# GW from eMD to RD Transition


## Di Liu

LAPTh Journal Club


## 28/03/2024

**Based on:**

Wands, arXiv: 0901.0989, Early Work

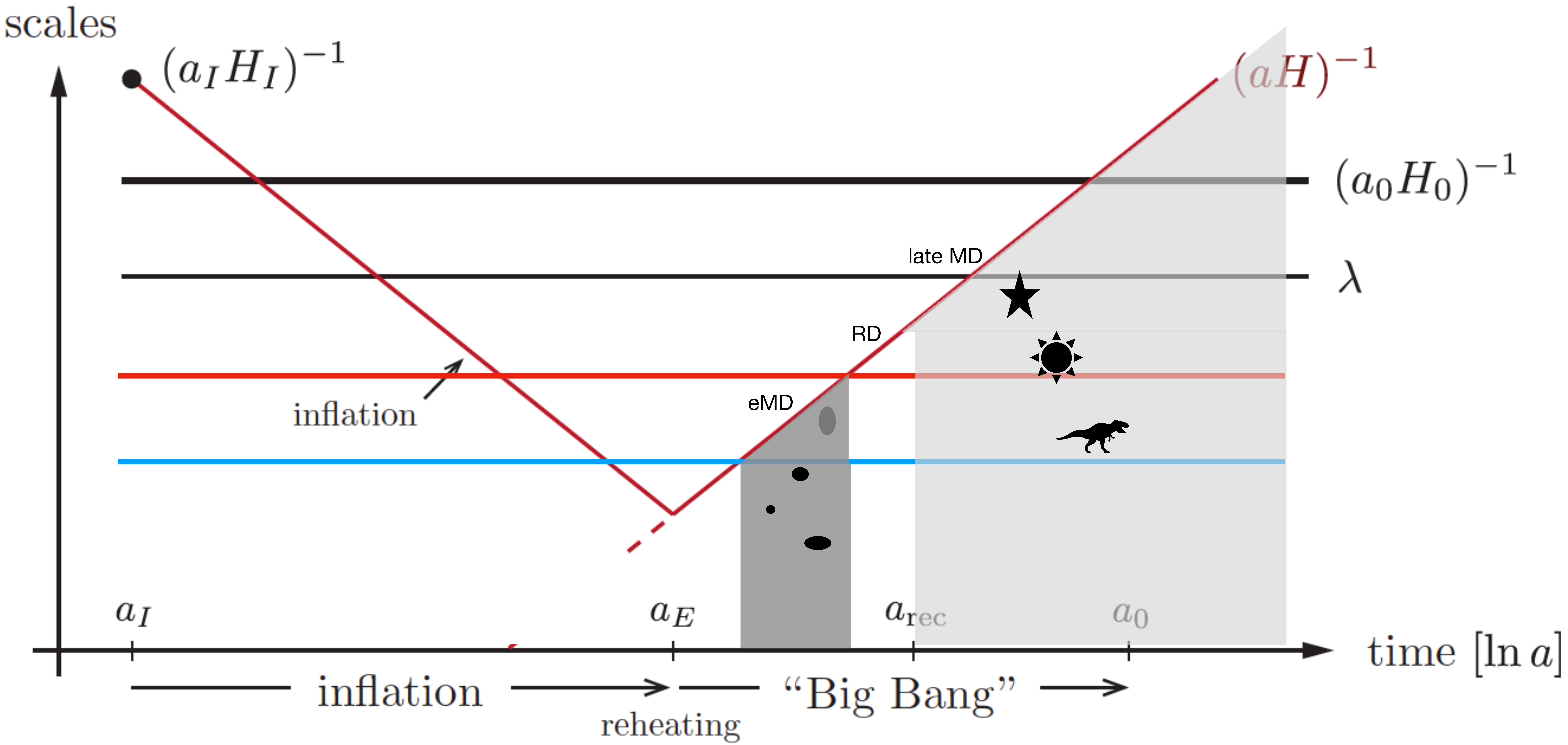
Inomata, arXiv:1904.12879, Rapid Transition

Inomata, arXiv:1904.12878, Gradual Transition

Graham, arXiv: 2311.12340, Intermediate Regime

Yanagida, arXiv: 2003.10455, pBH Domination

# Superhorizon Modes



# Outline

- Perturbed Metric Modes
- Power Spectrum and GW Relic
- Fluct. Evolution in eMD and RD
- Rapid and Gradual Transitions
- Concrete Models

# Primordial Curvature Perturbations

Perturbed FRW Metric

$$ds^2 = a^2(\eta)[-(1+2\Phi)d^2\eta + [(1-2\Psi)\delta_{ij} + 2F_{(i,j)} + h_{ij}]dx^i dx^j]$$

$\eta$: conformal time

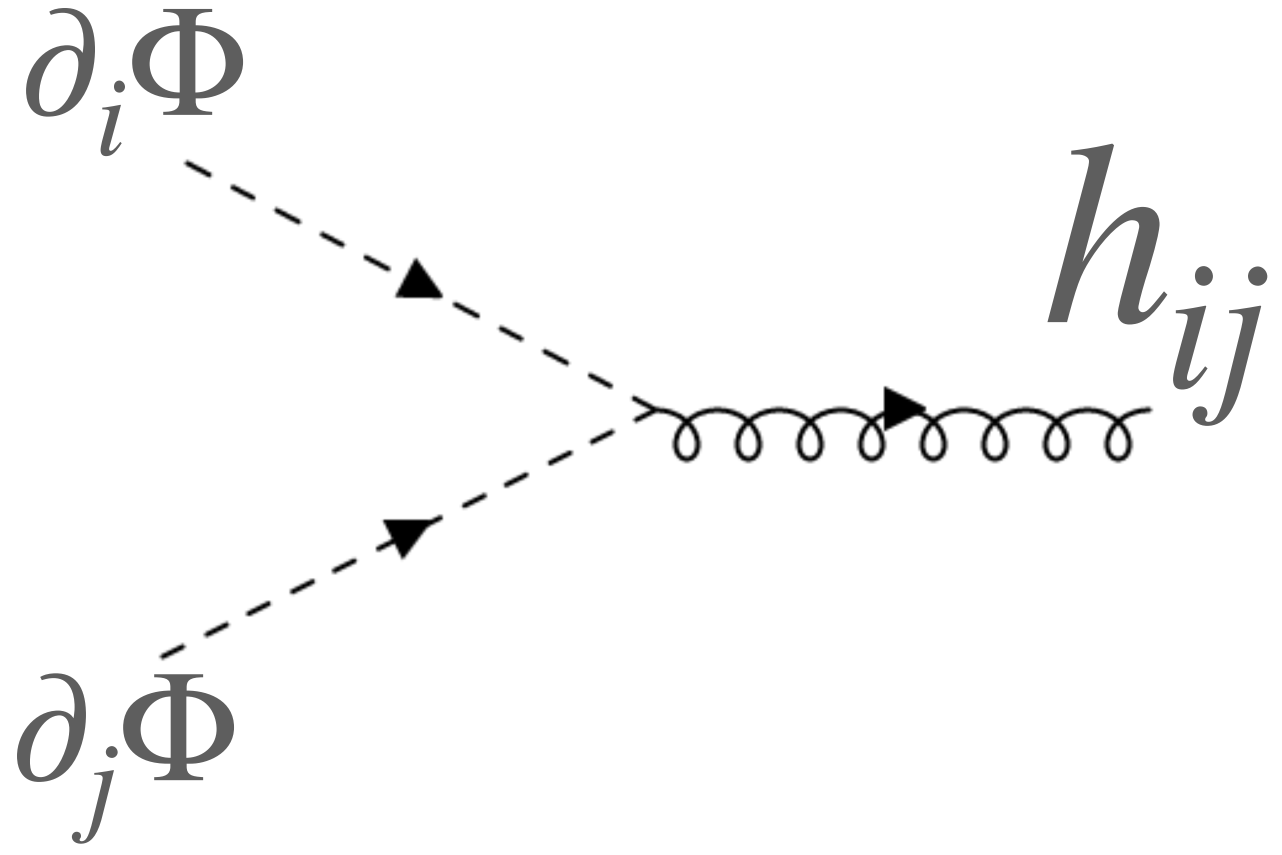
Linearized Einstein equation (First Order):

$$\nabla^2\Phi - 3\mathscr{H}\left(\Phi' + \mathscr{H}\Phi\right) = \frac{3\mathscr{H}^2}{2}\delta\,, \quad \delta = (\rho - \bar{\rho})/\bar{\rho} = \delta_m + \delta_r\,,$$

# Tensor ($h_{ij}$) Induced by Scalar ($\Phi$)

Second order

$$h''_{ij} + 2\mathscr{H} h'_{ij} + k^2 h_{ij} = S_{ij}^{TT}$$




$$S_{ij} = 4\Phi\partial_i\partial_j\Phi + 2\partial_i\Phi\partial_j\Phi - \frac{4}{3\mathscr{H}^2}\partial_i\left(\Phi' + \mathscr{H}\Phi\right)\partial_j\left(\Phi' + \mathscr{H}\Phi\right)$$

**Wands (2009)**

# Power Spectra

## Two-point function

$$\langle h_{\mathbf{k}}(\eta) h_{\mathbf{k}'}(\eta)\rangle = \left(\frac{40 g(k\eta)}{3(2\pi)^{3/2}}\right)^2 k^{-4} \int_0^{\mathbf{k}_{dom}} d^3\widetilde{\mathbf{k}}\, e(\mathbf{k},\widetilde{\mathbf{k}})\, d^3\widetilde{\mathbf{k}'}\, e(\mathbf{k}',\widetilde{\mathbf{k}'})\, \langle \Phi_{\mathbf{k}-\widetilde{\mathbf{k}}}\, \Phi_{\widetilde{\mathbf{k}}}\, \Phi_{\mathbf{k}'-\widetilde{\mathbf{k}'}}\, \Phi_{\widetilde{\mathbf{k}'}}\rangle$$

## Power Spectra

$$\langle h_{\mathbf{k}}(\eta) h_{\mathbf{k}'}(\eta)\rangle = \frac{\pi^2}{k^3}\delta^3(\mathbf{k}+\mathbf{k}')\mathcal{P}_h(k,\eta)$$

$$\langle \phi_{\mathbf{k}}(\eta) \phi_{\mathbf{k}'}(\eta)\rangle = \frac{2\pi^2}{k^3}\delta^3(\mathbf{k}+\mathbf{k}')\mathcal{P}_\zeta(k,\eta)$$

# GW Power Spectrum

$$\overline{\mathcal{P}_h(\eta,k)} = 4\int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{4v^2-(1+v^2-u^2)^2}{4vu}\right)^2 \overline{I^2(u,v,k,\eta,\eta_\mathrm{R})}\mathcal{P}_\zeta(uk)\mathcal{P}_\zeta(vk).$$

$I$ depends on $\Phi(\eta)$

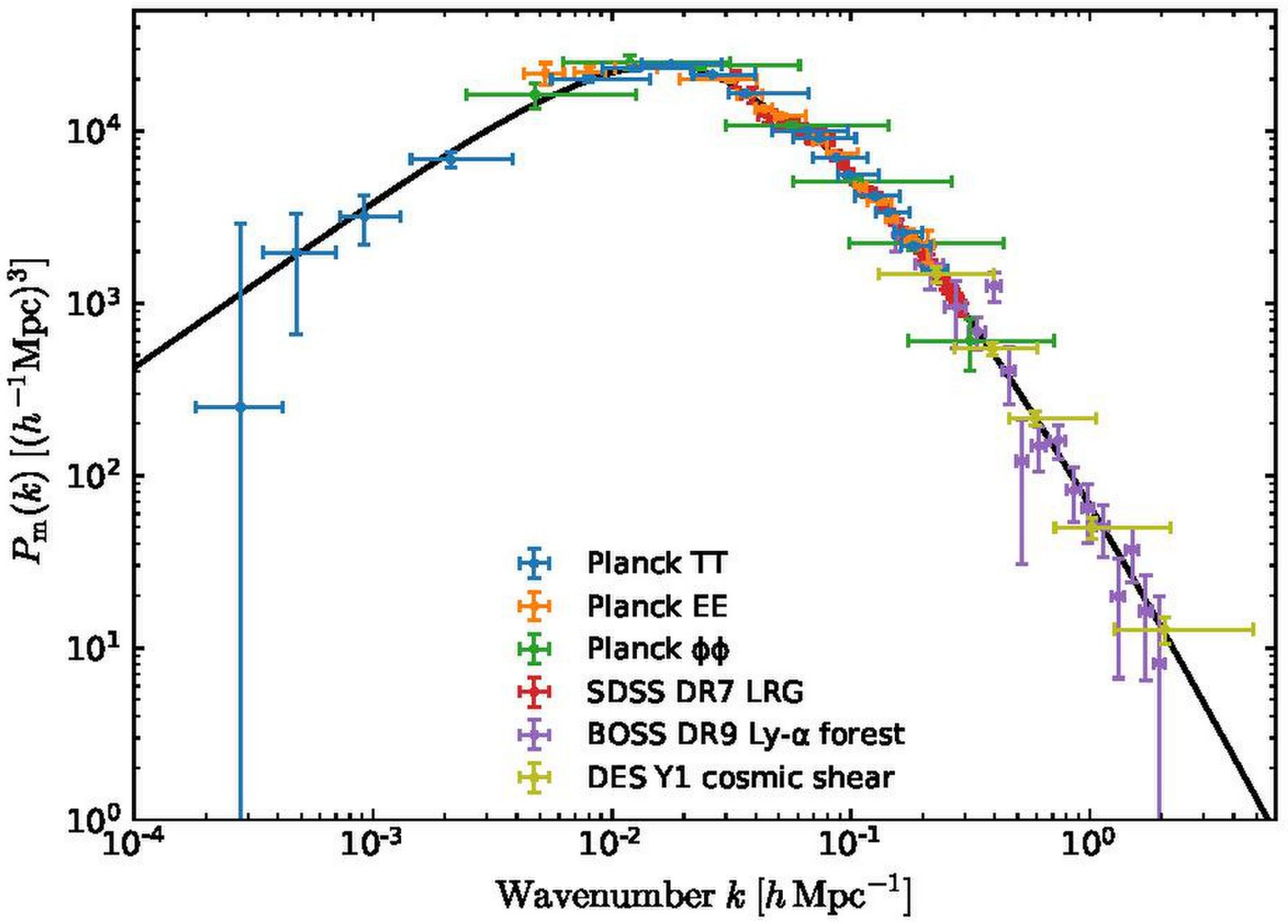
$$I(u,v,k,\eta,\eta_\mathrm{R}) = \int_0^x \mathrm{d}\bar{x}\frac{a(\bar{\eta})}{a(\eta)}kG_k(\eta,\bar{\eta})f(u,v,\bar{x},x_\mathrm{R}),$$

