ZTF-IN2P3 Quaterly Meeting

Reminder and update on EDRIS

Distance estimation: 3 major issues

1. Both magnitudes & standardization parameters (stretch, color, ...) have errors + errors are correlated

=> Difficulty to estimate the slopes (α, β, ...)

2. Some error on magnitudes is unexplained (intrinsic dispersion)

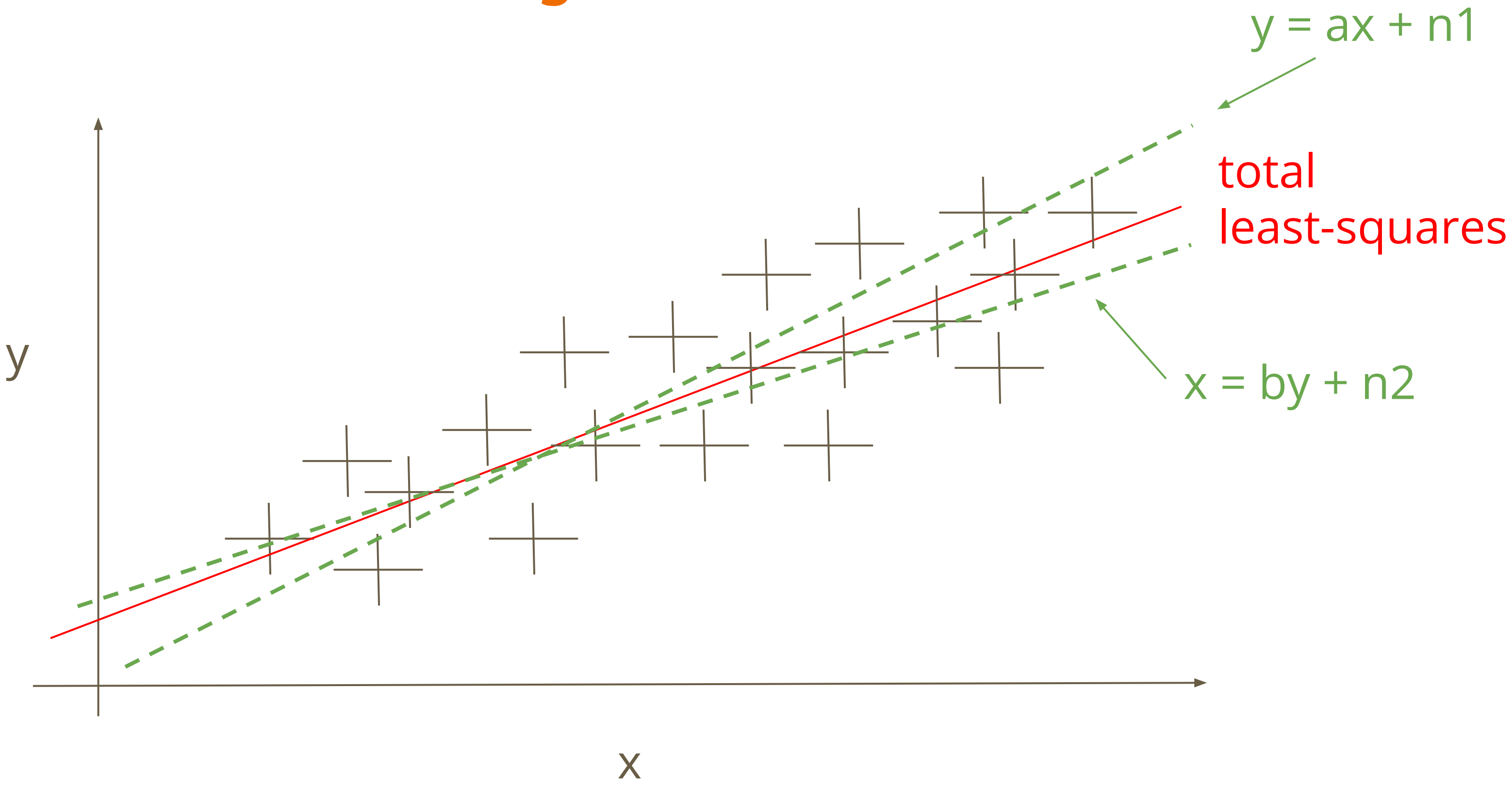
=> How to estimate this dispersion ?

