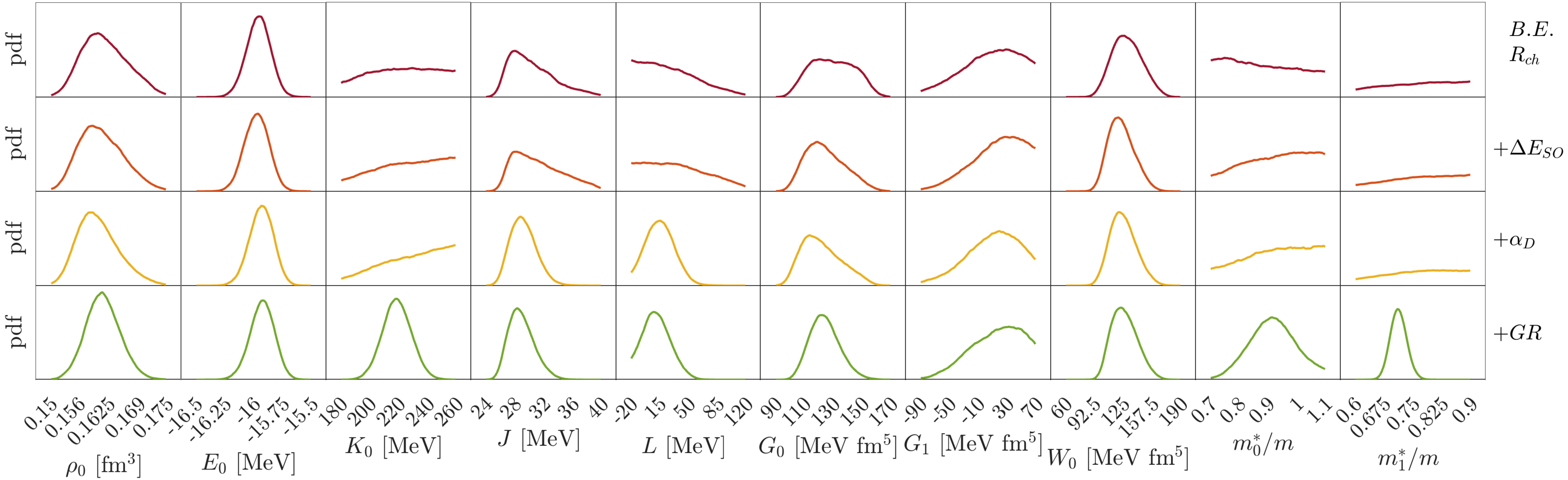
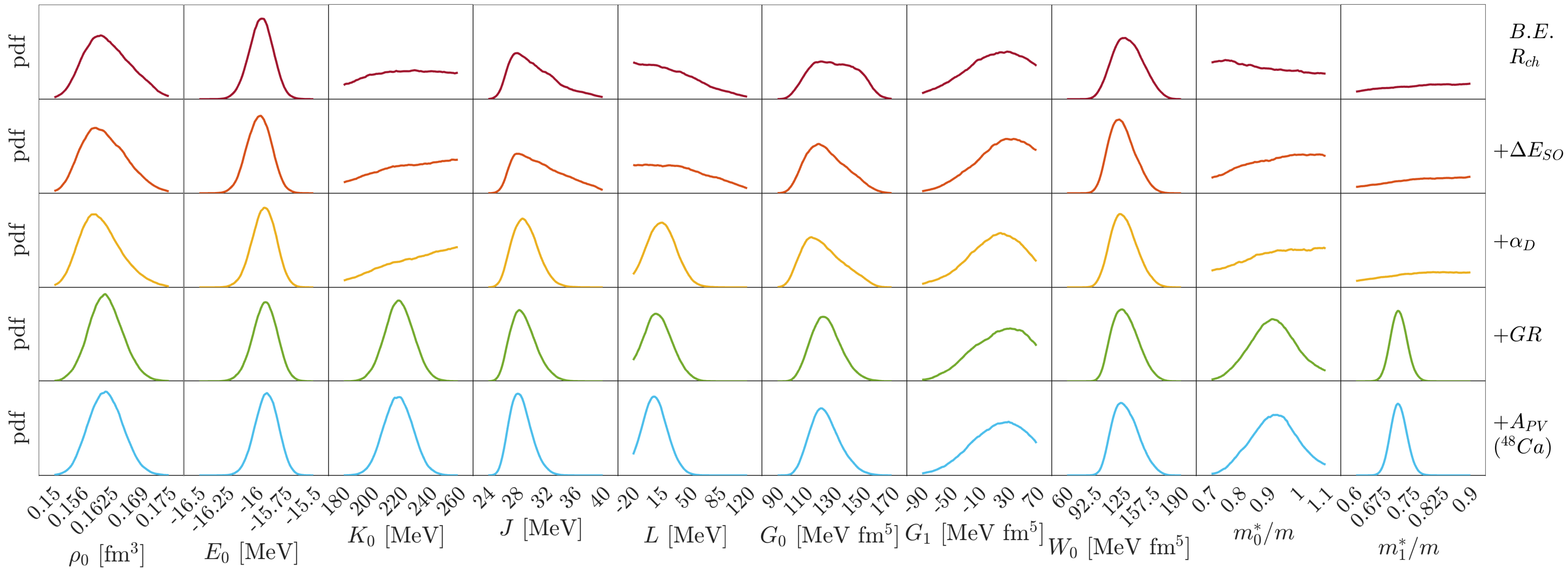