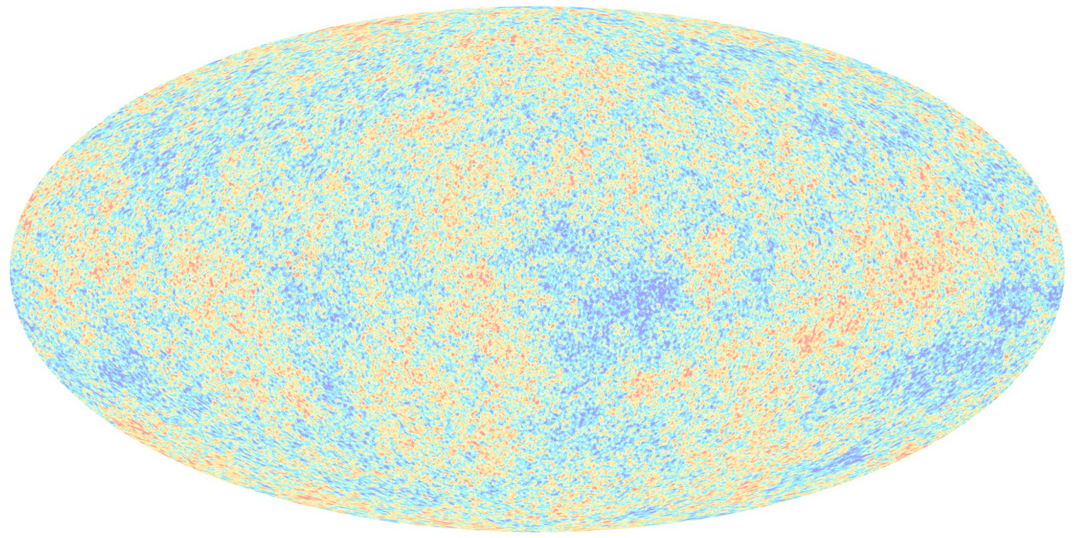
$$f(u,v,\bar{x},x_\mathrm{R}) = \frac{3\left(2(5+3w)\Phi(u\bar{x})\Phi(v\bar{x}) + 4\mathcal{H}^{-1}(\Phi'(u\bar{x})\Phi(v\bar{x}) + \Phi(u\bar{x})\Phi'(v\bar{x})) + 4\mathcal{H}^{-2}\Phi'(u\bar{x})\Phi'(v\bar{x})\right)}{25(1+w)}.$$

Green's function $G_k$

$$G_k''(\eta,\bar{\eta}) + \left(k^2 - \frac{a''(\eta)}{a(\eta)}\right)G_k(\eta,\bar{\eta}) = \delta(\eta-\bar{\eta}).$$

# Primordial Scalar Power Spectrum



$$\mathcal{P}_\zeta(k) = \frac{9}{25}\Delta^2_{\mathcal{R}}\left(\frac{k}{k_*}\right)^{n_s-1} \quad , \quad \Delta^2_{\mathcal{R}} = 5.3 \times 10^{-9}\ , n_s = 0.97$$

Planck 2018

# GW Relic Abundance

$$\rho_{GW} = \frac{1}{32\pi G}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle = \frac{k^2}{32\pi G a^2}\int d(\ln k)\ \mathcal{P}_h(k,\eta)\,.$$

## GW abundance from power spectrum

$$\Omega_{\rm GW}(\eta,k) = \frac{\rho_{\rm GW}(\eta,k)}{\rho_{\rm tot}(\eta)} = \frac{1}{24}\left(\frac{k}{\mathcal{H}(\eta)}\right)^2 \overline{\mathcal{P}_h(\eta,k)}$$

# Matter Domination (MD) and Radiation Domination (RD)

Time Evolution: $\Phi$

$$\Phi'' + 3\left(1 + c_s^2\right)\mathscr{H}\Phi' - c_s^2\nabla^2\Phi \approx 0$$

Sound speed: nonrelativistic matter $c_s = 0$, radiation $c_s = 1/3$

no pressure, constant

pressure, oscillation

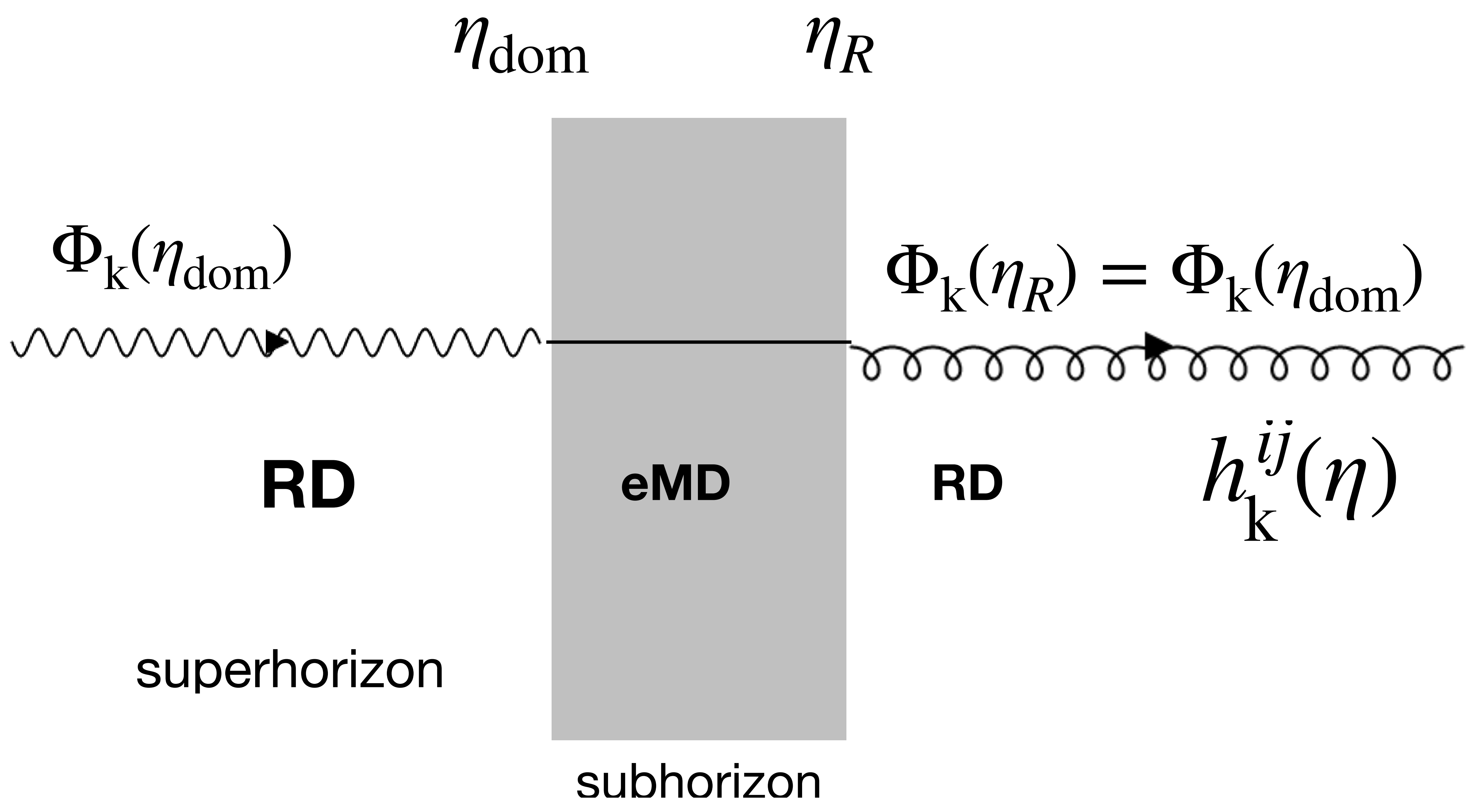
# MD and RD

Time Evolution: energy density fluctuation

For $k \gg aH$

| | MD | RD |
|---|---|---|
| $\delta_m$ | $a$ | $\log a$ |
| $\delta_r$ | $\cos(k\eta) + c$ | $\cos(k\eta)$ |

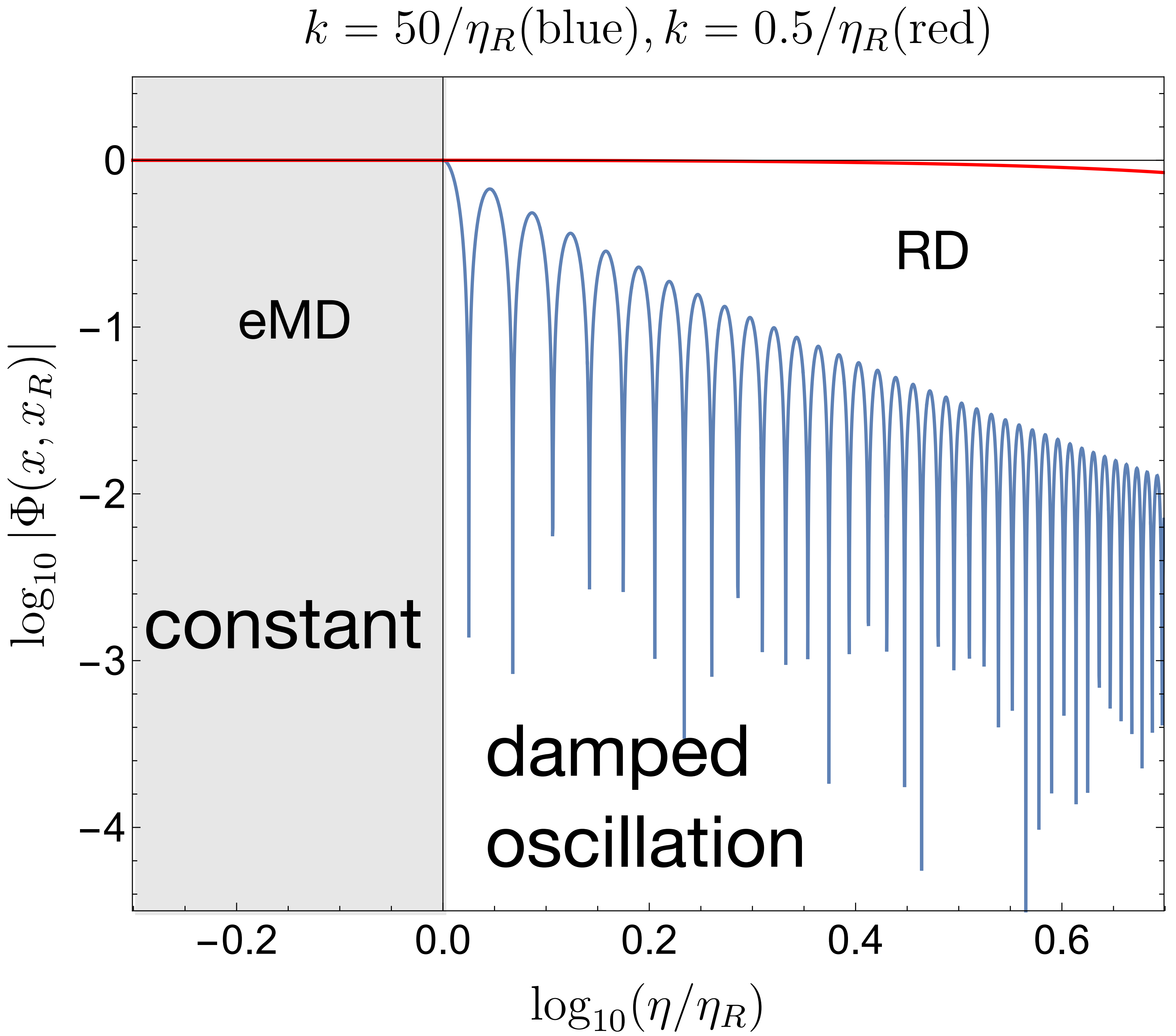
- $k = \eta_{dom}^{-1}$, $\Phi_k$ enters horizon




- In eMD, $\eta < \eta_R$, $\Phi_k$ remains const.

$$\eta_R : \rho_m = \rho_R$$

Rapid Transition

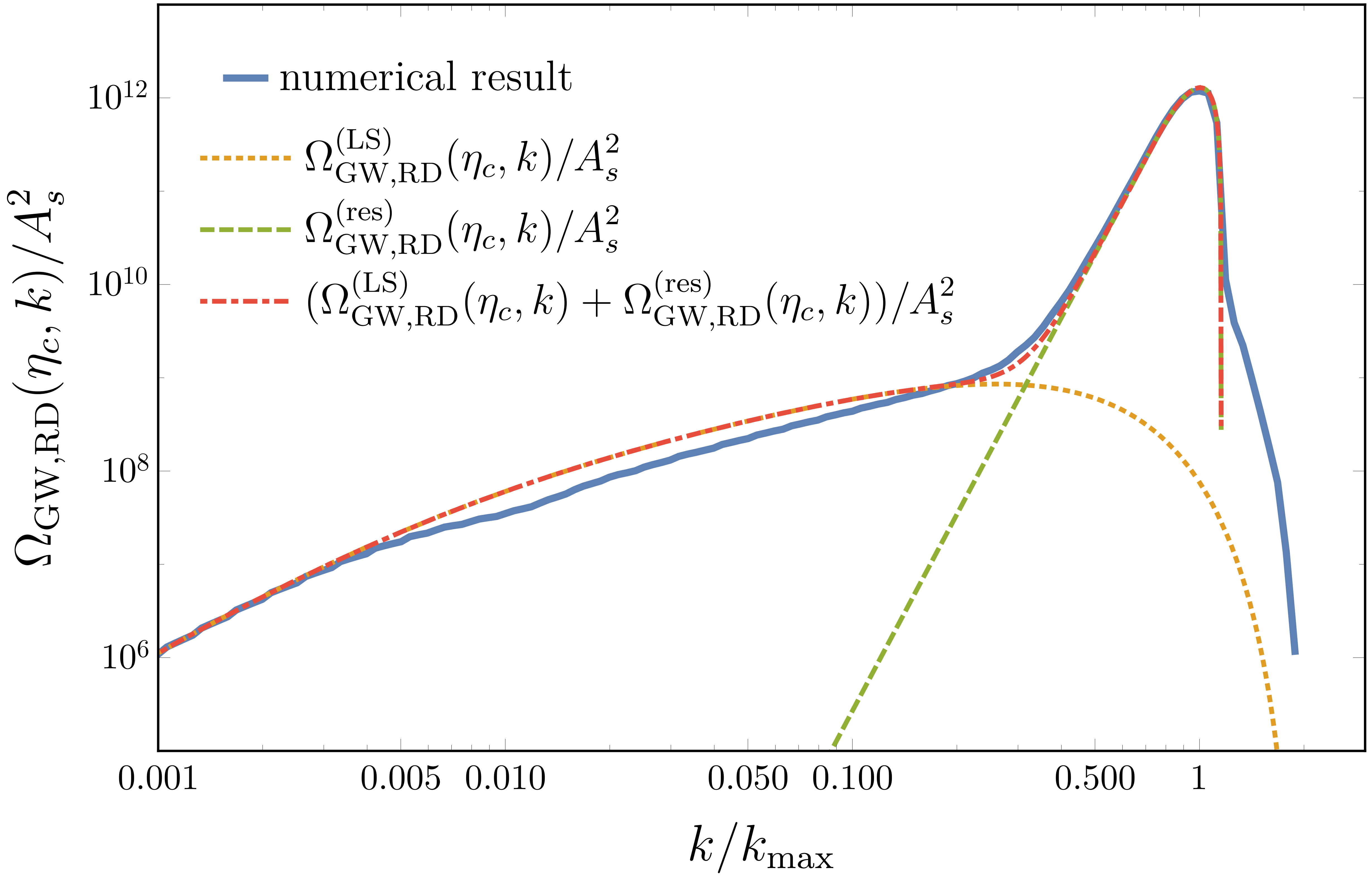



- In RD, $\eta > \eta_R$, $\Phi$ ocsillates

Rapid Transition

## Resonance Profile

$$h''_{\mathbf{k}} + \frac{4}{\eta} h'_{\mathbf{k}} + k^2 h_{\mathbf{k}} = \frac{1}{k^2 \eta^4} \mathcal{F}\left[\sin(k\tau), \cos(k\tau)\right]$$

