Heavy quarkonium production and High-Energy resummation

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SUBATECH
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I. A short introduction to NRQCD factorisation

Two central aspects of the problem

1. What is the (structural) difference between open heavy-flavour(HF) meson and quarkonium?
   - **For open HF mesons** the "naive" quark model receives large corrections:

     $$\left|D^0\right\rangle = c_0|(c\bar{u})_{\mathbf{1}}\rangle + c_1\left|(c\bar{u})_{\mathbf{8}}g\right\rangle + c_2\left|c\bar{u}d\bar{d}\right\rangle + \ldots,\ c_0 \sim c_1 \sim c_2 \sim \ldots$$

   - **For quarkonia** (*we hope*) the more complicated Fock-states are suppressed by relative velocity (v) of heavy-quarks in the bound state

     $$\begin{aligned} |J/\psi\rangle &= O(1)\left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle + O(v)\left|c\bar{c}\left[{}^3P_J^{(8)}\right] + g\right\rangle \\ &+ O(v^{3/2})\left|c\bar{c}\left[{}^1S_0^{(8)}\right] + g\right\rangle + O(v^2)\left|c\bar{c}\left[{}^3S_1^{(8)}\right] + gg\right\rangle + \ldots, \end{aligned}$$

2. How heavy quark (or $Q\bar{Q}$-pair) is produced in pp-collision? *Collinear Factorization + pQCD.* 3 regimes:
   - $p_T \sim M \ll \sqrt{S}$, where M is the meson mass ($\sim m_Q$ or $2m_Q$). **"fixed-order regime?"**
   - $p_T \gg M$, **"fragmentation regime"**
   - $p_T \ll M \ll \sqrt{S}$, **"TMD regime?"**

Quarkonium in the potential model

Cornell potential:

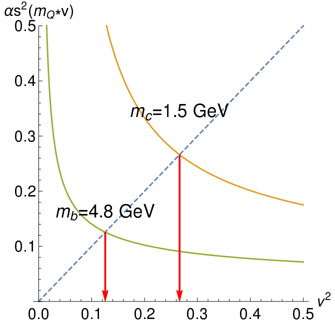
$$V(r) = -C_F \frac{\alpha_s(1/r)}{r} + \sigma r,$$

neglect linear part, because quarkonium is "small" (~ 0.3 fm) $\to$ Coulomb wavefunction (for effective mass $\frac{m_1 m_2}{m_1+m_2} = \frac{m_Q}{2}$):

$$R(r) = \frac{\sqrt{m_Q^3 \alpha_s^3 C_F^3}}{2} e^{-\frac{\alpha_s C_F}{2} m_Q r}$$

$$\langle v^2 \rangle = \frac{C_F^2 \alpha_s^2}{2}, \langle r \rangle = \frac{3}{2C_F} \frac{1}{m_Q v}$$

$$\Rightarrow \boxed{\alpha_s^2(m_Q v) \simeq v^2}$$



Non-relativistic QCD

The velocity-expansion for quarkonium eigenstate is carbon-copy of corresponding arguments from atomic physics (hierarchy of E-dipole/M-dipole with ΔS/M-dipole transitions):

$$\begin{aligned} |J/\psi\rangle &= O(1)\left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle + O(v)\left|c\bar{c}\left[{}^3P_J^{(8)}\right]+g\right\rangle \\ &+ O(v^{3/2})\left|c\bar{c}\left[{}^1S_0^{(8)}\right]+g\right\rangle + O(v^2)\left|c\bar{c}\left[{}^3S_1^{(8)}\right]+gg\right\rangle + \ldots, \end{aligned}$$

for validity of this arguments, we should work in *non-relativistic EFT*, dynamics of which conserves number of heavy quarks. In such EFT, $Q\bar{Q}$-pair is produced in a point, by local operator:

$$\mathcal{A}_{\rm NRQCD} = \langle J/\psi + X|\,\chi^\dagger(0)\kappa_n\psi(0)\,|0\rangle\,,$$

Different operators "couple" to different Fock states:

$$\chi^\dagger(0)\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1S_0^{(1)}\right]\right\rangle,\ \chi^\dagger(0)\sigma_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(1)}\right]\right\rangle,$$
$$\chi^\dagger(0)\sigma_i T^a\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^3S_1^{(8)}\right]\right\rangle,\ \chi^\dagger(0)D_i\psi(0) \leftrightarrow \left|c\bar{c}\left[{}^1P_1^{(8)}\right]\right\rangle, \ldots$$

squared NRQCD amplitude (=LDME):

$$\sum_X |\mathcal{A}|^2 = \langle 0|\underbrace{\psi^\dagger\kappa_n^\dagger\chi a^\dagger_{J/\psi}a_{J/\psi}\chi^\dagger\kappa_n\psi}_{\mathcal{O}_n^{J/\psi}}|0\rangle = \left\langle \mathcal{O}_n^{J/\psi}\right\rangle,$$

Non-relativistic QCD

Velocity-scaling of LDMEs follows from velocity-scaling of corresponding Fock states and of operators $\chi^\dagger \kappa_n \psi$:

	${}^1S_0^{(1)}$	${}^3S_1^{(1)}$	${}^1S_0^{(8)}$	${}^3S_1^{(8)}$	${}^1P_1^{(1)}$	${}^3P_0^{(1)}$	${}^3P_1^{(1)}$	${}^3P_2^{(1)}$	${}^1P_1^{(8)}$	${}^3P_0^{(8)}$	${}^3P_1^{(8)}$	${}^3P_2^{(8)}$
η_c	1		v^4	v^3					v^4			
J/ψ		1	v^3	v^4						v^4	v^4	v^4
h_c			v^2		v^2							
χ_{c0}				v^2		v^2						
χ_{c1}				v^2			v^2					
χ_{c2}				v^2				v^2				

Matching procedure between QCD and NRQCD:

$$v \to 0 : \mathcal{A}_{\text{QCD}}(gg \to Y_{Q\bar{Q}(v)}) = \sum_n f_n \left\langle Y_{Q\bar{Q}(v)} \right| \chi^\dagger(0)\kappa_n\psi(0) \left|0\right\rangle + O(v^\#),$$

$\Rightarrow$ NRQCD factorization formula ("theorem") [Bodwin, Braaten, Lepage 95'] :

$$\sigma(gg \to \mathcal{H} + X) = \sum_n \sigma(gg \to Q\bar{Q}[n] + X) \left\langle \mathcal{O}_n^{\mathcal{H}} \right\rangle.$$

Slides from M. Butenschön at **QaT-2021**:

3.2 Butenschön et al. LDMEs

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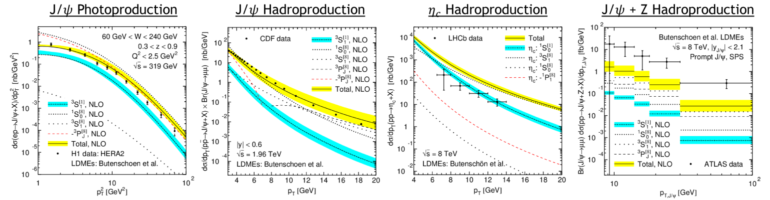

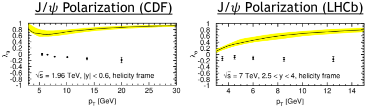

- Fit to 194 data points of J/ψ photo- and hadro-production, $\gamma\gamma$- and e^+e^- scattering
- $<O^{J/\psi}$(1S08)>=4.97±0.44, $<O^{J/\psi}$(3S18)>=0.22±0.06, $<O^{J/\psi}$(3P08)>=-1.61±0.20 [in 10^{-2} GeV^3 or 10^{-2} GeV^5]
- Ref.: [MB, Kniehl, PRD 84, 051501 (2011)]

- Data fitted to is described within scale uncertainties, other observables not.

Slides from M. Butenschön at **QaT-2021**:

3.3 Gong et al. LDMEs

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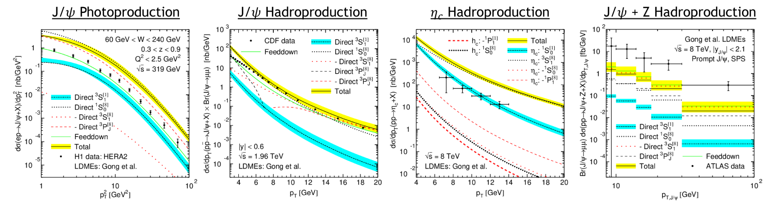

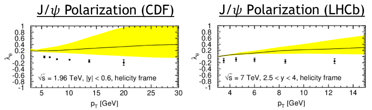

- Fit to J/$\psi,\psi(2S)$, χ_{cJ} hadroprod. yield with $p_T > 7$ GeV.
- $<O^{J/\psi}$(1S08)>=0.97 $\pm$0.09, $<O^{J/\psi}$(3S18)>=-0.46 $\pm$0.13, $<O^{J/\psi}$(3P08)>=-2.14 $\pm$0.56, $<O^{\psi(2S)}$(1S08)>=-0.01 $\pm$0.87, $<O^{\psi(2S)}$(3S18)>=0.34 $\pm$0.12, $<O^{\psi(2S)}$(3P08)>=0.95 $\pm$0.54, $<O^{\chi_{c0}}$(3S18)>=0.22 $\pm$0.05 [in 10^{-2} GeV^3 or 10^{-2} GeV^5]
- Ref.: [Gong, Wan, Wang, Zhang, PRL 110, 042002 (2013)]

- Data fitted to is described, other observables not.

