Study of nuclear symmetry energy from isospin transport in intermediate energy heavy-ion reactions

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Plan of the Talk:-

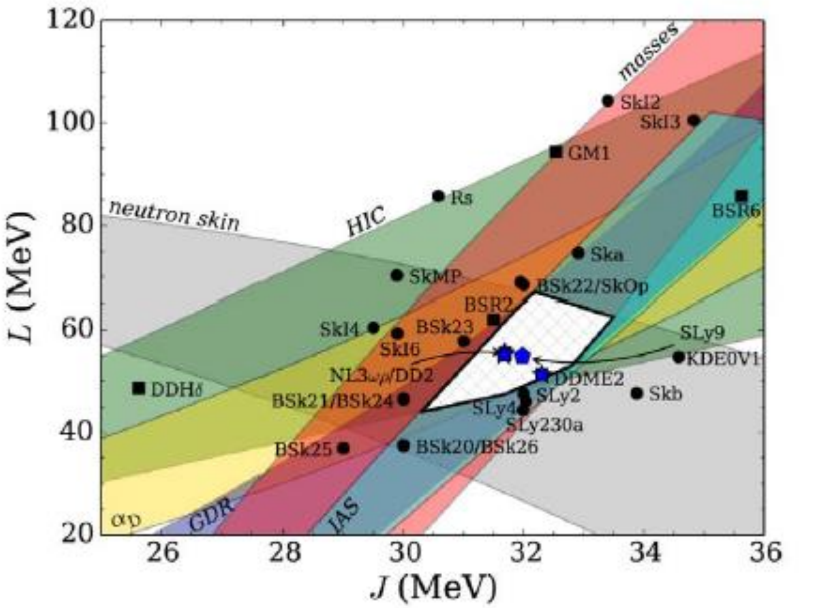
- Introduction
- BUU model for heavy ion reaction
- Isospin Transport
- Effect of EoS on isospin transport
- Isospin Current Density
- Summary

Nuclear Equation of state (EoS) from heavy ion reaction:-

Symmetry Energy

Nuclear Physics

Astrophysics



Ref: M. Fortin et. al., Phys . Rev. C 94, 035804 (2016)

Present Challenge:- Precise measurement of nuclear EoS.

Intermediate energy heavy ion reaction can be an useful probe!!

Symmetry energy measurement from Heavy ion collision :-

Important Observables:-

- Isoscaling
- π^-/π^+
- Isobaric yield ratio
- **Isospin tranport ratio**

……

Theoretical models of intermediate energy heavy-ion reactions:-

❑ *Statistical Models:-*

Basic Assumption: Equilibrium @ freeze-out

- ❖ Canonical Thermodynamical Model (CTM)
- ❖ Statistical Multifragmentation Model (SMM)
- ❖ Grand Canonical Model (GCM)

etc….

❑ *Dynamical Models:-*

Time evolution of projectile and target nucleons

- ❖ **Boltzmann Uehling Uhlenbeck (BUU) Model**
- ❖ Quantum Molecular Dynamics (QMD) Model
- ❖ Anti-symmetrized Molecular Dynamics (AMD) Model

etc….

❑ *Other Models:-*

- ❖ HIPSE
- ❖ Lattice Gas Model
- ❖ Percolation Model
- ❖ EPAX

etc….

Boltzmann-Uehling-Uhlenbeck model (BUU@VECC-McGill) for heavy ion collision :-

❖Based on the BUU equation,

$$\frac{\partial f_i}{\partial t} + \vec{v}_i.\vec{\nabla}_r f_i - \vec{\nabla}_r U.\vec{\nabla}_p f_i = \frac{1}{(2\pi)^6}\int d^3\vec{p}_j d^3\vec{p}_{j'} d\Omega \frac{d\sigma}{d\Omega} v_{ij}$$
$$\times\left\{f_{i'}f_{j'}(1-f_i)(1-f_j) - f_i f_j (1-f_{i'})(1-f_{j'})\right\}$$
$$\times(2\pi)^3\delta^3(\vec{p}_i + \vec{p}_j - \vec{p}_{i'} - \vec{p}_{j'})$$

where, $f_i \equiv f(\vec{r}_i, \vec{p}_i, t)$

❖ Can not be solved exactly, test particle method (N_{test}=100) is used for numerical calculation.

1. Initialization :-

❖Initial positions and momenta of the test particles are selected by Monte-Carlo simulations of initial phase space density (obtained from variational method over Myer's density profile).

$$\rho(r) = \rho_M\left[1-\left(1+\frac{R}{a}\right)\exp\left(-\frac{R}{a}\right)\frac{\sinh(r/a)}{r/a}\right] \qquad r\le R$$

$$\rho(r) = \rho_M\left[\left(\frac{R}{a}\right)\cosh\left(\frac{R}{a}\right)-\sinh\left(\frac{R}{a}\right)\right]\frac{\exp(-r/a)}{r/a} \qquad r>R$$

2. Time Evolution :-

2.a:Vlasov Propagation:-

Propagation of the test particles are calculated by,

$$\boxed{\frac{d\vec{r}_i}{dt} = \vec{v}_i}$$

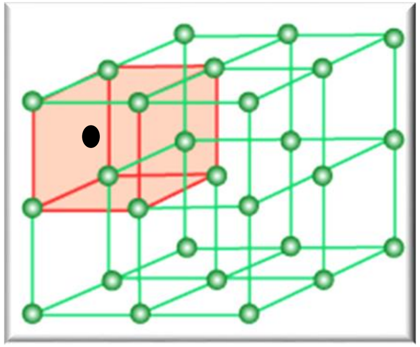
$$\boxed{\frac{d\vec{p}_i}{dt} = -\vec{\nabla}_r U\{\rho(\vec{r}(t))\delta(\vec{r}(t))\}}$$

$i=1,2,3.........N_{test}A_0$

Meta-modelling of the EoS

Ref: S. Mallik, G. Chaudhuri and F. Gulminelli, Phys. Rev. C. 100, 024611 (2019)

❖ Mean field potential is calculated accurately by using Lattice Hamiltonian Method.



Meta-modelling for the EoS (ELFc):-

$$e(\rho,\delta)=\left(E_{sat}+\frac{1}{2!}K_{sat}x^2+\frac{1}{3!}Q_{sat}x^3+\frac{1}{4!}Z_{sat}x^4+.....\right)$$
$$+\left(E_{sym}+L_{sym}x+\frac{1}{2!}K_{sym}x^2+\frac{1}{3!}Q_{sym}x^3+\frac{1}{4!}Z_{sym}x^4+.....\right)\delta^2$$

where, $x=(\rho-\rho_0)/3\rho_0 \qquad \delta=(\rho_n-\rho_p)/(\rho_n+\rho_p)$

Energy per particle of homogeneous nuclear matter at zero temperature in ELFc

$$e(\rho,\delta)=t(\rho,\delta)+v(\rho,\delta)$$
$$=\frac{t_0}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3}\left[(1+\kappa_0\frac{\rho}{\rho_0})f_1(\delta)+\kappa_{sym}\frac{\rho}{\rho_0}f_2(\delta)\right]$$
$$+\sum_{n=0}^{N}\frac{1}{n!}(v_n^{is}+v_n^{iv}\delta^2)x^n+(a_N^{is}+a_N^{iv}\delta^2)x^{N+1}\exp(-\frac{b\rho}{\rho_0})$$

where, $f_1=\{(1+\delta)^{5/3}+(1-\delta)^{5/3}\} \qquad f_2=\delta\{(1+\delta)^{5/3}+(1-\delta)^{5/3}\}$