Progressive marginalized posteriors



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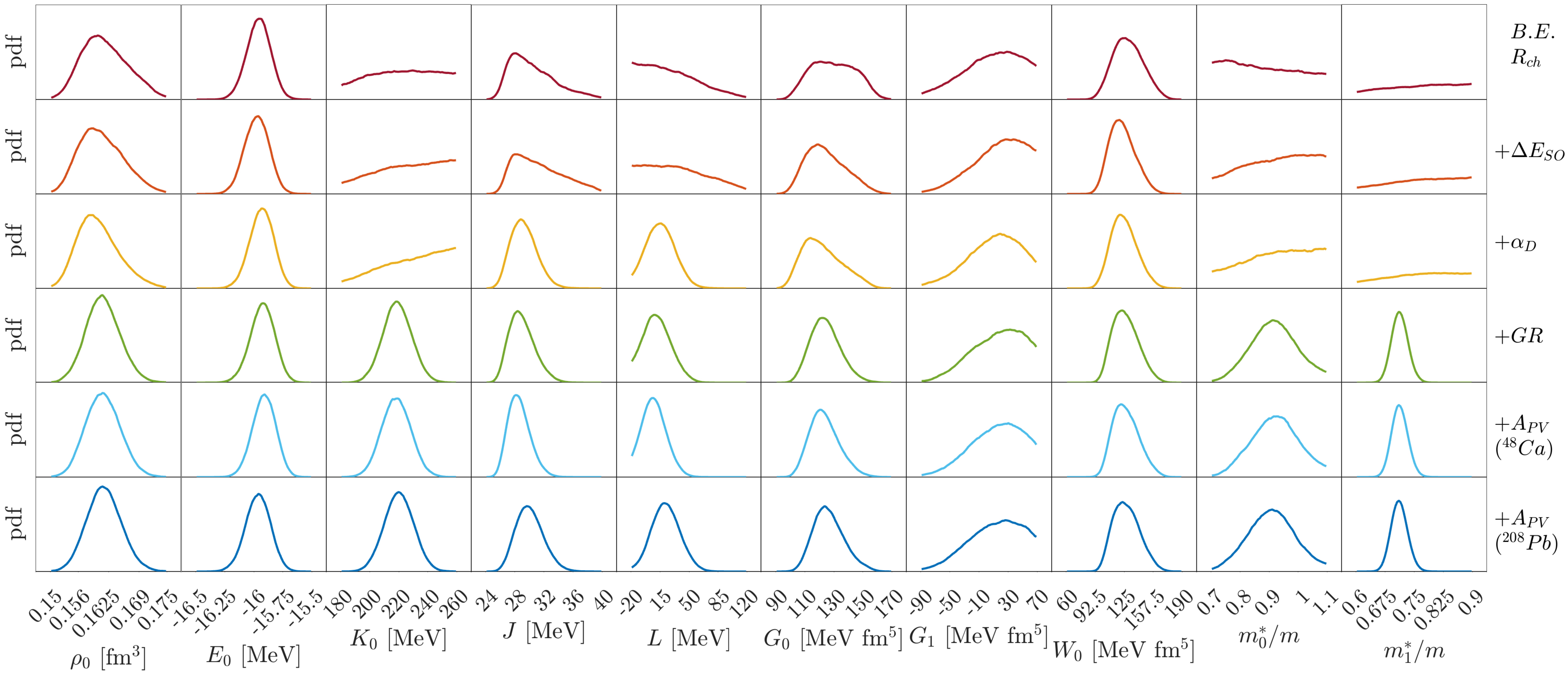

