Heavy-Ion Collisions and the low-density Neutron Star Equation of State: from the lab. to space.

"Valid treatment of the correlations and clusterization in low density matter"

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PHC
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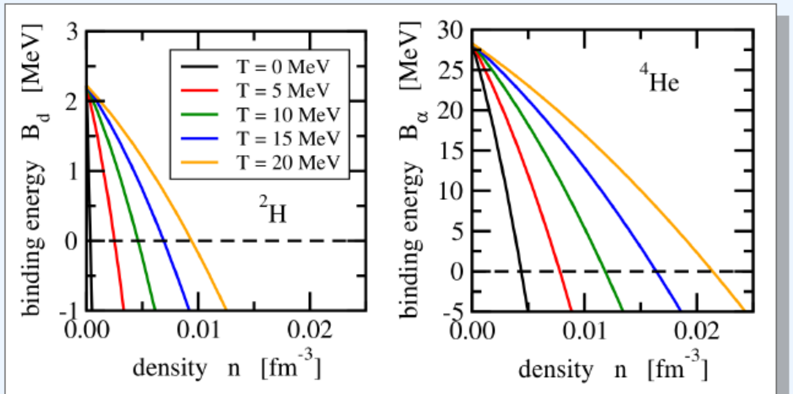
"Valid treatment of the correlations and clusterization in low density matter"

***In-medium* effects:**
Surrounding nuclear medium modify light cluster properties. Dissolution of clusters due to Pauli blocking (density).
G. Röpke publications
Implication for core-collapse supernovae dynamics:
modification of light clusters can affect the neutrinos and shock wave propagation
Arcones et al. PRC, 2008



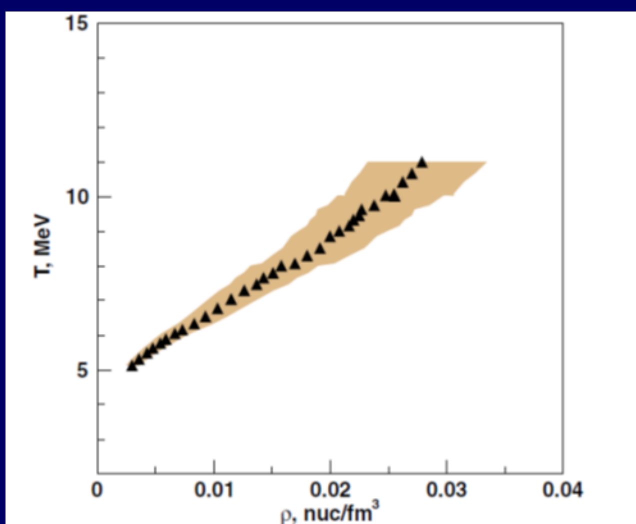
Typel, Phys. Rev. C **2010**

Many-body theory

Cluster formation modify the EOS at subsaturation density

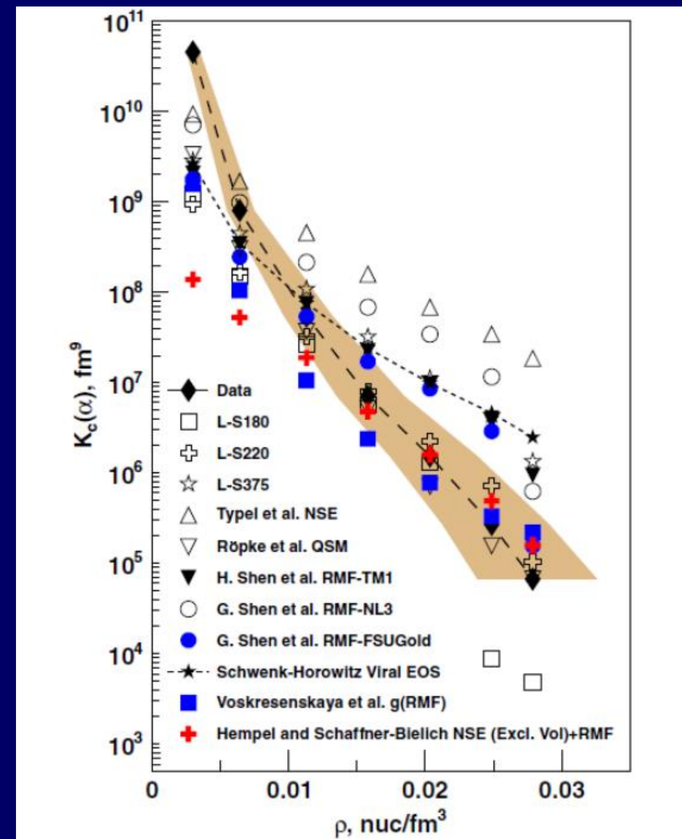
Texas A&M: equilibrium constant, K_c

Using Heavy-Ion collisions corresponding to central events and selecting mid-rapidity region and selecting high energetic particles:
It is possible to select events corresponding to different thermodynamical characteristics of a gas of nucleons and clusters (^2H, ^3H, ^3He and ^4He).



Low densities

L. Qin et al. ***PRL****108 (2012) 172701*

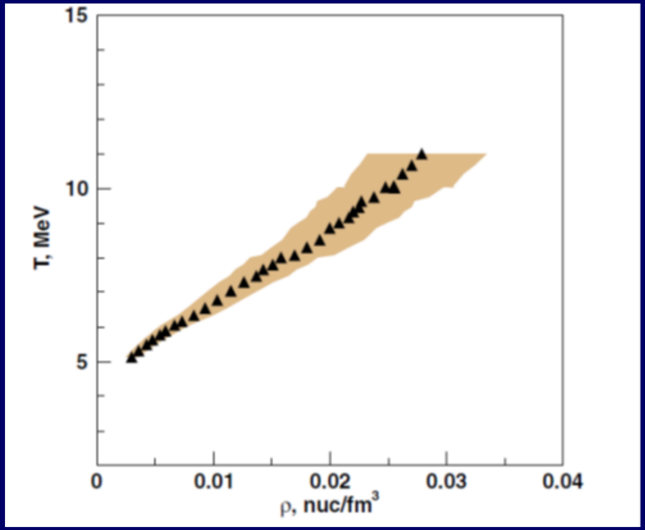


Data versus Model:
in-medium effects (the properties of nucleons in clusters do not correspond to the properties of free nucleons).

How to evaluate T and ρ ?

Equilibrium – Ideal gas

- **S. Das Gupta and A.Z. Mekjian Phys. Rep. 72 (1981) 131**
- **S. Albergo et al. Nuovo Cimento 89 (1985) 1**



L. Qin et al. ***PRL****108 (2012) 172701*

For each evolution interval (Coulomb corrected particle velocity):

1- Temperature: from Yields (^{2}H ^{4}He)/(^{3}H ^{3}He)

$$T = \frac{B(4,2)+B(2,1)-B(3,2)-B(3,1)}{\ln(\sqrt{9/8}(1.59\ R_{v_{surf}}))} MeV \text{ with } R_{v_{surf}} = \frac{M(2,1)M(4,2)}{M(3,1)M(3,2)}$$

2- Neutrons: from Yields (^{3}H/^{3}He)

$$(N/Z)_{free} = \frac{M(3,1)}{M(3,2)}\ e^{((B(3,2)-B(3,1))/T)}$$

3- Momentum space density Power law:

$$\frac{d^3M(A,Z)}{d^3p_A} = R_{np}^{N} \frac{(2s+1)\ e^{B(A,Z)/T}}{2^A} \left(\frac{h^3}{V_0}\right)^{A-1} \left(\frac{d^3M(1,1)}{d^3p}\right)^A$$

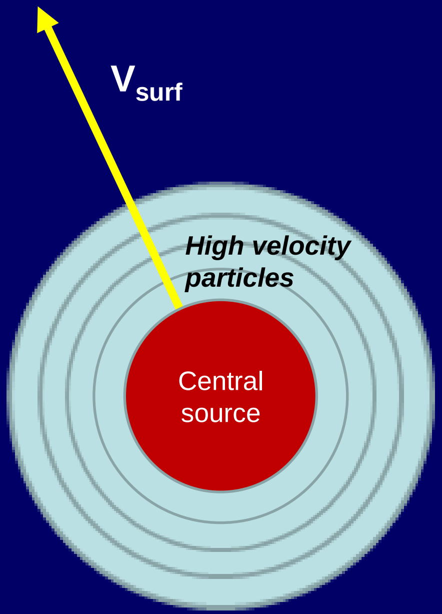
Cluster momentum spectrum versus (proton momentum spectrum)^A
(neutron spect. = proton spect., Coulomb correction)

VOLUME measurement ➔ DENSITY

Evolution intervals defined by V_{surf}

V_{surf}: surface velocity which is particle velocity corrected by Coulomb effects from « central » source.

V_{surf} slices: different ensembles (Temperature, density).



Chemical composition of V_{surf} slices: neutrons, protons, ^{2}H, ^{3}H, ^{3}He, ^{4}He (high velocity particles).

What is equilibrium constant, K_c ?

Law of mass action (Guldberg et Waage)

- **Equilibrium, same phase.**

$$\alpha A + \beta B \leftrightarrow \gamma C + \delta D$$

- **Constant Kc is relative to concentrations and stoichiometric coef.**

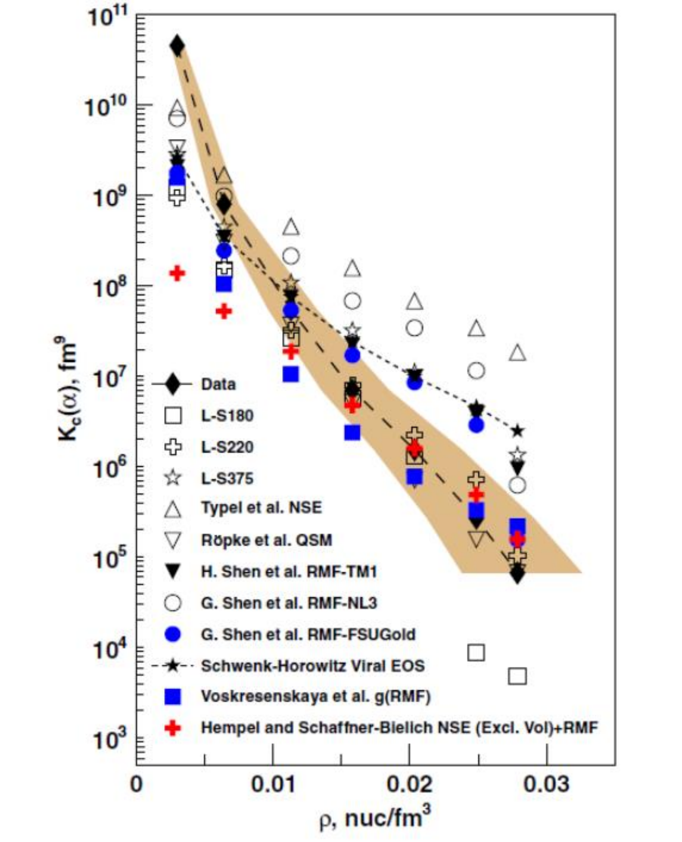
$$K_c = ([C]^{\gamma}.[D]^{\delta})/([A]^{\alpha}.[B]^{\beta})$$