Meta-modelling for the EoS (ELFc):-

Model parameters can be linked with a one-to-one correspondence to the usual EoS empirical parameters via

$$\boxed{v_0^{is} = E_{sat} - t_0(1+\kappa_0)}$$

$$\boxed{v_0^{iv} = E_{sym} - \frac{5}{9}t_0\left[1+(\kappa_0+3\kappa_{kym})\right]}$$

$$\boxed{v_1^{is} = -t_0(2+5\kappa_0)}$$

$$\boxed{v_1^{iv} = L_{sym} - \frac{5}{9}t_0\left[2+5(\kappa_0+3\kappa_{kym})\right]}$$

$$\boxed{v_2^{is} = K_{sat} - 2t_0(-1+5\kappa_0)}$$

$$\boxed{v_2^{iv} = K_{sym} - \frac{10}{9}t_0\left[-1+5(\kappa_0+3\kappa_{kym})\right]}$$

$$\boxed{v_3^{is} = Q_{sat} - 2t_0(4-5\kappa_0)}$$

$$\boxed{v_3^{iv} = Q_{sym} - \frac{10}{9}t_0\left[4-5(\kappa_0+3\kappa_{kym})\right]}$$

$$\boxed{v_4^{is} = Z_{sat} - 8t_0(-7+5\kappa_0)}$$

$$\boxed{v_4^{iv} = Z_{sym} - \frac{40}{9}t_0\left[-7+5(\kappa_0+3\kappa_{kym})\right]}$$

$$\left.\begin{array}{l} \rho_0, E_{sat}, K_{sat}, Q_{sat}, Z_{sat} \\ E_{sym}, L_{sym}, K_{sym}, Q_{sym}, Z_{sym} \end{array}\right\}$$

Different for different EoS

Ref: J. Margueron, R. H. Casali and F. Gulminelli, Phys. Rev. C. 97, 025805 (2018)

2.b: Collision:-

$$(\vec{r},\vec{p}_i,t),(\vec{r},\vec{p}_j,t)\rightarrow(\vec{r},\vec{p}_{i'},t),(\vec{r},\vec{p}_{j'},t)$$

❖ Collision Criteria:

$$\sqrt{(\Delta\vec{r})^2-\left(\frac{\Delta\vec{r}.\vec{p}}{p}\right)^2}\le\sqrt{\frac{\sigma_{nn}}{\pi}}$$

$$\left|\frac{\Delta\vec{r}.\vec{p}}{p}\right|\le\left(\frac{p}{\sqrt{p^2+m_1^2}}+\frac{p}{\sqrt{p^2+m_2^2}}\right)\frac{\delta t}{2}$$

Ref: G. F. Bertsch and S. Das Gupta , Phys. Rep. 160 (1988) 189

Fluctuation in BUU model:-

❖ If collision criteria is satisfied, then particle 1 and 2 will suffer momentum change Δp_1 and Δp_2 respectively (if the test particles are not Pauli blocked.).

❖ Since in scattering the whole nucleon moves, not part of it so, (N_{test}-1) test particles closest in phase space to 1 and (N_{test}-1) test particles closest in phase space to 2 will also suffer momentum change Δp_1 and Δp_2 respectively.

Ref: W. Bauer, G. F. Bertsch and S. Das Gupta, Phys. Rev. Lett. 58, 863 (1987)
S. Mallik, S. Das Gupta and G. Chaudhuri , Phys. Rev. C. 91, 034616 (2015)

3. Clusterization in BUU:-

Two test particles (of position $\vec{r}_i$ and $\vec{r}_j$) are part of the same cluster if

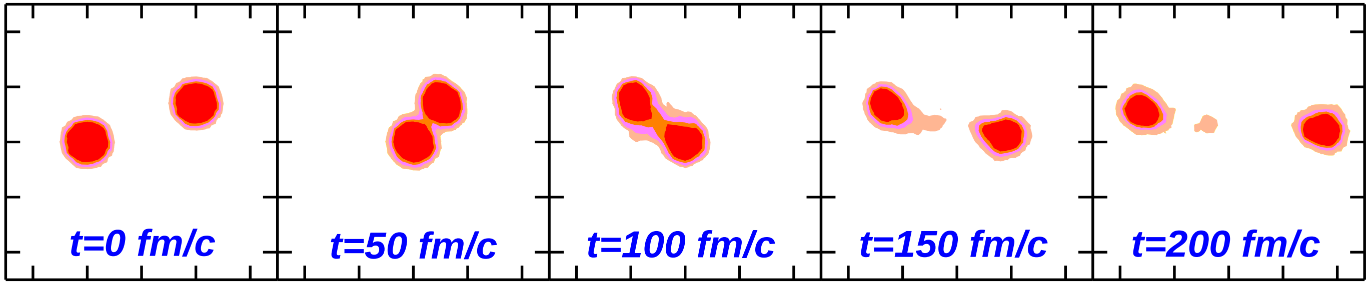
$$|\vec{r}_i-\vec{r}_j|\le 2\ \text{fm}$$

Isospin Transport :-

- *Transfer of isospin from more isospin asymmetric system to less isospin asymmetric system*

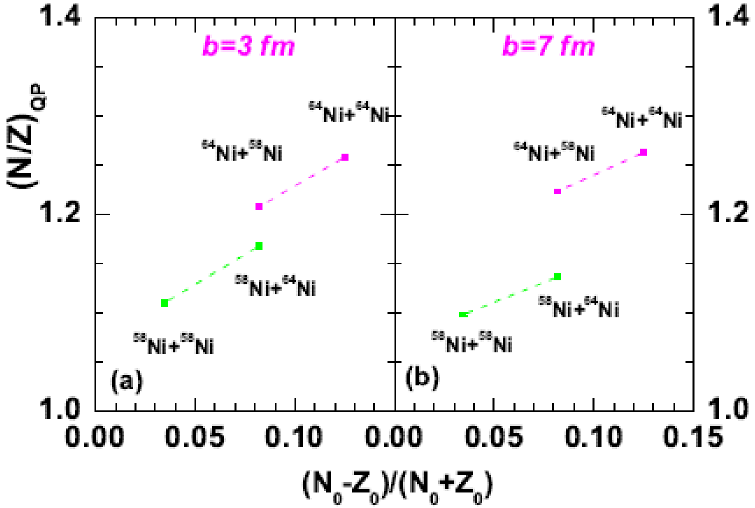
Time evolution :-

^{58}Ni+^{64}Ni E_p=52 MeV/nucleon b=7 fm (BUU@VECC-McGill Result)



N/Z of Quasiprojectile :-

Time=300 fm/c



Isospin Transport Ratio:-

$$R_i(x) = \frac{2x_i - x_{A+A} - x_{B+B}}{x_{A+A} - x_{B+B}}$$

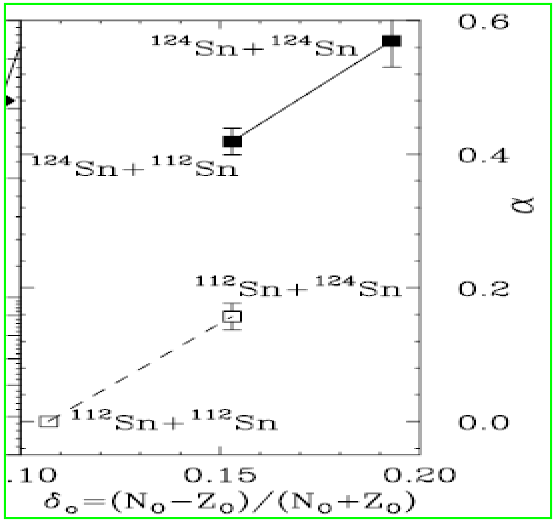
A and B are neutron rich and neutron deficient nucleus respectively.

