An overview on EOS constraints

Nuclear theoretical, experimental and astrophysical constraints

Koehn et al., arxiv: 2402.04172

Peter T. H. Pang

Utrecht University

Nikhef

Interdisciplinary collaboration

NMMA

Nuclear theoretical and experimental

Los Alamos
NATIONAL LABORATORY

Syracuse University

Universitetet i Stavanger

INDIANA UNIVERSITY

Astrophysical modelling and observations

Nikhef

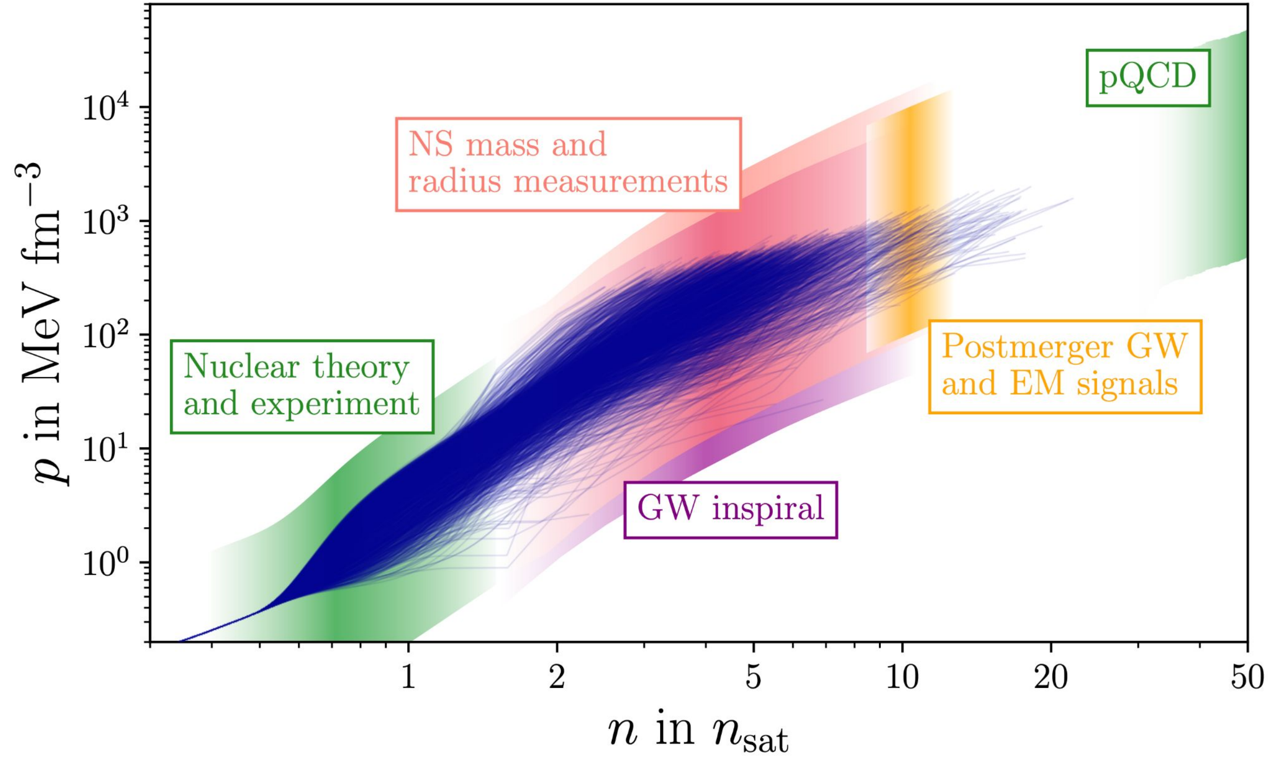
Universität Potsdam

Utrecht University

UNIVERSITY OF MINNESOTA

Università degli Studi di Ferrara

Neutron star equation-of-state



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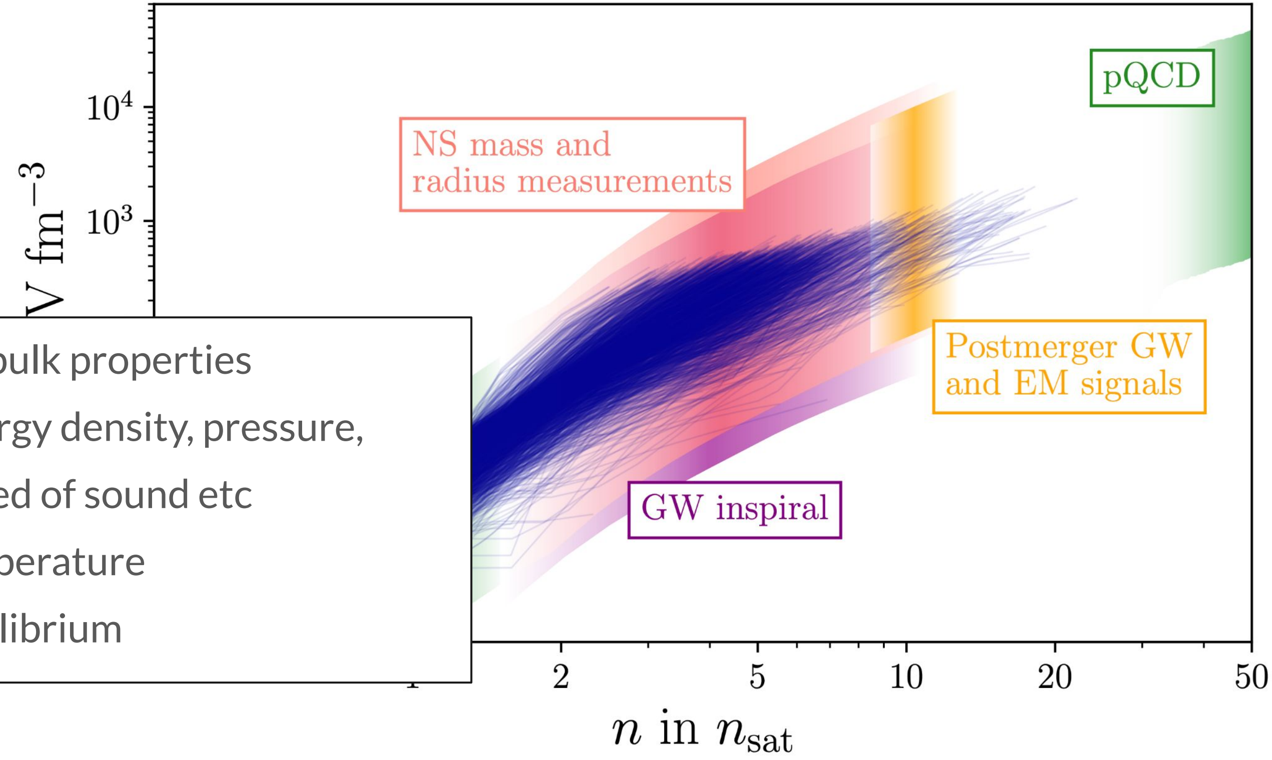


- Relating bulk properties
  - Energy density, pressure, speed of sound etc
- Zero temperature
- Beta equilibrium

Bayesian statistics

Posterior $\propto$ Likelihood × Prior

Bayesian statistics

Posterior $\propto$ Likelihood × Prior

- Astrophysical observations
- Experimental results
- Theoretical calculations

Bayesian statistics

Posterior $\propto$ Likelihood ×
Prior

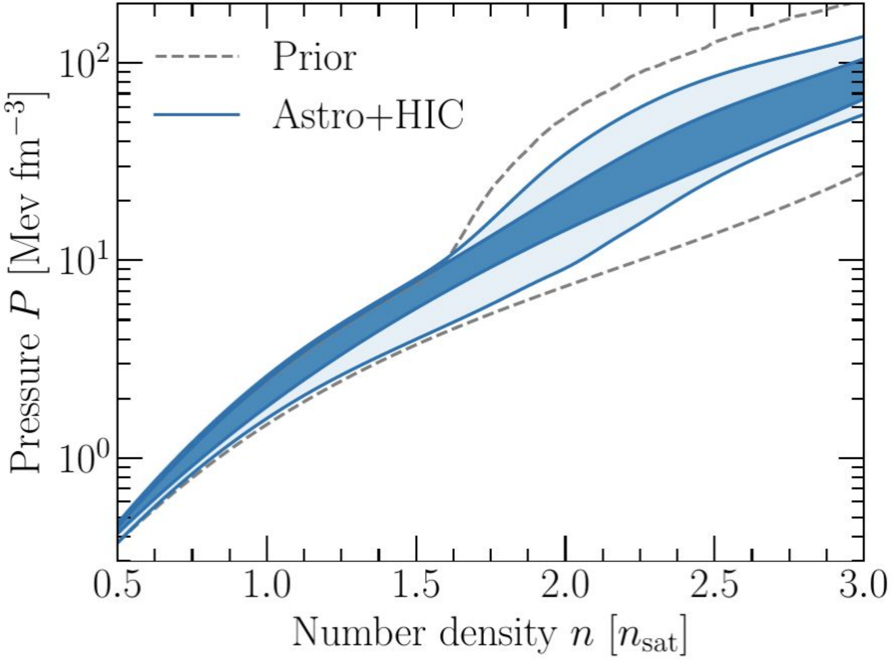
- Encode prior information / knowledge
- As generic as possible

Bayesian statistics

Posterior $\propto$ Likelihood × Prior

What we know about the EOS (so far)

Previous work



Huth & Pang et al., Nature 2022

- Chiral effective theory up to 1.5nsat
- Heavy-ion collision
- Gravitational waves
- Radio and X-ray pulsars

What can we do better?

Prior

- 100k EOS candidates
- Meta-Model up to 1 - 2 nsat
- 9 segments speed-of-sound extension up to nTOV

Parameter	Prior
$n_{\mathrm{break}}\ [n_{\mathrm{sat}}]$	$\mathcal{U}(1,2)$
$\mathrm{K_{sat}}\ [\mathrm{MeV}]$	$\mathcal{U}(150,300)$
$\mathrm{Q_{sat}}\ [\mathrm{MeV}]$	$\mathcal{U}(-500,1100)$
$\mathrm{Z_{sat}}\ [\mathrm{MeV}]$	$\mathcal{U}(-2500,1500)$
$\mathrm{E_{sym}}\ [\mathrm{MeV}]$	$\mathcal{U}(28,45)$
$\mathrm{L_{sym}}\ [\mathrm{MeV}]$	$\mathcal{U}(10,200)$
$\mathrm{K_{sym}}\ [\mathrm{MeV}]$	$\mathcal{U}(-300,100)$
$\mathrm{Q_{sym}}\ [\mathrm{MeV}]$	$\mathcal{U}(-800,800)$
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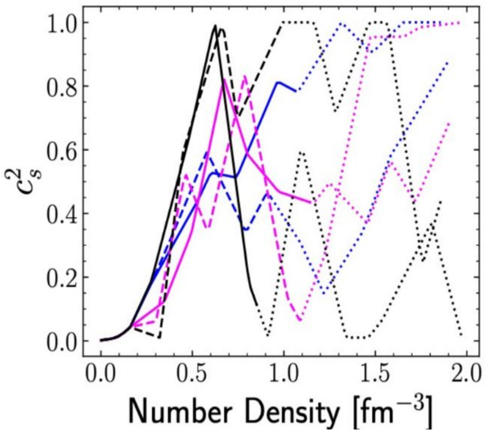

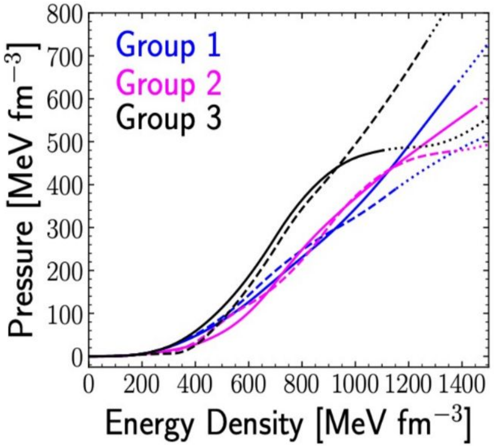

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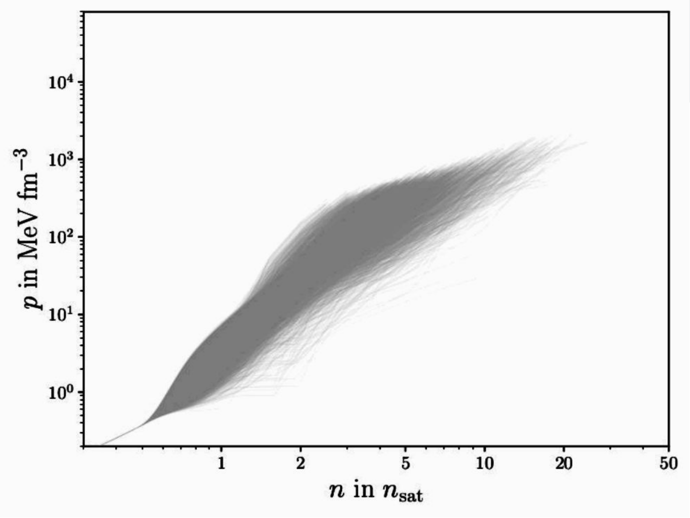

