# Toward a better characterization of the nuclear equation of state using central collisions around Fermi energy

## Methods and preliminary results

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# Outlines

- Introduction
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- Estimation of the simulation/experiment error
- Bayesian inference of $\sigma^*_{NN}$ in-medium nucleon-nucleon cross section
- Perspectives : density $\rho$

# Introduction

$$E(\delta, \rho) = E_{iso}(\rho) + E_{ivec}(\rho)\delta^2 + \mathcal{O}(\delta^3)$$

With:

$$E_{iso}(\rho) = E_{sat} + \frac{K_{sat}}{2}\left(\frac{\rho - \rho_0}{3\rho_0}\right)^2 + \mathcal{O}(\rho^3)$$

$$E_{ivec}(\rho) = E_{sym} + L_{sym}\left(\frac{\rho - \rho_0}{3\rho_0}\right) + \cdots$$

$$\cdots + \frac{K_{sym}}{2}\left(\frac{\rho - \rho_0}{3\rho_0}\right)^2 + \mathcal{O}(\rho^3)$$

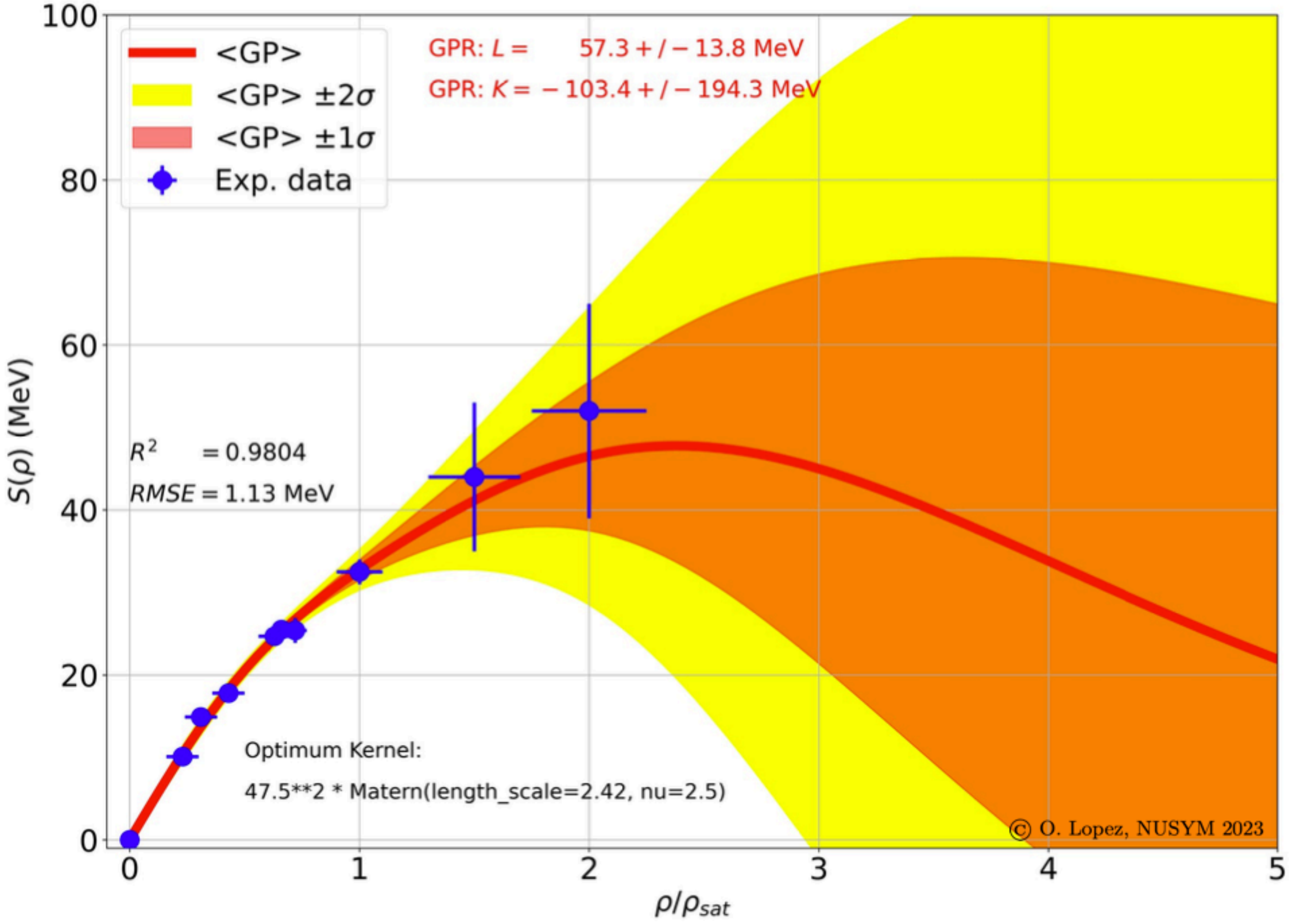
- The EOS relates the pressure, energy density, and other thermodynamic properties of nuclear matter. The equation is composed of two parts, the isoscalar and the isovector one.

- The first term (isoscalar) depends on density $\rho$. The second term (isovector) has dependencies in isospin $\delta$ and also density $\rho$.

- $E_{sat}$ is the saturation energy and $K_{sat}$ is the curvature of the isoscalar energy component. $E_{sym}$ is the symmetry energy, $L_{sym}$ the slope, $K_{sym}$ the curvature of the isovector part.

- The constraints of those parameters is the goal of my thesis. One can do it with a Machine-Learning approach...

# Motivations




- As an example, we can see the work of O. Lopez. It's a gaussian emulation of the symmetry energy on the experimental data gathered from the literature.
- The symmetry energy $S(\rho)$, which is the energy cost per nucleon for changing a symmetric (N=Z) nuclear system at a given baryonic density $\rho$ to a pure neutron matter (N=A) system.
- The more you explore the range of density, the better is the constraint on the parameter $L_{sym}$ and $K_{sym}$.
- My thesis aims to use Machine-Learning techniques and Bayesian inference on the INDRA-FAZIA datasets to improve and put constraints on the parameters of the EOS

# Nucleon-Nucleon Cross Sections and the EOS

- In-medium nucleon-nucleon (NN) cross sections $\sigma^*_{NN}$ describe the fundamental interaction between two nucleons (protons or neutrons) in the atomic nucleus. They determine the probability of these particles undergoing a collision or scattering when they encounter each other.

- By accurately modeling NN interactions from experimental scattering data, the energy and isospin contributions of the nuclear interaction within the medium can be determined.

- These energy contributions allow for the construction of the average nuclear interaction potential. This average potential is then used to solve the nuclear mean-field equation and obtain the equation of state.

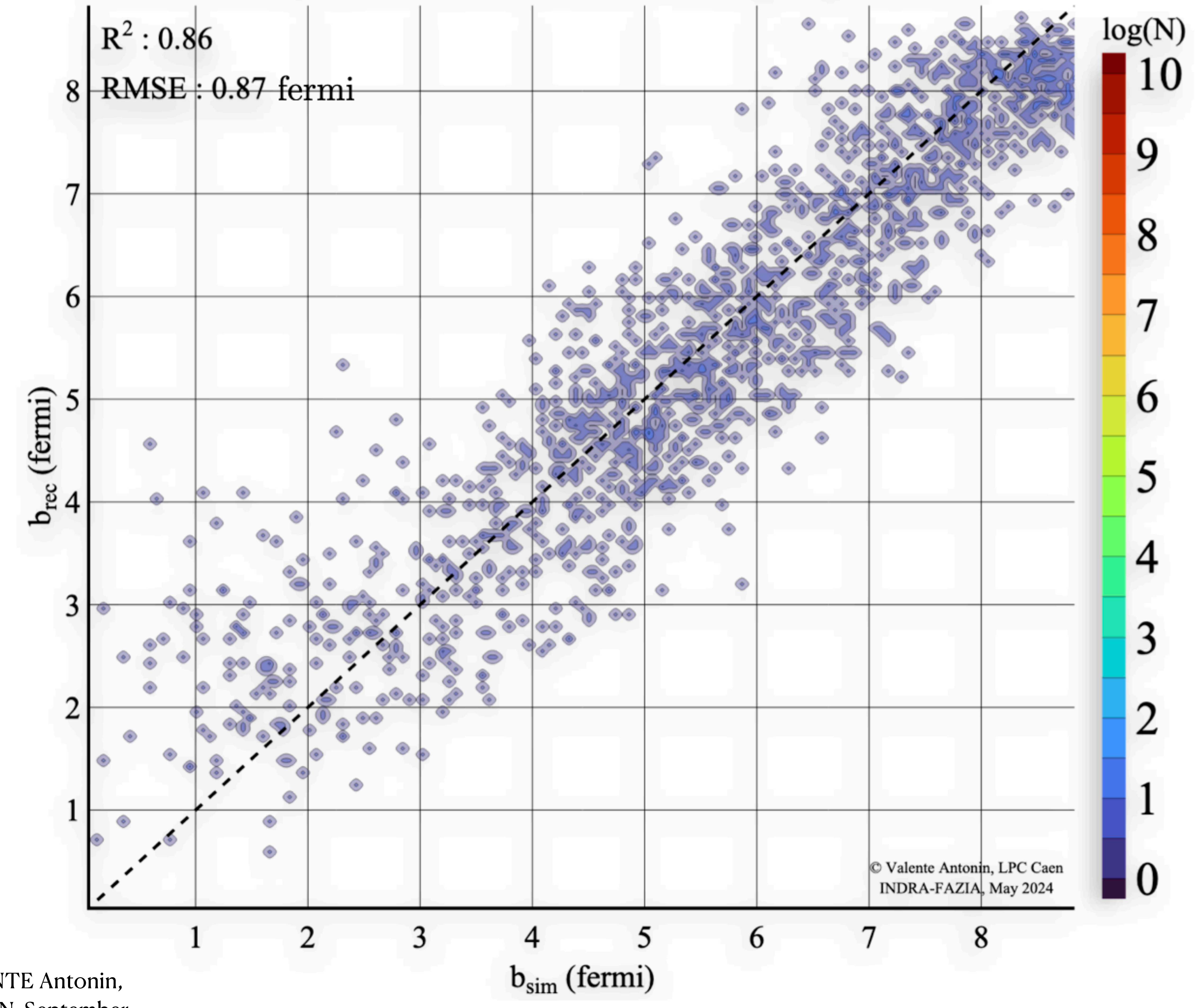
- Therefore, an accurate description of NN cross sections is crucial for correctly calculating the saturation properties of infinite nuclear matter (density, binding energy, symmetry energy, compressibility, etc.) predicted by the equation of state.

