NUSYM24 – XIIth International Symposium on Nuclear Symmetry Energy

A new paradigm in the consistent extraction of nuclear symmetry energy and related properties using the relativistic application of coherent density fluctuation model

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Introduction & need for Isospin based study

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- ✓ Exploration of nuclear properties close to the β-stability line can be easily performed using the traditional bulk properties.
- ✓ However, moving away from the stability line, the dominance of traditional observables begins to drop, and isospin-dependent quantities come into prominence.
- ✓ Isospin asymmetry ⇨ $\alpha = \frac{(N-Z)}{N+Z}$

*Phys. Rev. C **85**, 064319 (2012).*
*Phys. Rev. C **97**, 024322 (2018)*
*Chin. Phys. C **46**, 084101 (2022).*

Symmetry Energy

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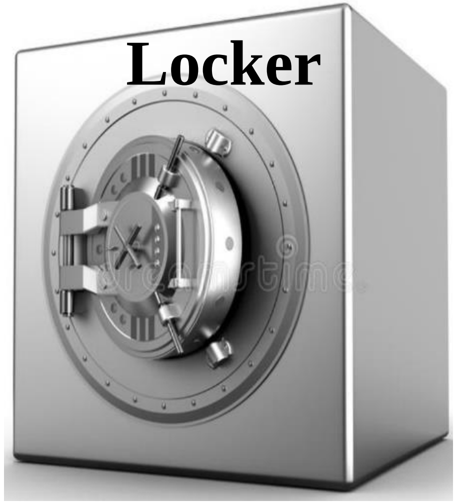
- Symmetry energy ⇨ variation of **B.E**. as the neutron to proton ratio of a nuclear system is varied

$$S^{NM}(\rho) = \frac{1}{2}\left(\frac{\partial^2 \varepsilon}{\partial \alpha^2}\right)$$

❖ **<u>Current aim to study finite nuclei using Symmetry energy</u>**

○ **Obstacle**

Symmetry energy is **not directly measurable**



Phys. Rev. C 97, 024322 (2018)

Finding the key

- CDFM : Density distribution of nuclear matter fluctuates around average distribution while maintaining spherical symmetry and uniformity.

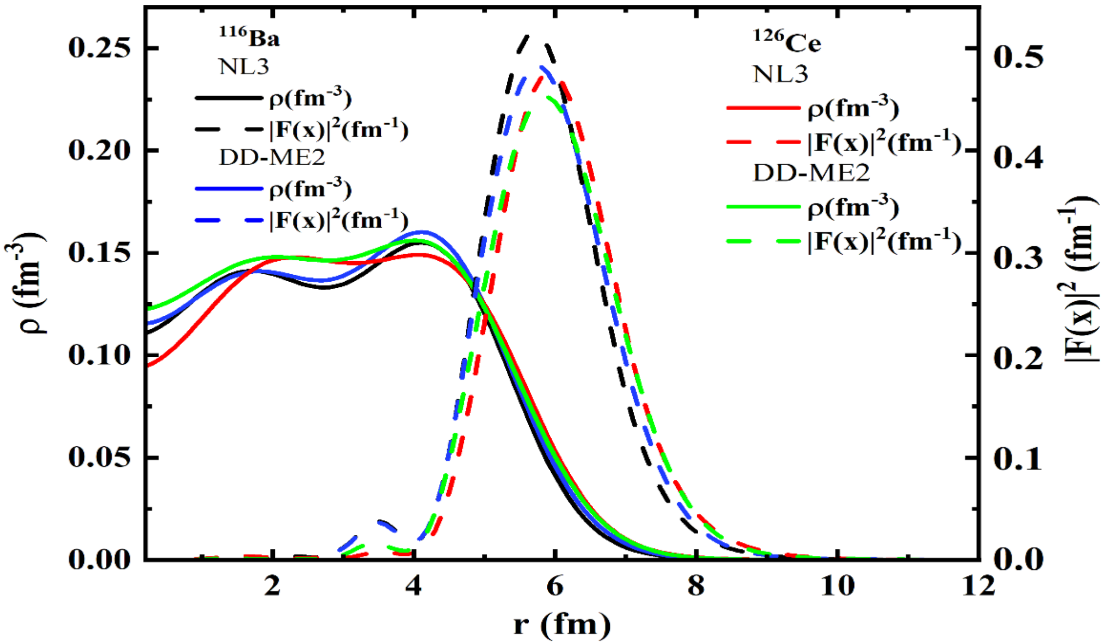
- The density **$\rho(r, r')$** of a finite nucleus can be rewritten as the coherent superposition of infinite number of one-body density matrix (OBDM) **$\rho_x(r, r')$** for spherical parts of nuclear matter called **Fluctons**.