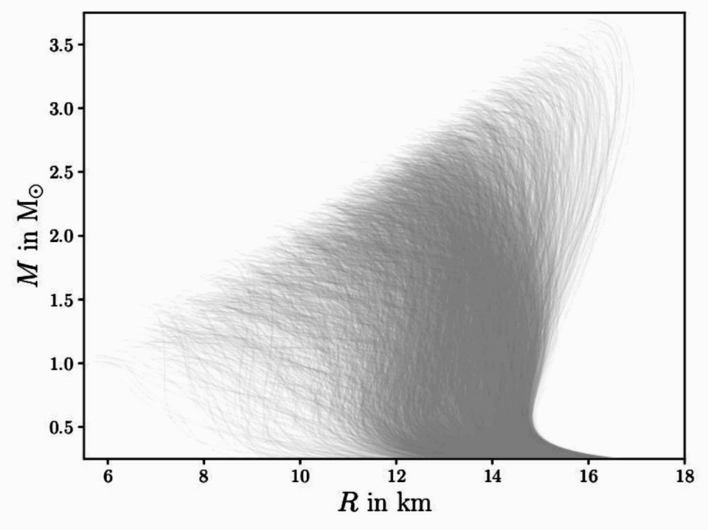

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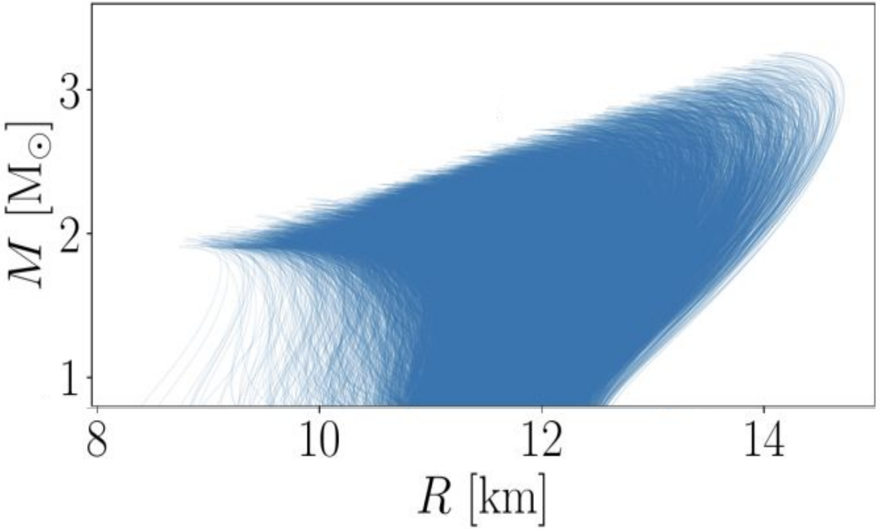
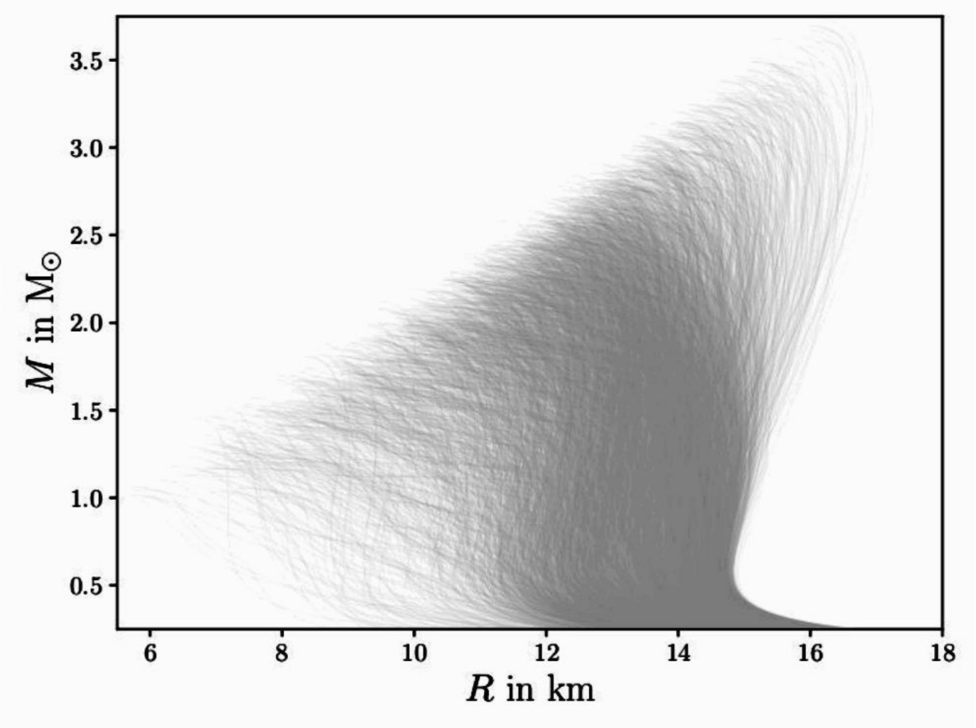


M [M$_\odot$]
3
2
1
8
10
12
14
R [km]



M in M$_\odot$
3.5
3.0
2.5
2.0
1.5
1.0
0.5
6
8
10
12
14
16
18
R in km

Likelihood

Nuclear experiment / theory	Isolated neutron star	Binary neutron star
Chiral EFT	Radio timing	GW170817 + AT2017gfo + GRB170817A
pQCD	NICER	
PREX-II	Black widow	GW190425
CREX	qLMXBs	GRB211211A
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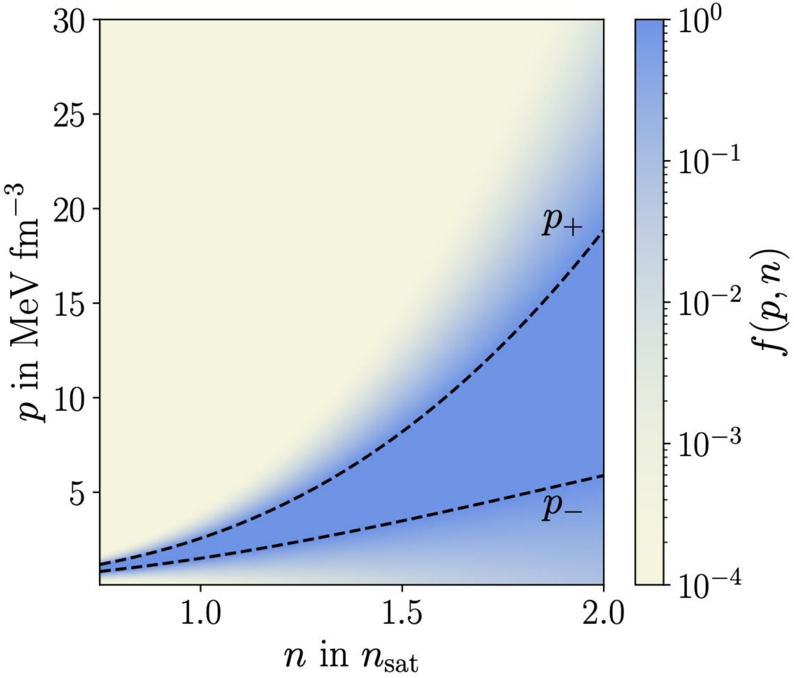
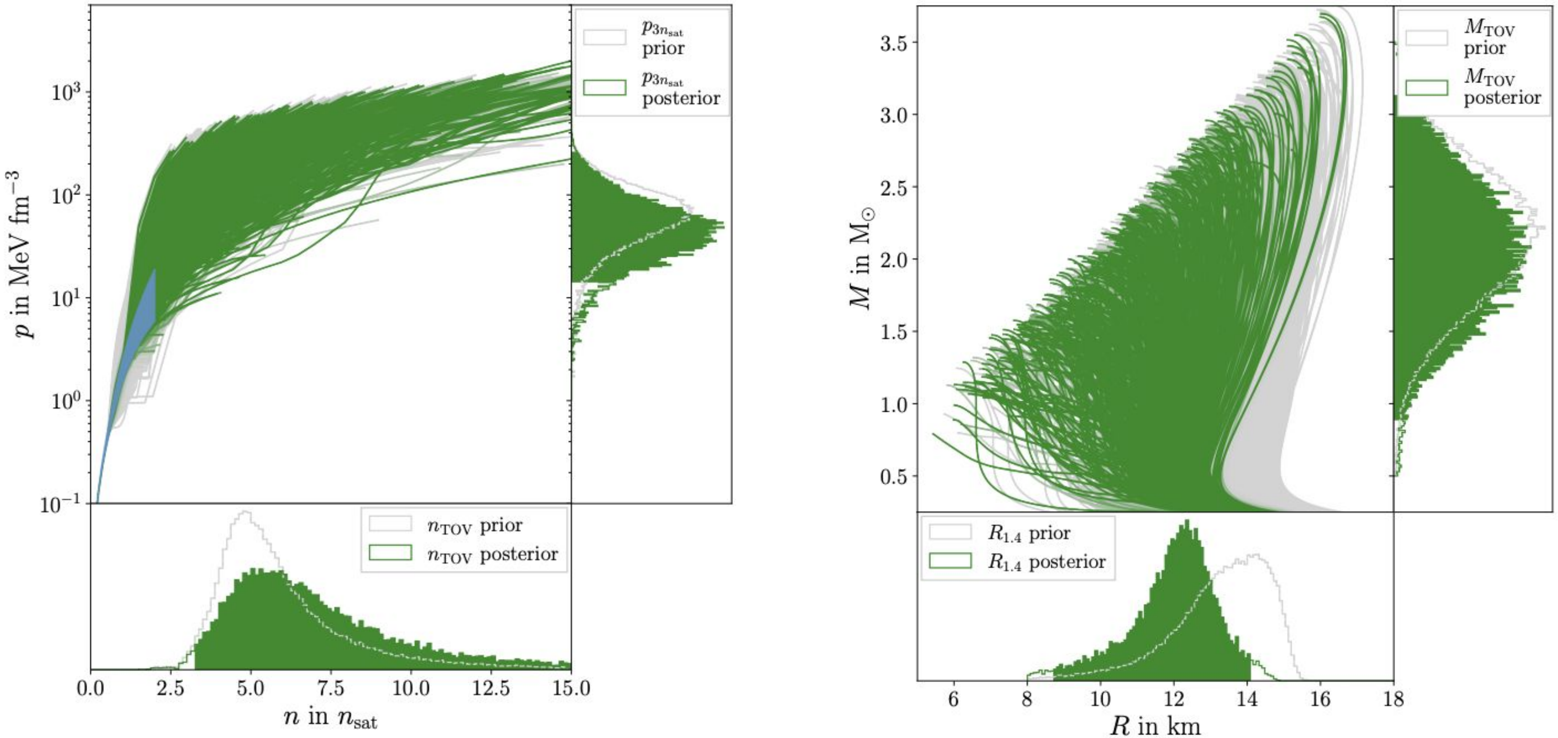
Chiral effective field theory

- CEFT calculation up to N^{2}LO (3$^{\text{rd}}$ order)
- Likelihood is designed that
  - j / (j + 1) -> 75% probability mass enclosed in band
  - see Furnstahl et al. Phys. Rev. C 92, 024005 (2015)
  - The EOS has to be in band across meta-model densities

$$f(p,n) = \begin{cases} \exp\left(-\beta \frac{p-p_+}{p_+-p_-}\right) & \text{if } p > p_+\,, \\ \exp\left(-\beta \frac{p_--p}{p_+-p_-}\right) & \text{if } p < p_-\,, \\ 1 & \text{else}\,. \end{cases}$$



Chiral effective theory



Perturbative QCD

- At ~40 nsat, QCD becomes perturbative
- Constraint on EOS at chemical potential of 2.6 GeV

$$\begin{aligned}\mathcal{L}(\mathrm{EOS}|\mathrm{pQCD}) = &\int dX\, dp_H\, dn_H \\ &\times P(\epsilon_L, p_L | n_L, \mu_H, n_H, p_H) \\ &\times P_{\mathrm{MHO}}(p_H, n_H | \vec{p}^{(j)}(\mu_H, X), \vec{n}^{(j)}(\mu_H, X)) \\ &\times P_{\mathrm{SM}}(X | \vec{p}^{(j)})\end{aligned}$$

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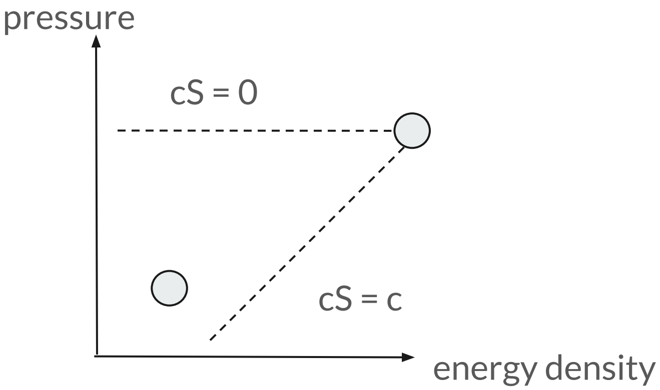
If the TOV density point can connect to the pQCD point



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\end{aligned}
$$

The pQCD point given the chemical potential and the renormalization scale

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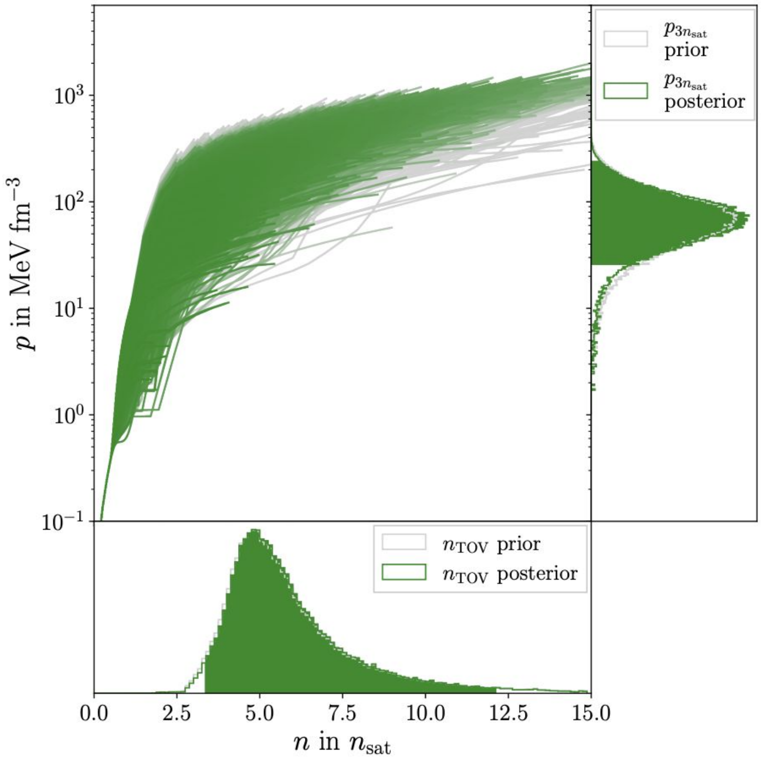
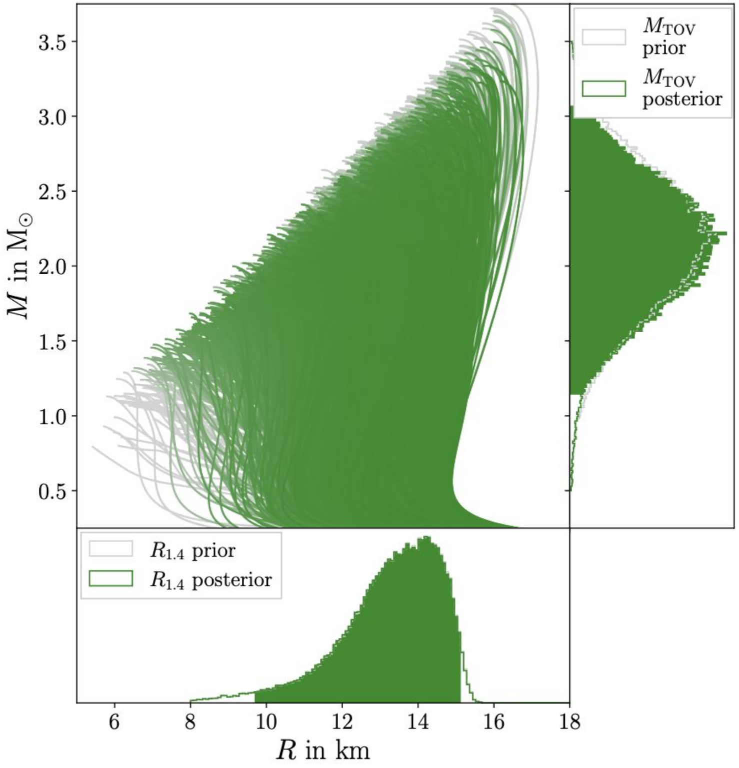
\end{aligned}
$$

