Probing the deep string spectrum

Chrysoula Markou

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Université de Mons

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Generalities of the spectrum

- two universal string parameters: α' and g_S

- infinitely many *physical* states

$$M^2 = \text{integer} \times \frac{1}{\alpha'}$$

  that are thought of as:

  1. mass eigenstates $\Rightarrow$ on–shell mass

  2. irreps of $SO(D-1)$ or $SO(D-2)$ $\Rightarrow$ TT

  1–particle states à la Bargmann and Wigner ?

- What does the spectrum look like? Is there a bigger *symmetry*?

The physicality condition

- bosonic strings: the string field can be expanded in *modes* α_n^μ with oscillator algebra: $[\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m+n}\eta^{\mu\nu} \;\Rightarrow$ Fock space

- string states: functions of $\alpha_{k<0}^\mu$

- energy–momentum tensor on the worldsheet $\Rightarrow$ *modes*:

$$L_n = \tfrac{1}{2} \sum_{m=-\infty}^{+\infty} : \alpha_{n-m} \cdot \alpha_m : \quad , \quad \alpha_0^\mu \sim p^\mu$$

  which satisfy the Virasoro algebra

$$[L_m, L_n] = (m-n)L_{m+n} + \tfrac{c}{12} m(m^2-1)\delta_{m+n}$$

- physical states must satisfy the Virasoro constraints
  three are sufficient

see e.g. Sasaki, Yamanaka 1985

$$(L_0 - 1)|\text{phys}\rangle = 0 \quad , \quad L_1|\text{phys}\rangle = 0 \quad , \quad L_2|\text{phys}\rangle = 0$$

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A covariant way: vertex operators

- state–operator correspondence (here for open bosonic strings):

$$\alpha_{-n_1}^{\mu_1} \dots \alpha_{-n_k}^{\mu_k} |0;p\rangle \quad \leftrightarrow \quad \partial^{n_1} X^{\mu_1} \dots \partial^{n_k} X^{\mu_k} \, e^{ip\cdot X}$$

  operator ingredients: primary ∂X and its descendants

$$V(z) = F\Big(\partial X^\mu, \partial^2 X^\mu, \dots, \partial^k X^\mu\Big)\, e^{ip\cdot X}$$

  impose the physical state condition:

$$[Q, V] \stackrel{!}{=} \text{tot. deriv.} \Rightarrow \quad h_V = 1\,, \quad \alpha' p^2 = 1 - N\,, \quad h_F = N \quad \& \quad \textbf{Wigner}$$

- first few levels:

$$\begin{aligned} N=0: \quad & V_{\text{tach}}(p,z) &=& \; e^{ip\cdot X} \\ N=1: \quad & V_\epsilon(p,z) &=& \; \epsilon_\mu \, \partial X^\mu \, e^{ip\cdot X} \quad \text{(T)} \\ N=2: \quad & V_B(p,z) &=& \; B_{\mu\nu} \, \partial X^\mu \, \partial X^\nu \, e^{ip\cdot X} \quad \text{(TT)} \end{aligned}$$

[g_o, T^a and normalizations in front of v.o. omitted for simplicity]

Example: the leading Regge trajectory

- *leading* Regge: highest spins ∀ level
  oscillator language:

$$\epsilon_{\mu_1\ldots\mu_s}(p)\,\alpha_{-1}^{\mu_1}\ldots\alpha_{-1}^{\mu_s}|p\rangle$$

  vertex operator language:

$$V(p,z) = F_1\,e^{ip\cdot X} = \sum_s \epsilon_{\mu_1\ldots\mu_s}(p)\,\partial X^{\mu_1}\ldots\partial X^{\mu_s}\,e^{ip\cdot X}$$

- Virasoro constraints:
  1. L_0 fixes p^2: $\alpha' p^2 = 1 - s$
  2. $L_1 \sim p\cdot\alpha_1 + \ldots$ checks transversality:

$$p^\nu \epsilon_{\nu\mu_2\ldots\mu_s} = 0 \quad \text{or} \quad p\cdot\frac{\delta F_1}{\delta\partial X} = 0$$

  3. $L_2 \sim \alpha_1\cdot\alpha_1 + \ldots$ checks tracelessness:

$$\eta^{\nu\sigma}\epsilon_{\nu\sigma\mu_3\ldots\mu_s} = 0 \quad \text{or} \quad \frac{\delta^2 F_1}{\delta\partial X\cdot\delta\partial X} = 0$$