Ref: F. Rami et. al, Phys. Rev. Lett. 84, 1120(2000)

Experimental measurement @MSU:-

Isoscaling: Ratio of yields of from two different reactions

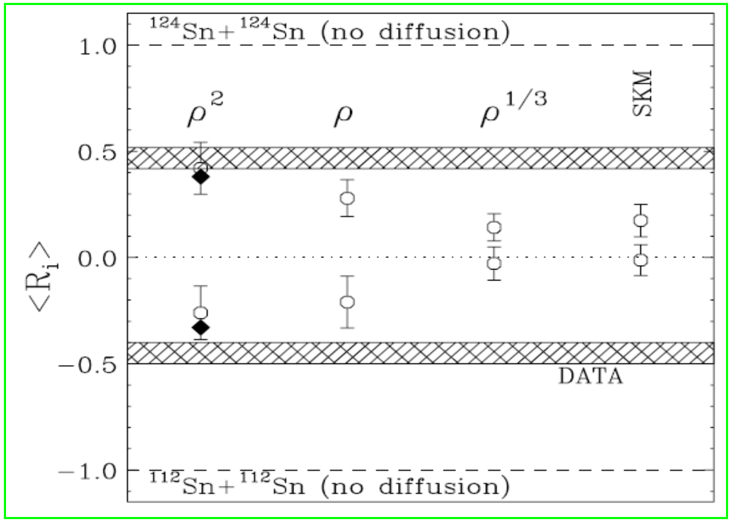
$$R_{21}(N,Z) = \frac{Y_2(N,Z)}{Y_1(N,Z)} = c\exp(\alpha N + \beta Z)$$



Theoretical Study:-

N/Z of projectile residue with stiff, moderate and soft EoS

$$\epsilon_{sym}(\rho) = \frac{C_{kin}}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3} + \frac{C_{pot}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$



Ref: M. B. Tsang et. al, Phys. Rev. Lett. 92, 062701 (2004)

Open question:-

- ***How R_i varies for different observables: Free nucleons, Quasiprojectile (QP)?***
- ***Dependence on different realistic EoS ?***

Identification of projectile like fragment (PLF):-

Studied Reaction:- ^{58}Ni+ ^{64}Ni @52 MeV/nucleon (b=7fm)

BUU calculation @ Projectile frame (Target is moving along –z direction)

Momentum Distribution of PLF and TLF:-

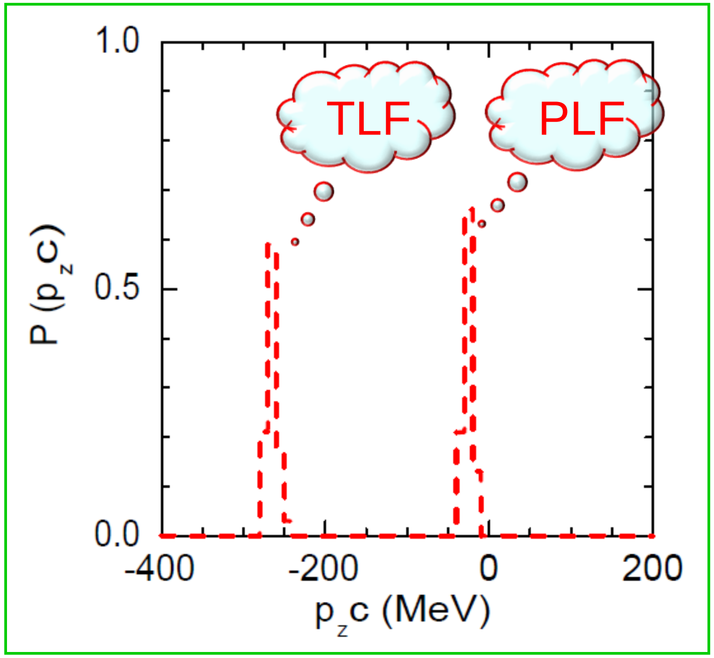
Isotropy of Momentum Distribution (I) of PLF:-

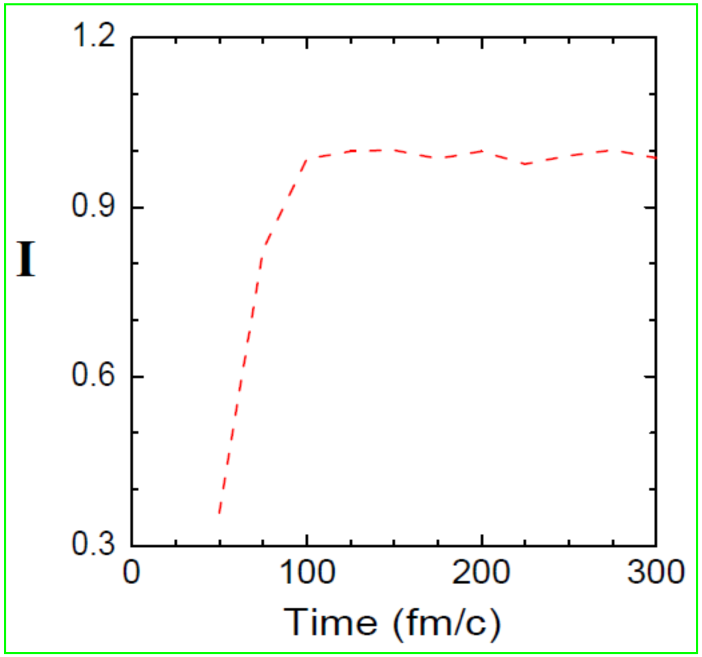
$$I = \frac{\frac{1}{N}\sum_{i=1}^{N}(p_{x_i} - P_x)^2 + \frac{1}{N}\sum_{i=1}^{N}(p_{y_i} - P_y)^2}{2\times\frac{1}{N}\sum_{i=1}^{N}(p_{z_i} - P_z)^2}$$

$$P_k = \frac{1}{N}\sum_{i=1}^{N} p_{k_i}$$

k=x,y and z

Time=150 fm/c





- I~1 i.e. thermalization for t≥100 fm/c.