Uncertainty on the renormalization scale

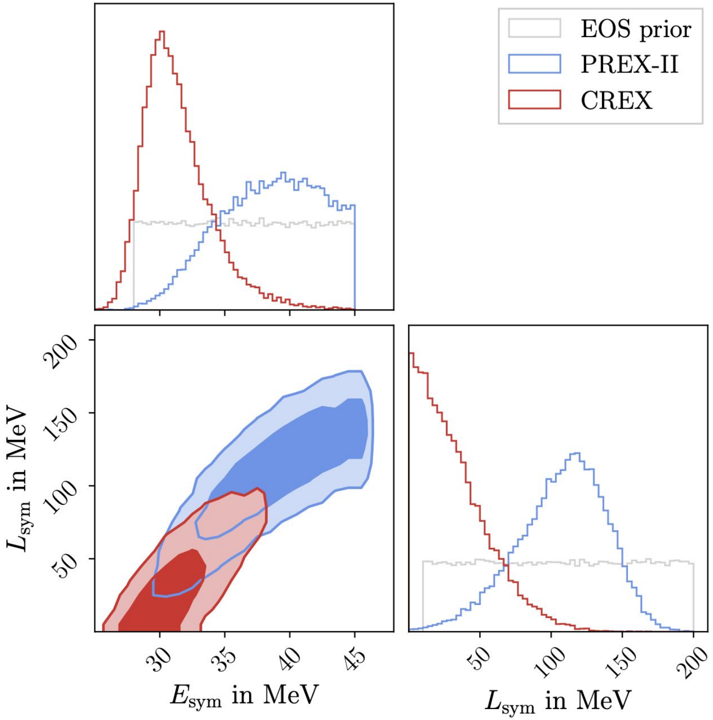
Perturbative QCD





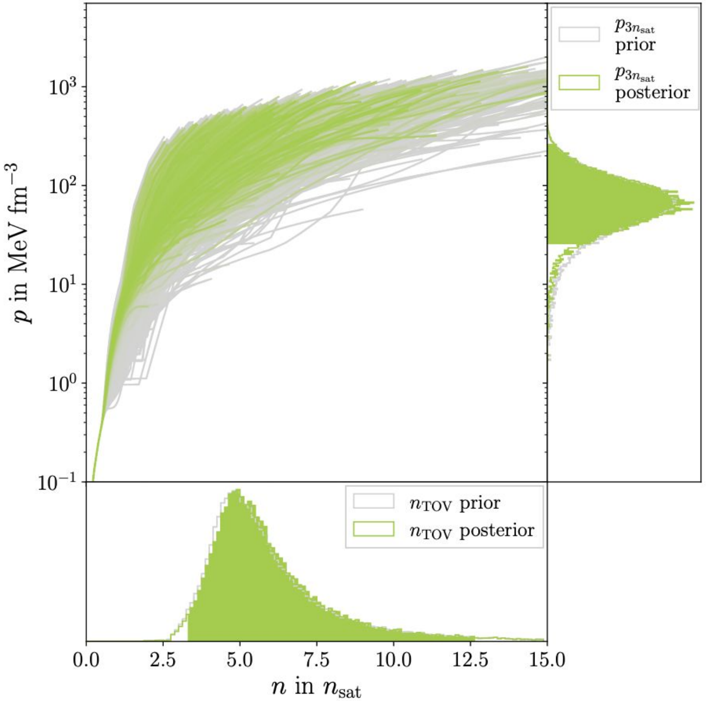
PREX-II and CREX

- Measurement on the neutron skin thickness
- Make use of the correlation between the thickness and Lsym
- Correlation between Esym and Lsym considered

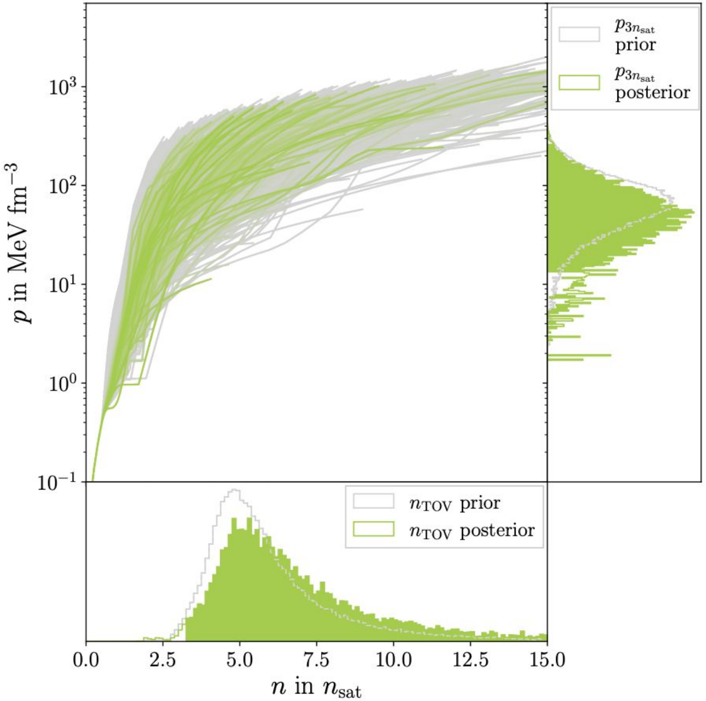


PREX-II and CREX

PREX-II



CREX



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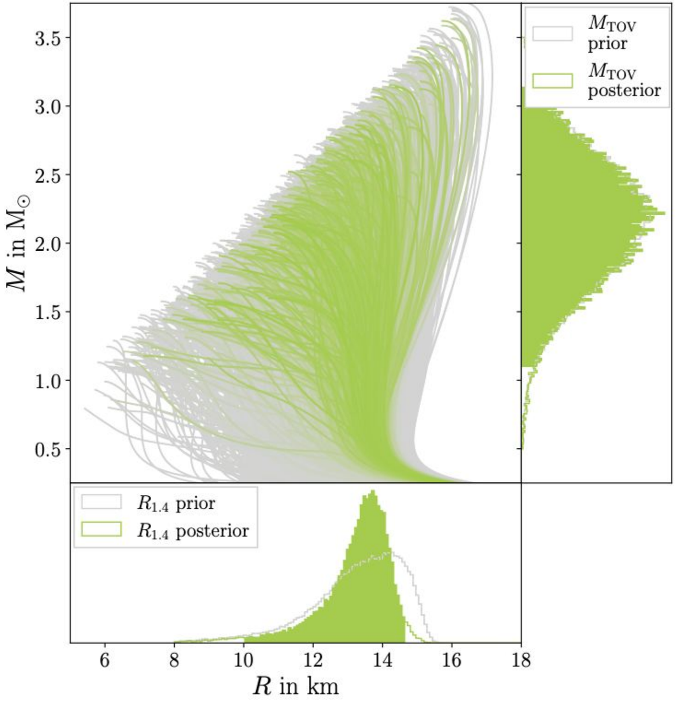
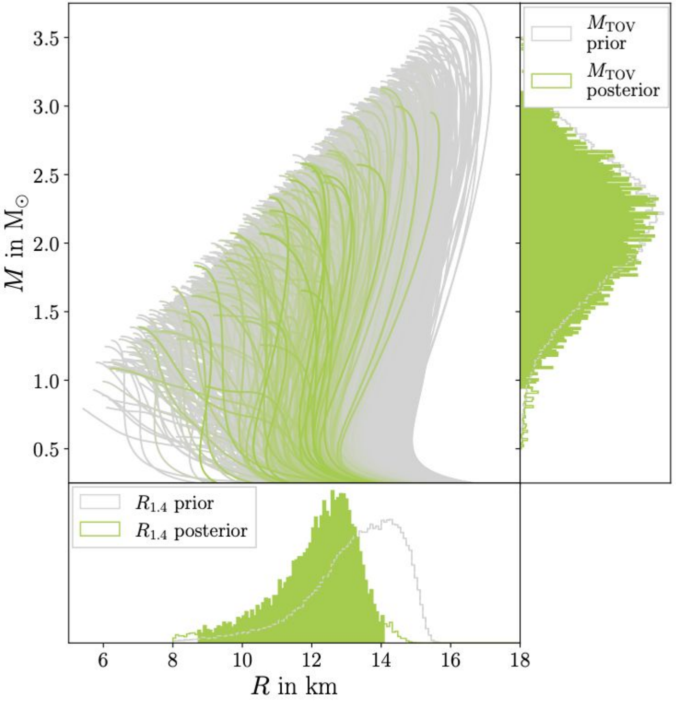


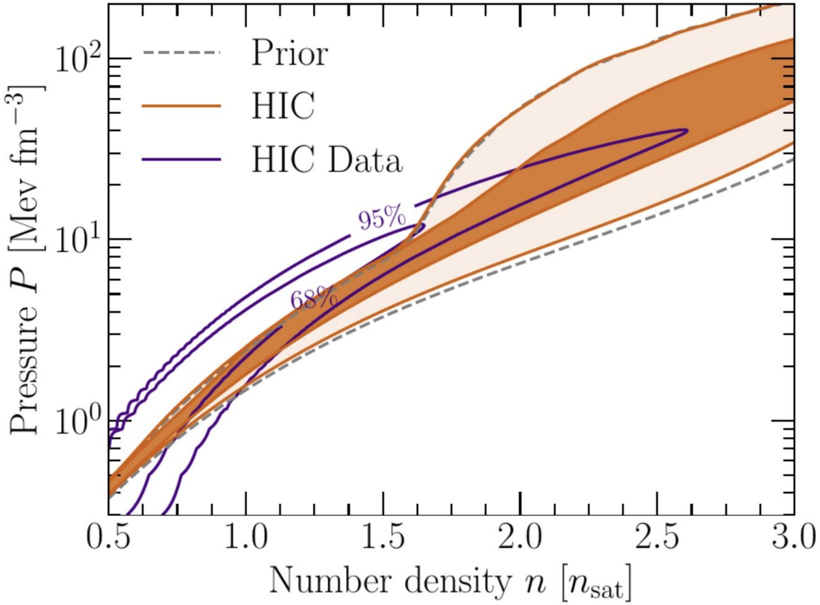
CREX



Heavy ion collision

- FOPI and ASY-EOS are considered
- Allow us to draw contour (purple) on the p-n plane

HIC experiments:



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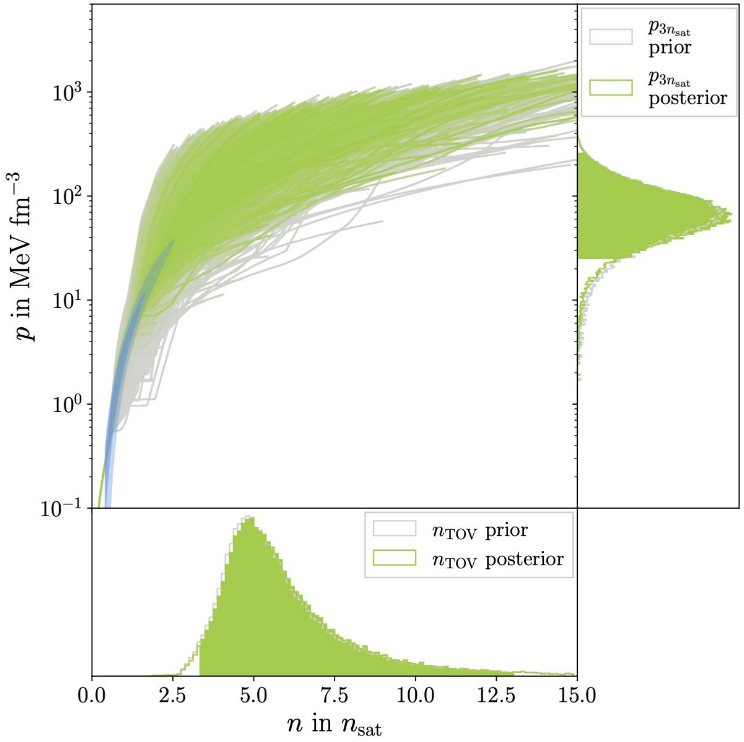
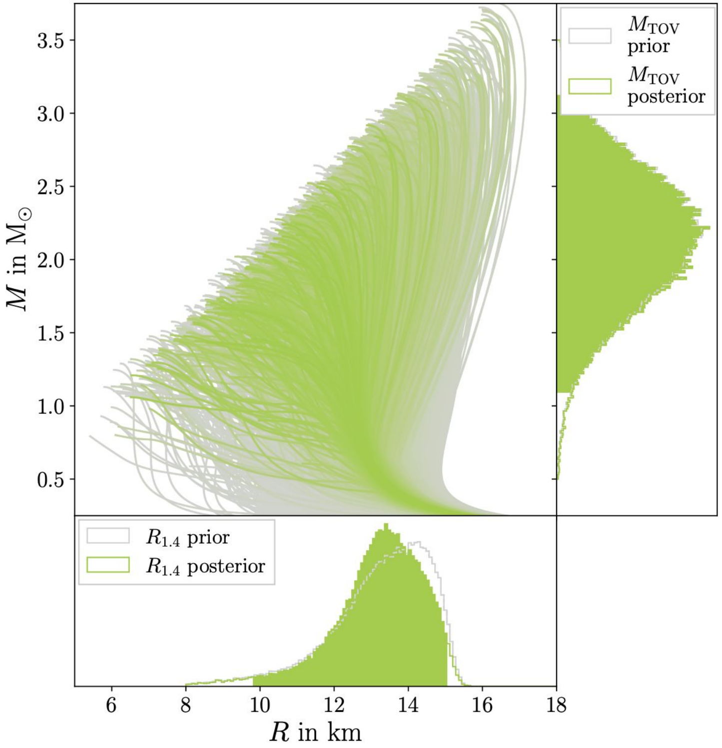
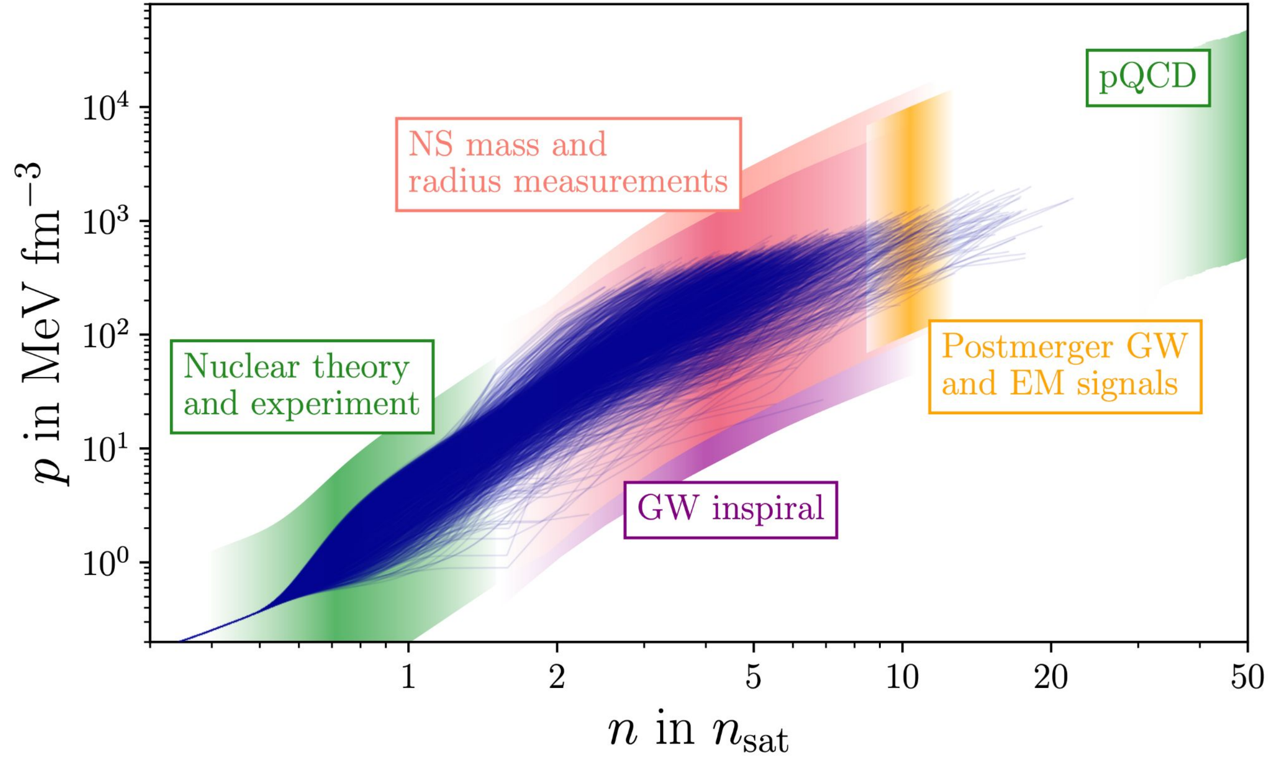




Neutron star equation-of-state



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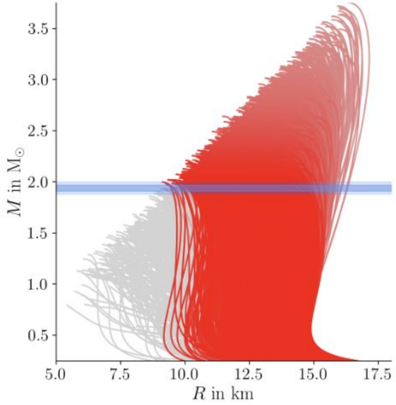
Heavy ion collision	Thermonuclear accretion bursts	Post-merger of GW170817

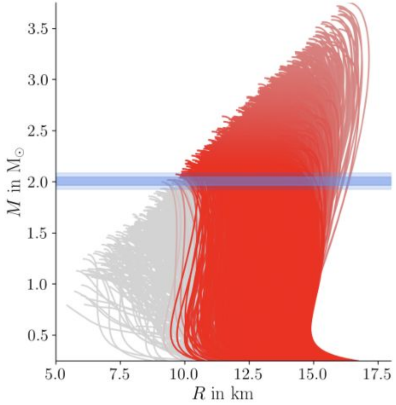
Radio timing

- The maximum support mass of NS depends on the EOS
- Heavy pulsars set the lower bound of it

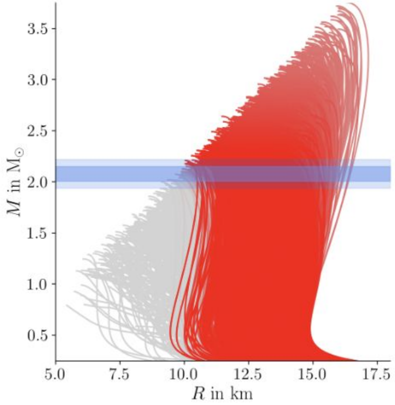
☐ **Radio timing J1614-2230** (Ref)



☐ **Radio timing J0348+0432** (Ref)



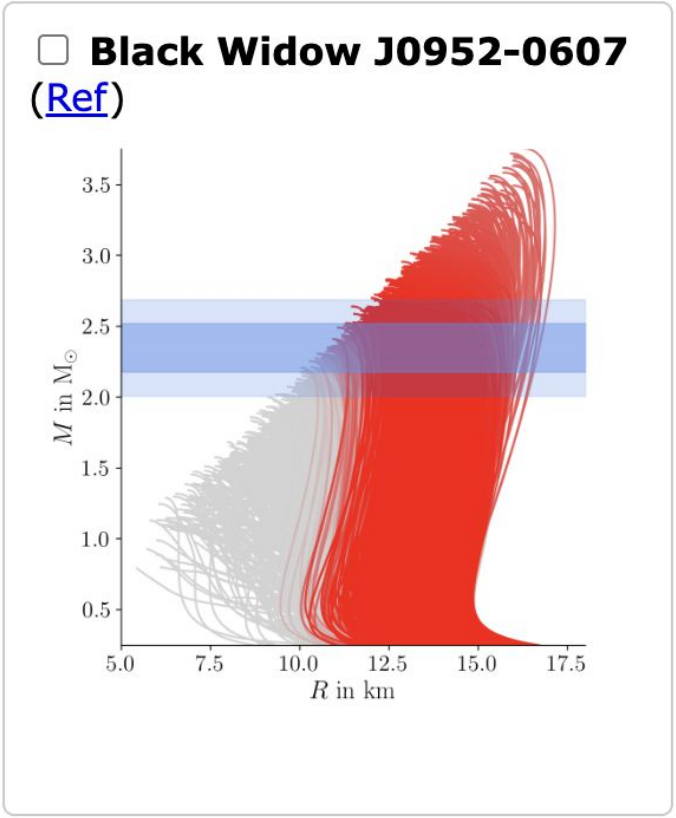
☐ **Radio timing J0740+6620** (Ref)



Black widow J0952-0607

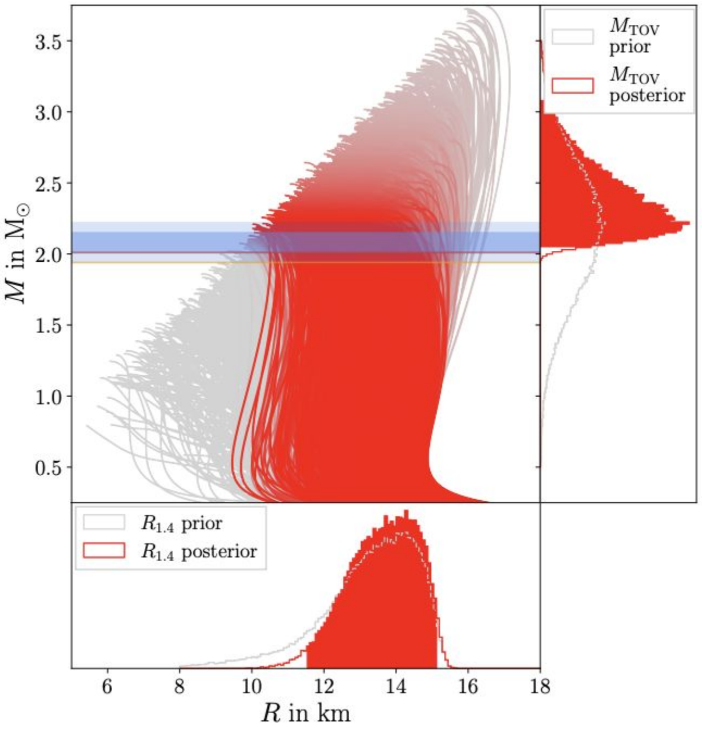
- Most massive neutron star ever observed
- Low-mass companion’s outer atmosphere is evaporated by the pulsar's radiation.
- Uncertainties from heat transport and temperature variations on the companion’s surface

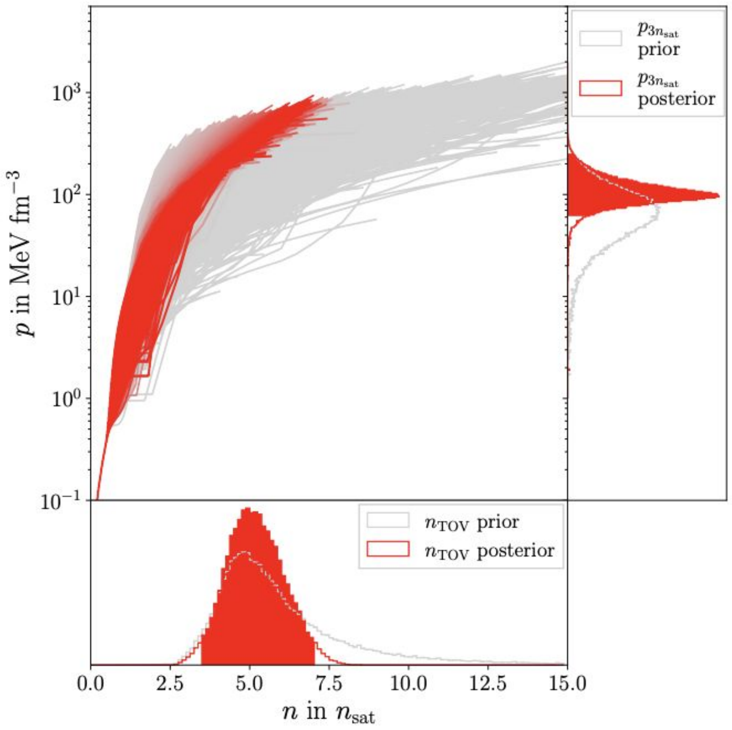
Black widow J0952-0607



☐ **Black Widow J0952-0607**
([Ref](Ref))

Mass measurement on neutron star



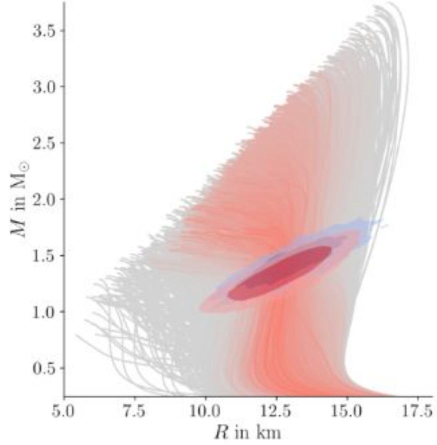


Mass-radius measurement on neutron star

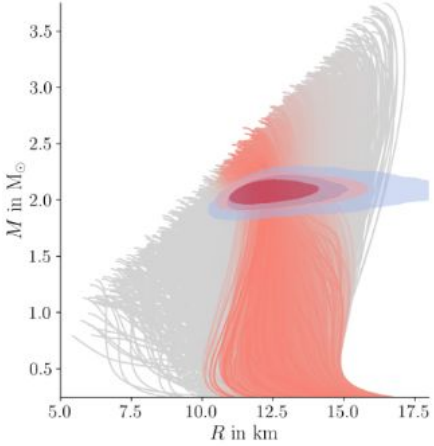
- The Neutron Star Interior Composition Explorer Mission (NICER)
  - Measure the X-ray pulsating profile
  - PSR J0030, PSR J0740+6620, J0437-4715
- Quiescent thermal X-ray spectra
  - qLMXBs (ω Centauri and X5 in Tucanae 47), HESS J1731-347
- Thermonuclear bursts
  - 4U 1702-429, J1808.8-3658

NICER

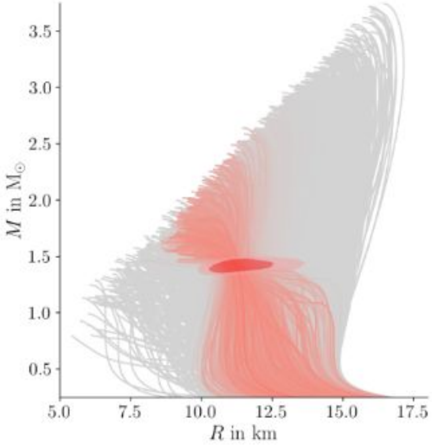
☐ **NICER J0030+0451** (Ref, Ref)



☐ **NICER J0740+6620** (Ref, Ref)



☐ **NICER J0437-4715** (Ref)