Sagnotti, Taronna '10

Building the spectrum 1

- Method 1: light–cone gauge
  - use transverse oscillators α^i_{-n}
  - *non*–covariant, leads to *superposition* or *fraction* of states

Goddard, Thorn 1972
Goddard, Goldstone, Rebbi, Thorn 1973

N	$gl(24)$ tensors	$so(24)$ irreps	little group irreps
0	$\vert k\rangle$ $\bullet$	$\bullet$	$\bullet$
1	$\alpha^i_{-1}\vert k\rangle$ $\yng(1)$	$\yng(1)$	$\yng(1)$
2	$\alpha^{i_1}_{-1}\alpha^{i_2}_{-1}\vert k\rangle$ $\quad$ $\alpha^i_{-2}\vert k\rangle$ $\yng(2)$ $\quad$ $\yng(1)$	$\yng(2) \oplus \yng(1) \oplus \bullet$	$\yng(2)$
3	$\alpha^{i_1}_{-1}\alpha^{i_2}_{-1}\alpha^{i_3}_{-1}\vert k\rangle$ $\quad$ $\alpha^{i_1}_{-2}\alpha^{i_2}_{-1}\vert k\rangle$ $\quad$ $\alpha^i_{-3}\vert k\rangle$ $\yng(3)$ $\quad$ $\yng(1) \otimes \yng(1)$ $\quad$ $\yng(1)$	$\yng(3) \oplus \yng(2) \oplus \yng(1) \oplus \bullet$ $\oplus \yng(1,1) \oplus \yng(1)$	$\yng(3)$ $\oplus \yng(1,1)$

see e.g. Blumenhagen, Lüst, Theisen book

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Building the spectrum 2

- Method 2: DDF
  - scatter k photons, each of momentum $-n_i q$, off of tachyon

$$\Rightarrow \text{generic state:} \quad \big(\epsilon_k \cdot A_{-n_k}\big) \dots \big(\epsilon_1 \cdot A_{-n_1}\big) e^{i\widetilde{p}\cdot X} \quad , \quad N = \sum_{i=1}^{k} n_i$$

    where A^μ_{-n}: DDF operators, with $A^i_{-n} \leftrightarrow \alpha^i_{-n}$

  - condition on the reference momentum: $\widetilde{p} \cdot q \stackrel{!}{=} 1$

    Del Giudice, Di Vecchia, Fubini 1972
    Brower 1972
    Skliros, Hindmarsh '11

  - disparate decays of states at very high N, *chaos*?

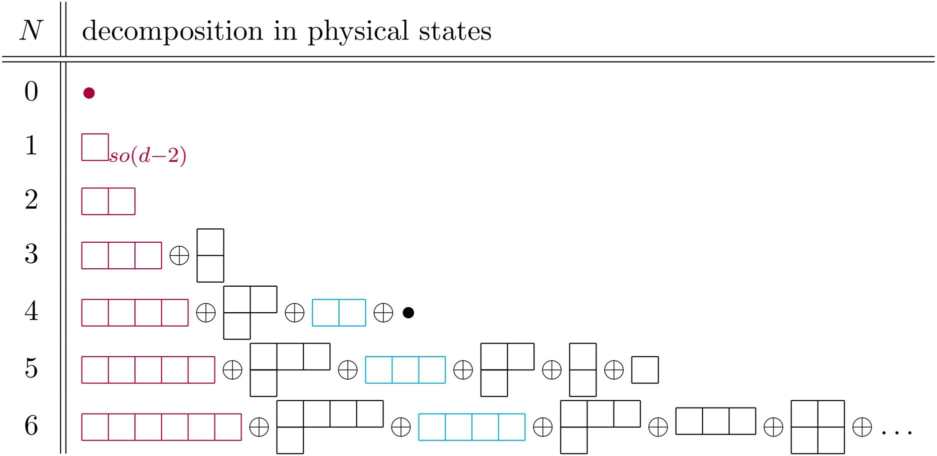
    Gross, Rosenhaus '21
    Rosenhaus '21
    Firrotta, Rosenhaus '22
    Bianchi, Firrotta, Sonnenschein, Weissman '22–'23

- Method 3: construct $SO(D-1)$ irreps from partition function

  Forcella, Hanany, J. Troost '10

Main challenge: how do excited states look like?

Visualisation: Regge *trajectories*



see e.g. Weinberg 1985, Mañes, Vozmediano 1989

beyond the leading Regge, the spectrum seems *repetitive*

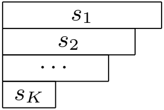
Is there a certain *pattern*?

