Studying real-time properties of gauge theories

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Subatech, Nantes, January 01, 2024

Talk based on (selection):

Complex Langevin: KB, Hotzy, Müller, 2212.08602, 2312.03063
$\hat{q}$, κ: KB, Kurkela, Lappi, Lindenbauer, Peuron, 2303.12595, 2303.12520

TECHNISCHE UNIVERSITÄT WIEN

INSTITUTE FOR THEORETICAL PHYSICS

FWF Der Wissenschaftsfonds.

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1. Motivation
2. Spectral functions in the quark-gluon plasma
3. Transport coefficients
4. Towards simulations of real-time lattice gauge theory
5. Conclusion

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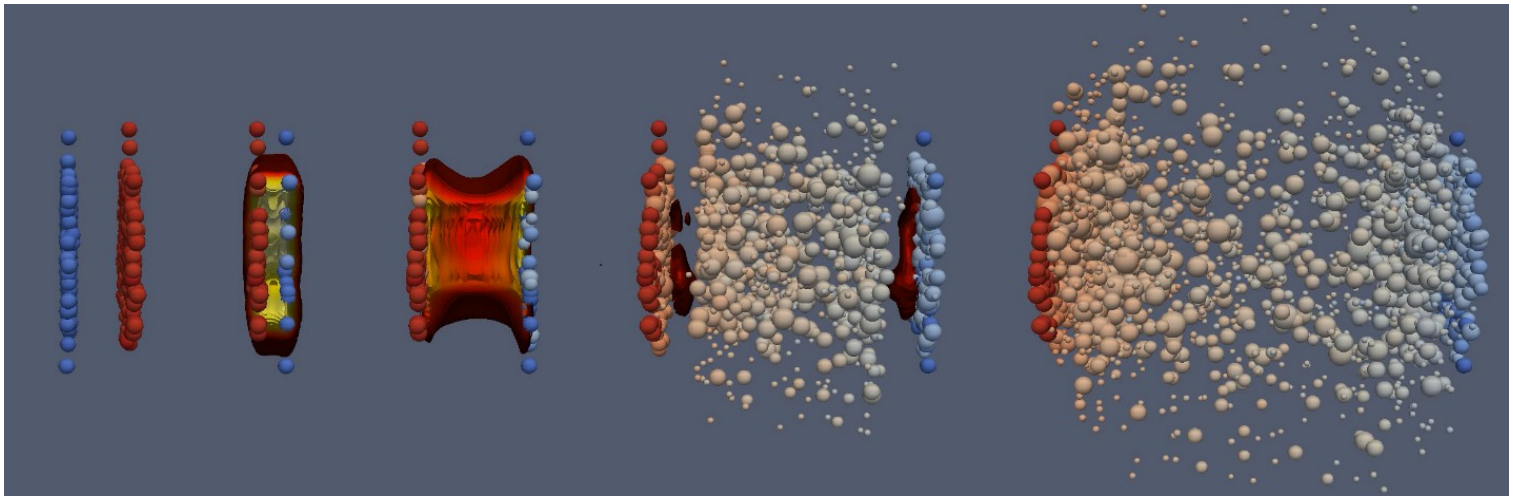
Stages in heavy-ion collisions



MADAI collaboration

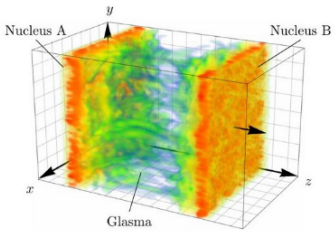
- High-energy collisions $\Rightarrow$ **Quark-Gluon plasma** (QGP) created
- Cooling during evolution, go through different **phases**
  $\Rightarrow$ pre-equilibrium QGP $\rightarrow$ fluid QGP $\rightarrow$ hadrons
- At early times, **pre-equilibrium QGP** phase (initial stages) involves large gluon fields $A \sim 1/g$, then quasiparticle excitations
  $\Rightarrow$ classical-statistical simulations $\Rightarrow$ QCD kinetic theory

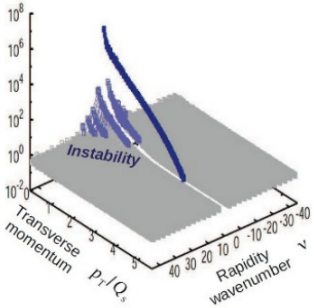
Goal

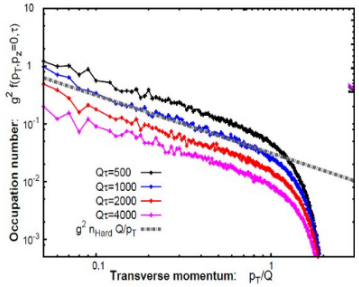
Learn about real-time properties of QCD

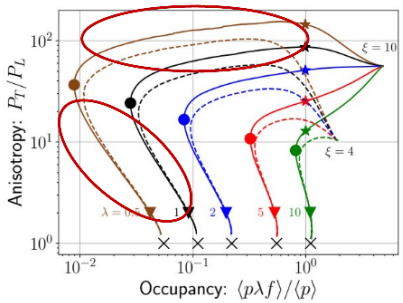
Initial stages in heavy-ion collisions (weak-g^2 perspective)

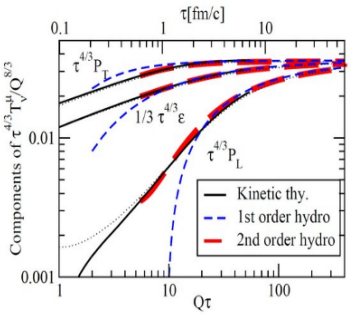
$Q_s\tau = 0^+$ | $Q_s\tau = \ln^2(\alpha_s^{-1})$ | $Q_s\tau = \alpha_s^{-3/2}$ | $Q_s\tau = \alpha_s^{-13/5}$











Glasma	Instabilities	Bottom-up stage 1	Bottom-up stages 2 and 3	Onset of hydrodynamics

$$\underbrace{D_\mu F^{\mu\nu} = J^\nu}_{\text{classical-statistical simulations}}$$

$$\underbrace{-\frac{\partial f_{\vec{p}}}{\partial \tau} = \mathcal{C}^{1\leftrightarrow 2}[f_{\vec{p}}] + \mathcal{C}^{2\leftrightarrow 2}[f_{\vec{p}}] - \frac{p_z}{\tau}\frac{\partial f_{\vec{p}}}{\partial p_z}}_{\text{QCD effective kinetic theory simulations}}$$

$$\underbrace{\qquad}_{\text{hydrodynamics ...}}$$

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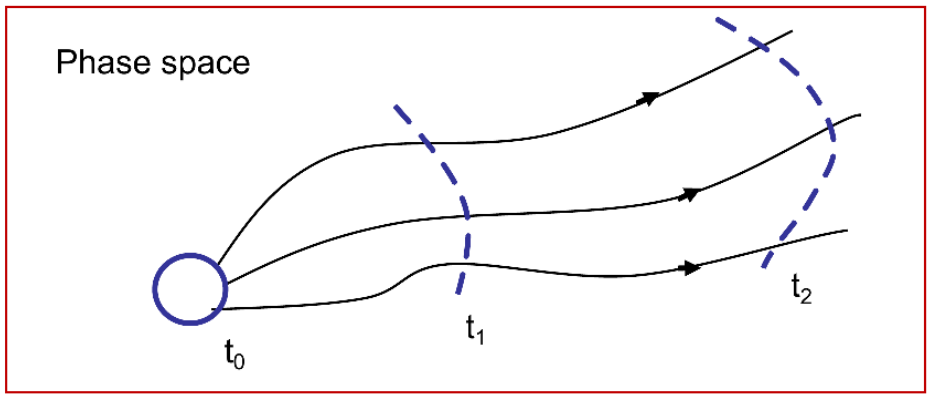
Classical-statistical simulations

- At initial time t_0 set (quantum) initial conditions (IC):

$\Rightarrow$ Choose $\langle AA \rangle$, $\langle EE \rangle$ in x or p space

- Approximate quantum dynamics with classical EOMs $D_\mu F^{\mu\nu} = 0$
  - $\Rightarrow$ Gauge co-variant lattice formulation using links $U_j(x) = e^{ig\, a_j\, A_j(x)}$
- Obtain observables at t by averaging over trajectories (same IC)

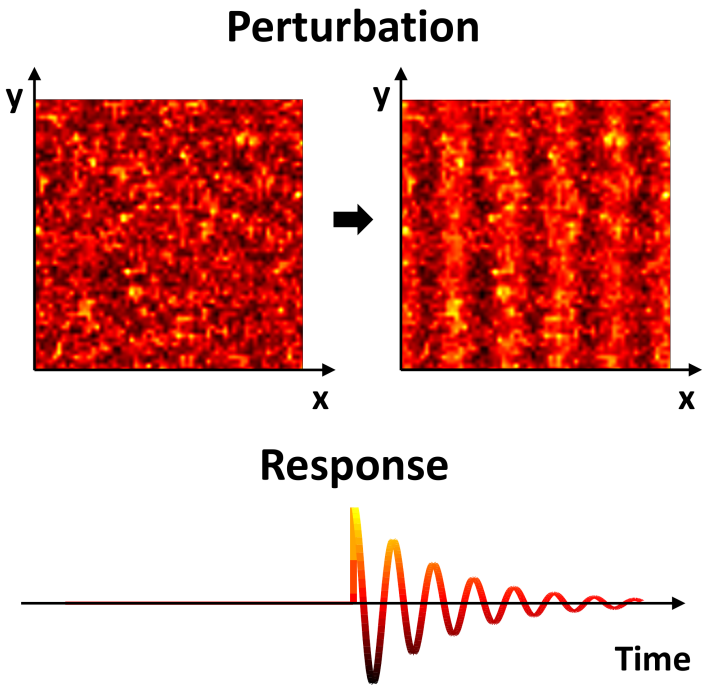


- Applicability limited to earliest times when $f(t, \vec{p}) \sim \langle EE \rangle / p \gg 1$
- Also kinetic theory, hydro, AdS/QCD are based on approximations

Quasiparticles? Extract gluon spectral function ρ

Classical-statistical $\mathrm{SU}(N_c)$ simulations + linear response theory

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*



- Similar algorithm for fermions
- Split $A(t,\vec{x}) \mapsto A(t,\vec{x}) + \delta A(t,\vec{x})$ at t, perturb with plane wave $j_0(\vec{p})\,\delta\,(t'-t)$
- Response $\langle \delta A(t',\vec{p}) \rangle = G_R(t',t,\vec{p})\, j_0(\vec{p})$
- Linearized EOM for $\delta A(t,\vec{x})$ such that Gauss law conserved (also in gauge-cov. formulation)

  Kurkela, Lappi, Peuron, *EUJC 76 (2016) 688*
- $\theta(t'-t)\,\boxed{\rho(t',t,p)} = G_R(t',t,p)$
- Fourier transform $\rho(\bar{t},\omega,p)$ $\quad (\bar{t} = \frac{1}{2}(t+t'))$

Very similar methods for scalars:

Aarts (2001); Piñeiro Orioli, Berges (2019); Schlichting, Smith, von Smekal (2020); KB, Piñeiro Orioli (2020); ...

What excitations drive the dynamics in the QGP?

Study microscopics of the Quark-Gluon plasma

Quasiparticles? Fluct.-diss. relation (FDR)? When is kinetic theory valid?

- Spectral function $\dot{\rho}(t, \omega, p) \sim \langle [\hat{A}, \hat{E}] \rangle$ encodes excitation spectrum!
- Statistical correlator $\ddot{F}(t, \omega, p) \sim \langle \{\hat{E}, \hat{E}\} \rangle$ appears related

What excitations drive the dynamics in the QGP?

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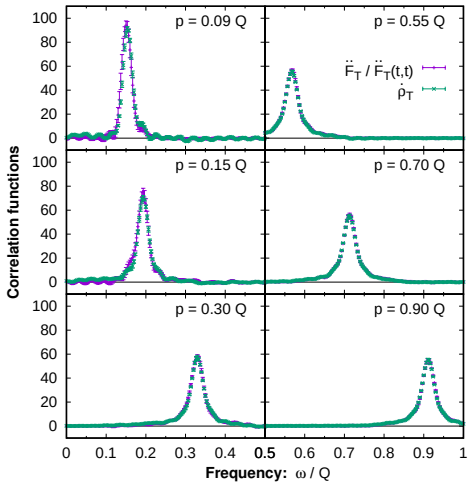
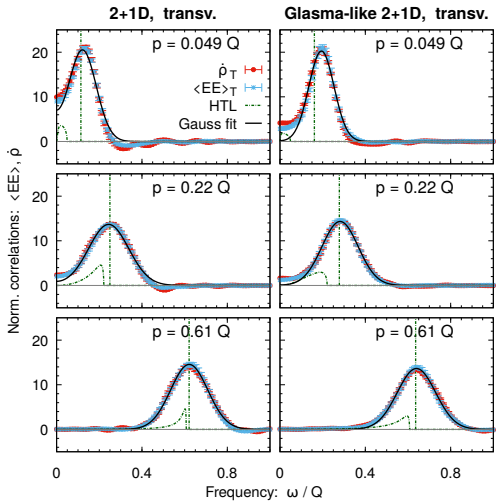
Quasiparticles? Fluct.-diss. relation (FDR)? When is kinetic theory valid?

- Spectral function $\dot{\rho}(t,\omega,p) \sim \langle[\hat{A},\hat{E}]\rangle$ encodes excitation spectrum!
- Statistical correlator $\ddot{F}(t,\omega,p) \sim \langle\{\hat{E},\hat{E}\}\rangle$ appears related

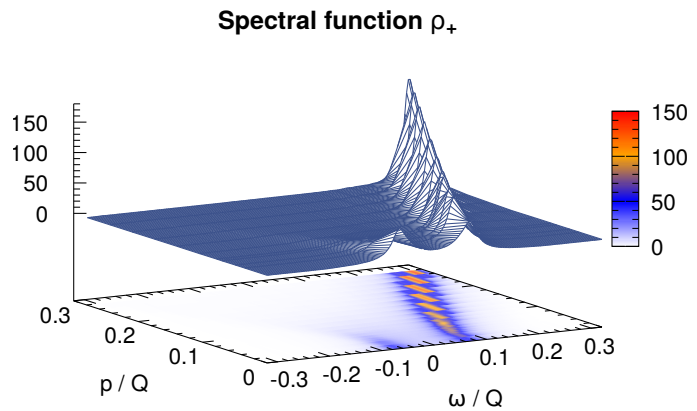
  $\Rightarrow$ generalized FDR: $\ddot{F}(t,\omega,p)/\ddot{F}(t,t,p) \approx \dot{\rho}(t,\omega,p)$



Gluonic 3+1D



Gluonic 2+1D



Fermionic 3+1D

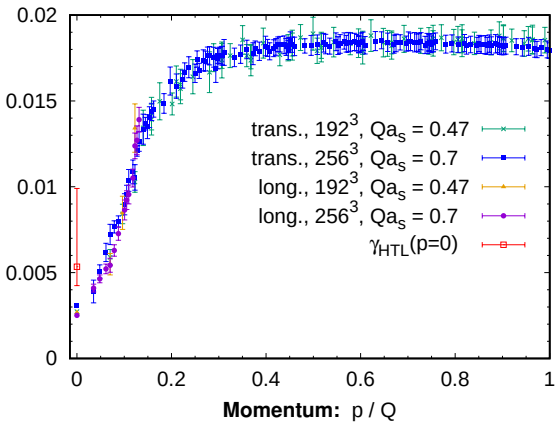
KB, Kurkela, Lappi, Peuron, PRD 98 (2018) 014006; PRD 100 (2019) 094022; JHEP 05 (2021) 225

KB, Lappi, Mace, Schlichting, PLB 827 (2022) 136963

Spectral functions: width of excitations (inverse lifetimes)

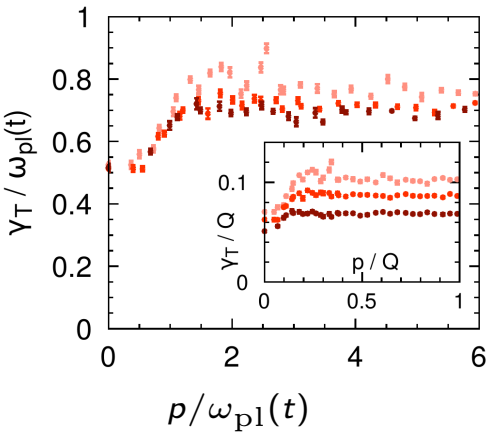
Examples: First principles' determination of damping rates $\gamma(t, p)$ in isotropic highly-occupied QGP (as on a slide before)

Gluonic 3+1D



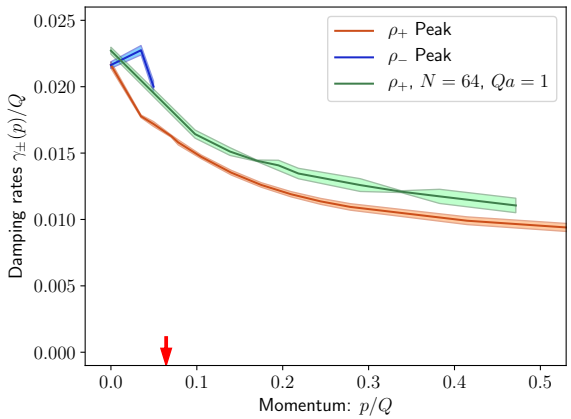
$\gamma \ll \omega_{\rm pl}$, increasing in p

Gluonic 2+1D



$\gamma \sim \omega_{\rm pl}$

Fermionic 3+1D



$\gamma \ll \omega_{\rm pl}$, decreasing in p

$\Rightarrow$ 3+1D: well described by perturbation theory (except fermions low p)

$\Rightarrow$ gluonic 2+1D: no quasiparticles for $p \lesssim \omega_{\rm pl}$; Anisotropic kinetics?