Isospin transport ratio for Ni+Ni reactions:-

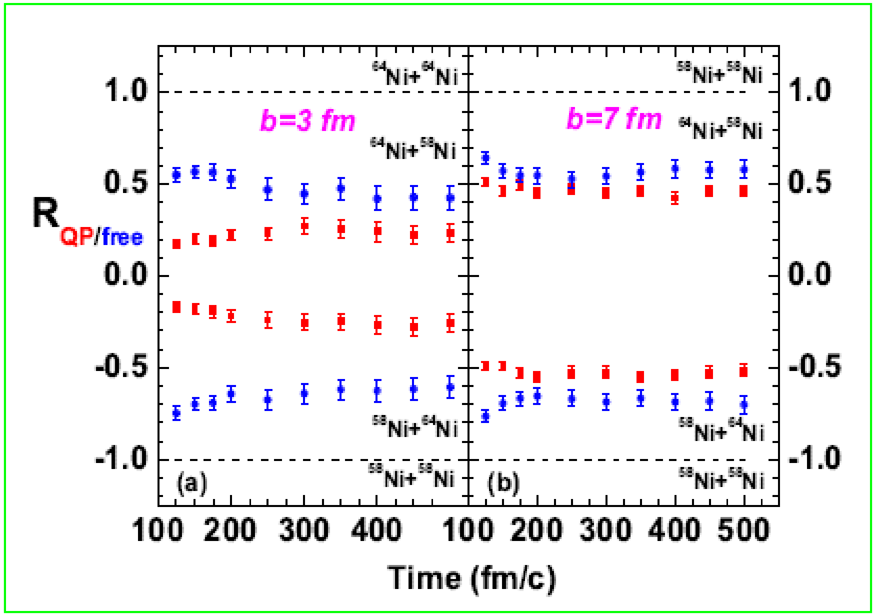
$$R_i = \frac{2x_i - x_{^{64}Ni+^{64}Ni} - x_{^{58}Ni+^{58}Ni}}{x_{^{64}Ni+^{64}Ni} - x_{^{58}Ni+^{58}Ni}}$$

Studied Reaction: ^{64}Ni+^{64}Ni, ^{64}Ni+^{58}Ni, ^{58}Ni+^{64}Ni, ^{58}Ni+^{58}Ni

(Transport calculation in projectile frame with Sly5 EoS)

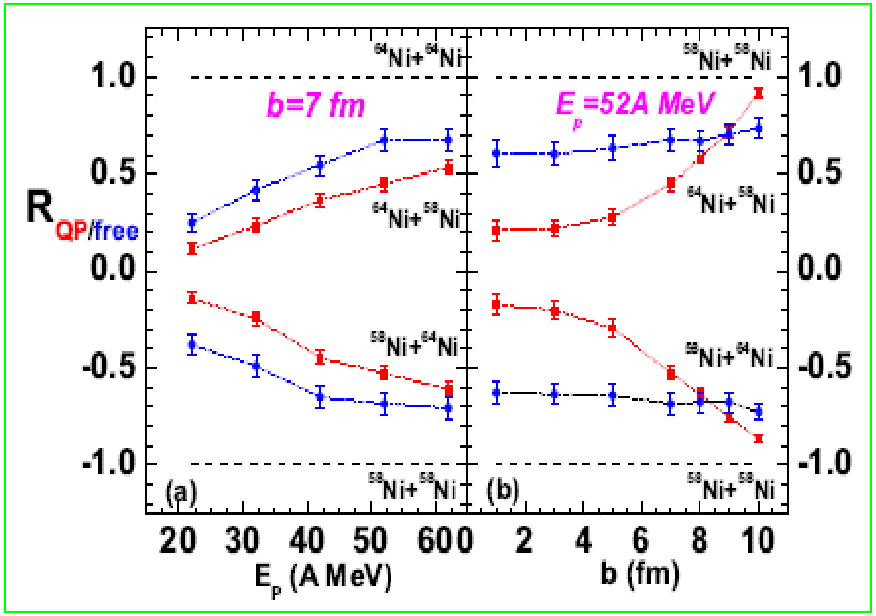
Time dependence:-

E_p=52 MeV/nucleon b=7 fm



Entrance channel dependence:-

t=300 fm/c



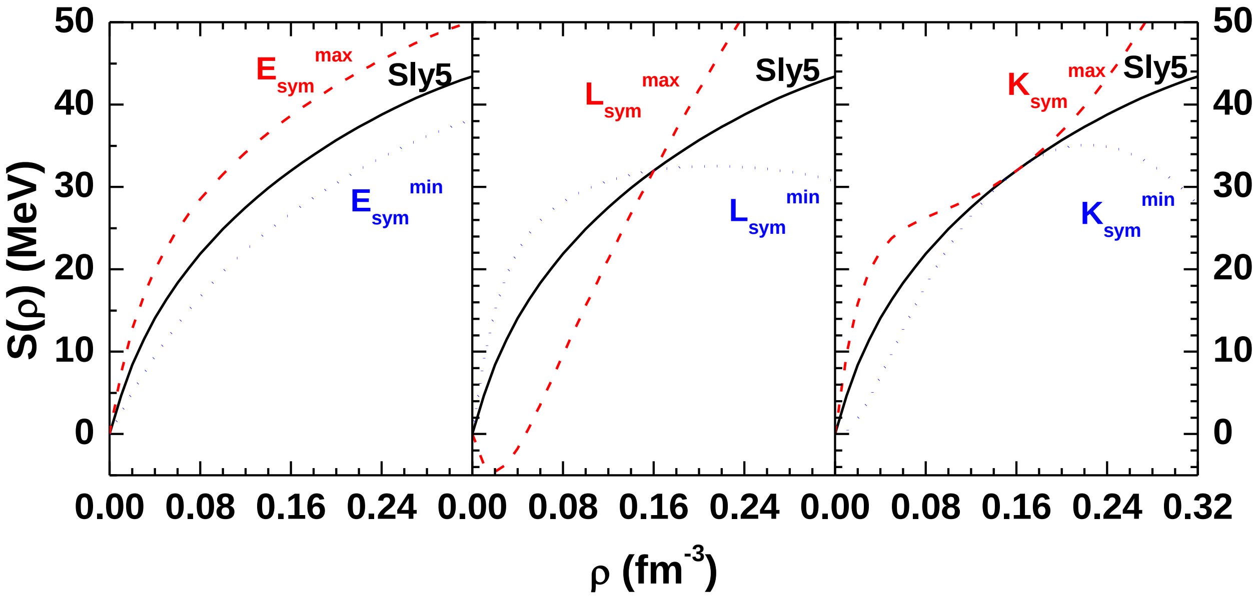
- ❖ For t≥150 fm/c isospin transport ratio is almost independent of time.
- ❖ R_{QP} and R_{fn} are different .