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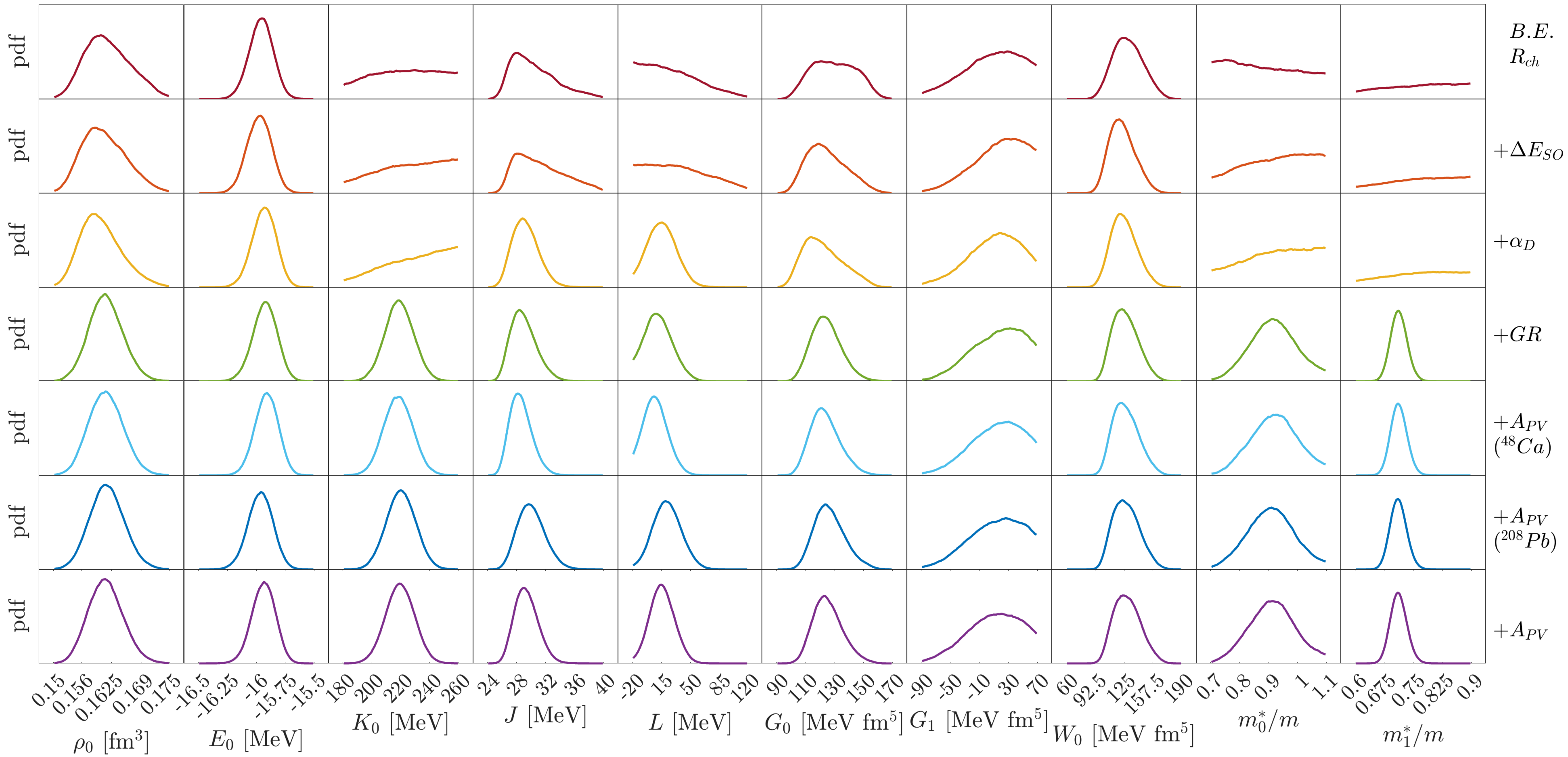
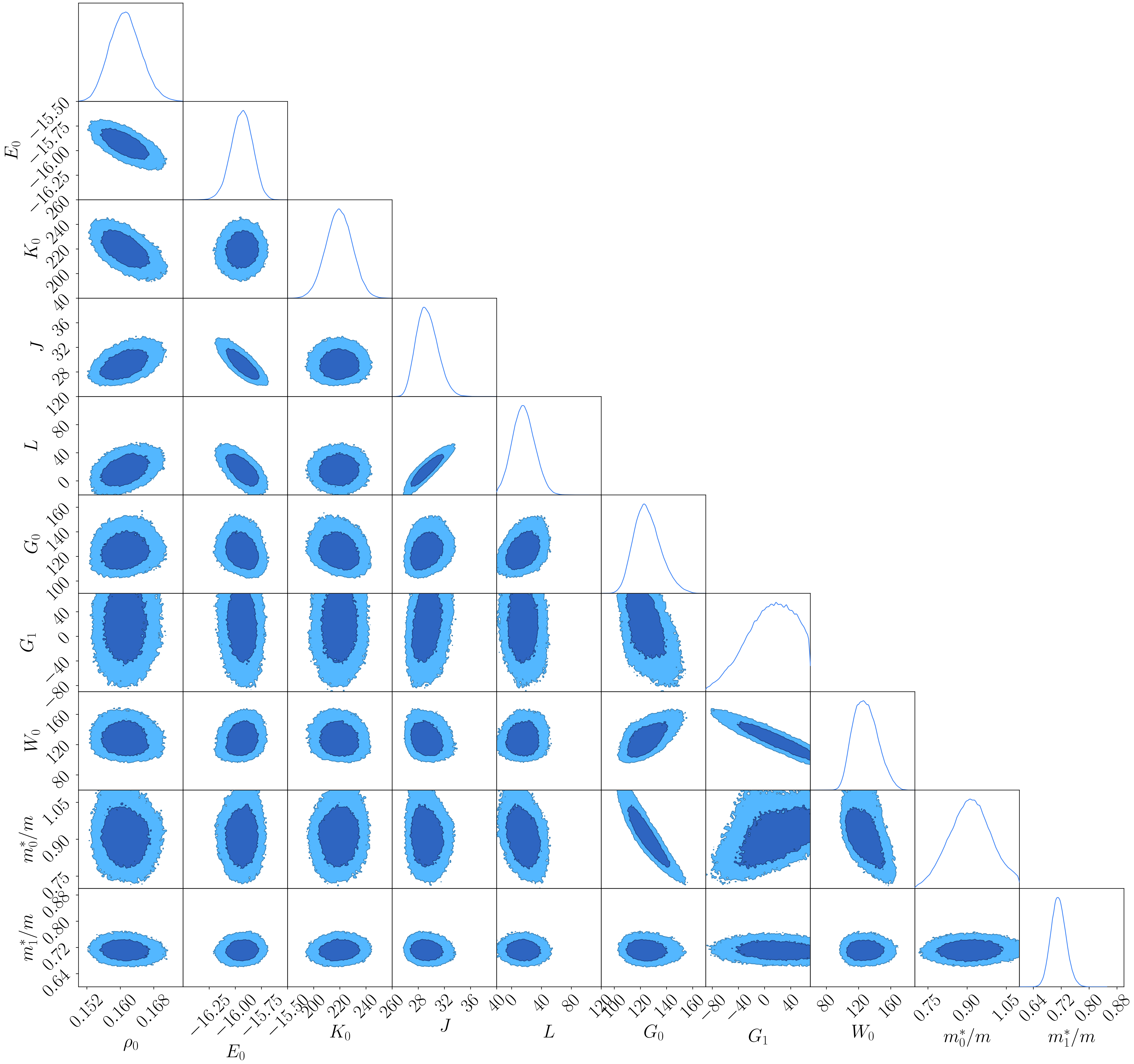


Corner plot and mean values



Parameter		μ	σ
ρ_0	[fm^3]	0.161	0.004
E_0	[MeV]	-15.938	0.102
K_0	[MeV]	219.483	10.007
J	[MeV]	29.378	1.626
L	[MeV]	16.136	14.732
G_0	[MeV fm^5]	125.470	10.210
G_1	[MeV fm^5]	9.439	35.735
W_0	[MeV fm^5]	128.719	14.848
m_0^*/m		0.913	0.079
m_1^*/m		0.712	0.021

Posterior observables means and uncertainties

$|\mu_{exp} - \mu_{theo}|$ in units of $\sigma_c = \sqrt{\sigma_{exp}^2 + \sigma_{theo}^2}$

(orange) : $[1,2)\ \sigma_c$

(red) : $[2,\infty)\ \sigma_c$

Inference

	Ground-state properties		
	$B.E.$ [MeV]	$R_{\rm ch}$ [fm]	$\Delta E_{\rm SO}$ [MeV]
^{208}Pb	1636 ± 1.8	5.49 ± 0.03	2.34 ± 0.16 (highlighted)
^{48}Ca	417 ± 1.2	3.51 ± 0.02	1.92 ± 0.20
^{40}Ca	342 ± 1.6	3.50 ± 0.02	-
^{56}Ni	482 ± 1.4 (highlighted)	-	-
^{68}Ni	590 ± 1.0	-	-
^{100}Sn	826 ± 1.6	-	-
^{132}Sn	1103 ± 1.7	4.71 ± 0.03	-
^{90}Zr	784 ± 1.3	4.27 ± 0.02	-