Bulg. J. Phys. ***6****, 151 (1979).*
Z. Phys. A ***304****, 239 (1982).*

Methodology of CDFM

- Spherical flucton density: $$\rho_o(x) = 3A/4\pi x^3$$

- Weight function: $$|\mathcal{F}(x)|^2 = -\frac{1}{\rho_o(x)} \frac{d\rho(r)}{dr}\bigg|_{r=x}$$



Total density distribution ρ and corresponding weight function $|F(x)|^2$ as a function of nuclear distance r

*Phys. Rev. C **85**, 064319 (2012).*
Phys. Rev. C97, 024322 (2018)

Methodology

Symmetry energy:

$$S = \int_0^\infty dx |\mathcal{F}(x)|^2 S^{NM}(\rho(x))$$

$$\kappa = \frac{S_V}{S_S}$$

Volume Symmetry Egy

Surface Symmetry Egy

$$S_V = S\left(1 + \frac{1}{\kappa A^{1/3}}\right)$$

$$S_S = \frac{S}{\kappa}\left(1 + \frac{1}{\kappa A^{1/3}}\right)$$

*Phys. Rev. C **85**, 064319 (2012).*

Phys. Rev. C97, 024322 (2018)

Choice of energy density functional

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- The original CDFM formalism makes use of Brüeckner's prescription.
- The studies based on Brüeckner's prescription have significantly improved our understanding of nuclear matter properties.
- The studies have successfully verified various experimental shell closures across various isotopic & isotonic chains.

Phys. Rev. C ***85****, 064319 (2012).*
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Using CDFM with Brueckner EDF