For a gas of protons & neutrons in equilibrium with clusters,

$$Z\ {}^1_1H + (A - Z)\ {}^1_0n \leftrightarrow {}^A_ZX$$

$$K_c(A, Z) = \frac{\rho(A, Z)}{\rho_p^Z \rho_n^{(A-Z)}}$$

The equilibrium constant is a universal characteristics

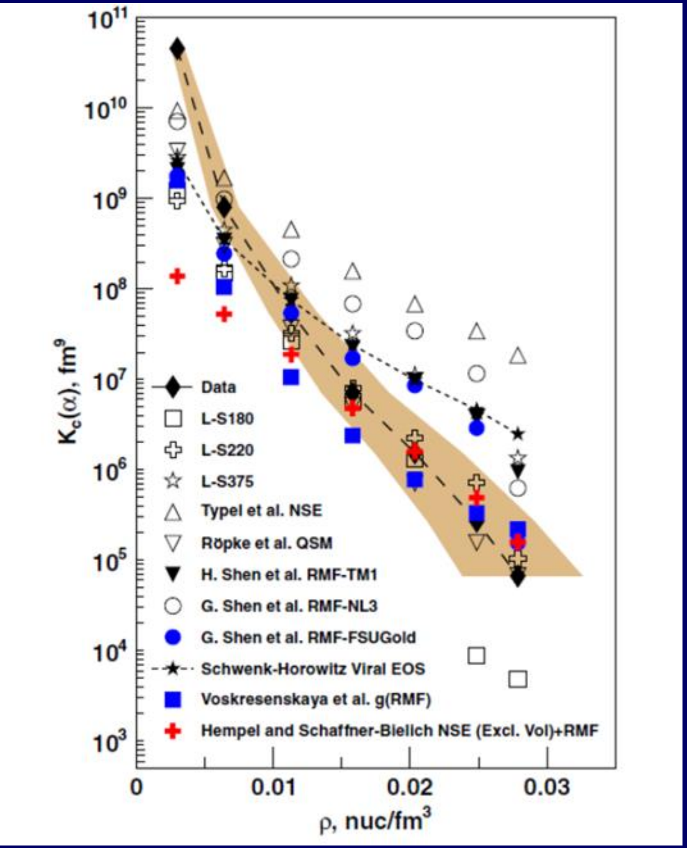


Equilibrium constant in terms of Mass Fractions and Volume

$$K_c(A, Z) = \frac{\omega_{AZ}}{A\omega_{11}^Z \omega_{10}^{A-Z}} \left(\frac{V_T}{A_T}\right)^{A-1}$$

What is wrong from our viewpoint

In-medium effects



L. Qin et al. ***PRL****108 (2012) 172701*

Equilibrium – Ideal gas

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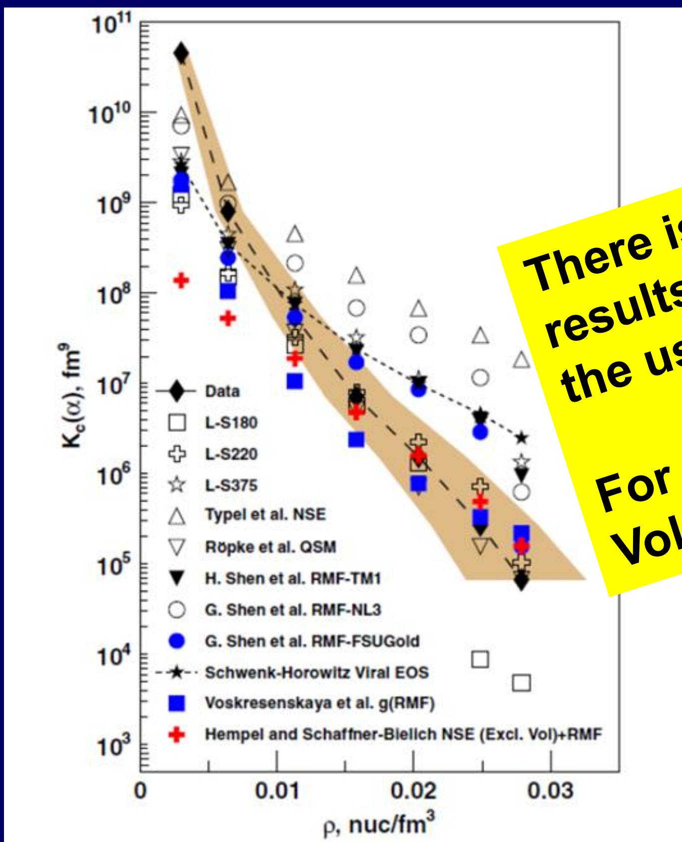
Cluster momentum spectrum versus (proton momentum spectrum)^A
(neutron spect. = proton spect., Coulomb correction)

VOLUME measurement → DENSITY

What is wrong from our viewpoint

In-medium effects



L. Qin et al. ***PRL****108 (2012) 172701*

Equilibrium – Ideal gas

1- Temperature: ... e)

There is a fundamental contradiction between the results indicating that there are in-medium effects and the use of ideal gas formulae.

For example, the used Binding Energies to extract the Volume values are the Vacuum Binding Energies

$$\frac{\cdots}{M(3,2)}\, e^{((B(3,2)-B(3,1))/T)}$$

...omentum space density Power law:

$$\frac{d^3M(A,Z)}{d^3p_A} = R_{np}^{N} \frac{(2s+1)\ e^{B(A,Z)/T}}{2^A} \left(\frac{h^3}{V_0}\right)^{A-1} \left(\frac{d^3M(1,1)}{d^3p}\right)^{A}$$

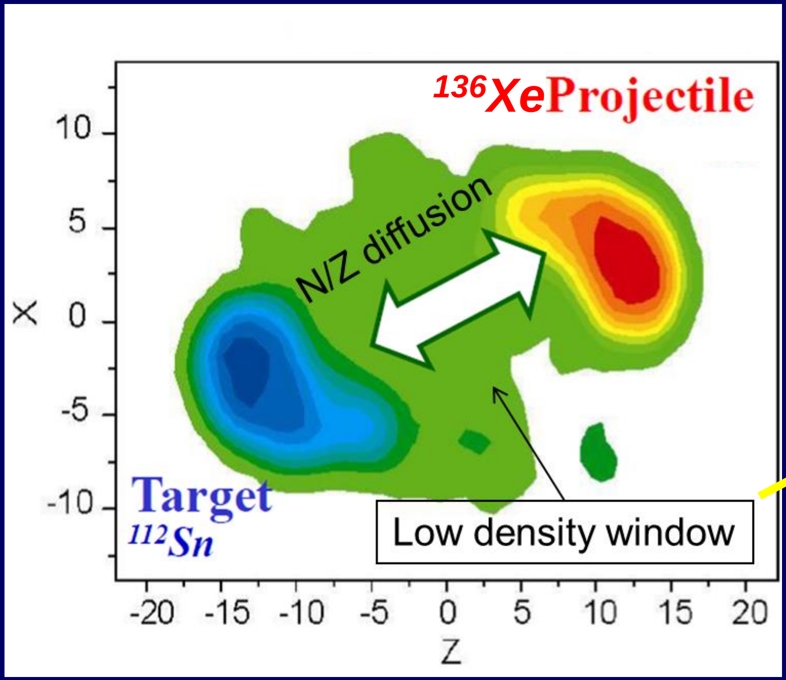
Cluster momentum spectrum versus (proton momentum spectrum)^A
(neutron spect. = proton spect., Coulomb correction)

VOLUME measurement ➔ DENSITY

New analysis with:

- Another set of data
- Relativistic Meam-Field Model because the only way to highlight in-medium effects is to use a model.

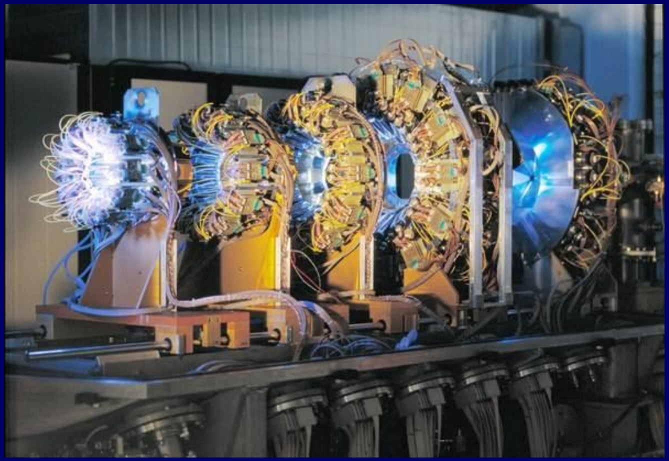
INDRA data



STUDY of a Gas composed of light clusters formed in central collisions

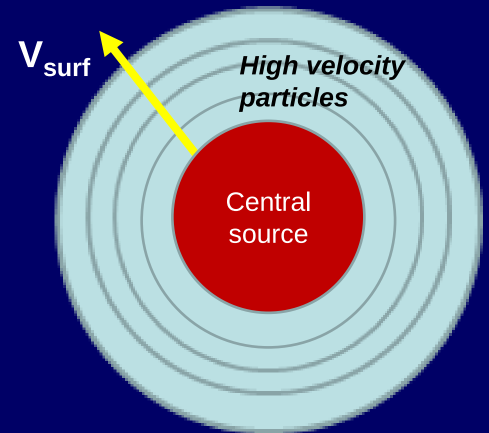
INDRA@GANIL

$^{136,124}Xe+^{124,112}Sn$ 32 A MeV



INDRA data:

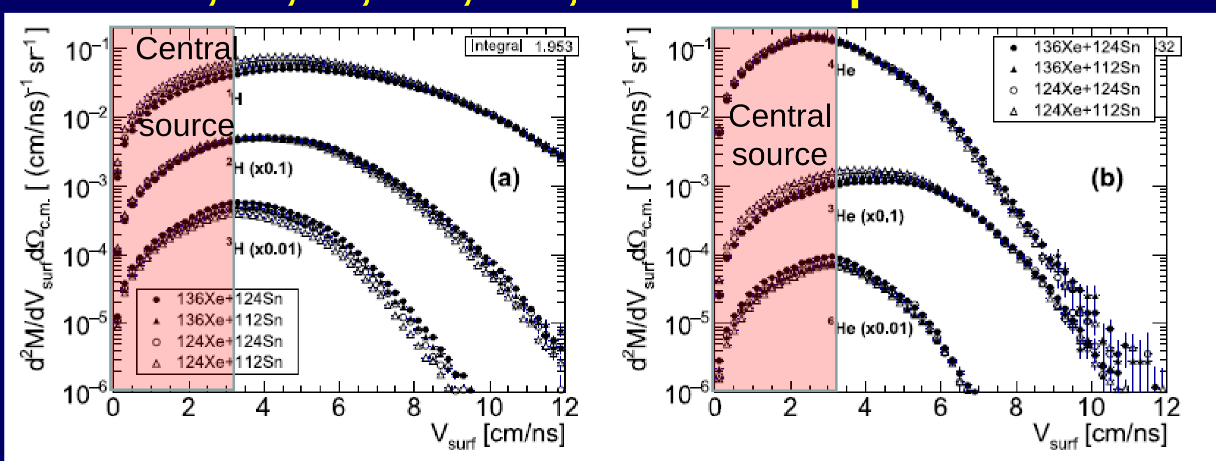
V_{surf}: surface velocity which is particle velocity corrected by Coulomb effects from « central » source.