# Central collisions : Neural Network approach of the impact parameter

## Correlation *HIPSE & ELIE* model

$^{58}Ni_{28}$ on $^{58}Ni_{28}$ at 52 Mev/Nucleon (INDRA, HIPSE))

*Correlation map of the impact parameter on ($b_{sim}$, $b_{rec}$) via MLP*




$^{58}Ni_{28}$ on $^{58}Ni_{28}$ at 52 Mev/Nucleon (INDRA, ELIE)

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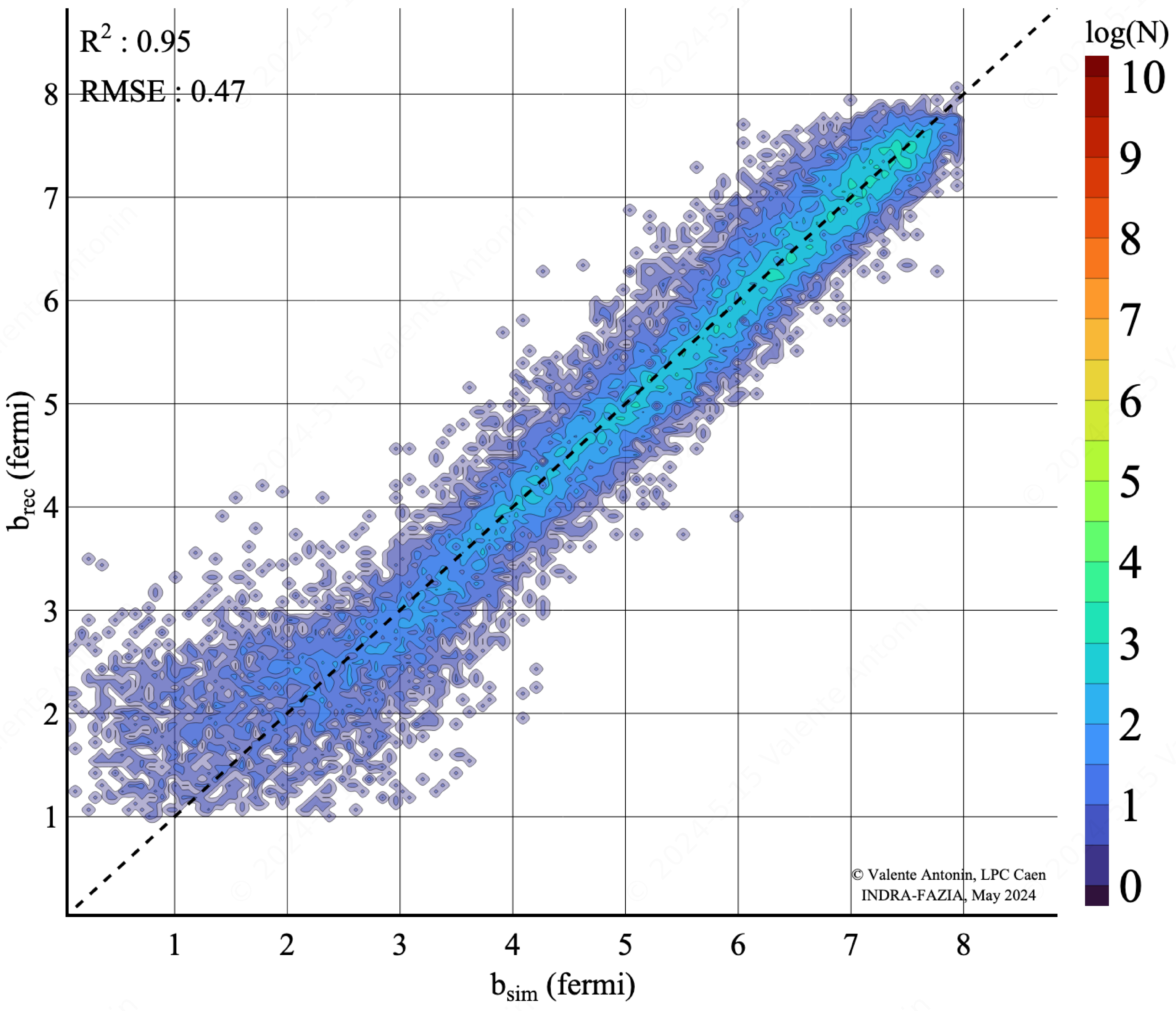
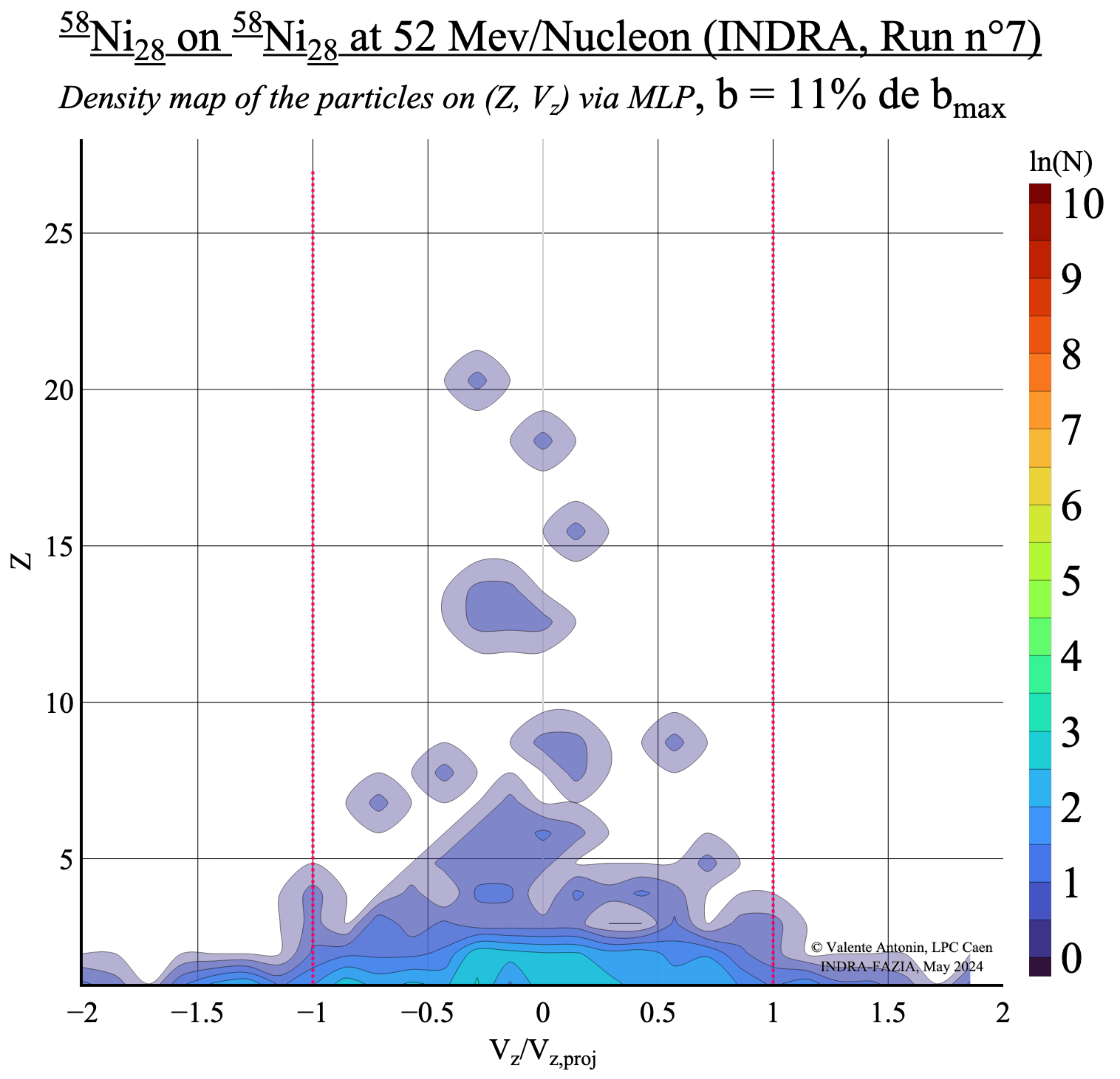
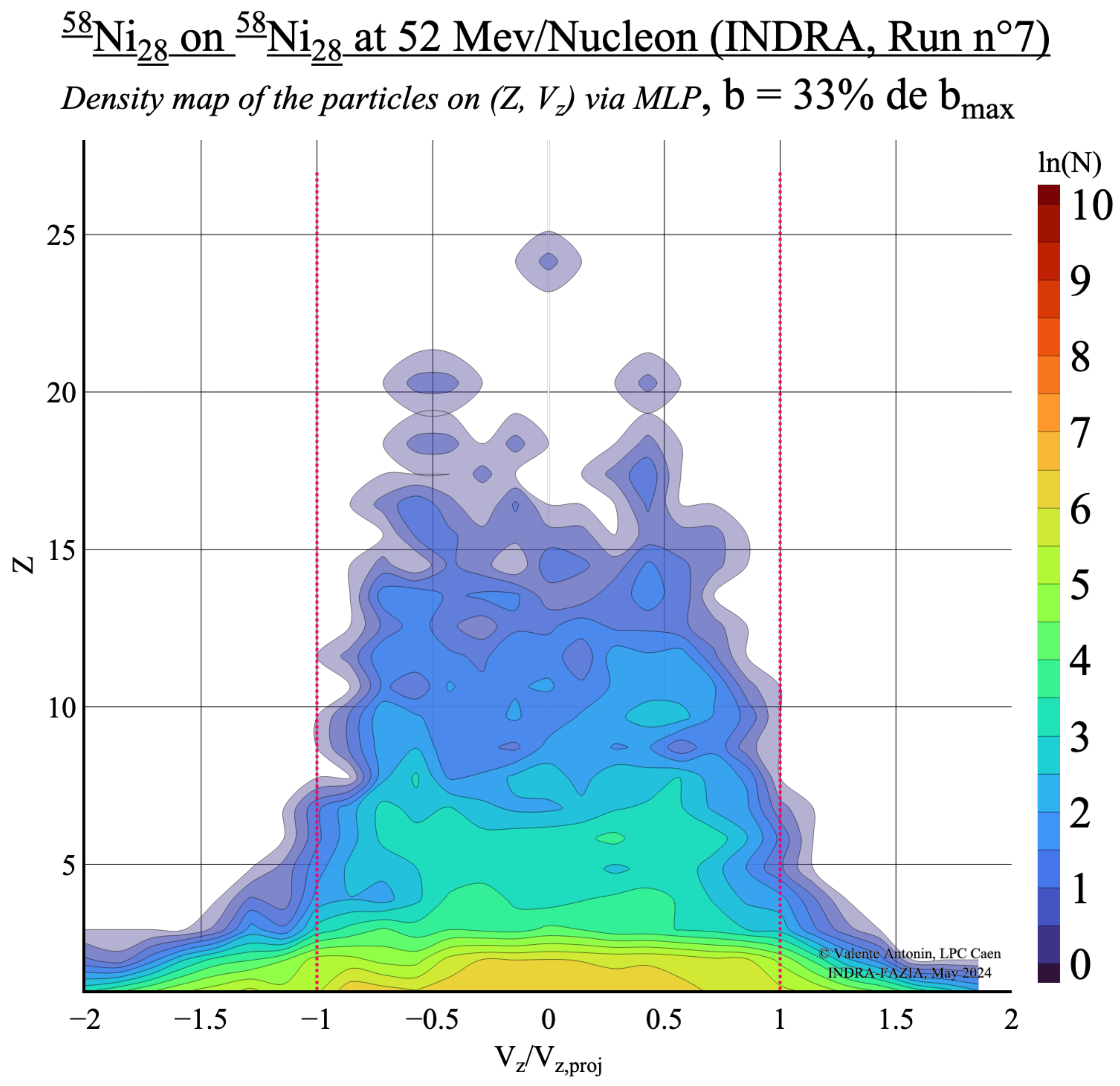