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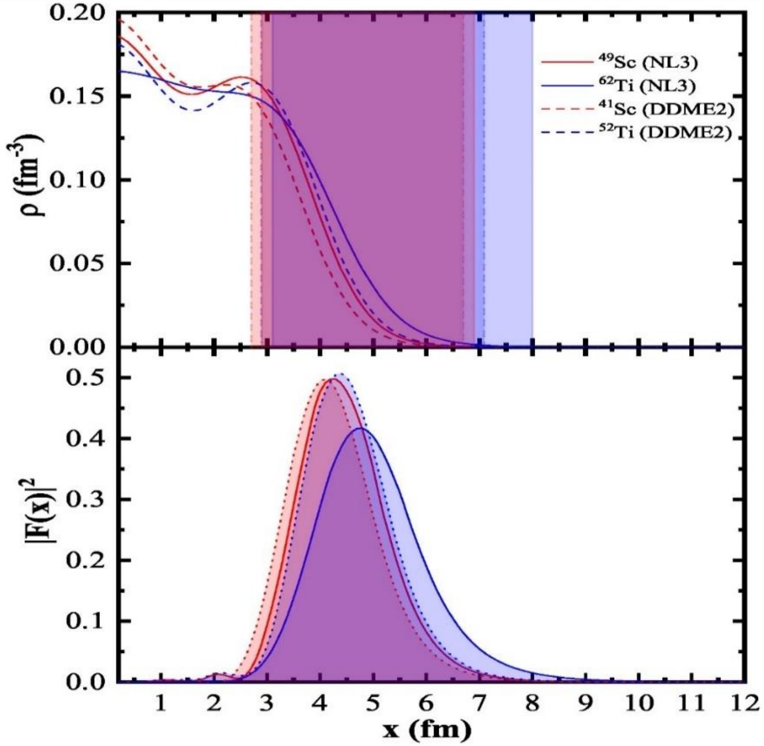
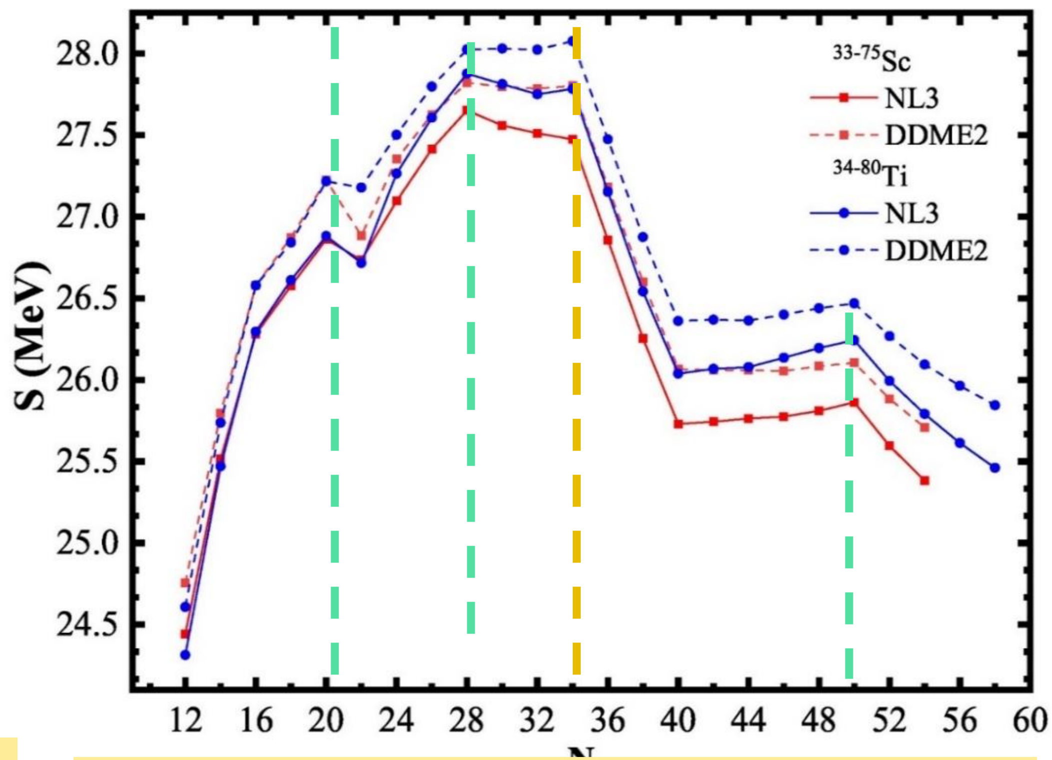
Verified Exp. Shell closure at N = 20, 28 & 50.
Predicted novel shell closure at N = 34.



Total density distribution ρ (upper panel) and corresponding weight function $|F(x)|^2$ (lower panel) as a function of nuclear distance x