Red squares → Quasiprojectile
Blue circles → Free nucleon with p_z>0 MeV/c

Sensitivity of symmetry energy to Isospin transport ratio:-

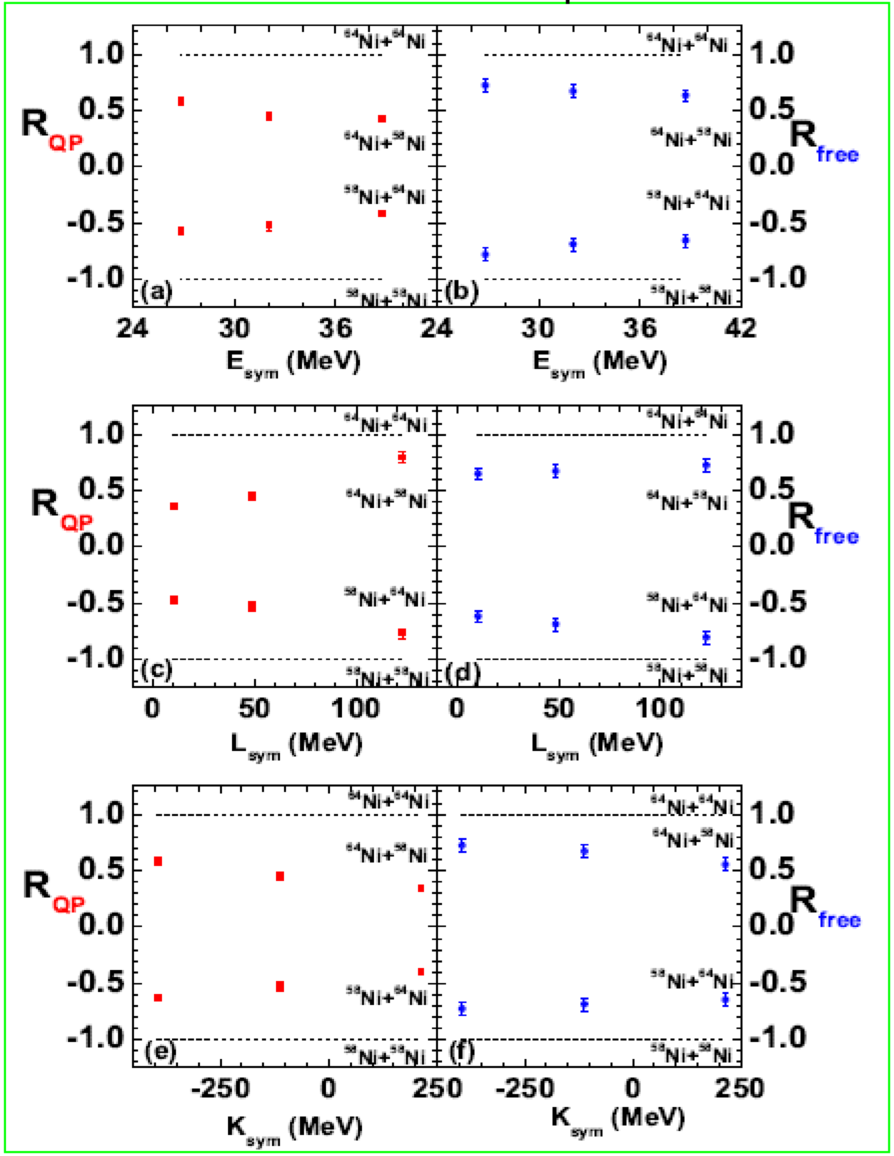
Varying Parameters			
Parameter	Minimum	SLY5	Maximum
E_{sym} (MeV)	26.83	32.03	38.71
L_{sym} (MeV)	29.2	48.3	122.7
K_{sym} (MeV)	-394	-112	213

Uncertainty due to E_{sym} L_{sym} and K_{sym}:-



Sensitivity of symmetry energy to Isospin transport ratio (Contd…) :-

E_p=52 MeV/nucleon b=7 fm

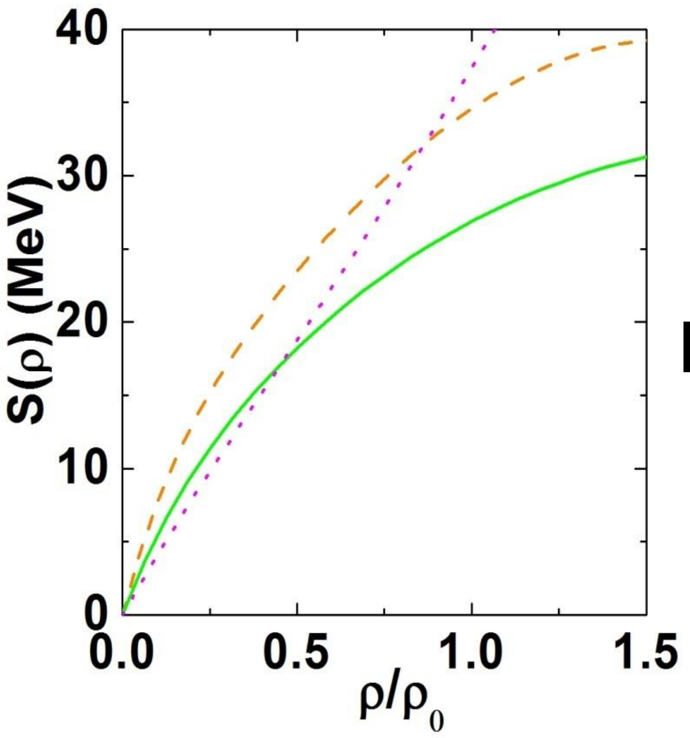


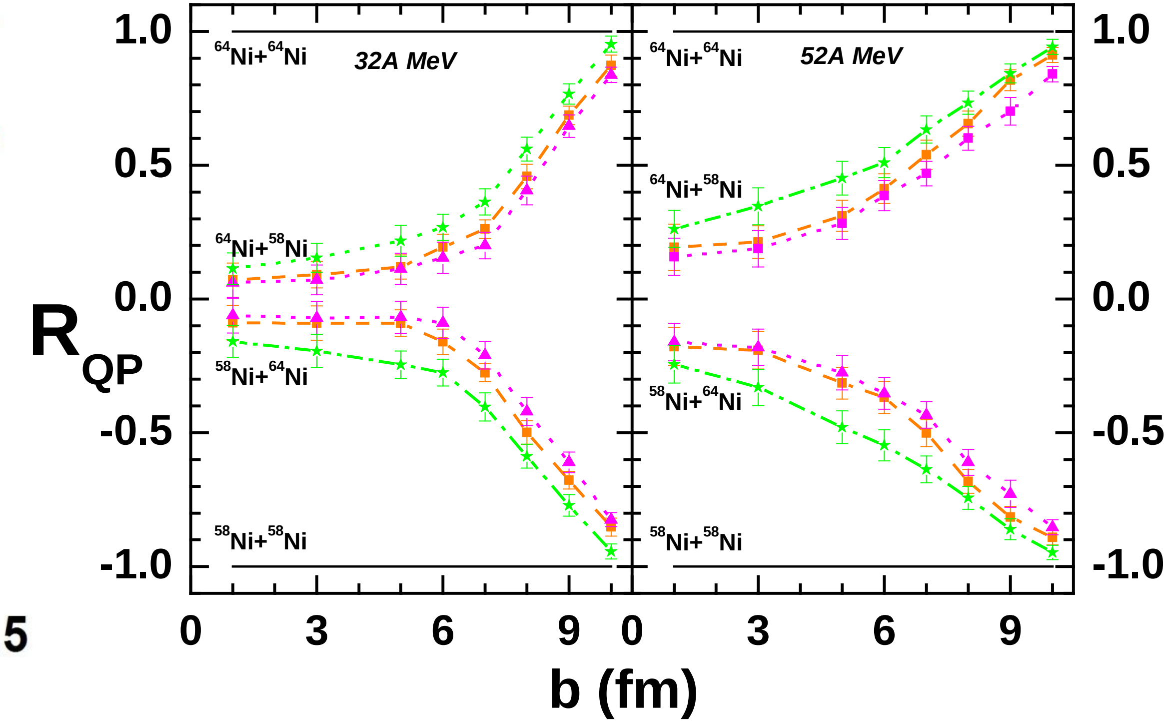
- ➢ Higher symmetry energy (can be achieved by high E_{sym} and K_{sym} or low L_{sym} at sub-saturation density) allows more isospin equilibration.
- ➢ R_{QP} and R_{free} values are widely different.
- ➢ R_{QP} is more sensitive to symmetry energy parameters compare to R_{free}.