Is the spectrum concealing a *bigger organizing symmetry*?

What does a state look like?

- *any* state has a polarization depicted by a Young diagram

$\epsilon^{\mu(s_1),\,\lambda(s_2),\ldots,\,\nu(s_K)}(p)$:



, $s_1 \geq s_2 \geq \ldots \geq s_K$

- for *physical* states: *dress* polarization by suitable polynomial

- notation: $X_\mu^{(k)} \equiv \partial^k X_\mu$, simplest physical example:

$$F = \epsilon^{\mu(s_1),\,\lambda(s_2),\ldots,\,\nu(s_K)}\, X^{(1)}_{\mu_1} \ldots X^{(1)}_{\mu_{s_1}} \ldots X^{(K)}_{\nu_1} \ldots X^{(K)}_{\nu_{s_K}}$$

lowest possible level: $N_{\min} = \sum_{i=1}^{K} s_i\, i$

- leading Regge: 1 row, spin–s at $N = s \Rightarrow$ simplest subexample

CM, Skvortsov '23

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Where does a state appear?

- there are *infinitely* many ways to render a given diagram physical
- *any* state appears at N_{min} and at higher levels $N = N_{\text{min}} + w$
  $\Rightarrow$ let's call w "*depth*"
- let's call trajectory a *fixed* number of rows at *fixed* w
  example: at $w = 0$, each value of K corresponds to a new trajectory ($K = 1$: leading Regge)
- example: *shifted* clone of leading Regge that starts at $N = 4$

trajectory	Young shape	spin	N	w	lowest member
leading Regge	s	$s \geq 0$	s	0	•
first "clone"	s	$s \geq 2$	$s + 2$	2	

CM, Skvortsov '23

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Let's reorganize the spectrum

- instead of N, let's **use w to organize the spectrum**

w	trajectories
0	$s \oplus \left(s-2 \oplus s-4 \oplus \dots \right) \oplus s-5 \oplus \dots$
1	$\left(s-3 \oplus s-5 \oplus \dots \right) \oplus \dots$
2	$s-2 \oplus 2^* \; s-4 \oplus \dots$
3	$s-3 \oplus \dots$
4	$3^* \; s-4 \oplus \dots$

- *complexity*: measured by w (and number of rows)
- remember: finite K for every trajectory!

CM, Skvortsov '23

All trajectories at once

- let's consider the *most general* vertex operator:

$$\mathbb{V}_F(z,p) = F\left[X^{(1)}, X^{(2)}, ...\right] e^{ip\cdot X^{(0)}}$$

- let's impose $[Q, \mathbb{V}_F(z,p)] =$ tot. deriv. $\Rightarrow$ obtain:
  - 1 on–shell condition

$$(L_0 - 1)F = \left(\sum_{n=0} n\, X^{(n)} \cdot \frac{\delta}{\delta X^{(n)}} + \alpha' p^2 - 1\right) F = 0$$

  - n differential constraints

$$L_{n>0}F \quad = \quad \Bigg[2\alpha'\, n!\, i\, p \cdot \tfrac{\delta}{\delta X^{(n)}} + \alpha' \sum_{m=1}^{m=n-1} m!(n-m)! \frac{\delta^2}{\delta X^{(m)} \cdot \delta X^{(n-m)}}$$

$$- \sum_{m=0} \frac{(n+m+1)!}{m!} X^{(m+1)} \cdot \frac{\delta}{\delta X^{(n+m+1)}}\Bigg] F = 0$$

- now leading Regge is the special case with no descendants

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A bigger organizing symmetry

- let's define the operators

$$T^k{}_l \equiv X^{(k)} \cdot \frac{\delta}{\delta X^{(l)}} \quad , \quad T_{kl} \equiv \frac{\delta^2}{\delta X^{(k)} \cdot \delta X^{(l)}} \quad , \quad T^{kl} \equiv X^{(k)} \cdot X^{(l)}$$

which check Young symmetry & tracelessness

$$T^k{}_l F \;\; = 0 \quad (k < l) \quad , \quad T_{kl} F = 0$$

and act on the k indices, whose range is *infinite*

- $\{T^k{}_l, T_{kl}, T^{kl}\}$ form $sp(2\bullet) \Rightarrow$ *Howe dual* to $so(D-1,1)$