Experiment

	Ground-state properties		
	$B.E.$[1] [MeV]	$R_{\rm ch}$[2] [fm]	$\Delta E_{\rm SO}$[3] [MeV]
^{208}Pb	$1636.4 \pm 1 \times 10^{-3}$	5.50 ± 0.001	1.96 ± 0.05
^{48}Ca	$416.0 \pm 2 \times 10^{-5}$	3.48 ± 0.002	1.72 ± 0.05
^{40}Ca	$342.1 \pm 4 \times 10^{-5}$	3.48 ± 0.002	-
^{56}Ni	$484.0 \pm 1 \times 10^{-3}$	-	-
^{68}Ni	$590.4 \pm 4 \times 10^{-4}$	-	-
^{100}Sn	825.2 ± 0.25	-	-
^{132}Sn	$1102.8 \pm 1 \times 10^{-3}$	4.71 ± 0.002	-
^{90}Zr	$783.9 \pm 1 \times 10^{-4}$	4.27 ± 0.001	-

[1]Meng Wang *et al* 2021 *Chinese Phys. C* **45** 030003

[2]Angeli et al., Atomic Data and Nuclear Data Tables 99 (2013)

[3]Zalewski et al., Phys. Rev. C **77**, 024316 (2008)

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Inference

Isoscalar resonances		
	$E^{\mathrm{IS}}_{\mathrm{GMR}}$ [MeV]	$E^{\mathrm{IS}}_{\mathrm{GQR}}$ [MeV]
^{208}Pb	13.5 ± 0.3	10.8 ± 0.4
^{90}Zr	17.8 ± 0.4	-

Experiment

Isoscalar resonances		
	$E^{\mathrm{IS}}_{\mathrm{GMR}}$ [1,2] [MeV]	$E^{\mathrm{IS}}_{\mathrm{GQR}}$ [3] [MeV]
^{208}Pb	13.5 ± 0.1	10.9 ± 0.3
^{90}Zr	17.7 ± 0.07	-

[1] D. Patel et al., Physics Letters B 726 (2013)

[2] Y.K. Gupta et al., Physics Letters B 760 (2016)

[3] Youngblood et al., Phys. Rev. C 69, 034315 (2004)

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(orange) : $[1,2)\ \sigma_c$

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Inference

	Isovector properties		
	α_D [fm^3]	$m(1)$ [MeV fm^2]	A_{PV} [p.p.b.]
^{208}Pb	19.5 ± 0.5	958 ± 22	589 ± 5
^{48}Ca	2.30 ± 0.08	-	2591 ± 54

Experiment

	Isovector properties		
	α_D [fm^3]	$m(1)$ [MeV fm^2]	A_{PV} (ppb)
^{208}Pb	19.60 ± 0.60	961 ± 22	550 ± 18
^{48}Ca	2.07 ± 0.22	-	2668 ± 113

[1]Tamii et al., PRL 107, 062502 (2011)

[2]Birkhan et al., PRL 118, 252501 (2017)

[3]S. GORIELY et al., Phys. Rev. C 102, 064309 (2020)

[4]PREX Collaboration, Phys. Rev. Lett. 126, 172502 (2021)

[5]CREX Collaboration, Phys. Rev. Lett. 129, 042501 (2022)

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Meta-Model nuclear equation of state

Meta-Model (M.M.): Taylor expansion of the nuclear equation of state around saturation[1]

M.M.

↓

Neutron star EoS

↓

Neutron star observables!

[1]Margueron et al., Phys. Rev. C **97**, 025805 (2018)

Second Bayesian inference: Parameters & Constraints

Parameters and prior distribution:

$\rho_0, E_0, K_0, J, L, m_0^*/m, m_1^*/m$

K_{sym}

$Q_0, Z_0, Q_{sym}, Z_{sym}$

Second Bayesian inference: Parameters & Constraints

Parameters and prior distribution:

$\rho_0, E_0, K_0, J, L, m_0^*/m, m_1^*/m$ Previous Posterior distribution

K_{sym}

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Second Bayesian inference: Parameters & Constraints

Parameters and prior distribution:

$\rho_0, E_0, K_0, J, L, m_0^*/m, m_1^*/m$

Previous Posterior distribution

K_{sym}

$K_{sym} = K_{sym}(\rho_0, E_0, K_0, \ldots)$

$Q_0, Z_0, Q_{sym}, Z_{sym}$

Second Bayesian inference: Parameters & Constraints

Parameters and prior distribution:

$\rho_0, E_0, K_0, J, L, m_0^*/m, m_1^*/m$ — Previous Posterior distribution

K_{sym} — $K_{sym} = K_{sym}(\rho_0, E_0, K_0, \ldots) \longrightarrow$ Not a free parameter!

$Q_0, Z_0, Q_{sym}, Z_{sym}$

Second Bayesian inference: Parameters & Constraints

Parameters and prior distribution:

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$Q_0, Z_0, Q_{sym}, Z_{sym}$	Uniform distribution

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Observational constraints:

- Maximum observed mass of Neutron Star;
- Ligo-Virgo-Collaboration tidal deformability results;
- NICER mission simultaneous mass-radius measurements
- Ab-initio computations of neutron matter at low density

