Constraints on EoS from study of light clusters and strange hadrons in heavy-ion collisions

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&

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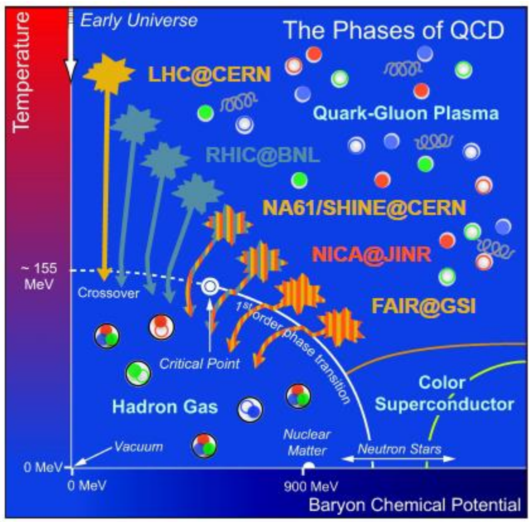
XIIth International Symposium on Nuclear Symmetry Energy (NUSYM 2024),
Caen, France, 9-14 September, 2024

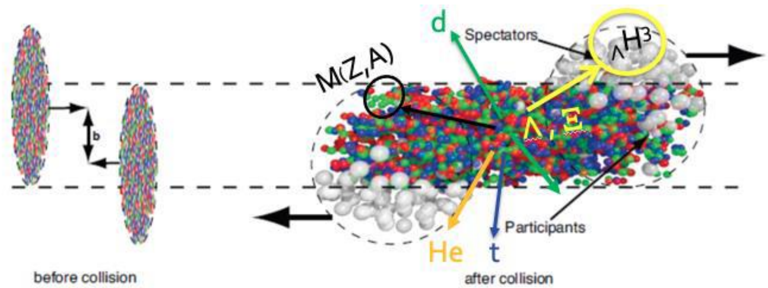
Cluster production in heavy-ion collisions

The phase diagram of QCD

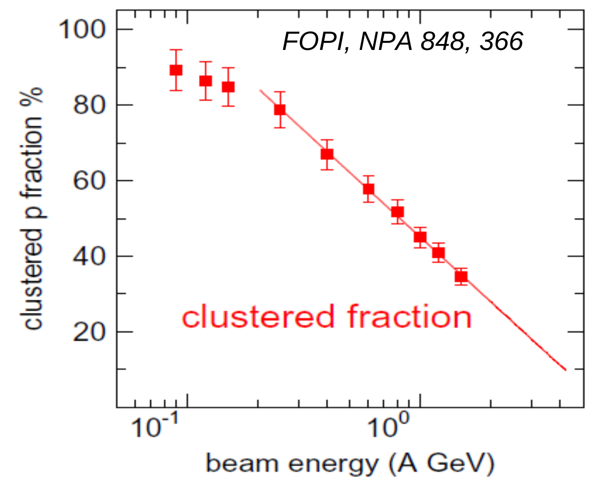


Clusters and (anti-) hypernuclei

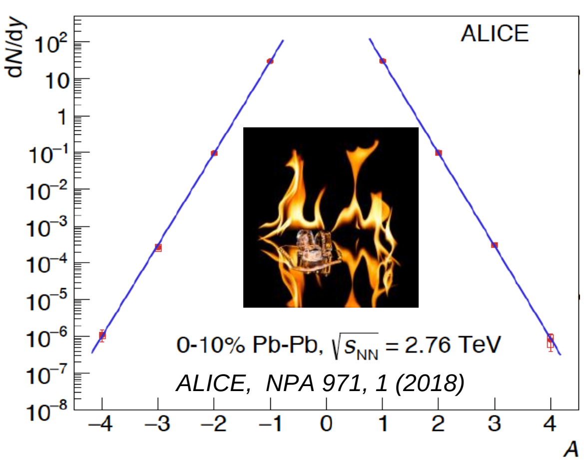
are observed experimentally at all energies



- **Clusters are very abundant at low energy**



Au+Au, central. midrapidity



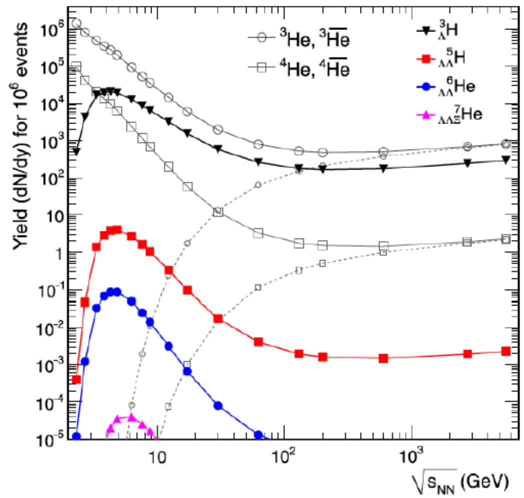
- **High energy HIC:** **‚Ice in a fire‘ puzzle:** **how the weakly bound objects can be formed and survive in a hot enviroment?!**

➔ **Mechanisms of cluster formation in strongly interacting matter are not well understood**

Modeling of cluster and hypernuclei formation

<u>Existing models for cluster formation:</u>

A. Andronic et al., PLB 697, 203 (2011)



❑ **statistical model:**
- **assumption of thermal equilibrium**

In order to understand the microscopic origin of cluster formation one needs a realistic model for the dynamical time evolution of the HIC

Dynamical Models:

I. cluster formation by coalescence mechanism
at a freeze-out time by coalescence radii in coordinate and momentum space

II. dynamical modeling of cluster formation based on interactions within microscopic transport models:

– potential‘ mechanism - via potential NN (NY) interactions (applied during the whole reaction time of HIC)

– ‘kinetic‘ mechanism - by hadronic scattering (hadronic reactions as NNN→ dN ; NNπ→ dπ, NN→ dπ)

PHQMD

PHQMD

PHQMD

PHQMD: a unified **n-body microscopic transport approach** for the description of heavy-ion collisions and **dynamical cluster formation** from low to ultra-relativistic energies

Realization: combined model PHQMD = (PHSD & QMD) + (MST/SACA)

Parton-Hadron-Quantum-Molecular Dynamics

PHSD mode

Initialization → propagation of baryons:
QMD (Quantum-Molecular Dynamics)

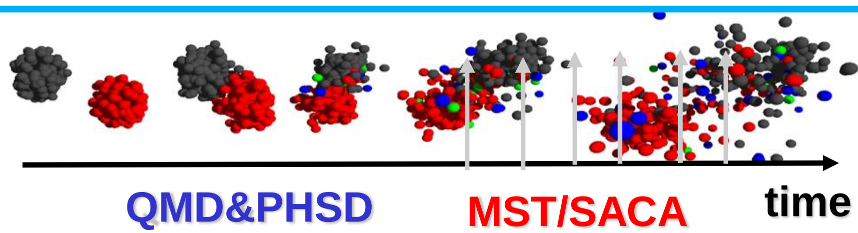
J.Aichelin, Phys. Rept. 202 (1991)

Propagation of partons (quarks, gluons) and mesons
+ collision integral = interactions of hadrons and partons (QGP)
from PHSD (Parton-Hadron-String Dynamics)