$\Rightarrow$ HTL / kinetic theory in 2+1D may require nonperturbative IR

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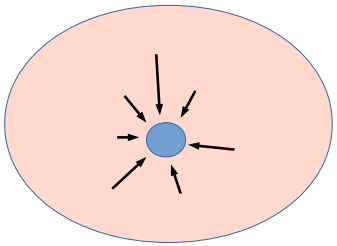
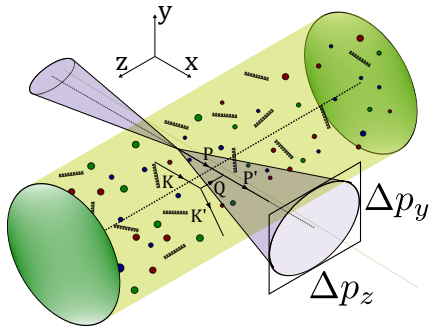
Transport coefficients κ, $\hat{q}$ from pre-equilibrium QGP

Jets, heavy quarks: potential for signatures of initial stages

medium interactions $\Rightarrow$ QGP properties encoded in observables

- Quarks/jets get 'kicks' $\dot{p}_i(\tau) = \mathcal{F}_i(\tau)$
- Heavy-quark diffusion coefficient $\kappa_i = \frac{\mathrm{d}}{\mathrm{d}\tau}\langle p_i^2 \rangle$
  $\Rightarrow$ heavy quark has small momentum $p \ll M$
- κ enters Lindblad eq. for quarkonium dynamics
  Brambilla, Escobedo, [Soto], [Strickland], Vairo, [v.d. Griend, Weber] (2016, 2021)
- Jet quenching parameter $\hat{q}_i = \frac{\mathrm{d}}{\mathrm{d}\tau}\langle p_{\perp,i}^2 \rangle$
  $\Rightarrow$ jet with high momentum $p \gg Q_s, T$
- Pre-equilibrium dynamics encoded in heavy-quark diffusion, quarkonia and jet quenching





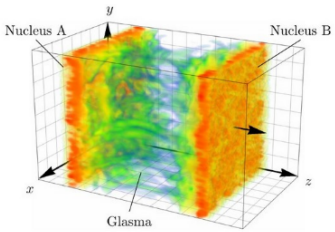
KB, Kurkela, Lappi, Lindenbauer, Peuron (2023)

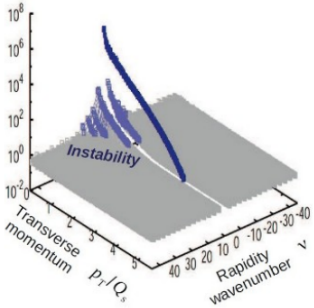
Mrowczynski (2018); Ruggieri, Das (2018); Sun, Coci, Das, Plumari, Ruggieri, Greco (2019); Ipp, Müller, Schuh (2020); KB, Kurkela, Lappi, Peuron (2020); Khowal, Das, Oliva, Ruggieri (2022); Carrington, Czajka, Mrowczynski (2020, 2022); Barata, Sadofyev, Wang (2023); Andres, Apolinário, Dominguez, Martinez, Salgado (2023); Avramescu, Baran, Greco, Ipp, Müller, Ruggieri (2023); KB, Kurkela, Lappi, Lindenbauer, Peuron (2023); Du (2023), ...

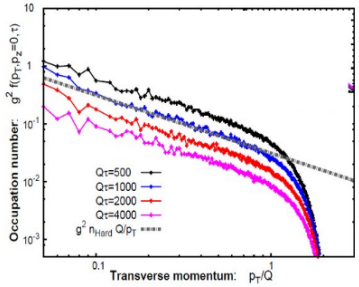
Initial stages in heavy-ion collisions (weak-g^2)

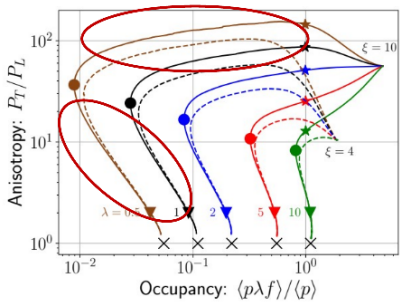
Reminder

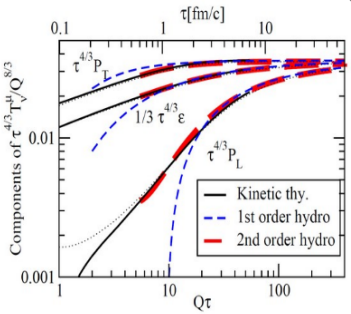
$Q_s\tau = 0^+$ | $Q_s\tau = \ln^2(\alpha_s^{-1})$ | $Q_s\tau = \alpha_s^{-3/2}$ | $Q_s\tau = \alpha_s^{-13/5}$











Glasma	Instabilities	Bottom-up stage 1	Bottom-up stages 2 and 3	Onset of hydrodynamics

$$D_\mu F^{\mu\nu} = J^\nu$$

classical-statistical simulations

$$-\frac{\partial f_{\vec{p}}}{\partial \tau} = \mathcal{C}^{1\leftrightarrow 2}[f_{\vec{p}}] + \mathcal{C}^{2\leftrightarrow 2}[f_{\vec{p}}] - \frac{p_z}{\tau}\frac{\partial f_{\vec{p}}}{\partial p_z}$$

QCD effective kinetic theory simulations

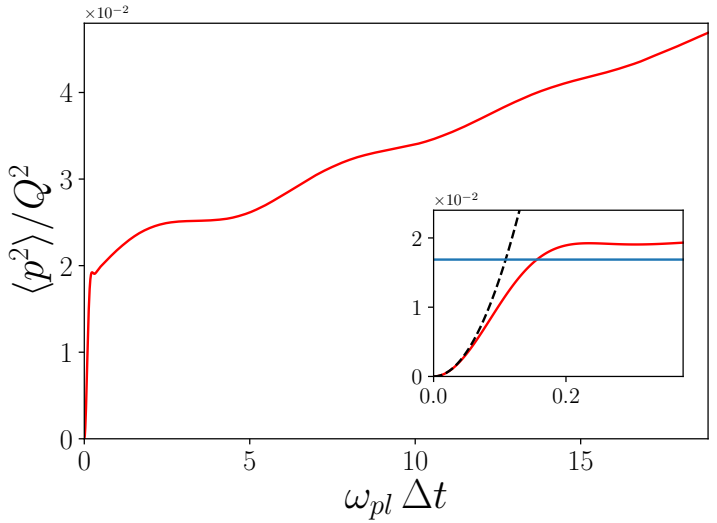
hydrodynamics ...

Heavy quark diffusion in a Glasma model

KB, Kurkela, Lappi, Peuron, JHEP 09, 077 (2020)

$$\langle p^2(t,\Delta t)\rangle = \frac{g^2}{2N_c}\int_t^{t+\Delta t} \mathrm{d}t' \int_t^{t+\Delta t} \mathrm{d}t''\, \langle EE\rangle(t',t'')$$

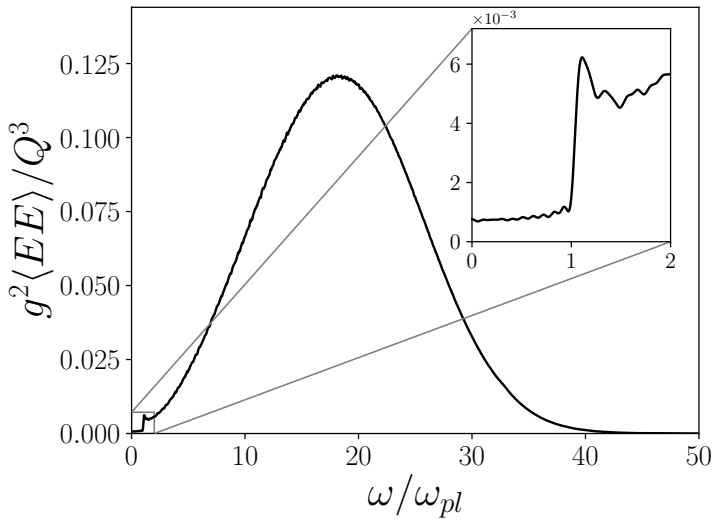
$$3\kappa\,(t,\Delta t) = \frac{\mathrm{d}}{\mathrm{d}\Delta t}\langle p^2(t,\Delta t)\rangle \approx \frac{g^2}{N_c}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\,\frac{\sin(\omega\,\Delta t)}{\omega}\,\langle EE\rangle(t,\omega)$$



Three features $\Rightarrow$ explained by $\langle EE\rangle$ or ρ

- quadratic initial growth
- oscillations from quasiparticles
- diffusion from Landau damping

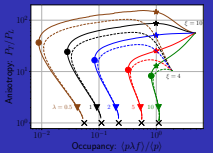
$\Rightarrow$ Knowledge of correlators essential!



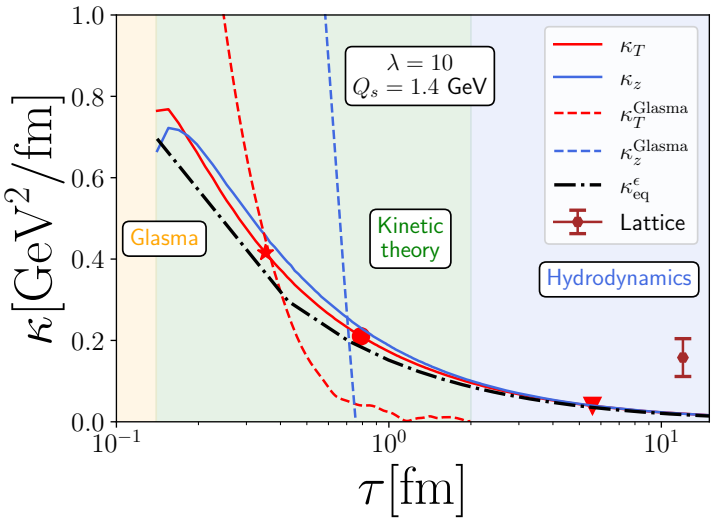
Also in the Glasma: large κ and $\kappa_z > \kappa_T$

Ipp, Müller, Schuh (2020); Carrington, Czajka, Mrowczynski (2022); Khowal, Das, Oliva, Ruggieri (2022); Avramescu et al. (2023)

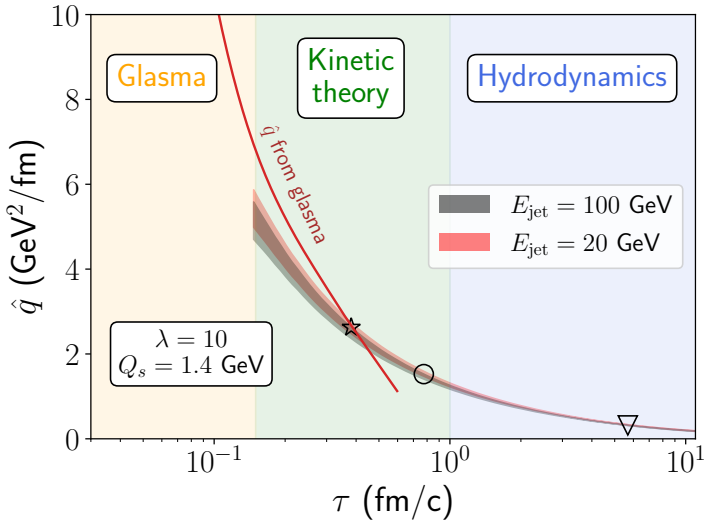
κ and $\hat{q}$ during bottom-up (kinetic evolution)



κ of heavy quarks



$\hat{q}$ of jets



KB, Kurkela, Lappi, Lindenbauer, Peuron; for κ [2303.12520], for $\hat{q}$ [2303.12595], [2312.00447], limiting attractors for κ & $\hat{q}$ [2312.11252]

- Close gap in κ and $\hat{q}$ between Glasma and hydro via **kinetic theory**
  - $\hat{q}$ smoothly connects Glasma and hydro, little sensitivity to details
  - κ_i less smooth (esp. κ_z), estimate $\langle p^2 \rangle = \int d\tau\, 3\kappa(\tau) \sim 1$ GeV^2
  - Mostly the same ordering $\kappa_z > \kappa_T$ and $\hat{q}_z > \hat{q}_y$
    - $\Rightarrow$ Observable via jet polarisation? (Hauksson, Iancu (2023))
    - $\Rightarrow$ Impact on jet energy loss?
    - $\Rightarrow$ Pheno. impact of $\kappa_i(\tau)$ on heavy quarks/quarkonia?

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Complex Langevin: towards ab-initio framework for QCD

This section is based on:

KB, Hotzy, Müller, *JHEP 06, 011 (2023) [2212.08602]*
KB, Hotzy, Müller, *2312.03063*


P. Hotzy


D.I. Müller

- Spectral functions, transport coefficients, viscosities are based on **real-time correlations**
- Such observables difficult to compute in **Lattice QCD** (Euclidean)
- **No general ab-initio** pre-equilibrium framework for QCD exists
- Existing methods (classical-statistical, kinetic theory, hydrodynamics) have **limited applicability**

Can we develop an ab-initio pre-equilibrium framework for QCD?

NP-hard sign problem due to complex action!