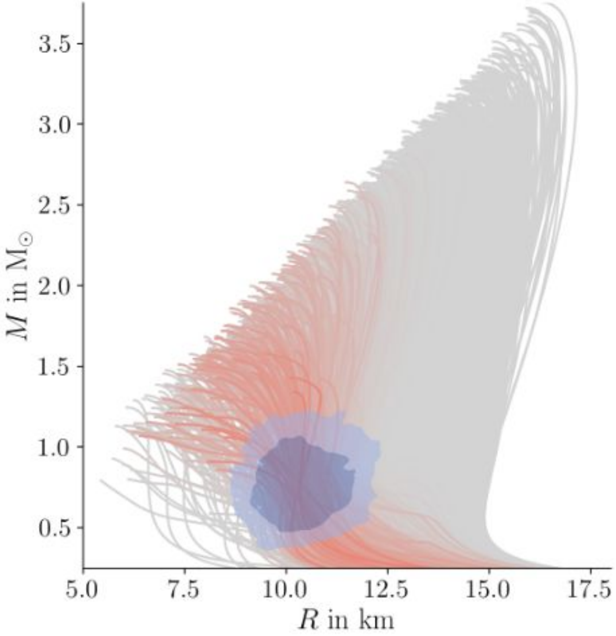


Quiescent thermal X-ray spectra

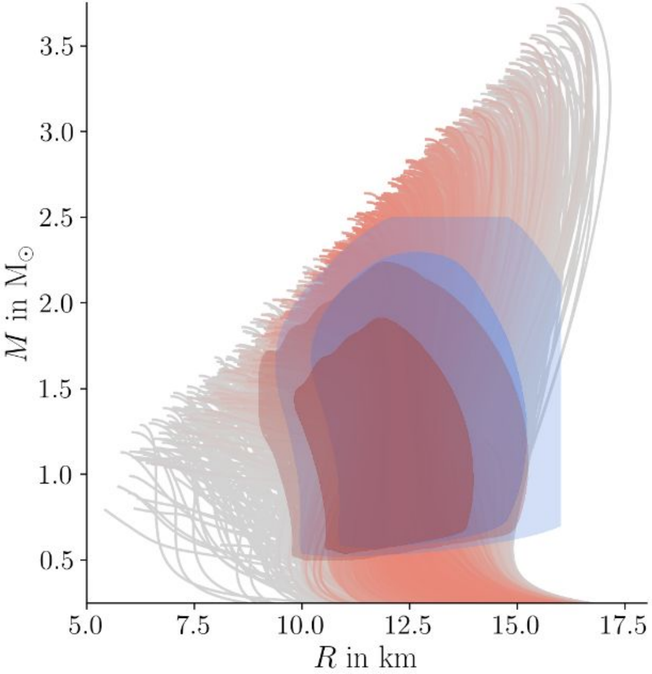
- Radius and mass can be deduced from X-ray spectra (thermal component)
    - Flux for informing the radius
    - Gravitational bending / redshift informing the compactness
- Systematics and uncertainty:
    - Distance estimates
    - Interstellar extinction
    - Non-thermal spectrum contributions
    - Surface emission models and atmospheric composition

Quiescent thermal X-ray spectra

☐ **HESS J1731-347** (Ref)

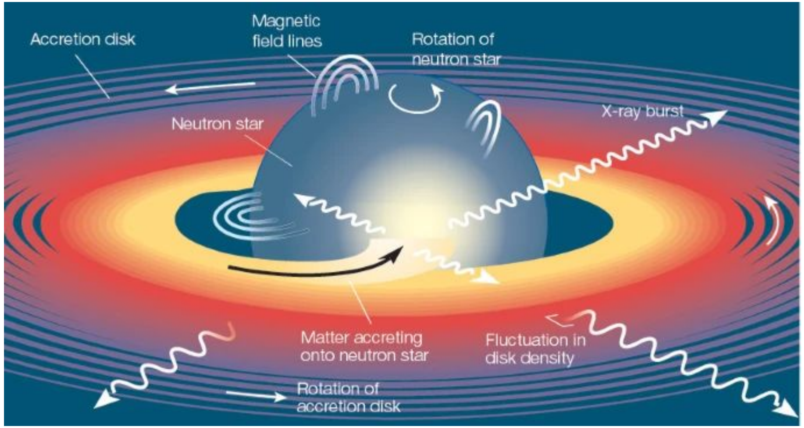


☐ **qLMXBs** (Ref)



Thermonuclear accretion burst

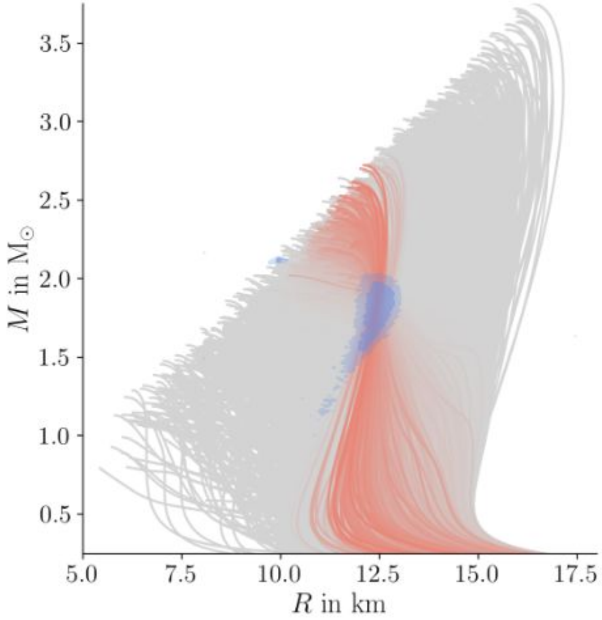
- Low-mass X-ray binaries (LMXBs)
    - Small orbital separation -> Roche limit -> accretion disk
    - Accretion can cause thermonuclear X-ray bursts (Type-I bursts)
    - Temperature, spin, mass, and radius
- Systematics and uncertainty:
    - Accretion environment
    - Incomplete burst observation



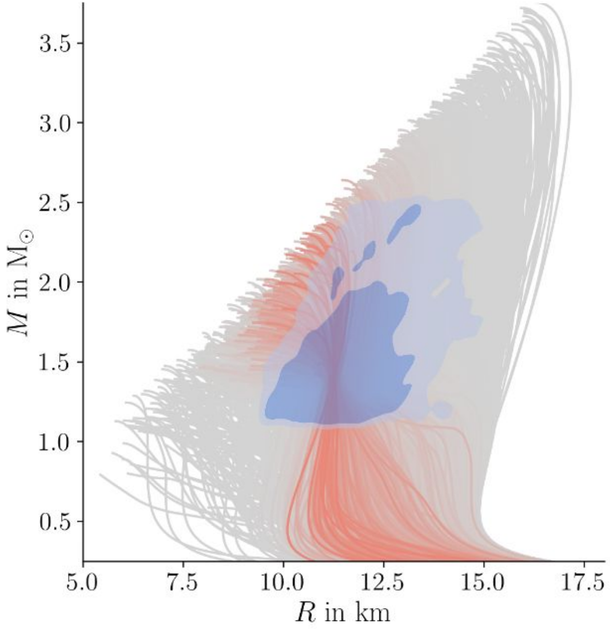
Wagoner, Nature 2003

Thermonuclear accretion burst

☐ **Burster 4U 1702-429** (Ref)



☐ **Burster J1808.8-3658** (Ref)



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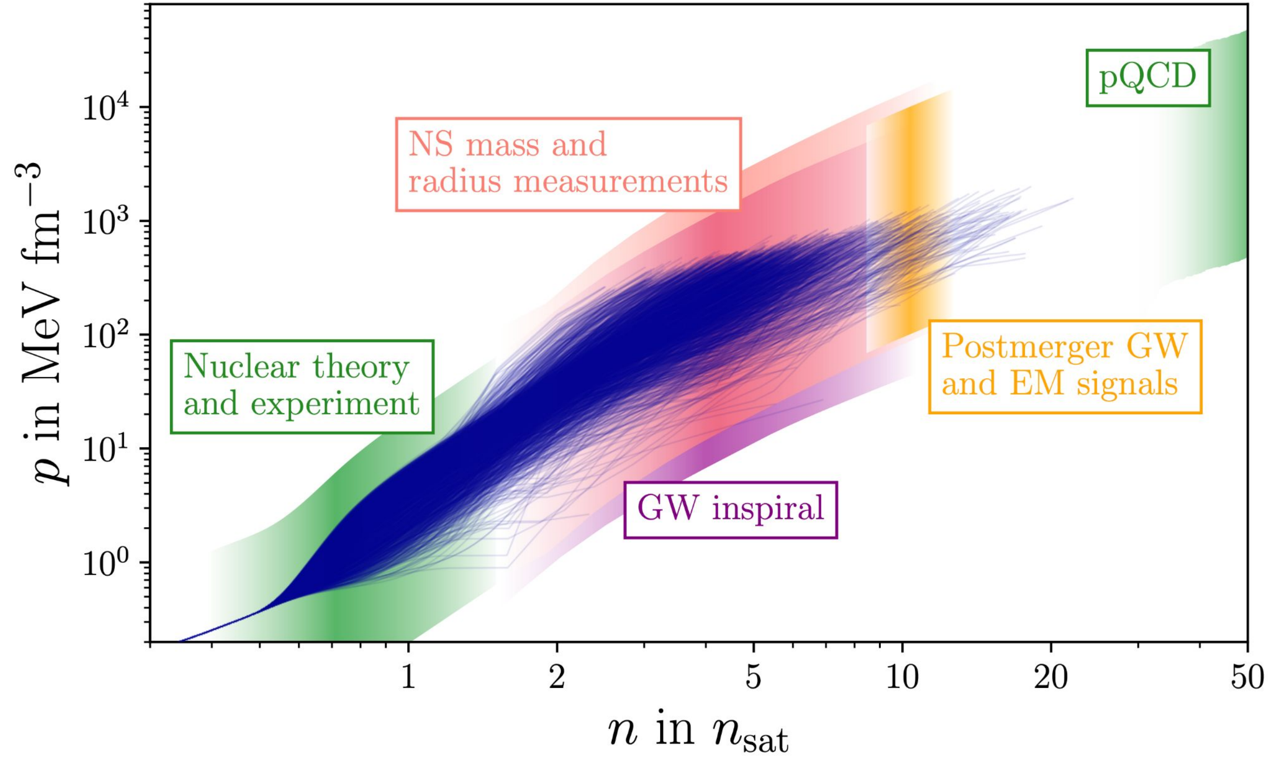

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Binary neutron star

Gravitational channel

- GW170817
- GW190425

Electromagnetic channel

- AT2017gfo
- GRB170817A
- GRB211211A

Remnant fate

- Spinning
- Non-spinning

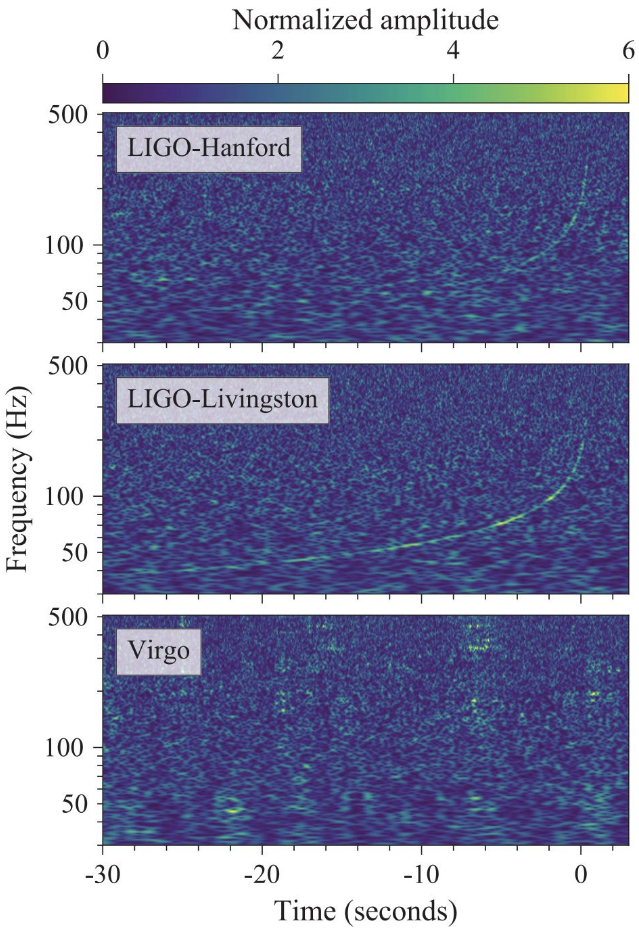
Gravitational waves

Gravitational wave of inspiral

- Encodes the masses of the binary
- Tidal deformation footprints

Reanalyse the gravitational wave data

- Ensure a full exploration
- Avoid the usage of importance sampling (KDE)