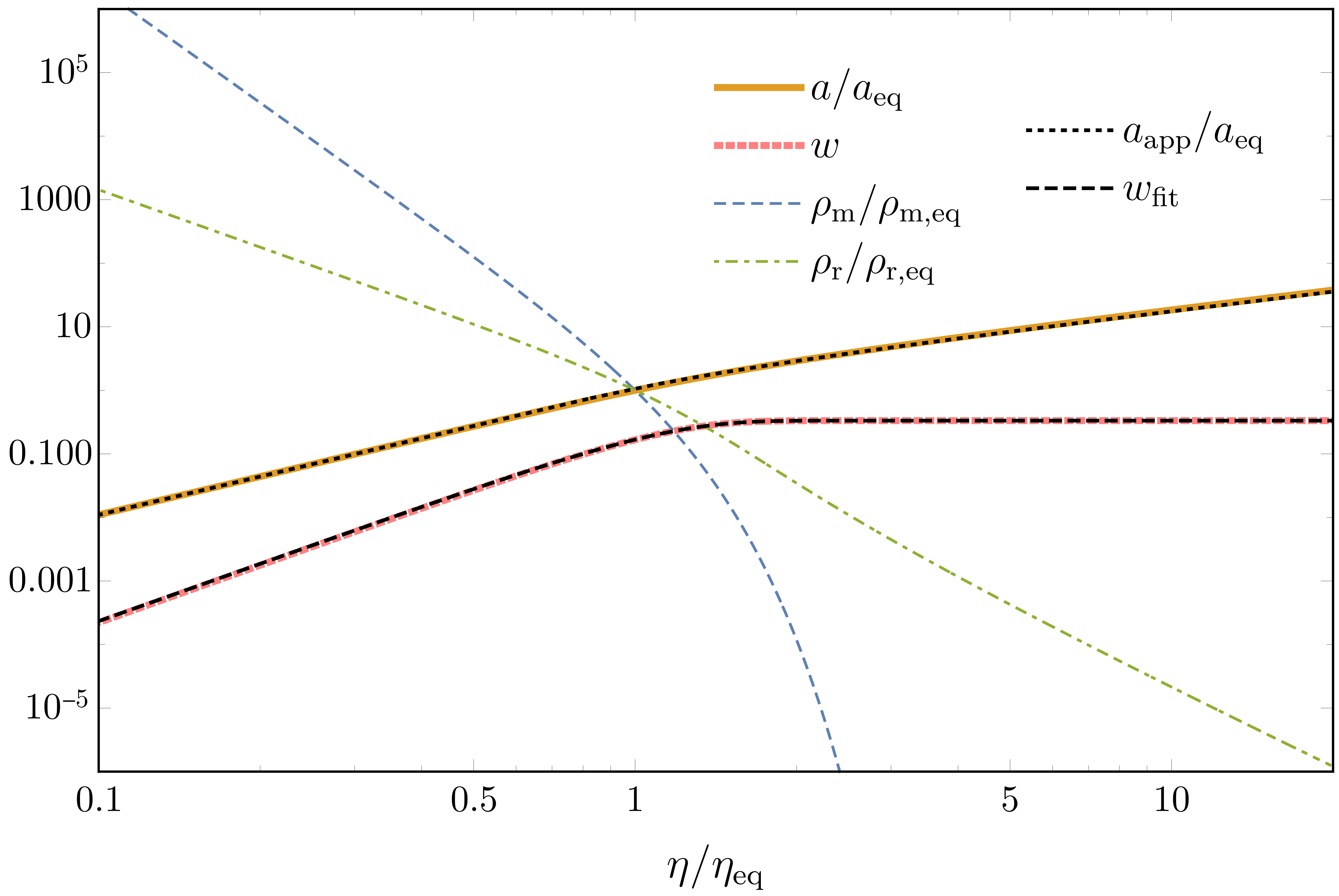



**Inomata(2019)**

eMD via unstable particle:

constant decay rate $\Gamma$

$$\rho_{\rm m}' = -(3\mathcal{H} + a\Gamma)\rho_{\rm m},$$
$$\rho_{\rm r}' = -4\mathcal{H}\rho_{\rm r} + a\Gamma\rho_{\rm m},$$




Gradual Transition

Matter to radiation transition rate $\Gamma/\mathscr{H} \sim \mathscr{O}(1)$

**Inomata(2019)**

## Gradual Transition

- During eMD, $\eta < \eta_R$, $\delta_m$ grows but $\delta_r$ does not.

$$\delta_m \gg \delta_r \text{ at } \eta = \eta_R$$

- After $\eta_R$, the evolution of $\Phi$ is dominated by $\rho_m \delta_m$ for a while.

oscillating

$$\Phi_k \simeq \frac{3\mathscr{H}}{2k^2}\left(\frac{\rho_m}{\rho}\delta_m + \frac{\rho_R}{\rho}\delta_r\right)$$

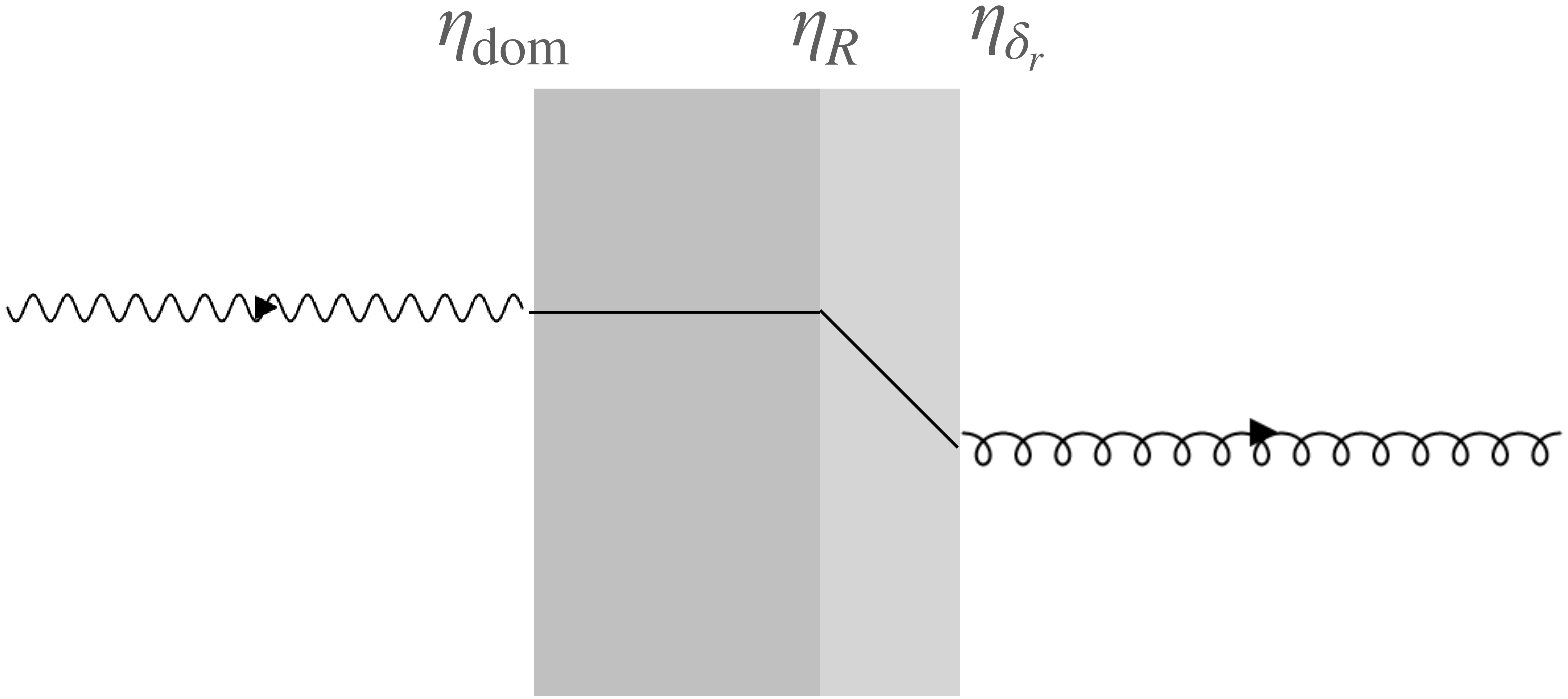
exponentially decreasing

# Gradual Transition

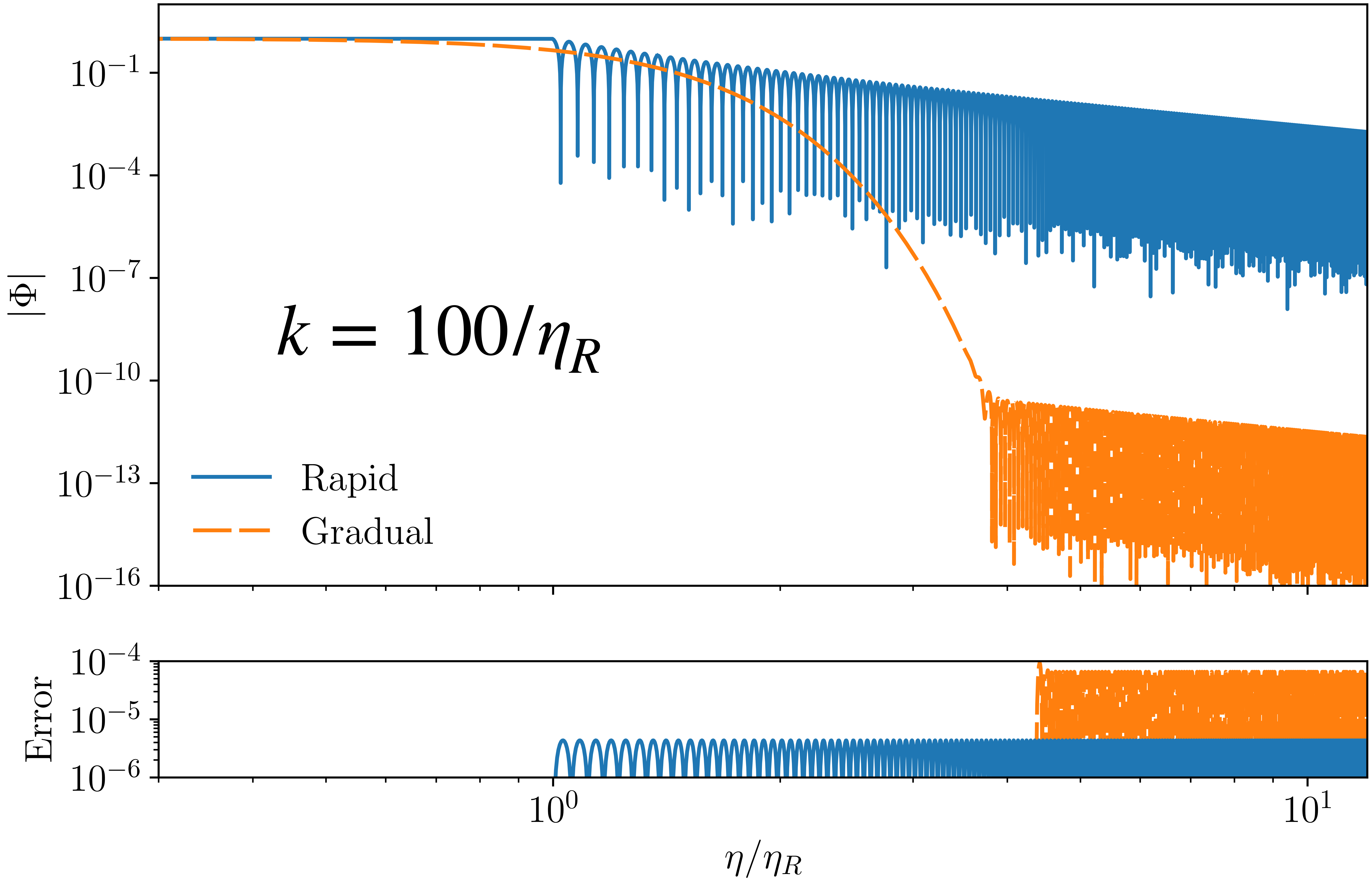
- $\eta_R < \eta < \eta_{\delta_r}$, $\Phi$ is proportional to $\rho_m \delta_m$ (exponentially decreasing).

$$\eta_{\delta_r} : \rho_m \delta_m = \rho_R \delta_r$$

- $\eta > \eta_{\delta_r}$, $\Phi$ is taken over by $\rho_R \delta_r$ (suppressed oscillation).