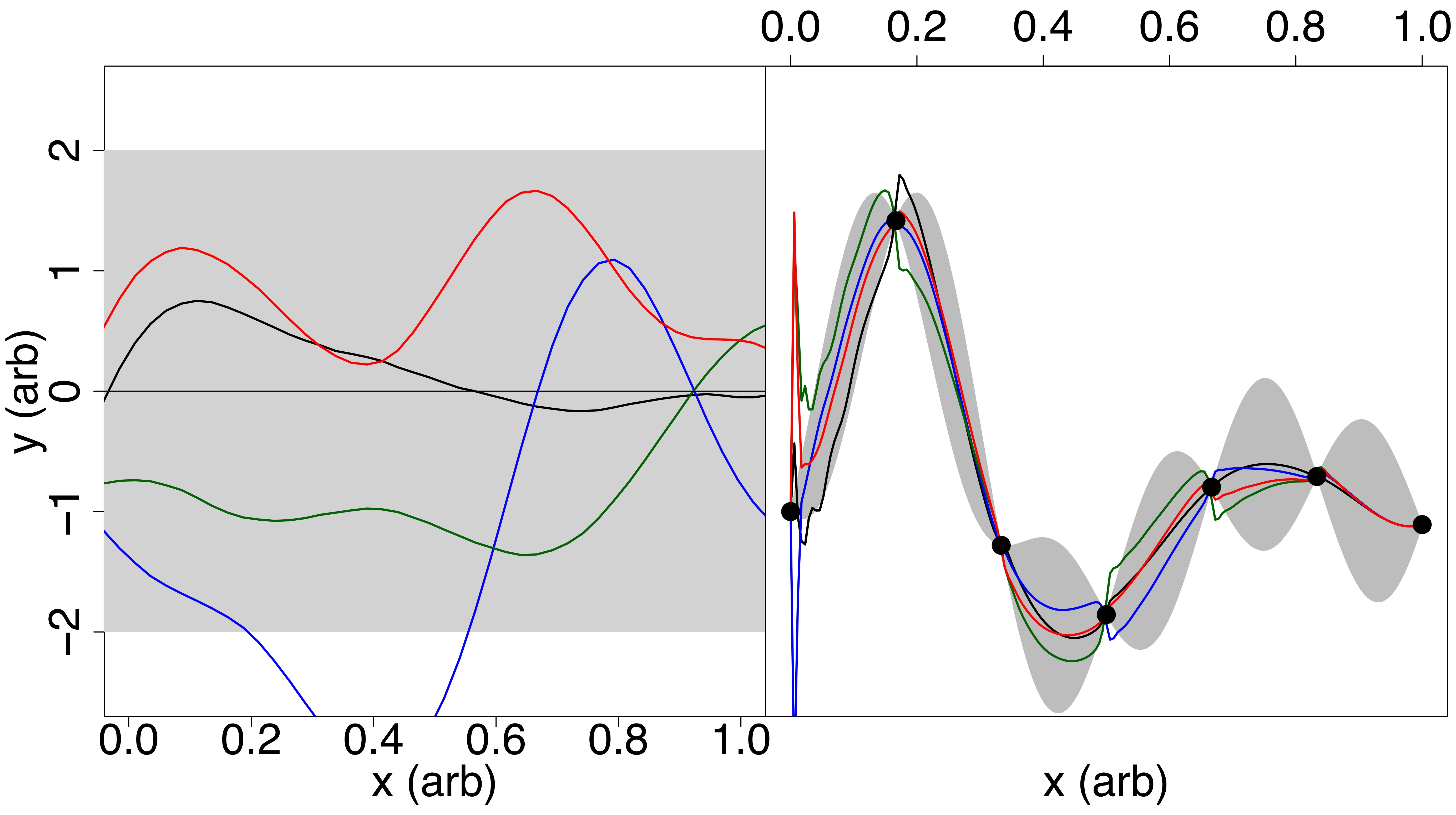
Marginalized posteriors

Conclusions

- Bayesian statistical analysis on nuclear matter parameters with nuclear experiments :
  - Skyrme ansatz
  - Fit with observables of different types (ground state, giant resonances,…)
  - Result: a robust posterior distribution of the (nuclear matter) parameters
  - Our protocol could describe the observables we chose; the only tension is with A_{PV} of ^{208}Pb

- Bayesian statistical analysis on nuclear matter parameters with neutron star observations:
  - Final distribution of parameters informed by both nuclear physics and neutron star observations!