Slides from M. Butenschön at **QaT-2021**:

3.4 Chao et al. LDMEs

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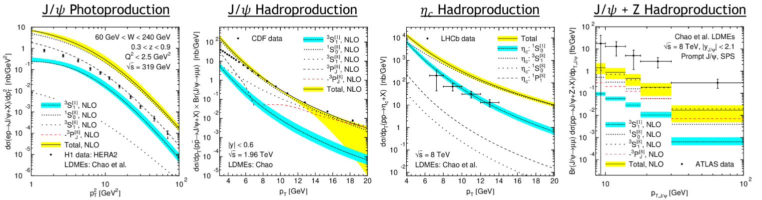

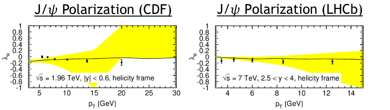

- Fit to CDF J/ψ yield and polarization with $p_T > 7$ GeV.
- $<O^{J/\psi}$(1S08)>=8.9 ±0.98, $<O^{J/\psi}$(3S18)>=0.30 ±0.12, $<O^{J/\psi}$(3P08)>=1.26 ±0.47 [in 10^{-2} GeV^3 or 10^{-2} GeV^5]
- Ref.: [Chao, Ma, Shao, Wang, Zhang, PRL 108, 242004 (2012)]

- Data fitted to is described, other observables not.

Slides from M. Butenschön at **QaT-2021**:

3.5 Chao et al. LDMEs: With η_c

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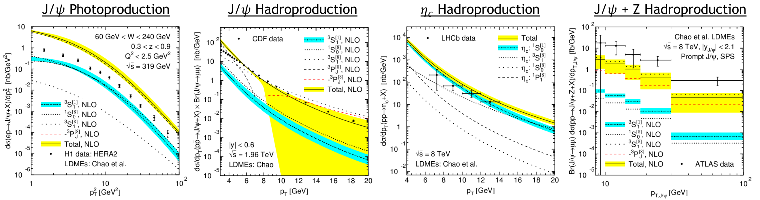
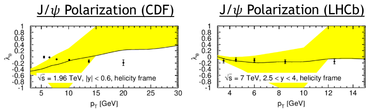




- Fit to CDF J/ψ yield $p_T > 7$ GeV (contraining two linear combinations M_0 and M_1) plus the LHCb η_c production data (constraining <$O^{J/\psi}$(1S08)>)
- <$O^{J/\psi}$(1S08)>=0.73 $\pm$0.73, <$O^{J/\psi}$(3S18)>=1.0 $\pm$0.3, <$O^{J/\psi}$(3P08)>=3.8 $\pm$1.1 [in 10^{-2} GeV^3 or 10^{-2} GeV^5]
- Ref.: [Han, Ma, Meng, Shao, Chao, PRL 114 (2015) 092005]

- Nontrivial: Largely unpolarized J/ψ compatible with data (although tensions to CDF data). But: J/ψ hadroproduction $p_T < 7$ GeV, J/ψ photo- and J/$\psi + Z$ production not described.

Upshot (for this talk)

- Colour-singlet contribution to η_c hadroproduction and J/ψ photoproduction is large ($O(50\%)$) or dominating
- Hadroproduction of J/ψ at $p_T \sim M \ll \sqrt{S}$ is not described by any fit

What will happen if we try to compute p_T-integrated cross sections?

II. Quarkonium production at high energy

In collaborqtion with Jean-Philippe Lansberg and Melih Ozcelik.
Based on JHEP **05** (2022) 083; `hep-ph/2306.02425` and ongoing work

Perturbative instability of quarkonium total cross sections

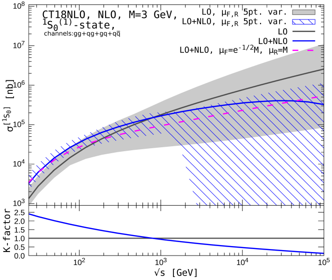
Inclusive η_c-hadroproduction (CSM)

[Mangano *et.al.*, '97, ..., Lansberg, Ozcelik, '20]

$$p+p \to c\bar{c}\left[{}^1S_0^{[1]}\right]+X,\ \text{LO: } g(p_1)+g(p_2) \to c\bar{c}\left[{}^1S_0^{[1]}\right],$$

$$\sigma(\sqrt{s_{pp}}) = f_i(x_1,\mu_F) \otimes f_j(x_2,\mu_F) \otimes \hat{\sigma}(z),$$

where $z = \frac{M^2}{\hat{s}}$ with $\hat{s} = (p_1+p_2)^2$.



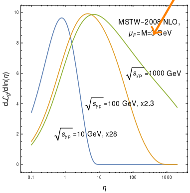
Inclusive J/ψ-photoproduction (CSM)

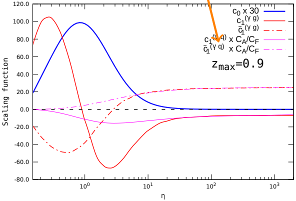
[Krämer, '96, ...,Colpani Serri *et.al*, '21]

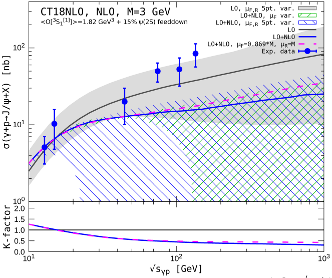
$$\gamma+p \to c\bar{c}\left[{}^3S_1^{[1]}\right]+X,\ \text{LO: } \gamma(q)+g(p_1) \to c\bar{c}\left[{}^3S_1^{[1]}\right]+g,$$

$$\sigma(\sqrt{s_{\gamma p}}) = f_i(x_1,\mu_F) \otimes \hat{\sigma}(\eta),$$

where $\eta = \frac{\hat{s}-M^2}{M^2}$ with $\hat{s} = (q+p_1)^2$, $z = \frac{pP}{qP}$.



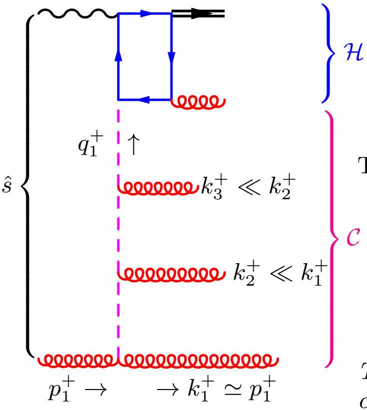




High-Energy Factorization (J/ψ photoproduction)

The **LLA** ($\sum_n \alpha_s^n \ln^{n-1}$) formalism [Collins, Ellis, '91; Catani, Ciafaloni, Hautmann, '91,'94]

Physical picture in the **LLA** for photoproduction:



The LLA in $\ln\frac{1}{\xi} = \ln\frac{p_1^+}{q_1^+} \sim \ln(1+\eta)$:

$$\hat{\sigma}_{\rm HEF}^{\ln(1/\xi)}(\eta) \propto \int\limits_{1/z}^{1+\eta} \frac{dy}{y} \int\limits_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{y}{1+\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \mathcal{H}(y, \mathbf{q}_{T1}^2),$$

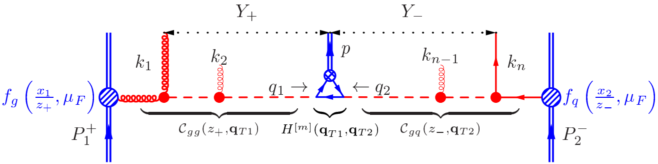
The **strict LLA** in $\ln(1+\eta) = \ln\frac{\hat{s}}{M^2}$:

$$\hat{\sigma}_{\rm HEF}^{\ln(1+\eta)}(\eta) \propto \int\limits_0^\infty d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{1}{1+\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \int\limits_{1/z}^\infty \frac{dy}{y} \mathcal{H}(y, \mathbf{q}_{T1}^2).$$

The LLA($\ln(1/\xi)$) contains some (N..)NLLA contributions relative to the LLA($\ln(1+\eta)$).

The coefficient function $\mathcal{H}$ has been calculated at LO[Kniehl, Vasin, Saleev, '06] and decreases as $1/y^2$ for $y \gg 1$.