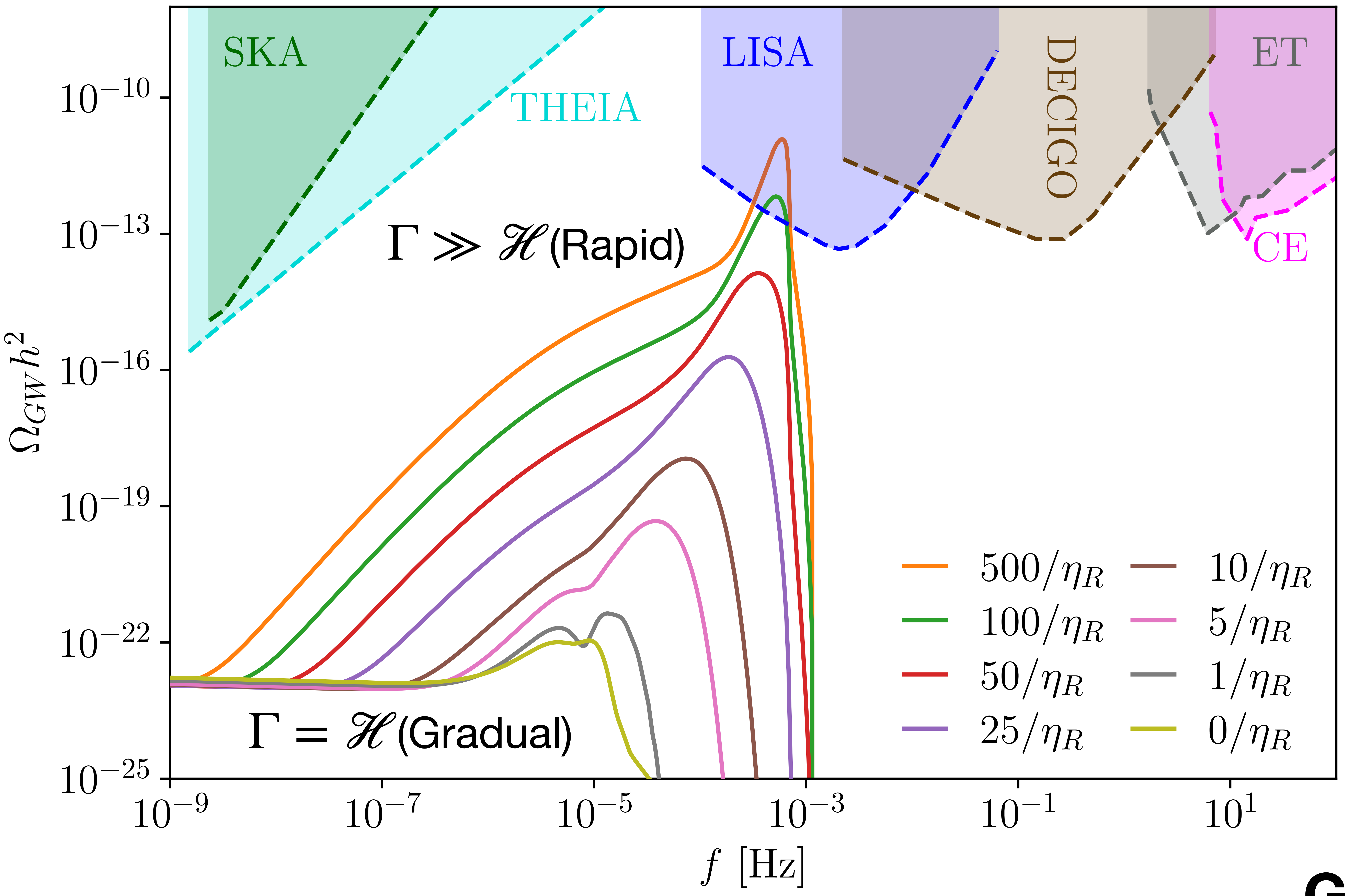


# Rapid and Gradual Transitions



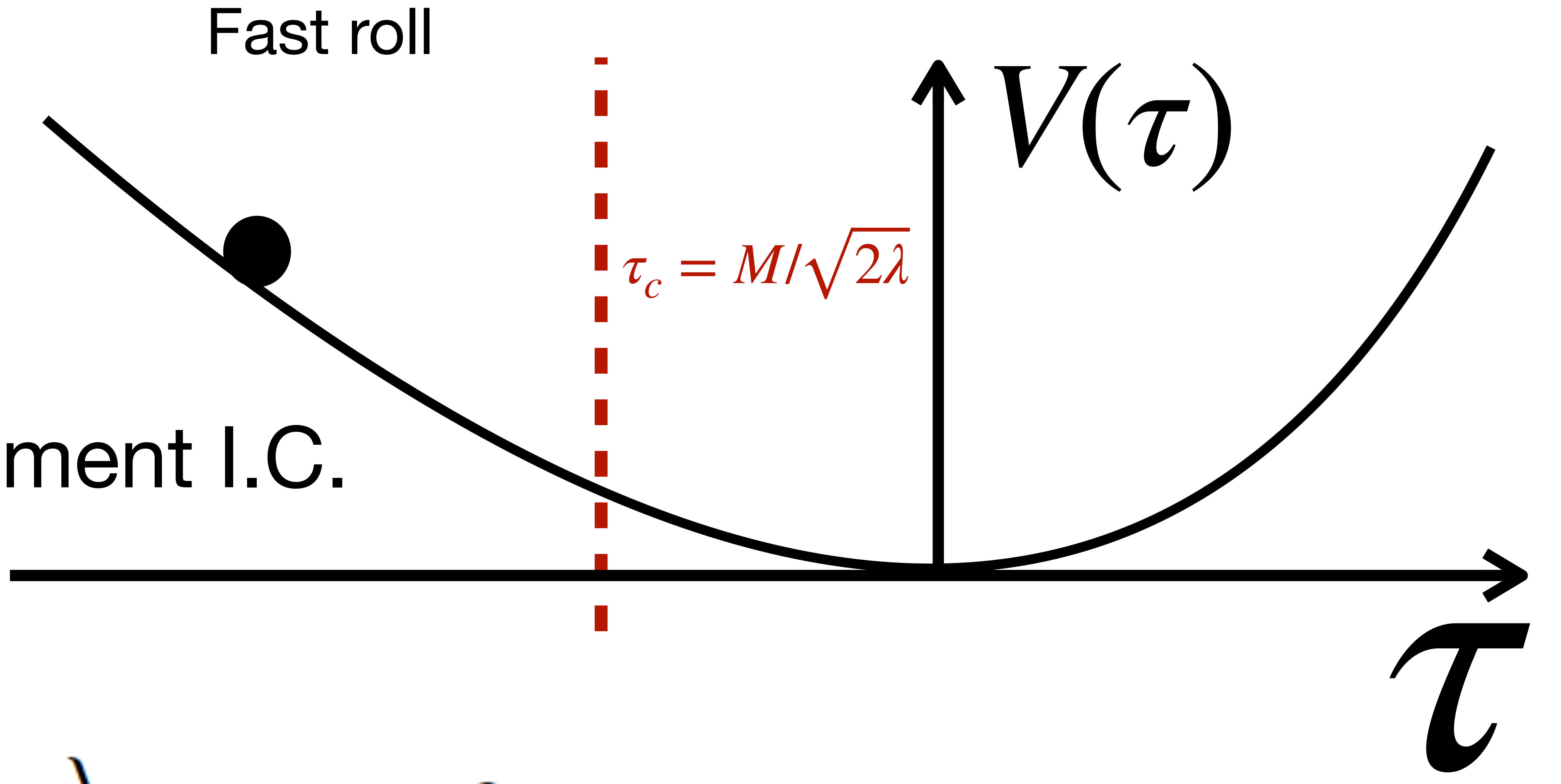
Intermediate Transition

GW signal from $T_R = 100\text{GeV}$




**Graham (2023)**

# Concrete Models




## 1) Time-varying Mass

parent particle: $\phi$

daughter particle: $\chi$

triggeron: $\tau$

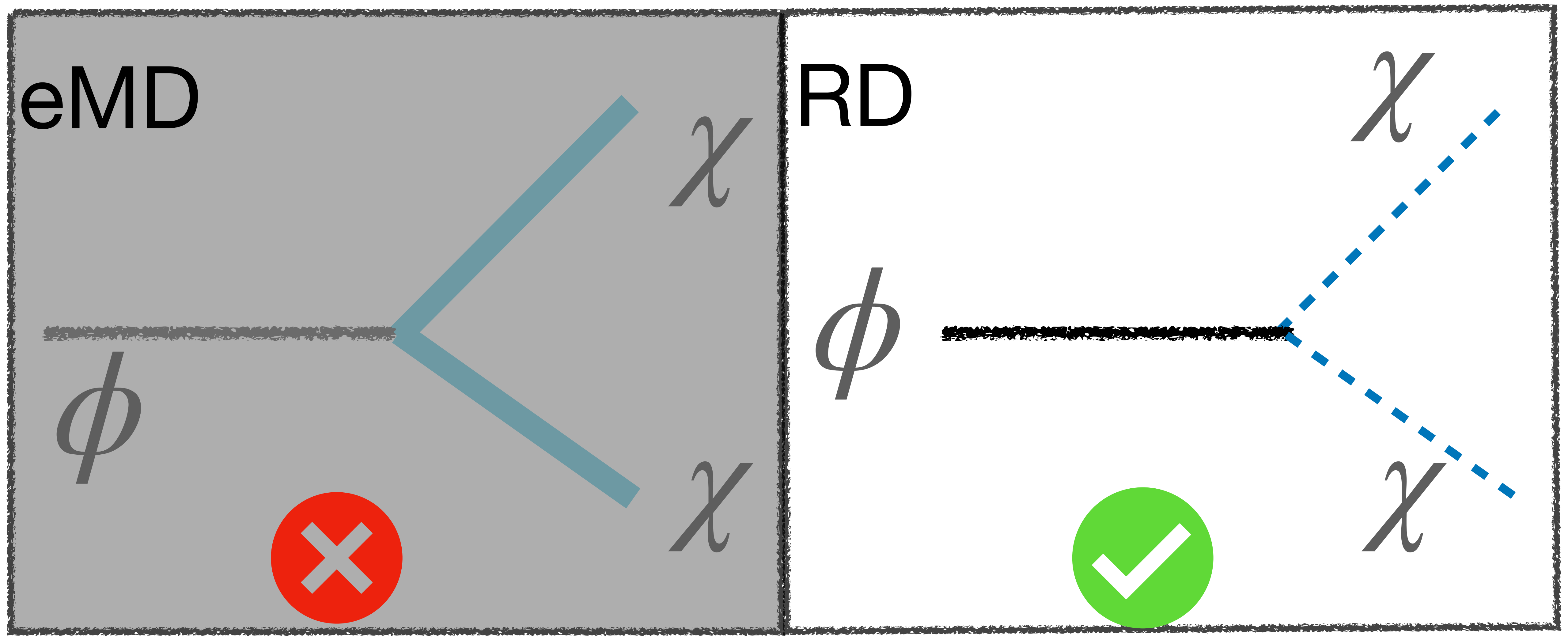
$$V = \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\tau^2 + \frac{\lambda}{4}\tau^2\chi^2 + \frac{c}{2}M\phi\chi^2$$

$\tau$-dependent mass for $\chi$

$m_{\chi,\mathrm{eff}}^2 = \langle \lambda\tau^2/2 \rangle$ is the effective mass squared of $\chi$