# Charge vs Velocity parallel of particles ordered by centrality classes with reconstructed impact parameter on experimental dataset

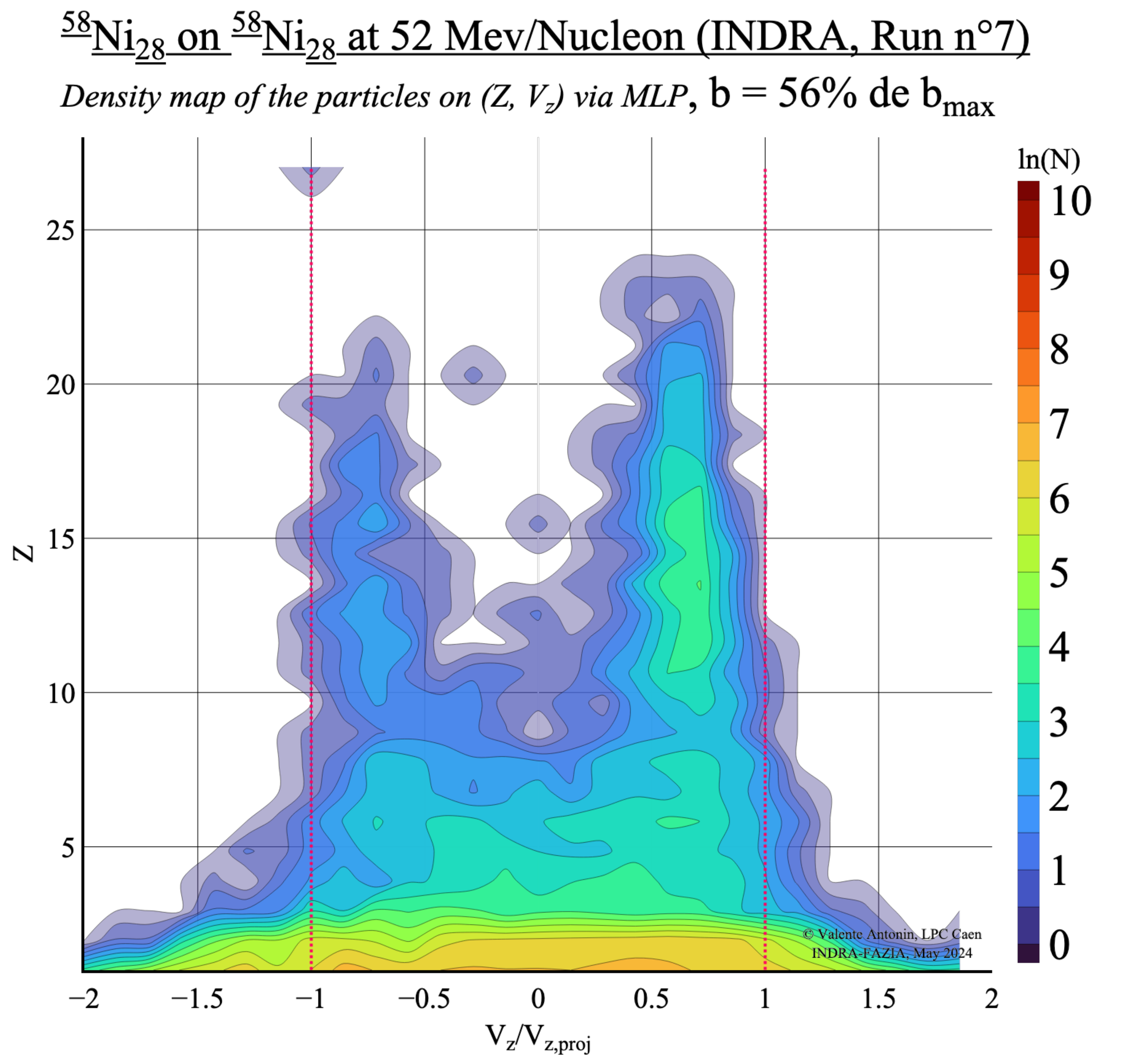
## Central



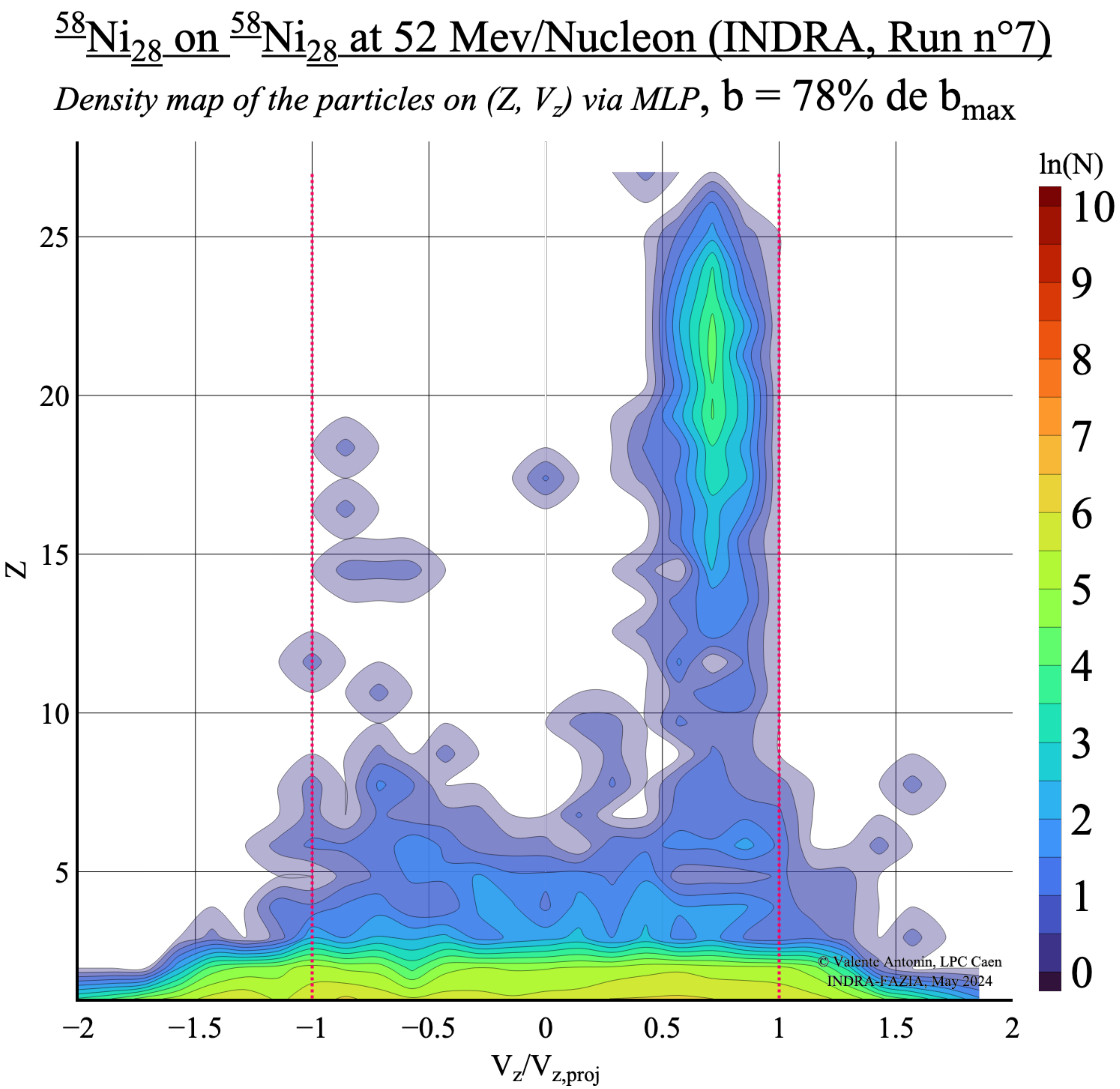

## Semi-Central



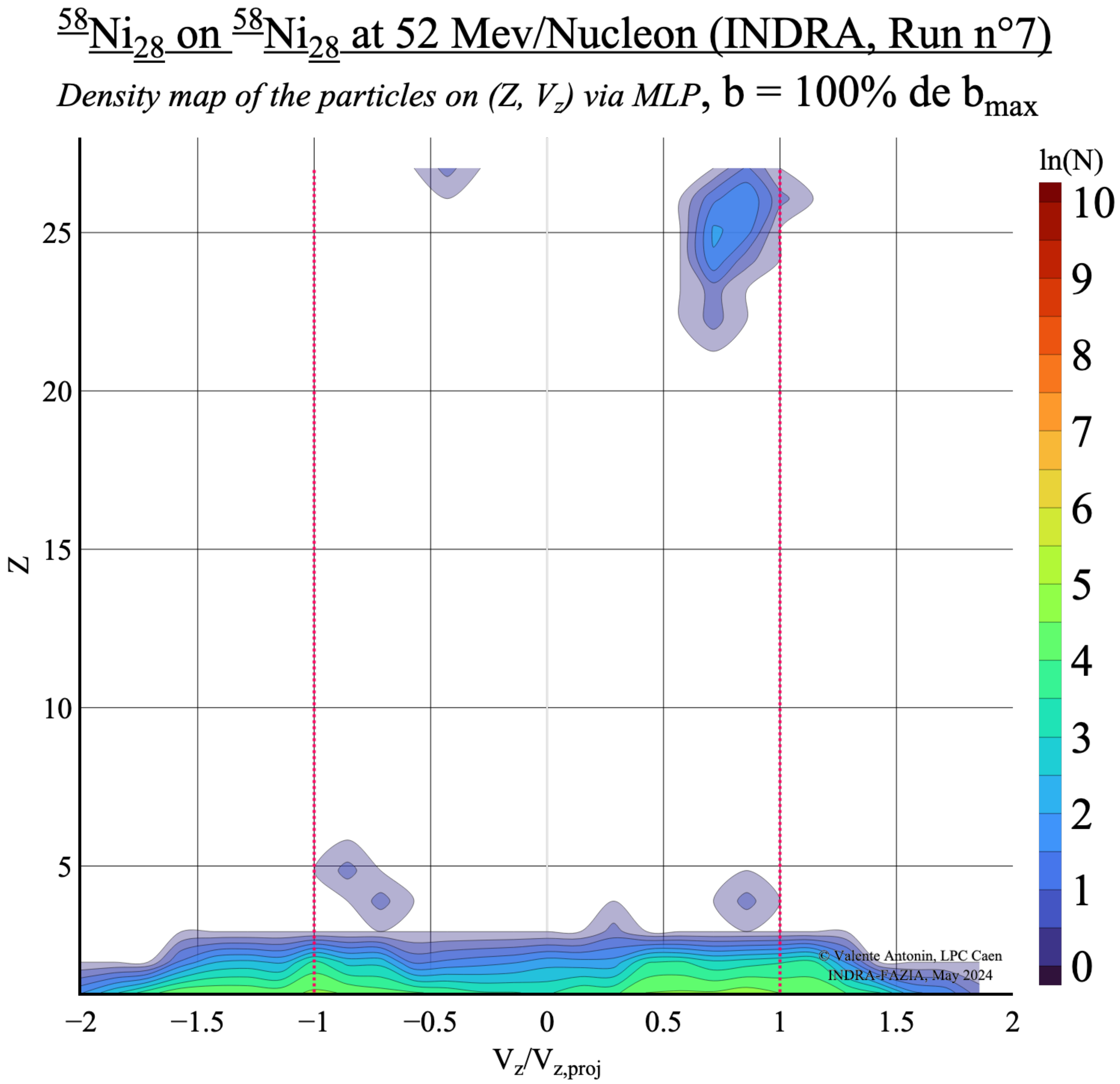

## Intermediate



## Semi-peripheral




## Peripheral



# Estimation of Simulation/Experiment error

- The Kolmogorov-Smirnov probability is a useful tool to evaluate whether two statistical samples come from the same probability distribution, based on the comparison of their empirical cumulative distribution functions.