Variation of Symmetry Energy S as a function of neutron number N

Chin. Phys. C ***46****, 084101 (2022).*

Relativistic energy density functional

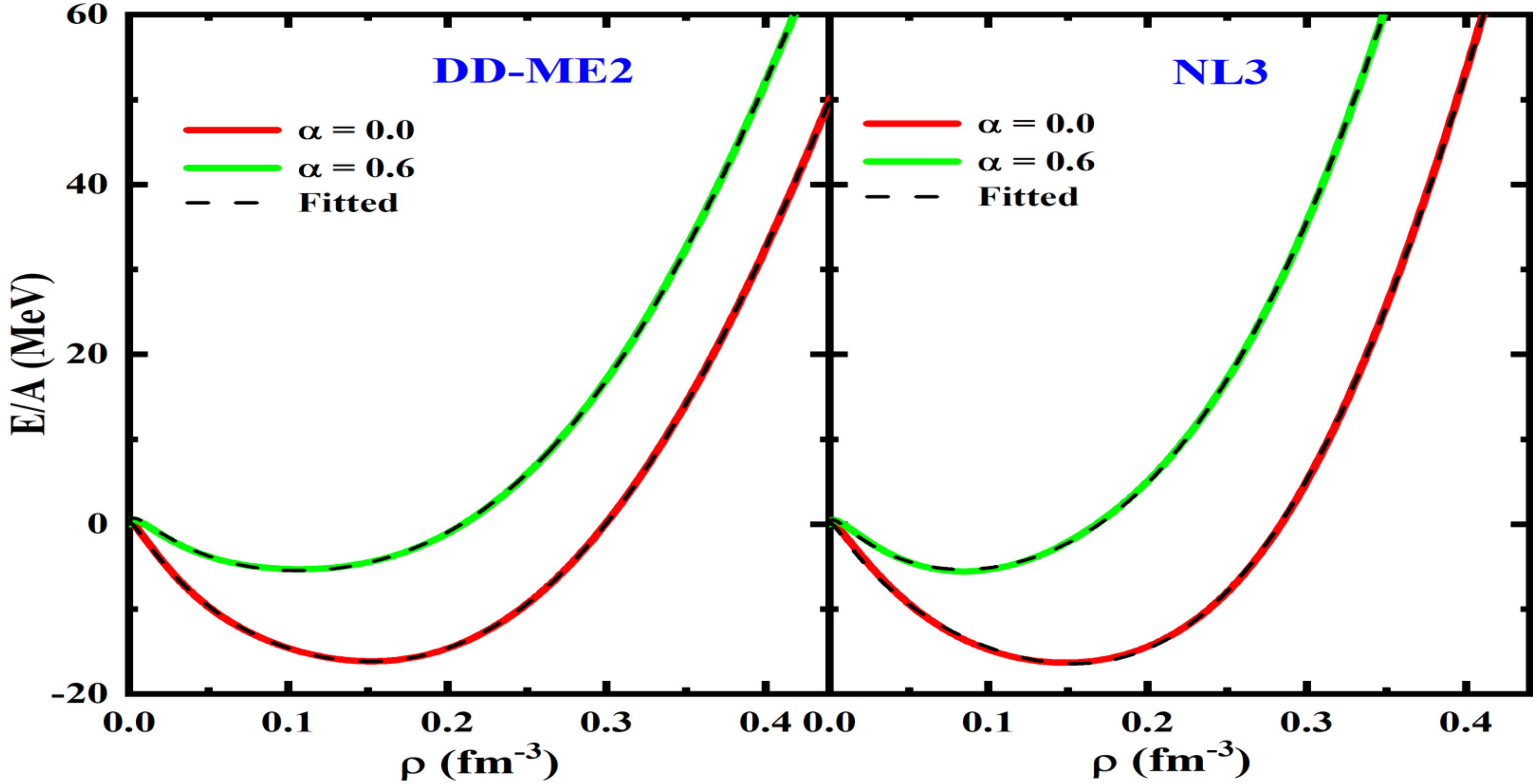
- The next step is to include relativistic prescription within the CDFM, i.e. make use of Relativistic energy density functional (R-EDF).

- Recently, we have introduced a parameterization of R-EDF in Ref: *[Europhys. Lett., **146**, 14001 (2024).]*

- The newly fitted R-EDF with CDFM *[Europhys. Lett., **146**, 14001 (2024)]* inherently counters the Coester-band problem.

- This novel parametrization based on R-EDF provides precise results in the study of nuclear structure at drip-line & possible correlations of isospin properties with the nuclear bulk properties.