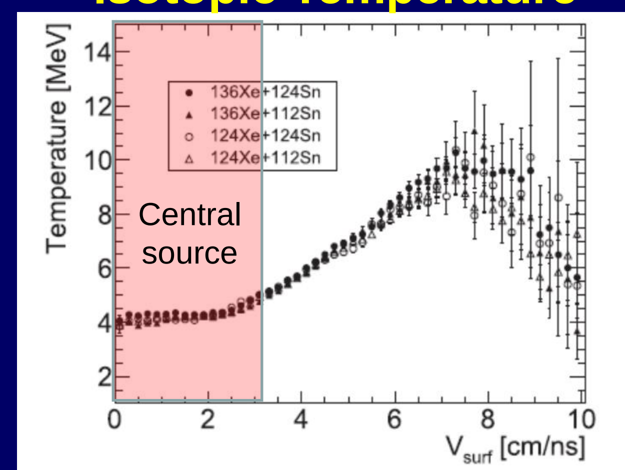
V_{surf} slices: different ensembles (Temperature, density).

Chemical composition of V_{surf} slices: neutrons, protons, ^2H, ^3H, ^3He, ^4He, ^6He (high velocity particles).

^1H, ^2H, ^3H, ^3He, ^4He, ^6He Vsurf spectra



Isotopic Temperature



Relativistic Mean-Field with clusters

RMF formalism

- **With nucleons and light clusters as independent quasi-particles**
- **In-medium effets of light clusters are taken into account.**
- **The interactions are mediated by the exchange of virtual mesons: the isoscalar-scalar σ-meson, the isoscalar-vector ω-meson, the isovector-vector ρ-meson.**

$$\mathcal{L} = \sum_{\substack{j=n,p,\\ {}^2\mathrm{H},{}^3\mathrm{H},\\ {}^3\mathrm{He},{}^4\mathrm{He}}} \mathcal{L}_j + \sum_{m=\sigma,\omega,\rho} \mathcal{L}_m + \mathcal{L}_{\omega\rho}$$

Lagrangian:

1. **n,p and clusters mesons interaction**
2. **Meson fields**
3. **Mixed meson term (ω and ρ mesons)**

The meson-cluster couplings are:

$$g_{\omega j} = A_j \, g_{\omega N}$$

$$g_{\sigma j} = x_s A_j g_{\sigma N}$$

Cluster « j » relative to Nucleon couplings:

- **A_j is cluster Mass**
- **X_s, the coupling ratio, measures the in-medium modification of the cluster properties.**

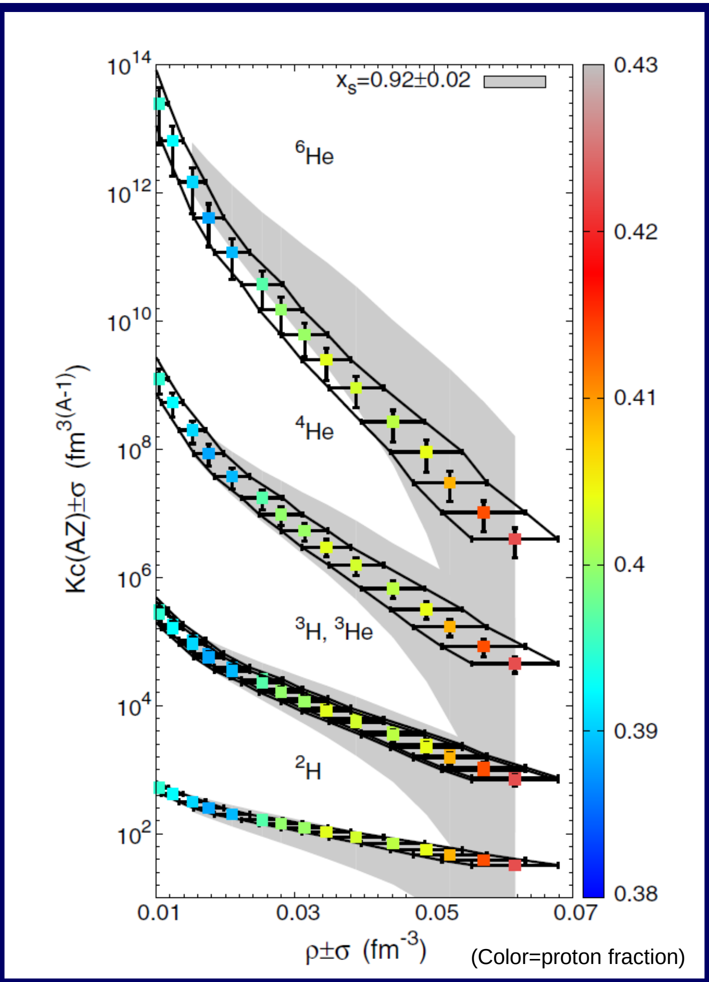
$0<X_s<1$ meams in-medium effects.

X_s(density, Temperature) is calibrated on experimental data.

H. Pais et al. PRC97, 045805 (2018) – H. Pais et al. PRC99, 055806 (2019)

Result of the analysis

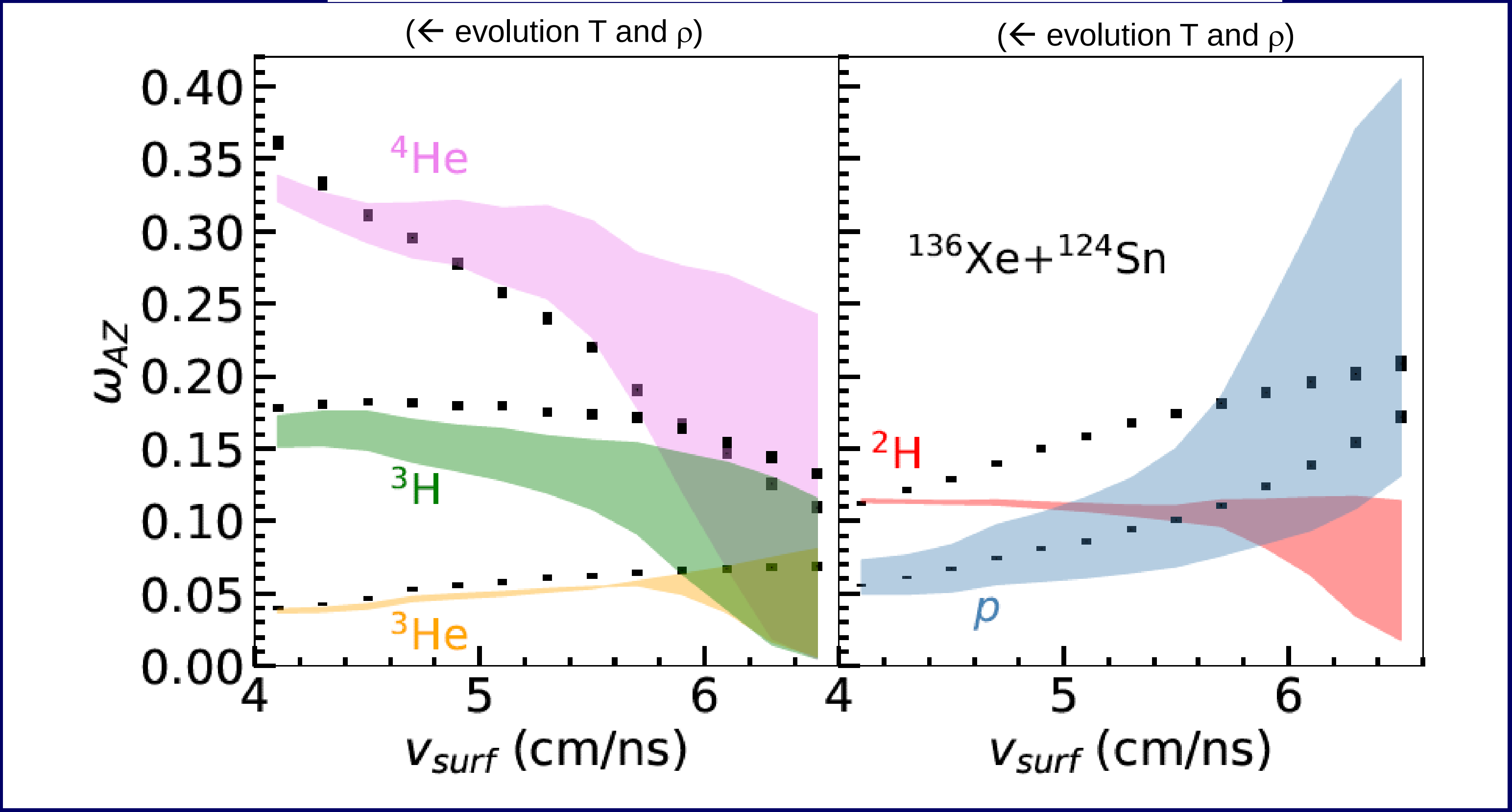
INDRA (points) versus RMF (grey)



H. Pais et al. ***J. Phys. G*** *47 (2020) 105204*
H. Pais et al. ***PRL*** *125 (2020) 012701*

But the Mass Fractions are not well reproduced

INDRA (points) versus RMF (color areas)



Big disagreement for ^{2}H, disagreement for ^{4}He, ^{3}H

T. Custodio, A. Rebillard-Soulié et al. ***submitted*** *(2024)*

Back to experimental data

We used measured mass fractions and RMF predictions

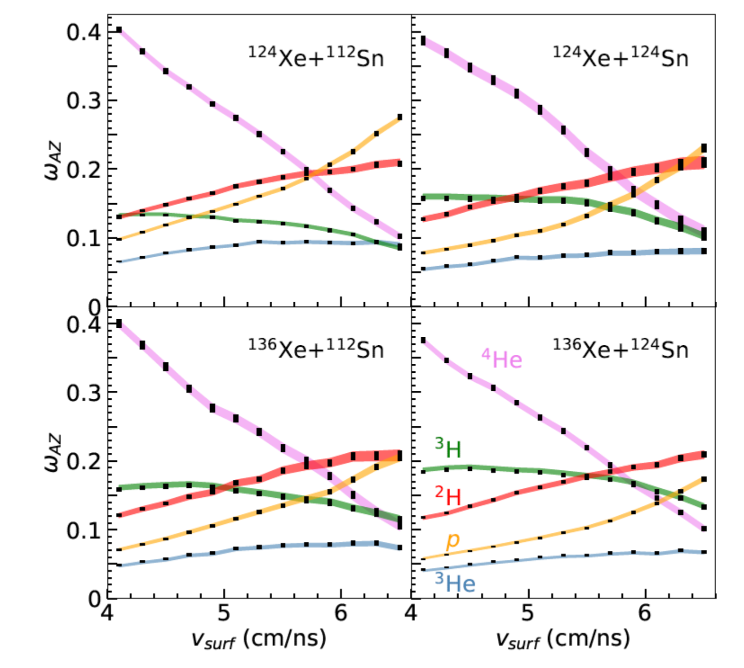
For each evolution (T,ρ) bin (V_{surf}) and each system ($^{124,136}Xe+^{124,112}Sn$), independent Bayesian inferences on the measured mass fractions were carried out.
Independent posterior distributions of the model parameters θ = (T, ρ, x_s) were obtained.