High-Energy Factorization (η_c hadroproduction)



Small parameter: $z=\frac{M^2}{\hat{s}}$, LLA in $\alpha_s^n \ln^{n-1}\frac{1}{z}$:

$$\hat{\sigma}_{ij}^{[m],\ \mathrm{HEF}}(z,\mu_F,\mu_R)=\int\limits_{-\infty}^{\infty} d\eta \int\limits_{0}^{\infty} d\mathbf{q}_{T1}^2 d\mathbf{q}_{T2}^2\, \mathcal{C}_{gi}\left(\frac{M_T}{M}\sqrt{z}e^{\eta},\mathbf{q}_{T1}^2,\mu_F,\mu_R\right)$$

$$\times \mathcal{C}_{gj}\left(\frac{M_T}{M}\sqrt{z}e^{-\eta},\mathbf{q}_{T2}^2,\mu_F,\mu_R\right)\int\limits_0^{2\pi}\frac{d\phi}{2}\,\frac{H^{[m]}(\mathbf{q}_{T1}^2,\mathbf{q}_{T2}^2,\phi)}{M_T^4}$$

The coefficient functions $H^{[m]}$ are known at LO in α_s [Hagler *et.al*, 2000; Kniehl, Vasin, Saleev 2006] for $m={}^1S_0^{(1,8)},\ {}^3P_J^{(1,8)},\ {}^3S_1^{(8)}$.
The $H^{[m]}$ is a tree-level “squared matrix element” of the $2\rightarrow 1$-type process:

$$R_+(\mathbf{q}_{T1},q_1^+)+R_-(\mathbf{q}_{T2},q_2^-)\rightarrow c\bar{c}[m].$$

LLA evolution w.r.t. $\ln 1/\xi$

In the LL($\ln 1/\xi$)-approximation, the $Y = \ln 1/\xi$-evolution equation for *collinearly un-subtracted* $\tilde{\mathcal{C}}$-factor has the form:

$$\tilde{\mathcal{C}}(\xi, \mathbf{q}_T) = \delta(1-\xi)\delta(\mathbf{q}_T^2) + \hat{\alpha}_s \int\limits_{\xi}^{1} \frac{dz}{z} \int d^{2-2\epsilon}\mathbf{k}_T K(\mathbf{k}_T^2, \mathbf{q}_T^2) \tilde{\mathcal{C}}\left(\frac{\xi}{z}, \mathbf{q}_T - \mathbf{k}_T\right)$$

with $\hat{\alpha}_s = \alpha_s C_A/\pi$ and

$$K(\mathbf{k}_T^2, \mathbf{q}_T^2) = \frac{1}{\pi(2\pi)^{-2\epsilon}\mathbf{k}_T^2} + \delta^{(2-2\epsilon)}(\mathbf{k}_T)\ 2\omega_g(\mathbf{q}_T^2),$$

where $\omega_g(\mathbf{q}_T^2)$ – 1-loop Regge trajectory of a gluon. It is convenient to go from $(z, \mathbf{q}_T)$-space to $(N, \mathbf{x}_T)$-space:

$$\tilde{\mathcal{C}}(N, \mathbf{x}_T) = \int d^{2-2\epsilon}\mathbf{q}_T\ e^{i\mathbf{x}_T\mathbf{q}_T} \int\limits_0^1 dx\ x^{N-1}\ \tilde{\mathcal{C}}(x, \mathbf{q}_T),$$

because:

- Mellin convolutions over z turn into products: $\int \frac{dz}{z} \to \frac{1}{N}$
- Large logs map to poles at $N = 0$: $\boxed{\alpha_s^{k+1} \ln^{k} \frac{1}{\xi} \to \frac{\alpha_s^{k+1}}{N^{k+1}}}$
- All *collinear divergences* are contained inside $\mathcal{C}$ in $\mathbf{x}_T$-space.

Exact LL solution and the DLA

In $(N, \mathbf{q}_T)$-space, subtracted $\mathcal{C}$, which resums all terms $\propto (\hat{\alpha}_s/N)^n$ (complete LLA) has the form [Collins, Ellis, '91; Catani, Ciafaloni, Hautmann, '91,'94]:

$$\mathcal{C}(N, \mathbf{q}_T, \mu_F) = R(\gamma_{gg}(N, \alpha_s)) \frac{\gamma_{gg}(N, \alpha_s)}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2} \right)^{\gamma_{gg}(N, \alpha_s)},$$

where $\gamma_{gg}(N, \alpha_s)$ is the solution of [Jaroszewicz, '82]:

$$\frac{\hat{\alpha}_s}{N} \chi(\gamma_{gg}(N, \alpha_s)) = 1, \text{ with } \chi(\gamma) = 2\psi(1) - \psi(\gamma) - \psi(1 - \gamma),$$

where $\psi(\gamma) = d \ln \Gamma(\gamma)/d\gamma$ – Euler's ψ-function. The first few terms:

$$\gamma_{gg}(N, \alpha_s) = \underbrace{\underbrace{\frac{\hat{\alpha}_s}{N}}_{\text{DLA [Blümlein, '95]}} + 2\zeta(3) \frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5) \frac{\hat{\alpha}_s^6}{N^6} + \dots}_{\text{LLA}}$$

$\frac{\hat{\alpha}_s}{N} \leftrightarrow P_{gg}(z \to 0) = \frac{2C_A}{z} + \dots$

The function $R(\gamma)$ is

$$R(\gamma_{gg}(N, \alpha_s)) = 1 + O(\alpha_s^3).$$

Fixed-order asymptotics from HEF

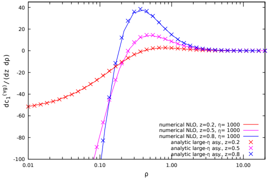
When expanded up to $O(\alpha_s)$ the HEF resummation should predict the $\hat{s} \gg M^2$ asymptotics of the CF coefficient function $\hat{\sigma}$

For the $g+g \to c\bar{c}\left[{}^1S_0^{(1)}, {}^3P_0^{(1)}, {}^3P_2^{(1)}\right]$ the NLO and NNLO($\alpha_s^2 \ln(1/z)$) terms in $\hat{\sigma}$ are predicted [M.N., Lansberg, Ozcelik '22]:

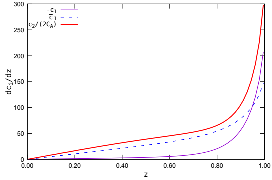
State	$A_0^{[m]}$	$A_1^{[m]}$	$A_2^{[m]}$	$B_2^{[m]}$
${}^1S_0$	1	**-1**	$\frac{\pi^2}{6}$	$\frac{\pi^2}{6}$
${}^3S_1$	0	1	0	$\frac{\pi^2}{6}$
${}^3P_0$	1	**$-\frac{43}{27}$**	$\frac{\pi^2}{6}+\frac{2}{3}$	$\frac{\pi^2}{6}+\frac{40}{27}$
${}^3P_1$	0	$\frac{5}{54}$	$-\frac{1}{9}$	$-\frac{2}{9}$
${}^3P_2$	1	**$-\frac{53}{36}$**	$\frac{\pi^2}{6}+\frac{1}{2}$	$\frac{\pi^2}{6}+\frac{11}{9}$

$$\hat{\sigma}_{gg}^{[m]}(z\to 0) = \sigma_{\rm LO}^{[m]}\left\{A_0^{[m]}\delta(1-z) + \frac{\alpha_s}{\pi}2C_A\left[A_1^{[m]} + A_0^{[m]}\ln\frac{M^2}{\mu_F^2}\right] + \left(\frac{\alpha_s}{\pi}\right)^2 \ln\frac{1}{z}\cdot C_A^2\left[2A_2^{[m]} + B_2^{[m]} + 4A_1^{[m]}\ln\frac{M^2}{\mu_F^2} + 2A_0^{[m]}\ln^2\frac{M^2}{\mu_F^2}\right] + O(\alpha_s^3)\right\},$$

For the $\gamma + g \to c\bar{c}\left[{}^3S_1^{(1)}\right] + g$ we have computed $\eta\to\infty$ limit of the z and $\rho = \mathbf{p}_T^2/M^2$-differential NLO "scaling functions" in closed analytic form,



and obtained numerical results for NNLO "scaling function" c_2 in front of $\alpha_s \ln(1+\eta)$.



Inverse Error Weighting (InEW) matching

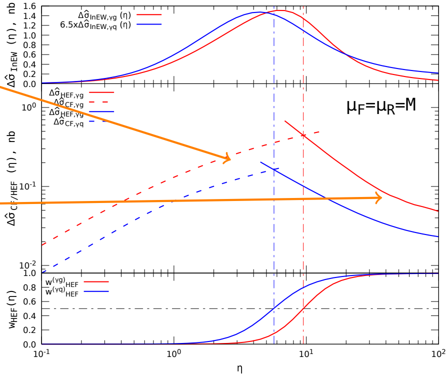
Development of an idea from [Echevarria *et al.*, 18'] :

$$\hat{\sigma}(\eta) = w_{\rm CF}(\eta)\hat{\sigma}_{\rm CF}(\eta) + (1 - w_{\rm CF}(\eta))\hat{\sigma}_{\rm HEF}(\eta),$$

the weights are determined through the estimates of “errors”:

$$w_{\rm CF}(\eta) = \frac{\Delta\hat{\sigma}^{-2}_{\rm CF}(\eta)}{\Delta\hat{\sigma}^{-2}_{\rm CF}(\eta) + \Delta\hat{\sigma}^{-2}_{\rm HEF}(\eta)}, \quad w_{\rm HEF}(\eta) = 1 - w_{\rm CF}(\eta).$$

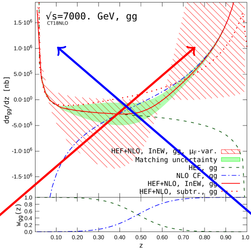
- $\Delta\hat{\sigma}_{\rm CF}(\eta)$ is due to **missing higher orders and large logarithms,** it can be estimated from the α_s expansion of $\hat{\sigma}_{\rm HEF}(\eta)$.
- $\Delta\hat{\sigma}_{\rm HEF}(\eta)$ is (mostly) due to **missing power corrections in** $1/\eta$: $\Delta\hat{\sigma}_{\rm HEF}(\eta) \sim A\eta^{-\alpha_{\rm HEF}}$. We determine A and $\alpha_{\rm HEF}$ from behaviour of $\hat{\sigma}_{\rm CF}(\eta) - \hat{\sigma}_{\rm CF}(\infty)$ at $\eta \gg 1$.