Observables in the real-time formalism

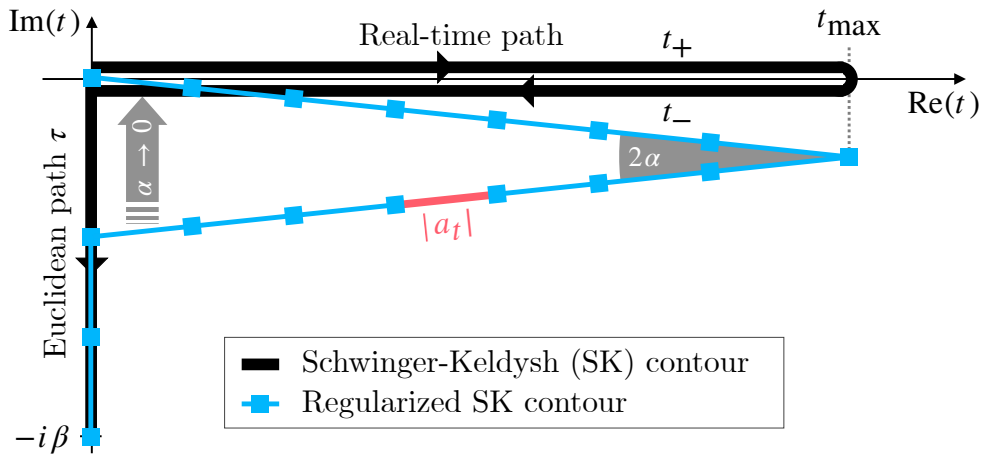
- **Path integral expression** for expectation values:

$$\langle \mathscr{O}[A] \rangle = \frac{1}{Z} \int \mathcal{D}A_E \, e^{-S_E[A_E]} \int \mathcal{D}A_+ \, \mathcal{D}A_- \, e^{iS[A_+,A_-]} \, \mathcal{O}[A_+, A_-, A_E]$$

- **Yang-Mills action:**

$$S_{\mathrm{YM}} = -\frac{1}{4} \int_{\mathscr{C}_\pm, \mathscr{C}_E} d^4x F_a^{\mu\nu} F^a_{\mu\nu}$$

- **Correlation functions of $T^{\mu\nu}$:**
  - speed of sound
  - bulk- and shear-viscosity
  - spectral functions



First: (Real) Langevin method

- **Langevin** method for Euclidean Yang-Mills theory

$$\partial_\theta A^a_\mu(\theta, x) = -\frac{\delta S_E}{\delta A^a_\mu(\theta, x)} + \eta^a_\mu(\theta, x),$$
$$\langle \eta \rangle = 0, \quad \langle \eta(\theta)\eta(\theta') \rangle \propto 2\delta(\theta - \theta').$$

- Equivalent to **Fokker-Planck equation** via Ito's Lemma

$$\partial_\theta P(A, \theta) = \delta_A \left(\delta_A + \delta_A S_E\right) P(A, \theta)$$

- Converges to **stationary** solution $P(A, \theta \to \infty) = \frac{1}{Z} \exp\left[-S_{\mathrm{E}}[A]\right]$.

Expectation value by sampling at late θ

$$\langle \mathcal{O}[A] \rangle == \frac{1}{Z} \int \mathcal{D}A_E \, e^{-S_E[A_E]} \mathcal{O}[A(\theta)] \approx \lim_{\theta_0 \to \infty} \frac{1}{T} \int_{\theta_0}^{\theta_0 + T} d\theta \, \mathcal{O}[A(\theta)]$$

Now: Complex Langevin for real-time Yang-Mills theory

- **Complex Langevin (CL)** method for Yang-Mills theory

$$\frac{\partial A^a_\mu(\theta,x)}{\partial\theta} = i\frac{\delta S_{\rm YM}}{\delta A^a_\mu(\theta,x)} + \eta^a_\mu(\theta,x),$$

$$\langle\eta^a_\mu(\theta,x)\rangle = 0, \quad \langle\eta^a_\mu(\theta,x)\eta^b_\nu(\theta',y)\rangle = 2\delta(\theta-\theta')\delta^{(4)}(x-y)\delta^{ab}\delta_{\mu\nu}.$$

- **Complexification** of Lie algebra: $\mathrm{SU}(N) \to \mathrm{SL}(N,\mathbb{C})$
- If process converges correctly, $A^a_\mu(\theta\to\infty,x)$ is described by

$$\rho[A] = \mathcal{N}\exp\left[iS_{\rm YM}[A]\right]$$

CL bypasses the sign problem by sampling at late θ

$$\langle\mathcal{O}[A]\rangle = \int\mathcal{D}A\,\rho[A]\mathcal{O}[A] \approx \lim_{\theta_0\to\infty}\frac{1}{T}\int_{\theta_0}^{\theta_0+T} d\theta\,\mathcal{O}[A(\theta)]$$

Complex Langevin on the lattice

Technical details

- Link variables and plaquette variables

$$U_{x,\mu} \simeq \exp\left[iga_\mu A_\mu\left(x+\hat{\mu}/2\right)\right] \in \mathrm{SU}(N_c) \rightsquigarrow \mathrm{SL}(N_c,\mathbb{C}),$$
$$U_{x,\mu\nu} = U_{x,\mu}U_{x+\hat{\mu},\nu}U^{-1}_{x+\hat{\nu},\mu}U^{-1}_{x,\nu}$$

- Wilson plaquette action: $S_{\mathrm{W}}[U] = \frac{1}{2N_c}\sum_{x,\mu\neq\nu}\beta_{\mu\nu}\mathrm{Tr}\left[U_{x,\mu\nu}-1\right]$
- Coupling constants: $\beta_{0i} = -\frac{2N_c}{g^2}\frac{a_s}{a_{t,k}}, \quad \beta_{ij} = +\frac{2N_c}{g^2}\frac{\bar{a}_{t,k}}{a_s}$
- Averaged lattice spacing (time reversability): $\bar{a}_{t,k} = (a_{t,k}+a_{t,k+1})/2$

Update step for link variables

$$U_{x,\mu}(\theta+\epsilon) = \exp\left[it^a\left(i\Gamma_\mu\epsilon\, D^a_{x,\mu}S_{\mathrm{W}} + \sqrt{\Gamma_\mu\epsilon}\,\eta^a_{x,\mu}(\theta)\right)\right]U_{x,\mu}(\theta)$$

$\Rightarrow$ Field independent kernel function Γ_μ leaves stationary solution intact but can be utilised to **improve convergence and stability!**

New anisotropic kernel

- CL suffers from two types of **instabilities:** (Aarts, James, Seiler, Stamatescu (2011))
  - *Runaways*: numerical blowup of the solution
  - *Wrong convergence*: wrong results (assumptions violated)
- Several methods introduced to mitigate these issues (Backup): adaptive step size, gauge cooling, dynamical stabilization, *kernels*, ...

Anisotropic kernel

- We introduce an anisotropic kernel to alleviate instabilities

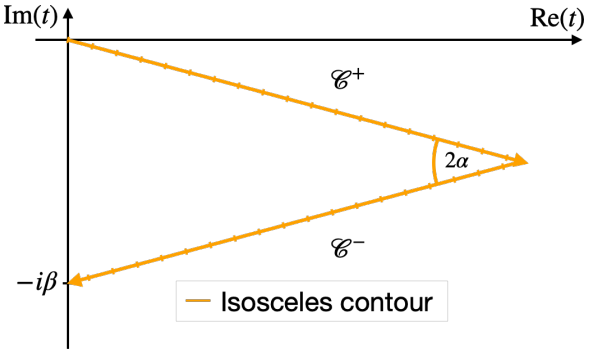
$$\Gamma_t = |a_t|^2/a_s^2, \ \Gamma_s = 1$$

- *Motivation*: Based on a careful rederivation of the CL equation on complex time contours. (Backup)
- Previous studies used $\Gamma_\mu = 1$ (Berges, Borsanyi, Sexty, Stamatescu (2007))

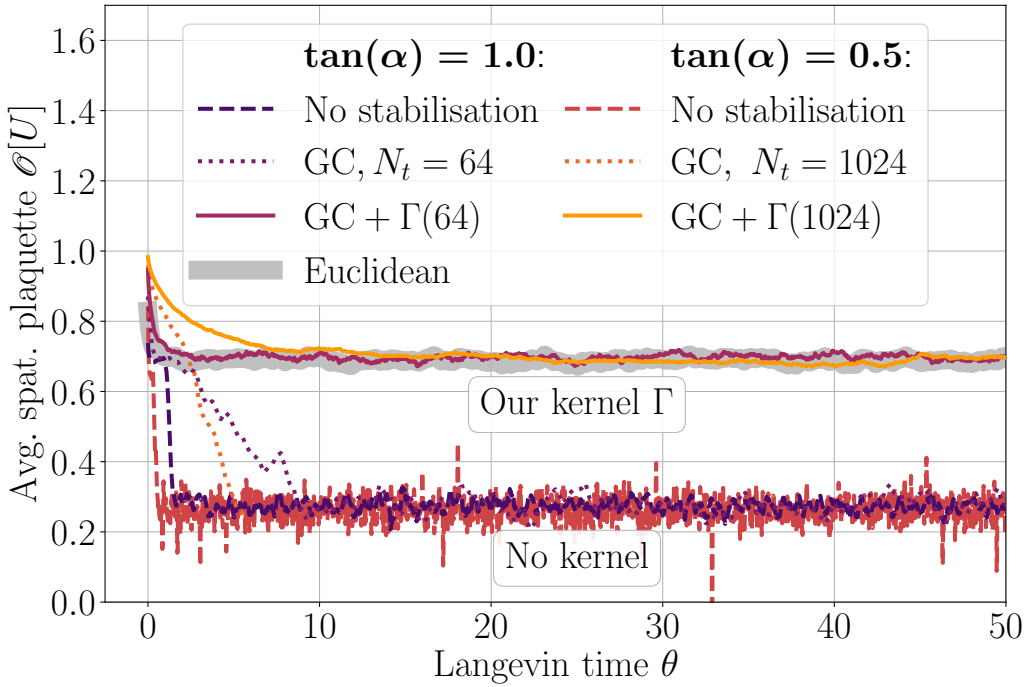
Stabilization (correct convergence) using Γ

Lattice parameters:
$L = N_t \times 4^3$
$g = 1, \beta = 1/T = 4$

- Reproduce SU(2) results of
  (Berges, Borsanyi, Sexty, Stamatescu (2007))
- for $\mathcal{O}[U] = \frac{1}{6 N_c N_x} \sum_{x, i \neq j} \mathrm{Tr}\left[U_{x,ij}\right]$
- on isosceles contours



- Thermal (Euclidean)
  t-independent $\longrightarrow$ **compare**



- We find **metastable θ-region**

☒ Not-stabilised simulation:
Quickly wrong convergence

☑ Stabilised simulation:
Our kernel with GC reproduce Euclidean results. ($N_t|a_t|$ fixed)

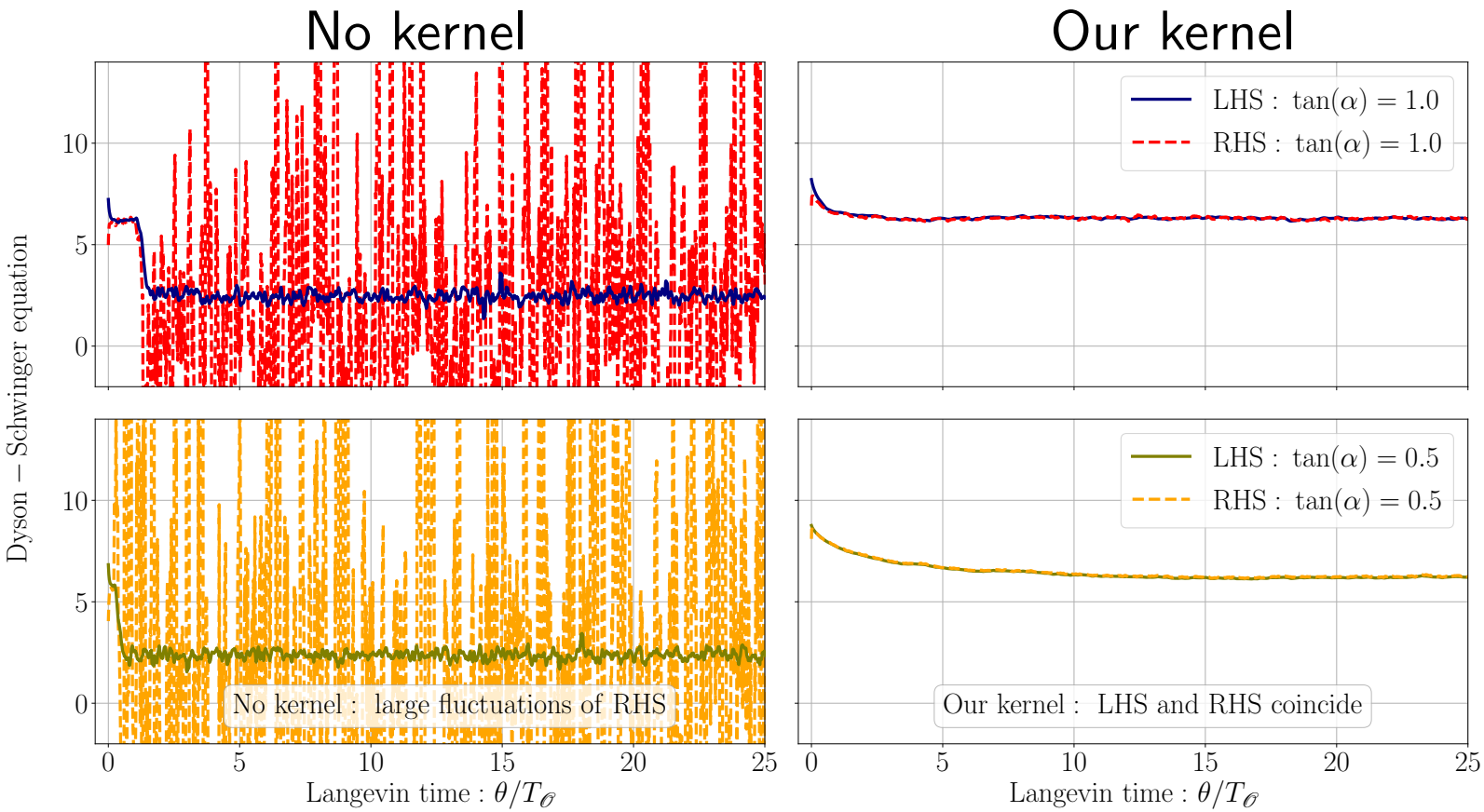
$\Rightarrow$ *Existing methods insufficient, our kernel stabilizes!*

Dyson-Schwinger equations as probe for stability

- Self-consistency check of link configuration

$$\frac{2(N_c^2-1)}{N_c}\,\langle \mathrm{ReTr}(U_{x,ij})\rangle = \frac{i}{2N_c}\sum_{|\rho|\neq i}\beta_{i\rho}\left\langle \mathrm{ReTr}\left[(U_{x,i\rho}-U_{x,i\rho}^{-1})U_{x,ij}\right]\right\rangle$$

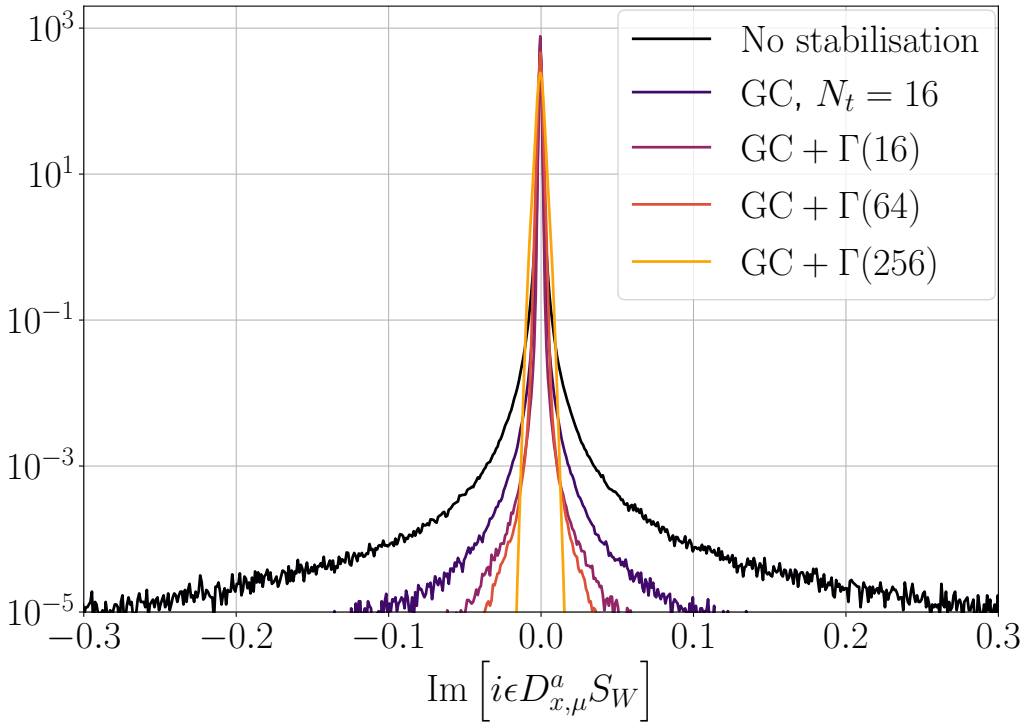
- RHS is very sensitive $\rightarrow$ probe for stability