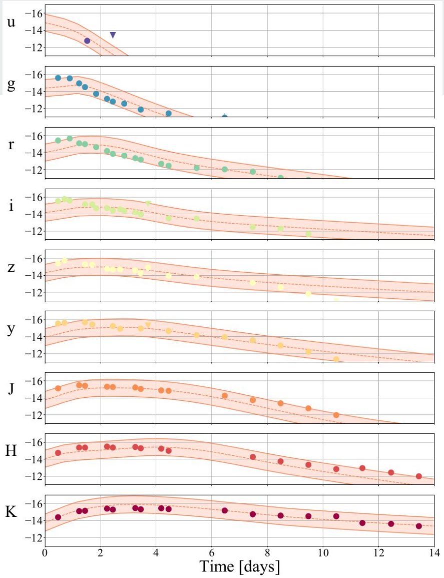


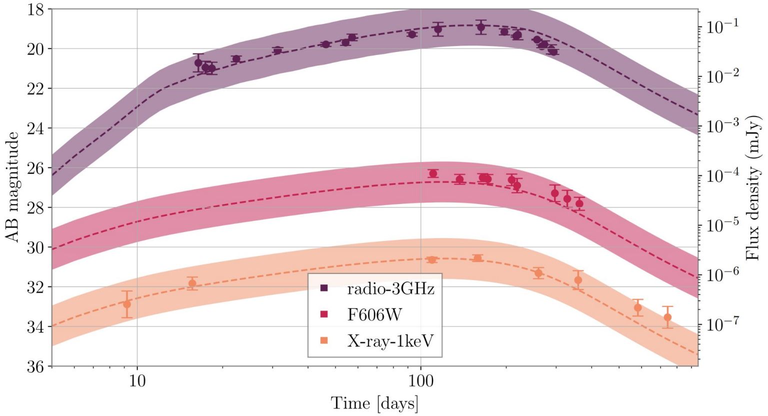
Electromagnetic signal

Electromagnetic signal

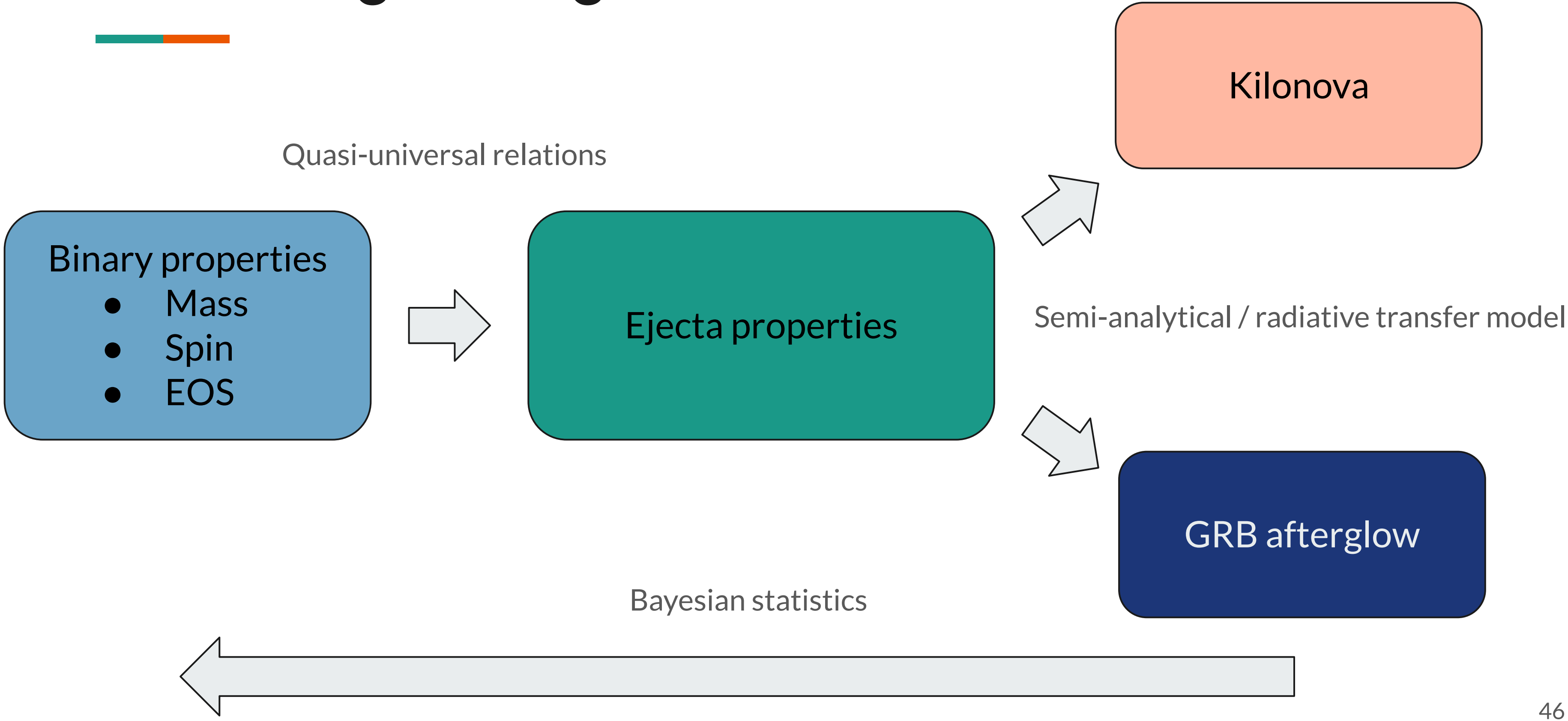
Electromagnetic signal

- Kilonova
  - Driven by r-process
  - Inform us about ejecta properties
- GRB afterglow
  - Energy of the central engine
  - Precise viewing angle measurement





Electromagnetic signal



Quasi-universal relations

Binary properties
- Mass
- Spin
- EOS

Ejecta properties

Semi-analytical / radiative transfer model

Kilonova

GRB afterglow

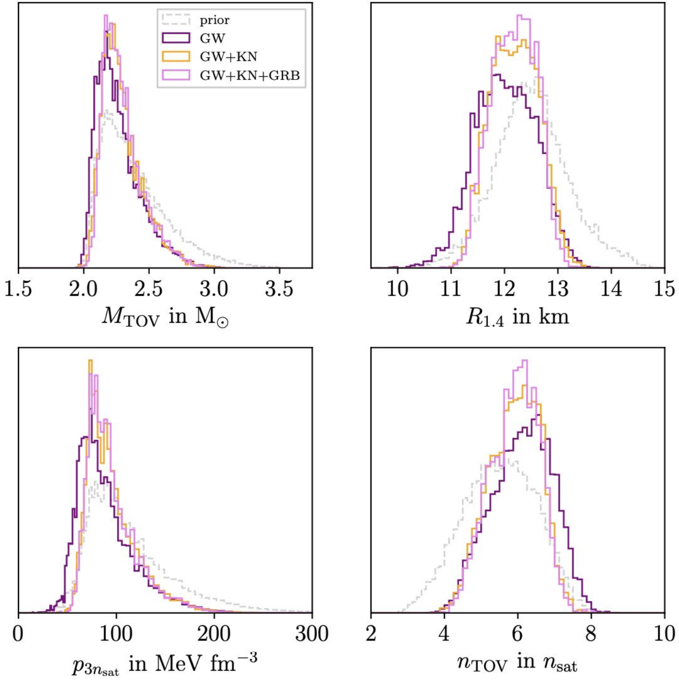
Bayesian statistics

Electromagnetic signal

github.com/nuclear-multimessenger-astronomy/nmma

Make use of NMMA

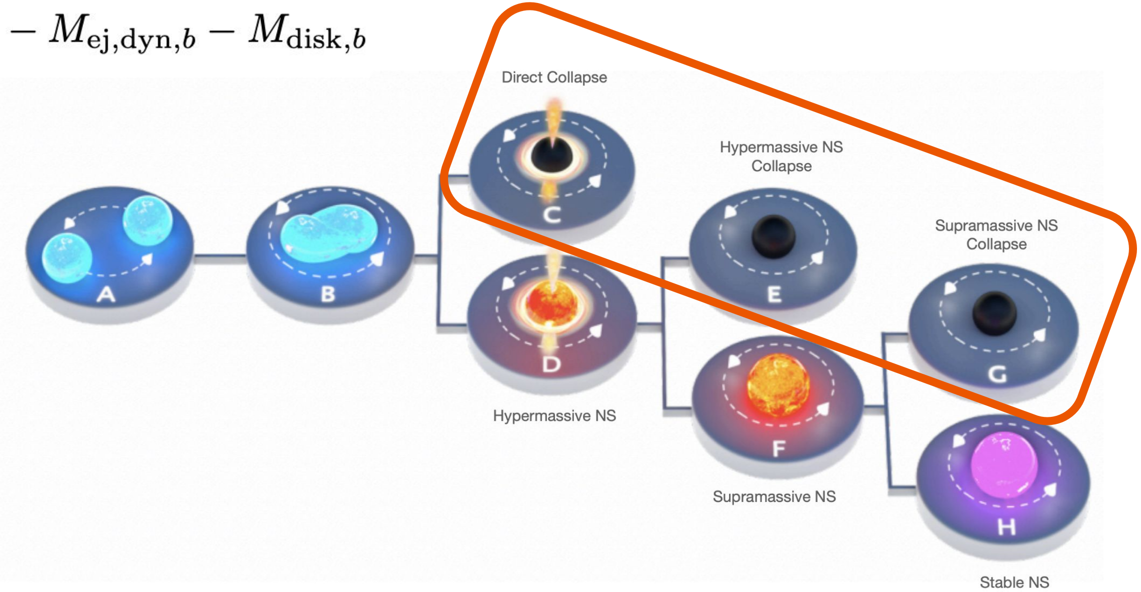
- GW + KN + GRB all-at-once
  - ~ 22 - 24 dimension + 60 auxiliary parameters
- Fully incorporate underlying correlation



Fate of remnant

GW170817 collapse into a black hole

- Place an upper bound on the maximum mass
- $M_{\mathrm{rem},b} = M_{1,b} + M_{2,b} - M_{\mathrm{ej,dyn},b} - M_{\mathrm{disk},b}$
- $M_{\mathrm{rem},b} > M_{\mathrm{coll},b}$



Likelihood

Nuclear experiment / theory	**Isolated neutron star**	**Binary neutron star**
Chiral EFT	Radio timing	GW170817 + AT2017gfo + GRB170817A
pQCD	NICER	
PREX-II	Black widow	GW190425
CREX	qLMXBs	GRB211211A
Heavy ion collision	Thermonuclear accretion bursts	Post-merger of GW170817

Likelihood

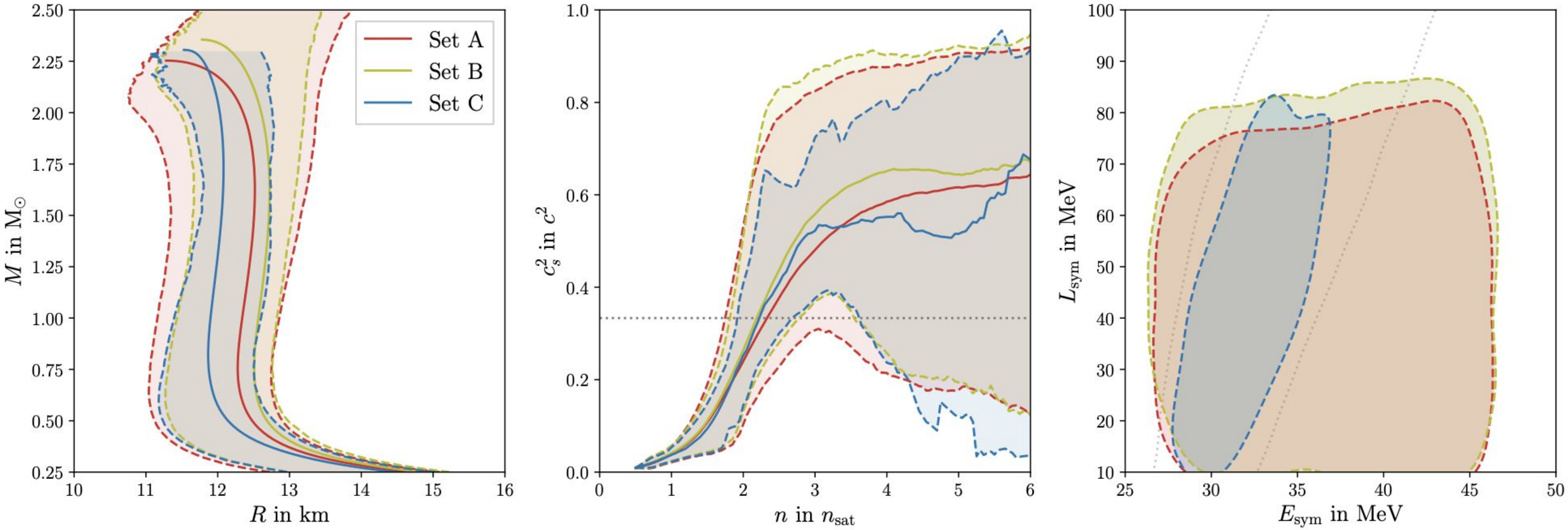
Nuclear experiment / theory	Isolated neutron star	Binary neutron star
Chiral EFT	Radio timing	GW170817
		fo
		17A
		25
CREX	qLMXBs	GRB211211A
Heavy ion collision	Thermonuclear accretion bursts	Post-merger of GW170817

SYSTEMATICS

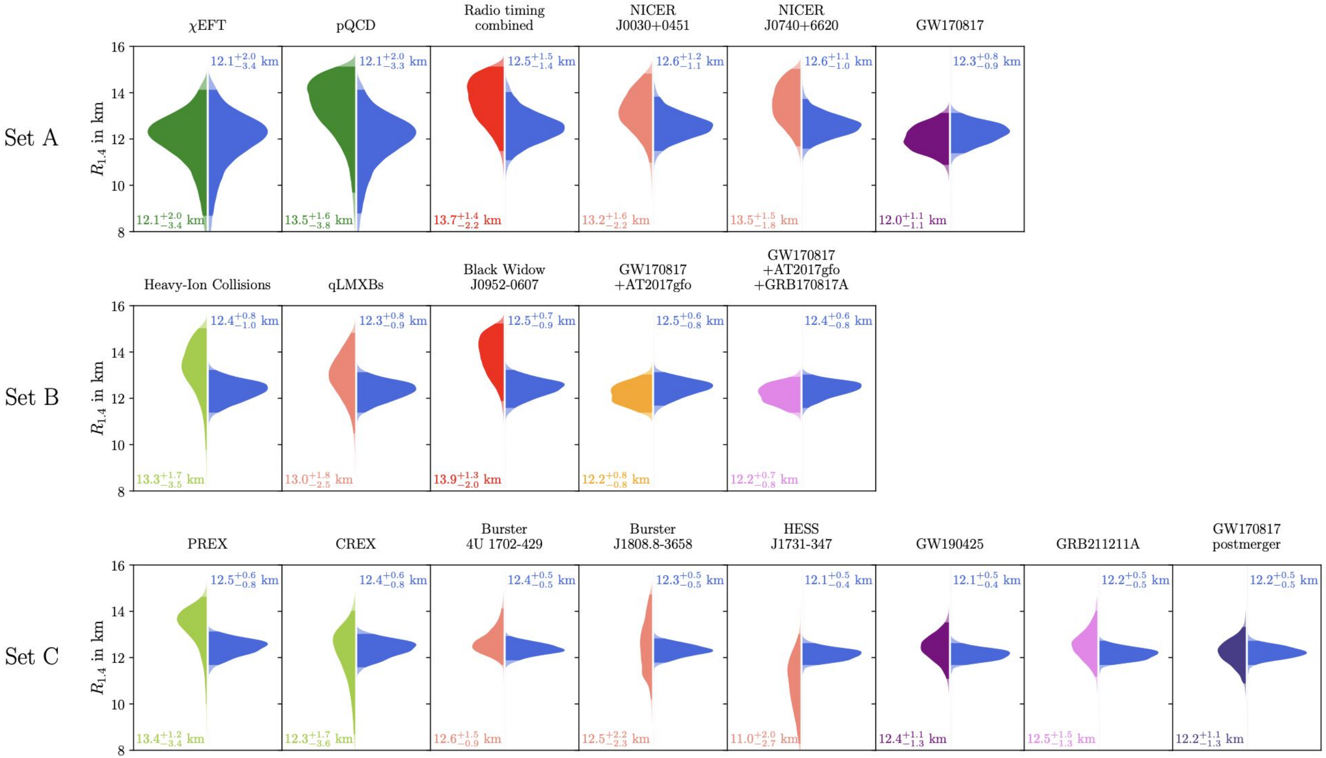
Constraints

Set A (Conservative)	Set B (Middle ground)	Set C (Aggressive)
Chiral EFT	Set A	Set B
pQCD	Heavy ion collision	PREX-II + CREX
Radio timing	Black widow	GW190425
NICER J0030+0451 J0740+6620	qLMXBs	Brusters, HESS, GRB211211A
GW170817	GW170817 + KN + GRB	Post-merger of GW170817