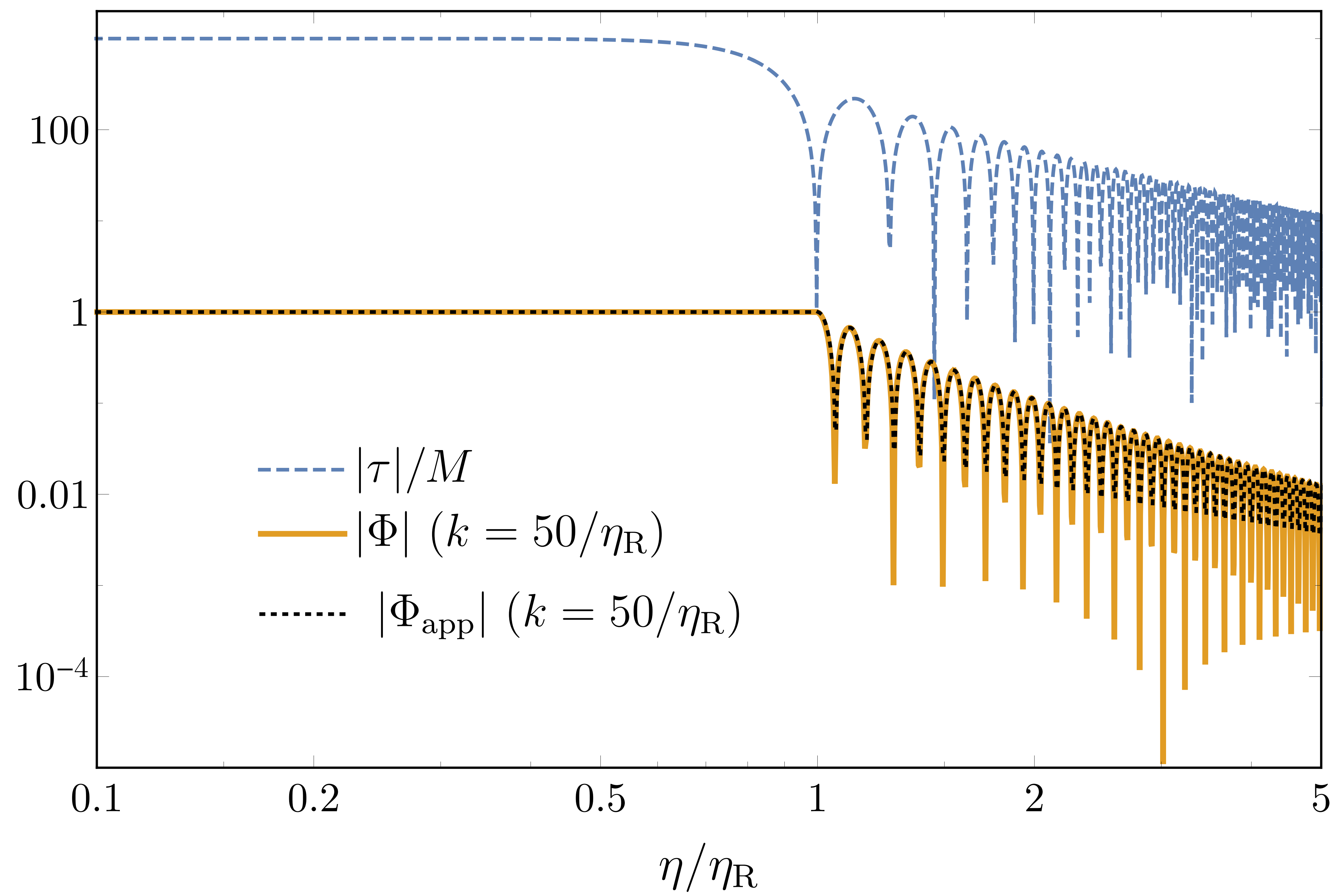
**Inomata(2019)**

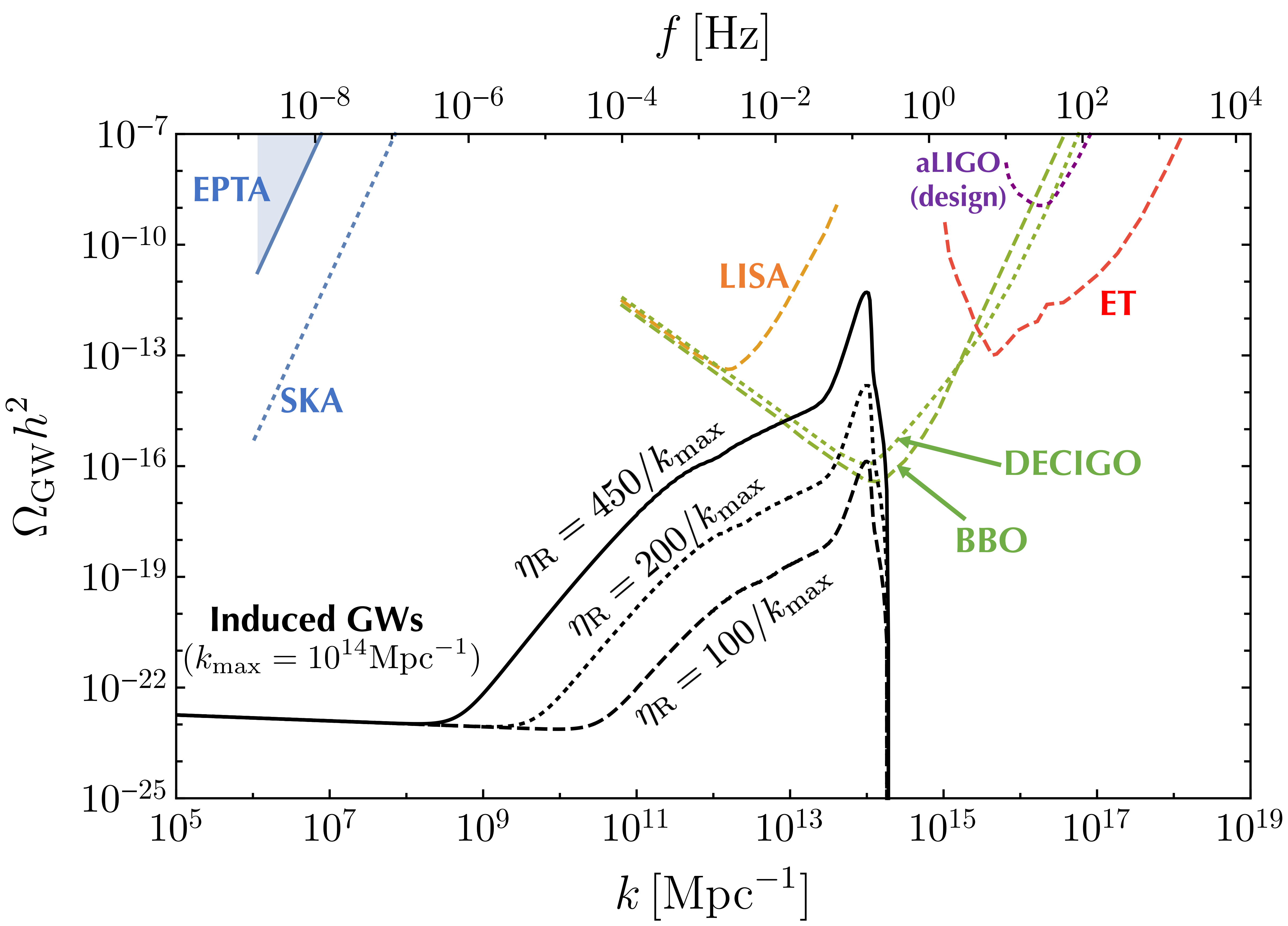
# 1) Time Varying Mass




$$\Gamma = \frac{c^2 M}{32\pi}\sqrt{1 - \frac{m_{\chi,\text{eff}}^2}{(M/2)^2}}\,\Theta\left(M^2 - 4m_{\chi,\text{eff}}^2\right)$$

# 1) Time Varying Mass



## 2) pBH Domination

pHB completes the evaporation at $t_{\mathrm{eva}}$

$$\Gamma \equiv -\frac{1}{M_{\mathrm{PBH}}}\frac{\mathrm{d}M_{\mathrm{PBH}}}{\mathrm{d}t} = \frac{1}{3\,(t_{\mathrm{eva}} - t)},$$

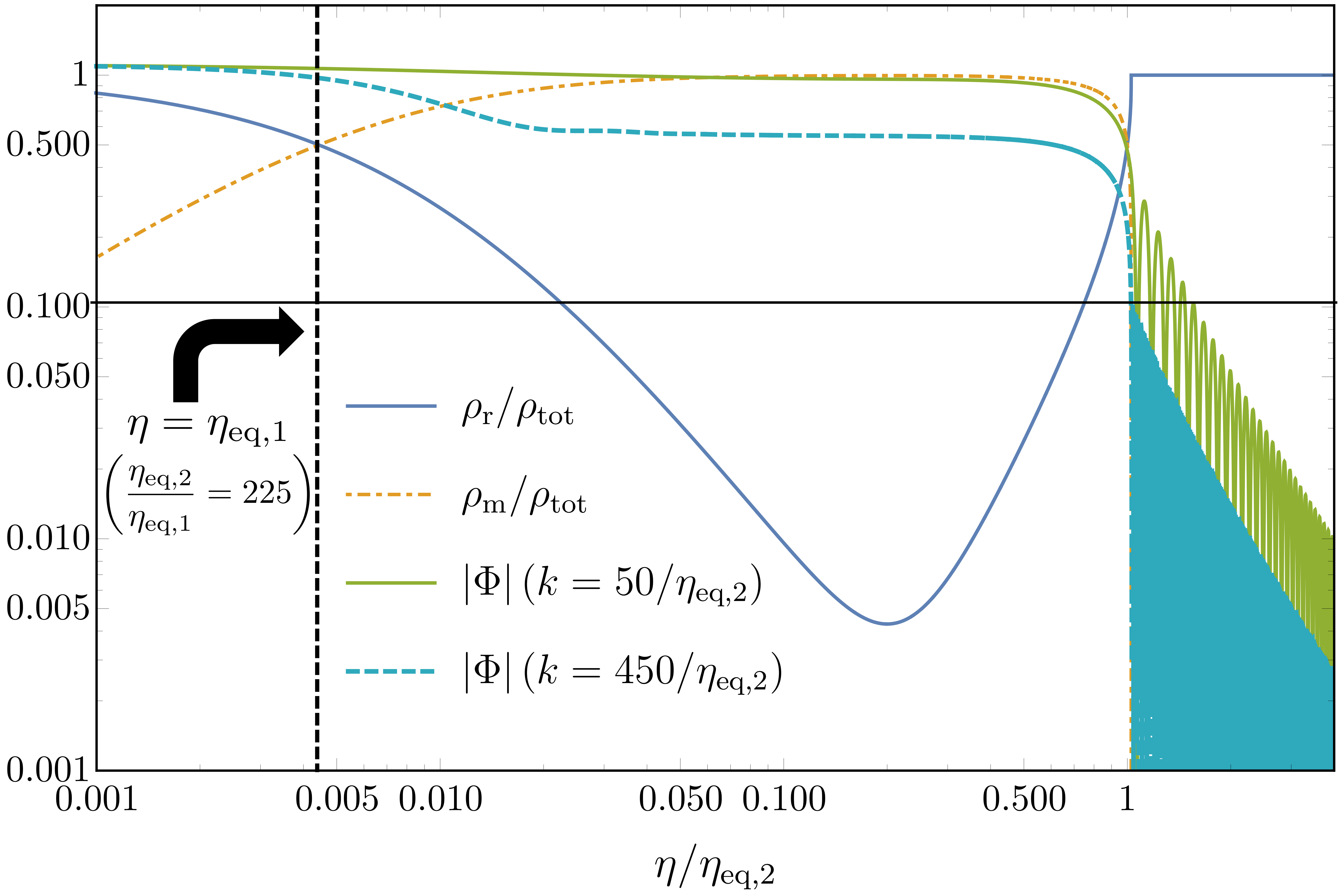
$\Phi_{\mathrm{pBH}} =$ Suppression Factor (S) $\times$ Rapid Decay ansatz

**Yanagida (2000)**

# 2) pBH Domination

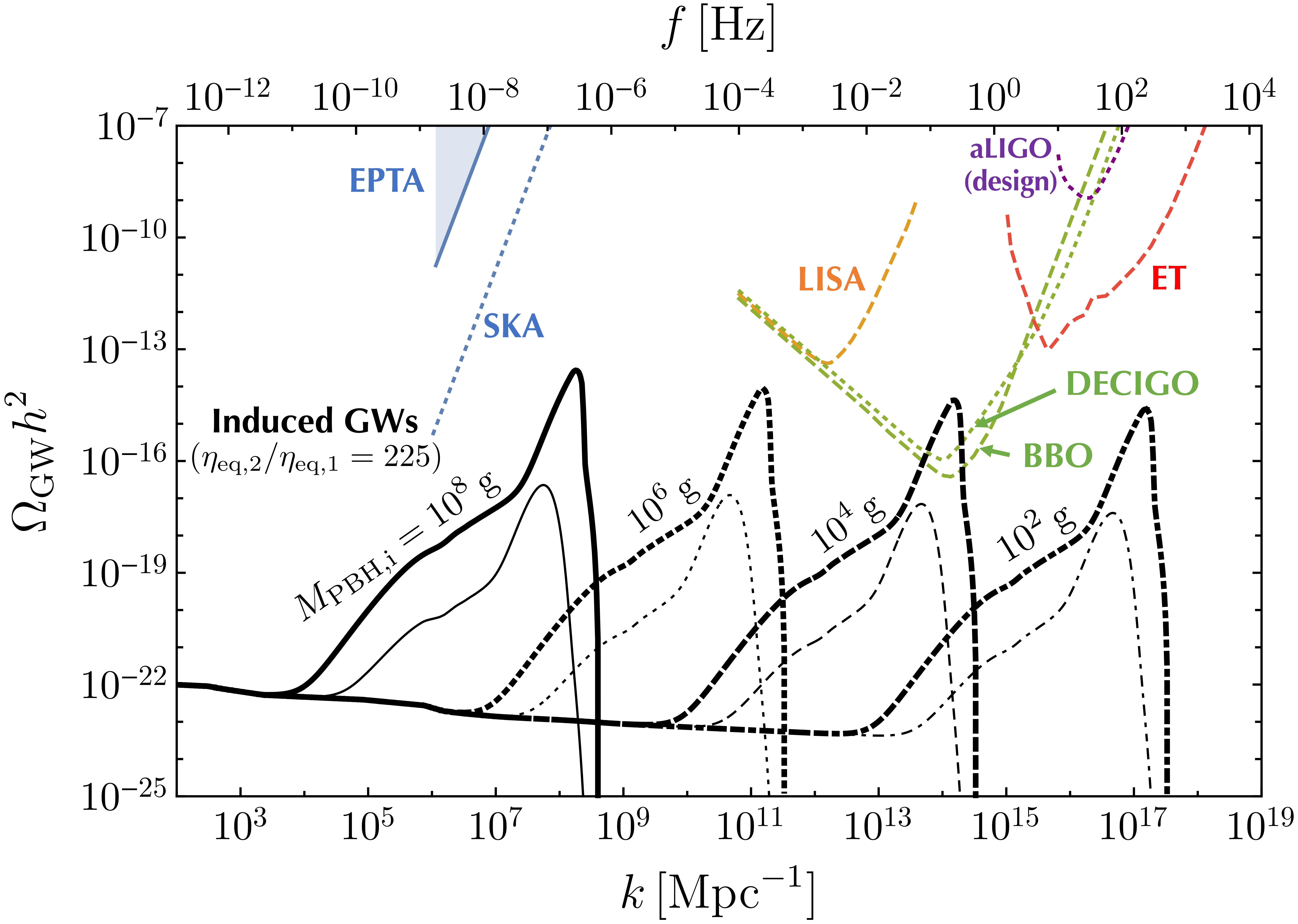
$$\Phi_{\mathrm{osc,fit}}(x, x_0) = S\left(A(x_0)\mathcal{J}(x, x_0) + B(x_0)\mathcal{Y}(x, x_0)\right)$$

$$S \sim \mathcal{O}(0.1)$$



# 2) pBH Domination

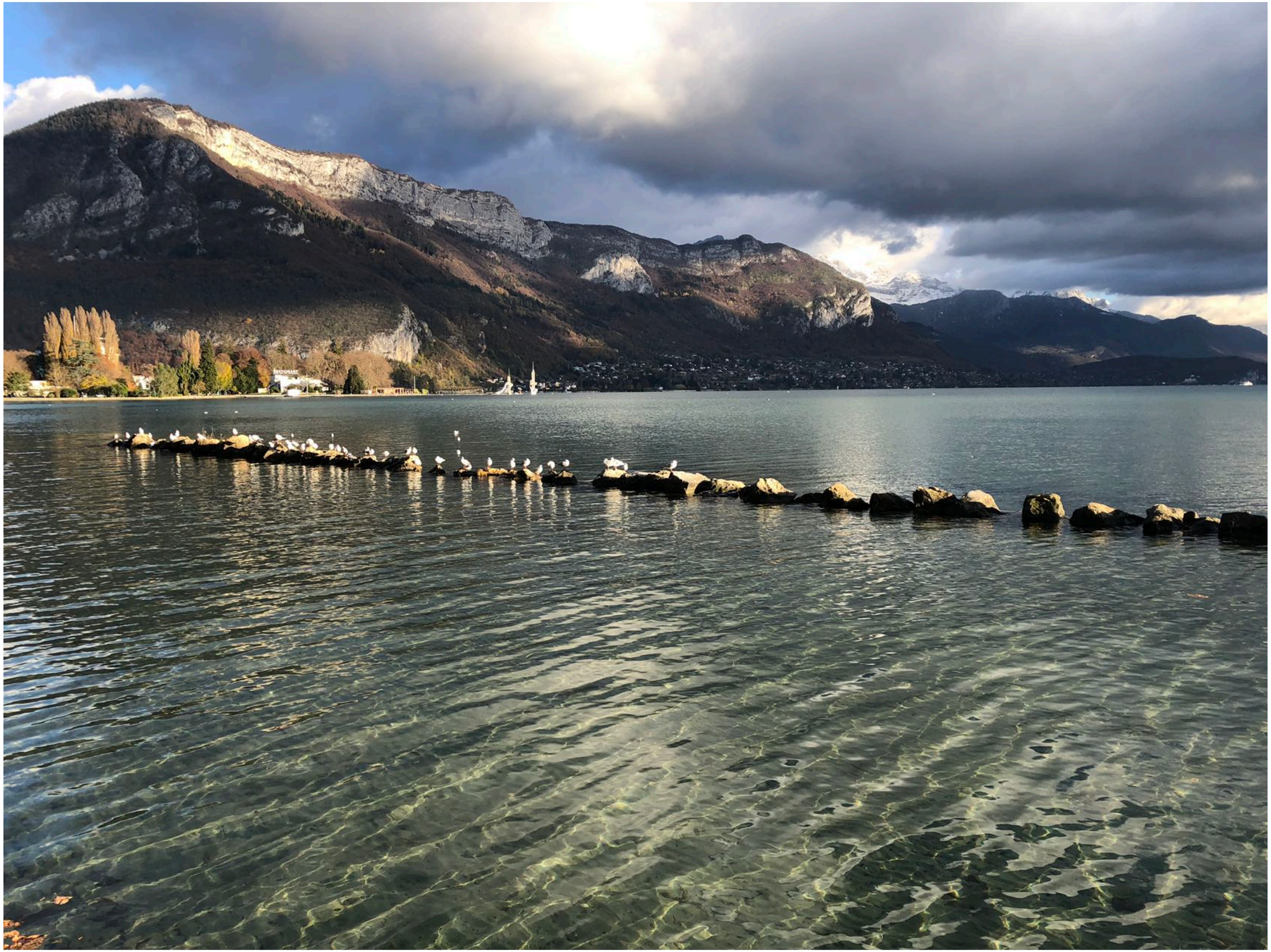
$$\mathscr{P}_\zeta(k) \to S^2(k)\mathscr{P}_\zeta(k)$$