for the duality of commuting algebras see Howe 1989

idea: use this *bigger* symmetry to construct trajectories

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- simplification: transverse subspace is sufficient
$\Rightarrow p$ appears only within the transverse metric

see e.g. Mañes, Vozmediano 1989

A bigger organizing symmetry

- let's rewrite our constraints as

$$(L_0 - 1)F = \Big(\textstyle\sum_{n=0} n\, T^n{}_n + \alpha' p^2 - 1 \Big) F = 0$$

$$L^{\perp}_{n>0} F = \left[\alpha' \sum_{m=1}^{n-1} m!(n-m)!\, T^{\perp}_{m,n-m} - \sum_{m=0} \frac{(n+m+1)!}{m!} T^{m+1}{}_{n+m+1} \right] F = 0$$

- on any function F need only

$$(L_0 - 1)F = 0 \quad , \quad L^{\perp}_1 F = 0 \quad , \quad L^{\perp}_2 F = 0$$

- Howe duality now implies that:

  the lowest weight states of $sp(2\bullet)$ *solve* the Virasoro constraints

  - example: $w = 0$

$$F = \epsilon^{\mu(s_1),\, \lambda(s_2), \ldots,\, \nu(s_K)}\, X^{(1)}_{\mu_1} \ldots X^{(1)}_{\mu_{s_1}} \ldots X^{(K)}_{\nu_1} \ldots X^{(K)}_{\nu_{s_K}}$$

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Building the spectrum: a new technology

- can distinguish 2 kinds of embeddings:
  "*principal*" ($w = 0$) and "*non–principal*" ($w > 0$)
- **idea**: use $sp(2\bullet)$ creation operators to construct *deeper* trajectories

$$F^f_{w>0} \equiv f(T_\perp^{mn}, T^k{}_l)\, F_{w=0} \quad , \quad k > l \, ,$$

  where f: *trajectory–shifting* operator of weight w

- example: take leading Regge ($w = 0$)

$$F \equiv \epsilon^{a(s)} X^{(1)}_{a(s)} \quad , \quad T^\perp_{11} F = 0$$

  and dress it to create a subleading trajectory at $w = 2$

$$F^f = \left[\frac{\gamma_1}{\alpha'} T_\perp^{11} + \gamma_2 T^3{}_1 + \gamma_3 (T^2{}_1)^2 \right] F$$

$$\boxed{\quad s \quad} = f \times \boxed{\quad s \quad}$$

CM, Skvortsov '23

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Building the spectrum: a new technology

- for the trajectory $\boxed{\;s\;}$ at $w = 2$, solve the Virasoro constraints:

$$\Rightarrow \quad \gamma_2 = \gamma_1 \frac{D + 2s - 1}{3s} \quad , \quad \gamma_3 = -\gamma_1 \frac{D + 2s - 1}{2(s-1)}$$

1 free parameter (other than spin)
spin is not fixed ⇒ full trajectory is known!

- singularity at $s \to 1$: determines lightest member–state

- example: the lightest member–state $\boxed{\;\;}\boxed{\;\;}$ of the clone

$$\begin{aligned} F_B &= B_{\mu\nu}\, i\partial X^\mu i\partial X^\nu \\ F_{\widetilde{B}} &= \widetilde{B}_{\mu\nu}\left[-\tfrac{3}{\alpha'}\, i\partial X^\mu i\partial X^\nu i\partial X^\kappa i\partial X_\kappa + 29\, i\partial X^\mu i\partial^3 X^\nu - 87\, i\partial^2 X^\mu i\partial^2 X^\nu\right] \end{aligned}$$

- amplitudes accessible! e.g. scattering of tachyon off a spin–s

$$\mathcal{A}^{s00}_{w=2} \sim \mathcal{A}^{s00}_{w=0}(-D + 4s + 31) \sim (p_2 \cdot \epsilon)^s (-D + 4s + 31)$$

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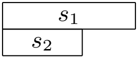
Building the spectrum: a new technology

- 2–row example:

$$F = \epsilon^{\mu(s_1),\,\nu(s_2)} X^{(1)}_{\mu_1} \ldots X^{(1)}_{\mu_{s_1}} X^{(2)}_{\nu_1} \ldots X^{(2)}_{\nu_{s_2}}$$

$$f_{w=1} \;:\; {T^2}_1\,, \quad {T^3}_2$$

$$f_{w=2} \;:\; T^{11}_{\perp}\,, \quad ({T^2}_1)^2\,, \quad {T^2}_1 {T^3}_2\,, \quad {T^3}_2 {T^3}_2\,, \quad {T^3}_1\,, \quad {T^4}_2$$

Young shape	w	N	possible f terms	# of free parameters	lightest member
	0	$s_1 + 2s_2$	trivial	1	
	1	$s_1 + 2s_2 + 1$	2	1	
	2	$s_1 + 2s_2 + 2$	6	3	

- multiplicity = number of free parameters (other than spin)

- other deep trajectories available

CM, Skvortsov '23

Is the *complete* spectrum within reach?