Gaussian process (GP) emulator

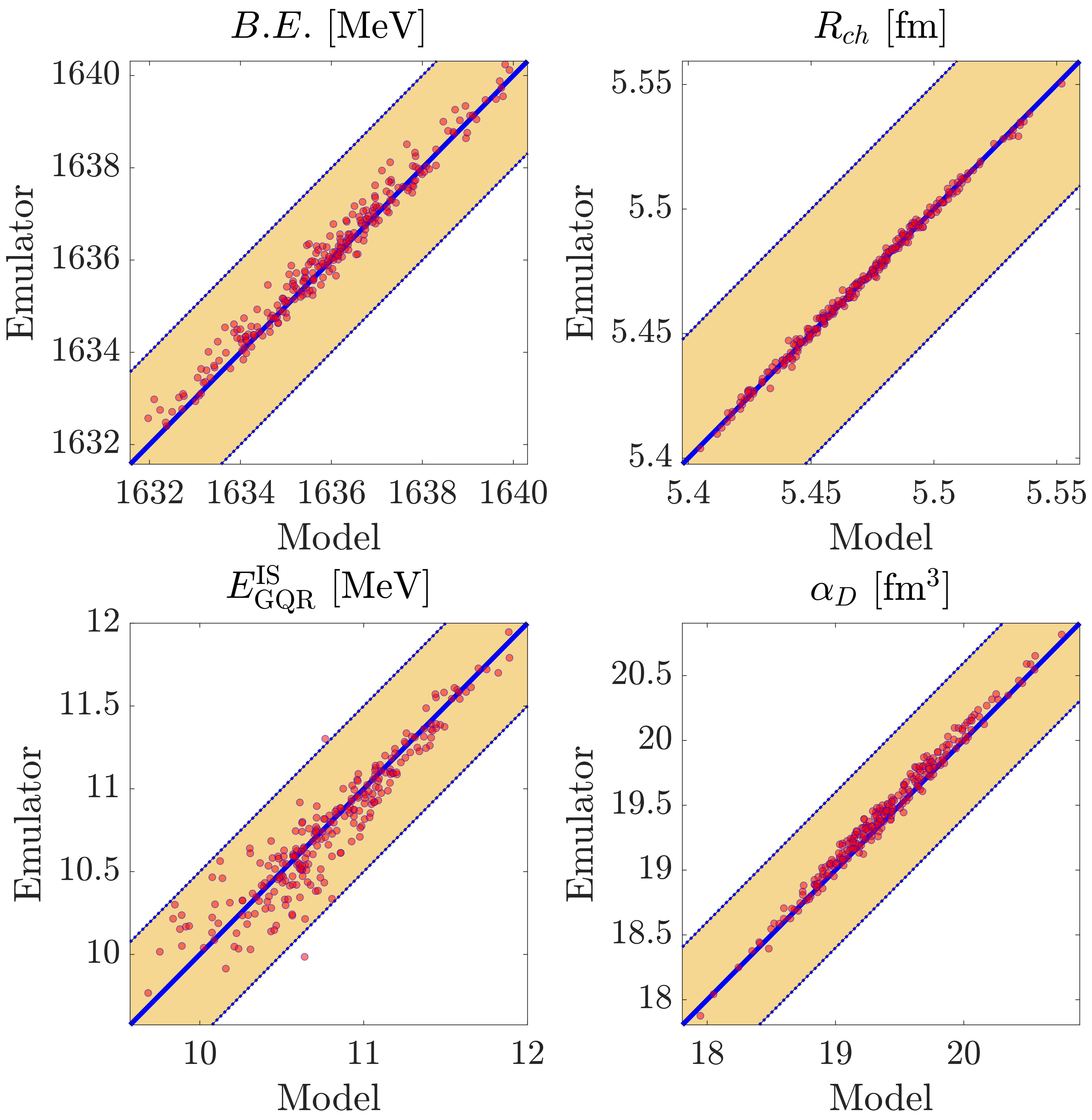


From MADAI user manual

The MADAI package:

- was built for GP applied to bayesian inference
- given the parameters prior distributions, it automatically builds the grid
- it does a MCMC to estimate the posterior distribution
- it extracts parameters sample following the posteriors