Marginalised posterior obtained by integrating on T, ρ and x_s

$$p_i\left(\theta|\{\omega_{AZ}\}\right) = \frac{p_\theta}{\mathcal{Z}}\mathcal{L}_{\rm g}\left(\{\omega_{AZ}\}_i|\theta\right)$$

where p_θ is a flat prior and L_g is a gaussain likehood.

INDRA (points) vs RMF (color area)

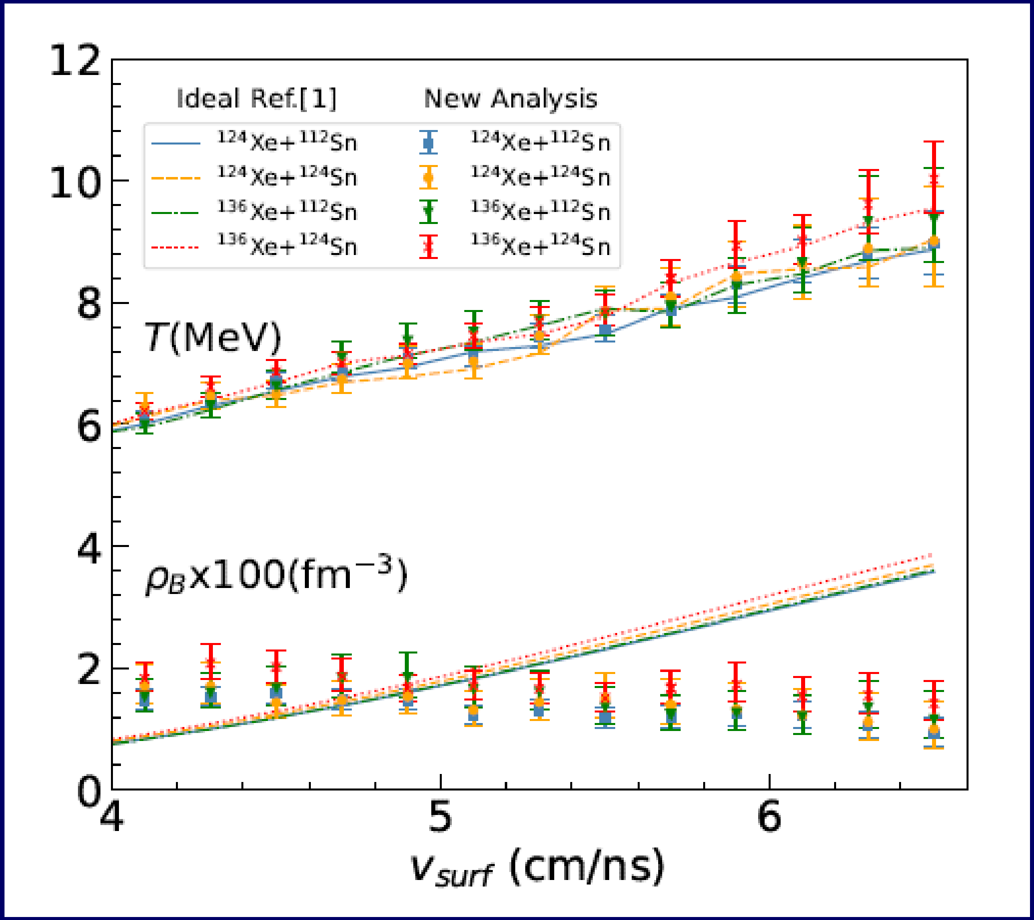


Calibration using Mass Fractions
Marginalised posteriors versus INDRA data
(2 σ uncertainties)

T. Custodio, A. Rebillard-Soulié et al. ***submitted*** *(2024)*

Bayesian inference results: T and ρ

Mean values
(points: Bayesian, lines Ideal Gas)



Conclusions:

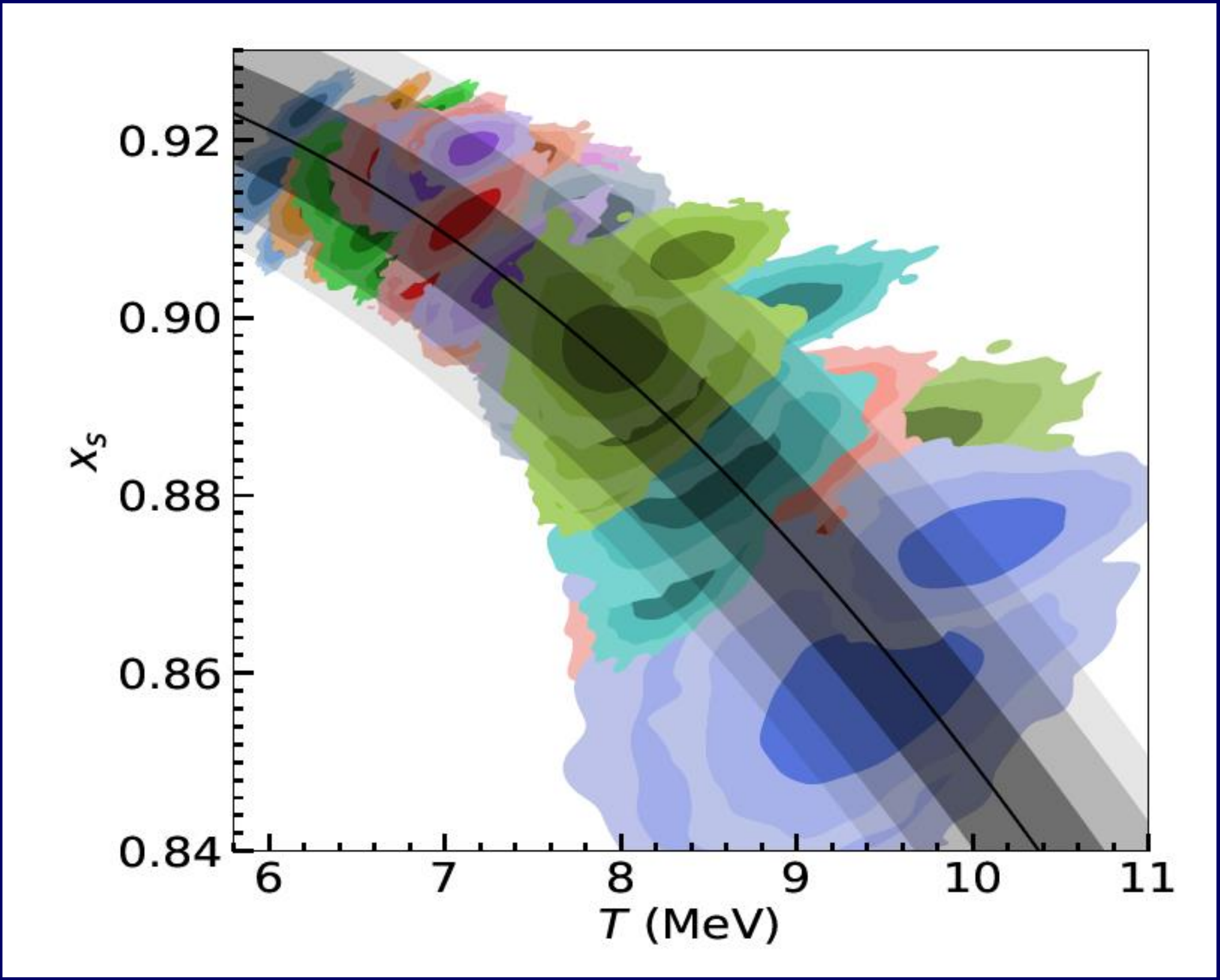
- **Temperature using Ideal Gas formula is ok** (in-medium effects disappear as a result of the subtraction of binding energies)

$$T = \frac{B(4,2)+B(2,1)-B(3,2)-B(3,1)}{\ln(\sqrt{9/8}(1.59\ R_{v_{surf}}))} MeV \text{ with } R_{v_{surf}} = \frac{M(2,1)M(4,2)}{M(3,1)M(3,2)}$$

- **Density is almost constant (0.015 fm^{-3}) contrary to previous analysis (Ideal gas).**

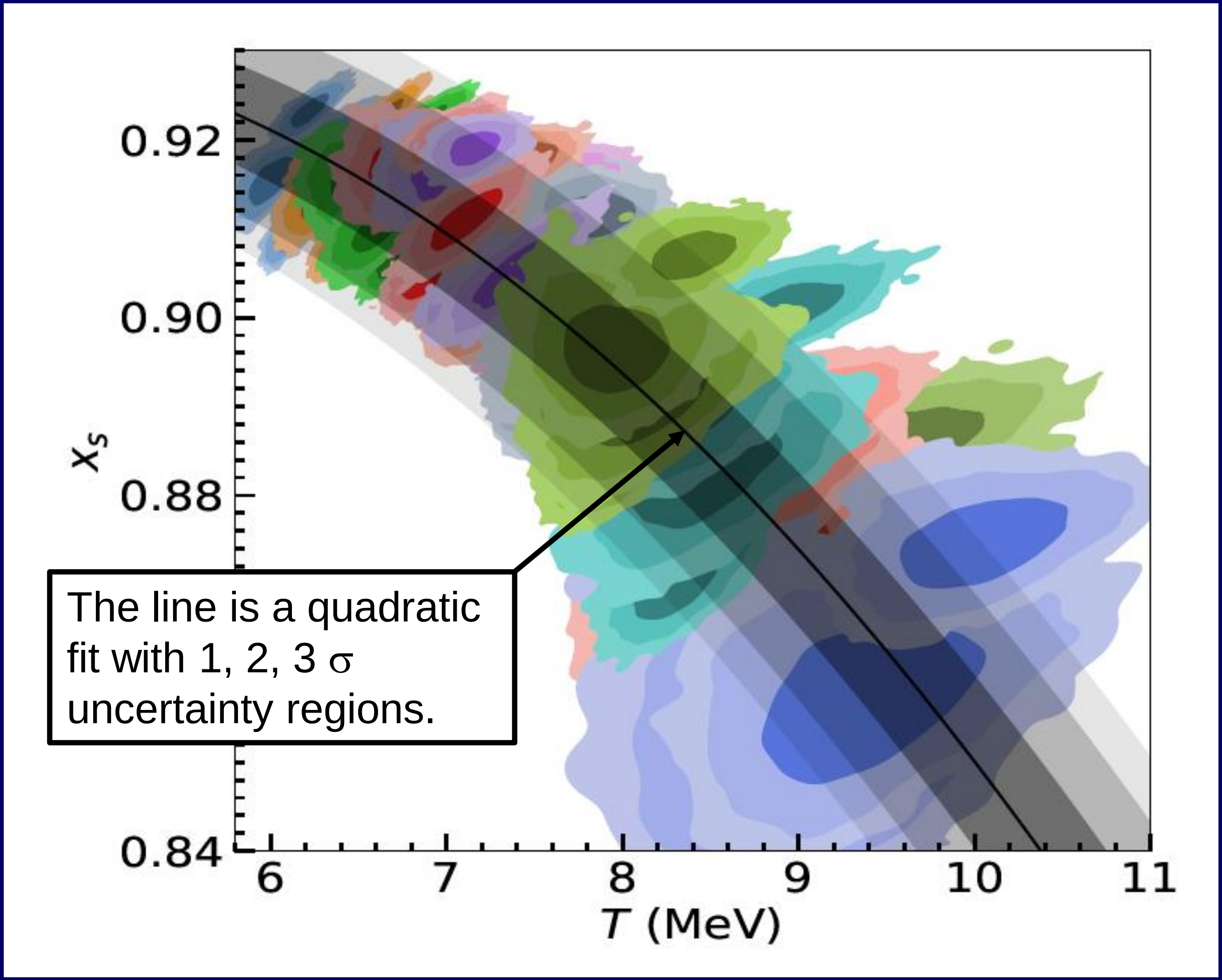
T. Custodio, A. Rebillard-Soulié et al. ***submitted*** *(2024)*