3. SNIa surveys suffers from selection bias (Malmquist bias)

=> How to take into account the truncation of surveys ?

1. Estimation of the slopes without bias

Illustration of the issue: generic linear case



Model with latent parameters

Noise
- intrinsic dispersion
- covariance

Standardization coefficients

Distances

$$\begin{pmatrix} m_i \\ Y_{1,i} \\ Y_{2,i} \\ \vdots \\ Y_{n,i} \end{pmatrix} = \begin{pmatrix} \mu_i(z,\theta) \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \cdots & \alpha_n \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X^\star_{1,i} \\ X^\star_{2,i} \\ X^\star_{3,i} \\ \vdots \\ X^\star_{n,i} \end{pmatrix} + \begin{pmatrix} \epsilon_i \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \eta_i$$

$$\epsilon_i \sim \mathcal{N}(0, \sigma^2)$$

$$\eta \sim \mathcal{N}(0, Cov(m, Y))$$

LCs parameters

Latent parameters

Associated negative log-likelihood function

$$r = d - (\mu + AX^{\star})$$

Classic likelihood for multivariate normal distributions

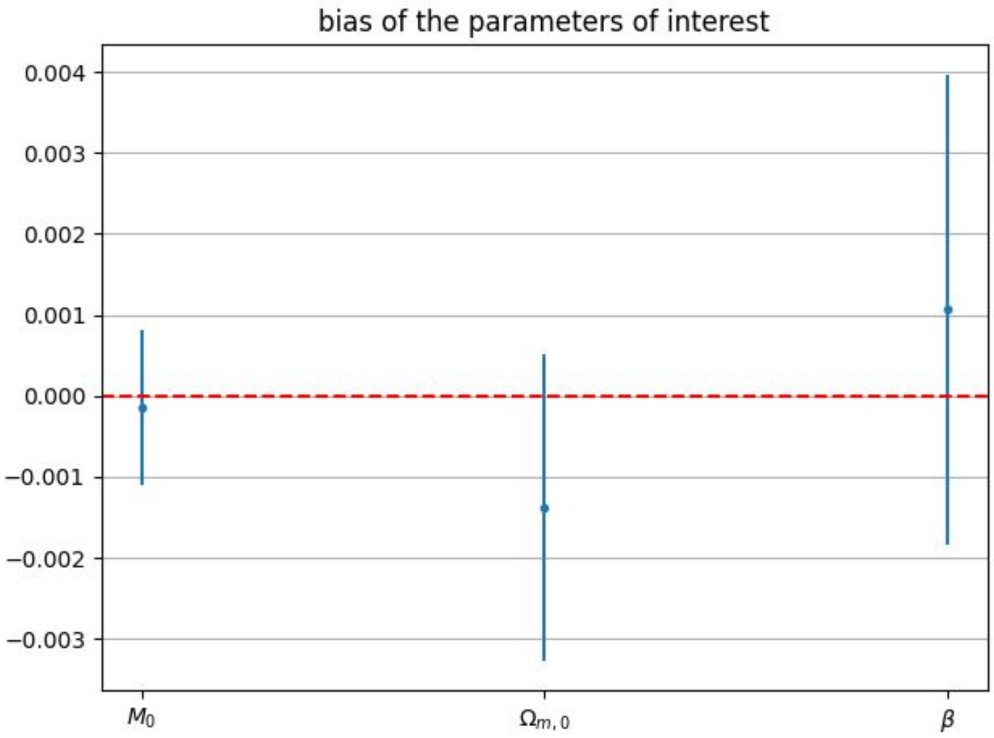
$$\Gamma = -\ln\left(|W|\right) + r^{\dagger} W r$$

Useful to estimate σ (see later)

$$W = \left(Cov(m, Y) + \begin{pmatrix} \sigma^2 I_N & 0 \\ 0 & 0 \end{pmatrix}\right)^{-1}$$