Localized histogram of the drift terms

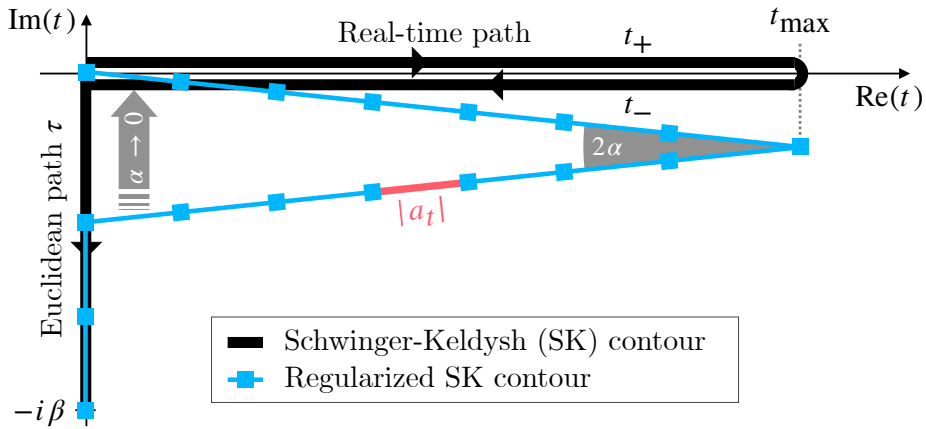
- Localised histograms of $\mathrm{Im}\,[iDS_W]$ indicate **correct convergence**
- **No skirts/tails** using our kernel with sufficiently large N_t
- **Systematic improvement** using finer lattices (larger N_t)

- Gauge cooling helps, but skirts are still present
- Increasing N_t without our kernel does *not* improve the stability



Our simulation approach for real-time Yang Mills theory

- SK-contour needs regularization
  - $\Rightarrow$ we **tilt the SK-contour**
  - $\Rightarrow$ instabilities more severe for smaller tilt angles
- **Kernel alleviates** instabilities
- **Metastability**: Region of correct convergence metastable. Increasing N_t systematically improves life time.
- **Observables** $\langle O \rangle$ calculated by averag. over region, then $\alpha \to 0$



Unequal-time correlator (of B^2, clover leaf) **for first time:**

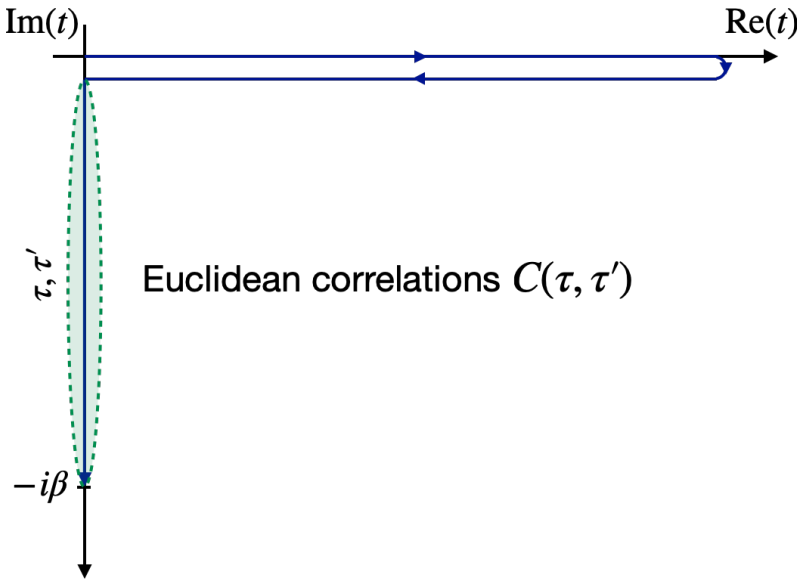
$$C(t,t') = \frac{1}{N_s^3} \sum_x \langle B^2(t,x) B^2(t',x) \rangle - \langle B^2 \rangle^2$$

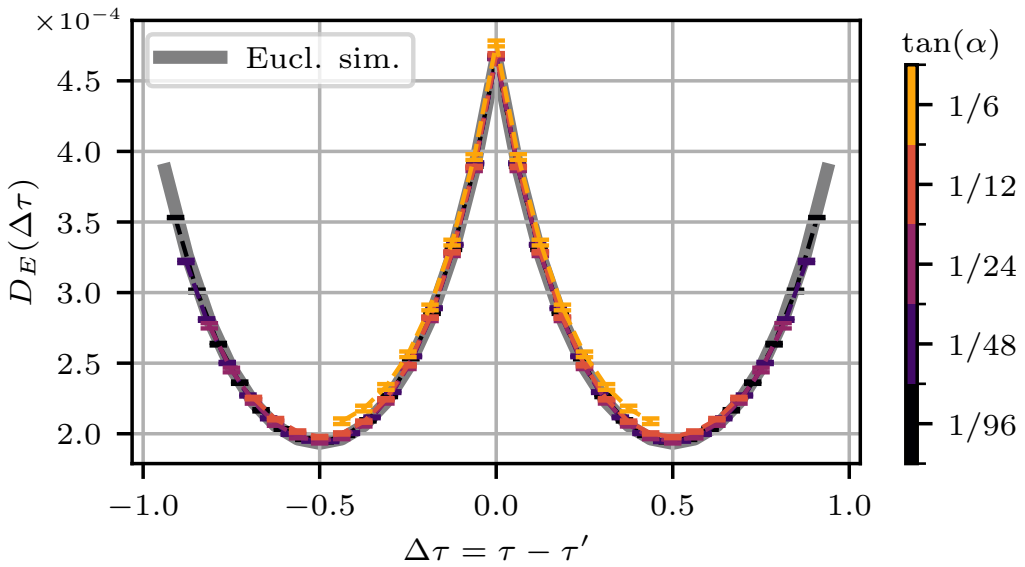
Euclidean correlation function

Lattice parameters:
$L = 64 \times 16^3$
$g = 0.5, \beta = 1/T = 1$

- Euclidean correlation function $C(\tau, \tau')$ along thermal path
- Coinciding with correlation function from Euclidean simulation

$\Rightarrow$ **Independence** of tilt angle α

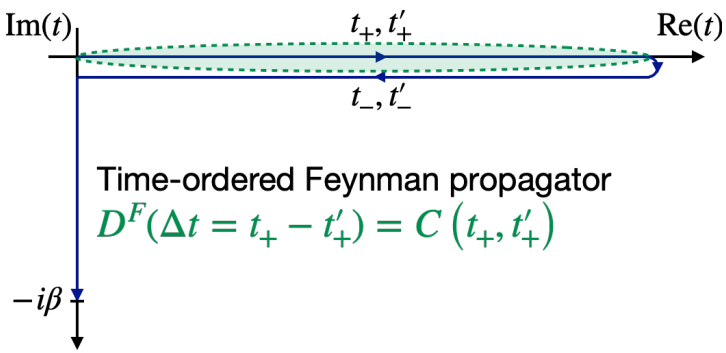




Limit towards Schwinger-Keldysh-contour $\alpha \to 0$

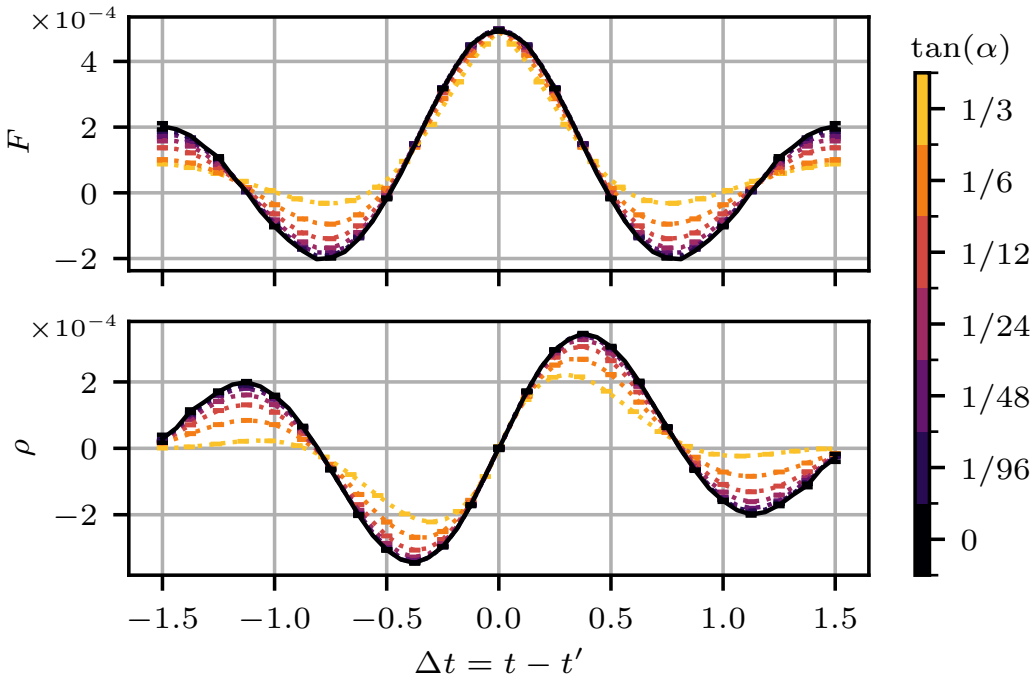
- Decreasing tilt angle leads to "converging" Feynman propagator D^F.

$\Rightarrow$ **Justification** for extrapolation $\alpha \to 0$.



Statistical & spectral functions

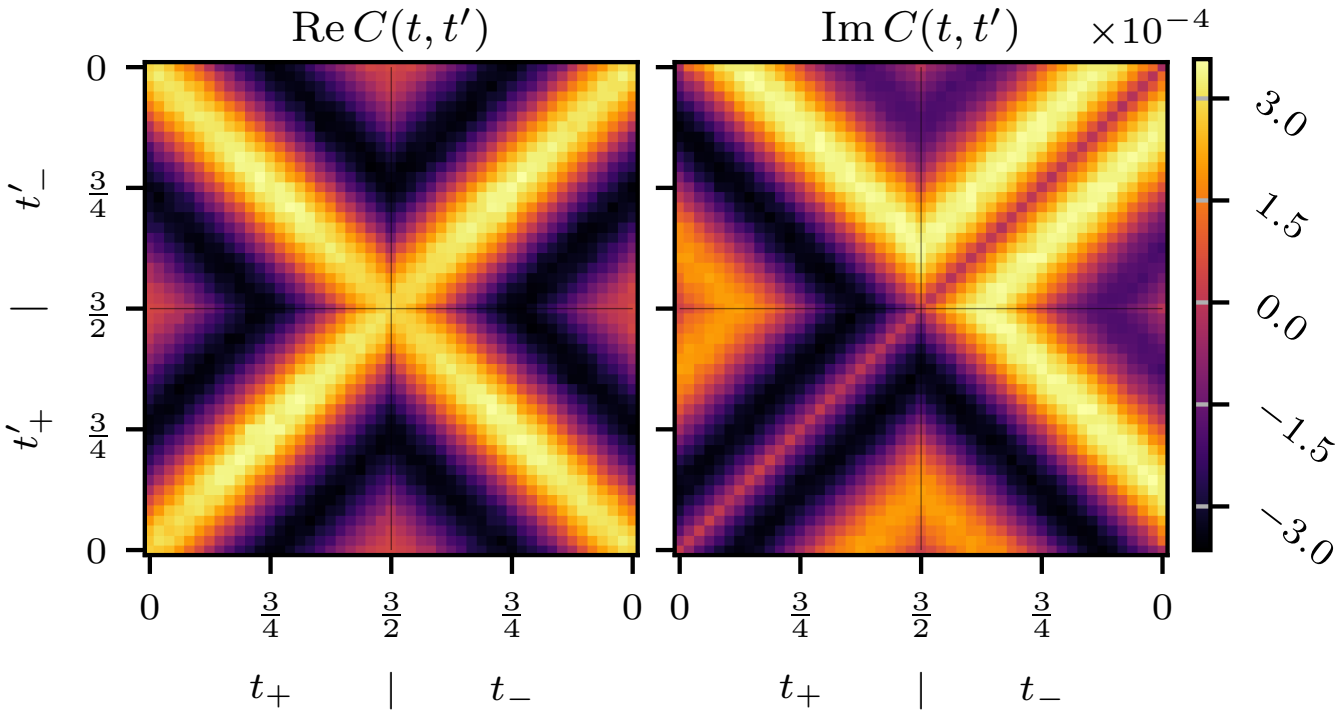
$$F = \mathrm{Re}\, D^F$$
$$\rho = -\mathrm{sgn}(\Delta t)\mathrm{Im}\, D^F$$



Extrapolated to the Schwinger-Keldysh contour

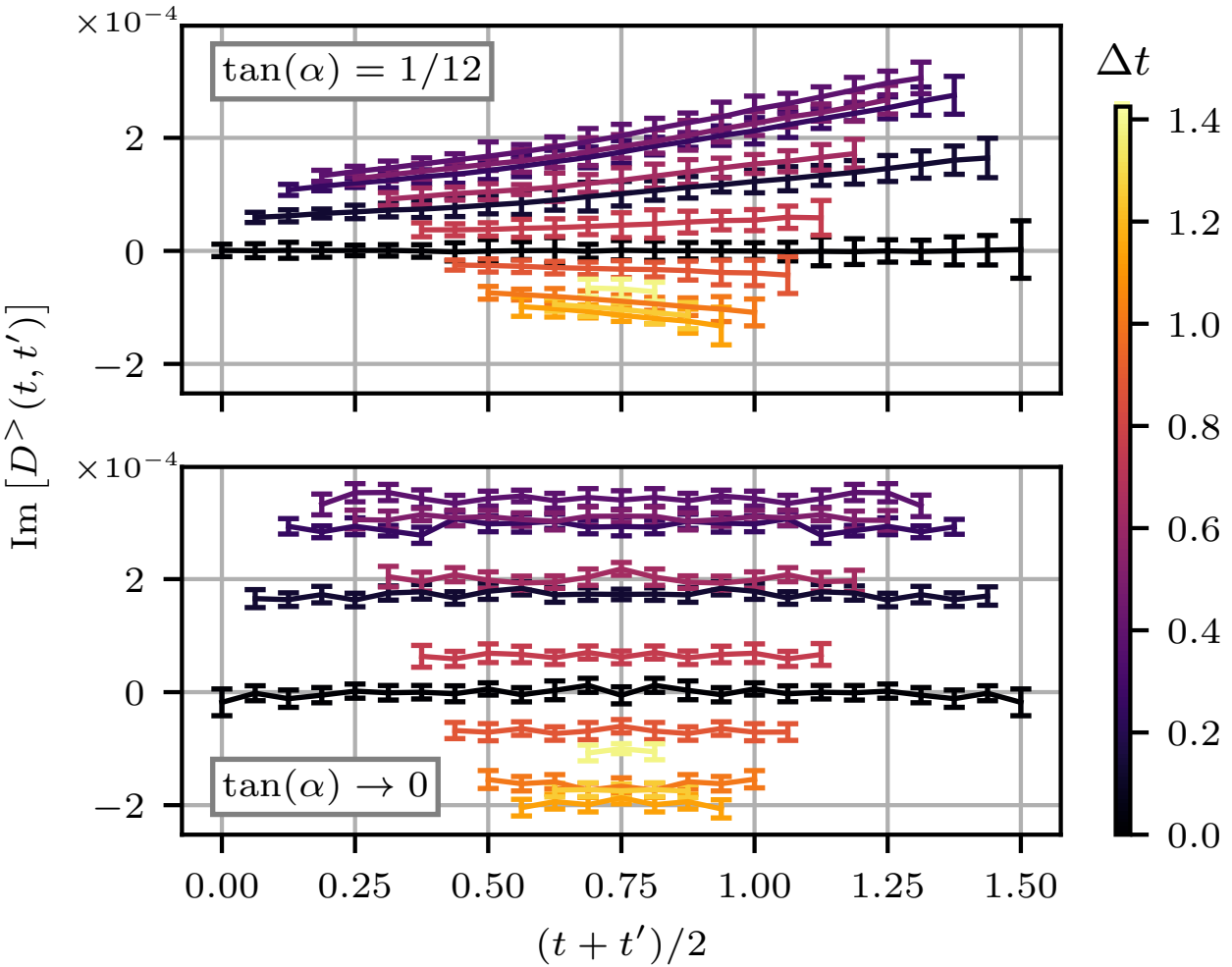
- **Real-time correlations** for SU(2) Yang-Mills theory in 3+1D
- Quadrants correspond to different propagators (Feynamn, Wightman)

$$D^{<}(t,t') = C(t_+, t'_-) \qquad D^{\bar{F}}(t,t') = C(t_-, t'_-)$$
$$D^{F}(t,t') = C(t_+, t'_+) \qquad D^{>}(t,t') = C(t_-, t'_+)$$