Matching with NLO

The HEF is valid in the **leading-power** in $M^2/\hat{s}$, so for $\hat{s} \sim M^2$ we match it with NLO CF by the *Inverse-Error Weighting Method* [Echevarria *et.al.*, 18'].

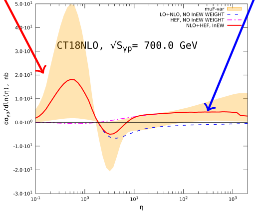
η_c**-hadroproduction**, $z = M^2/\hat{s}$:

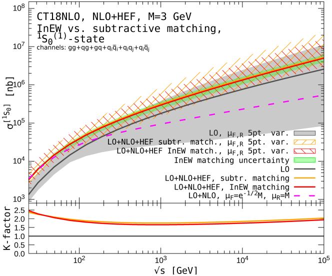


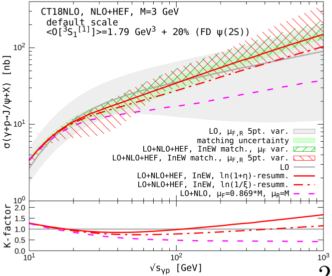
NLO

HEF

J/ψ**-photoproduction**, $\eta = (\hat{s} - M^2)/M^2$:





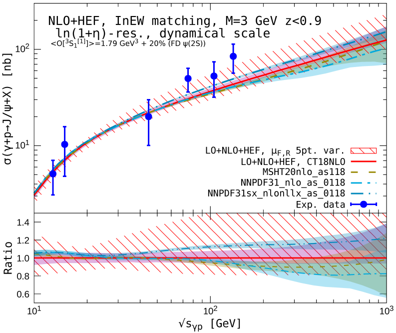


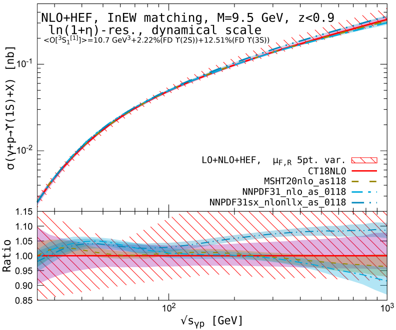
Vector quarkonium photoproduction: dynamical scale

Matched results for J/ψ photoproduction can be further improved by noticing that in the LO process:

$$\gamma(q) + g(p_1) \to Q\bar{Q}\left[{}^3S_1^{[1]}\right] + g,$$

the emitted gluon can not be soft, so that $\langle \hat{s} \rangle_{\rm LO}$ (~ 25 GeV^2 at high $\sqrt{s_{\gamma p}}$ for J/ψ) rather than M^2 can be taken as a default value of μ_F^2 and μ_R^2:





Exclusive J/ψ photoproduction

$$p(P) + \gamma(q) \to J/\psi(p) + p(P'),\ q^2 \simeq 0,$$

Kinematics (skewness):

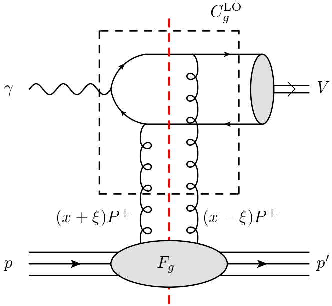
$$\xi = \frac{p^+}{2P^+} = \frac{M}{4E_p} e^{y(J/\psi)},$$

Factorisation formula ($P \simeq P'$):

$$A = \int_{-1}^{1} \frac{dx}{x} F_g(x, \xi, \mu_F) C_g(x),$$

$$\sigma \propto |A|^2.$$

Figure from
hep-ph/1507.06942



Exclusive J/ψ photoproduction at NLO

Partonic energy ($t = (P - P')^2 \simeq 0$):

$$\hat{s} = M^2 \frac{x+\xi}{2\xi} \gg M^2 \text{ if } \xi \ll x \ll 1,$$

NLO γg amplitude at $\xi \ll 1$ [Ivanov, Schaefer, Szymanowsky; Gracey, Jones, Teubner] :

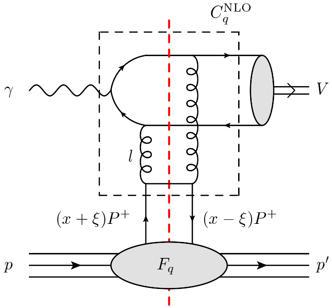
$$\mathrm{Im} A_{\mathrm{NLO}} \sim \hat{\alpha}_s \ln \frac{M^2}{4\mu_F^2} \int_\xi^1 \frac{dx}{x} H_g(x, \xi, \mu_F) + \ldots$$

The GPD H is relatively flat as function of x so $\int_\xi^1 \frac{dx}{x} \sim \ln 1/\xi$.

Also in Mellin space

$$\hat{\alpha}_s \int_\xi^1 \frac{dx}{x} \to \frac{\hat{\alpha}_s}{N}.$$

Figure from hep-ph/1507.06942



Treatment of the instability

Work in torgress, together with Jean-Philippe Lansberg, Chris Flett, Saad Nabebacus and Jakub Wagner.

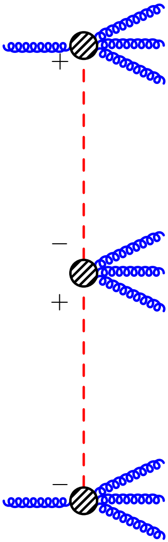
- Simplest solution: choose $\hat{\mu}_F = \frac{M}{2}$ (+ some other less conventional tricks...) [Jones, Martin, Ryskin, Teubner, 2016; ...]
- HEF resummation of $\hat{\alpha}_s^n/N^n$ corrections [Ivanov 2007]
- **My proposal:** one have to do matching of the HEF-resummed $C(x)$ at $\xi \ll x \ll 1$ and NLO CPM at $x \sim 1$.
- The closed formula for for the coefficient function at $x \ll 1$ can be derived in DLA:

$$\frac{2C_{\perp g}^{\mathrm{HEF}}(x)}{-i\pi\hat{\alpha}_s F_{\mathrm{LO}}} = \frac{1}{|x|}\sqrt{\frac{L_\mu}{L_x}}\left\{ I_1\left(2\sqrt{L_x L_\mu}\right) - 2\sum_{k=1}^{\infty} \mathrm{Li}_{2k}(-1)\left(\frac{L_x}{L_\mu}\right)^k I_{2k-1}\left(2\sqrt{L_x L_\mu}\right)\right\},$$

where $L_\mu = \ln[M^2/(4\mu_F^2)]$, $L_x = \hat{\alpha}_s \ln 1/|x|$ and Bessel functions $I_n(2\sqrt{L_\mu L_x})$ turn into $J_n(2\sqrt{-L_\mu L_x})$ if $L_\mu < 0$.

II. Beyond DLA: one-loop corrections to quarkonium impact-factors

The Gauge-Invariant EFT for Multi-Regge processes in QCD



- Reggeized gluon fields $R_\pm$ carry $(k_\pm, \mathbf{k}_T, k_\mp = 0)$: $\partial_\mp R_\pm = 0$.
- **Induced interactions** of particles and Reggeons [Lipatov '95, '97; Bondarenko, Zubkov '18]:

$$L = \frac{i}{g_s} \mathrm{tr} \left[R_+ \partial_\perp^2 \partial_- \left(W[A_-] - W^\dagger[A_-] \right) + (+ \leftrightarrow -) \right],$$

with $W_{x_\mp}[x_\pm, \mathbf{x}_T, A_\pm] = P \exp \left[\frac{-i g_s}{2} \int\limits_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, \mathbf{x}_T) \right] = \left(1 + i g_s \partial_\pm^{-1} A_\pm\right)^{-1}$.

- Expansion of the Wilson line generates **induced vertices**:

$$\mathrm{tr} \left[R_+ \partial_\perp^2 A_- + (-i g_s)(\partial_\perp^2 R_+)(A_- \partial_-^{-1} A_-) + (-i g_s)^2 (\partial_\perp^2 R_+)(A_- \partial_-^{-1} A_- \partial_-^{-1} A_-) + O(g_s^3) + (+ \leftrightarrow -) \right].$$

- The *Eikonal propagators* $\partial_\pm^{-1} \to -i/(k^\pm)$ lead to **rapidity divergences**, which are regularized by tilting the Wilson lines from the light-cone [Hentschinski, Sabio Vera, Chachamis *et. al.*, '12-'13; M.N. '19]:

$$n_\pm^\mu \to \tilde{n}_\pm^\mu = n_\pm^\mu + r n_\mp^\mu, \; r \ll 1: \; \tilde{k}^\pm = \tilde{n}^\pm k.$$

Prescription for the Eikonal poles

The interpretation of Eikonal poles $\partial_\pm^{-1} \to -i/(k^\pm)$ comes from the **Hermitian** form of the Lagrangian[Lipatov '97; Bondarenko, Zubkov '18]:
$iR_+\partial_\perp^2\partial_- \left(W\left[A_-\right] - W^\dagger\left[A_-\right]\right)/g_s.$