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215

Cluster recognition:
MST (Minimum Spanning Tree)
or SACA (Simulated Annealing Clusterization Algorithm)
or coalescence mechanism

R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266



PHQMD:
J. Aichelin et al., PRC 101 (2020) 044905;
S. Gläßel et al., PRC 105 (2022) 1;
V. Kireyeu et al., PRC 105 (2022) 044909;
G. Coci et al., PRC 108 (2023) 1, 014902

QMD propagation (EoM)

- **Generalized Ritz variational principle**: $$\delta \int_{t_1}^{t_2} dt < \psi(t)| i\frac{d}{dt} - H|\psi(t) >= 0.$$

Many-body wave function:

$$\psi(t) = \prod_{i=1}^{N} \psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t)$$

Ansatz:
Gaussian trial wave function (with width L) centered at r_{i0}, p_{i0}

$$\psi(\mathbf{r}_i, \mathbf{r}_{i0}, \mathbf{p}_{i0}, t) = C\ \mathrm{e}^{-\frac{1}{4L}\left(\mathbf{r}_i - \mathbf{r}_{i0}(t) - \frac{\mathbf{p}_{i0}(t)}{m}t\right)^2} \cdot \mathrm{e}^{i\mathbf{p}_{i0}(t)(\mathbf{r}_i - \mathbf{r}_{i0}(t))} \cdot \mathrm{e}^{-i\frac{\mathbf{p}_{i0}^2(t)}{2m}t}$$

L=4.33 fm²

- **Equations-of-motion (EoM) for Gaussian centers in coordinate and momentum space:**

$$\dot{r}_{i0} = \frac{\partial\langle H\rangle}{\partial p_{i0}} \qquad \dot{p}_{i0} = -\frac{\partial\langle H\rangle}{\partial r_{i0}}$$

Many-body Hamiltonian: $$H = \sum_i H_i = \sum_i (T_i + V_i) = \sum_i (T_i + \sum_{j\neq i} V_{i,j})$$

[Aichelin, Phys. Rept. 202 (1991)]

- **Nucleon-nucleon local two-body potential:**

$$V_{ij} = V(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_{i0}, \mathbf{r}_{j0}, \mathbf{p}_{i0}, \mathbf{p}_{j0}, t) = V_{\mathrm{Skyrme\ loc}} + V_{\mathrm{mom}} + V_{\mathrm{Coul}}$$

momentum dependent potential

➔ **Single-particle potential <V> :**

1) Skyrme potential ('static') :

$$\langle V_{Skyrme}(\mathbf{r_{i0}}, t)\rangle = \alpha\left(\frac{\rho_{int}(\mathbf{r_{i0}}, t)}{\rho_0}\right) + \beta\left(\frac{\rho_{int}(\mathbf{r_{i0}}, t)}{\rho_0}\right)^{\gamma}$$

with relativistic extended interaction density:

$$\rho_{int}(\mathbf{r_{i0}}, t) \rightarrow C\sum_j \left(\frac{4}{\pi L}\right)^{3/2} \mathrm{e}^{-\frac{4}{L}(\mathbf{r^T_{i0}}(t) - \mathbf{r^T_{j0}}(t))^2} \times \mathrm{e}^{-\frac{4\gamma_{cm}^2}{L}(\mathbf{r^L_{i0}}(t) - \mathbf{r^L_{j0}}(t))^2},$$

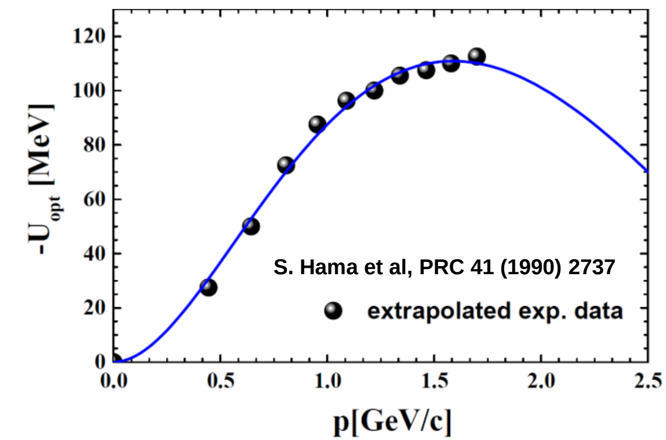
Momentum dependent potential → EoS in PHQMD

2) Momentum dependent potential :

$$V(\mathbf{r}_1, \mathbf{r}_2, \mathbf{p}_{01}, \mathbf{p}_{02}) = (a\Delta p + b\Delta p^2)\ exp[-c\sqrt{\Delta p}]\ \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

$$\Delta p = \sqrt{(\mathbf{p}_{01} - \mathbf{p}_{02})^2}$$

Parameters a, b, c are fitted to the "optical" potential (Schrödinger equivalent potential U_{SEP}) extracted from elastic scattering data in pA: $U_{SEQ}(p) = \frac{\int^{p_F} V(\mathbf{p} - \mathbf{p}_1) dp_1^3}{\frac{4}{3}\pi p_F^3}$



- **In infinite matter a potential corresponds to the EoS:**

$$E/A(\rho) = \frac{3}{5}E_F + V_{Skyrme\ stat}(\rho) + V_{mom}(\rho)$$

$$V_{Skyrme} = \alpha\frac{\rho}{\rho_0} + \beta\frac{\rho}{\rho_0}^{\gamma}$$

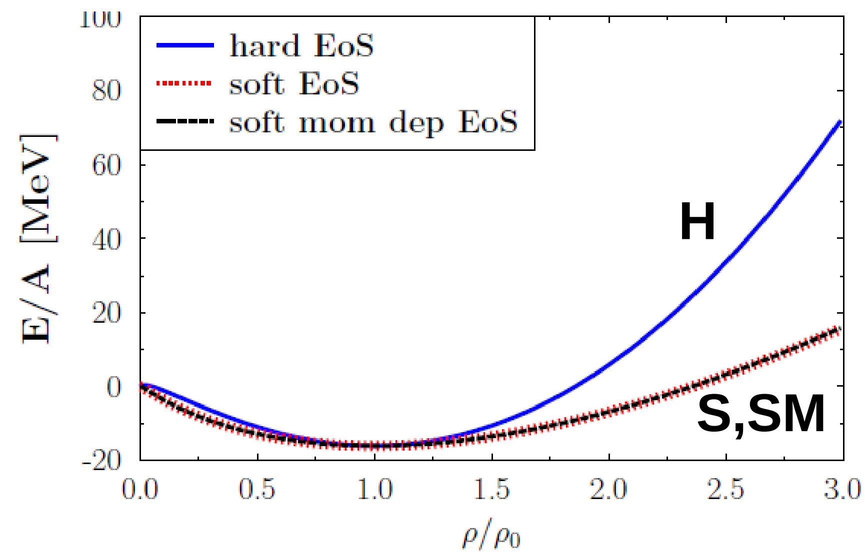
compression modulus K of nuclear matter:

$$K = -V\frac{dP}{dV} = 9\rho^2\frac{\partial^2(E/A(\rho))}{(\partial\rho)^2}|_{\rho=\rho_0}.$$