Emergent time translation invariance for $\alpha \to 0$

- Thermal equilibrium: correlations independent of $(t + t')/2$
- Independence found numerically only in limit $\alpha \to 0$



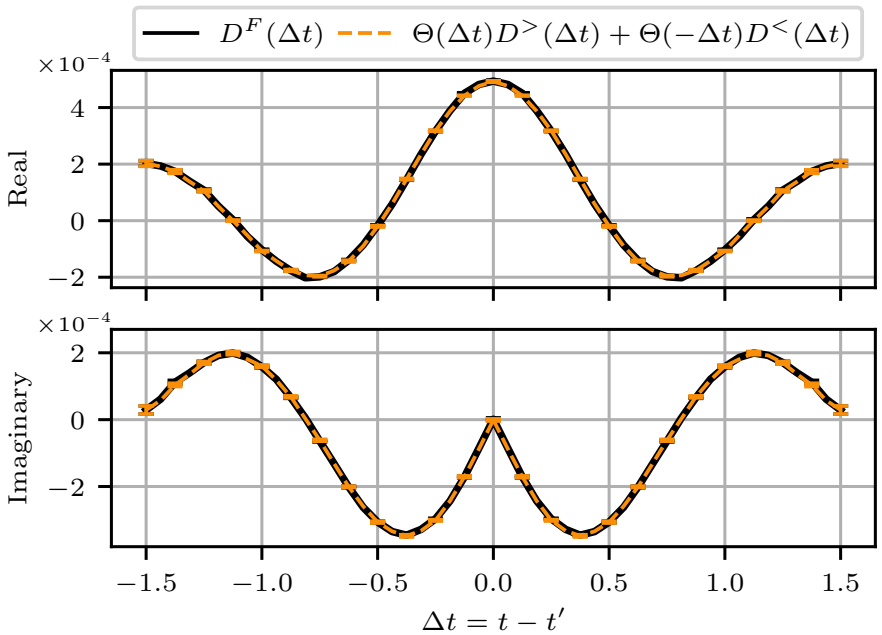
Consistency among propagators only for $\alpha \to 0$

- Feynman propagator and Weightman functions **analytically related**

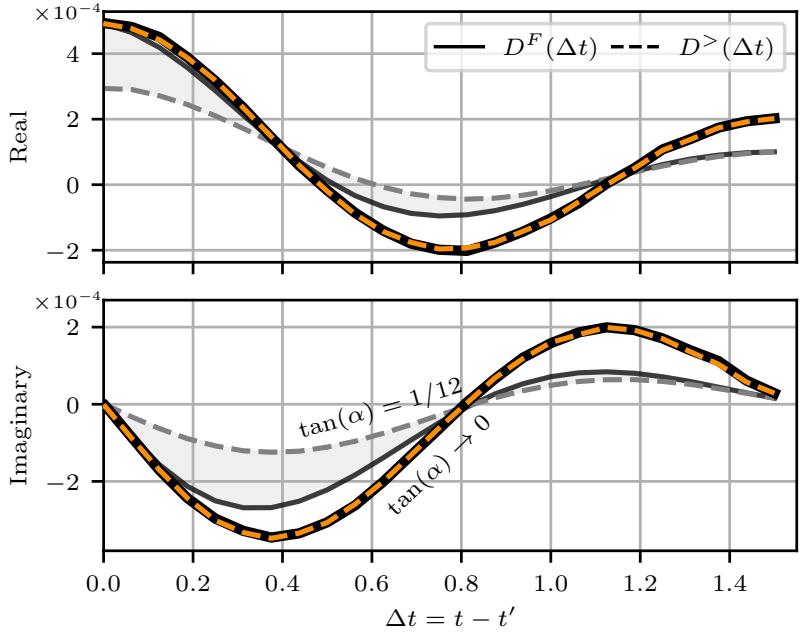
$$D^F(t-t') = \theta(t-t')D^>(t,t') + \theta(t'-t)D^<(t,t')$$

- **Relation established** only in the limit $\alpha \to 0$

Limit $\alpha \to 0$



$\alpha \to 0$ vs. finite α



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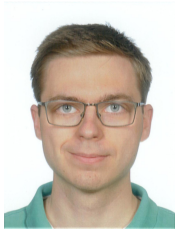
1. Motivation
2. Spectral functions in the quark-gluon plasma
3. Transport coefficients
4. Towards simulations of real-time lattice gauge theory
5. Conclusion

Research group

- *Postdoc:*
  David Müller

- *PhDs:*
  Paul Hotzy, Florian Lindenbauer

- *Master thesis, Master project:*
  Alois Altenburger, Tobias Angelli, Liane Backfried, Fabian Helmberger




Conclusion

- Initial stages of the QGP $\Rightarrow$ class.-stat., kinetics, hydro
- Spectral functions, unequal-time correlators
  - $\Rightarrow$ Reveal excitations, explain medium properties
  - $\Rightarrow$ Nonperturbative computation in pre-equilibrium QGP
- Transport coefficients κ, $\hat{q}$
  - $\Rightarrow$ encode dynamics of QGP
  - $\Rightarrow$ different features, anisotropic $\kappa_z > \kappa_T$, $\hat{q}$ smooth transition
- Complex Langevin (CL) $\Rightarrow$ towards direct real-time simulations
  - $\Rightarrow$ First direct calculation of (t, t') correlators (3+1D lattice SU(2))

Outlook

- Impact of medium $\kappa_i(\tau)$, $\hat{q}_i(\tau)$ on heavy quarks, quarkonia, jets?
- CL method currently limited $\Rightarrow$ larger g, $t_{\max}$? renormalization?
- Application of SU(N) out of equilibrium?

Thank you for your attention!

Backup slides

Gluon ρ

Gluon unequal-time correlations

in 3+1D:

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018) [1804.01966]*

in 2+1D:

KB, Kurkela, Lappi, Peuron, *JHEP 05, 225 (2021) [2101.02715]*

- Spectral function ($\dot{\rho} = \partial_t \rho,\ E = \partial_t A$)

$$\rho(x', x) = \frac{i}{N_c^2 - 1} \left\langle \left[\hat{A}(x'), \hat{A}(x) \right] \right\rangle$$

- Statistical correlator $\langle EE \rangle$ ($\equiv \ddot{F}$), in general independent of $\dot{\rho}$

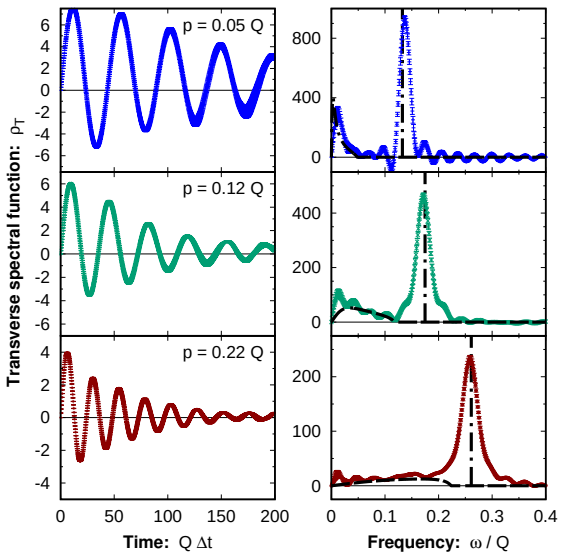
$$\langle EE \rangle(x', x) = \frac{1}{2(N_c^2 - 1)} \left\langle \left\{ \hat{E}(x'), \hat{E}(x) \right\} \right\rangle$$

- Classical-statistical: $\langle EE \rangle(t', t, p) = \frac{1}{N_c^2 - 1} \langle E(t', \vec{p}) E^*(t, \vec{p}) \rangle$

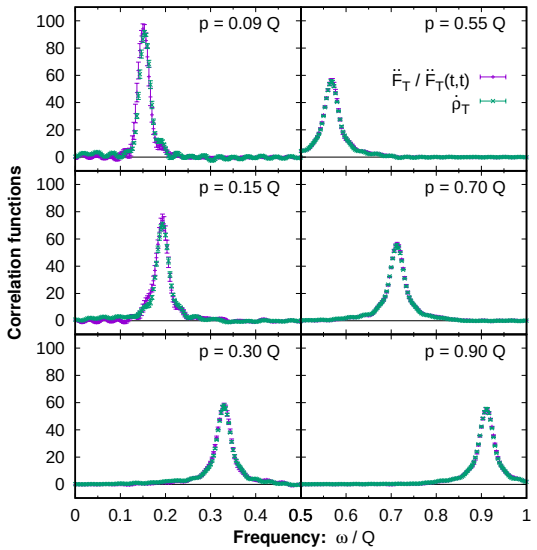
Gluon ρ in 3+1D: compare with HTL perturbation theory

KB, Kurkela, Lappi, Peuron, PRD 98, 014006 (2018)

$\rho_T(\omega, p)$



$\dot{\rho}_T(\omega, p) \approx \omega \rho_T(\omega, p)$



- Narrow Lorentzian q.p. peaks
  (position $\omega(p)$, width $\gamma(p)$)
- HTL at LO (black dashed) describes main features well
- Landau cut ($\omega < p$) and q.p. peak distinguishable

- Generalized fluctuation dissipation relation (FDR)

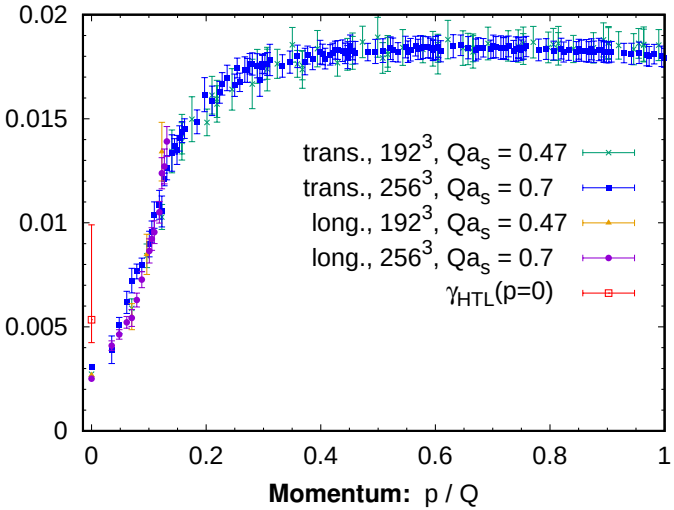
$$\frac{\langle EE \rangle_\alpha(t, \omega, p)}{\langle EE \rangle_\alpha(t, \Delta t{=}0, p)} \approx \frac{\dot{\rho}_\alpha(t, \omega, p)}{\dot{\rho}_\alpha(t, \Delta t{=}0, p)}$$

$\ddot{F} \equiv \langle EE \rangle$, $\alpha = T, L$ polarizations

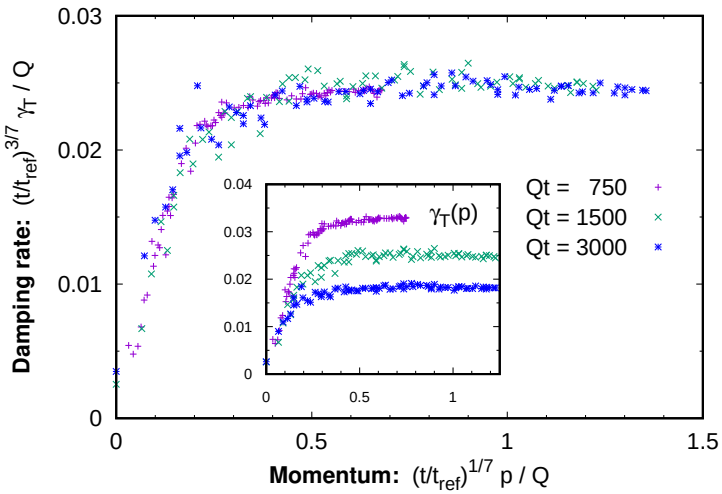
Gluon ρ in 3+1D: Damping rates (quasiparticle width)

KB, Kurkela, Lappi, Peuron, PRD 98, 014006 (2018)

Width $\gamma_T(p)$



$\propto (Qt)^{3/7}\gamma_T(t,p)$

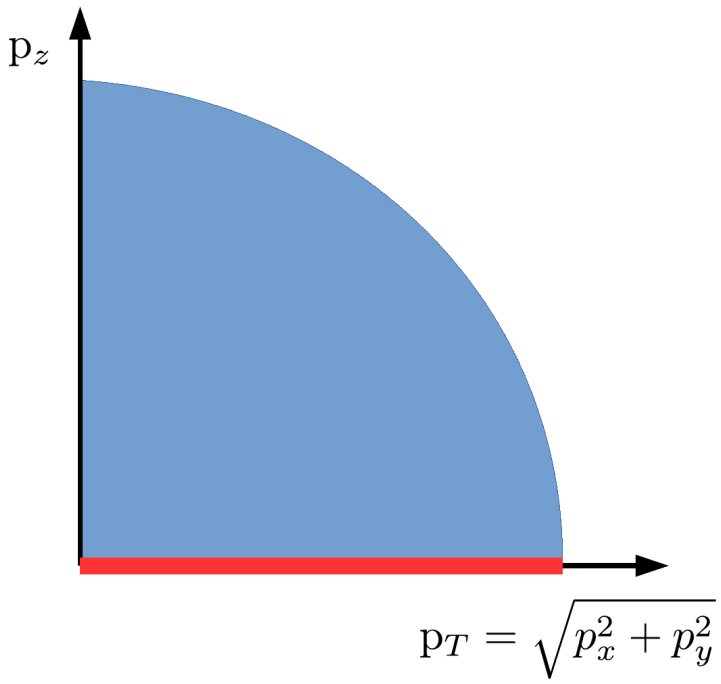


- Peak width $\gamma(t,p) \ll \omega(t,p)$ beyond HTL at LO
- $\gamma_T(t,p)$ increases with p (different from fermion $\gamma_+(t,p)$)

  (vs. HTL prediction $\gamma(p{=}0)$, $\gamma(p{\to}\infty)$, Braaten, Pisarski (1990); Pisarski (1992))
- Self-similar scaling $\Rightarrow \frac{\gamma_\alpha(t,p)}{\omega(t,p{=}0)} \sim (Qt)^{-2/7}$ decreases as expected

  (different from fermions where $\frac{\gamma_+(t,p{=}0)}{\omega_+(t,p{=}0)} \approx const$)