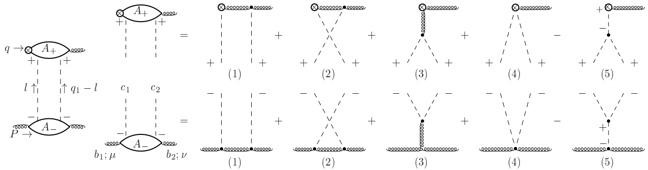
- For the Rgg vertex leads to the PV-prescription for the pole:

$$\frac{1}{[\tilde{k}^\pm]} = \frac{1}{2}\left(\frac{1}{\tilde{k}^\pm + i\varepsilon} + \frac{1}{\tilde{k}^\pm - i\varepsilon}\right),$$

  for $Rggg\dots$ vertices – more complicated, mixes colour and kinematics.
- Enforces $(-)$-signature of 1R exchange. Important for obtaining correct 2-loop Regge trajectory [Hentchinski, Sabio-vera, '13]

Two interpretations of $3R$ vertices: $R_\pm R_\mp R_\mp$

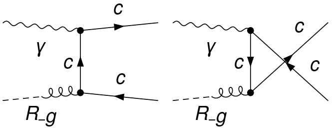
- As subtraction terms, leading to bootstrap [Hentchinski PhD-thesis; M.N. '19] :



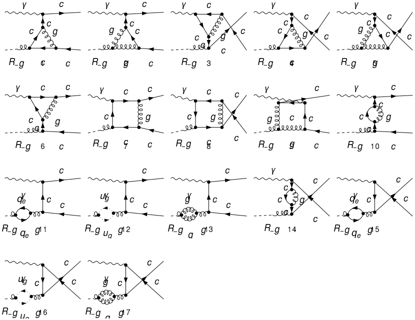
- If $\int \frac{dk^\pm}{[k^\pm]} = 0$, then RRR, $RRRRR$, ..., vertices are **zero** (*Gribov's signature conservation rule*).

$R\gamma \to c\bar{c}\left[{}^1S_0^{[8]}\right]$ @ 1 loop

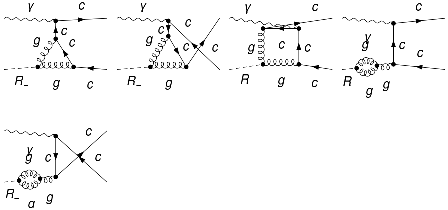
Interference with LO:



Rg-coupling diagrams:

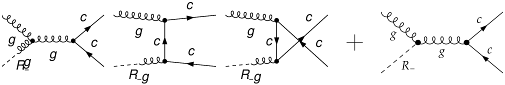


Induced *Rgg* coupling diagrams:

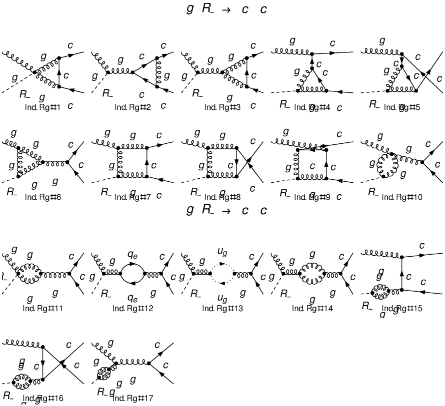


- Diagrams had been generated using custom **`FeynArts`** model-file, projector on the $c\bar{c}\left[{}^1S_0^{(8)}\right]$-state is inserted
- heavy-quark momenta $= p_{\mathcal{Q}}/2 \Rightarrow$ need to resolve linear dependence of quadratic denominators in some diagrams before IBP
- IBP reduction to master integrals has been performed using **`FIRE`**

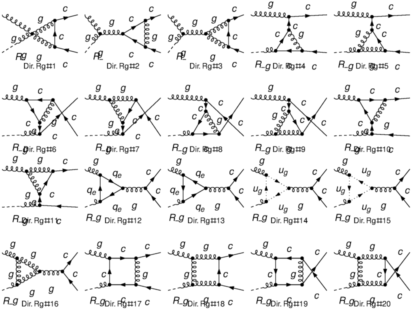
$Rg \to c\bar{c}\left[{}^1S_0^{[1]}\right]$ and $c\bar{c}\left[{}^3S_1^{[8]}\right]$ @ 1 loop

Interference with LO: 

Induced Rgg coupling diagrams:

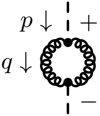


Some Rg-coupling diagrams:



and so on...

Rapidity divergences and regularization.

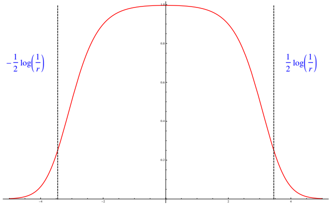


$$= g_s^2 C_A \delta_{ab} \int \frac{d^d q}{(2\pi)^D} \frac{\left(\mathbf{p}_T^2 (n_+ n_-)\right)^2}{q^2 (p-q)^2 q^+ q^-}, \quad \int \frac{dq^+ dq^-}{q^+ q^-} = \int_{y_1}^{y_2} dy \int \frac{dq^2}{q^2 + \mathbf{q}_T^2}$$

the regularization by explicit cutoff in rapidity was originally proposed [Lipatov, '95] ($q^\pm = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$, $p^+ = p^- = 0$):

$$\delta_{ab} \mathbf{p}_T^2 \times \underbrace{C_A g_s^2 \int \frac{\mathbf{p}_T^2 d^{D-2} \mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2}}_{\omega^{(1)}(\mathbf{p}_T^2)} \times (y_2 - y_1) + \text{finite terms}$$

The square of regularized Lipatov vertex:



$$\Gamma_{+\mu-} \Gamma_{+\nu-} P^{\mu\nu} = \frac{16 \mathbf{q}_{T1}^2 \mathbf{q}_{T2}^2}{\mathbf{k}_T^2} f(y),$$

$$\longleftarrow f(y) = \frac{1}{(re^{-y} + e^y)^2 (re^y + e^{-y})^2},$$

$$\int_{-\infty}^{+\infty} dy \; f(y) = -1 - \log r + O(r)$$

Rapidity divergences at one loop

Only log-divergence $\sim \log r$ (Blue cells in the table) is related with Reggeization of particles in t-channel.
Integrals which **do not** have log-divergence may still contain the power-dependence on r:

- $r^{-\epsilon} \to 0$ for $r \to 0$ and $\epsilon < 0$.
- $r^{+\epsilon} \to \infty$ for $r \to 0$ and $\epsilon < 0$ – **weak-power divergence** (Pink cells in the table)
- $r^{-1+\epsilon} \to \infty$ – **power divergence.** (Red)

(# LC prop.) \ (# quadr. prop.)	1	2	3	4
1	$A_{[-]}$	$B_{[-]}$	$C_{[-]}$	...
2	$A_{[+-]}$	$B_{[+-]}$	$C_{[+-]}$	...
3	...	...	...	...

The **weak-power** and **power-divergences** cancel between Feynman diagrams describing one region in rapidity, so only log-divergences are left.

Scalar integrals with power RDs.

Notation: $\left\{\frac{\mu}{k}\right\}^{2\epsilon} = \frac{1}{2}\left[\left(\frac{\mu}{k-i\varepsilon}\right)^{2\epsilon} + \left(\frac{\mu}{-k-i\varepsilon}\right)^{2\epsilon}\right].$

Tadpoles:

$$
\begin{aligned}
A_{[-]}(p) &= -\frac{\tilde{p}^{-}\; r^{-1+\epsilon}}{\cos(\pi\epsilon)}\frac{1}{2\epsilon(1-2\epsilon)}\left\{\frac{\mu}{\tilde{p}^{-}}\right\}^{2\epsilon}, \\
A_{[--]}(p) &= \frac{1}{\tilde{p}_{-}}A_{[-]}(p).
\end{aligned}
$$

Bubbles:

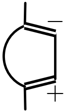
$$
\begin{aligned}
B_{[-]}(p) &= \frac{1}{p^{-}\epsilon^{2}}\left(\frac{\mu^{2}}{-p^{2}}\right)^{\epsilon} + \frac{1-2\epsilon}{\epsilon}\frac{r\cdot A_{[-]}(p)}{\tilde{p}_{-}^{2}} + \Delta B_{[-]}(-p^{2},p_{-}) + O(r), \\
B_{[--]}(p) &= \frac{2}{\tilde{p}_{-}}B_{[-]}(p),
\end{aligned}
$$

where:

$$
\Delta B_{[-]}(-p^{2},p_{-}) = -\frac{1}{p_{-}}\left(\frac{p_{-}^{2}\mu^{2}}{(-p^{2})^{2}}\right)^{\epsilon}\frac{\Gamma^{2}(1-2\epsilon)\Gamma(1+2\epsilon)\cdot r^{-\epsilon}}{2\epsilon^{2}\Gamma^{2}(1-\epsilon)}.
$$

Logarithmic RDs

- $[+-]$-bubble in transverse kinematics $p^- = p^+ = 0$:



$$B_{[+-]}(\mathbf{p}_T) = \frac{1}{\mathbf{p}_T^2}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon}\frac{i\pi + 2\log r}{\epsilon},$$

- $[+-]$-bubble in $p^- = 0$ kinematics:

$$\begin{aligned} B_{[+-]}(\mathbf{p}_T, p^+) &= \frac{1}{\mathbf{p}_T^2}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon}\frac{\Gamma^2(1+\epsilon)\Gamma(2+\epsilon)\sin(\pi\epsilon)}{\pi\epsilon^2} \\ &\times \left[i\pi + \log r - \log\frac{p_+^2}{\mathbf{p}_T^2} - \psi(1+\epsilon) + \psi(1)\right] + O(r^{1/2}) \end{aligned}$$

- $[+-]$-bubble in light-like kinematics $p^2 = 0$:

$$B_{[+-]}(\mathbf{p}_T^2, p^2 = 0) = \int \frac{[d^d l]}{l^2(l+p)^2[l^+][l^- + p^-]} = \frac{-2\Gamma(1-\epsilon)\Gamma(1+\epsilon)}{\mathbf{p}_T^2\epsilon^2}\left(\frac{\mu^2}{\mathbf{p}_T^2}\right)^{\epsilon}.$$

Triangle integrals, logarithmic RD

Result for $Q^2 = 0$:

$$C_{[-]}(t_1, 0, q^-) = \frac{1}{q^- t_1}\left(\frac{\mu^2}{t_1}\right)^{\epsilon} \frac{1}{\epsilon}\left[\log r + i\pi - \log\frac{|q_-|^2}{t_1}\right.$$
$$\left. -\psi(1+\epsilon) - \psi(1) + 2\psi(-\epsilon)\right] + O(r^{1/2}),$$

coincides with the result of [G. Chachamis, *et. al.*, '12] .