- The objective is therefore to have, on a set of statistical observables, an agreement between the simulation and the experiment. We quantify this by calculating the geometric mean of all the Kolmogorov-Smirnov probabilities.

# On the data :

The systems used are :

- $^{58}Ni/^{58}Ni$ at 52, 74 and 90 $MeV/nucleon$

ELIE is the simulation used and the experimental data is from INDRA Second Campaign (Ni+Ni).

# Bayesian inference
# Nucleon-Nucleon Cross Sections

We do a sampling of NN-cross section, with a uniform probability law during the simulation :

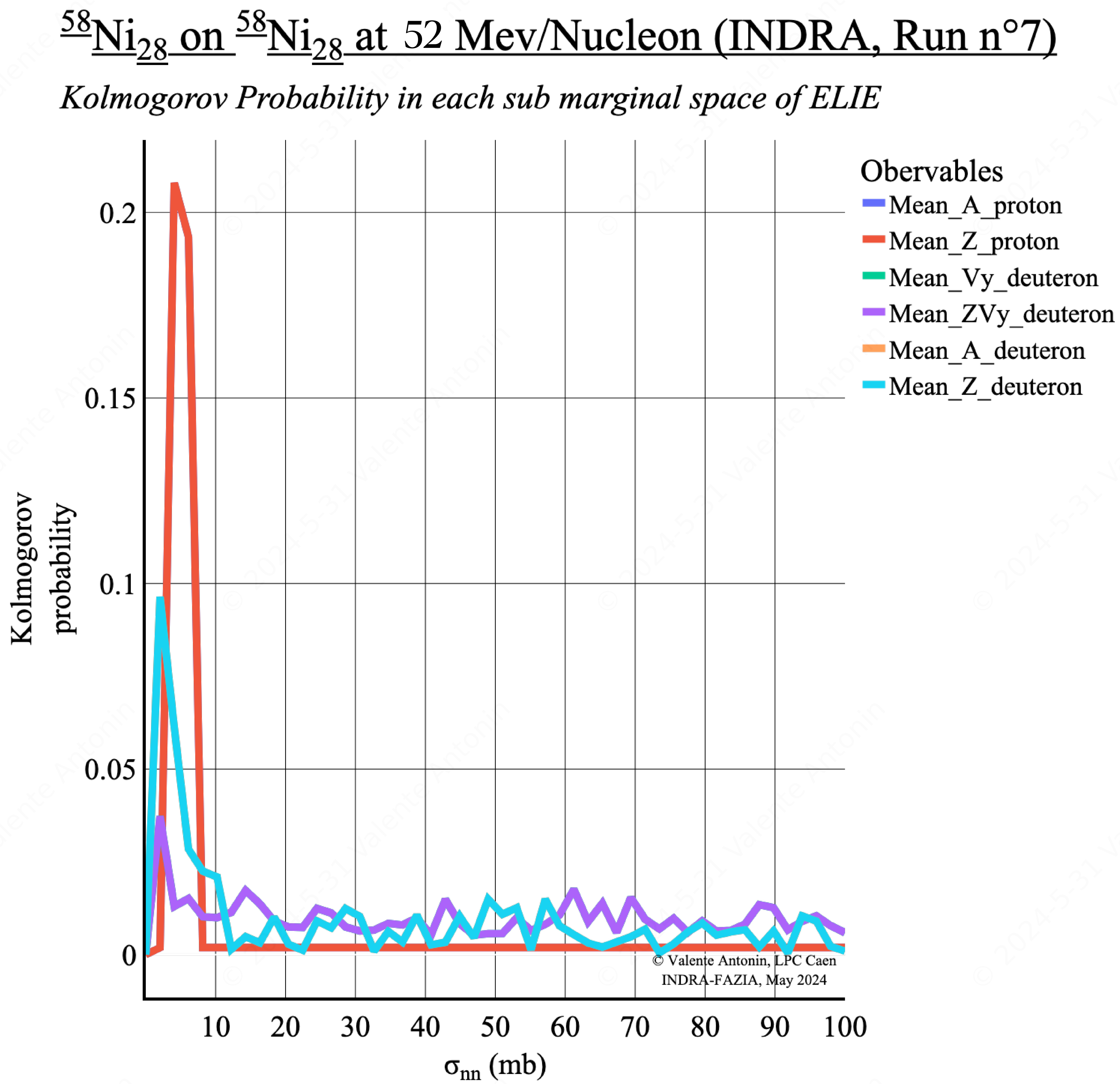
$$P(x) = \begin{cases} \frac{1}{\sigma_{max} - \sigma_{min}} & \text{if } x \in [\sigma_{min}, \sigma_{max}] \\ 0 & \text{if } x \notin [\sigma_{min}, \sigma_{max}] \end{cases}$$

