The confrontation between the muon magnetic moment and the Standard Model: recent results and perspectives

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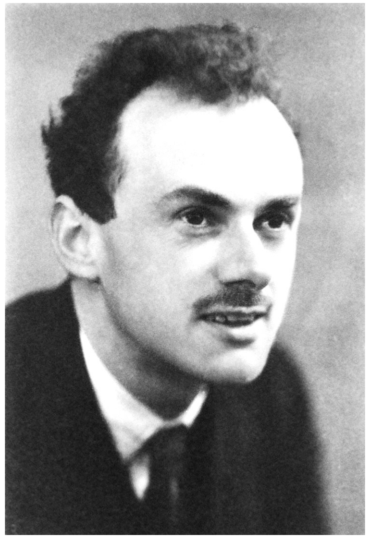
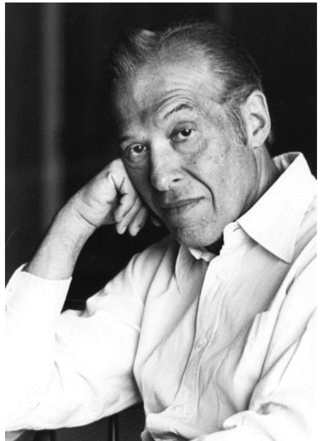
The electron g-2 early history



- Dirac's relativistic theory of the electron (1928) naturally accounted for quantized particle spin, and described elementary spin-1/2 particles (and their anti-particles)
- In the classical limit, one finds the Pauli equation with a magnetic moment:

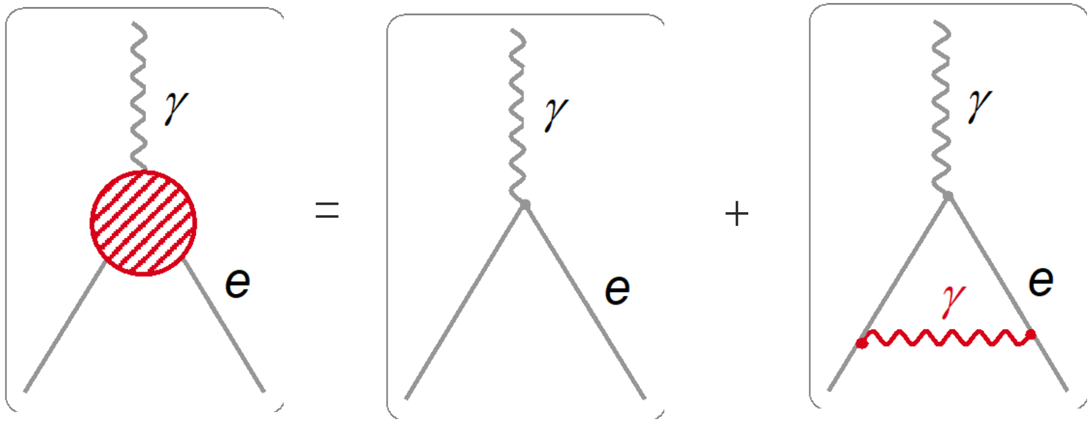
$$\vec{\mu} = -g_e \frac{e}{2m_e} \vec{S} \qquad \text{with } |\boldsymbol{g_e}| = 2 \text{ is the gyromagnetic factor}$$

- Dirac's prediction confirmed to 0.1% by Kinsler & Houston in 1934 studying the Zeeman effect in neon. Deviation from $g_e = 2$ established by Nafe, Nels & Rabi in 1947 by comparing the hyperfine structure of hydrogen and deuterium spectra
- First precision measurement of $g_e = 2.00238 \pm 0.00010$ by Kusch & Foley in 1947 using Rabi's atomic beam magnetic resonance technique
- New physics? Understood with QED (Schwinger 1948)



$$a_e^{\text{QED}} = \frac{\alpha}{2\pi} + \cdots = 0.001\,161\ \ldots$$

magnetic anomaly a = (g–2) / 2



Why measure the muon g-2 ?