E.o.S.	$\alpha[MeV]$	$\beta[MeV]$	γ	K [MeV]
S	-383.5	329.5	1.15	200
H	-125.3	71.0	2.0	380
SM	-478.87	413.76	1.10	200

	a $[MeV^{-1}]$	b$[MeV^{-2}]$	c$[MeV^{-1}]$
	236.326	-20.73	0.901

EoS for infinite cold nuclear matter at rest



Mechanisms for cluster production in PHQMD:

I. potential interactions

(recongnized by MST)

&

II. kinetic reactions

III. Coalescence (to compare with I+II)

PHQMD

I. Cluster recognition: Minimum Spanning Tree (MST)

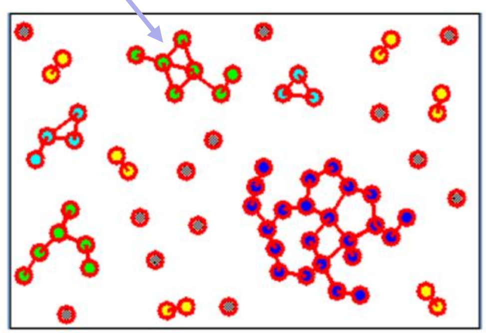
R. K. Puri, J. Aichelin, J.Comp. Phys. 162 (2000) 245-266

The Minimum Spanning Tree (MST) is a cluster recognition method applicable for the (asymptotic) final states where coordinate space correlations may only survive for bound states.
The MST algorithm searches for accumulations of particles in coordinate space:

1. **Two particles are 'bound' if their distance in the cluster rest frame fulfills**

$$|\vec{r_i} - \vec{r_j}| \leq 4 \text{ fm} \quad \text{(range of NN potential)}$$

2. **Particle is bound to a cluster if it binds with at least one particle of the cluster**

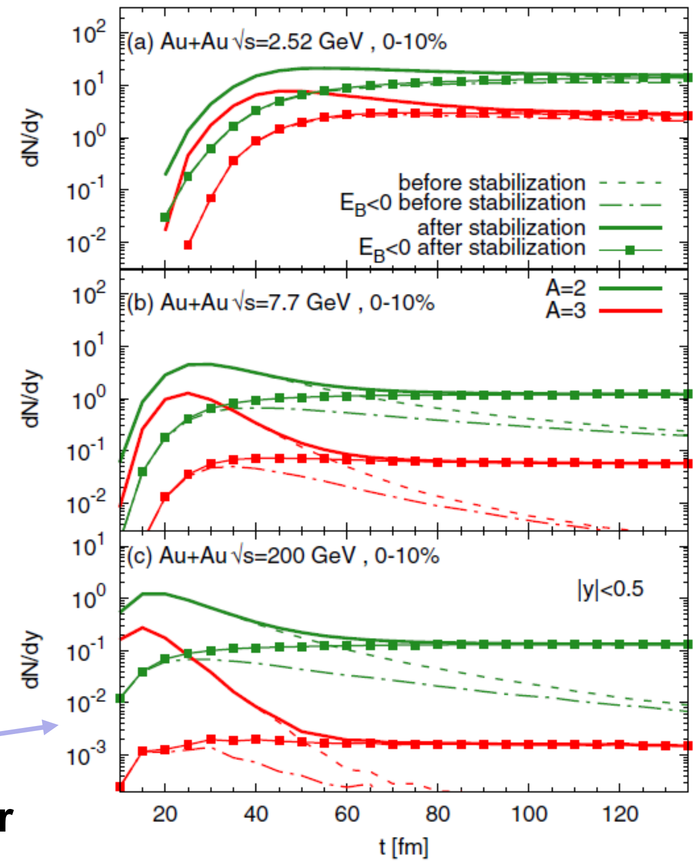


* Remark: inclusion of an additional momentum cut (coalescence) leads to small changes: particles with large relative momentum are almost never at the same position (V. Kireyeu, Phys.Rev.C 103 (2021) 5)

Advanced MST (aMST)

- ❑ **MST + extra condition: $E_B<0$**
  negative binding energy for identified clusters

- ❑ **Stabilization procedure – to correct artifacts of the semi-classical QMD:**
  recombine the final "lost" nucleons back into cluster if they left the cluster without rescattering



G. Coci et al., Phys.Rev.C 108 (2023) 014902

II. Deuteron production by hadronic reactions

"Kinetic mechanism"
1) hadronic inelastic reactions NN ⟷ dπ , πNN ⟷ dπ , NNN ⟷ dN
2) hadronic elastic π+d, N+d reactions

SMASH: D. Oliinychenko et al., PRC 99 (2019) 044907;
J. Staudenmaier et al., PRC 104 (2021) 034908
AMPT: R.Q. Wang et al. PRC 108 (2023) 3

- **Collision rate** for hadron "*i*" is the number of reactions in the **covariant volume** $d^4x = dt*dV$
- With **test particle ansatz the transition rate for 3→2 reactions**:

W. Cassing, NPA 700 (2002) 618

$$\frac{\Delta N_{coll}[3+4+5 \to 1(d)+2]}{\Delta N_3 \Delta N_4 \Delta N_5} = P_{3,2}(\sqrt{s})$$

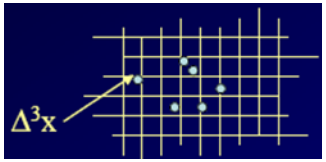
$$P_{3,2}(\sqrt{s}) = F_{spin} F_{iso} P_{2,3}(\sqrt{s}) \frac{E_1^f E_2^f}{2E_3 E_4 E_5} \frac{R_2(\sqrt{s}, m_1, m_2)}{R_3(\sqrt{s}, m_3, m_4, m_5)} \frac{1}{\Delta V_{cell}}$$

Energy and momentum of final particles

2,3-body phase space integrals [Byckling, Kajantie]

$$P_{2,3}\left(\sqrt{s}\right) = \sigma_{tot}^{2,3}(\sqrt{s}) v_{rel} \frac{\Delta t}{\Delta V_{cell}}$$

➔ solved by **stochastic method**



- Numerically tested in **"static" box:** PHQMD provides a good agreement with analytic solutions from rate equations and with SMASH for the same selection of reactions
- **New in PHQMD: π+N+N⟷ d+π** inclusion of **all possible isospin channels** allowed by total isospin T conservation ➔ enhancement of the d production

$$\pi^{\pm,0} + p + n \leftrightarrow \pi^{\pm,0} + d$$
$$\pi^- + p + p \leftrightarrow \pi^0 + d$$
$$\pi^+ + n + n \leftrightarrow \pi^0 + d$$
$$\pi^0 + p + p \leftrightarrow \pi^+ + d$$
$$\pi^0 + n + n \leftrightarrow \pi^- + d$$