Bayesian inference results: x_s



T. Custodio, A. Rebillard-Soulié et al. ***submitted*** *(2024)*

Bayesian inference results: x_s

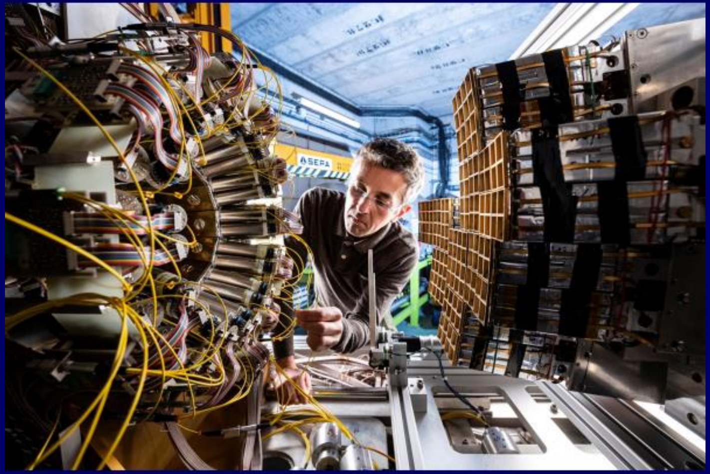


T. Custodio, A. Rebillard-Soulié et al. ***submitted*** *(2024)*

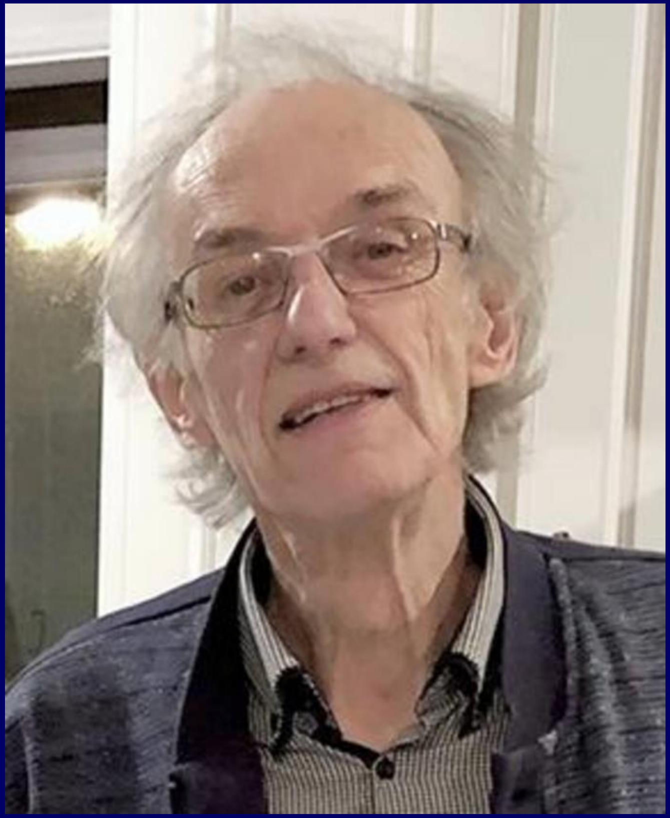
Conclusions

- The INDRA data give information on a single value of the baryonic density (0.015 fm^{-3}).
- The INDRA data are then compatible with the « freeze-out » picture with selected ensembles corresponding to different temperatures.
- The cluster-σ-meson coupling is temperature dependent: weaker when the temperature increases in agreement with microscopic quantum statistical calculations.

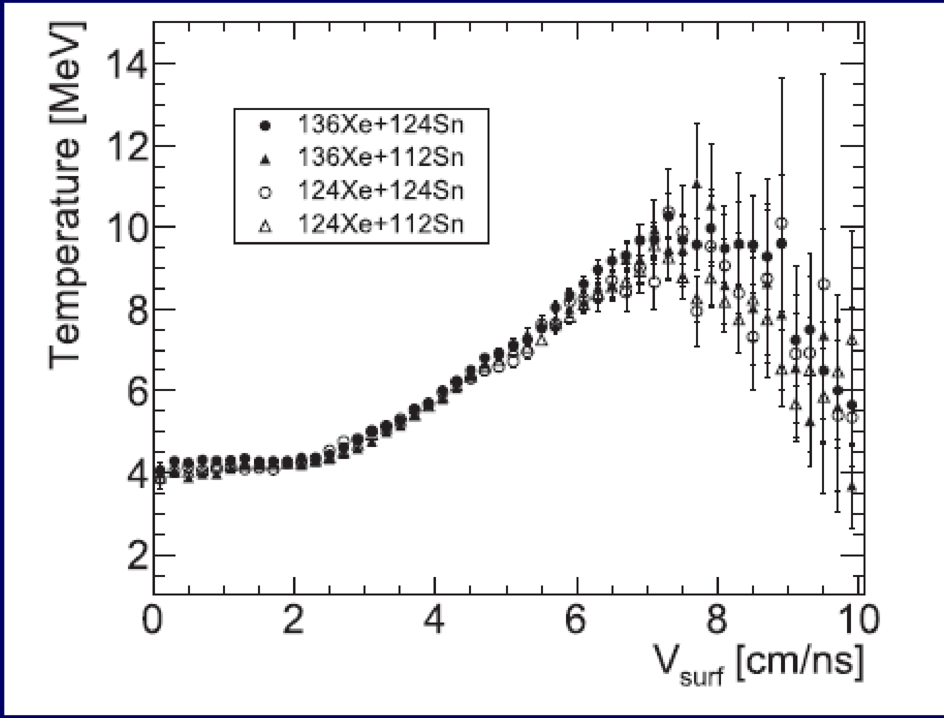
A new experiment has been performed (INDRA/FAZIA) to validate our conclusions with new data corresponding to quasi-projectile vaporization using Ar+Ni 74 A MeV collisions.
The results will be available soon.



I would like to dedicate my talk to René Roy (Professor at Laval University, Québec, Canada), who passed away in May 2024.



RESERVES

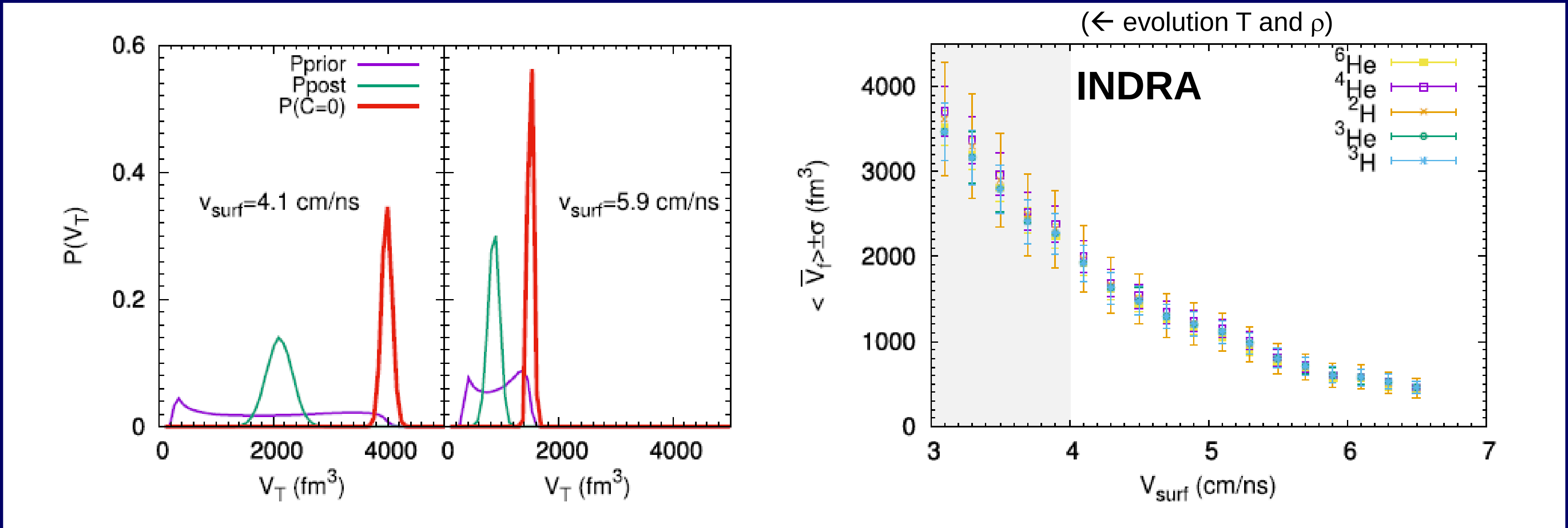


Temperature [MeV]
14
12
10
8
6
4
2
136Xe+124Sn
136Xe+112Sn
124Xe+124Sn
124Xe+112Sn
0
2
4
6
8
10
V surf [cm/ns]

Attempt to resolve the contradiction

Correction factor for the Volume formulae (4 parameters):