- 3 families of fermions (leptons and quarks) with universal coupling strengths to electroweak interactions
- The 3 charged leptons $l \equiv (e, \mu, \tau)$ differ only by their own leptonic quantum numbers and their masses

  m_e = 0.511 MeV m_μ = 105.7 MeV m_τ = 1776.9 MeV
- e stable, μ and τ are unstable and decay through the weak interaction with lifetimes 2.2 μs and 390 fs

- sensitivity of a_l to new physics at energy scale Λ goes like m_l^2 / Λ^2
- Muon more sensitive by large factor $(m_\mu / m_e)^2 \sim 43000$, but measurement limited by short lifetime
- Measurement for τ lepton not practical at the moment

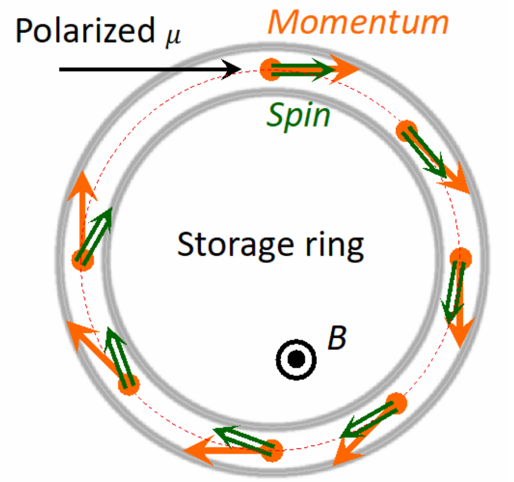
Principle of muon g-2 measurement (CERN 1960-80)

1. Parity violation polarizes muons in pion decay

spin orientation

$$\overset{\longleftarrow}{\bar{\nu}_\mu} \longleftarrow \pi^- \longrightarrow \overset{\longrightarrow}{\mu^-_{\text{polarized}}}$$

2. Anomalous frequency proportional to a_μ



- Very uniform magnetic field
- Focusing with electrostatic quadrupoles

3. Magic γ to cancel β×E effect:

$$\vec{\omega}_a = \frac{e}{m_\mu c}\left[a_\mu \vec{B} - \cancel{\left(a_\mu - \frac{1}{\gamma^2 - 1}\right)} \vec{\beta} \times \vec{E}\right] \approx \frac{e}{m_\mu c} a_\mu \vec{B}$$