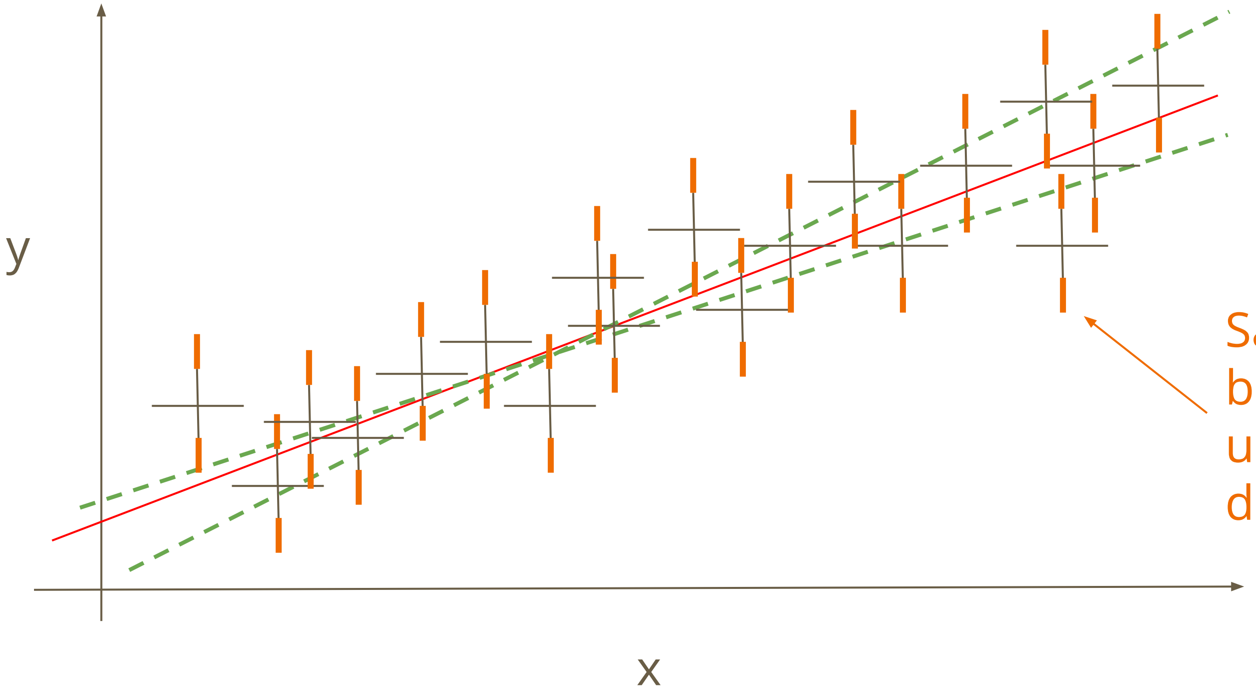
Validation of the estimator: Monte-Carlo simulation

- 100 draws
- 3 parameters (1 nuisance, 1 cosmo, 1 slope)
- No bias as expected



2. Joint estimation of usual parameters & intrinsic dispersion

Illustration of the issue: generic linear case



Same case as before but with unexplained dispersion

Complications encountered

Model and likelihood function do not change

But:

- W is a (3N, 3N) matrix
- W contains σ => need to invert it at each step of minimization
- The estimator of variance is biased

$$\mathbb{E}\left(\hat{\sigma^2}\right) = \frac{N-k}{N}\sigma^2$$

N: number of data
k: number of parameters

- The estimator of standardization coefficients is biased when $\alpha_i \sigma_{X_i^\star} \sim \sigma$

Fast inversion of the covariance matrix

Schur complement inverse technique

$$W = \begin{pmatrix} C_{mm} + \sigma^2 I_N & C_1 \\ C_1^\dagger & C_2 \end{pmatrix}^{-1} \Rightarrow W = \begin{pmatrix} S^{-1} & -S^{-1}C_1C_2^{-1} \\ -C_2^{-1}C_1^\dagger S^{-1} & C_2^{-1} + C_2^{-1}C_1^\dagger S^{-1}C_1C_2^{-1} \end{pmatrix}$$