Systems in 3+1D $\rightarrow$ 2+1D

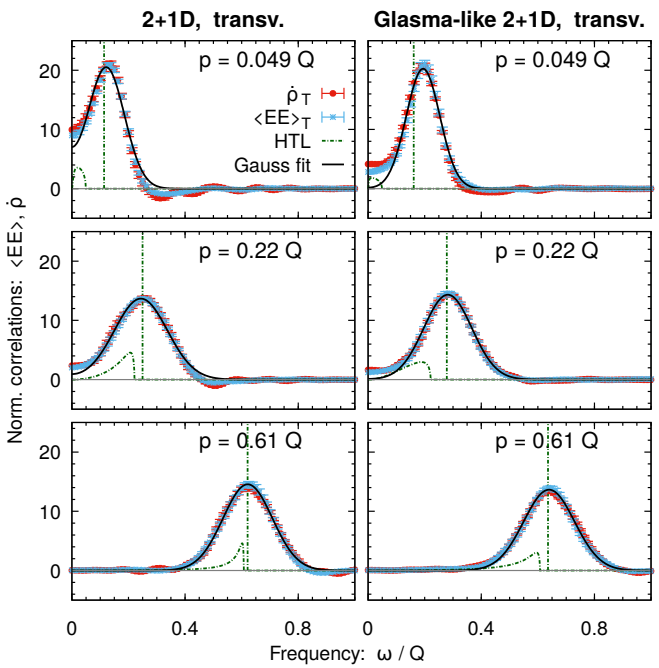


- Isotropic 3+1D: $f(t, p)$
- 2+1D: $f(t, p_T, p_z{=}0)$
- can be regarded as extreme momentum anisotropy

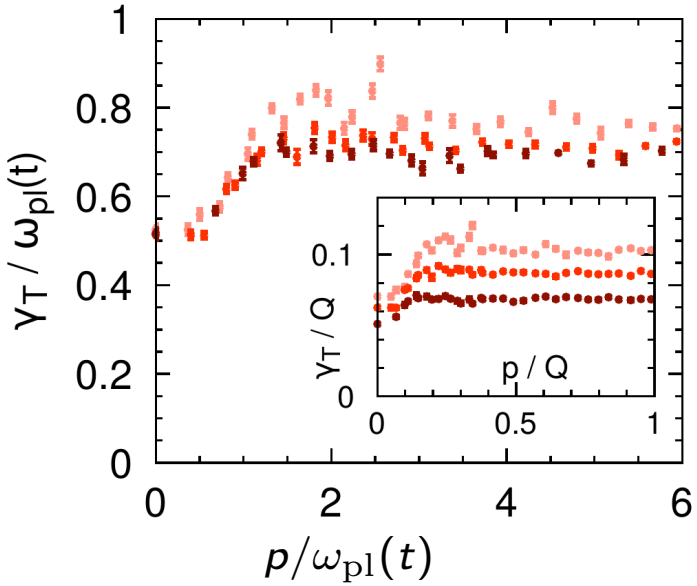
Gluon ρ in 2+1D

KB, Kurkela, Lappi, Peuron, JHEP 05, 225 (2021)

$\dot{\rho}_T(\omega, p) \approx \omega \rho_T(\omega, p)$



Width $\gamma_T(t, p)/\omega_{\rm pl}(t)$



✓ Generalized FDR, but:

✗ Broad non-Lorentzian peaks

✗ HTL curves (green) agree poorly

✗ Landau cut and q.p. peak not distinguishable

- Peak width $\gamma_T(t, p) \sim \omega_{\rm pl}(t)$

  $(\omega_{\rm pl} \equiv \omega_T(p{=}0))$

$\Rightarrow$ no quasiparticles for $p \lesssim \omega_{\rm pl}$!

Fermion ρ

Fermion spectral function

KB, Lappi, Mace, Schlichting, *PLB 827, 136963 (2022) [2106.11319]*

- $\mathrm{SU}(N_c)$ gauge theory (simulations: $N_c = 2$, $U_j \approx \exp\left(ig\, a_s A_j\right)$)

$$
\begin{aligned}
H_{YM} &= \frac{1}{g^2 a_s} \sum_{\vec{x},i} \mathrm{Tr}[E_i(t',\vec{x})^2] + \frac{1}{2} \sum_j \mathrm{ReTr}[1 - U_{ij}(t',\vec{x})] \\
\hat{H}_W &= \frac{1}{2} \sum_{\vec{x}} \left[\hat{\psi}^\dagger(t',\vec{x}), \gamma^0 \left(-i \not{D}_s[U] + m\right) \hat{\psi}(t',\vec{x})\right]
\end{aligned}
$$

- Neglect fermionic backreaction (suppressed), temporal gauge $A_0 = 0$, mode expansion, plane waves at reference time t

  See also: Aarts, Smit (1999); Kasper, Hebenstreit, Berges (2014); Mace, Mueller, Schlichting, Sharma (2016); ...

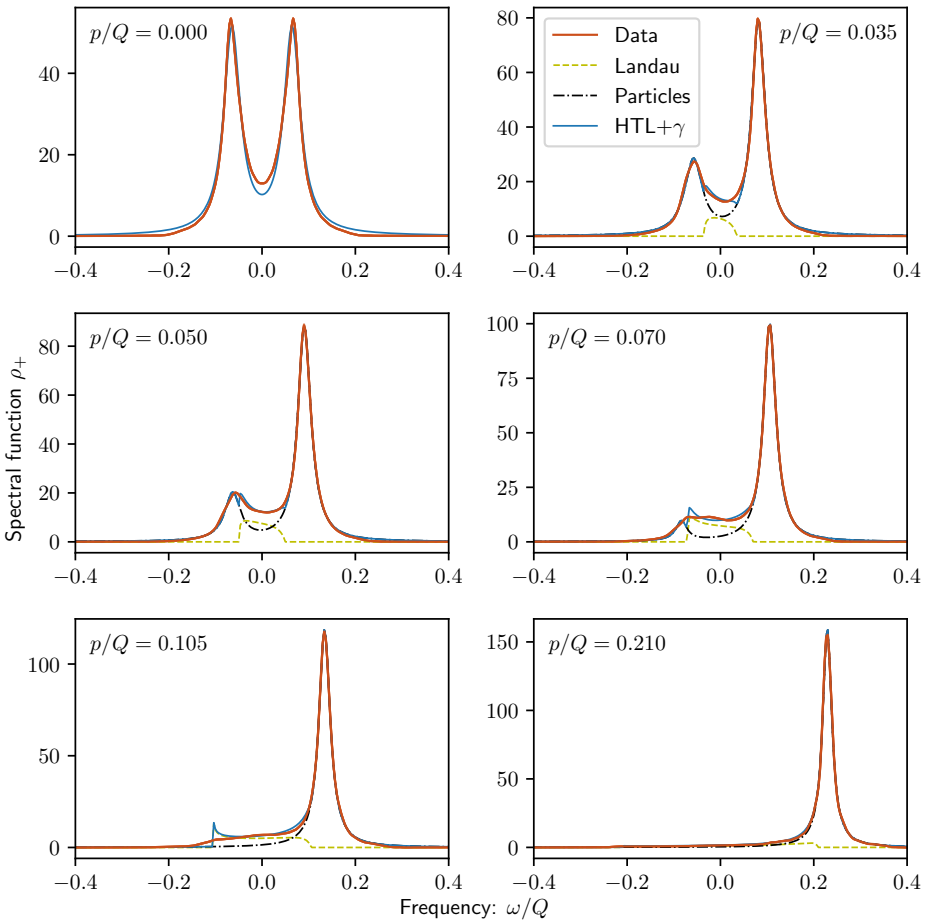
- Definition $\rho^{\alpha\beta}(x',x) = \left\langle \left\{ \hat{\psi}^\alpha(t',\vec{x}'), \hat{\bar{\psi}}^\beta(t,\vec{x}) \right\} \right\rangle$

  $\Rightarrow$ Fourier transform $\rho^{\alpha\beta}(t,\omega,p)$ via $t'-t$ and $\vec{x}'-\vec{x}$

Fermion ρ in 3+1D: Comparison with HTL

$$\rho_+(t,\omega,p) \equiv \rho_V^0 + \rho_V$$

$$(\rho_V^0 = \tfrac{1}{4}\mathrm{Tr}(\rho\gamma^0),\ \rho_V = -\frac{E_{\vec{p}}\, p^j}{4\, p^2}\,\mathrm{Tr}(\rho\gamma^j))$$



- HTL+γ extension
  (Landau cut + Lorentzian q.p. peaks:)
  Braaten, Pisarski (1992); Rebhan (1992); Mrowczynski, Thomas (2000); Blaizot, Iancu (2002); ...

$$\rho_+^{\mathrm{HTL}}(\omega,p) = 2\pi\,\beta_+(\omega/p,p) + \frac{2Z_+(p)\gamma_+(p)}{(\omega-\omega_+(p))^2+\gamma_+^2(p)} + \frac{2Z_-(p)\gamma_-(p)}{(\omega+\omega_-(p))^2+\gamma_-^2(p)}$$

- HTL dispersions $\omega_\pm(p)$ and residues $Z_\pm(p)$ agree with data

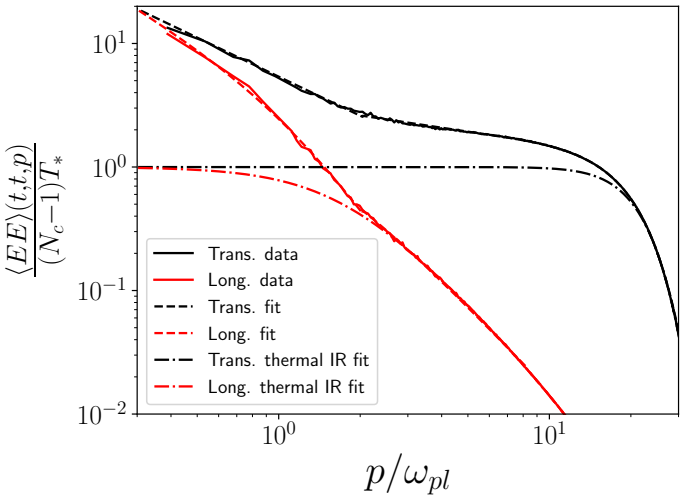
- Lattice 256^3, $Qa_s = 0.75$, $m = 0.003125\,Q$, $Qt = 1500$

Deep infrared and transport

Deep infrared (IR) of 3+1D gluonic plasmas

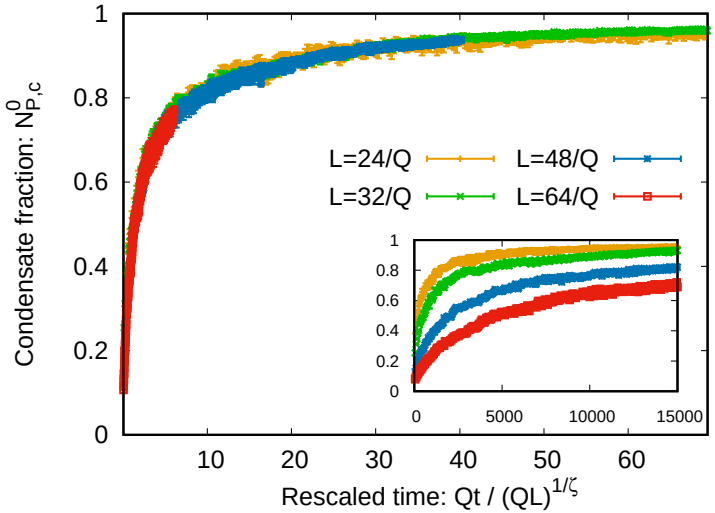
Can we understand the infrared ($p \ll Q$) of the pre-equilibrium QGP?

Excess of IR modes in $\langle EE \rangle$ correlators for $p \lesssim \omega_{\rm pl}$



KB, Kurkela, Lappi, Peuron, JHEP 09 (2020) 077

Gauge-invariant condensation via spatial Wilson and Polyakov loops



Berges, KB, Mace, Pawlowski, PRD 102 (2020) 034014

Berges, KB, Butler, de Bruin, Pawlowski, 2307.13669

- Excess of IR: observable in gauge-invariant κ (see below)
- Gauge-inv. cond. similar to Bose cond. in scalars $\Rightarrow$ connection?

Far-from-equ. condensation in scalars: Berges, Sexty (2012); Piñerio Orioli, KB, Berges (2015)

How to observe properties of the pre-equilibrium QGP?

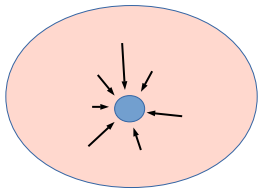
E.g., via impact on transport coefficients

As an example here, IR excess of gluons affects κ evolution

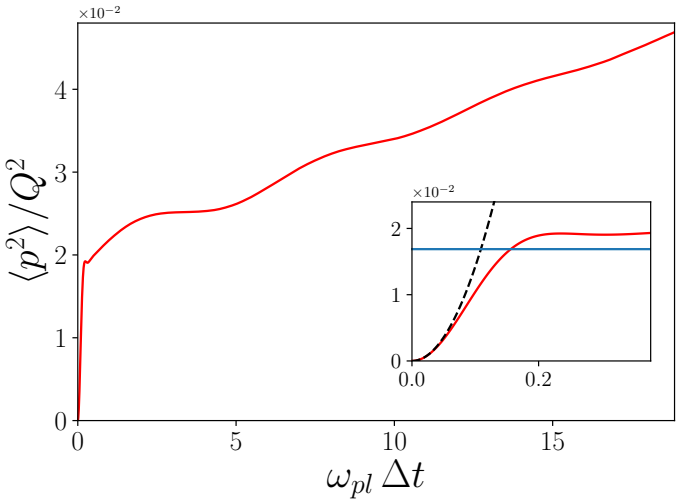
KB, Kurkela, Lappi, Peuron, *JHEP 09, 077 (2020)*

- Heavy quarks 'kicked' by the QGP medium

$$\dot{p}_i(t) = \mathcal{F}_i(t)$$



Momentum broadening



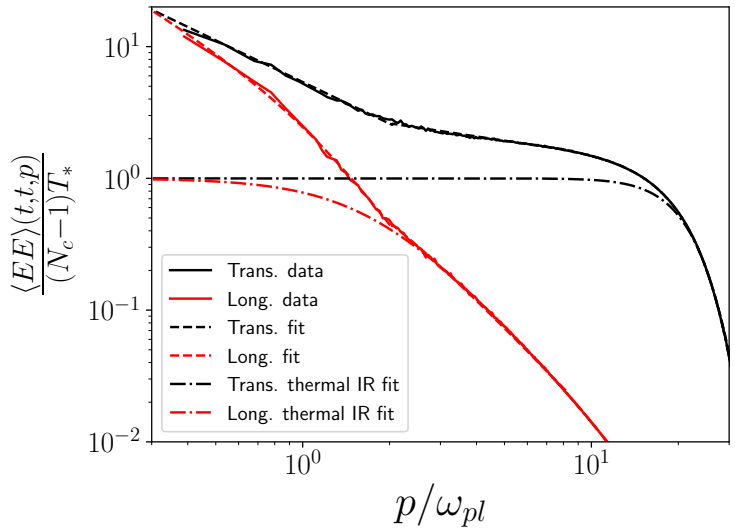
- Heavy-quark diffusion coefficient
  $3\kappa\,(t, \Delta t) = \frac{\mathrm{d}}{\mathrm{d}\Delta t}\langle p^2(t, \Delta t)\rangle$
- κ for bottomonium suppression

  Brambilla, Escobedo, Soto, Vairo (2016)
- Impact on κ from IR excess

Approach: reconstruct κ using correlators $\dot{\rho}$ and $\langle EE \rangle$

$$3\kappa(t,\Delta t) \approx \frac{g^2}{N_c}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\,\frac{\sin(\omega\,\Delta t)}{\omega} \times\left[2\langle EE\rangle_T(t,t,p)\frac{\dot{\rho}_T(t,\omega,p)}{\dot{\rho}_T(t,t,p)}+\langle EE\rangle_L(t,t,p)\frac{\dot{\rho}_L(t,\omega,p)}{\dot{\rho}_L(t,t,p)}\right]$$

- use extracted equal-time and spectral functions in computation
- use $\langle EE\rangle_{T/L}(t,t,p)$ with IR excess ('data') & without ('thermal HTL')
- in $\dot{\rho}_{T/L}(t,\omega,p)$ Landau ($\omega < p$) and q.p. terms can be distinguished