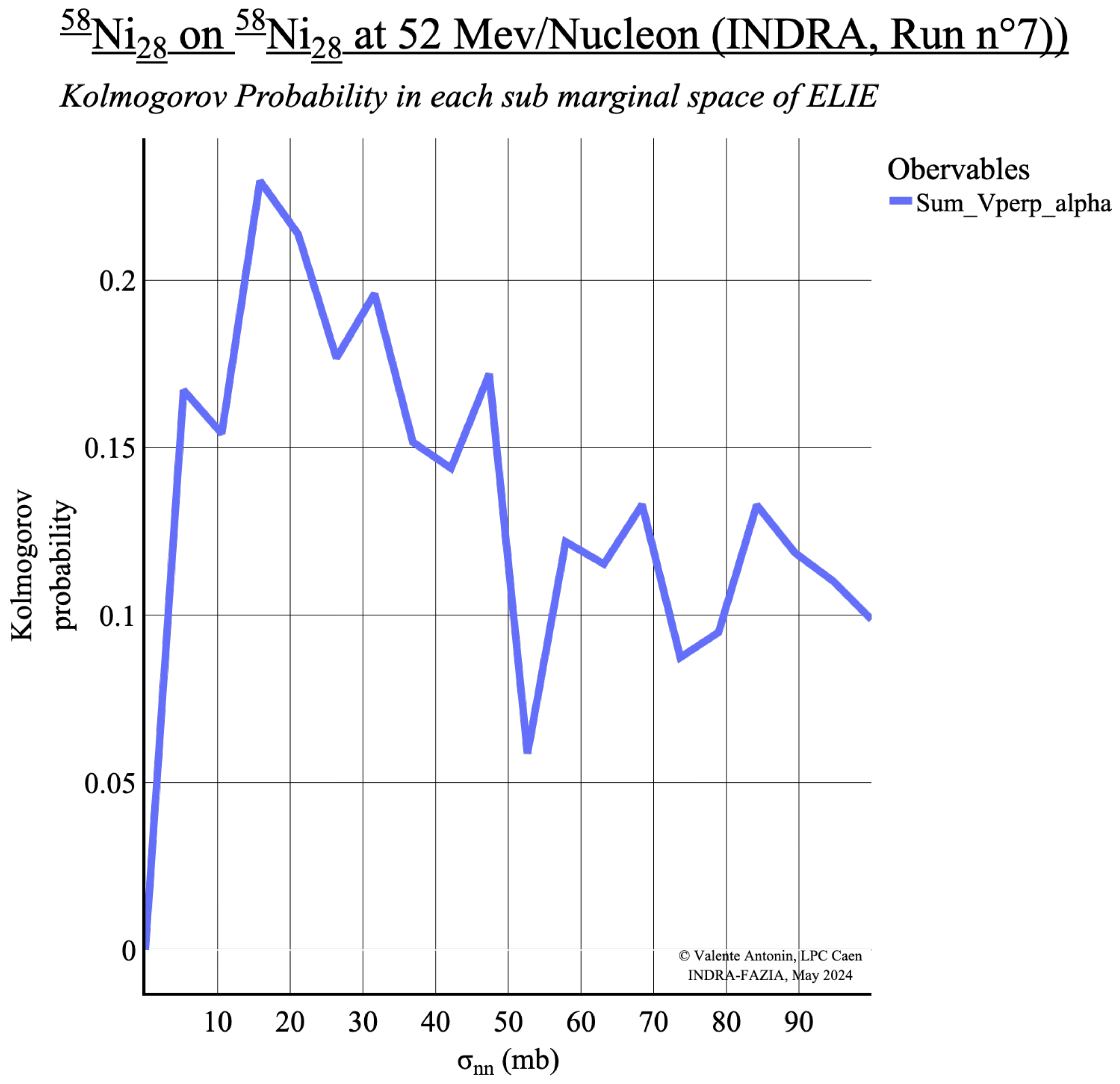
We then cut the interval $[\sigma_{\text{min}}; \sigma_{max}]$ into 20 bins and we calculate a Kolmogorov probability for each bin.

The explored range is from 0 to 100 mb.