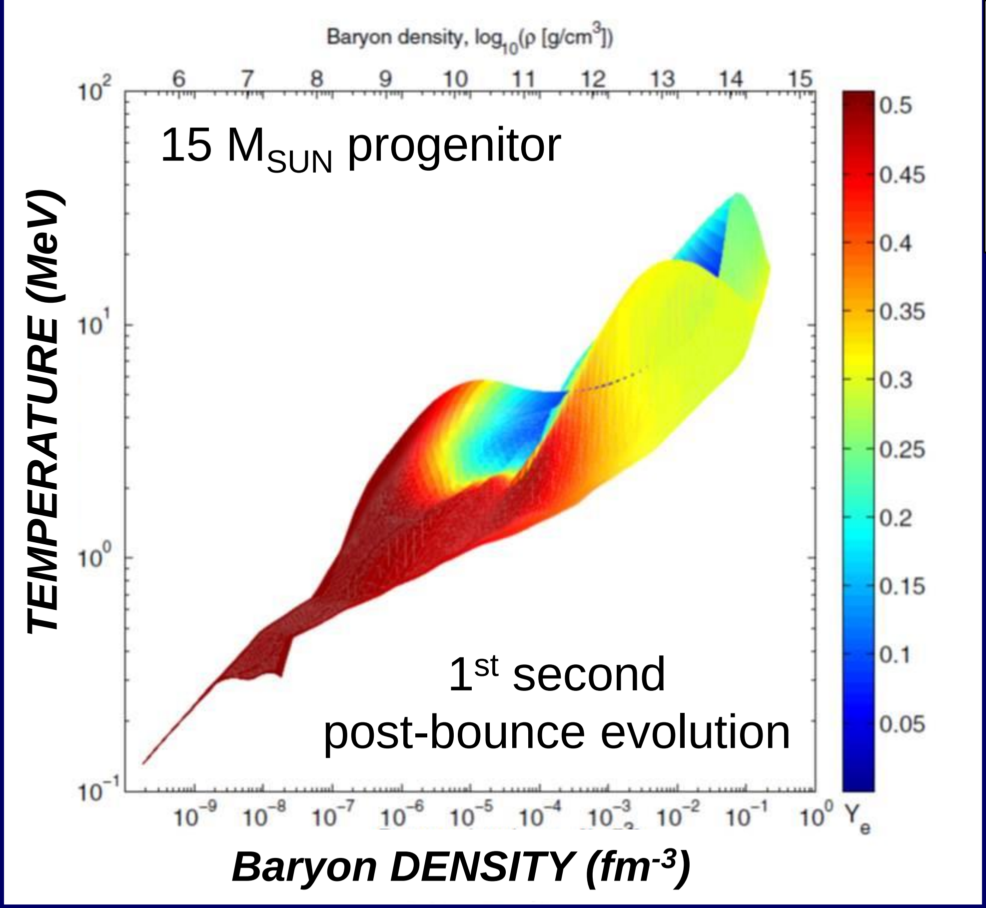
$$C_{AZ}(\rho_B, y_p, T) = \exp\left[-\frac{a_1 A^{a_2} + a_3 |I|^{a_4}}{T_{\mathrm{HHe}}(A-1)}\right]$$



Four parameters: Bayesian analysis whose goal is to obtain identical Volumes for the isotopes. Analysis converges.

H. Pais et al. ***J. Phys. G*** *47 (2020) 105204*
H. Pais et al. ***PRL*** *125 (2020) 012701*

Astrophysics: supernova modelisation



Phase space covered in Core-Collapse Supervova simulations

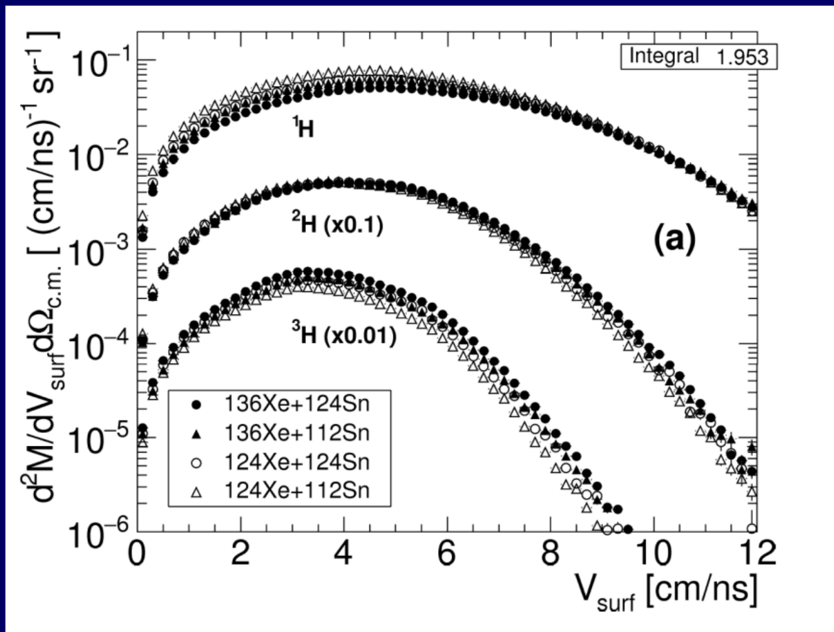
Color: electron fraction
- From Symmetric matter (0.5) red
- To Neutron matter (0) blue

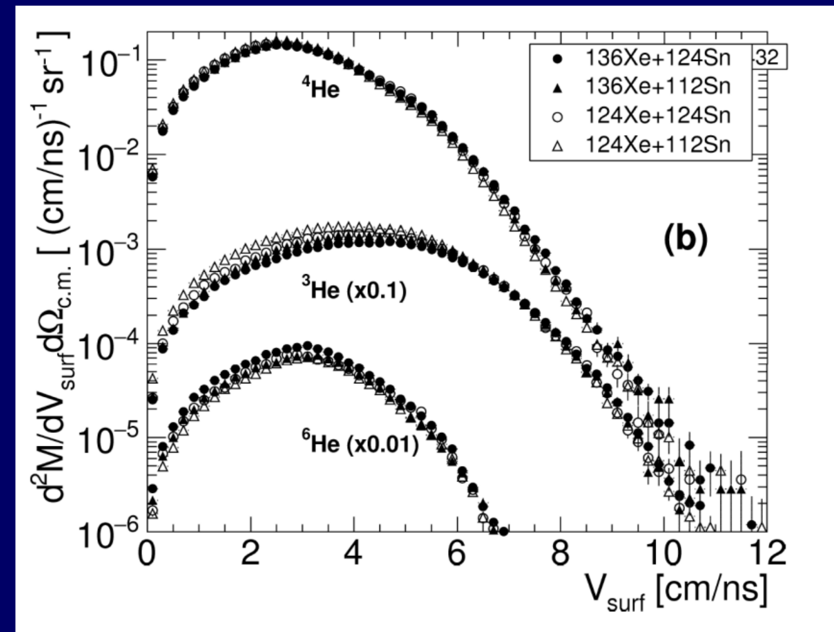
T. Fischer et al. Astro. Phys. Journal 194:39 (2011)

Questions for nuclear physics: what is the chemical composition at these densities and temperatures & measure in medium effects.

Original velocity spectra at cluster creation time

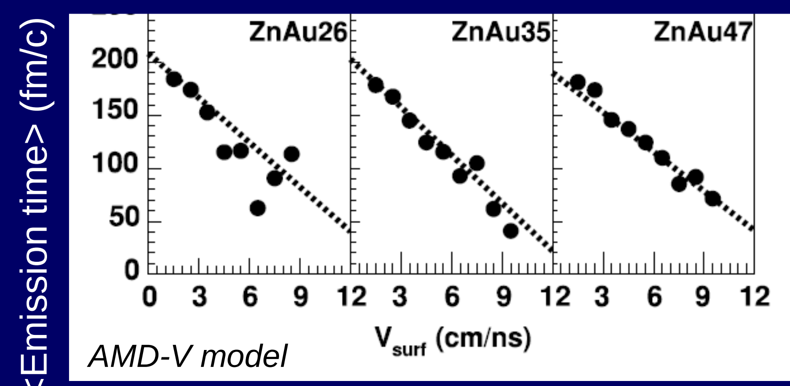
1- Coulomb correction





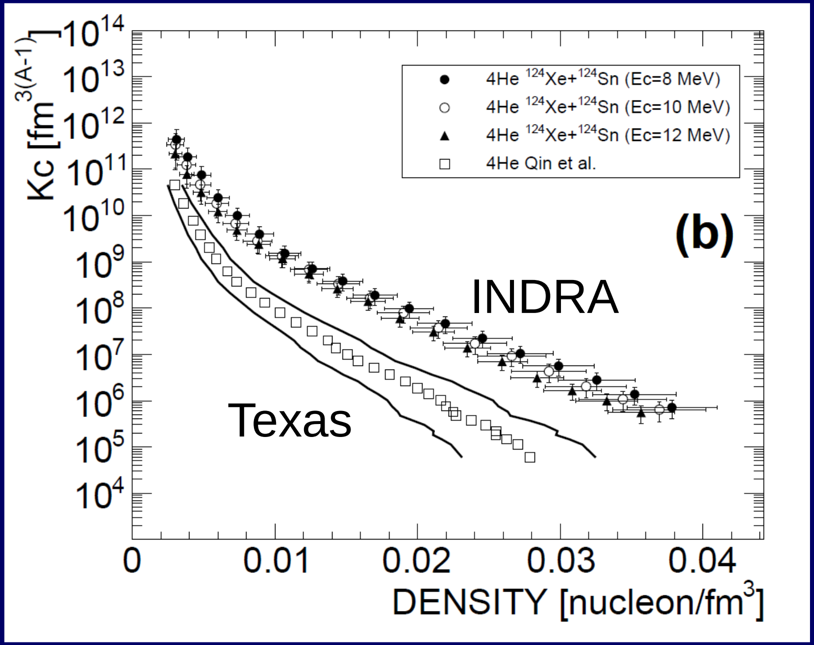
2- Hot expanding source

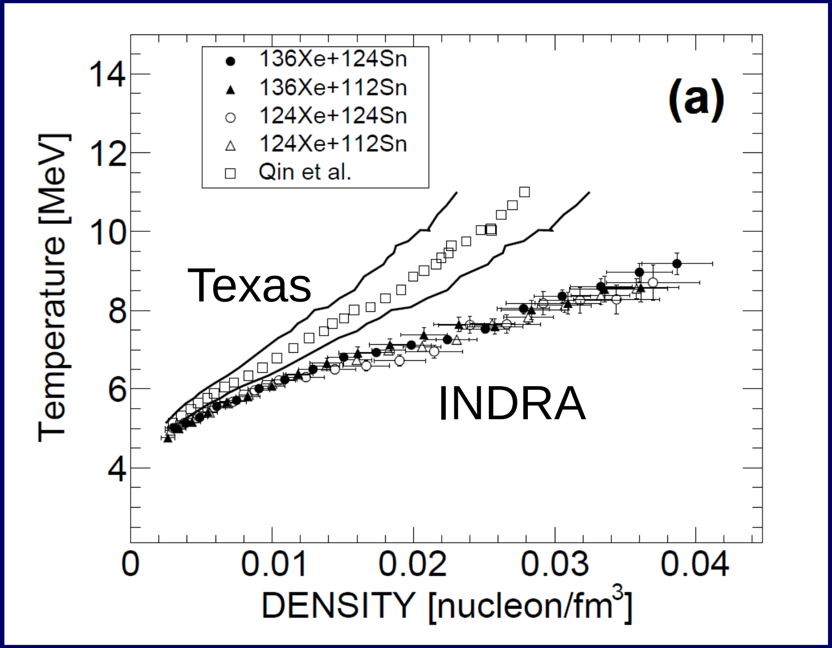
The velocity is a clock: each velocity bin represents the state of the evolving source at a given time.