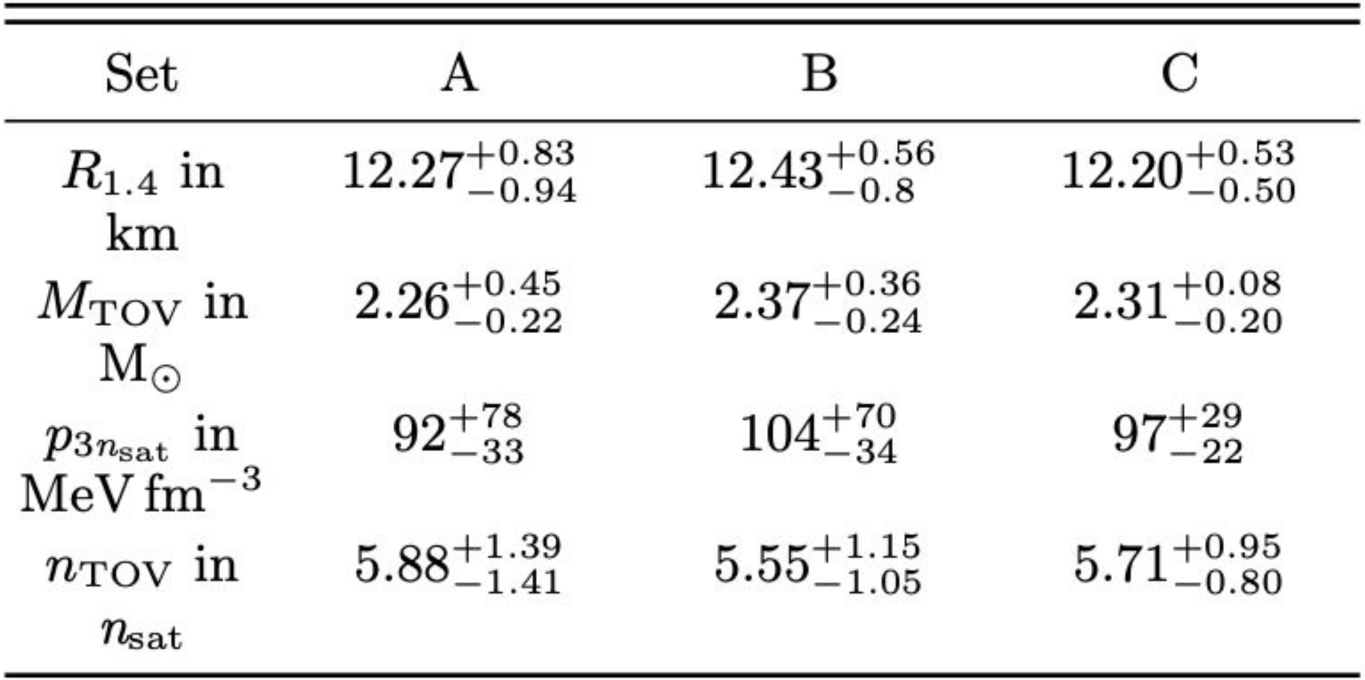
Combined


2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
M in $M_\odot$
10
11
12
13
14
15
16
R in km
Set A
Set B
Set C
1.0
0.8
0.6
0.4
0.2
0.0
c_s^2 in c^2
0
1
2
3
4
5
6
n in n_{sat}
100
90
80
70
60
50
40
30
20
10
L_{sym} in MeV
25
30
35
40
45
50
E_{sym} in MeV

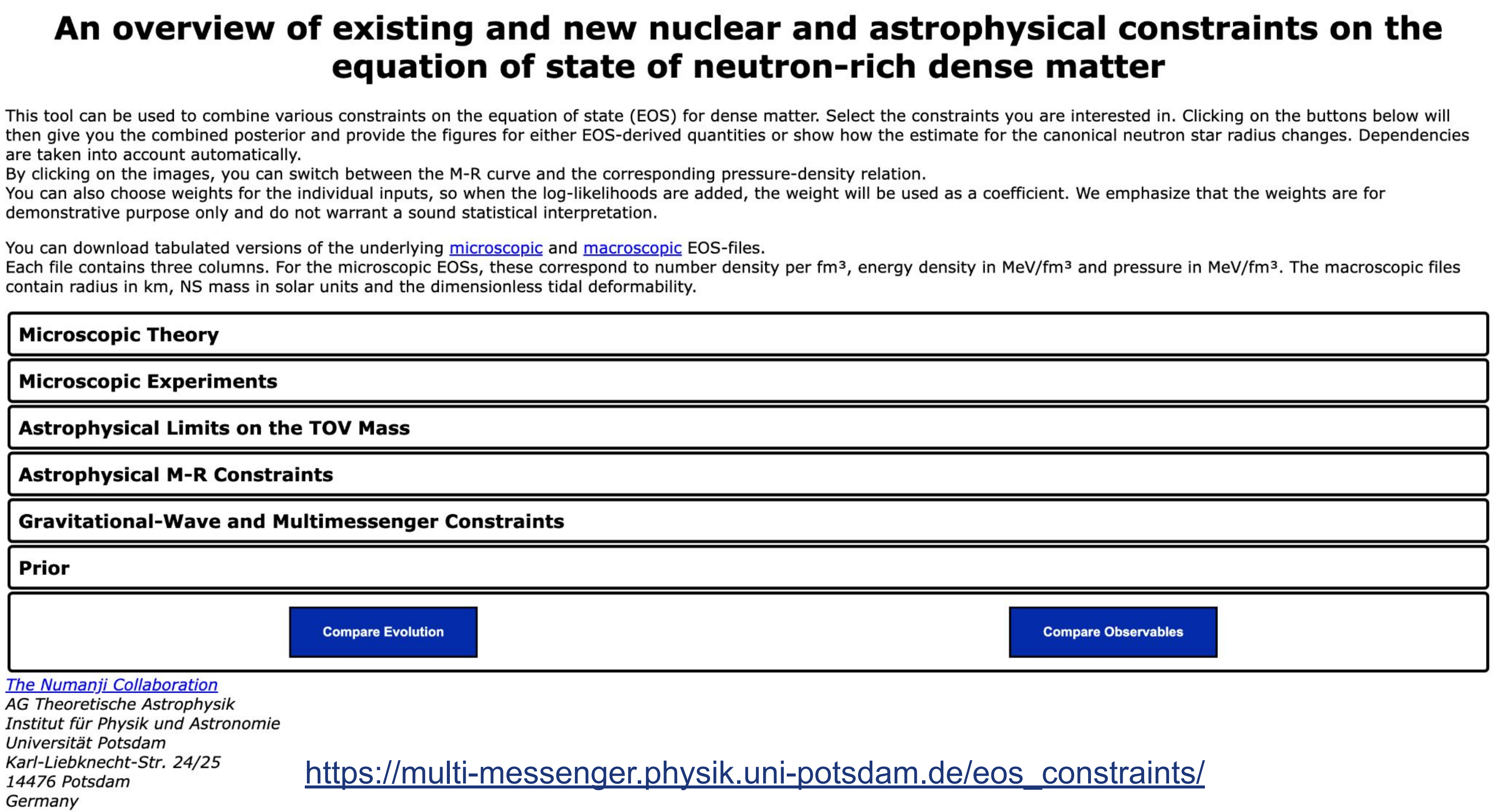
Combined


Set A
$R_{1.4}$ in km
16
14
12
10
8
χEFT
$12.1^{+2.0}_{-3.4}$ km
$12.1^{+2.0}_{-3.4}$ km
pQCD
$12.1^{+2.0}_{-3.3}$ km
$13.5^{+1.6}_{-3.8}$ km
Radio timing combined
$12.5^{+1.5}_{-1.4}$ km
$13.7^{+1.4}_{-2.2}$ km
NICER J0030+0451
$12.6^{+1.2}_{-1.1}$ km
$13.2^{+1.6}_{-2.2}$ km
NICER J0740+6620
$12.6^{+1.1}_{-1.0}$ km
$13.5^{+1.5}_{-1.8}$ km
GW170817
$12.3^{+0.8}_{-0.9}$ km
$12.0^{+1.1}_{-1.1}$ km
Set B
$R_{1.4}$ in km
16
14
12
10
8
Heavy-Ion Collisions
$12.4^{+0.8}_{-1.0}$ km
$13.3^{+1.7}_{-3.5}$ km
qLMXBs
$12.3^{+0.8}_{-0.9}$ km
$13.0^{+1.8}_{-2.5}$ km
Black Widow J0952-0607
$12.5^{+0.7}_{-0.9}$ km
$13.9^{+1.3}_{-2.0}$ km
GW170817 +AT2017gfo
$12.5^{+0.6}_{-0.8}$ km
$12.2^{+0.8}_{-0.8}$ km
GW170817 +AT2017gfo +GRB170817A
$12.4^{+0.6}_{-0.8}$ km
$12.2^{+0.7}_{-0.8}$ km
Set C
$R_{1.4}$ in km
16
14
12
10
8
PREX
$12.5^{+0.6}_{-0.8}$ km
$13.4^{+1.2}_{-3.4}$ km
CREX
$12.4^{+0.6}_{-0.8}$ km
$12.3^{+1.7}_{-3.6}$ km
Burster 4U 1702-429
$12.4^{+0.5}_{-0.5}$ km
$12.6^{+1.5}_{-0.9}$ km
Burster J1808.8-3658
$12.3^{+0.5}_{-0.5}$ km
$12.5^{+2.2}_{-2.3}$ km
HESS J1731-347
$12.1^{+0.5}_{-0.4}$ km
$11.0^{+2.0}_{-2.7}$ km
GW190425
$12.1^{+0.5}_{-0.4}$ km
$12.4^{+1.1}_{-1.3}$ km
GRB211211A
$12.2^{+0.5}_{-0.5}$ km
$12.5^{+1.5}_{-1.3}$ km
GW170817 postmerger
$12.2^{+0.5}_{-0.5}$ km
$12.2^{+1.1}_{-1.3}$ km

Combined



Set	A	B	C
$R_{1.4}$ in km	$12.27^{+0.83}_{-0.94}$	$12.43^{+0.56}_{-0.8}$	$12.20^{+0.53}_{-0.50}$
M_{TOV} in $\mathrm{M}_{\odot}$	$2.26^{+0.45}_{-0.22}$	$2.37^{+0.36}_{-0.24}$	$2.31^{+0.08}_{-0.20}$
$p_{3n_{\mathrm{sat}}}$ in $\mathrm{MeV\,fm}^{-3}$	92^{+78}_{-33}	104^{+70}_{-34}	97^{+29}_{-22}
n_{TOV} in n_{sat}	$5.88^{+1.39}_{-1.41}$	$5.55^{+1.15}_{-1.05}$	$5.71^{+0.95}_{-0.80}$



An overview of existing and new nuclear and astrophysical constraints on the equation of state of neutron-rich dense matter

This tool can be used to combine various constraints on the equation of state (EOS) for dense matter. Select the constraints you are interested in. Clicking on the buttons below will then give you the combined posterior and provide the figures for either EOS-derived quantities or show how the estimate for the canonical neutron star radius changes. Dependencies are taken into account automatically.
By clicking on the images, you can switch between the M-R curve and the corresponding pressure-density relation.
You can also choose weights for the individual inputs, so when the log-likelihoods are added, the weight will be used as a coefficient. We emphasize that the weights are for demonstrative purpose only and do not warrant a sound statistical interpretation.

You can download tabulated versions of the underlying microscopic and macroscopic EOS-files.
Each file contains three columns. For the microscopic EOSs, these correspond to number density per fm³, energy density in MeV/fm³ and pressure in MeV/fm³. The macroscopic files contain radius in km, NS mass in solar units and the dimensionless tidal deformability.