Diagonalization of the Schur complement

$$S^{-1} = Q\left(\Lambda + \sigma^2 I_N\right)^{-1} Q^\dagger$$

Fast computation of the likelihood

Thus, writing $r = (r_1 \quad r_2)$ to match the structure of W

Only matrix-to-vector products

$$r^\dagger W r = r_1^\dagger S^{-1} r_1 - 2 r_1^\dagger S^{-1} C_1 C_2^{-1} r_2 + r_2^\dagger C_2^{-1} r_2 + r_2^\dagger C_2^{-1} C_1^\dagger S^{-1} C_1 C_2^{-1} r_2$$

$$-\ln\left(|W|\right) = \ln\left(|C_2|\right) + \sum_i \ln\left(\Lambda_i + \sigma^2\right)$$

=> computation in $O(N^2)$

Reason for bias on β and σ

Well known statistical result: the estimator of the variance is biased

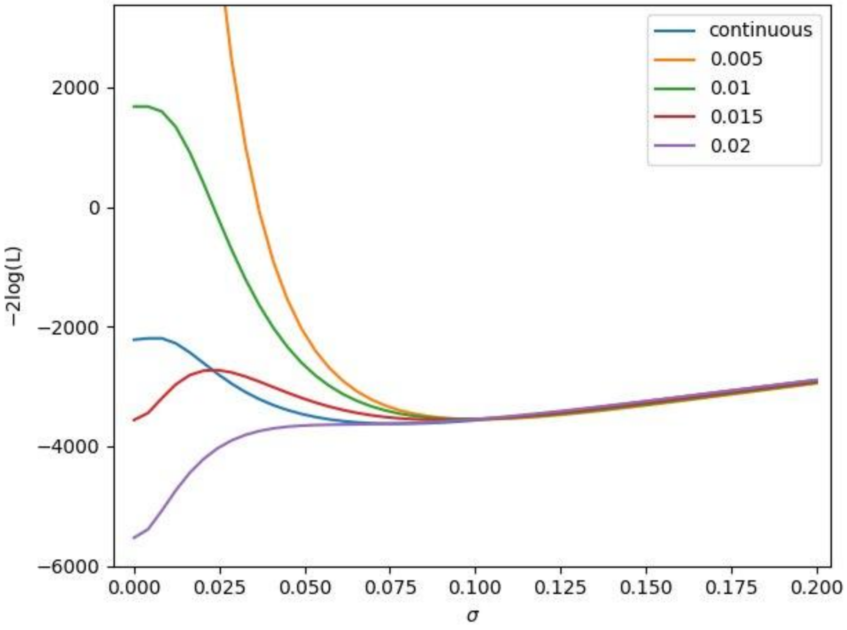
$$\mathbb{E}\left(\hat{\sigma^2}\right) = \frac{N-k}{N}\sigma^2$$

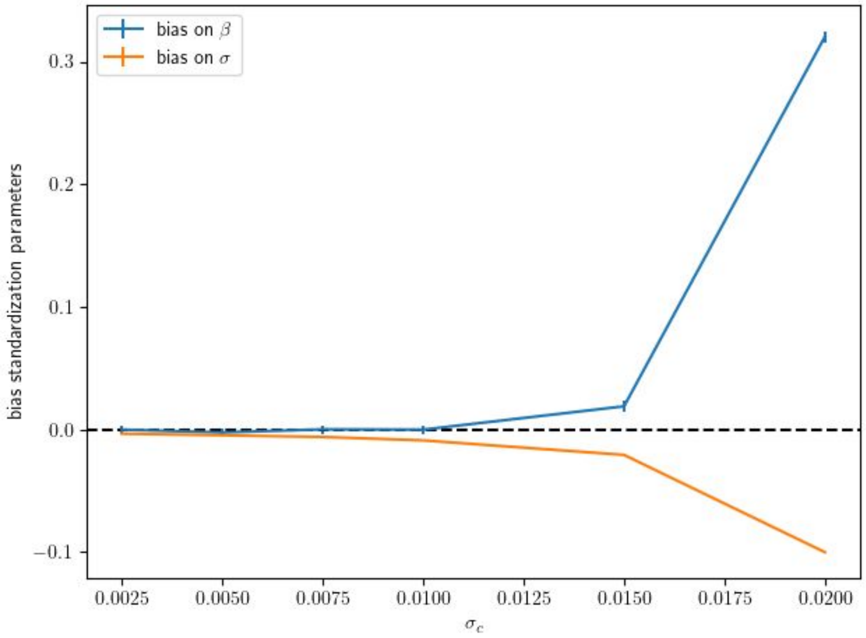
N: number of data
k: number of parameters