Result for $Q^2 \neq 0$ [M.N., '19] :

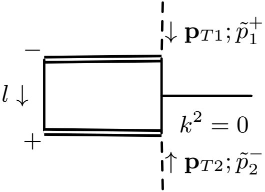
$$C_{[-]}(t_1, Q^2, q_-) = C_{[-]}(t_1, 0, q_-) + \left(\frac{\mu^2}{t_1}\right)^{\epsilon} \frac{I(Q^2/t_1)}{q_- t_1} - \frac{1}{t_1}\Delta B_{[-]}(Q^2, q_-),$$

where

$$\begin{aligned} I(X) &= -\frac{2X^{-\epsilon}}{\epsilon^2} - \frac{2}{\epsilon}\int\limits_0^X \frac{(1 - x^{-\epsilon})dx}{1-x} \\ &= -\frac{2X^{-\epsilon}}{\epsilon^2} + 2\left[-\mathrm{Li}_2(1-X) + \frac{\pi^2}{6}\right] + O(\epsilon). \end{aligned}$$

Triangle with two light-cone propagators

Usual one-loop Feynman integrals with more than 4 propagators are reducible to more simple integrals up to terms $O(\epsilon)$.



We apply method of [Bern, Dixon, Kosower, '92]. The $O(\epsilon)$ remnant is proportional to $(d-4)I^{(d+2)}$ and integral $I^{(6)}$ is finite.
The resiut in Euclidean region ($p_1^+ > 0$, $-p_2^- > 0$, $\mathbf{p}_{T1,2}^2 > 0$):

$$C_{[+-]}(\mathbf{p}_{T1}^2, \mathbf{p}_{T2}^2, p_1^+, -p_2^-) = \frac{(-1)}{2\mathbf{p}_{T1}^2\mathbf{p}_{T2}^2\mathbf{k}_T^2} \times$$
$$\left\{\mathbf{p}_{T1}^2(\mathbf{p}_{T2}^2 - \mathbf{p}_{T1}^2 + \mathbf{k}_T^2)\left[B_{[+-]}(\mathbf{p}_{T1}^2, p_1^+) + (-p_2^-)C_{[-]}(\mathbf{p}_{T1}^2, \mathbf{p}_{T2}^2, -p_2^-)\right]\right.$$
$$+\mathbf{p}_{T2}^2(\mathbf{p}_{T1}^2 - \mathbf{p}_{T2}^2 + \mathbf{k}_T^2)\left[B_{[+-]}(\mathbf{p}_{T2}^2, -p_2^-) + p_1^+ C_{[+]}(\mathbf{p}_{T2}^2, \mathbf{p}_{T1}^2, p_1^+)\right]$$
$$\left.-\mathbf{k}_T^2(\mathbf{p}_{T1}^2 + \mathbf{p}_{T2}^2 - \mathbf{k}_T^2)B_{[+-]}(\mathbf{k}_T^2, k^2 = 0)\right\},$$

where $\mathbf{k}_T^2 = p_1^+(-p_2^-)$.
The $\log r$-divergence cancels within square brackets, as expected.

Integrals with massive internal lines

In presence of the linear denominator the massive propagator can be converted to the massless one:

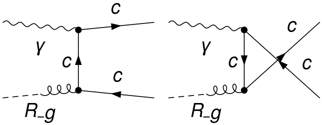
$$\frac{1}{((\tilde{n}_+ l) + k_+)(l^2 - m^2)} = \frac{1}{((\tilde{n}_+ l) + k_+)(l + \kappa \tilde{n}_+)^2} + \frac{2\kappa \left[(\tilde{n}_+ l) + \frac{m^2 + \tilde{n}_+^2 \kappa^2}{2\kappa} \right]}{((\tilde{n}_+ l) + k_+)(l + \kappa \tilde{n}_+)^2 (l^2 - m^2)}$$

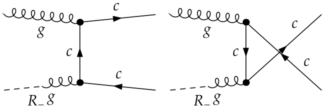
$\Rightarrow$ all the masses can be moved to integrals with **only quadratic propagators**. New **massless** scalar integrals with RDs arise($k^2 = 0$, $p^2 = 4m^2$, $p = q + q_1$, $q^2 = 0$):

$$\begin{aligned}
B_{[+]}(-k, k-q) &= \int \frac{d^D l}{[\tilde{l}^+](l-k)^2 (l+k-q)^2}, \\
C_{[+]}(0, -k, k-q) &= \int \frac{d^D l}{[\tilde{l}^+] l^2 (l-k)^2 (l+k-q)^2}, \\
B_{[+]}(p, k) &= \int \frac{d^D l}{[\tilde{l}^+](l+p)^2 (l+k)^2}, \\
C_{[+]}(p, k, q_1) &= \int \frac{d^D l}{[\tilde{l}^+](l+p)^2 (l+k)^2 (l+q_1)^2},
\end{aligned}$$

but they have the same complexity as already encountered ones.

Results: $R\gamma \to c\bar{c}\left[{}^1S_0^{[8]}\right]$ vs. $Rg \to c\bar{c}\left[{}^1S_0^{[1]}\right]$ @ 1 loop





Results for $2\Re\left[\frac{H_{\mathbf{1L\ x\ LO}}(\mathbf{q}_T) - \textbf{(On-shell mass CT)}}{(\alpha_s/(2\pi))H_{\mathrm{LO}}(\mathbf{q}_T)}\right]$:

$$^1S_0^{[8]}: \left(\frac{\mu^2}{\mathbf{q}_T^2}\right)^{\epsilon}\frac{1}{\epsilon}\left[N_c\left(\ln\frac{\mathbf{q}_T^2}{M^2} + \ln\frac{q_-^2}{\mathbf{q}_T^2 r} + \frac{19}{6}\right) - \frac{2n_F}{3} - \frac{3}{2N_c}\right] + F_{^1S_0^{[8]}}(\mathbf{q}_T^2/M^2)$$

$$^1S_0^{[1]}: \left(\frac{\mu^2}{\mathbf{q}_T^2}\right)^{\epsilon}\left\{-\frac{N_c}{\epsilon^2} + \frac{1}{\epsilon}\left[N_c\left(\ln\frac{q_-^2}{\mathbf{q}_T^2 r} + \frac{25}{6}\right) - \frac{2n_F}{3} - \frac{3}{2N_c}\right]\right\} + F_{^1S_0^{[1]}}(\mathbf{q}_T^2/M^2)$$

$$F_{^1S_0^{[1]}}(\tau) = -\frac{10}{9}n_F + \Re[C_F F^{(C_F)}_{^1S_0^{[1]}}(\tau) + C_A F^{(C_A)}_{^1S_0^{[1]}}(\tau)],$$

$$F^{(C_F)}_{^1S_0^{[1]}}(\tau) = F^{(C_F)}_{^1S_0^{[8]}}(\tau),$$

while $F^{(C_A)}_{^1S_0^{[1]}}(\tau) \neq F^{(C_A)}_{^1S_0^{[8]}}(\tau)$.