Validation

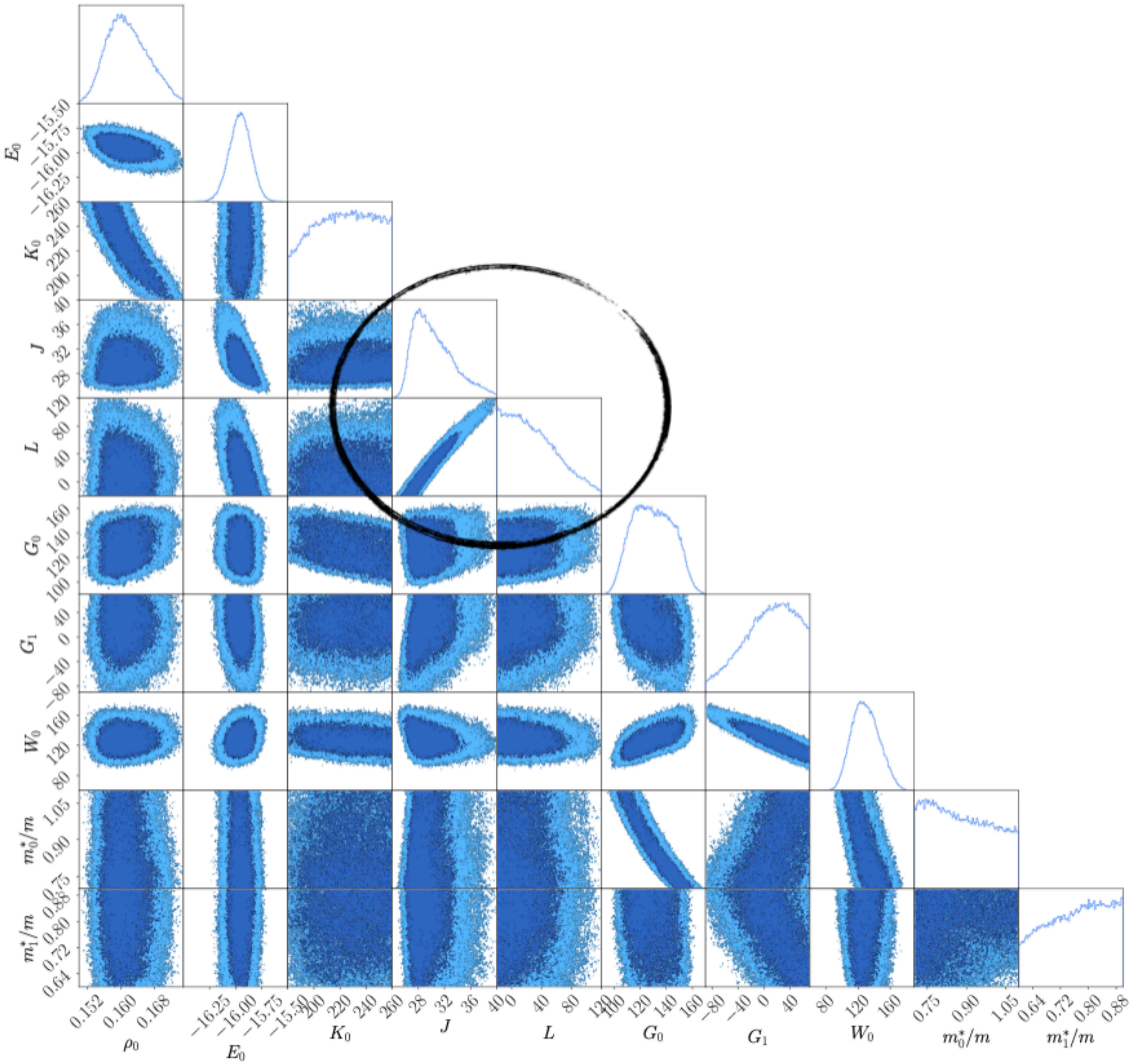


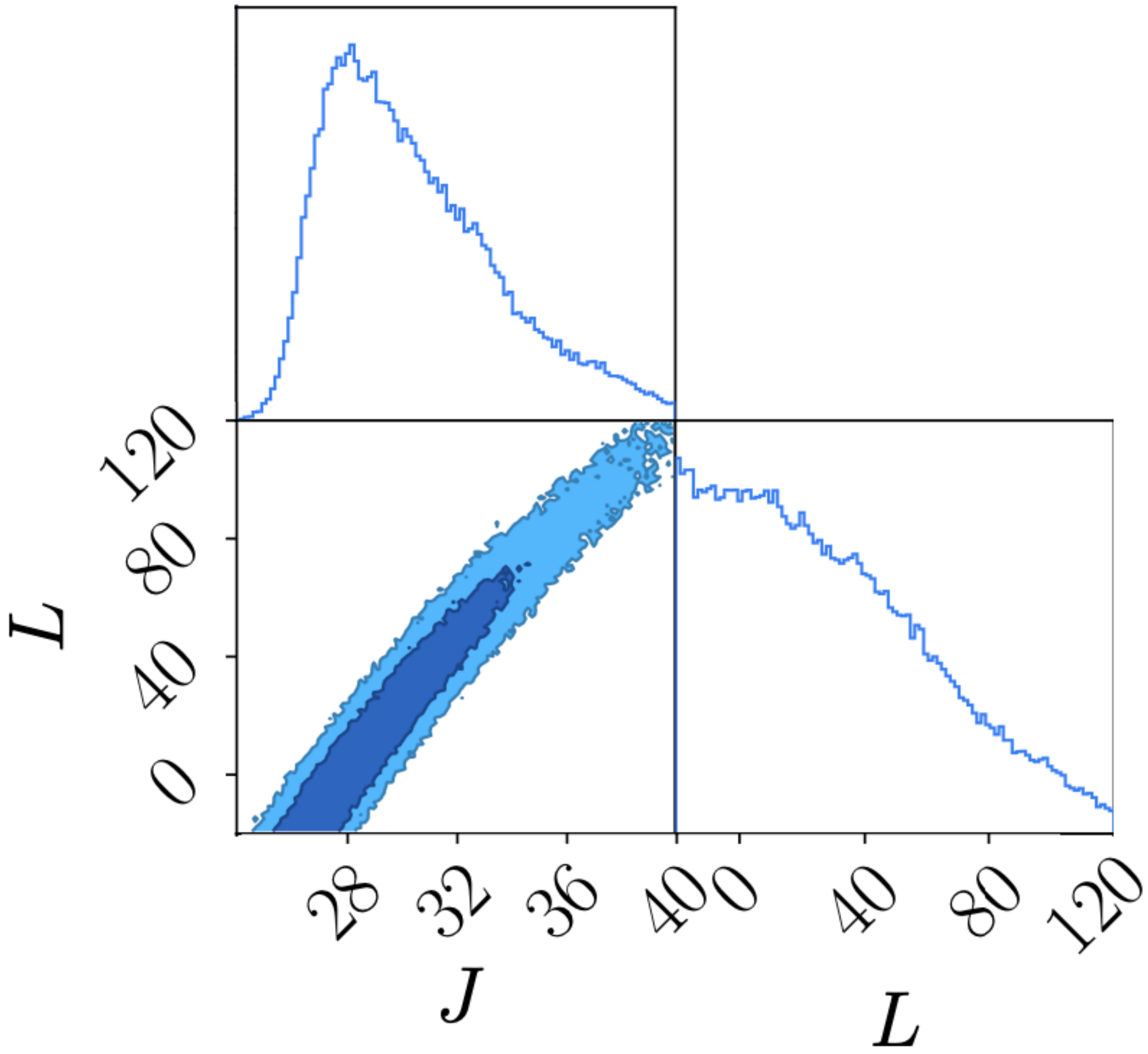
Ground-state properties						
	Discrepancy			Corr. coefficient		
	$B.E.$	$R_{\rm ch}$	$\Delta E_{\rm SO}$	$B.E.$	$R_{\rm ch}$	$\Delta E_{\rm SO}$
^{208}Pb	0 %	0 %	0 %	0.993	1.000	0.997
^{48}Ca	0 %	0 %	0 %	0.998	0.999	0.998
^{40}Ca	0 %	0 %	-	0.999	0.999	-
^{56}Ni	0 %	-	-	0.996	-	-
^{68}Ni	0 %	-	-	0.994	-	-
^{100}Sn	0 %	-	-	0.994	-	-
^{132}Sn	0 %	0 %	-	0.992	1.000	-
^{90}Zr	0 %	0 %	-	0.996	1.000	-

Isoscalar resonances				
	Discrepancy		Corr. coefficient	
	$E^{\rm IS}_{\rm GMR}$	$E^{\rm IS}_{\rm GQR}$	$E^{\rm IS}_{\rm GMR}$	$E^{\rm IS}_{\rm GQR}$
^{208}Pb	0 %	1.0 %	1.000	0.904
^{90}Zr	0 %	-	1.000	-