# Summary

- Tensor fluctuations induced by scalar modes
- GW from primordial scalar fluctuations in eMD
- GW encodes eMD to RD information:
  Rapid (enhance peak), Gradual (suppressed)
- Models: Time-varying decay, pBH domination

# Thank You for Your Attention!



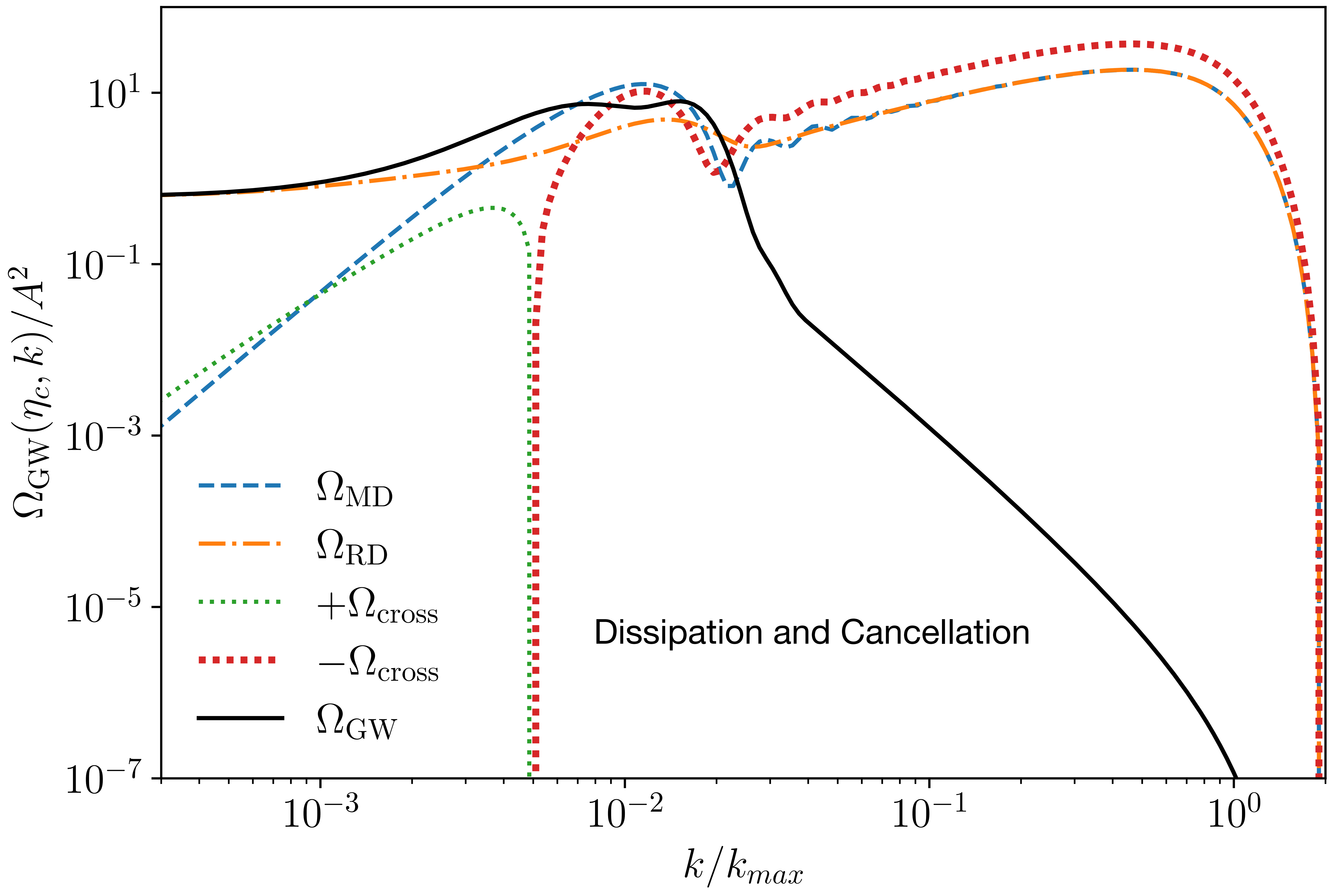
# Backups

$$
\begin{aligned}
I(u,v,x,x_\mathrm{R}) = &\int_0^{x_\mathrm{R}} \mathrm{d}\bar{x} \left(\frac{1}{2(x/x_\mathrm{R})-1}\right)\left(\frac{\bar{x}}{x_\mathrm{R}}\right)^2 \\
&\qquad\qquad \times kG_k^{\mathrm{eMD}\to\mathrm{RD}}(\eta,\bar{\eta}) f(u,v,\bar{x},x_\mathrm{R}) \\
&+ \int_{x_\mathrm{R}}^{x} \mathrm{d}\bar{x} \left(\frac{2(\bar{x}/x_\mathrm{R})-1}{2(x/x_\mathrm{R})-1}\right) \\
&\qquad\qquad \times kG_k^{\mathrm{RD}}(\eta,\bar{\eta}) f(u,v,\bar{x},x_\mathrm{R}) \\
\equiv & I_\mathrm{eMD}(u,v,x,x_\mathrm{R}) + I_\mathrm{RD}(u,v,x,x_\mathrm{R}),
\end{aligned}
$$

$$
\Phi_\mathbf{k}(\eta) = \begin{cases} 1\,, & x < x_R \ (\mathrm{MD}) \\ A(x_R)\mathcal{J}(x) + B(x_R)\mathcal{Y}(x)\,, & x \geq x_R \ (\mathrm{RD}) \end{cases}
$$

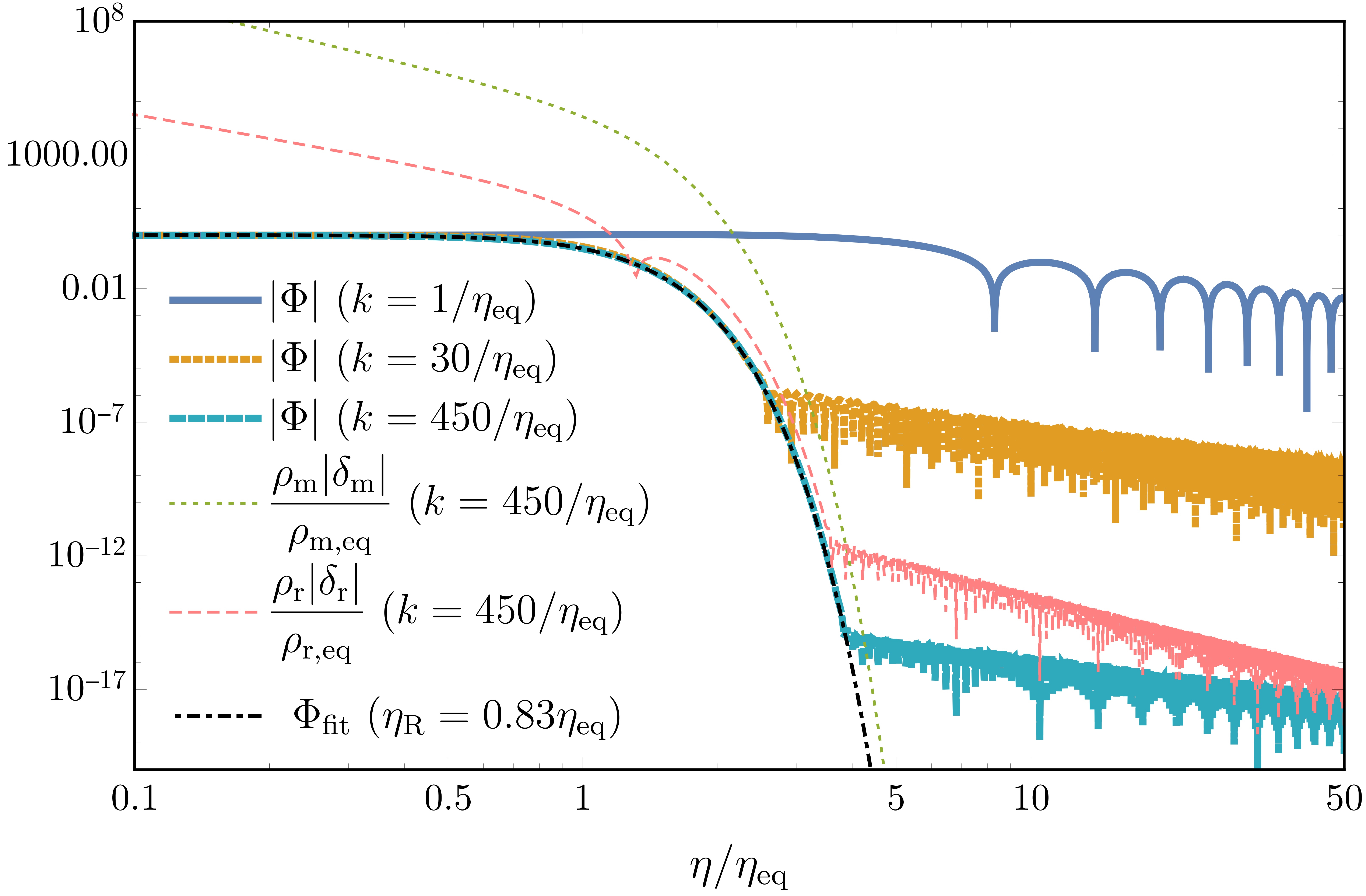
where $x = k\eta$, $x_R = k\eta_R$, $\mathcal{J}(x)$ and $\mathcal{Y}(x)$ are defined from the first and second spherical Bessel functions, Gradual Transition

## $\Omega_{\rm GW}$ from MD, RD and the cross term




**Inomata (2019)**
**Graham (2023)**

# $\Phi$ and the induced GW is suppressed if eMD to RD transition is slow




$$\Phi \sim \exp\left(-\int^{\eta} d\bar{\eta}\, a(\bar{\eta})\Gamma\right) = \begin{cases} \exp\left(-\frac{2}{3}\left(\frac{\eta}{\eta_{\rm R}}\right)^3\right) & (\eta < \eta_{\rm R}) \\ \exp\left(-2\left(\left(\frac{\eta}{\eta_{\rm R}}\right)^2 - \frac{\eta}{\eta_{\rm R}} + \frac{1}{3}\right)\right) & (\eta \geq \eta_{\rm R}) \end{cases} \equiv \Phi_{\rm fit},$$

exponential decay

Suppression factor

$S \sim \mathcal{O}\left(10^{-14}\right)$

# Power Spectra and GW

Two Points function

$$\langle h_{\mathbf{k}}(\eta) h_{\mathbf{k}'}(\eta)\rangle = \left(\frac{40 g(k\eta)}{3(2\pi)^{3/2}}\right)^2 k^{-4} \int_0^{\mathbf{k}_{dom}} d^3\widetilde{\mathbf{k}}\, e(\mathbf{k},\widetilde{\mathbf{k}})\, d^3\widetilde{\mathbf{k}'}\, e(\mathbf{k}',\widetilde{\mathbf{k}'})\, \langle \Phi_{\mathbf{k}-\widetilde{\mathbf{k}}}\, \Phi_{\widetilde{\mathbf{k}}}\, \Phi_{\mathbf{k}'-\widetilde{\mathbf{k}'}}\, \Phi_{\widetilde{\mathbf{k}'}}\rangle$$

Power Spectrum

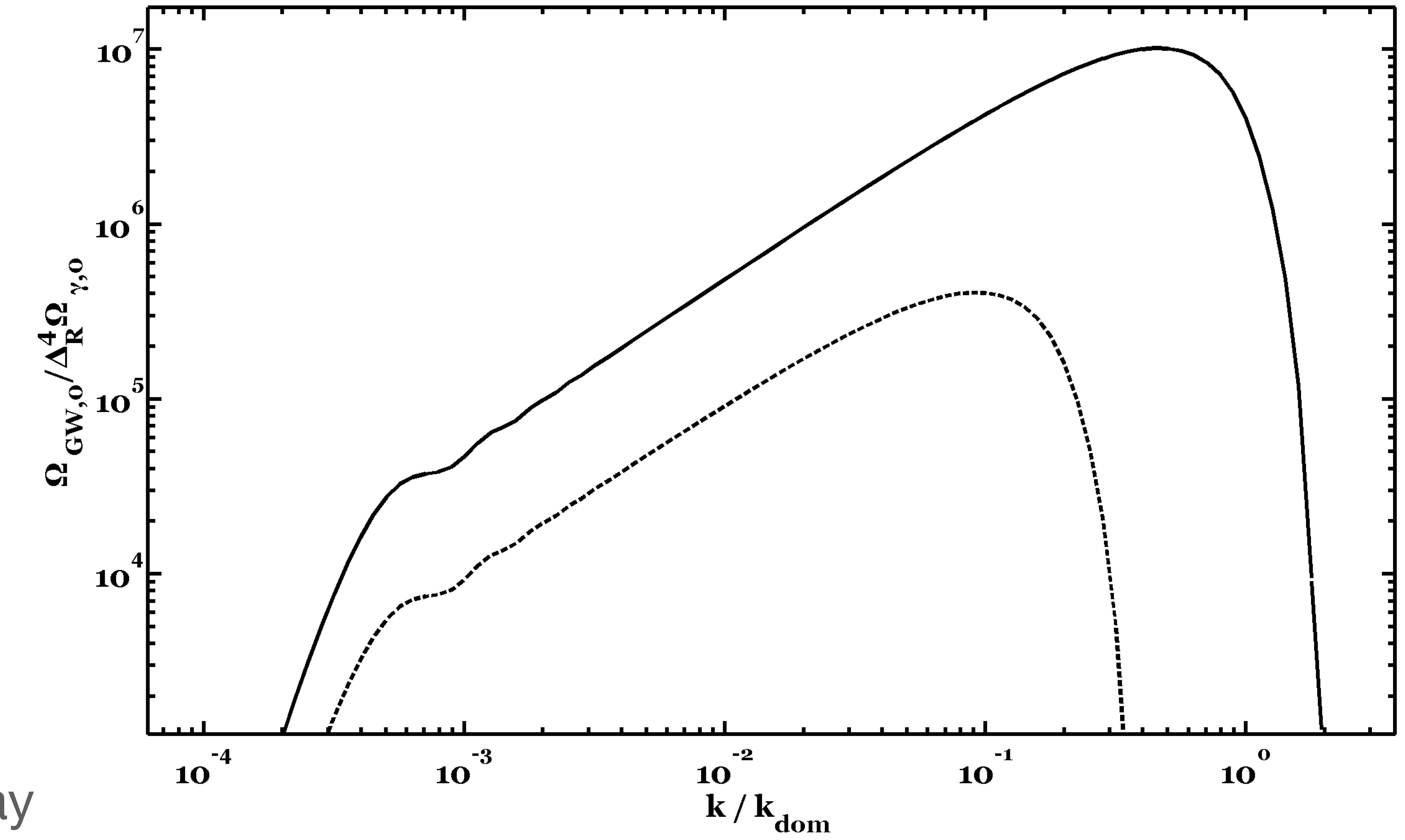
$$\langle h_{\mathbf{k}}(\eta) h_{\mathbf{k}'}(\eta)\rangle = \frac{\pi^2}{k^3}\delta^3(\mathbf{k}+\mathbf{k}')\mathcal{P}_h(k,\eta)$$

$$\langle \phi_{\mathbf{k}}(\eta) \phi_{\mathbf{k}'}(\eta)\rangle = \frac{2\pi^2}{k^3}\delta^3(\mathbf{k}+\mathbf{k}')\mathcal{P}_\zeta(k,\eta)$$