G. Coci et al., Phys.Rev.*C* 108 (2023) 014902

Modelling finite-size effects in kinetic mechanism

G. Coci et al., PRC 108 (2023) 014902

How to account for the **quantum nature of deuteron**, i.e. for

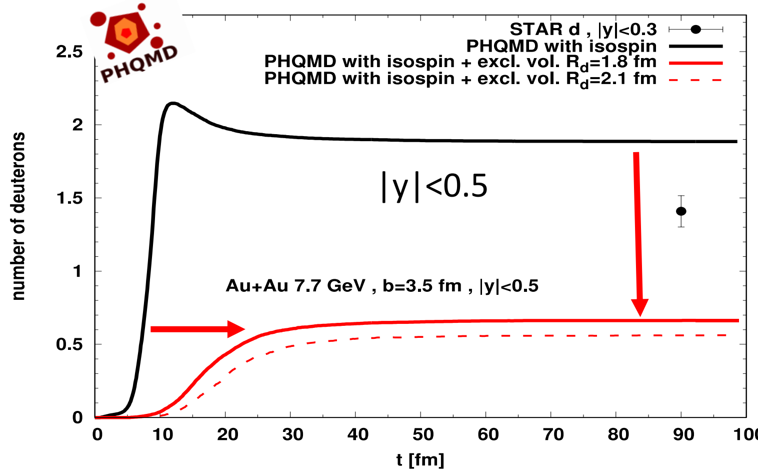
1) the **finite-size of *d* in coordinate space** (*d* is not a point-like particle) – for in-medium d production
2) the **momentum correlations of *p* and *n*** inside *d*

Realization:

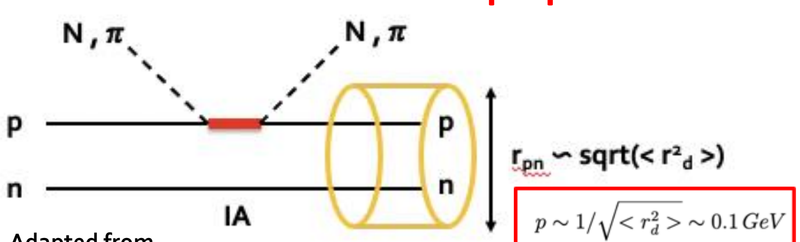
1) assume that a deuteron can not be formed in a high density region, i.e. if there are other particles (hadrons or partons) inside the 'excluded volume':

Excluded-Volume Condition: $|\vec{r}(i)^* - \vec{r}(d)^*| < R_d$

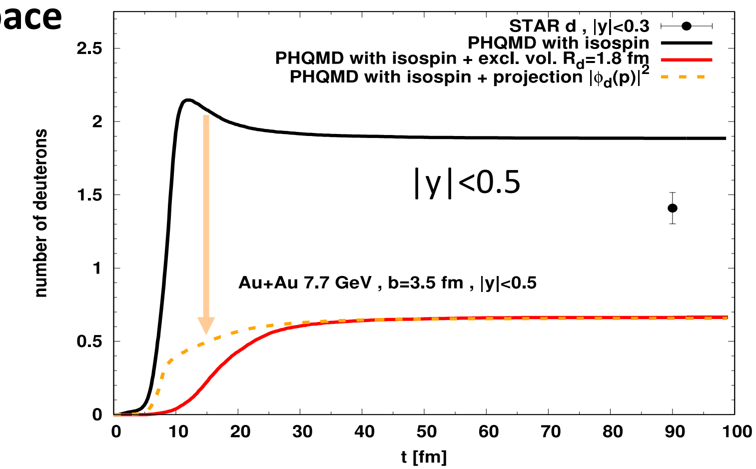
- ❑ **Strong reduction of d production**
- ❑ p_T slope is not affected by excluded volume condition



2) QM properties of deuteron must be also in momentum space
→ **momentum correlations of pn-pair**



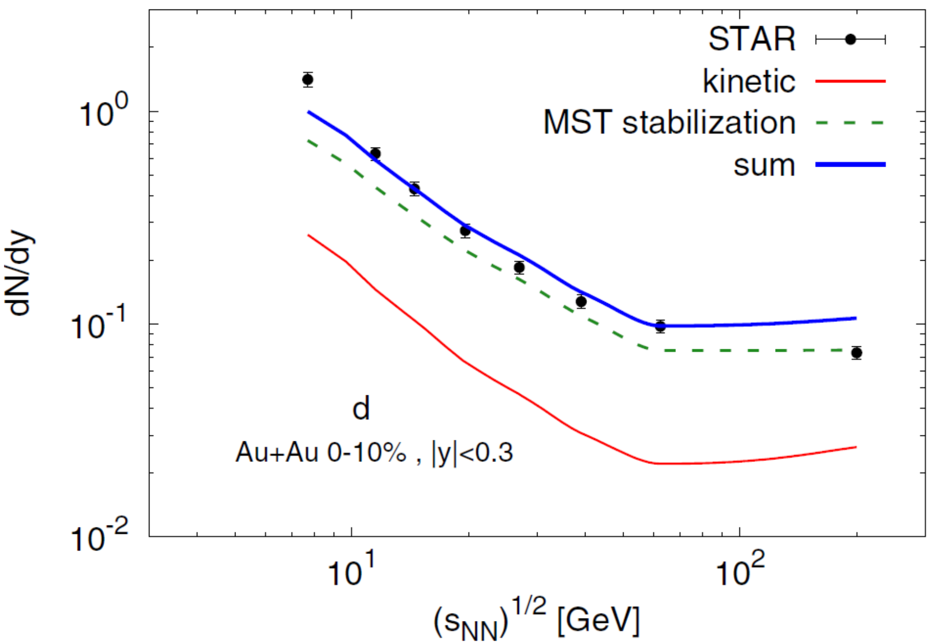
Adapted from
[Haidelbauer, Uzikov PLB 562(2003)]
[Hoftiezer et al. PRC23 (1981)]
Same spirit as AMPT [K.-J. Sun, R. Wang, C.-M. Ko et al., 2106.12742]



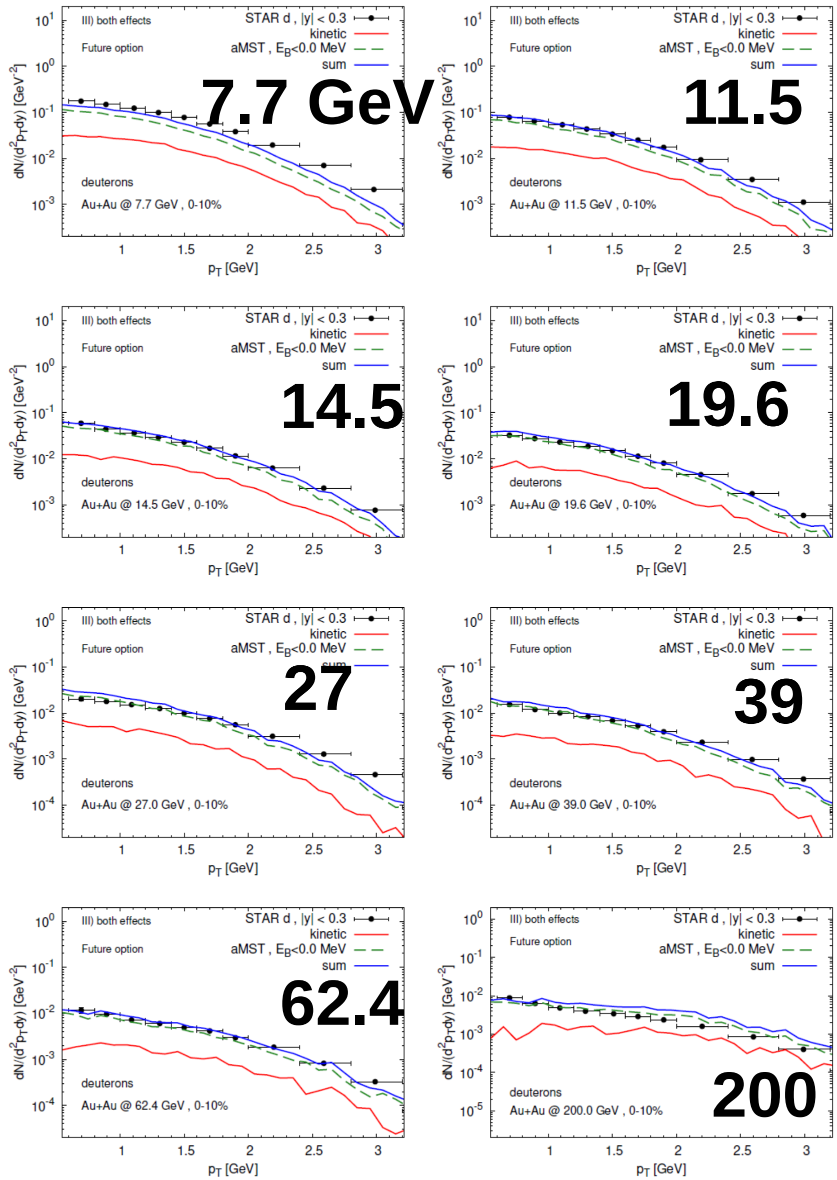
- ❑ **Strong reduction of d production** by projection on DWF $|\phi_d(p)|^2$