What can the complete spectrum teach us about fields and strings and, ultimately, about quantum gravity?

Other examples of subleading states

- let's look at massive spin–2 superstring states in 4D

- *lightest* such brane state: leading

$$V^{(-1)}_{M,\text{open}}(z,p) = g_o\, T^a\, e^{-\phi}\, M_{\mu\nu}\, \partial X^\mu \psi^\nu\, e^{ipX} \quad , \quad p^2 = -\tfrac{1}{\alpha'}$$

- (some of the) *lightest* such bulk states: subleading

$$V^{(-1,-1)}_{M,\text{closed}}(z,\bar{z},k) = g_c\, e^{-\phi-\widetilde{\phi}}\, M_{\mu\nu}\, \mathcal{J}\, \widetilde{\mathcal{J}}\, \psi^\mu\, \widetilde{\psi}^\nu\, e^{ik\cdot X} \quad , \quad k^2 = -\tfrac{4}{\alpha'}$$

  $\Rightarrow$ *mimics* graviton but is *not* highest spin of supermultiplet
  ($\mathcal{J}$: Kac–Moody current due to compactification)

- can compute e.g. 3–point string amplitudes
  bulk massive spin–2: manifest *double copy* structure

Lüst, CM, Mazloumi, Stieberger '21, '23

Other examples of subleading interactions

- $\exists$ consistent *field* theory of massless and massive spin–2 states: bimetric theory around flat backgrounds (kinetic term for massive spin–2: Einstein–Hilbert)

de Rham, Gabadadze, Tolley '10
Hassan, Rosen '11
Hassan, Schmidt–May, von Strauss '12
Babichev, Marzola, Raidal, Schmidt–May, Urban, Veermäe, von Strauss '16

- extract "effective" vertices from our *finite cubic* amplitudes, e.g.:

$$\mathcal{L}^{\text{eff}}_{\text{M}^3} \quad = \quad \frac{g_c}{\alpha'} \Big\{ y\, [M^3] + 2\alpha' M^{\mu\nu} \big[\partial_\mu M_{\rho\sigma} \partial_\nu M^{\rho\sigma} - x\, \partial_\nu M_{\rho\sigma} \partial^\sigma M^\rho_\mu \big] \Big\}$$

$$(x, y) = \begin{cases} (2,0) & \text{if} \quad M_{\mu\nu} \quad \text{is a bulk state of} \quad \mathcal{N} = \widetilde{4}, \widetilde{8} \\ (3,1) & \text{if} \quad M_{\mu\nu} \quad \text{is a brane state} \end{cases}$$

- *match* for bulk state with 1–parameter subfamily of (on–shell, cubic) bimetric theory

Lüst, CM, Mazloumi, Stieberger '21 and '23

curiously same family as:
Bonifacio, Hinterbichler, Joyce, Rosen '17
Momeni, Rumbutis, Tolley '20 and Johnson, Jones, Paranjape '20
Engelbrecht, Jones, Paranjape '22

Concluding remarks

- massive strings and fields: only cubic level similarities

Sagnotti, Taronna '10
Lüst, CM, Mazloumi, Stieberger '21, '23

- we have a new technology that excavates entire trajectories
  - key observation: Howe duality between $sp(2\bullet)$ and $so(D-1,1)$
  - idea: use this bigger than the Virasoro symmetry
  - gearwheel: $sp(2\bullet)$ creation operators

CM, Skvortsov '23

- field–theory amplitudes for spinning black holes

Guevara, Ochirov, Vines '18
Maybee, O'Connell, Vines '19

"massive" higher–spin symmetry $\Rightarrow$ 3–point Kerr amplitudes!

Cangemi, Chiodaroli, Johansson, Ochirov, Pichini, Skvortsov '22, '23

leading Regge: no BH features

Pichini, Cangemi '22

- Can strings yield Kerr amplitudes? can we treat the *entire* spectrum? chaos? holography?