J. Wang et al. PRC72 (2005) 024603

INDRA versus Texas A&M: K_c (^4He)

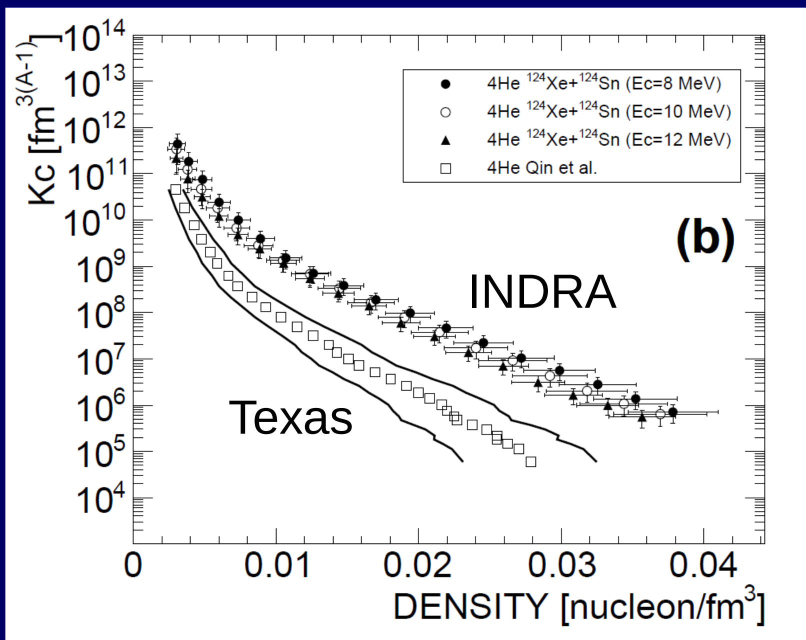


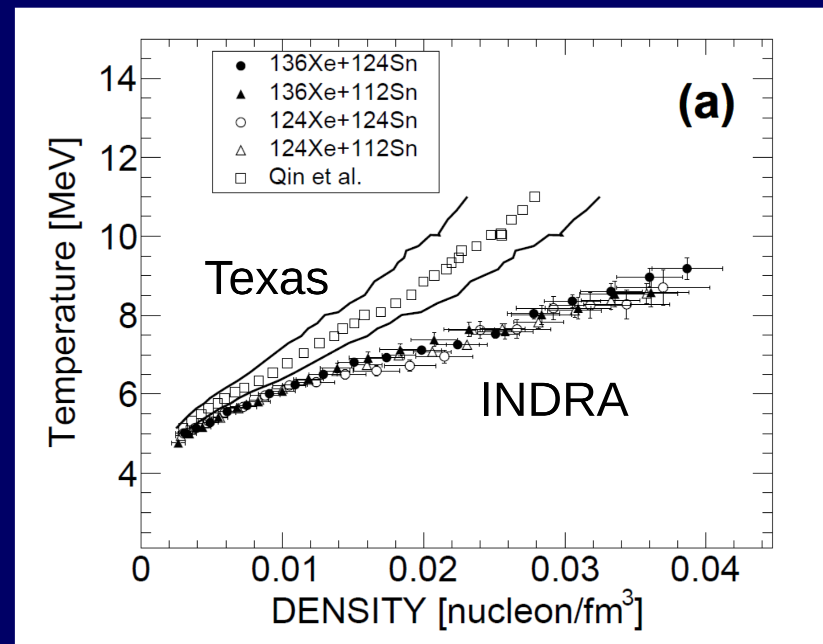


Equilibrium constant values are different
but
the thermodynamical paths are different

R. Bougault et al. ***J. Phys. G*** *47 (2020) 025103*

INDRA versus Texas A&M: K_c (^4He)



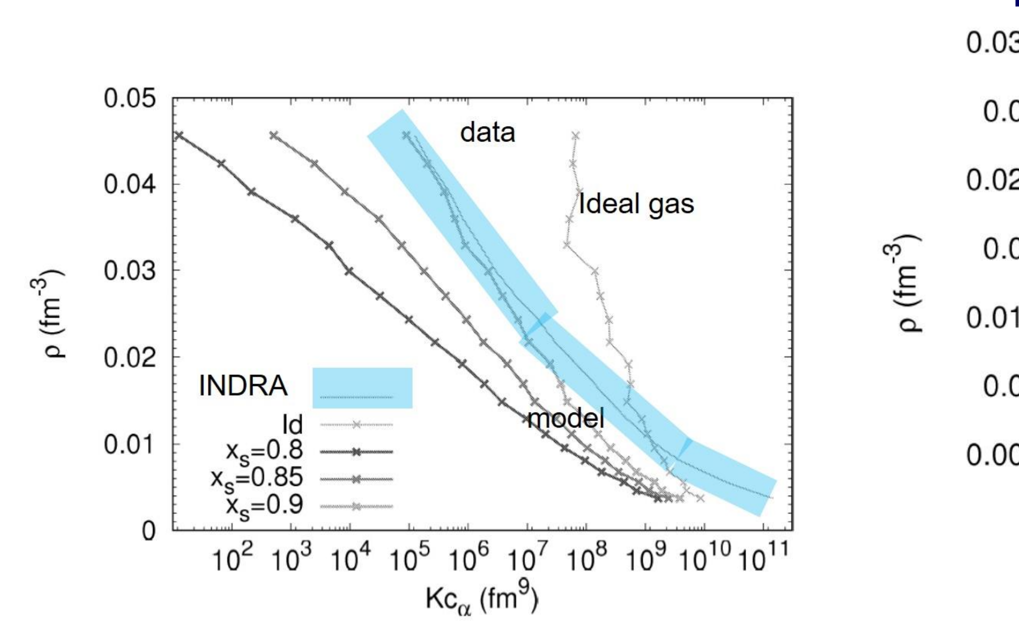


The only way to compare the two sets of data is to use a model.

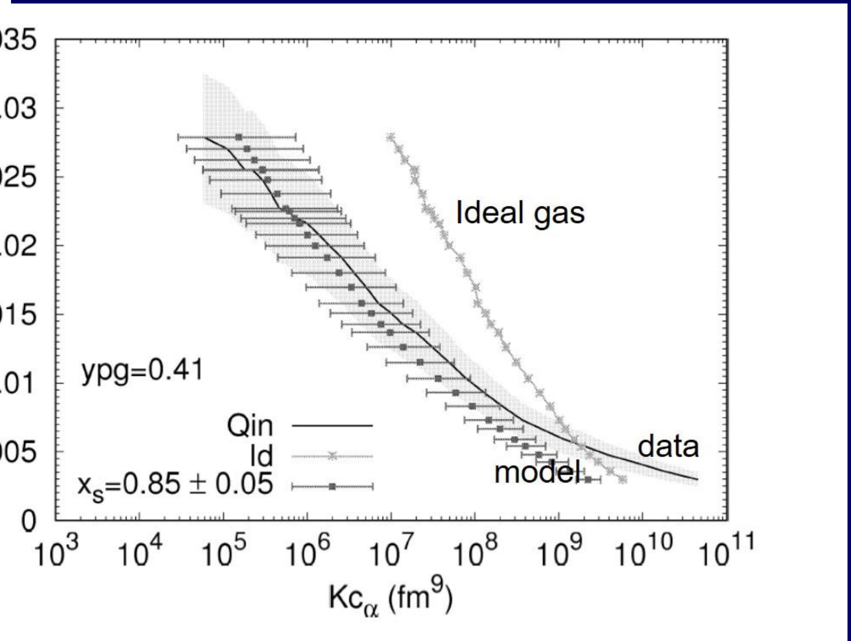
Moreover, the only way to highlight in-medium effects is also to use a model (the data cannot speak for itself).

Relativistic Meam-Field versus DATA

INDRA & RMF



Texas A&M & RMF

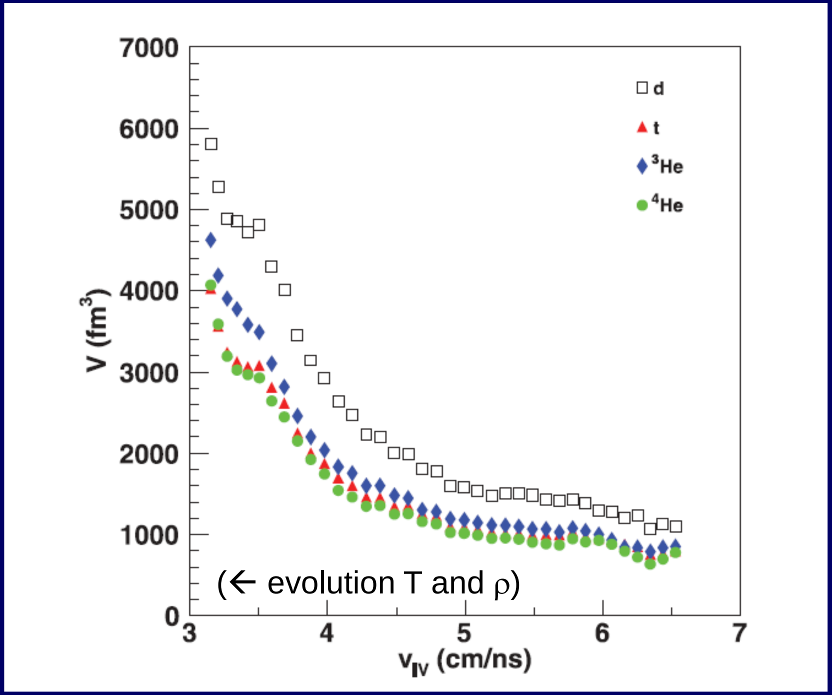


1) Clear deviations from Ideal gas: **in medium effects are present**
2) Some **deviations** data/RMF calculations **at very low densities**
3) indra Xs=0.9 while Texas A&M Xs=0.85

What is wrong for our point of view

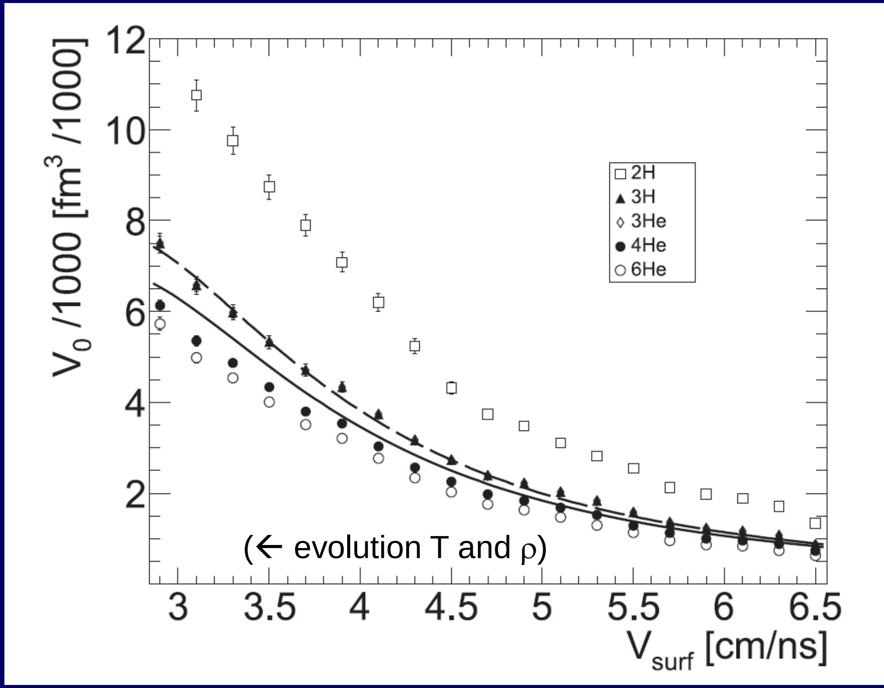
For both experiments, the value of the volume depends on the isotope

Texas A&M



R. Wada et al. ***PRC*** *85 (2012) 064618*

INDRA



R. Bougault et al. ***J. Phys. G*** *47 (2020) 025103*

The value used is the average for A>2.