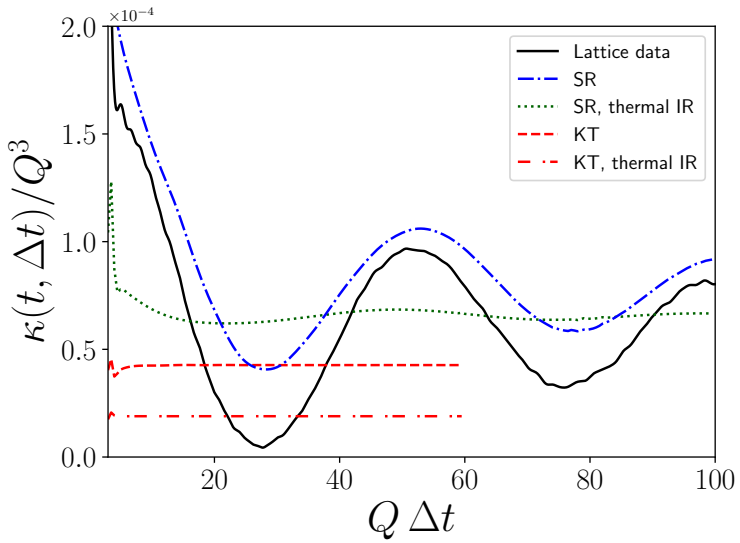


Gauge-invariant observation of IR gluon excess in κ

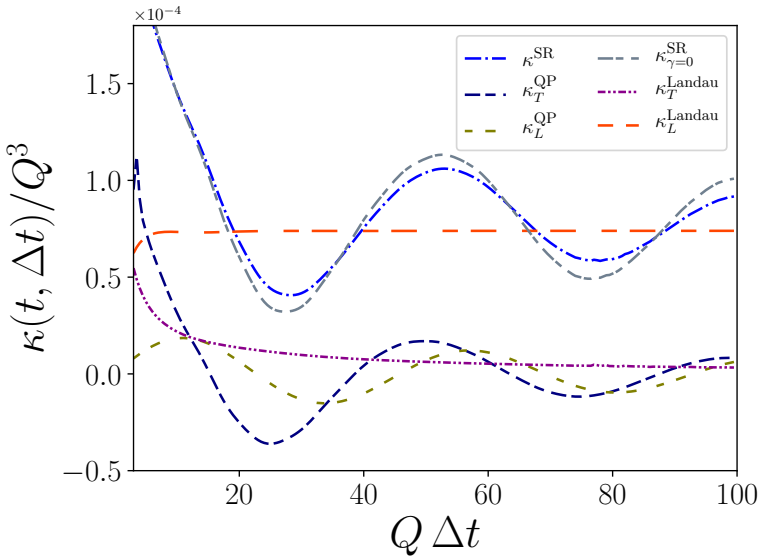
KB, Kurkela, Lappi, Peuron, JHEP 09 (2020) 077

$$\kappa(t,\Delta t) \approx \frac{g^2}{3N_c}\int\frac{\mathrm{d}^3p}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}\,\frac{\sin(\omega\,\Delta t)}{\omega} \times\left[2\langle EE\rangle_T(t,t,p)\frac{\dot\rho_T(t,\omega,p)}{\dot\rho_T(t,t,p)}+\langle EE\rangle_L(t,t,p)\frac{\dot\rho_L(t,\omega,p)}{\dot\rho_L(t,t,p)}\right]$$

Total $\kappa(t,\Delta t)\equiv\sum_i\kappa_i(t,\Delta t)$



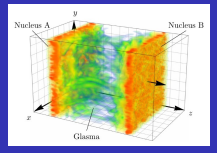
Components $\kappa_i(t,\Delta t)$



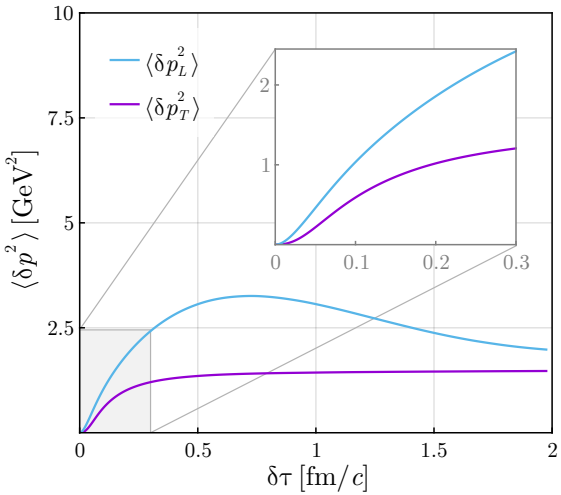
- Nonperturbative effects of $\langle EE\rangle_\alpha(t,t,p)$ and $\dot\rho_\alpha(t,\omega,p)$ visible!
- Oscillations with ω_{pl} due to QP excitations, sign of IR excess
- Heavy quarks, quarkonia, jets encode nonthermal dynamics of QGP

κ during initial stages

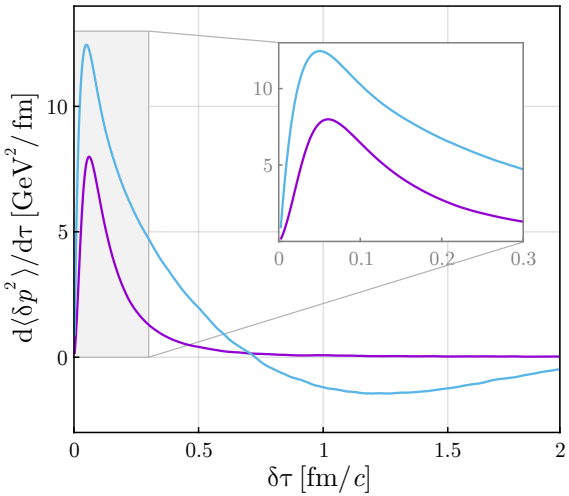
κ during Glasma phase



$\langle p_i^2 \rangle$ of beauty quarks

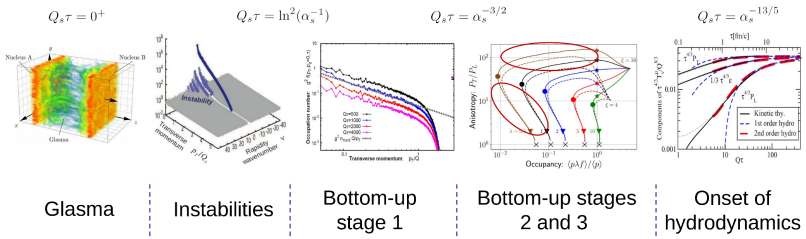


κ_i of beauty quarks



Avramescu, Baran, Greco, Ipp, Müller, Ruggieri PRD 107, 114021 (2023); 2307.07999

- Classical-statistical simulations of hard probes in the Glasma
  - Extraction of κ_i and $\hat{q}_i$ in Glasma phase
    Ipp, Müller, Schuh (2020); KB, Kurkela, Lappi, Peuron (2020); Carrington, Czajka, Mrowczynski (2022); Khowal, Das, Oliva, Ruggieri (2022); Avramescu et al. (2023)
  - Large values, anisotropic coefficients $\kappa_z > \kappa_T$ (z is beam direction)
- What about kinetic regime?



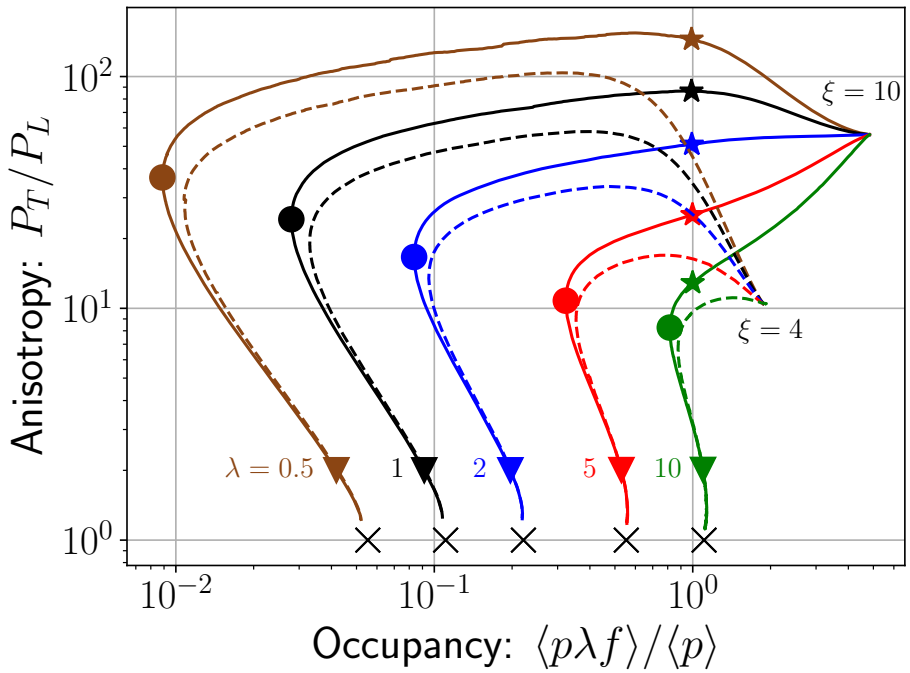
Kinetic theory ⇒ Bottom-up thermalization scenario

- Bottom-up scenario

  Baier, Mueller, Schiff, Son, PLB (2001)

- Consists of three stages
  1. Classical attractor
  2. Anisotropy freezes
  3. Radiational breakup

- Different bottom-up stages separated by markers $(\lambda = g^2 N_c)$
  - ★ large pressure anisotropy $P_T \gg P_L$, occupancy $f \sim 1/\lambda$
    - minimum (mean) occupancy f
  - ▽ close to isotropic $P_T/P_L = 2$



Kurkela, Zhu (2015); *version from:*
KB, Kurkela, Lappi, Lindenbauer, Peuron (2023)

- Pressure $P_{T,L} \sim \int d^3p \frac{p_{\perp,z}^2}{p} f$
- Mean $\langle O\rangle = \int d^3p\, f(p)\, O(p)$

- Thermalization time scale $\tau_{\rm BMSS} = \alpha_s^{-13/5}/Q_s$, initial momentum Q_s

QGP description: effective kinetic theory (EKT)

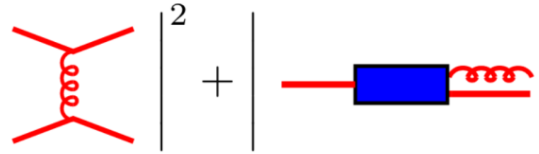
- When quasiparticles have formed: Kinetic theory applicable

  Note: Assumes narrow excitations in spectral functions, which may not be true at low momenta for strong anisotropy (Backup in 'ρ': 'Gluonic 2+1D')

  KB, Kurkela, Lappi, Peuron (2018, 2019, 2021)

- Time evolution described by Boltzmann equation at LO

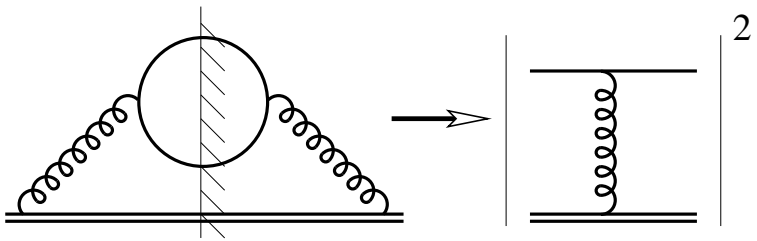
$$(\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla})f = \left|\ \right|^2 + \left|\ \right|^2$$



$$\frac{\partial f_{\vec{p}}}{\partial \tau} - \frac{p_z}{\tau}\frac{\partial f_{\vec{p}}}{\partial p_z} = -\mathcal{C}^{2\leftrightarrow 2}[f_{\vec{p}}] - \mathcal{C}^{1\leftrightarrow 2}[f_{\vec{p}}]$$

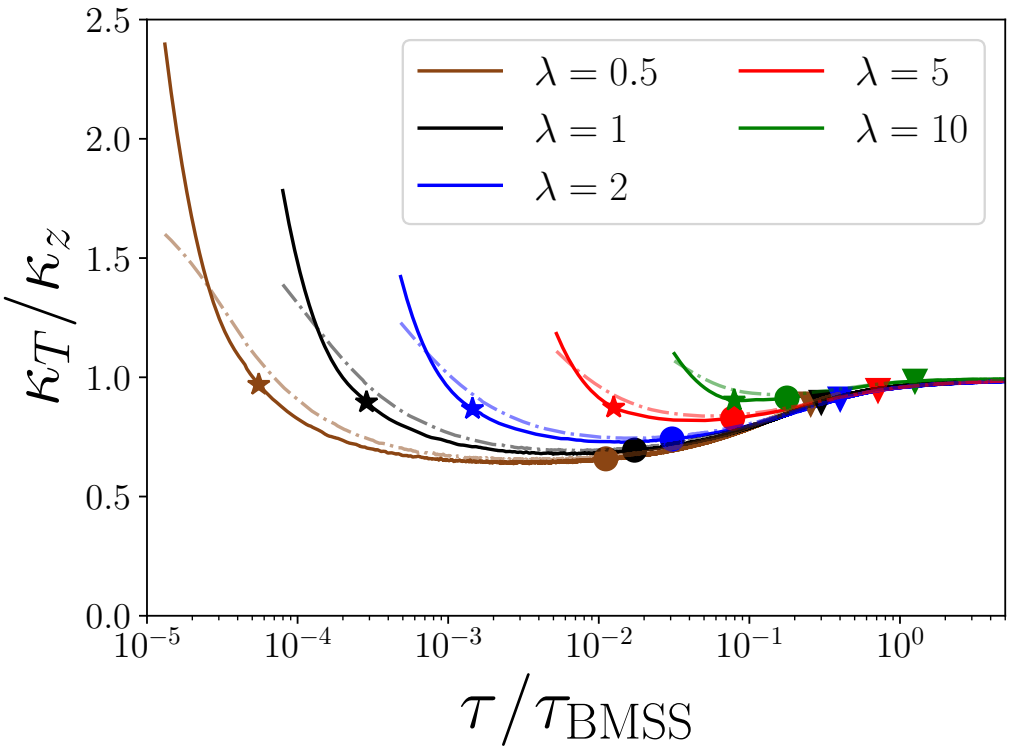
Arnold, Moore, Yaffe, JHEP 01, 030 (2003)

- Heavy-quark coefficient κ_i: Moore, Teaney (2005); Caron-Huot, Moore (2008)



Transverse vs. longitudinal diffusion

KB, Kurkela, Lappi, Lindenbauer, Peuron, 2303.12520



- $\kappa_T > \kappa_z$ during over-occupied stage (z is longitudinal/beam axis)
- $\kappa_T < \kappa_z$ after $\star$ due to momentum anisotropy and low occupancy
- Most of the time $\kappa_T < \kappa_z$, anisotropy larger at weak coupling

Complex Langevin

Parametrization of the complex time contour

Unambiguous CL formalism

Introduce curve parameter λ for $t(\lambda)$ leads to

$$\partial_\theta A^a_\mu(\theta,\lambda,\vec{x}) = i\,\frac{\delta S_{\mathrm{YM}}}{\delta A^a_\mu(\lambda,\vec{x})}\bigg|_{A=A(\theta)} + \eta^a_\mu(\theta,\lambda,\vec{x}),$$

$$\langle \eta^a_\mu(\theta,\lambda,\vec{x})\eta^b_\nu(\theta',\lambda',\vec{x}')\rangle = 2\delta_{\mu\nu}\delta^{ab}\delta(\theta-\theta')\delta(\lambda-\lambda')\delta^{(3)}(\vec{x}-\vec{x}')$$

- Complex time contours leads to **ambiguous noise correlator** expression (δ-distribution for complex arguments)
  - ✓ Ambiguities are resolved by a **path parameter** CL formulation

$$t : [a,b] \mapsto \mathbb{C}, \qquad t(a) = 0, \quad t(b) = -i\beta$$

  - ✓ Noise correlator in terms of λ

$$\delta(t(\lambda) - t(\lambda')) \rightsquigarrow \delta(\lambda - \lambda')$$

- Solution **independent** of chosen parametrization (kernel freedom)