Red squares → Quasiprojectile
Blue circles → Free nucleon with p_z>0 . MeV/c

Ref: S. Mallik, F. Gulminelli and D. Gruyer; Jour. Phys. G. 49, 015102 (2021)

Isospin transport ratio from BUU calculation with different EoS:-





High S(ρ) → **More isospin diffusion** → **More equilibration**

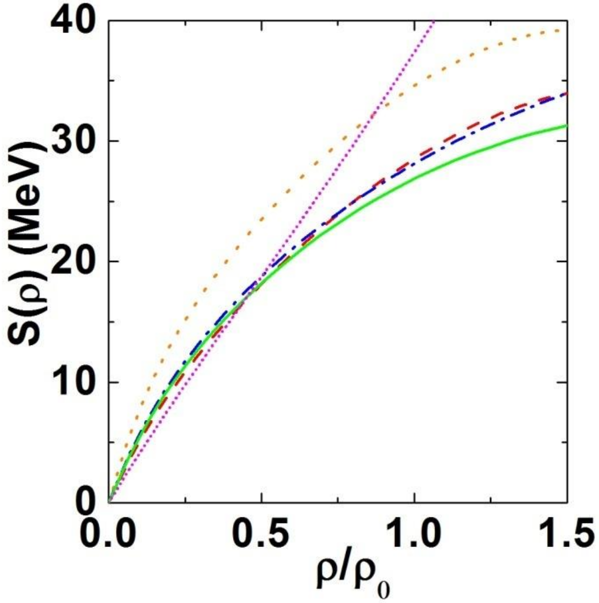
Ab-initio: C. Drischler et.al., Phys. Rev. C 93, 054314 (2016).
SGII: Nguyen Van Giai et. al, Phys. Lett. B106, 379 (1981).
NL3: G. A. Lalazissis et. al., Phys. Rev. C 55, 540 (1997).
SAMI: X. Roca-Maza et. al., Phys. Rev. C 86, 031306(R) (2012).

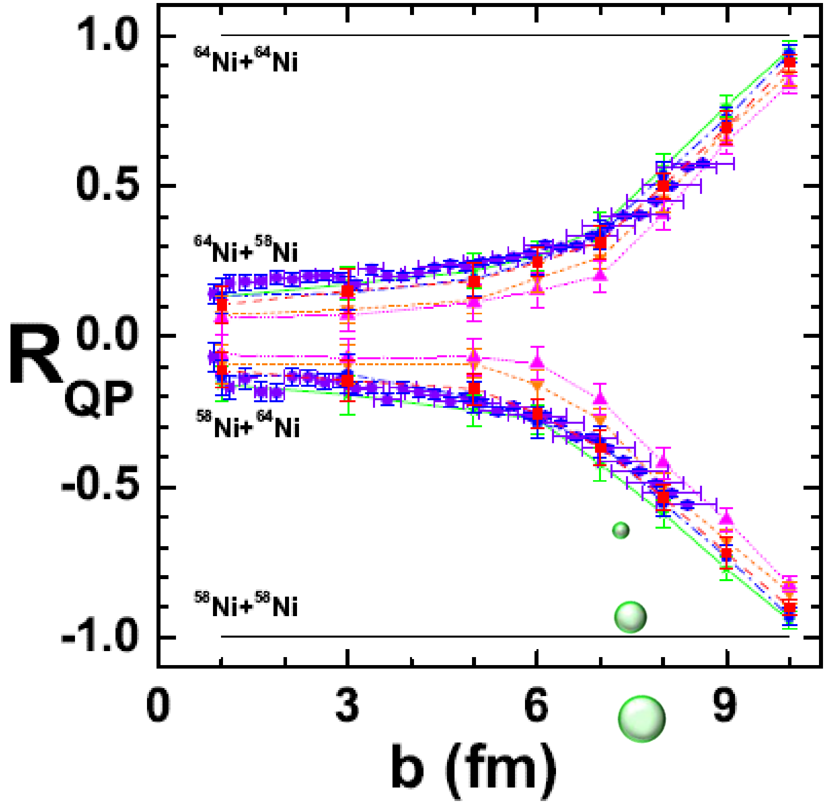
Orange dashed line→ abinitio-1 EoS
Green solid line → SGII EoS
Magenta dotted line → NL3 EoS

Isospin transport ratio: Comparison with experimental data:-

In collaboration with INDRA-FAZIA

Studied Reaction: ^{64,58}Ni+^{64,58}Ni@E_p=32 MeV/nucleon





Violet Circles → Experimental data
Orange dotted line → ab-initio-1 EoS
Red dashed line → ab-initio-7 EoS
Blue dash dotted line → SAMI EoS
Green solid line → SGII EoS
Magenta short dotted line → NL3 EoS

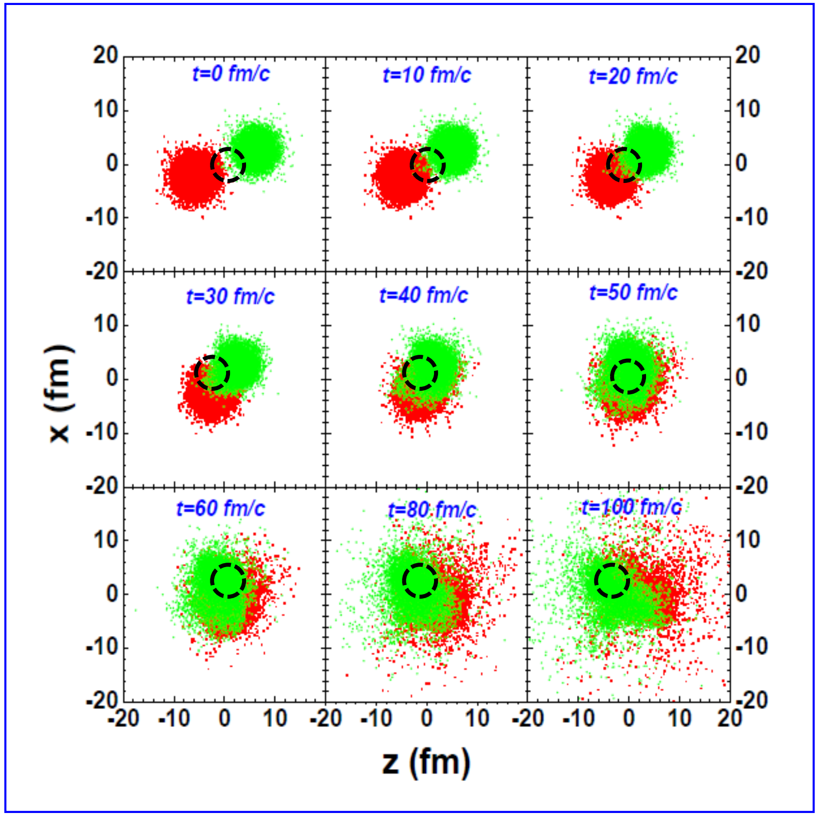
PRELIMINARY

Which density range is probed during isospin transport ??