Microscopic Theory

Microscopic Experiments

Astrophysical Limits on the TOV Mass

Astrophysical M-R Constraints

Gravitational-Wave and Multimessenger Constraints

Prior

The Numanji Collaboration
AG Theoretische Astrophysik
Institut für Physik und Astronomie
Universität Potsdam
Karl-Liebknecht-Str. 24/25
14476 Potsdam
Germany

https://multi-messenger.physik.uni-potsdam.de/eos_constraints/

Conclusions

- An overview of multi-messenger constraint on neutron star EOS
  - Nuclear experiments
  - Nuclear theoretical calculations
  - Astrophysical observations
- Largely extension from previous work
- Proposed novel and statistically robust likelihood function
- Interactive portal for both nuclear physicists and astrophysicists
  - https://multi-messenger.physik.uni-potsdam.de/eos_constraints/

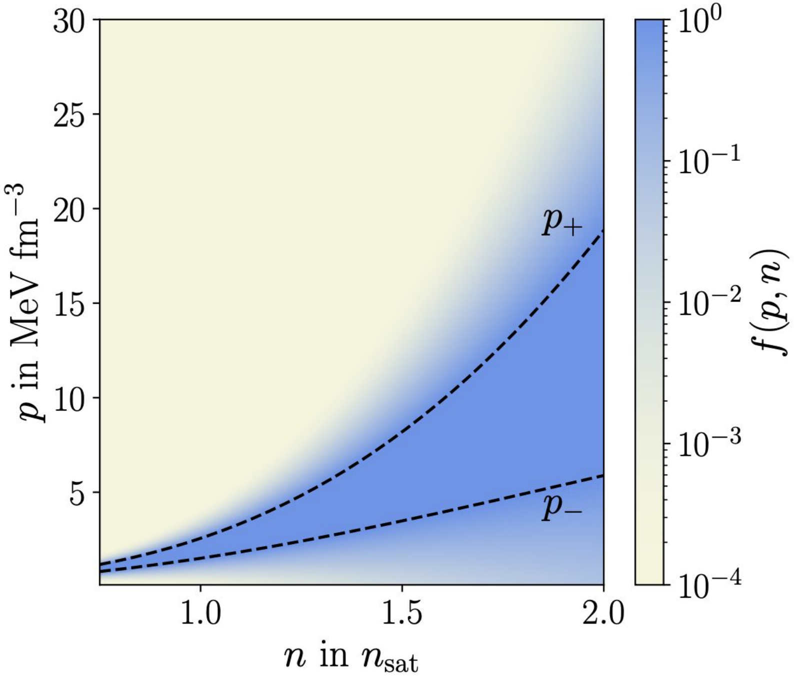
Chiral effective theory

$$f(p,n) = \begin{cases} \exp\left(-\beta \frac{p-p_+}{p_+-p_-}\right) & \text{if } p > p_+\,, \\ \exp\left(-\beta \frac{p_--p}{p_+-p_-}\right) & \text{if } p < p_-\,, \\ 1 & \text{else}\,. \end{cases}$$

- CEFT calculation up to N^{2}LO
- Based on Furnstahl et al. Phys. Rev. C 92, 024005 (2015).
  - 75% probability mass enclosed in band

$$\mathcal{L}(\mathrm{EOS}|\chi\mathrm{EFT}) \propto \prod_j f(p(n_j, \mathrm{EOS}), n_j)$$

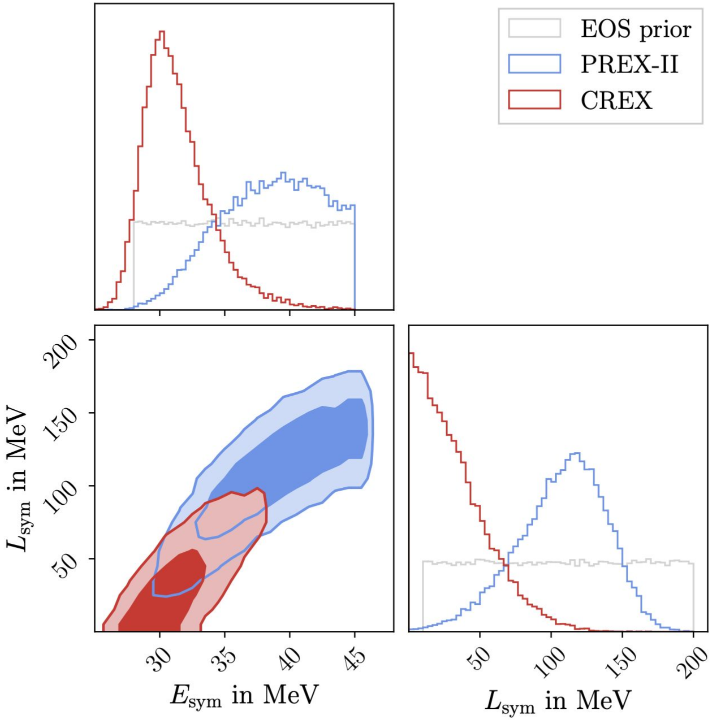
$$\mathcal{L}(\mathrm{EOS}|\chi\mathrm{EFT}) \propto$$
$$\exp\left(\int_{0.75\,n_{\mathrm{sat}}}^{n_{\mathrm{break}}} \frac{\log f(p(n,\mathrm{EOS}),n)}{n_{\mathrm{break}} - 0.75\,n_{\mathrm{sat}}}\,dn\right)$$



PREX-II and CREX

- Measurement on the neutron skin thickness
- Make use of the correlation between the thickness and Lsym
- Correlation between Esym and Lsym considered

$$\log \mathcal{L}(E_{\rm sym}, L_{\rm sym}|\text{PREX-II}) = -\frac{1}{2}\left(\frac{(\mu - R_{\rm skin}^{208,\rm fit}(L_{\rm sym}))^2}{\sigma^2} + \frac{(L_{\rm sym} - L_{\rm sym}^{\rm fit}(E_{\rm sym}))^2}{\sigma_{\rm fit}^2}\right)$$



Heavy ion collision

- FOPI and ASY-EOS are considered
- Allow us to draw contour (purple) on the p-n plane

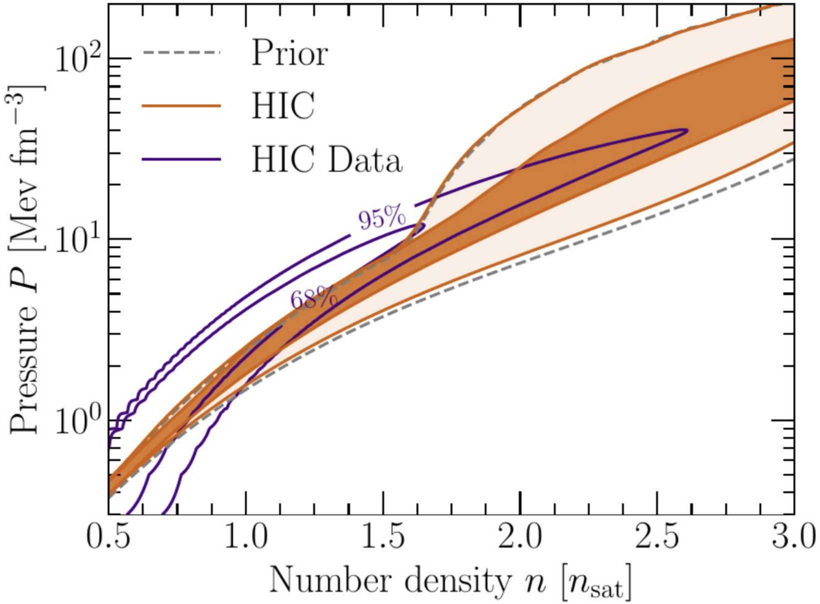
$$\frac{E}{A}(n,\delta) \approx e_{\mathrm{sat}}(n) + e_{\mathrm{sym}}(n)\delta^2 + \dots$$

$$e_{\mathrm{sat}}(n) = \frac{3}{5}\left(\frac{n}{n_{\mathrm{sat}}}\right)^{2/3} E_F + \frac{\alpha}{2}\left(\frac{n}{n_{\mathrm{sat}}}\right) + \frac{\beta}{\gamma+1}\left(\frac{n}{n_{\mathrm{sat}}}\right)^{\gamma}$$

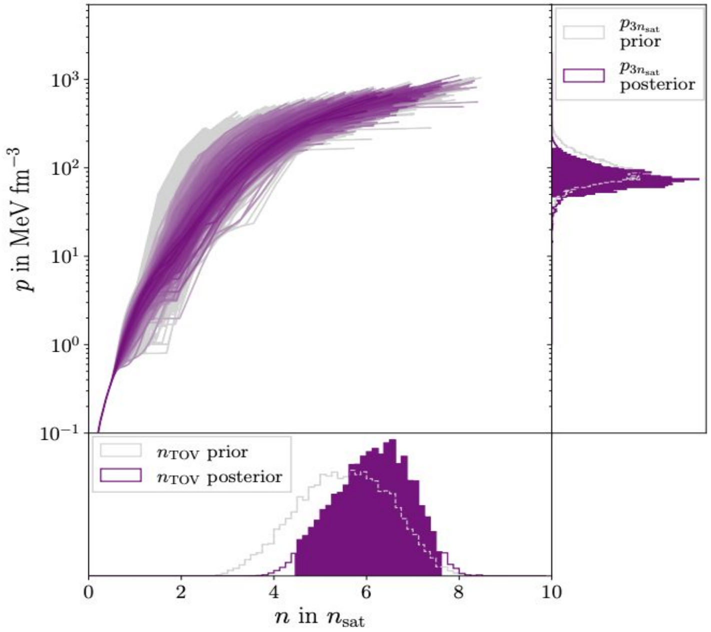
$$e_{\mathrm{sym}}(n) = E_{\mathrm{kin},0}\left(\frac{n}{n_{\mathrm{sat}}}\right)^{2/3} + E_{\mathrm{pot},0}\left(\frac{n}{n_{\mathrm{sat}}}\right)^{\gamma_{\mathrm{asy}}}$$

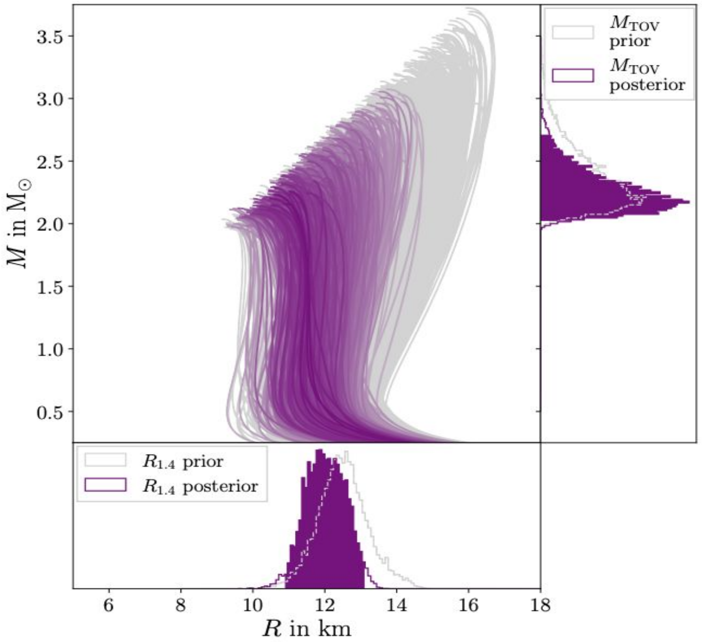
$$\mathcal{L}(\mathrm{EOS}|\mathrm{HIC}) = \int dn\ P(p(n,\mathrm{EOS}), n|\mathrm{HIC})\ C(n)$$

HIC experiments:

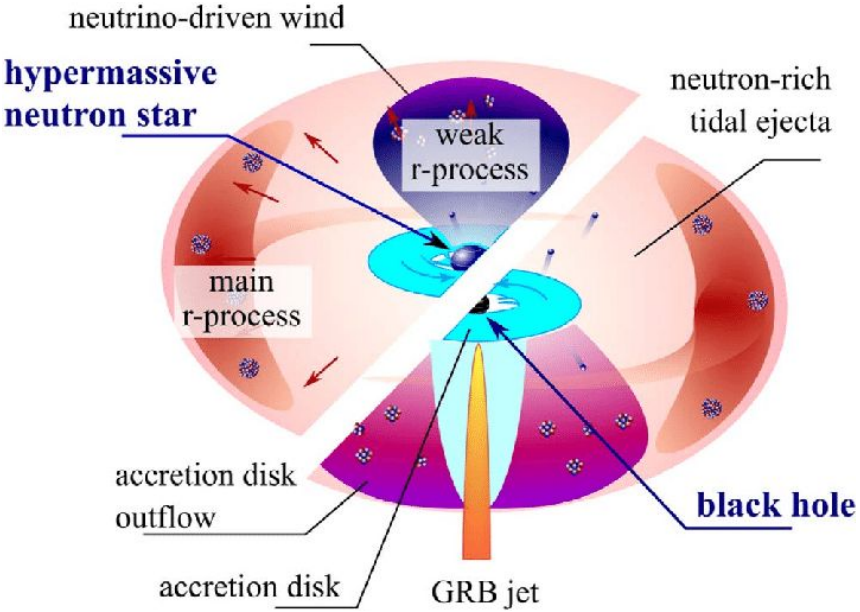


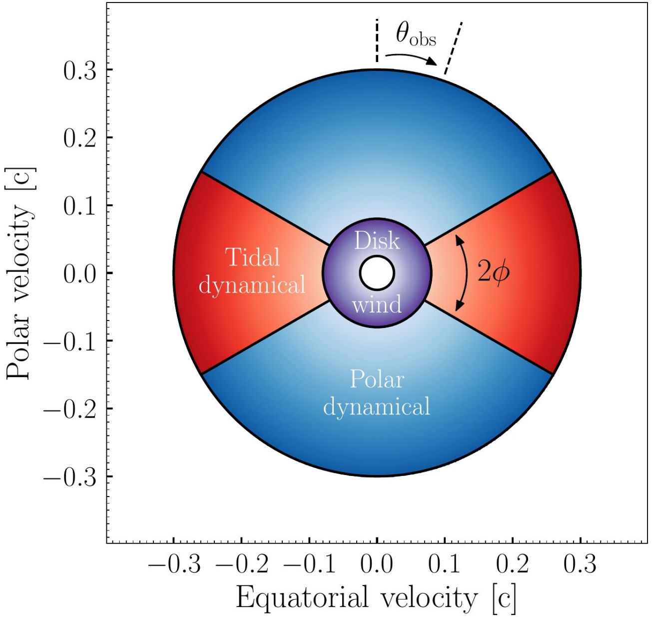
Gravitational waves





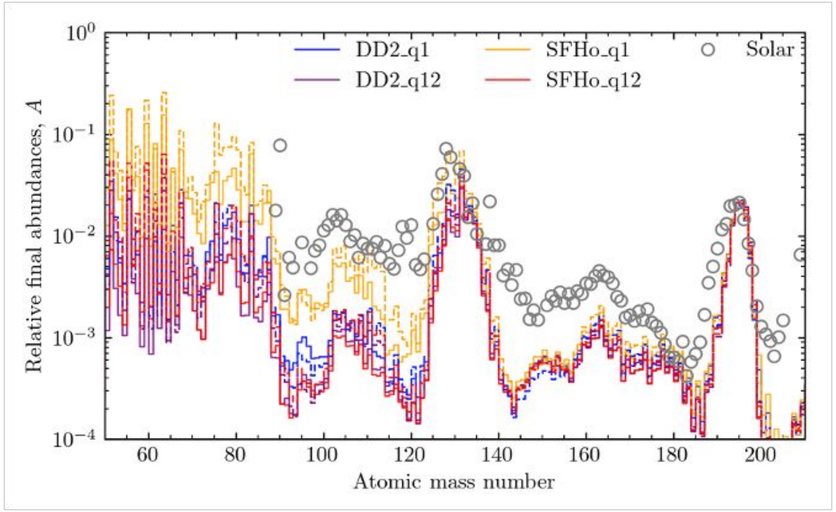
Kilonova and GRB





Kilonova

- neutron rich ejecta produce heavy r-process elements
- pseudo-black body radiation from r-process elements
- mergers are major sites for the formation of heavy elements



Schianchi et al., Phys. Rev. D 2024

Kilonova

1. Compute lightcurves for a set (grid) of ejecta properties with a radiative transfer code
2. interpolate within this grid through Gaussian Process Regression or a Neural Network