The C_F coefficient

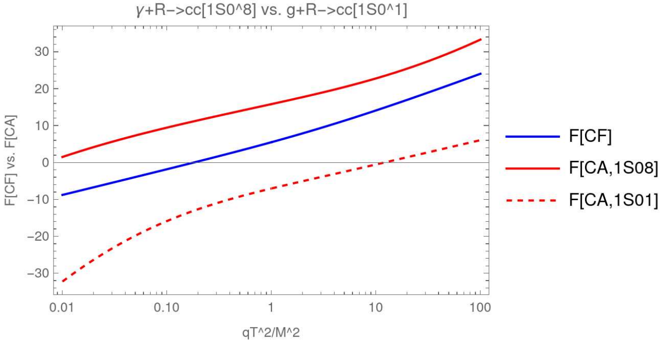
$$F^{(C_F)}_{{}^1S_0^{[8]}}(\tau) = \frac{\mathcal{L}_2 + \mathcal{L}_7(1-2\tau)}{\tau+1}$$

$$+\frac{1}{6(\tau+1)(2\tau+1)^2}\{144L_1\tau^2 + 144L_1\tau + 36L_1 - 16\pi^2\tau^3 - 72\tau^3 + 72\tau^3\log(2)$$

$$-156\tau^2 + 12\tau^2\log^2(2\tau+1) + 168\tau^2\log(2) - 24\left(3\tau^2+5\tau+2\right)\tau\log(\tau+1)$$

$$+12\pi^2\tau - 108\tau + 12\tau\log^2(2\tau+1) + 3\log^2(2\tau+1) + 132\tau\log(2)$$

$$+18(\tau+1)(2\tau+1)^2\log(\tau) + 4\pi^2 - 24 + 36\log(2)\}$$



The C_A coefficient for $R\gamma \to c\bar{c}\left[{}^1S_0^{[8]}\right]$

$$
\begin{aligned}
F^{(C_A)}_{{}^1S_0^{[8]}}(\tau) \quad = \quad & \frac{1}{2(\tau-1)(\tau+1)^3}\{(\tau+1)^2\left(-4\mathcal{L}_4\left(\tau^2-1\right)+\mathcal{L}_2(\tau+1)(2\tau+1)+\mathcal{L}_7\tau(2\tau-3)+\mathcal{L}_7\right) \\
& +2\mathcal{L}_6(\tau(\tau((\tau-4)\tau-6)-4)+1)\} \\
+ \quad & \frac{1}{36(\tau-1)(\tau+1)^3(2\tau+1)}\{-216L_1\tau^4-324L_1\tau^3+108L_1\tau^2+324L_1\tau+108L_1 \\
& +120\pi^2\tau^5+608\tau^5-36\tau^5\log^2(\tau+1)+36\tau^5\log^2(2\tau+1)-36\tau^5\log^2(2) \\
& -72\tau^5\log(2)\log(\tau+1)+216\tau^5\log(\tau+1)+72\tau^5\log(2)+228\pi^2\tau^4+1520\tau^4 \\
& -306\tau^4\log^2(\tau+1)+144\tau^4\log^2(2\tau+1)-306\tau^4\log^2(2) \\
& +252\tau^4\log(2)\log(\tau+1)+432\tau^4\log(\tau+1)+360\tau^4\log(2)+84\pi^2\tau^3+608\tau^3 \\
& -360\tau^3\log^2(\tau+1)+225\tau^3\log^2(2\tau+1)-360\tau^3\log^2(2)+576\tau^3\log(2)\log(\tau+1) \\
& +72\tau^3\log(\tau+1)+72\tau^3\log(2)-120\pi^2\tau^2-1216\tau^2-108\tau^2\log^2(\tau+1) \\
& +171\tau^2\log^2(2\tau+1)-108\tau^2\log^2(2)+504\tau^2\log(2)\log(\tau+1)-360\tau^2\log(\tau+1) \\
& -360\tau^2\log(2)-72(\tau+1)^3\left(2\tau^2-\tau-1\right)\log(\tau-1)(\log(2)-\log(\tau+1)) \\
& +36(2\tau+1)\log(\tau)\left[-\tau^4+\tau^4\log(8)-6\tau^2\log(2)+\left(-\tau^3+4\tau^2+6\tau+4\right)\tau\log(\tau+1)\right. \\
& \left.-8\tau\log(2)-\log(2\tau+2)+1\right]-18\left(2\tau^5+17\tau^4+20\tau^3+6\tau^2-6\tau-3\right)\log^2(\tau) \\
& -84\pi^2\tau-1216\tau+108\tau\log^2(\tau+1)+63\tau\log^2(2\tau+1)+108\tau\log^2(2) \\
& +54\log^2(\tau+1)+9\log^2(2\tau+1)+72\tau\log(2)\log(\tau+1)-288\tau\log(\tau+1) \\
& -144\tau\log(2)-36\log(2)\log(\tau+1)-72\log(\tau+1)-12\pi^2-304+54\log^2(2)\}
\end{aligned}
$$

The C_A coefficient for $Rg \to c\bar{c}\left[{}^1S_0^{[1]}\right]$

$$
\begin{aligned}
F^{(C_A)}_{{}^1S_0^{[1]}}(\tau) \quad = \quad & \frac{1}{(\tau-1)(\tau+1)^3}\{2\mathcal{L}_1\left(\tau^2+\tau-2\right)(\tau+1)^3+\tau\Big[2\mathcal{L}_5\left(\tau(\tau+1)\left(\tau^2-2\right)+1\right) \\
& -\mathcal{L}_7\left(\tau^2+\tau-1\right)-\left(\mathcal{L}_2(\tau+2)(\tau+1)^2\right)+\mathcal{L}_6(\tau(\tau(6-(\tau-4)\tau)+4)-1)\Big] \\
& +2\mathcal{L}_3(\tau-1)(\tau+1)^3+2\mathcal{L}_5+\mathcal{L}_7\} \\
- \quad & \frac{1}{18(\tau-1)(\tau+1)^3}\{6\pi^2\tau^5-36\tau^5\log(2)\log(\tau+1)+36\tau^5\log(\tau+1)\log(\tau+2)+63\pi^2\tau^4 \\
& -98\tau^4-63\tau^4\log^2(\tau+1)+9\tau^4\log^2(2\tau+1)-63\tau^4\log^2(2)+54\tau^4\log(2)\log(\tau+1) \\
& -36\tau^4\log(\tau+1)+36\tau^4\log(\tau+1)\log(\tau+2)+36\tau^4\log(2)+138\pi^2\tau^3-196\tau^3 \\
& -72\tau^3\log^2(\tau+1)+36\tau^3\log^2(2\tau+1)-72\tau^3\log^2(2)+144\tau^3\log(2)\log(\tau+1) \\
& -36\tau^3\log(\tau+1)-72\tau^3\log(\tau+1)\log(\tau+2)-36\tau^3\log(2)+18\pi^2\tau^2 \\
& -18\tau^2\log^2(\tau+1)+45\tau^2\log^2(2\tau+1)-18\tau^2\log^2(2)+108\tau^2\log(2)\log(\tau+1) \\
& +36\tau^2\log(\tau+1)-72\tau^2\log(\tau+1)\log(\tau+2)-36\tau^2\log(2) \\
& -18\left(4\tau^4+5\tau^3+\tau^2-3\tau-1\right)\log^2(\tau)+18\log(\tau)\Big[\tau^5\log(2)-\tau^4(\log(4)-2) \\
& -\tau^3\log(4)-2\tau^2(1+\log(4))-\left(\tau^4-4\tau^3-6\tau^2-4\tau+1\right)\tau\log(\tau+1)-\tau\log(8)-\log(4)\Big] \\
& -120\pi^2\tau+196\tau+36\tau\log^2(\tau+1)+18\tau\log^2(2\tau+1)+36\tau\log^2(2)+9\log^2(\tau+1) \\
& -36\tau\log(2)\log(\tau+1)+36\tau\log(\tau+1)+36\tau\log(\tau+1)\log(\tau+2)+36\tau\log(2) \\
& -36(\tau-1)(\tau+1)^3\log(\tau-1)(\log(2)-\log(\tau+1))-18\log(2)\log(\tau+1) \\
& +36\log(\tau+1)\log(\tau+2)-69\pi^2+98+9\log^2(2)\}
\end{aligned}
$$

$$L_1 = \sqrt{\tau(1+\tau)}\ln\left[1+2\tau+2\sqrt{\tau(1+\tau)}\right],$$

$$\mathcal{L}_1 = \mathrm{Li}_2\left(\frac{1}{\tau}+1\right)$$

$$\mathcal{L}_2 = \mathrm{Li}_2\left(\frac{1}{-2\tau-1}\right)$$

$$\mathcal{L}_3 = \mathrm{Li}_2\left(\frac{1}{\tau}\right)+\mathrm{Li}_2\left(\frac{\tau-1}{\tau+1}\right)-\mathrm{Li}_2\left(\frac{\tau+1}{2\tau}\right)+\frac{\mathrm{Li}_2\left(\frac{1}{4}\right)}{2}+\mathrm{Li}_2(-2)$$

$$\mathcal{L}_4 = \mathrm{Li}_2\left(1+\frac{1}{\tau}\right)+\mathrm{Li}_2\left(\frac{1}{\tau}\right)+\mathrm{Li}_2\left(\frac{\tau-1}{\tau+1}\right)-\mathrm{Li}_2\left(\frac{\tau+1}{2\tau}\right)+\frac{\mathrm{Li}_2\left(\frac{1}{4}\right)}{2}+\mathrm{Li}_2(-2)$$

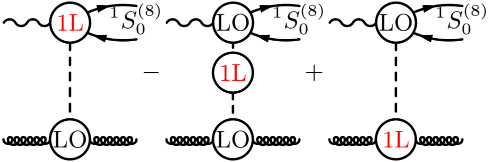
$$\mathcal{L}_5 = \mathrm{Li}_2\left(-\frac{1}{\tau+1}\right)-\mathrm{Li}_2(\tau+2)+\frac{1}{2}\mathrm{Li}_2\left(\frac{2\tau+1}{2\tau+2}\right)$$

$$\begin{aligned}\mathcal{L}_6 = & -\mathrm{Li}_2\left(-\frac{2\tau+1}{\tau^2}\right)+\mathrm{Li}_2\left(-\frac{-2\tau^2+\tau+1}{2\tau^2}\right)+\mathrm{Li}_2\left(\frac{1}{2}-\frac{\tau}{2}\right)+\mathrm{Li}_2\left(-\frac{1}{\tau}\right)\\ & -\mathrm{Li}_2\left(\frac{\tau-1}{2\tau}\right)-\mathrm{Li}_2(-\tau)+\mathrm{Li}_2\left(\frac{1-\tau}{\tau+1}\right)\end{aligned}$$