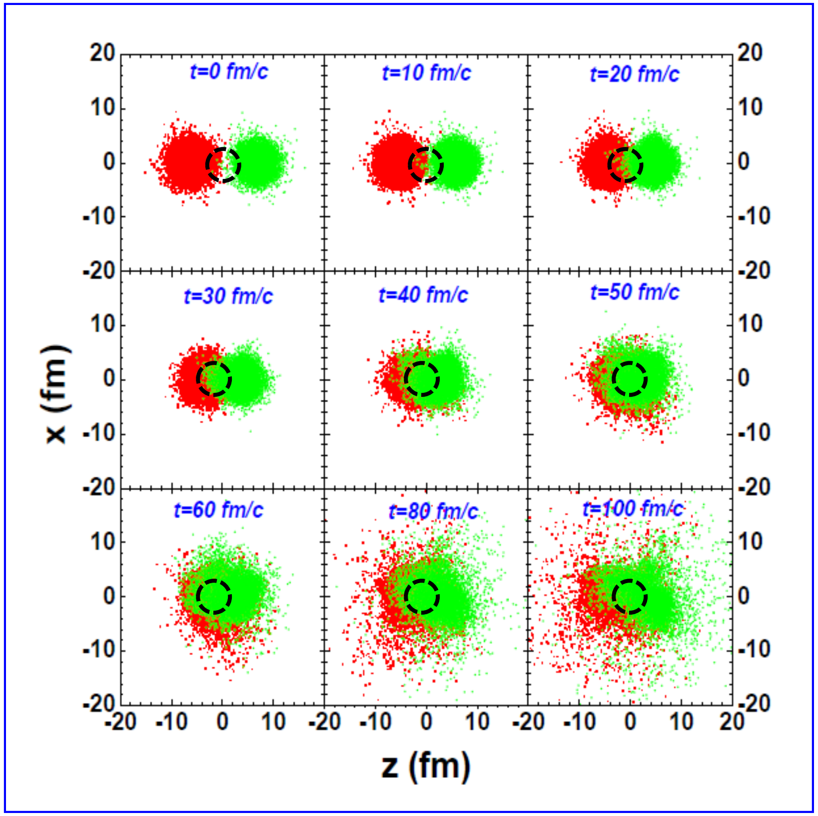
Isospin current density:-

Studied Reaction: ^{64}Ni+^{58}Ni@E_p=32 MeV/nucleon b=5 fm

Time evolution in Centre of mass frame



Time evolution in principal axis frame



Neutron and proton current density inside a sphere in neck region (@principal axis frame)

$$J^n_w = \frac{1}{N_{grid}} \sum_{i=1}^{N_{grid}} \rho_{i,n} v^{pa}_{i,w}$$

$$J^p_w = \frac{1}{N_{grid}} \sum_{i=1}^{N_{grid}} \rho_{i,p} v^{pa}_{i,w}$$

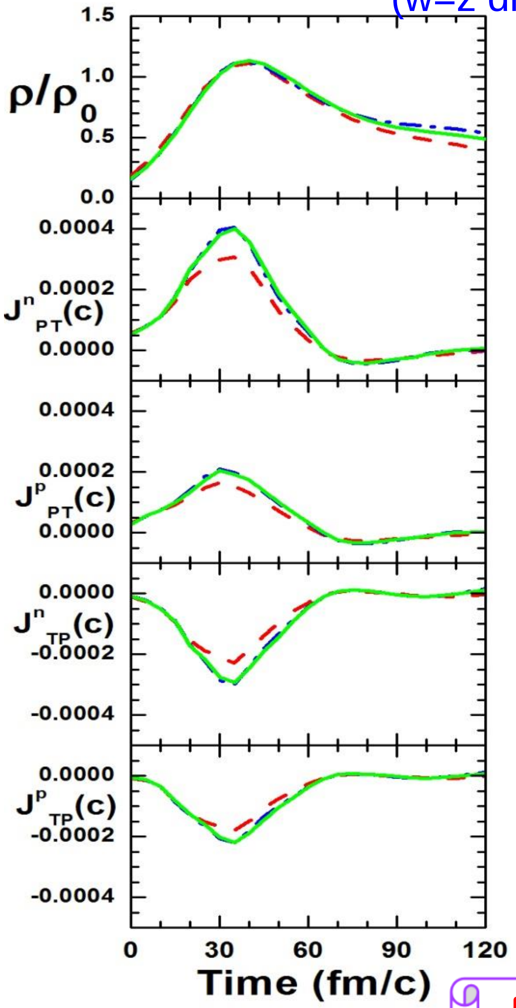
N_{grid}=No. of grid points inside the sphere w=x,y and z direction

Isospin current density (Contd..):-

Studied Reaction: ^{64}Ni+^{58}Ni@E_p=32 MeV/nucleon b=5 fm

Neutron and proton current density:-

(w=z direction)



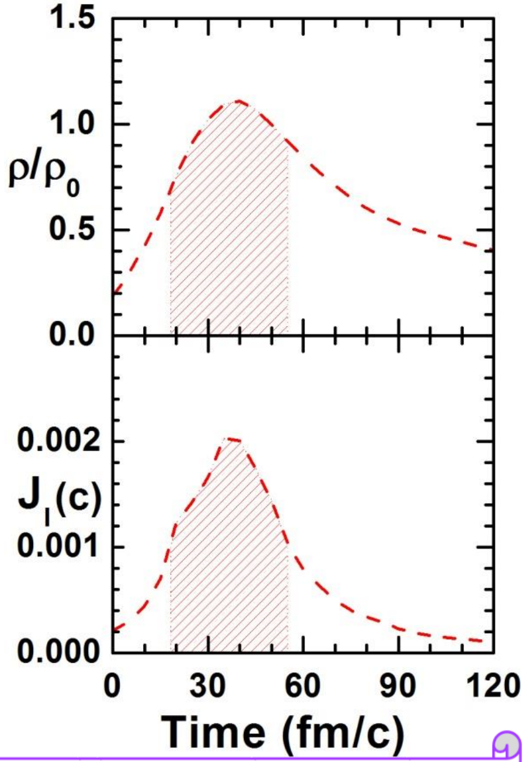
PT → Projectile to target
TP → Target to projectile

Isospin current density:-

$$\begin{aligned} J_{I,w} &= J_w^n - J_w^p \\ &= (j^n_{w,PT} - j^n_{w,TP}) - (j^p_{w,PT} - j^p_{w,TP}) \end{aligned}$$

$$J_I = \sqrt{J_{I,x}^2 + J_{I,y}^2 + J_{I,z}^2}$$

w=x,y and z direction



Isospin transport occurs around $0.65\rho_0 \leq \rho \leq 1.1\rho_0$

Red solid lines → ab-initio-7 EoS
Blue dash dotted lines → SAMI EoS
Green dotted lines → SGII EoS

PRELIMINARY

Conclusions:-

- Isospin transport ratio calculated from quasiprojectile as well as free nucleon are not identical but both are affected by the density dependence of symmetry energy.

- Sensitivity of nuclear EoS on quasiprojectile is more compare to free nucleon.

- In order to reduce the error bar of nuclear EoS, BUU results with different realistic EoS are being compared with INDRA-FAZIA data of isospin transport ratio of the quasiprojectile.

- Current densities are being calculated for estimating the precise region of sub-saturation densities responsible for isospin diffusion in heavy-ion reactions around the Fermi energy domain.

The work is in progress…….