When the color is not well constrained ($\beta\sigma_c \sim \sigma$): no effect on distances as long as there is no selection effect

But intrinsic dispersion is absorbed in color: β increases & σ is estimated to 0

Bias of the estimator: likelihood profile





Similar effect when we increase error on the stretch

Which regime for a realistic covariance matrix ?

- Mock of the ZTF and SNLS5 surveys with *skysurvey*

  https://github.com/MickaelRigault/skysurvey

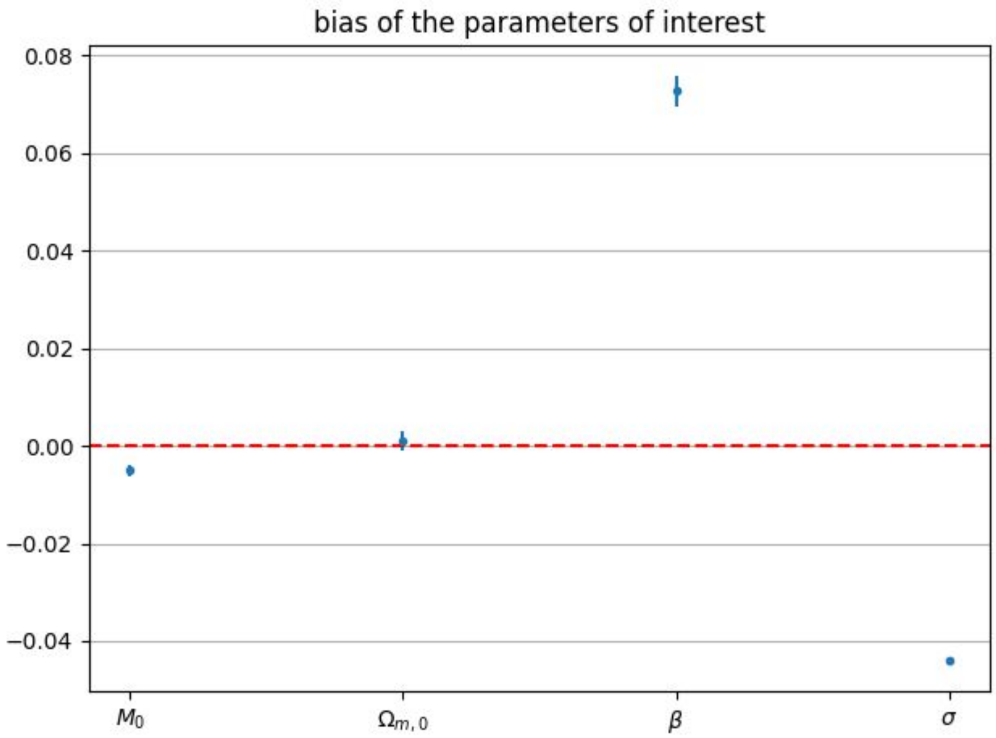
- Cov(m,c) obtained through a variation of SALT1D