Kinetic vs. potential deuteron production

Excitation function dN/dy of deuterons at midrapidity



p_T – spectra (BES RHIC)

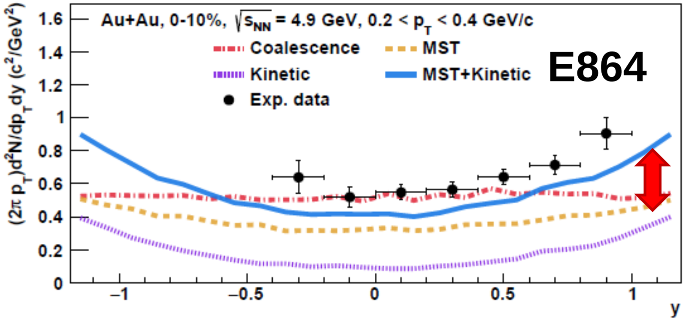


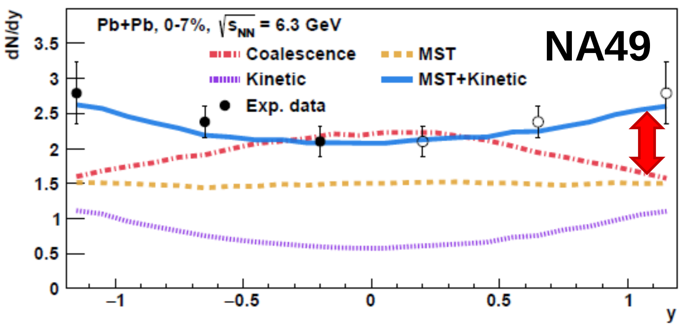
- PHQMD provides a good description of STAR data
- Functional forms of y- and p_T-spectra are slightly different for kinetic and potential deuterons
- **The potential mechanism is dominant for d production at all energies!**

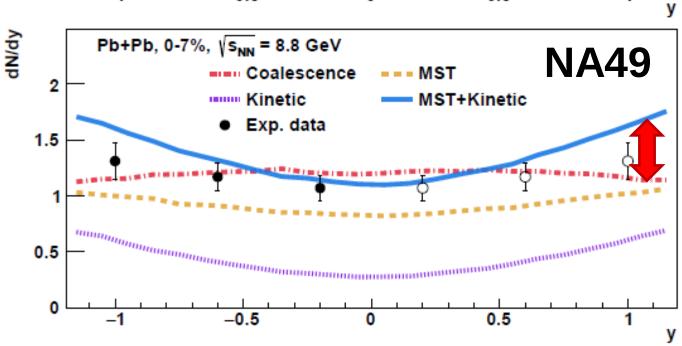
G. Coci et al., Phys.Rev.*C* 108 (2023) 014902

Mechanism for deuteron production: coalescence and MST+kinetic ⟺ experimental data

PHQMD

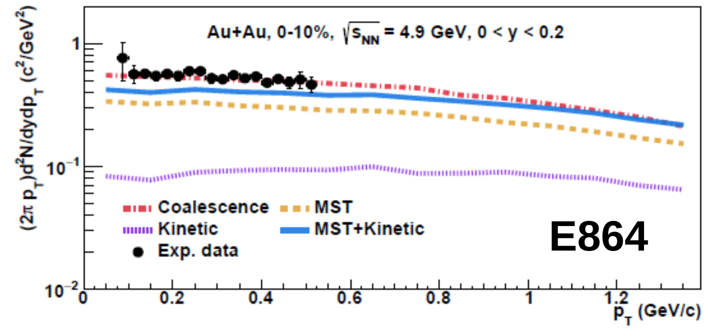


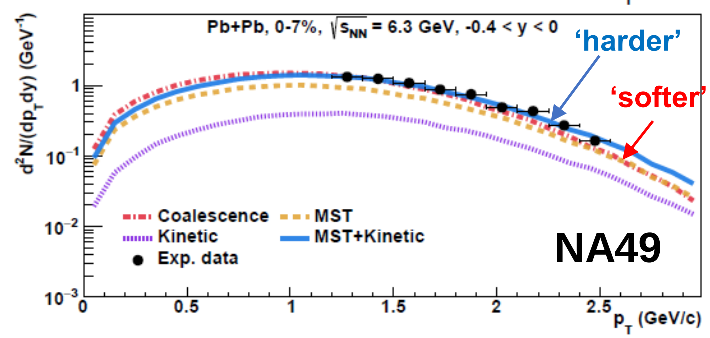


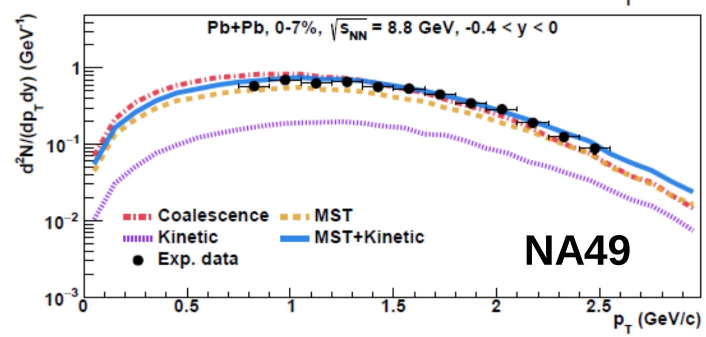


p_T -distributions have a different slope for coalescence/ MST+kinetic mechanisms

y- distributions show differences







➔ The analysis of the presently available data points tentatively to the MST + kinetic scenario but further experimental data are necessary to establish the cluster production mechanism