Thank you…

Mean Field potential from meta-modelling for the EoS:-

Mean field potential for neutron/proton (N=4)

$$
\begin{aligned}
U_{\frac{n}{p}} &= (v_0^{is} + v_0^{iv}\delta^2) + \sum_{n=1}^{4} \frac{n+1}{n!}(v_n^{is} + v_n^{iv}\delta^2)x^n \\
&+ \frac{1}{3}\sum_{n=1}^{4} \frac{1}{(n-1)!}(v_n^{is} + v_n^{iv}\delta^2)x^{n-1} \pm 2\delta(1\mp\delta)\sum_{n=1}^{4}\frac{1}{n!}v_n^{iv}x^n x^{N+1} \\
&+ \left[(a_4^{is} + a_4^{iv}\delta^2)\{\frac{5}{3}x^4 + (6-b)x^5 - 3bx^6\} \pm 2\delta(1\mp\delta)a_4^{iv}x^5 \right] \\
&\times \exp\{-b(1+3x)\} + \frac{3c}{\rho_0^{2/3}}\nabla^2 x
\end{aligned}
$$

Mean field term due to density dependence of the effective mass

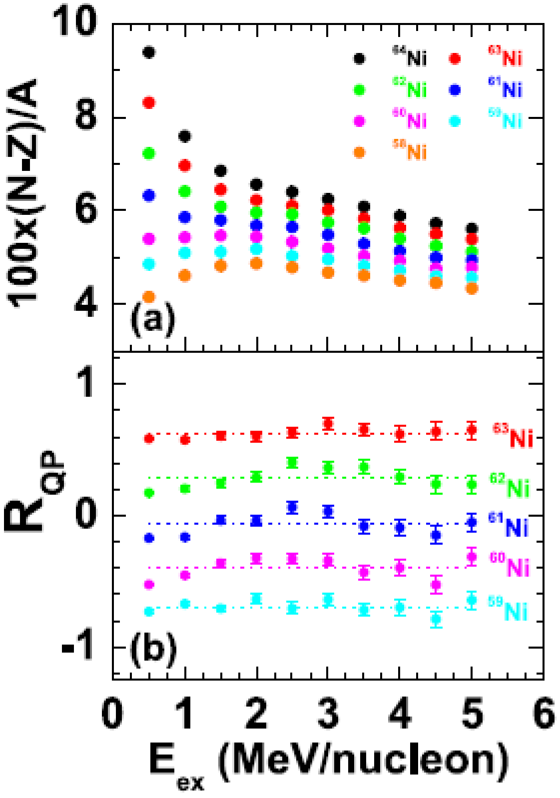
$$
U_q^{eff} = \sum_{q=n,p} \tau_q \frac{\partial}{\partial \rho_q}\left(\frac{m_q}{m_q^*}\right) = \tau_q \frac{\kappa_0 + \kappa_{sym}}{\rho_0} + \tau_q^{'} \frac{\kappa_0 - \kappa_{sym}}{\rho_0}
$$

- 10 EoS empirical parameters

$$
\begin{aligned}
&\rho_0, E_{sat}, K_{sat}, Q_{sat}, Z_{sat} \\
&E_{sym}, L_{sym}, K_{sym}, Q_{sym}, Z_{sym}
\end{aligned}
$$

- 2 parameters for density dependence of effective mass κ_0, κ_{sym}

Effect of Secondary decay:-



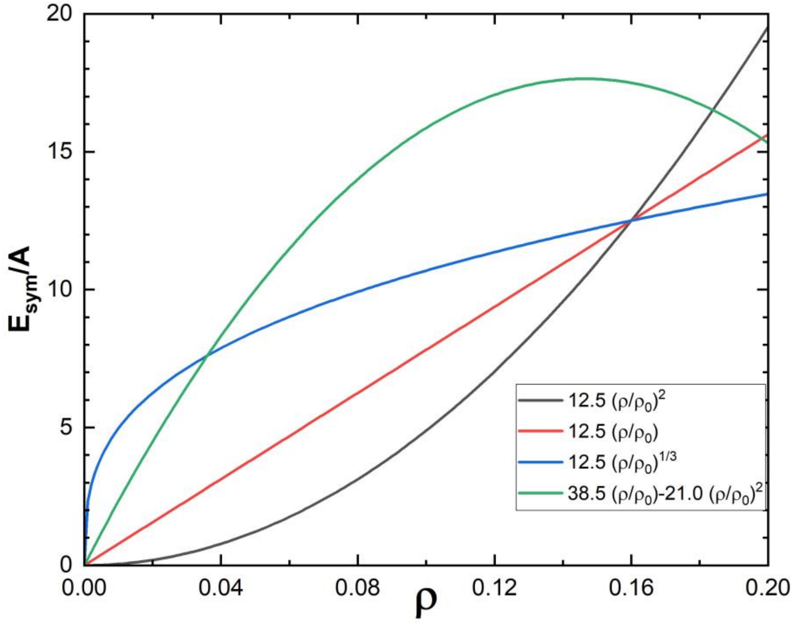
GEMINI++ Calculation by D. Gruyer

MSU Work:-

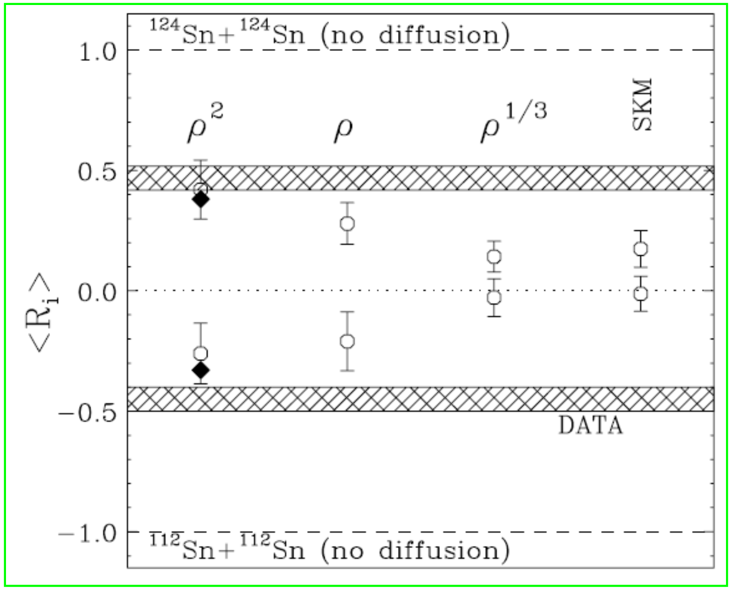
Energy parameterization:-

$$\epsilon_{sym}(\rho)=\frac{C_{kin}}{2}\left(\frac{\rho}{\rho_0}\right)^{2/3}+\frac{C_{pot}}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

Symmetry energy behavior:-



Isospin transport ratio:-



Ref: M. B. Tsang et. al, Phys. Rev. Lett. 92, 062701 (2004)

Nuclear EoS Parameters

EoS	ρ_0 (fm^{-3})	E_{sat} (MeV)	K_{sat} (MeV)	Q_{sat} (MeV)	Z_{sat} (MeV)	E_{sym} (MeV)	L_{sym} (MeV)	K_{sym} (MeV)	Q_{sym} (MeV)	Z_{sym} (MeV)
abinitio-1	0.1890	-16.92	241	-125	1281	34.57	48.5	-224	-311	-1974
abinitio7	0.14	-13.23	192	-139	901	28.53	43.9	-144	-95	-2149
SAMI	0.1587	-15.93	245	-339	1331	28.16	43.7	-120	372	-2179
NL3	0.1480	-16.24	271	198	9302	37.35	118.3	101	182	-3981
SGII	0.1583	-15.59	215	-381	1742	26.83	37.6	-146	330	-1891
Sly5	0.1604	-15.98	230	364	1592	32.03	48.3	-112	501	-3087