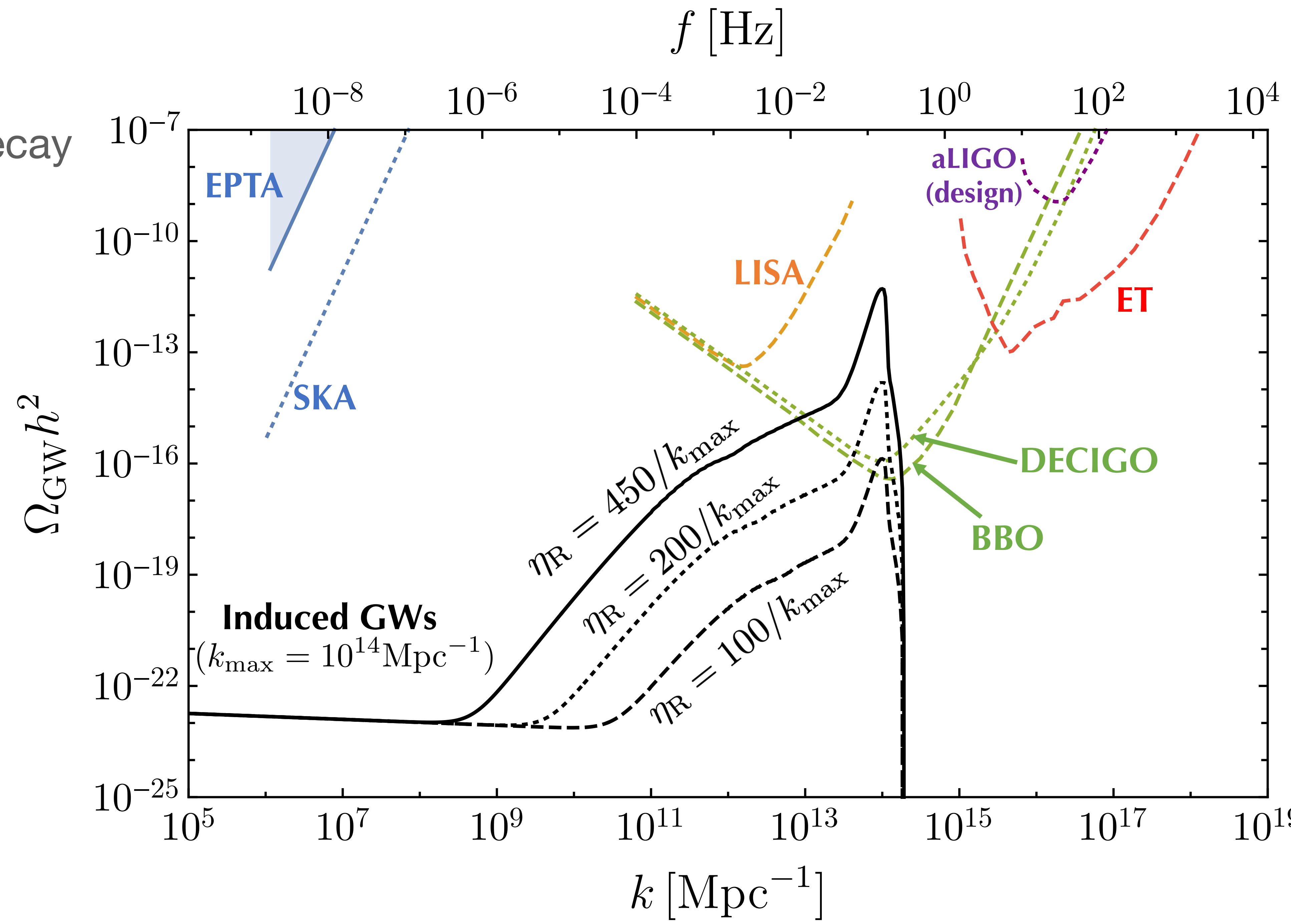
Simple expression for eMD alone

$$2\pi k \mathcal{P}_h(k) = \left(\frac{40 g(k\eta)}{3}\right)^2 \int d^3\tilde{\mathbf{k}} e(\mathbf{k},\tilde{\mathbf{k}}) \left[e(\mathbf{k}',\tilde{\mathbf{k}}) + e(\mathbf{k}',\mathbf{k}-\tilde{\mathbf{k}})\right] \mathcal{P}_\zeta(\mathbf{k}-\tilde{\mathbf{k}})\mathcal{P}_\zeta(\tilde{\mathbf{k}})$$

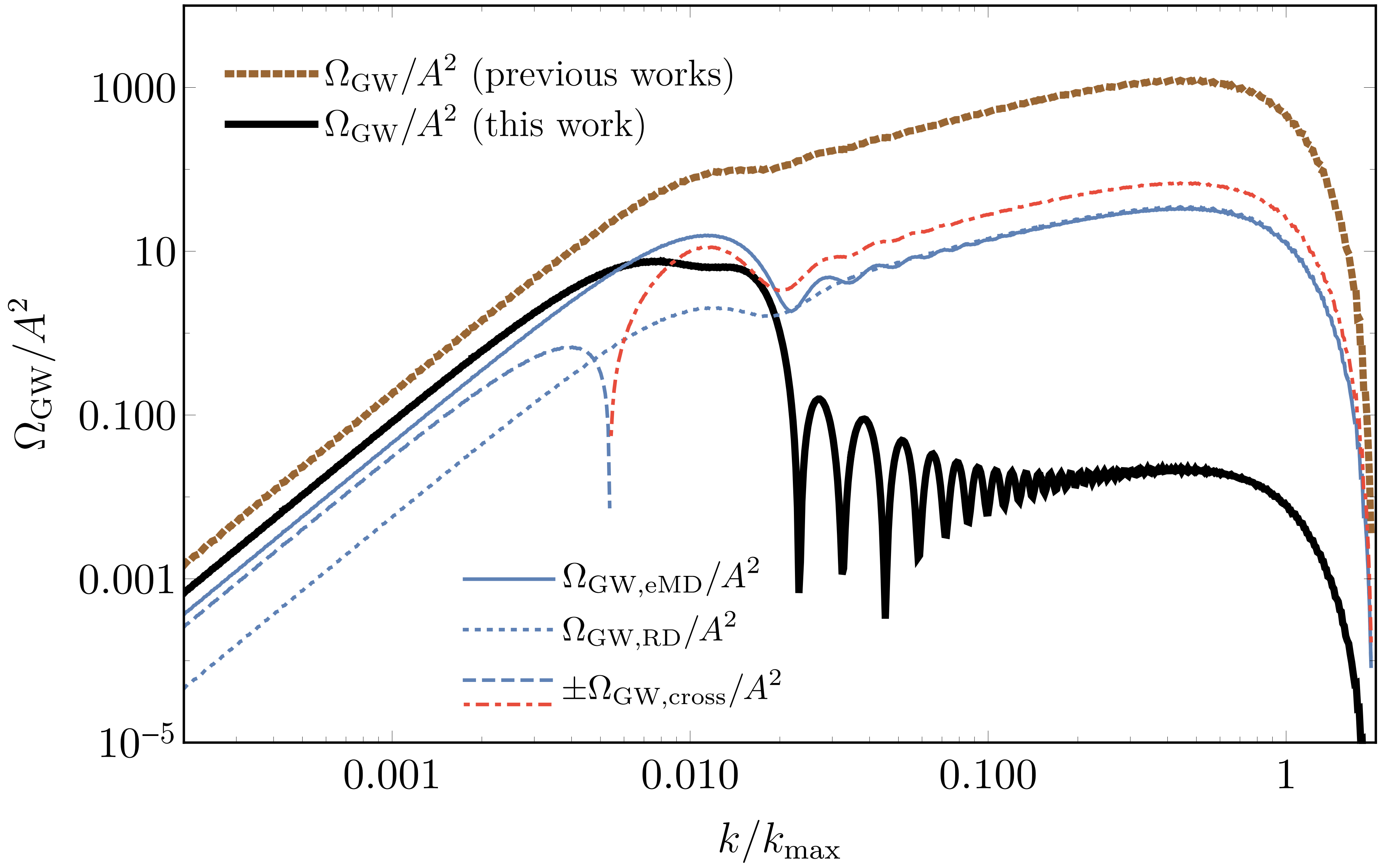
Orignial, 2009



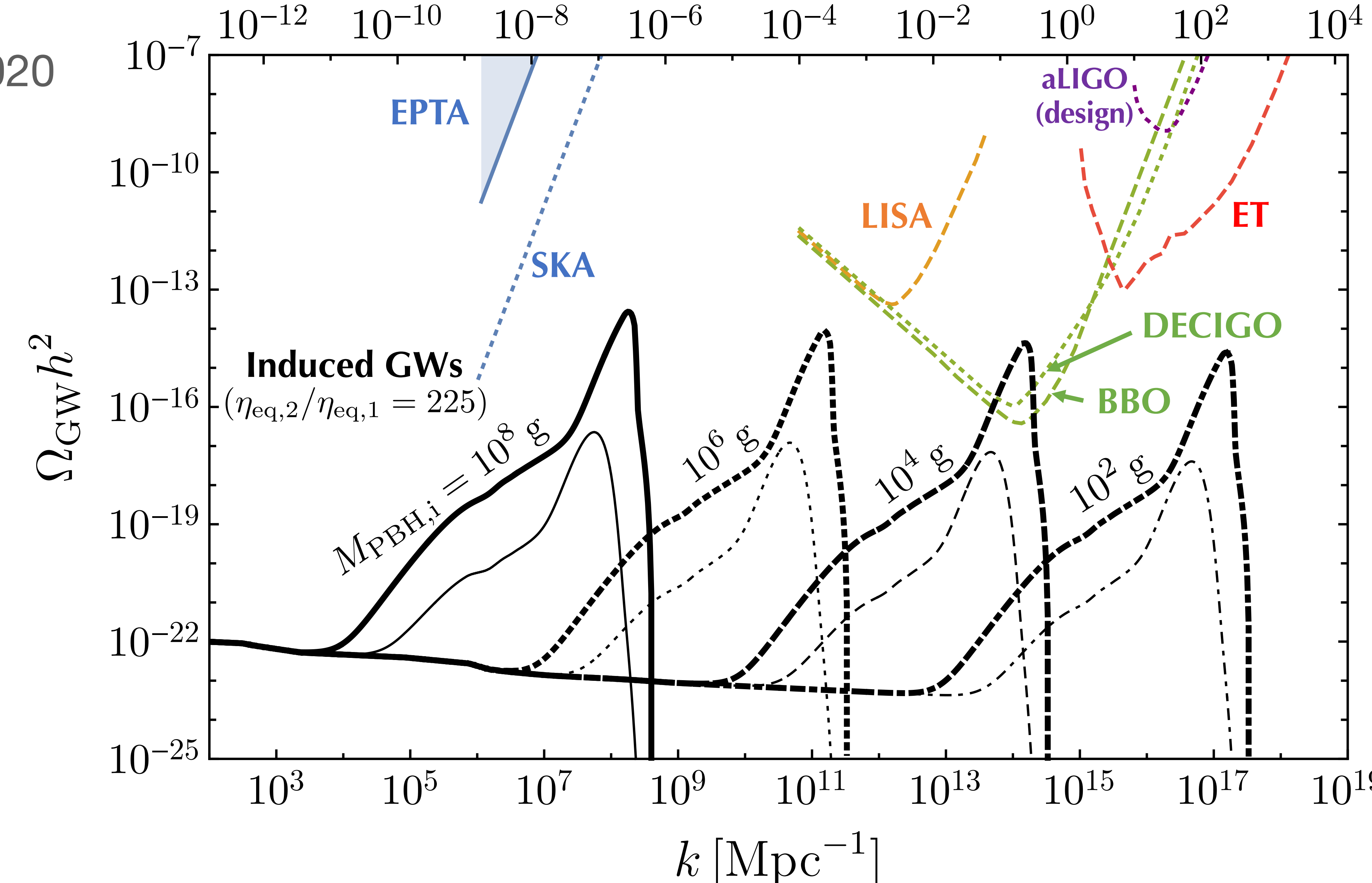

Sudden Decay
2019




Gradual Decay
2019




pBH, 2020



# Evolution of gravitational Potential

$$
\begin{aligned}
\delta'_{\mathrm{m}} &= -\theta_{\mathrm{m}} + 3\Phi' - a\Gamma\Phi, \\
\theta'_{\mathrm{m}} &= -\mathcal{H}\theta_{\mathrm{m}} + k^2\Phi, \\
\delta'_{\mathrm{r}} &= -\frac{4}{3}(\theta_{\mathrm{r}} - 3\Phi') + a\Gamma\frac{\rho_{\mathrm{m}}}{\rho_{\mathrm{r}}}(\delta_{\mathrm{m}} - \delta_{\mathrm{r}} + \Phi), \\
\theta'_{\mathrm{r}} &= \frac{k^2}{4}\delta_{\mathrm{r}} + k^2\Phi - a\Gamma\frac{3\rho_{\mathrm{m}}}{4\rho_{\mathrm{r}}}\left(\frac{4}{3}\theta_{\mathrm{r}} - \theta_{\mathrm{m}}\right),
\end{aligned}
$$

where $\delta$ and $\theta$ denote the energy density perturbation and the velocity divergence