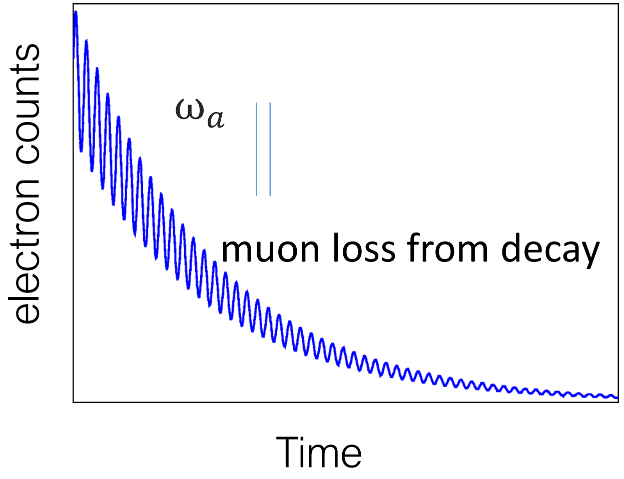
P_μ = 3.09 GeV/c

4. Again parity violation in muon decay

$$\mu^-_{\text{polarized}} \longrightarrow e^- + \bar{\nu}_e + \nu_\mu$$

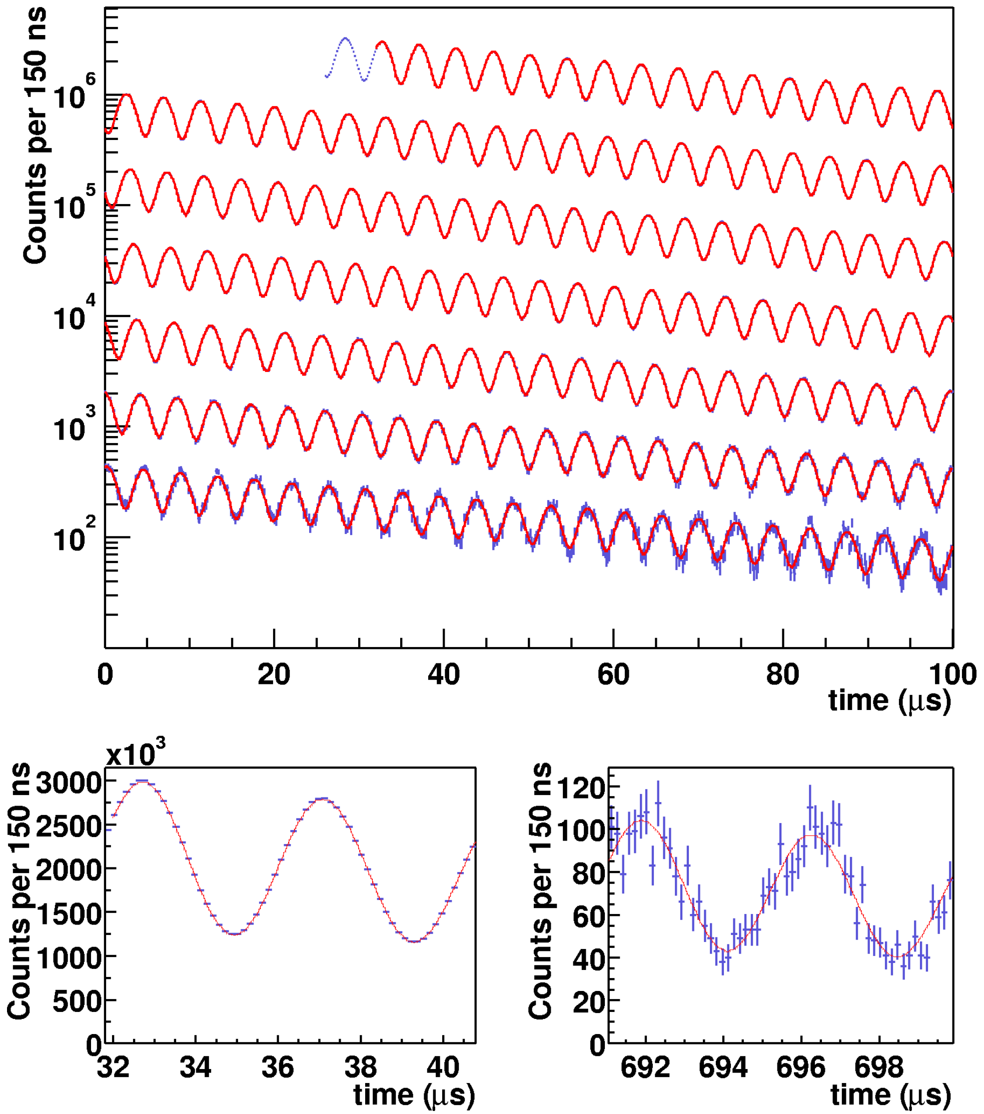
fast electron emitted in direction opposite to muon spin

Double miracle by virtue of P violation !



Muon g-2 measurement (Brookhaven 1990-2006)

E821 (g –2), hep-ex/0202024



Observed positron rate in successive 100 μs periods
~150 polarisation rotations during measurement period