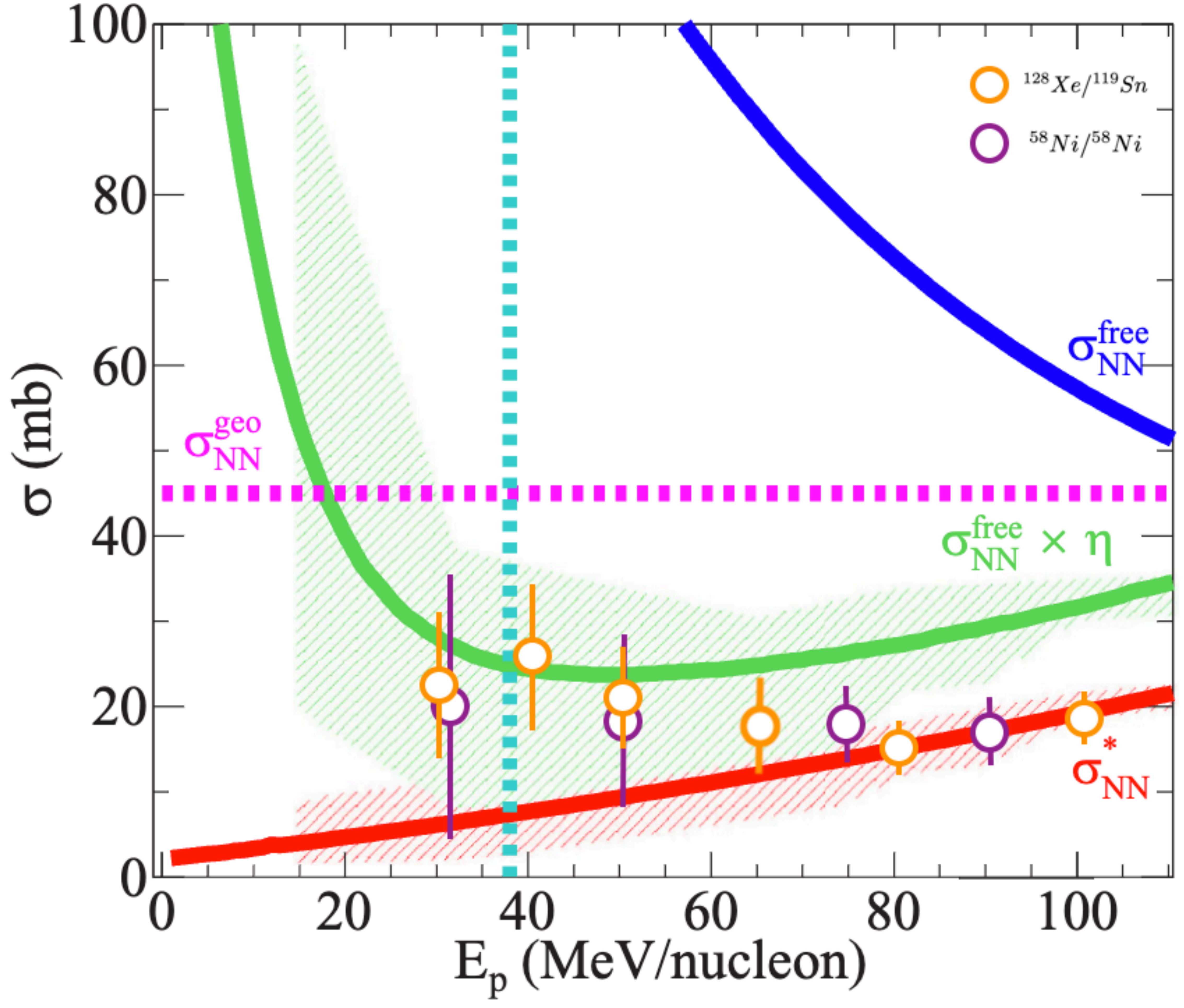
## Very sensitive observables

## Less-sensitive observables



# Nucleon-Nucleon Cross inference results

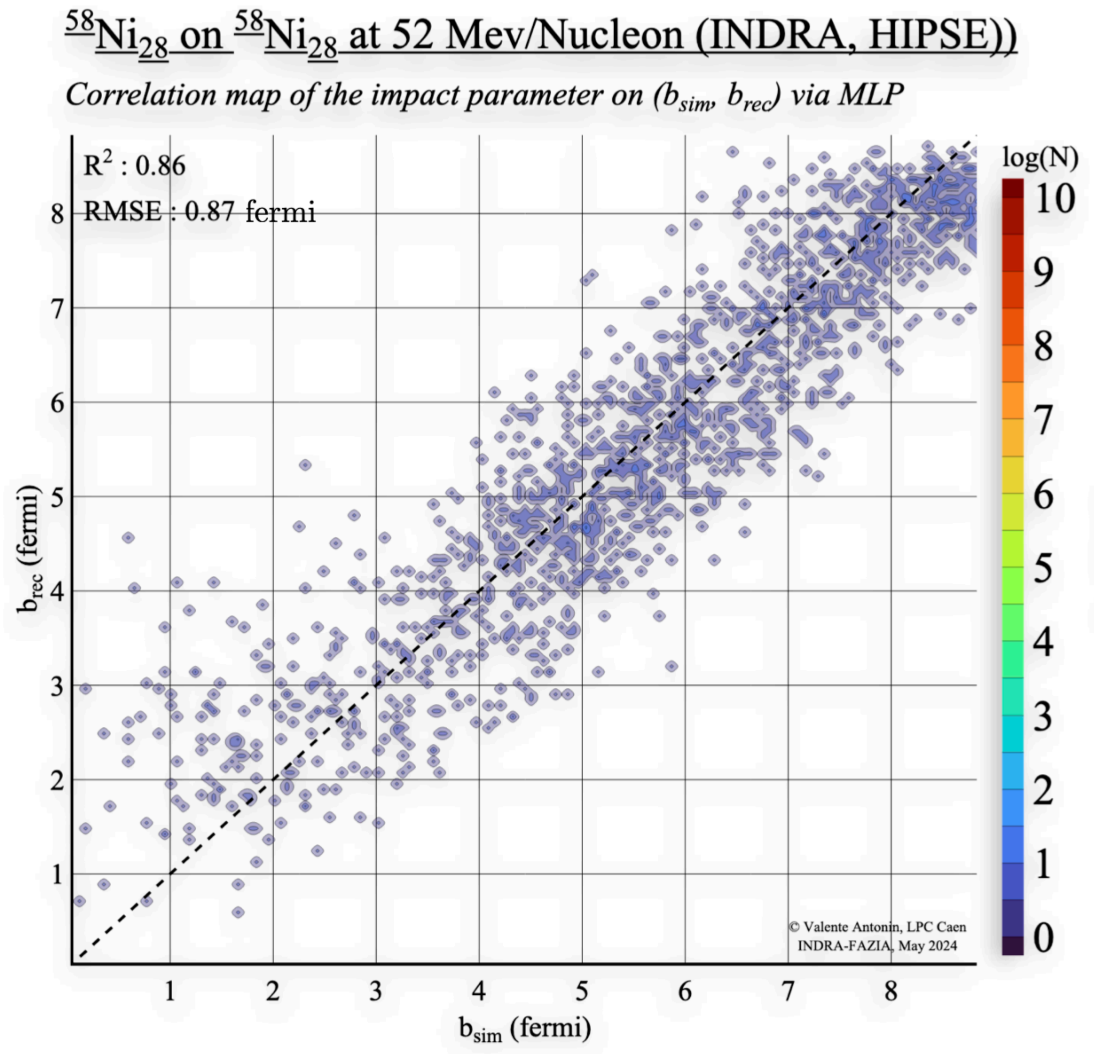


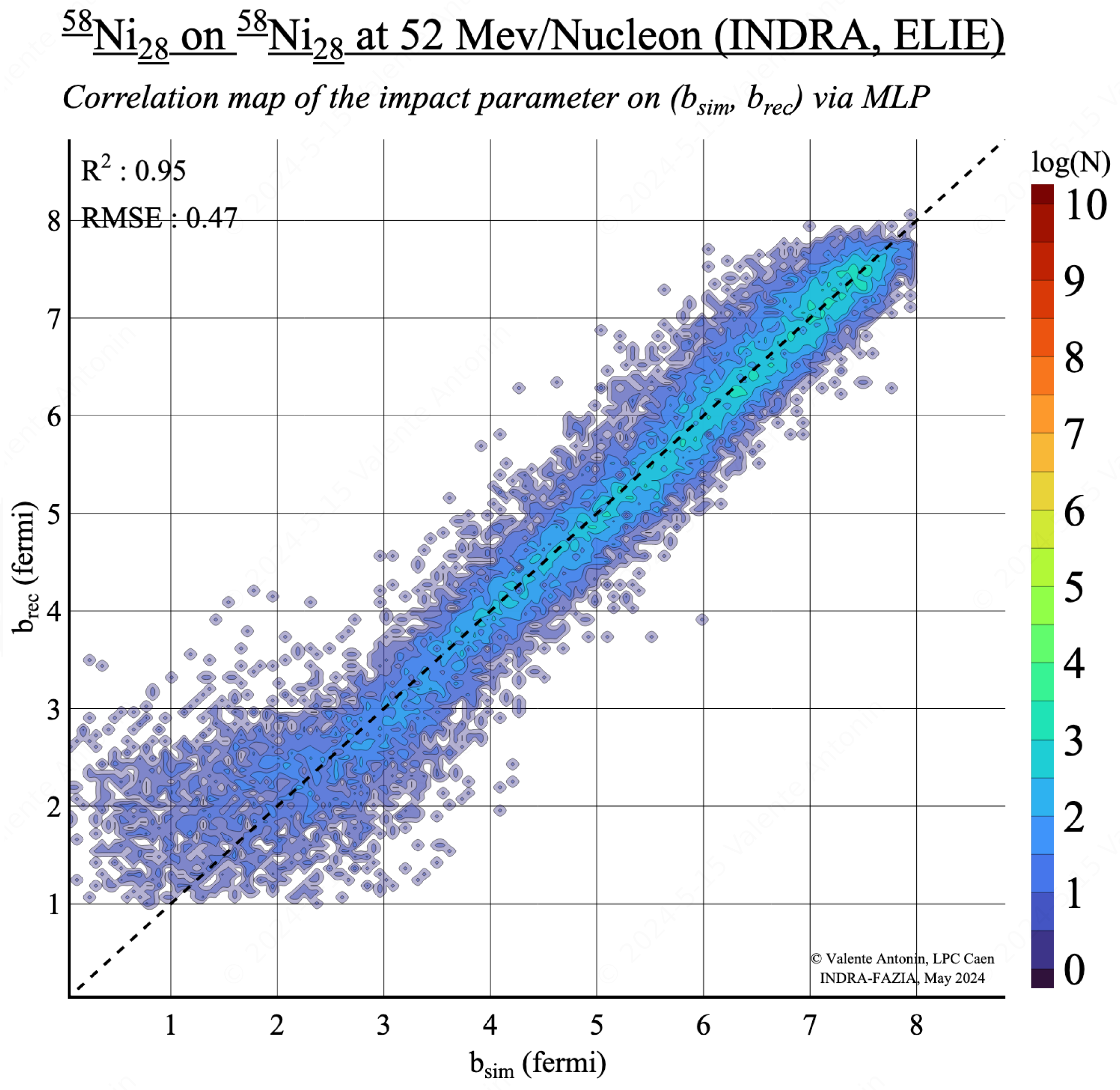

- The solid curves are from M. Henri & al. (PRC 101, 064622 (2020)). The first curve represent the free nucleon-nucleon cross section and the second one represent the first curve corrected with a Pauli blocking effect. The third, is the nucleon-nucleon cross section with in-medium effect proposed by D.D.S Coupland & al. (PRC 84, 054603 (2011)).

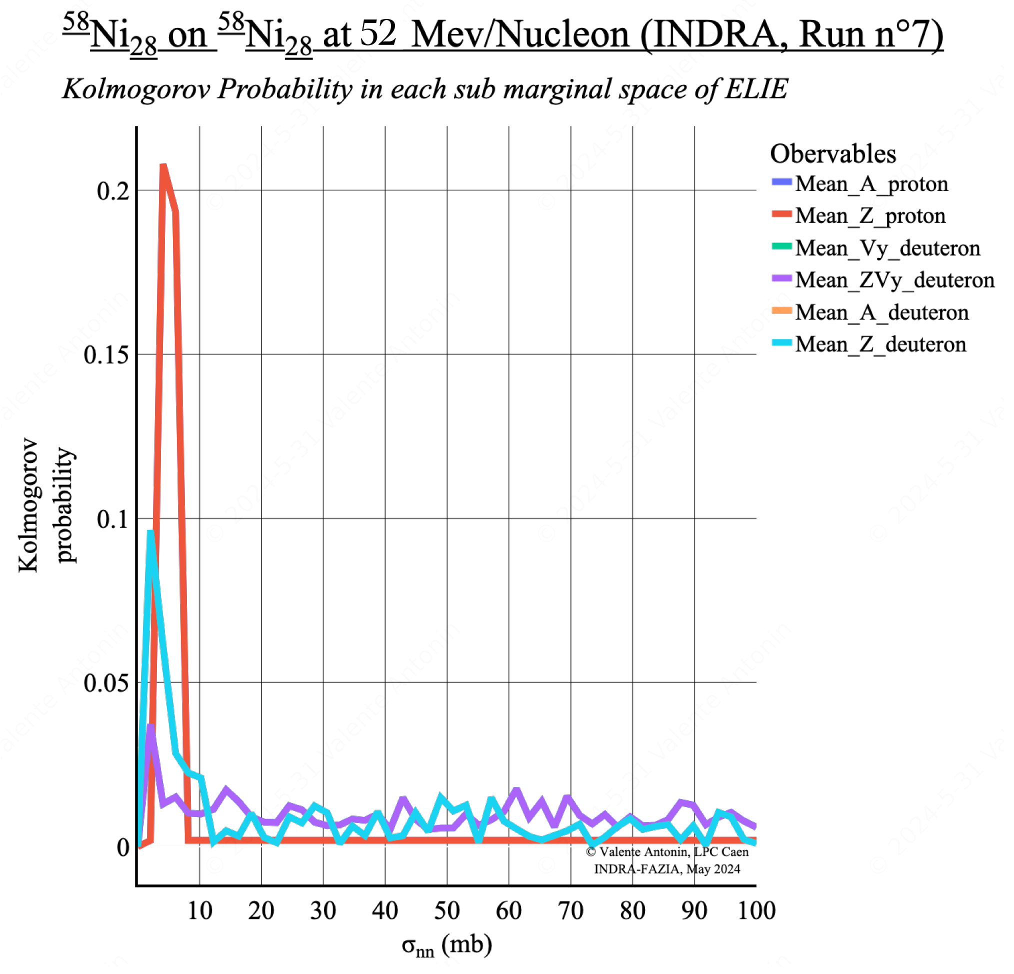
- On it, I put my results of the Bayesian analysis. The $^{58}Ni/^{58}Ni$ and $^{129}Xe/^{119}Sn$ are shown here. Those results are in good agreement with in-medium effect on the cross section for the upper range in energy.