  see François HAZENBERG thesis, 2019

- Seem to be in an intermediate regime

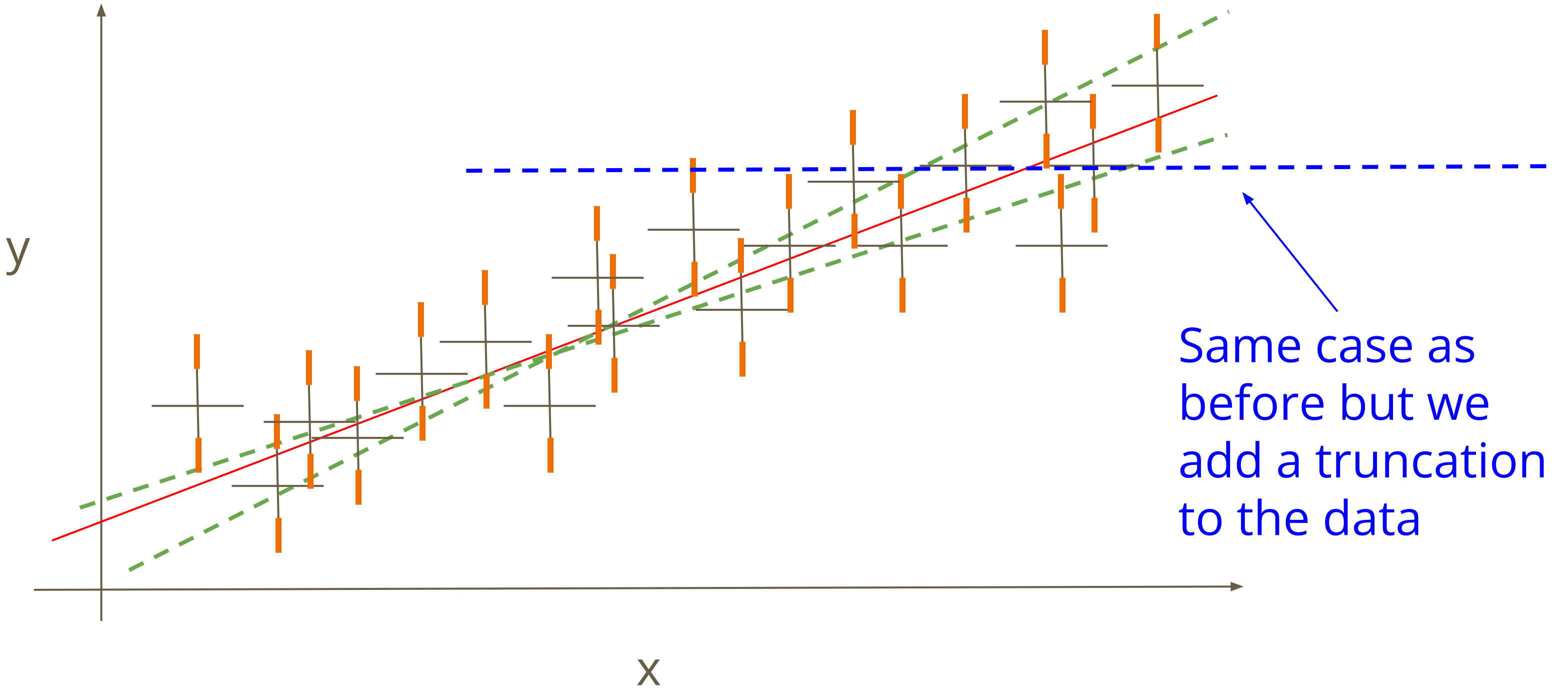
- Working on ReMLE implementation

  strongly inspired by Harville, 1977



3. Estimation of distances for truncated SNIa surveys

Illustration of the issue: generic linear case



Same case as before but we add a truncation to the data

Malmquist bias on a toy model

$$m_i^\star = M^\star + \mu_i + \epsilon_i \text{ with } \epsilon_i \sim \mathcal{N}(0,\, \sigma^2)$$

Truncation

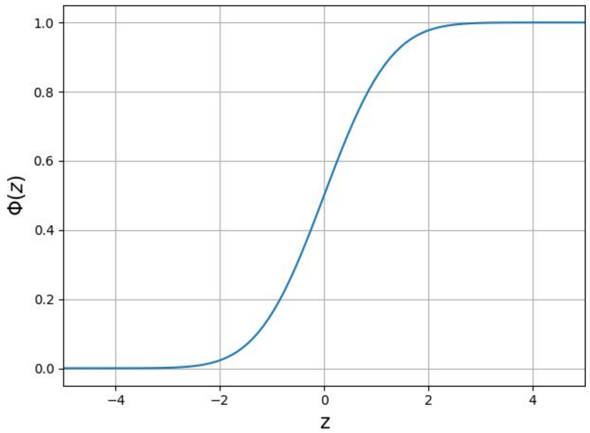
$$m_i = m_i^\star \text{ if } m_i^\star \leqslant m_{lim} + \kappa_i \text{ with } \kappa_i \sim \mathcal{N}(0, \sigma_d^2)$$