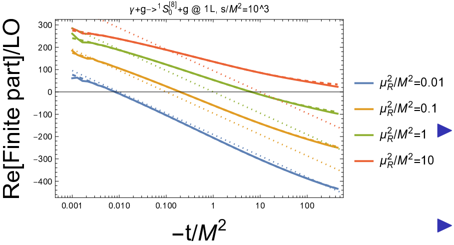
$$\mathcal{L}_7 = \mathrm{Li}_2(-2\tau-1)-\mathrm{Li}_2\left(\frac{2\sqrt{\tau}}{\sqrt{\tau}-\sqrt{\tau+1}}\right)-\mathrm{Li}_2\left(\frac{2\sqrt{\tau}}{\sqrt{\tau}+\sqrt{\tau+1}}\right)$$

$R\gamma \to c\bar{c}\left[{}^1S_0^{[8]}\right]$ @ 1 loop, cross-check

In the combination of 1-loop results in the EFT:



the $\ln r$ cancels and it should reproduce the the Regge limit($s \gg -t$) of the *real part* of the $2 \to 2$ 1-loop QCD amplitude:



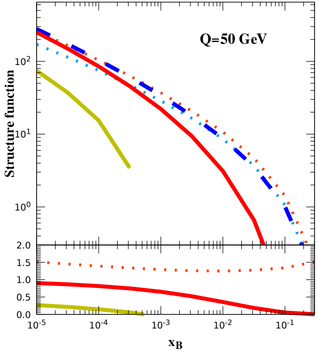
Solid lines – QCD, dashed lines – EFT, dotted lines – $-2C_A \ln(-t/\mu_R^2)\ln(s/M^2)$

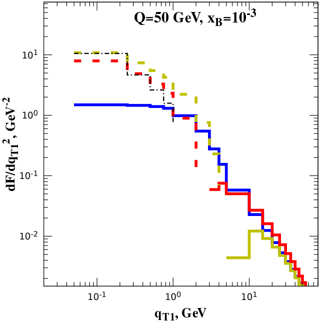
$$\gamma + g \to c\bar{c}\left[{}^1S_0^{(8)}\right] + g.$$

- The $2 \to 2$ QCD 1-loop amplitude can be computed numerically using `FormCalc` (with some tricks, due to Coulomb divergence)
- The Regge limit of $1/\epsilon$ divergent part agrees with the EFT result
- For the finite part agreement within few % is reached, need to push to higher s

“Monster logs” in the coefficient function

In standard k_T-factorisation (or CGC/saturation) computations, where the $\sigma = \Phi(x, \mathbf{q}_T^2) \otimes \mathcal{H}(y, \mathbf{q}_T^2)$ the appearance of $\alpha_s \ln^{1,2}(\mathbf{q}_T^2/\mu^2)$ for $\mathbf{q}_T^2 \ll \mu^2$ at NLO for $\mathcal{H}$ is a serious problem. Example, Higgs-DIS in the $m_t \to \infty$ limit [M.N. ’20]:





(look at yellow curves – standard MRK computation, red curves – computation with modified-MRK $\simeq$ “kinematic constraint”)

$$\mathbf{q}_T^2 \ll Q^2 : \mathcal{H}(y, \mathbf{q}_T^2) \sim -\frac{\alpha_s C_A}{2\pi} \ln^2 \frac{\mathbf{q}_T^2}{Q^2} + \text{(single-log terms)}.$$

The "Monster logs" at small $\mathbf{q}_T$ are not scary for the matching computation

$$\hat{\sigma}_{\rm HEF}(\eta) \propto \int_0^{1+\eta} \frac{dy}{y} \int_0^{\infty} d\mathbf{q}_{T1}^2 \mathcal{C}\left(\frac{y}{1+\eta}, \mathbf{q}_{T1}^2, \mu_F, \mu_R\right) \mathcal{H}(y, \mathbf{q}_{T1}^2).$$

At NLO for $\mathcal{H}$ one typically encounters corrections $\propto \alpha_s \ln^n \frac{M^2}{\mathbf{q}_T^2}$ at $\mathbf{q}_T^2 \ll M^2$ with $n = 1, 2$. Let's study their effect in N-space (note that $\gamma_N = \hat{\alpha}_s/N$):

$$\int_0^{\mu_F^2} d\mathbf{q}_T^2\, \mathcal{C}_{\rm DLA}(N, \mathbf{q}_T^2, \mu_F^2) \times \hat{\alpha}_s \ln^n \frac{\mu_F^2}{\mathbf{q}_T^2} = \hat{\alpha}_s \gamma_N \int_0^{\mu_F^2} \frac{d\mathbf{q}_T^2}{\mathbf{q}_T^2} \left(\frac{\mathbf{q}_T^2}{\mu_F^2}\right)^{\gamma_N} \ln^n \frac{\mu_F^2}{\mathbf{q}_T^2}$$

$$= \hat{\alpha}_s \frac{(-1)^n n!}{\gamma_N^n} = \begin{cases} -N & \text{for } n = 1 \\ \frac{2N^2}{\hat{\alpha}_s} & \text{for } n = 2 \end{cases} \xrightarrow[\text{Mellin transform}]{} \begin{cases} -\delta'(\eta) & \text{for } n = 1 \\ \frac{2}{\hat{\alpha}_s}\delta''(\eta) & \text{for } n = 2 \end{cases}$$

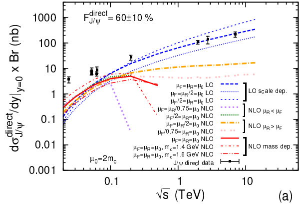
So these contributions *do not belong to NLA in* $\eta = (\hat{s} - M^2)/M^2 \gg 1$ and *will be removed by the matching!*

Conclusions and outlook

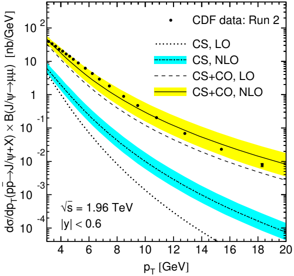
- The perturbative instability of p_T-integrated quarkonium production cross sections at NLO comes from the region $\hat{s} \gg M^2$. The problem can be solved via matching of NLO calculation at $\hat{s} \sim M^2$ and LLA HEF calculation at $\hat{s} \gg M^2$
- The LLA HEF has to be truncated down to DLA for resummation factors, to be consistent with NLO DGLAP evolution
- The inclusive η_c hadroproduction and J/ψ photoproduction have been considered as examples
- The *next-to-DLA* calculation is needed to further reduce scale-uncertainties. Both virtual and real corrections to HEF coefficient function can be computed within the *High-Energy EFT* formalism
- **The virtual corrections to $\gamma R \to c\bar{c}[^1S_0^{[8]}]$, $gR \to c\bar{c}[^1S_0^{[1]}]$ and $gR \to c\bar{c}[^3S_1^{[8]}]$ IFs has been computed**
- The logarithms $\ln M^2/\mathbf{q}_T^2$ for $\mathbf{q}_T^2 \ll M^2$ in the NLO HEF coefficient function will not be a problem for the matching calculation!

There is a lot to do even in DLA+NLO!

[Y.Feng, *et al.*, '15]



[Butenschön, He, Kniehl, '11]



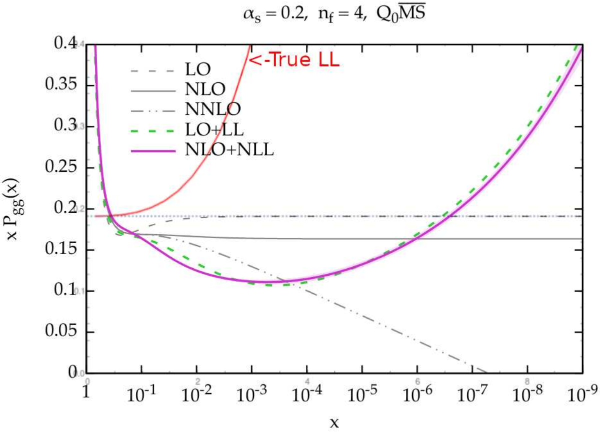
Thank you for your attention!

Backup: DGLAP P_{gg} at small z

LO: $P_{gg}(z) = \frac{2C_A}{z} + \ldots \Leftrightarrow \gamma_N = \frac{\hat{\alpha}_s}{N}$

Plot from hep-ph/1607.02153 with my curve (in red) for the **strict LLA:**

$\frac{\hat{\alpha}_s}{N}\chi_{LO}(\gamma_{gg}(N)) = 1 \Rightarrow \gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \ldots$



The "LO+LL" and "NLO+NLL" curves represent a form of matching between DGLAP and BFKL expansions, in a scheme by Altarelli, Ball and Forte which is more complicated than the **strict LL or NLL approximation**.

Effect of anomalous dimension beyond LO

Effect of taking **full LLA** for $\gamma_{gg}(N) = \frac{\hat{\alpha}_s}{N} + 2\zeta(3)\frac{\hat{\alpha}_s^4}{N^4} + 2\zeta(5)\frac{\hat{\alpha}_s^6}{N^6} + \ldots$ together with NLO PDF.