Parameterization of EDF



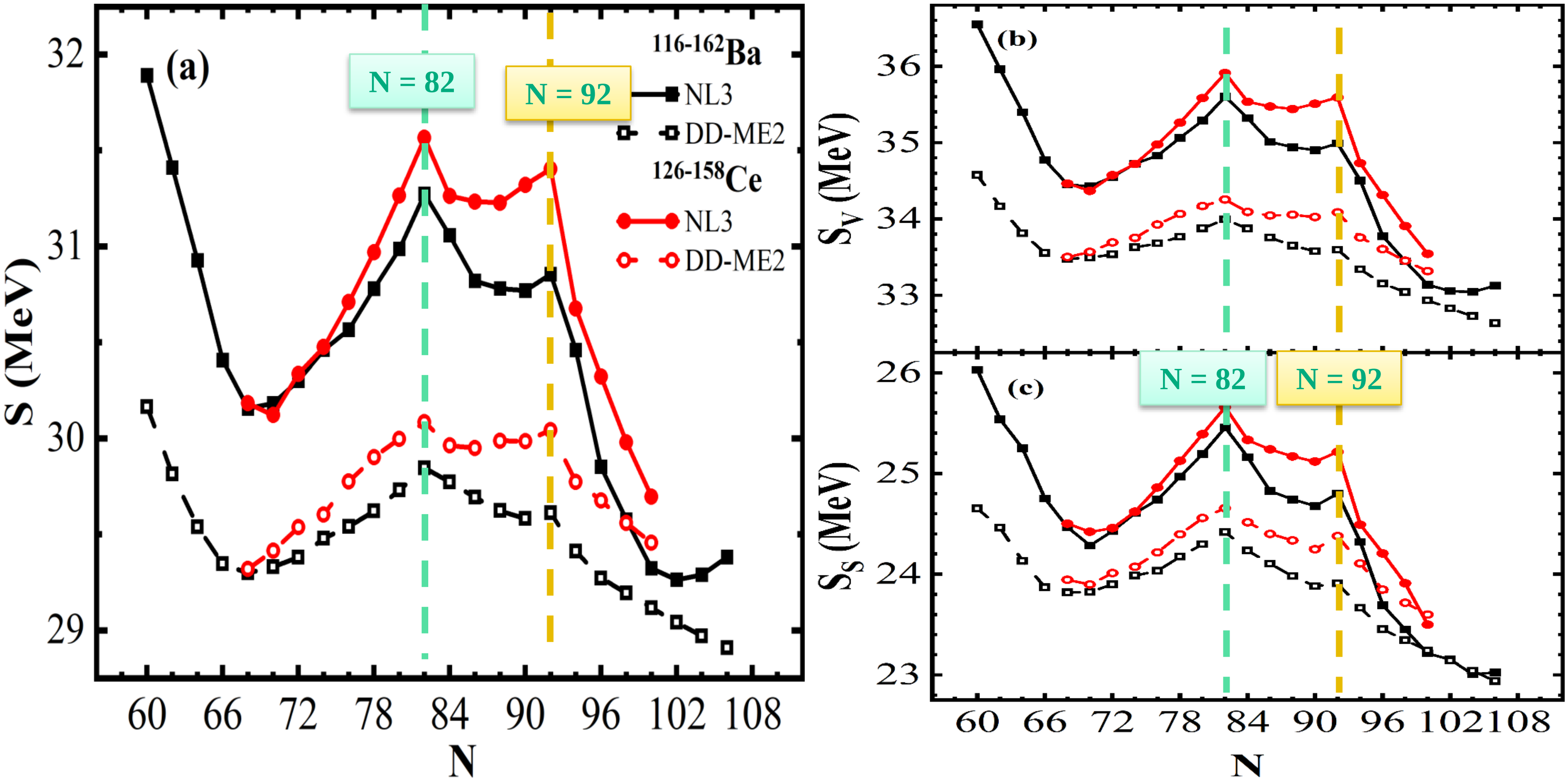
The energy per nucleon (E/A) as a function of total number density (ρ) of DD-ME2 and NL3 parameter sets for different asymmetry

- ✓ The parametrization resulted in the root-mean-square deviation of E/A for DD-ME2 and NL3 as 0.07704 and 0.19146 MeV, respectively.