$$m_i \text{ is unobserved otherwise}$$

$$\Gamma = \sum_i 2\ln(\sigma) + \frac{1}{\sigma^2} r^\dagger r + 2\ln\left(\Phi\left(\frac{m_{lim} - M^\star - \mu_i}{\sqrt{\sigma_d^2 + \sigma^2}}\right)\right) - 2\ln\left(\Phi\left(\frac{m_{lim} - m_i}{\sigma_d}\right)\right)$$

Instrument related

Fluctuations of observation conditions



$$\Phi(z) = \frac{1}{2}\left(1 + \text{erf}\left(\frac{z}{\sqrt{2}}\right)\right)$$

More realistic model

Addition of standardization & covariance

$$\begin{pmatrix} m_i^\star \\ Y_{1,i}^\star \\ Y_{2,i}^\star \\ \vdots \\ Y_{n,i}^\star \end{pmatrix} = \begin{pmatrix} \mu_i(z,\theta) \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 & \cdots & \alpha_n \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \vdots & \vdots & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_{1,i}^\star \\ X_{2,i}^\star \\ X_{3,i}^\star \\ \vdots \\ X_{n,i}^\star \end{pmatrix} + \begin{pmatrix} \epsilon_i \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad \epsilon_i \sim \mathcal{N}(0,\sigma^2)$$

Fluctuations

$$Y_i = Y_i^\star + \eta_i \text{ with } \eta \sim \mathcal{N}(0, Cov(m,Y)) \text{ if } m_i^\star \leqslant m_{lim} + \kappa_i \text{ with } \kappa_i \sim \mathcal{N}(0,\sigma_d^2)$$

Y_i is unobserved otherwise

Associated negative log-likelihood function

$$\Gamma = \Gamma_{MLE} + \sum_i \left[2\ln\left(\Phi\left(\frac{m_{lim} - \mu_i - \alpha_1 X^\star_{1,i} - \cdots - \alpha_n X^\star_{n,i}}{\sqrt{\sigma_d^2 + \sigma^2}}\right)\right) - 2\ln\left(\Phi\left(\frac{m_{lim} - m_i}{\sqrt{\sigma_d^2 + f(C_i)}}\right)\right)\right]$$

Takes into account the truncation effects

$$\Phi(z) = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{z}{\sqrt{2}}\right)\right)$$

Parameters: $\begin{pmatrix} \theta \\ \alpha \\ X^\star \\ \sigma \\ m_{lim} \\ \sigma_d \end{pmatrix}$

Conclusion

What is missing ?

1. Remove standardization coefficients & intrinsic dispersion bias with ReMLE (non bias intrinsic dispersion is needed to accurately measure parameters of the selection function)
2. Add the ReMLE term to the estimator with truncation term: estimation of the selection function => validation of the complete estimator with Monte-Carlo simulations
3. Confront full estimator with:
 - deviation from initial hypothesis (more complex selection functions, …)
 - sources of systematics

Goal: methodology paper for DR2.5

Backup slides

Description of the SALT1D model

Inspired by François HAZENBERG thesis, 2019

For a supernovae in a specific band b:

$$m_b = m_B - 2.5\log_{10}(1+z) + P\left(\frac{\overline{\lambda_b} + \delta\lambda_b}{1+z}\right) + cQ\left(\frac{\overline{\lambda_b} + \delta\lambda_b}{1+z}\right) + Z_b + \delta Z_b$$

$$Cov(m_B, c, P, Q, \delta\lambda, \delta Z) = \left(\frac{1}{2}H_\Gamma\right)^{-1}$$

P: mean spectra (cubic BSpline), Q: color law (degree 3 polynomial function)