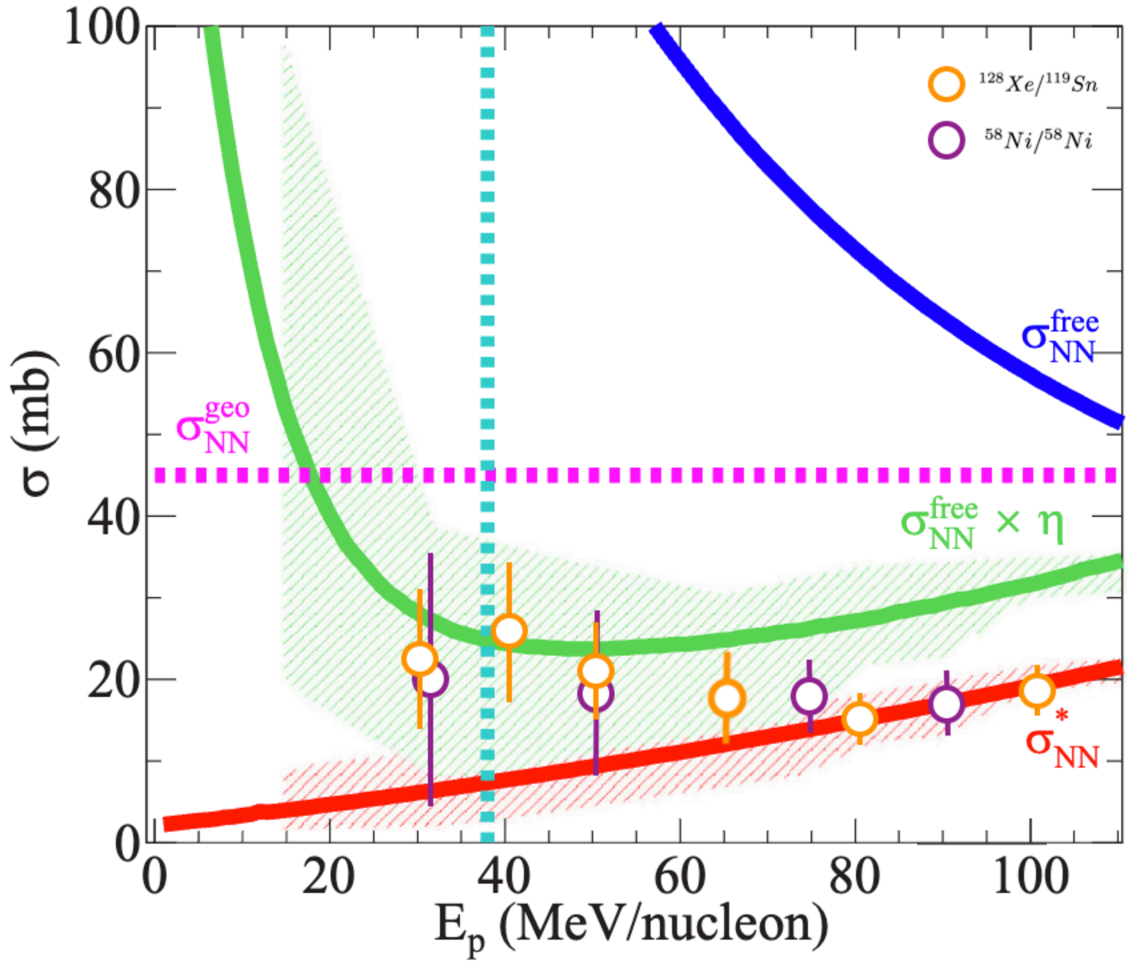
- For the lower range the uncertainties are increasing and we cannot conclude if the cross section is governed by only Pauli blocking or with in-medium effect. We suspect that our model is not well suited for this range of energy.

# Summary



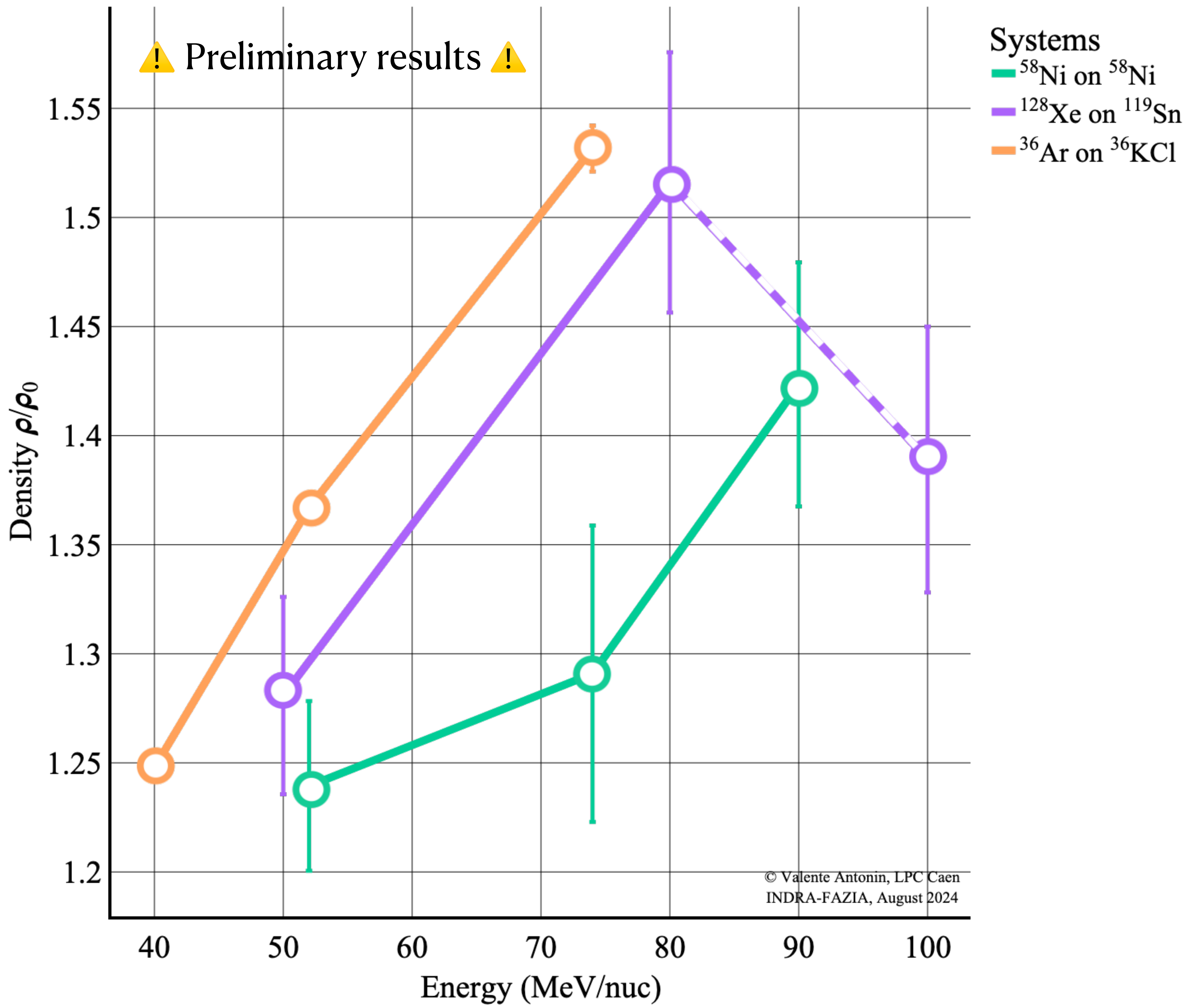




- Inference via Machine Learning on impact parameter : selection of central and peripheral events. The Impact parameter is inferred < 1 fermi error.
- Inference of in-medium nucleon-nucleon cross section with ELIE model. The results show that we are agreeing with M. Henri results and we also provide mean values and error bars based on experimental data. To be continued with other system.
- We will also pursue those inferences with the density to determine EOS parameters in future works.

# Perspectives

## Bayesian analysis of INDRA data

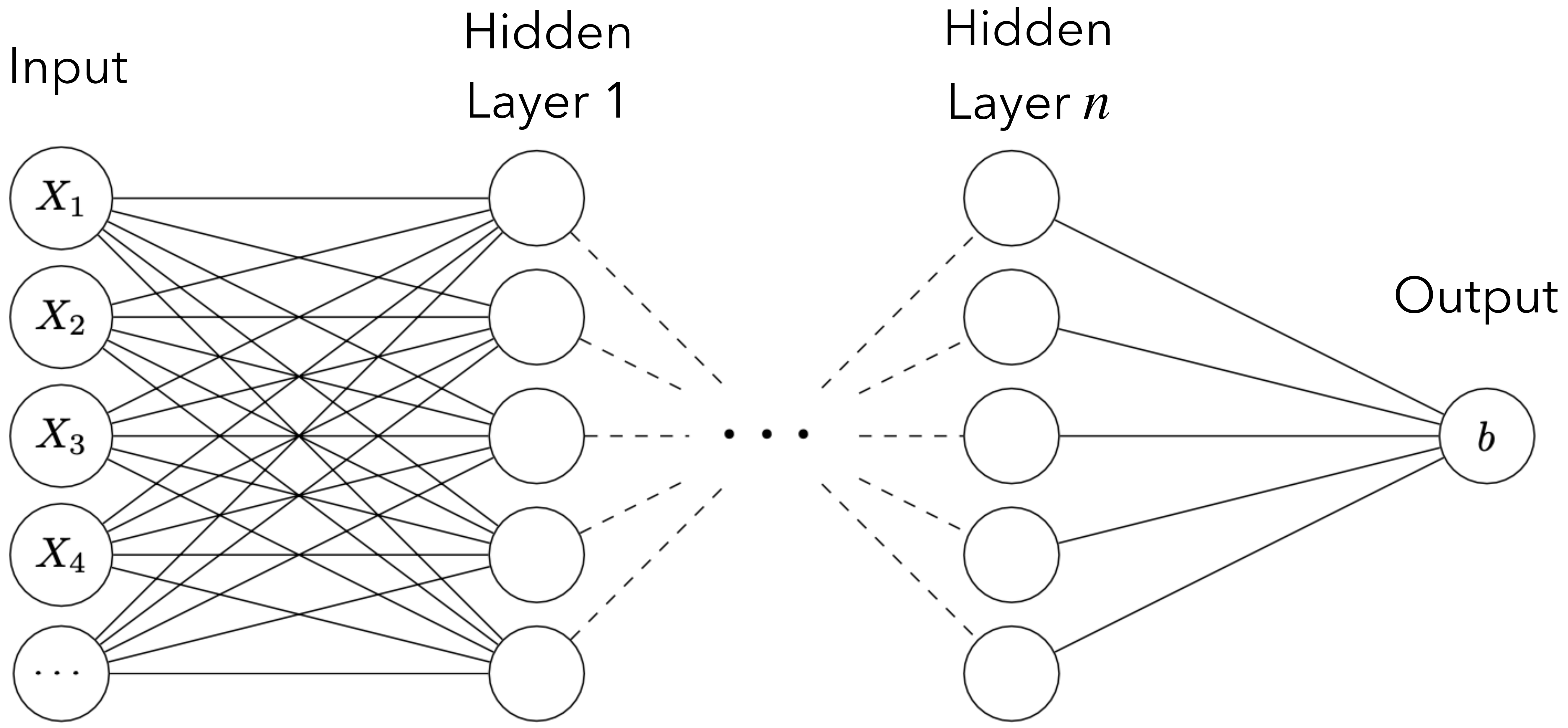
*Results of the Bayesian Inference of the Density* $\rho$ (ELIE)