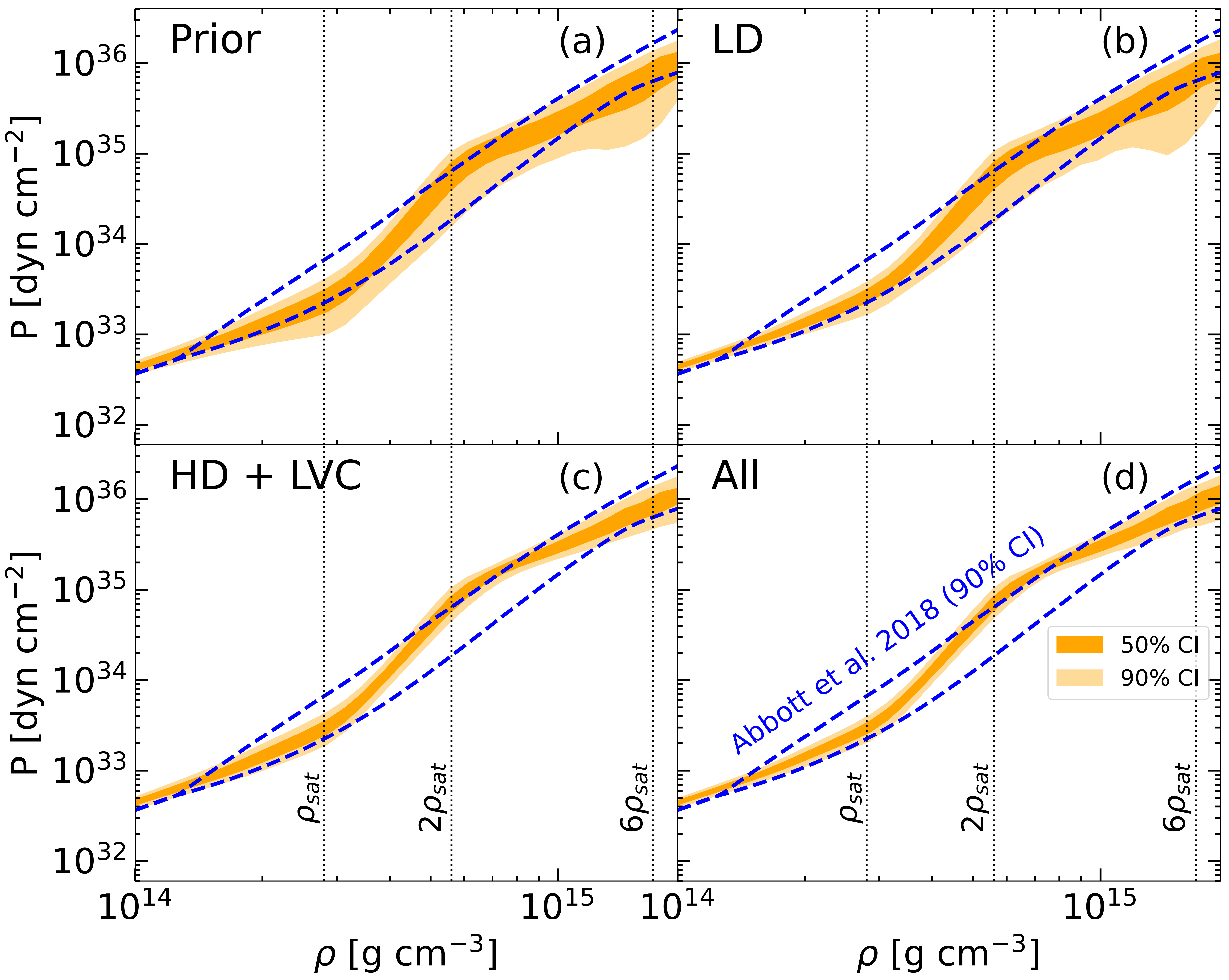
Isovector properties						
	Discrepancy			Corr. coefficient		
	α_D	$m(1)$	A_{PV}	α_D	$m(1)$	A_{PV}
^{208}Pb	0 %	0 %	0 %	0.988	0.9999	0.998
^{48}Ca	0 %	-	0 %	0.990	-	0.9992

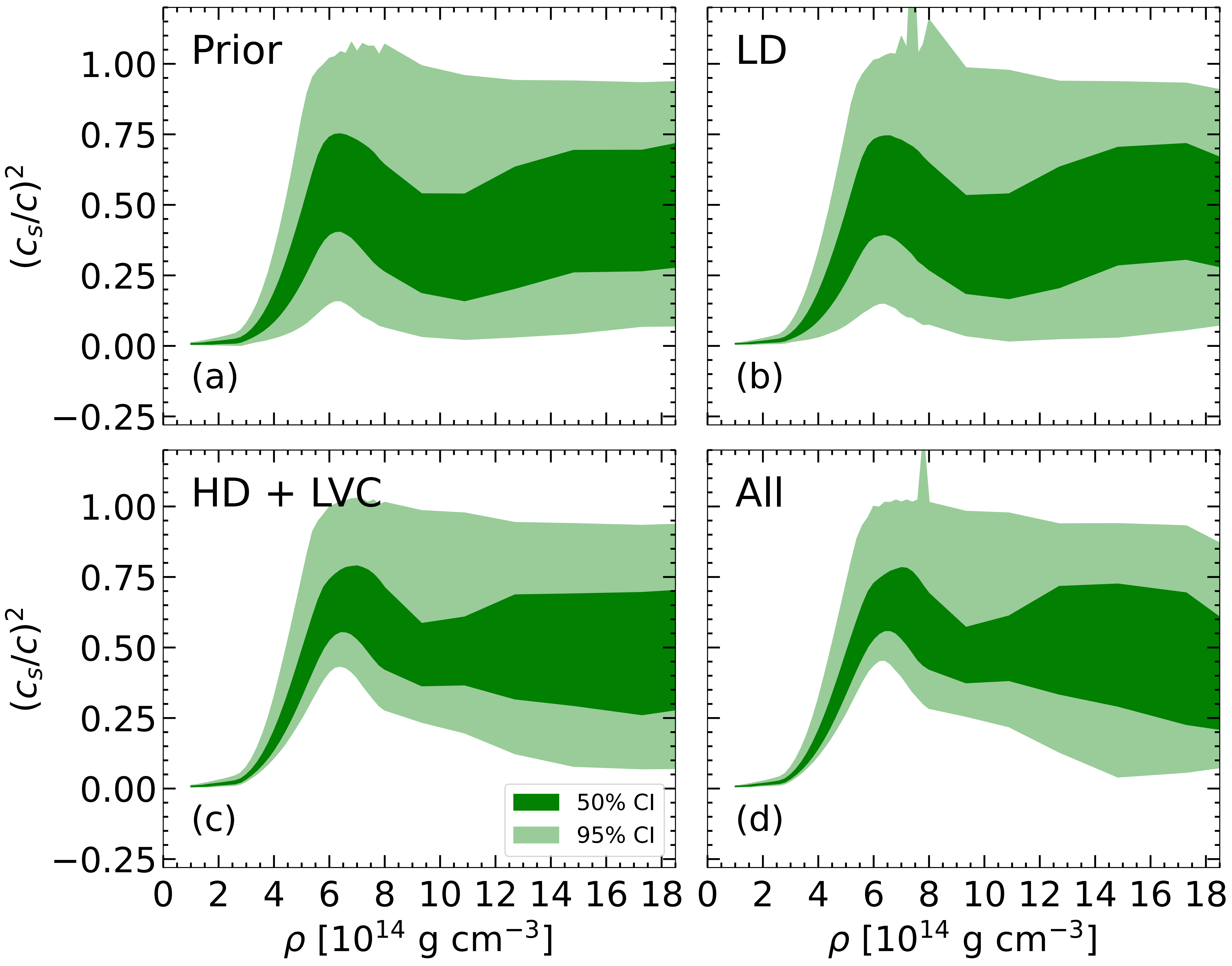
$B.E., R_{ch}$ only corner plot





Equation of state and sound speed posterior distributions





B4

Mass-Radius relation and $\Lambda_{1.4M_\odot}$, $\Lambda_{2.0M_\odot}$ posterior distributions