## GW from $I_{\rm eMD}$ Wands 2009

LSS-> power law primordial scalar curvature perturbation [Planck 2018],

$$\mathcal{P}_\zeta(k) = \frac{9}{25}\Delta^2_{\mathcal{R}}\left(\frac{k}{k_*}\right)^{n_s-1} \quad , \quad \Delta^2_{\mathcal{R}} = 5.3\times 10^{-9}\,, n_s = 0.97$$

$$\Longrightarrow \quad \mathcal{P}_h(k,\eta) = 2\left(\frac{25g(k\eta)}{5}\right)^2 \Delta^4_{\mathcal{R}}\left(\frac{k_{dom}}{k}\right) I_1(k/k_{dom})$$

where $I_1$ is an integral with $0 < I_1 \leq 16/15$

In the Fourier space,

$$h_{\rm k} = \frac{g(k\eta)S_{\rm k}}{k^2} = \frac{40g(k\eta)}{3(2\pi)^{3/2}}\int_0^{k_{dom}} d^3\tilde{\rm k}\, e({\rm k},\tilde{\rm k})\Phi_{{\rm k}-\tilde{\rm k}}\Phi_{\tilde{\rm k}}$$

$$e(\mathbf{k},\widetilde{\mathbf{k}}) = e^{ij}(\mathbf{k})\widetilde{k}_i\widetilde{k}_j = \widetilde{k}^2\sin^2\theta\,,$$

$$g(k\eta) = 1 + 3\left(\frac{k\eta\cos(k\eta) - \sin(k\eta)}{k^3\eta^3}\right)$$

# Present density of GW from eMD part

$$\rho_{GW} = \frac{1}{32\pi G}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle = \frac{k^2}{32\pi G a^2}\int d(\ln k)\ \mathcal{P}_h(k,\eta)\,.$$

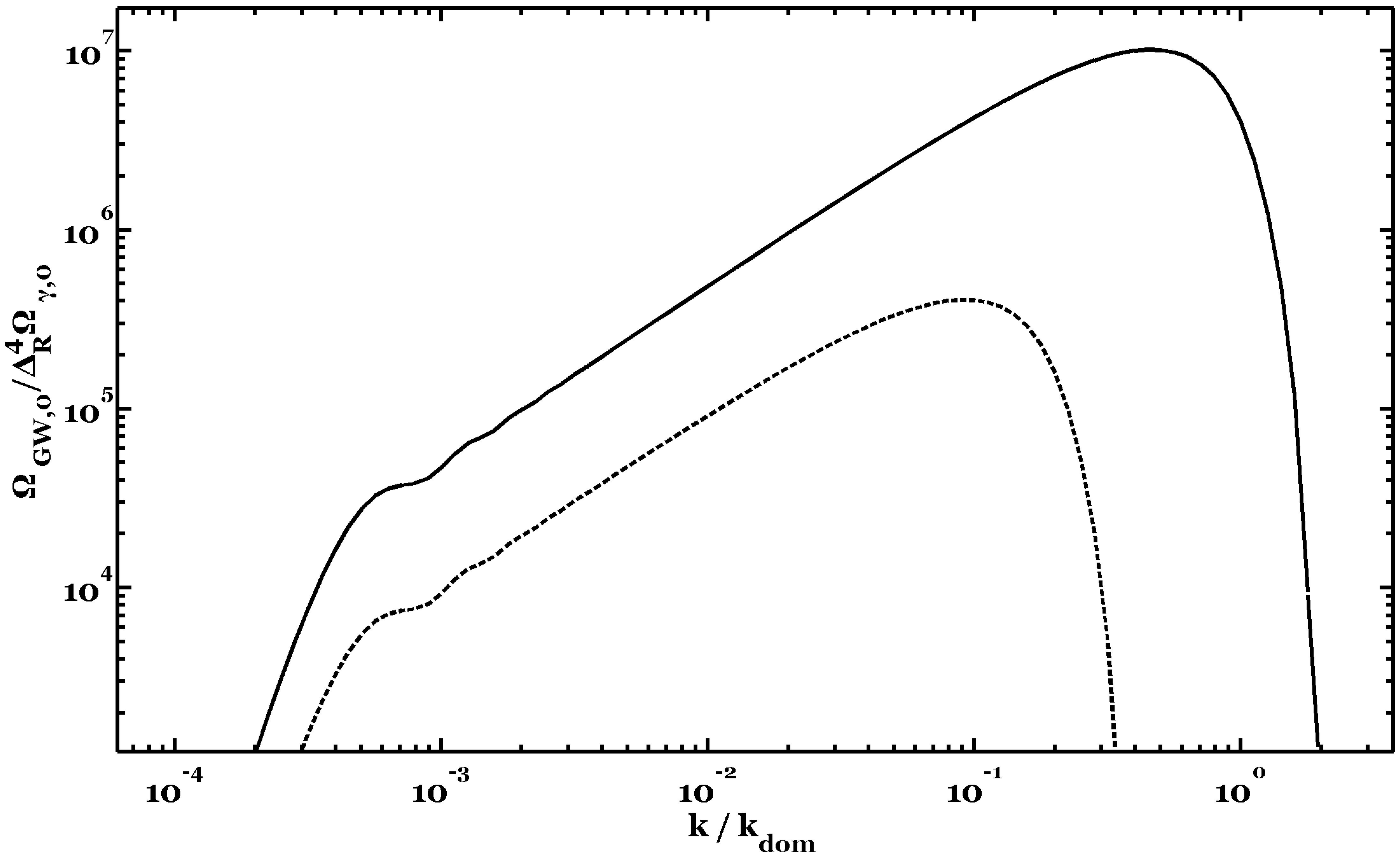
$$\Omega_{GW}(k,\eta) = \frac{1}{12}\left(\frac{k}{\mathcal{H}}\right)^2 \mathcal{P}_h(k,\eta)$$

$$\longrightarrow \quad \Omega_{GW,0}(k) \simeq \frac{\Omega_{\gamma,0}}{12}\left(\frac{k}{k_{\rm dec}}\right)^2 \mathcal{P}_h(k,\eta_{\rm dec})$$

where the present density of photons is $\Omega_{\gamma,0} \simeq 1.2\times 10^{-5}$,

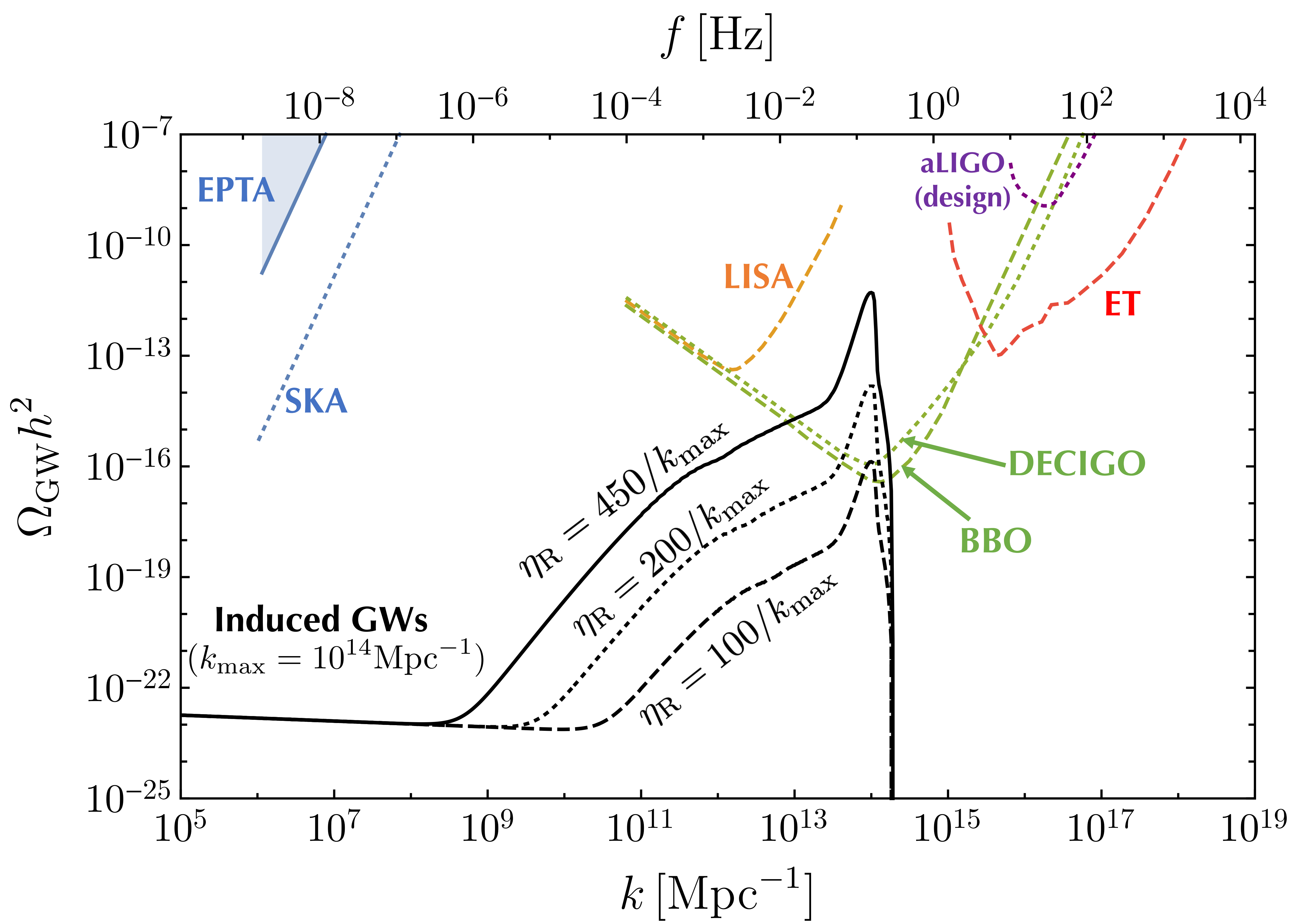
# GW Relic Abundance

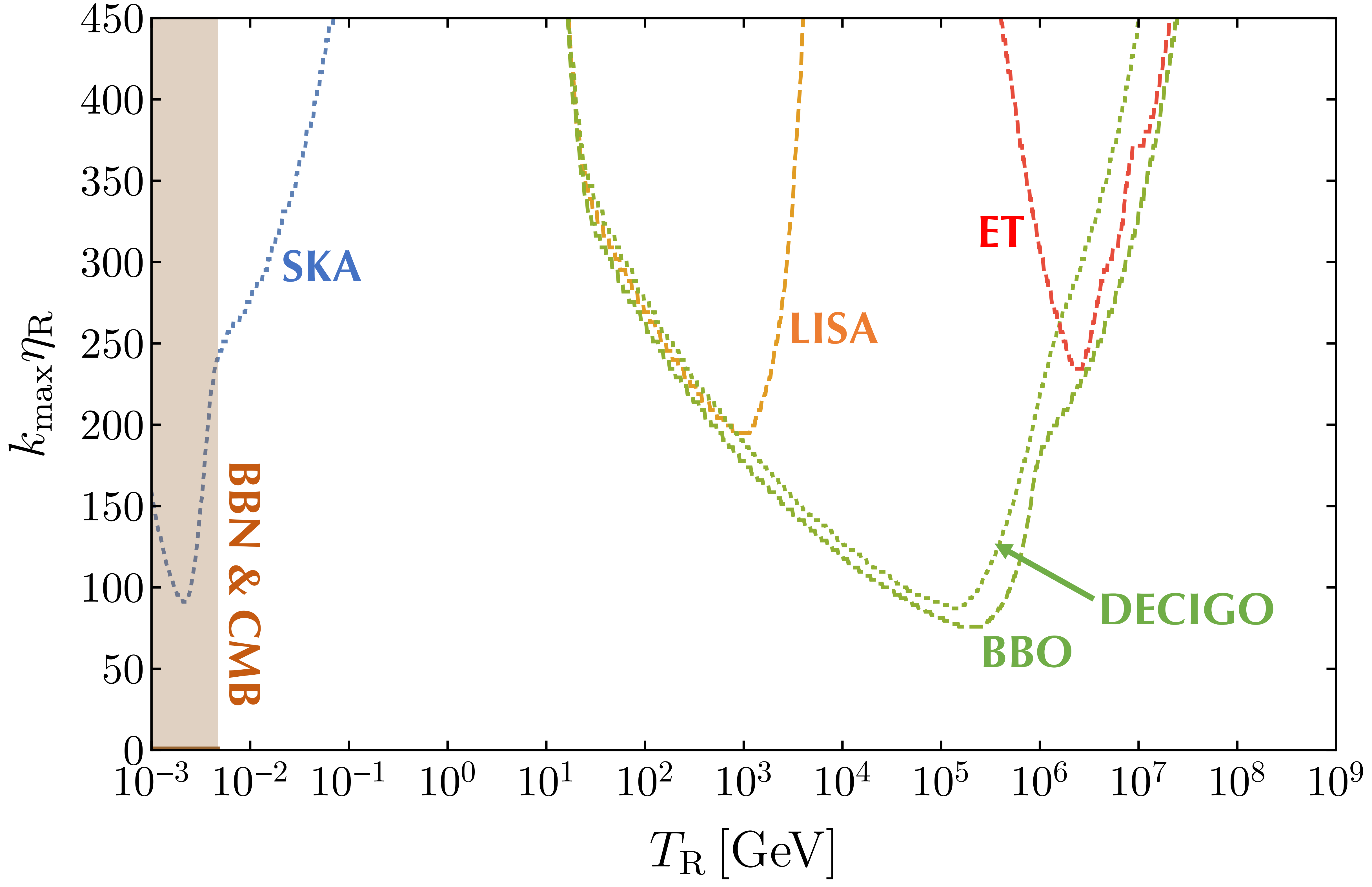
Wands, 2009



$$I_{\rm eMD} + I_{\rm RD}$$

cutoff scale multiplied by $\eta_R$ that can be probed by the future observations




Effective sensitivities to stochastic GW of current and future experiments

Key ingredient:

Sudden eMD to RD transition

# Another sudden model

### 2. Another sudden-reheating scenario realized by a field that experiences a first order phase transition

Suppose that $\phi$ is protected by a symmetry from decaying, without any decay channels of $\phi$ to lighter particles. Let us further assume that $\tau$ is charged under the symmetry and is too heavy for $\phi$ to decay into. There may be an interaction term of the form

$$\mathcal{L} = c\tau\phi\chi\chi + \ldots, \tag{A10}$$

where $c$ is a coupling constant. Suppose that initially the field value of $\tau$ is zero, to be contrasted with the previous model. Then the decay of $\phi$ becomes possible once $\tau$ acquires a finite vacuum expectation value, thereby spontaneously breaking the symmetry.

Such a symmetry-breaking phase transition can occur suddenly if the phase transition is first order. The transition occurs through the tunneling effect, and the tunneling rate is exponentially sensitive to the cosmic temperature (to be more precise, the temperature of the thermal bath to which $\tau$ is coupled), and hence such a transition is sudden [66]. After the transition, $\phi$ becomes able to decay into $\chi$ particles. Provided that this decay rate is much larger than the Hubble parameter, the decay completes within a timescale much shorter than the Hubble time at that time. Associated with the decay of $\phi$, the temperature increases, which may restore the symmetry temporarily. Thus, the importance of the backreaction to the decay of $\phi$ requires a further study. Eventually, the temperature decreases, and $\tau$ settles to the symmetry-breaking vacuum.

One way to suppress the backreaction may be to assume that the initial thermal bath is made up of a hidden sector with $\tau$ being a portal to the visible sector. Then, the increase in the temperature felt by $\tau$ would not be significantly affected by the decay of $\phi$.