- Nuclear density is a fundamental variable in the EOS, as it determines the overall behavior of nuclear matter under different conditions.
- At low densities (below saturation density), the EOS is well constrained by experimental data from nuclear masses, radii, and low-energy scattering experiments. This regime is relevant for the description of ordinary nuclei and the outer layers of neutron stars.
- At higher densities (above saturation), the EOS becomes increasingly uncertain due to the lack of experimental data. This regime is crucial for understanding the core of neutron stars and the properties of supernova remnants.
- An accurate determination of the EOS across a wide range of densities is essential for modeling astrophysical phenomena like neutron star structures, supernova explosions, and neutron star mergers, which can potentially provide insight into the nature of dense nuclear matter.

# Thanks for you
# for your attention !

# Illustration of the Neural Network




- The architecture is a 4 hidden layer neural network with 256, 128, 64 and 32 neuron is this order.

- The activation function choosen is the EULU function.

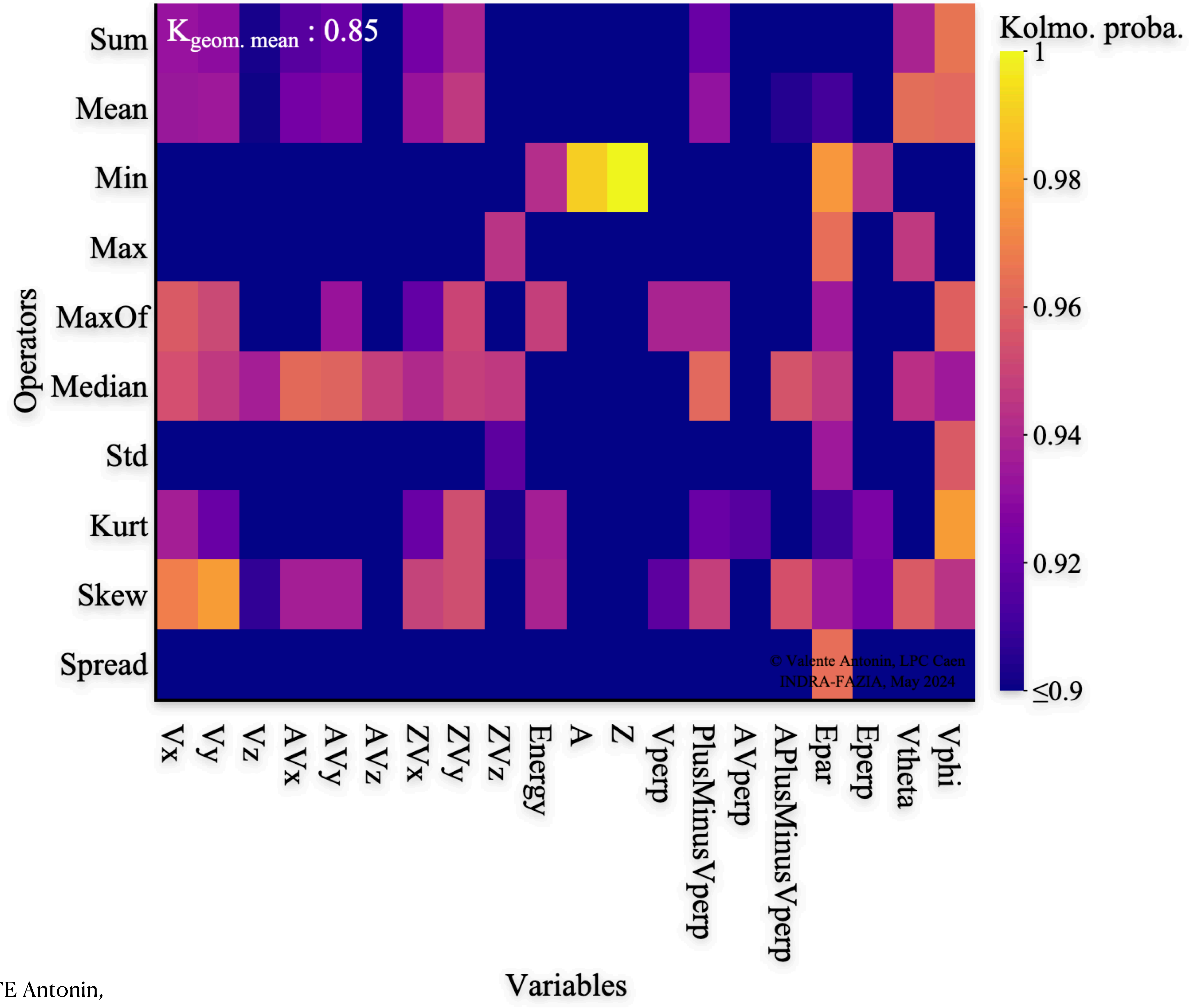
- Training dataset 80% and test dataset 20% with a backprogation and gradient descent for the regression.

- The Mean-Square-Error is the estimator of the error to minimize.

# Kolmogorov Matrices

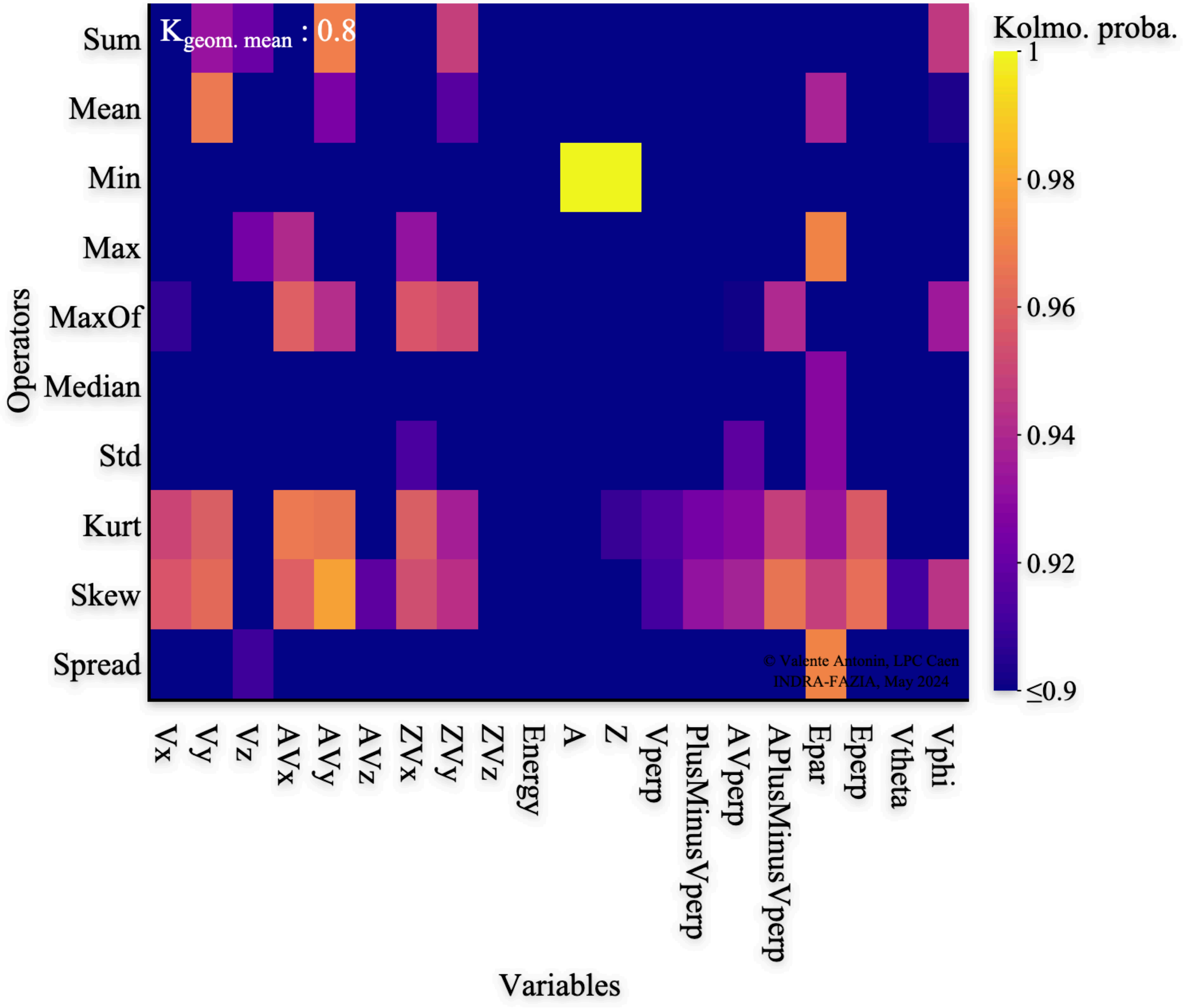
$^{58}Ni_{28}$ on $^{58}Ni_{28}$ at 52 Mev/Nucleon (INDRA, Run n°7))

*Matrix of Kolmogorov of the events in HIPSE*




$^{58}Ni_{28}$ on $^{58}Ni_{28}$ at 52 Mev/Nucleon (INDRA, Run n°7))

*Matrix of Kolmogorov of the events in ELIE*



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