Europhys. Lett., ***146***, *14001 (2024)*

Results using Relativistic-EDF within CDFM

Verified existing & predicted novel shell closure



The calculated symmetry energy (S), volume (S_V) & surface symmetry (S_S) energy using the parameterized Relativistic-EDF within the CDFM formalism.

Summary / Take away

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- CDFM formalism can be successfully used to study finite nuclei across nuclear chart.

- Rather than using Brueckner-EDF, with the incorporation of Relativistic prescription (Relativsitic-EDF) we can avoid the Coester band problem.

- This method can successfully verify exiting experimental shell closure & predict novel ones.


Thank
you

Extra Sildes

Following the literature, as detailed in Ref. [Z Physik A **297**, 257–260 (1980), JINR-E–2-11282, and Z Physik **304**, 239–243 (1982)] we can derive the expression of the weight function.

The mixed density, whose Fourier transformation is connected with the momentum distribution, can be written quite generally in the form:

$$\rho(\mathbf{r}, \mathbf{r}') = \int_0^\infty |\mathcal{F}(x)|^2 \rho_x(\mathbf{r}, \mathbf{r}') dx. \quad (1)$$

Here, $|\mathcal{F}(x)|^2$ is the weight function for the different uniform distributions in the average density distribution. Moreover, ρ_x is the density matrix for A nucleons uniformly distributed in the sphere with radius x and spherical flucton density $\rho_o(x) = 3A/4\pi x^3$, expressed as:

$$\rho_x(\mathbf{r}, \mathbf{r}') = \quad 3\rho_0(x) \frac{J_1(k_F(x)|\mathbf{r}-\mathbf{r}'|)}{(k_F(x)|\mathbf{r}-\mathbf{r}'|)}. \quad (2)$$

Here, J_1 is a Bessel function of first-order, and $k_F(x)$ is known as the Fermi momentum of the nucleons given as:

$$k_F(x) = \left(\frac{3\pi^2}{2}\rho_o(x)\right)^{1/3} = \left(\frac{9\pi A}{8}\right)^{1/3}. \quad (3)$$

It is important to note that Eq. 1 relates to a general CDFM assertion that the density distribution of nuclear matter fluctuates around average distribution while maintaining spherical symmetry and uniformity. Following the definition of density matrix, one-particle density $\rho(\mathbf{r})$ is given by its diagonal elements as:

$$\rho(\mathbf{r}) = \rho(\mathbf{r}, \mathbf{r}')\big|_{\mathbf{r}' \to \mathbf{r}}. \quad (4)$$

Using Eqs. 1 and 4 gives

$$\rho(\mathbf{r}) = \int_0^\infty |\mathcal{F}(x)|^2 \rho_x(\mathbf{r}) dx, \quad (5)$$

which is a decomposition of ρ in terms of Θ-like densities as:

$$\rho_x(\mathbf{r}) = \rho_0(x)\theta(x - \mathbf{r}), \quad \theta(y) = \begin{cases} 1, & y \geq 0 \\ 0, & y < 0 \end{cases}. \quad (6)$$

Consequently, after the differentiation of Eq. 5 results in:

$$|\mathcal{F}(x)|^2 = -\frac{1}{\rho_o(x)} \frac{d\rho(r)}{dr}\bigg|_{r=x}, \quad (7)$$

Why RMF?

- Main advantage: Takes account of spin-orbit interactions automatically.
- Coester band problem (Nuclear saturation & binding energy) is solved by relativistic models.
- Empirical saturation point of symmetric nuclear matter that is $E/A \approx -16$ MeV at $\rho \approx 0.2$ fm−3 instead of $\rho \approx 0.15$ fm−3, commonly known as the Coaster-Band problem.
- Provides reasonably good description of finite nuclei & infinite nuclear matter

Prog. Part. Nucl. Phys. **37**, 193 (1996), Phys. Rev. C **97**, 024322 (2018).