Attempt to resolve the contradiction

Correction factor for the Ideal Gas Volume formulae:

$$V_f = h^3 R_{np}^{(A-Z)/(A-1)} C_{AZ} \times \exp\left[\frac{B_{AZ}}{T(A-1)}\right] \left(\frac{g_{AZ}}{2^A} \frac{\tilde{Y}_{11}^A(\vec{p})}{\tilde{Y}_{AZ}(A\vec{p})}\right)^{1/(A-1)}$$

Cluster momentum spectrum divided by (proton momentum spectrum)^A

Previously, C_{AZ}=1 (Ideal Gas). Now C_{AZ} will depends on (A,Z):

$$C_{AZ}(\rho_B, y_p, T) = \exp\left[-\frac{a_1 A^{a_2} + a_3 |I|^{a_4}}{T_{\mathrm{HHe}}(A-1)}\right]$$

- **The correction factor C_{AZ} is a modification of the cluster binding energies due to the presence of the medium and is set so that $V_f(^6He)= V_f(^4He)= V_f(^3He)=V_f(^3H)=V_f(^2H)$ (which is not the case for Texas A&M)**
- **C_{AZ} has very general four parameters expression depending on Mass and I = (2Z-A)/2.**

H. Pais et al. ***J. Phys. G*** *47 (2020) 105204*
H. Pais et al. ***PRL*** *125 (2020) 012701*

Back to experimental data

We used measured mass fractions and RMF predictions

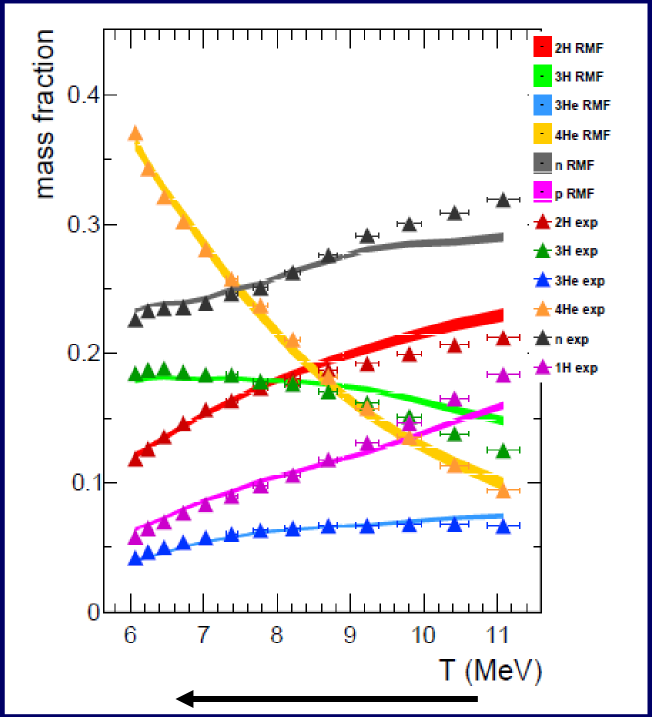
For each system ($^{124,136}Xe+^{124,112}Sn$), independent Bayesian inferences on the measured mass fractions were carried out with T and ρ parametrisations as a function of V_{surf} (the sorting variable):

- $\rho(V_{surf}) = a_1 V_{surf}^2 + a_2 V_{surf} + a_3$
- $T(V_{surf}) = b_1 V_{surf}^2 + b_2 V_{surf} + b_3$

Independent posterior distributions of the parameters $\theta = (a_1, a_2, a_3, b_1, b_2, b_3, x_s)$ were obtained.

(X_s is the coupling ratio of RMF which measures the in-medium modification of the cluster properties).

INDRA (points) vs RMF (color area)



V_{surf}

Calibration using Mass Fractions
Marginalised posteriors versus INDRA data
(2 σ uncertainties)

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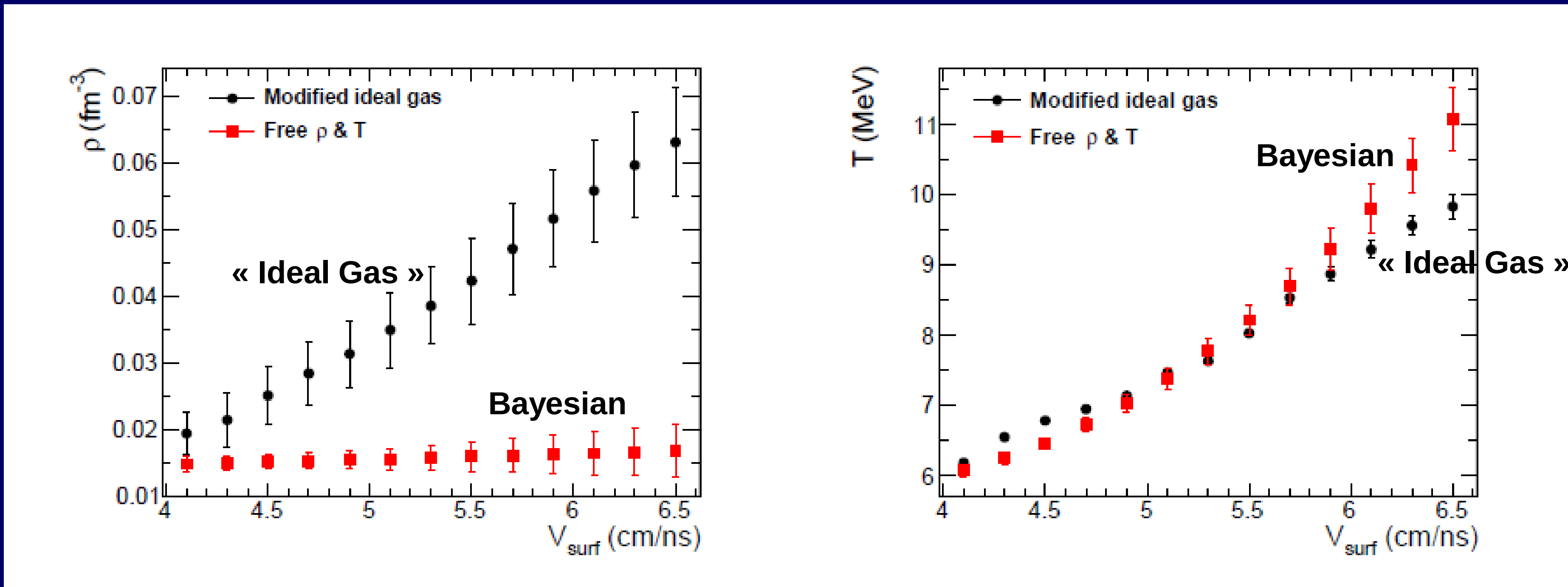
Bayesian inference results: T and ρ



Conclusions:

- **Temperature using Ideal Gas formula is ok** (in-medium effects disappear as a result of the subtraction of binding energies)

$$T = \frac{B(4,2)+B(2,1)-B(3,2)-B(3,1)}{\ln(\sqrt{9/8}(1.59\ R_{v_{surf}}))} MeV \text{ with } R_{v_{surf}} = \frac{M(2,1)M(4,2)}{M(3,1)M(3,2)}$$

- **Density is almost constant (0.015 fm^{-3}) contrary to previous analysis (« Ideal gas »).**

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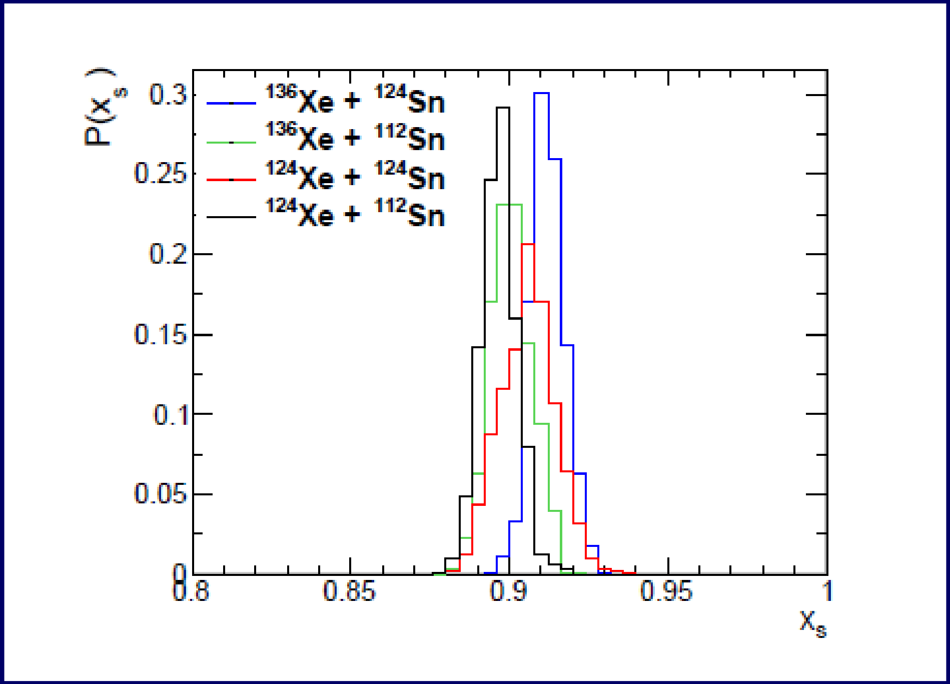
Bayesian inference results: X_s

Xs <1 means in-medium effects



We have not fixed a temperature dependency for Xs in this analysis therefore this is a mean value.

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