V. Kireyeu et al., PRC109 (2024), 044906

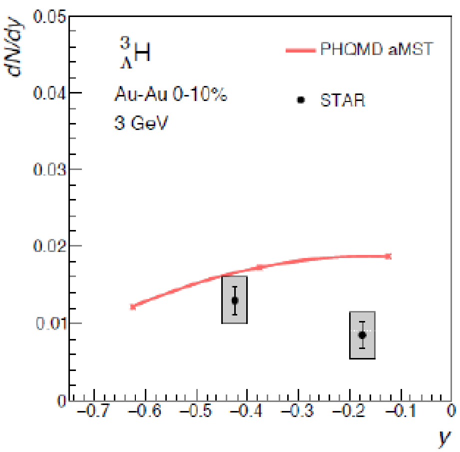
Hypernuclei production at STAR : $s^{1/2}$=3 GeV

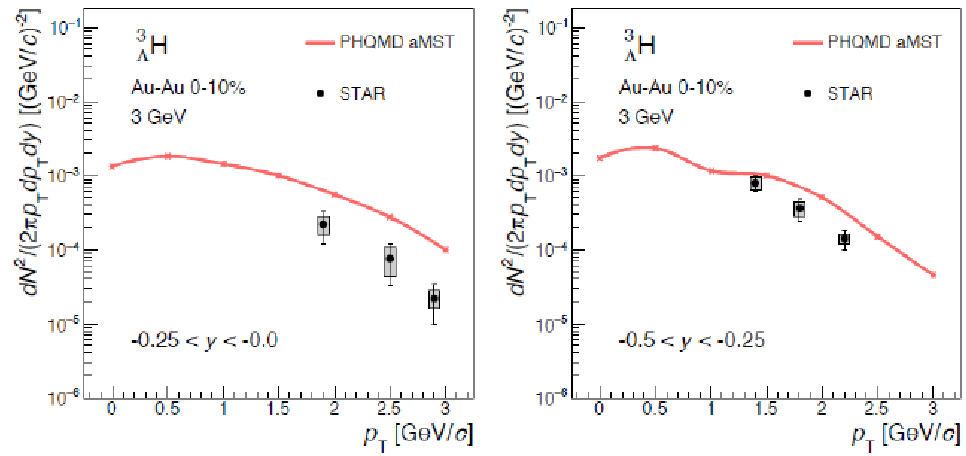
PHQMD

y – spectra (extracted)

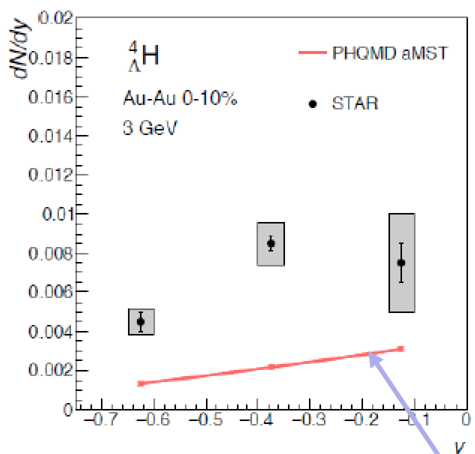
p_T – spectra (measured)

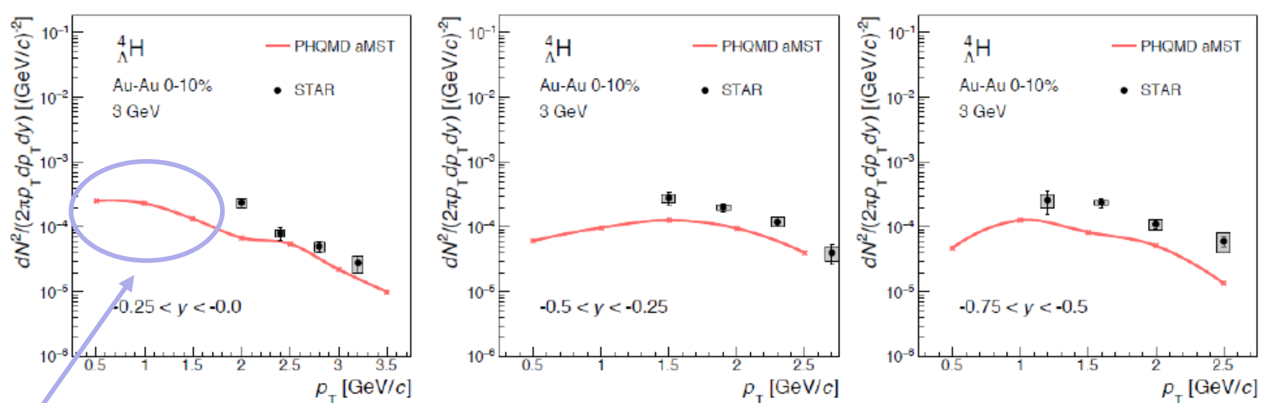
$^{3}_{\Lambda}H$





$^{4}_{\Lambda}H$





➔ **Low p_T – exp. data are needed for reliable estimation of y-spectra**

➔ **Hypernuclei production is sensitive to the hyperon dynamics**

PHQMD: S. Gläßel et al., Phys. Rev. C 105 (2022) 1

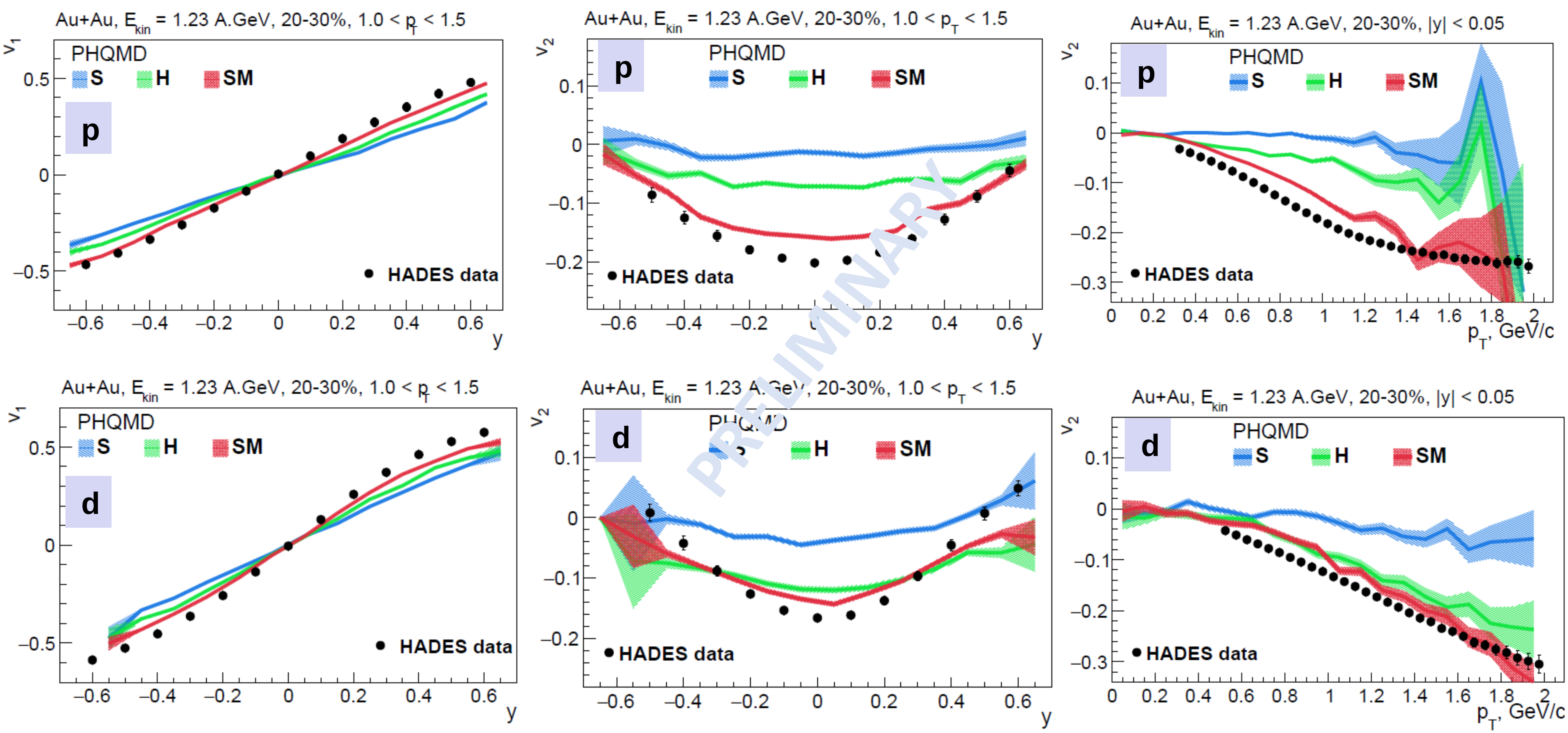
How to learn about EoS from clusters:

→ spectra and v_1, v_2 of light clusters with different EoS in PHQMD: hard, soft, momentum dependent potential

Cf. talks by Jörg Aichelin (Monday) and Arnaud Le Fevre (Tuesday)

EoS dependence of $v_1(y)$, $v_2(y)$, $v_2(p_T)$ at SIS energies: p,d

$v_1(y)$ $v_2(y)$ $v_2(p_T)$



PHQMD5.2W: S= soft EoS, H=hard EoS, MS = soft momentum dependent EoS

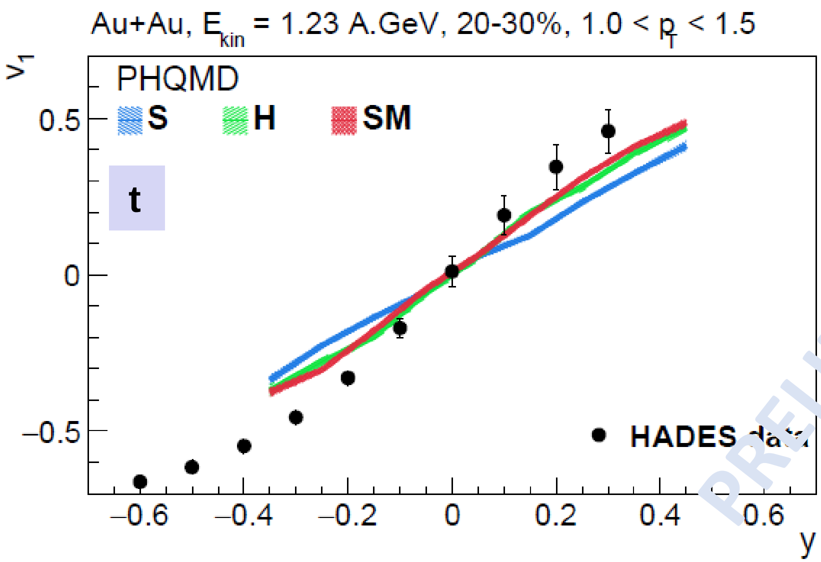
HADES data: of v_1, v_2 at high p_T: 1.0 < p_T < 1.5 GeV/c **[HADES: Eur. Phys. J. A59 (2023) 80]**

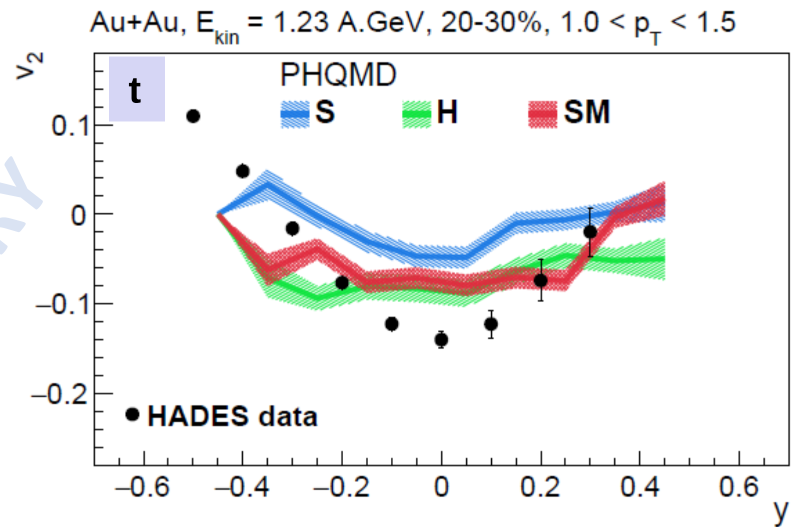
- ❑ **Strong EoS dependence of $v_1(y)$, $v_2(y)$ and $v_2(p_T)$ of protons and deuterons**
- ❑ **HADES data favor a soft momentum dependent potential (MS)**

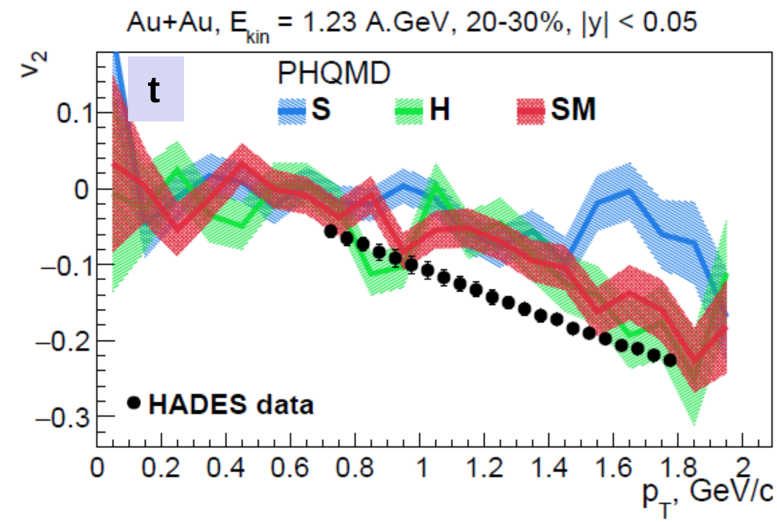
Viktar Kireyeu, in progress

EoS dependence of $v_1(y)$, $v_2(y)$, $v_2(p_T)$ at SIS energies: triton

PHQMD







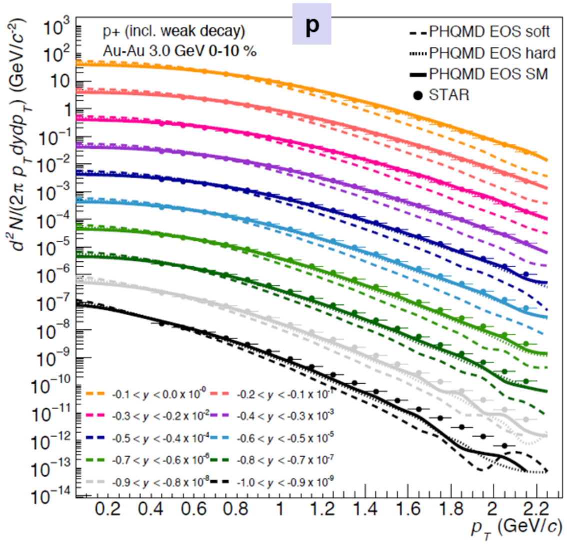
PHQMD5.2W: S= soft EoS, H=hard EoS,
MS = soft momentum dependent EoS
HADES data: of v_1, v_2 at high p_T:
$1.0 < p_T < 1.5$ GeV/c [HADES: Eur. Phys. J. A59 (2023) 80]