Euler-Maruyama for numerical solution of the CLE

- Jacobian of parametrization leads to additional factors
- Lattice spacing a_λ of parametrization for contour

$$U_{x,t}(\theta+\epsilon) = \exp\left(it^a \left[i\epsilon \frac{a_\lambda}{a_s} \left. \frac{\delta S_{\mathrm{W}}}{\delta \tilde{A}^a_{x,t}} \right|_\theta + \sqrt{\epsilon} \sqrt{\frac{a_\lambda}{a_s}} \eta^a_{x,\lambda}(\theta) \right] \right) U_{x,t}(\theta)$$

$$U_{x,i}(\theta+\epsilon) = \exp\left(it^a \left[i\epsilon \frac{a_s}{\bar{a}_\lambda} \left. \frac{\delta S_{\mathrm{W}}}{\delta \tilde{A}^a_{x,i}} \right|_\theta + \sqrt{\epsilon} \sqrt{\frac{a_s}{\bar{a}_\lambda}} \eta^a_{x,i}(\theta) \right] \right) U_{x,i}(\theta)$$

CL update step for link variables on a SK-contour

- New update resolves ambiguities of complex time contour
- But introduces a divergence for $a_\lambda \to 0$
- Solution: rescale ϵ by a_s/a_λ or $a_s/\bar{a}_\lambda$ (kernel freedom)

Existing CL stabilization techniques

- **Gauge cooling (GC):** Exploit gauge freedom to minimize non-unitarity measured by a functional $F[U]$ Seiler, Sexty, Stamatescu (2013)

$$U_{x,\mu} \mapsto U^V_{x,\mu} = V_x U_{x,\mu} V^{-1}_{x+\mu}, \qquad F[U] \geq F[U^V]$$

- **Adaptive stepsize (AS):** Regulate large drift terms $K^a_{x,\mu} \equiv \frac{\delta S_{\mathrm{W}}}{\delta \tilde{A}^a_{x,\mu}}$ which lead to instabilities (Aarts, James, Seiler, Stamatescu (2010))

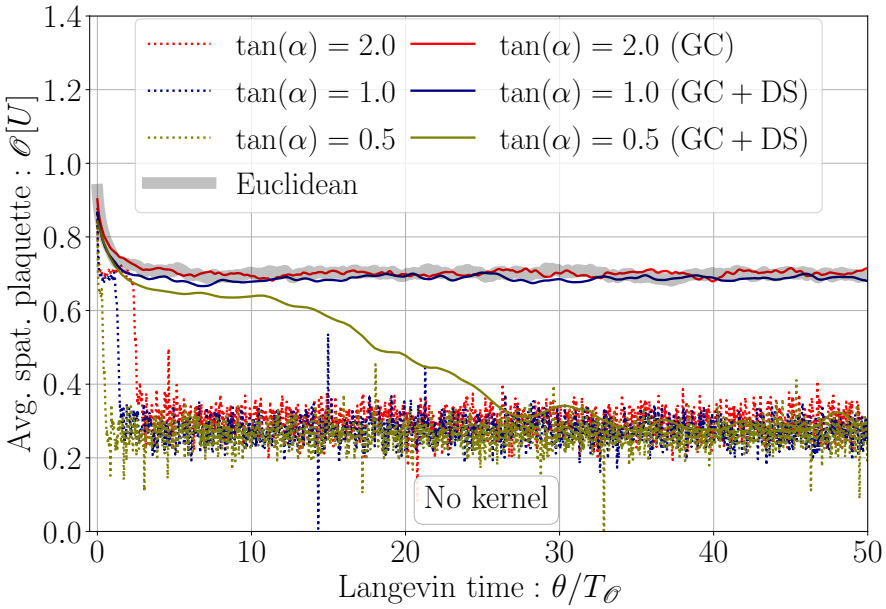
$$\epsilon \mapsto \tilde{\epsilon} = \epsilon \min\left(1, \frac{B}{\max\limits_{x,\mu,a} |K^a_{x,\mu}|}\right)$$

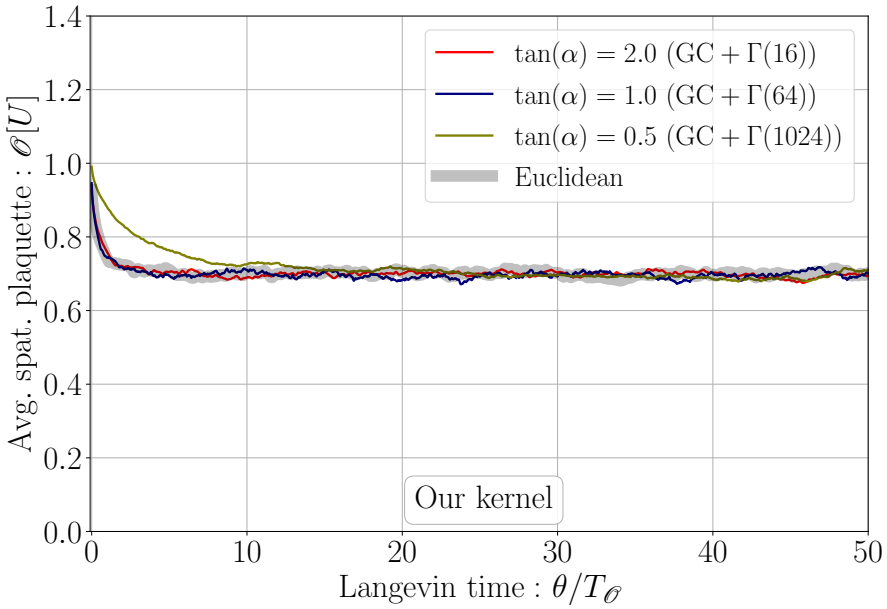
- **Dynamical stabilization (DS):** Penalize drift terms depending on the local non-unitary of the configuration Attanasio, Jäger (2019)

$$K^a_{x,\mu} \mapsto \tilde{K}^a_{x,\mu} = K^a_{x,\mu} + i\alpha_{\mathrm{DS}} M^a_x,$$

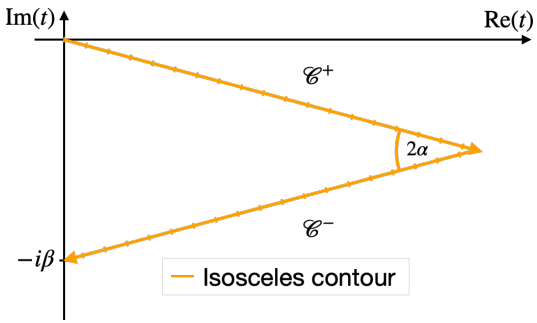
$$M^a_x = b^a_x \left(\sum_c b^c_x b^c_x\right), \quad b^a_x = \sum_\mu \mathrm{Tr}[t^a U_{x,\mu} U^\dagger_{x,\mu}]$$

Anisotropic kernel comparison I

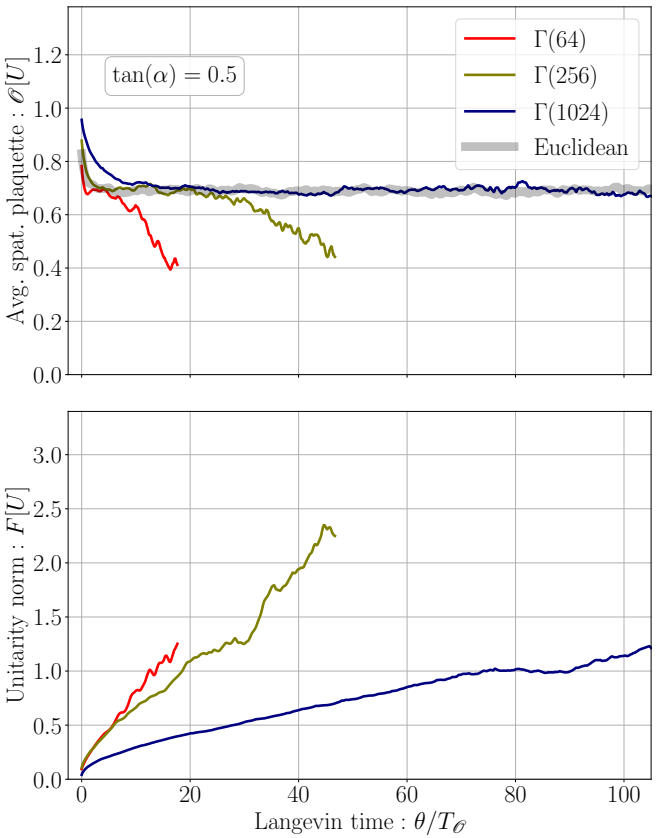


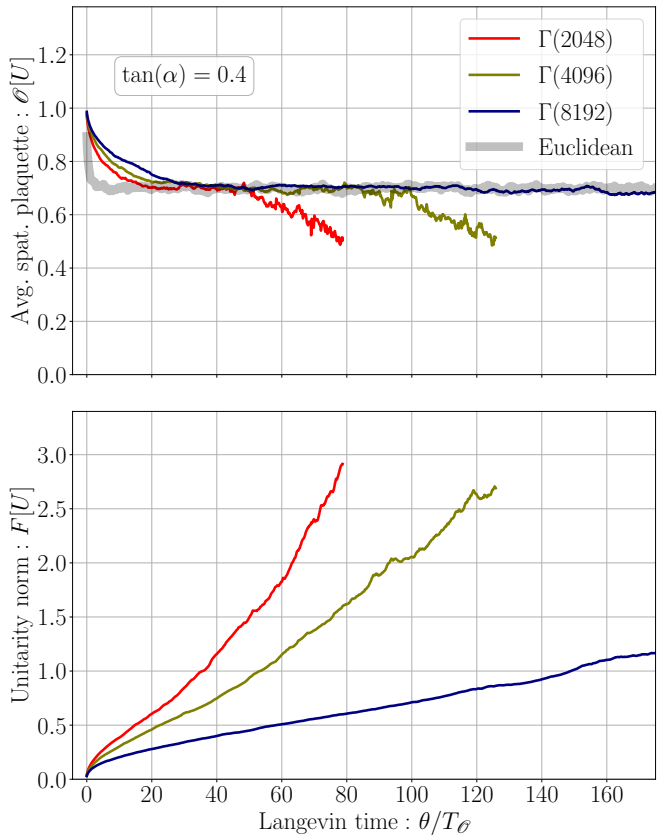


- Simulations on isosceles (see main part of the talk)
- Stabilization methods used:
  - GC: Gauge cooling
  - AS: Adaptive step size (everywhere)
  - DS: Dynamical stabilization

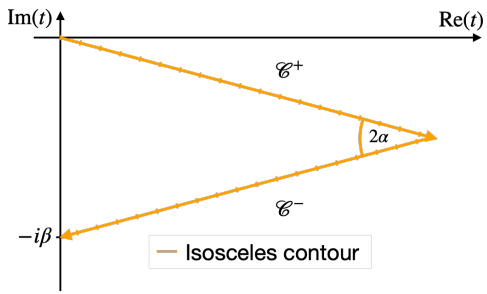


Anisotropic kernel comparison II

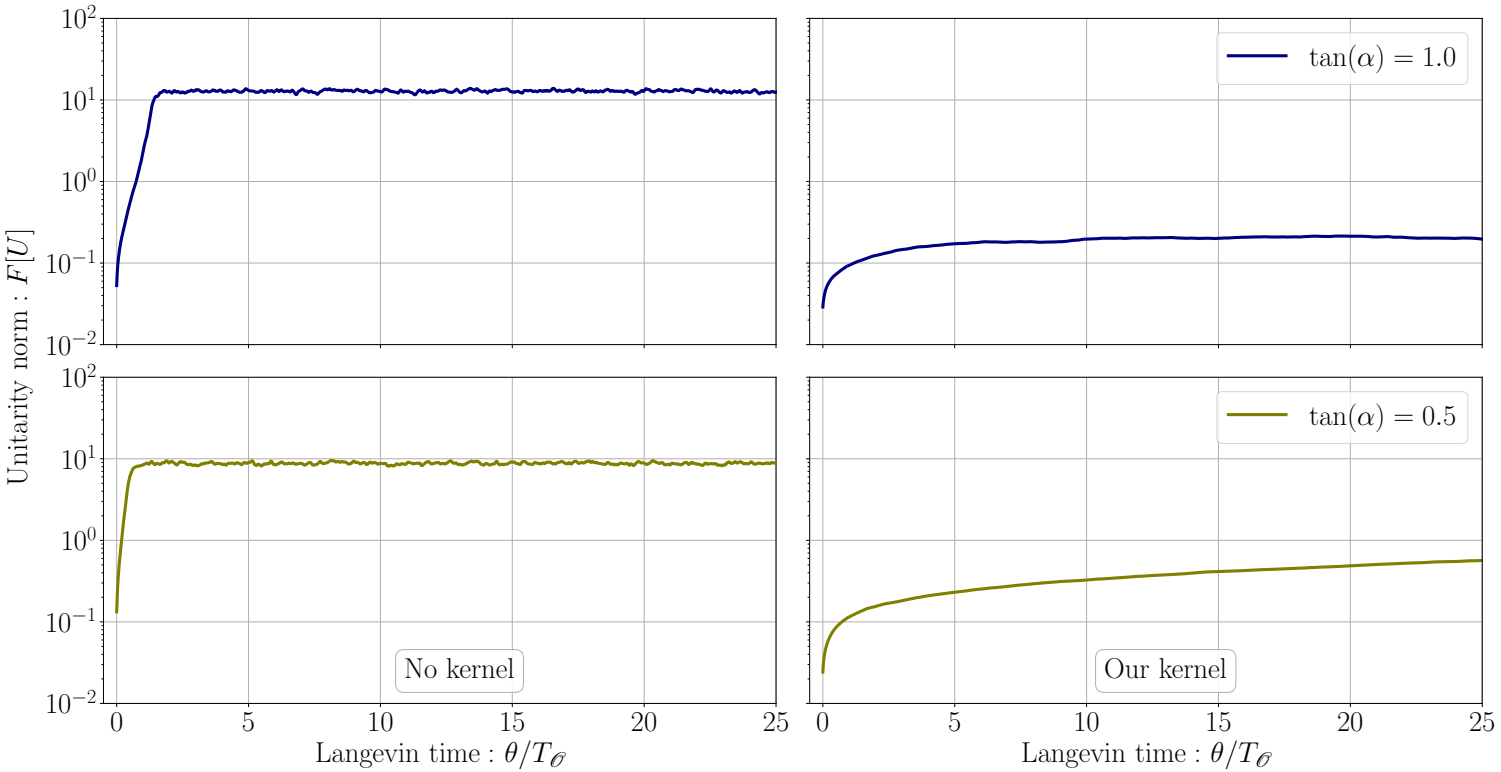




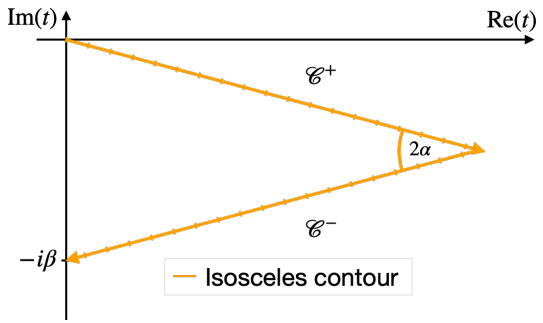
- Simulations on isosceles
- Systematic improvement on finer discretizations



Anisotropic kernel comparison III



- Simulations on isosceles
- $F[U] = \frac{1}{4N_t N_s^3} \sum_{x,\mu} \mathrm{Tr}\left[(U_{x,\mu} U_{x,\mu}^\dagger - 1)^2\right] \geq 0$
- Measure of how 'non-unitary' links are



Unitarity norm for unequal-time correlator simulations

- For different tilt angles of regularized SK contour
- Gray band indicates measurement region