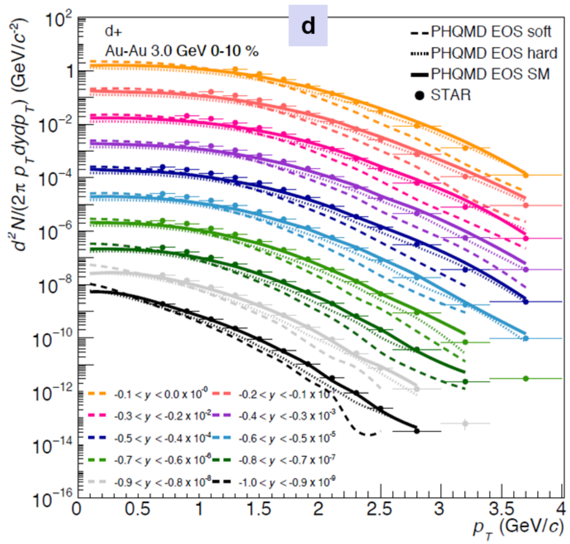
- **Strong EoS dependence of $v_1(y)$, $v_2(y)$ and $v_2(p_T)$ of tritons**
- **HADES data favor a soft momentum dependent potential (MS)**

Viktar Kireyeu, in progress

EoS dependence of p_T-spectra at STAR : $s^{1/2}$=3 GeV

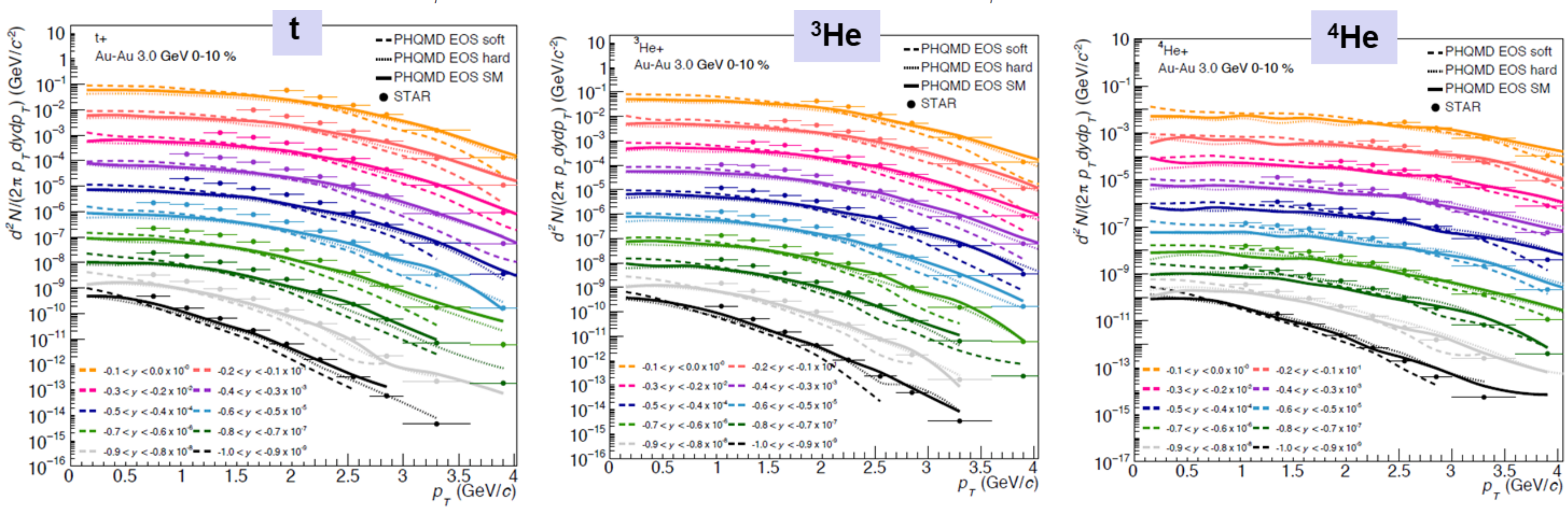
Susanne Gläßel, in progress





S= soft EoS,
H=hard EoS,
SM = soft momentum dependent EoS

PRELIMINARY



- ❑ Visible dependence of p_T spectra of p, d, t, ^{3}He, ^{4}He on EoS
- ❑ STAR p_T data favor a hard or soft-momentum dependent potential (H/SM)

STAR:H. Liu et al., 2208.04650, Acta Phys.Polon.Supp. 16 (2023) 1, 148

Summary

The PHQMD is a microscopic n-body transport approach for the description of heavy-ion dynamics and cluster and hypernuclei formation identified by Minimum Spanning Tree model

combined model PHQMD = (PHSD & QMD) & (MST | SACA)

Clusters are formed dynamically

1) by potential interactions among nucleons and hyperons
   Novel development: momentum dependent potential with soft EoS

2) by kinetic mechanism for d : hadronic inelastic reactions NN ↔ dπ , πNN ↔ dπ , NNN ↔ dN
   with inclusion of all possible isospin channels which enhance d production
   + accounting of quantum properties of d, modelled by the finite-size excluded volume effect in coordinate space and projection of relative momentum of p+n pair on d wave-function in momentum space which leads to a strong reduction of d production

- The PHQMD reproduces cluster and hypernuclei data on dN/dy and dN/dp_T as well as ratios d/p and $\bar{d}/\bar{p}$ for heavy-ion collisions from SIS to top RHIC energies.
- Measurement of dN/dy beyond mid-rapidity will allow to distinguish the mechanisms for cluster production: coalescence versus dynamical cluster production recognized by MST + kinetic mechanism for deuterons

- Strong dependencee of y- and p_T-spectra and v_1,v_2 on EoS - soft, hard, soft-mom. dependent - at SIS energies
- The influence of U(p) decreases with increasing collision energy since the modelled U_{SEP}(p) has a maximum at energy 1.5 GeV and decreases for large p ← no exp. data for extrapolation of U_{SEP}(p) to large p!
- HADES data data on v_1,v_2 favour a soft momentum dependent potential (SM)
- STAR data at 3 GeV favour a hard EoS or SM

- Stable clusters are formed shortly after elastic and inelastic collisions have ceased and behind the front of the expanding energetic hadrons (similar results within PHQMD and UrQMD)
  → since the 'fire' is not at the same place as the 'ice', cluster can survive