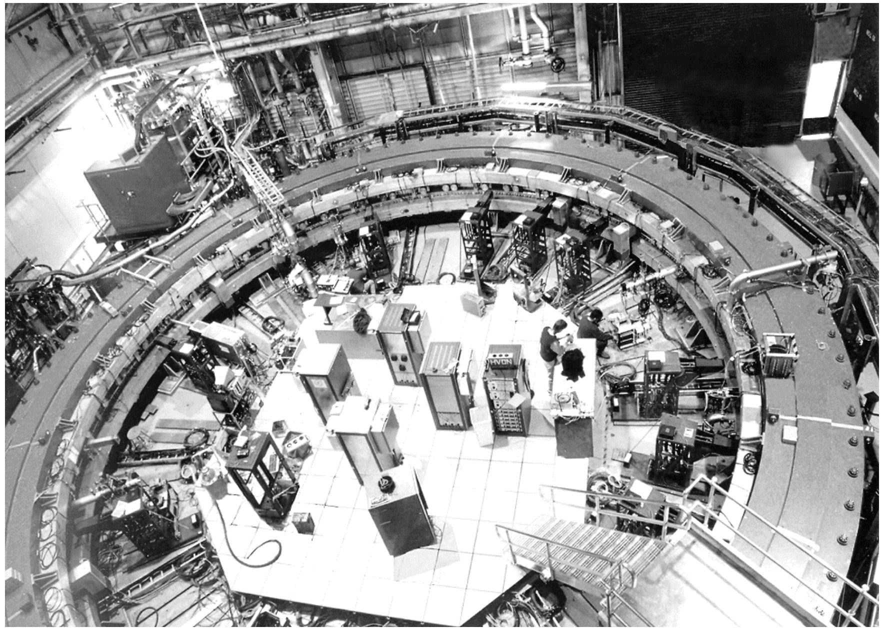
$$\omega_a \approx \frac{e}{m_\mu c} a_\mu B$$

obtained from time-dependent fit $N(t) = N_0 e^{-t/\gamma\tau}[1 - A \cdot \sin(\omega_a t - \phi)]$

In blue: fit parameters

B field measured with Hall probes with RMN frequency as reference

⇒ a_μ obtained as ratio of 2 frequencies (double blind analysis)

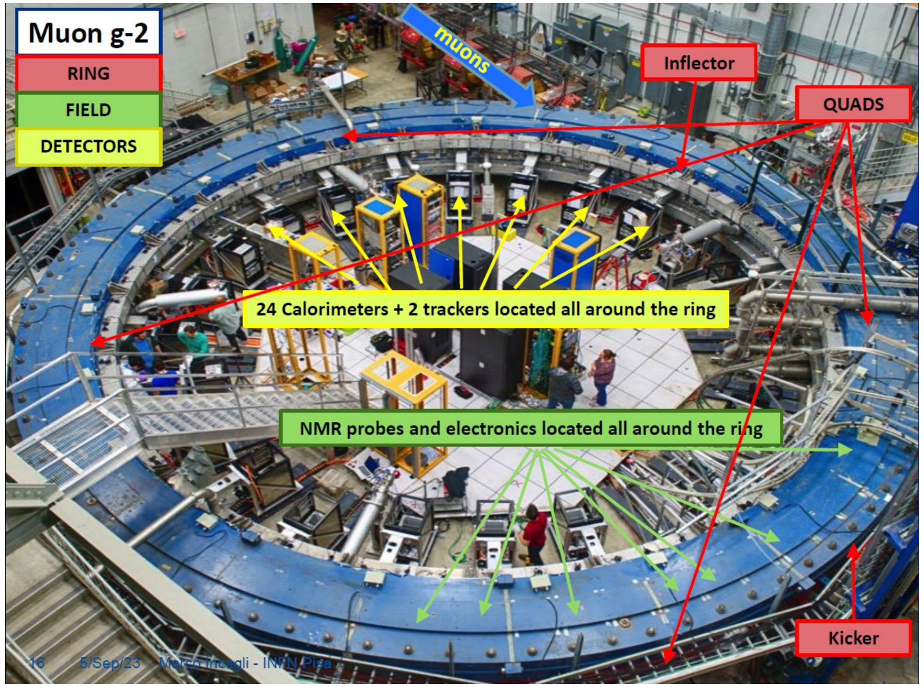


$$a_\mu = 11\,659\,209.1\,(5.4)(3.3)\ \cdot 10^{-10}$$

stat syst

The muon g-2 Fermilab experiment (2018-2025)

- Brookhaven experiment limited by statistics, systematic effects well understood, could be improved with more intense (x 20) and pure muon beam at Fermilab
- Goal: reduce final uncertainty by a factor of 4 (over several years)
- Enlarged collaboration
- Experiment completely redesigned (beam instrumentation, detectors, electronics), only superconducting magnet kept and shipped

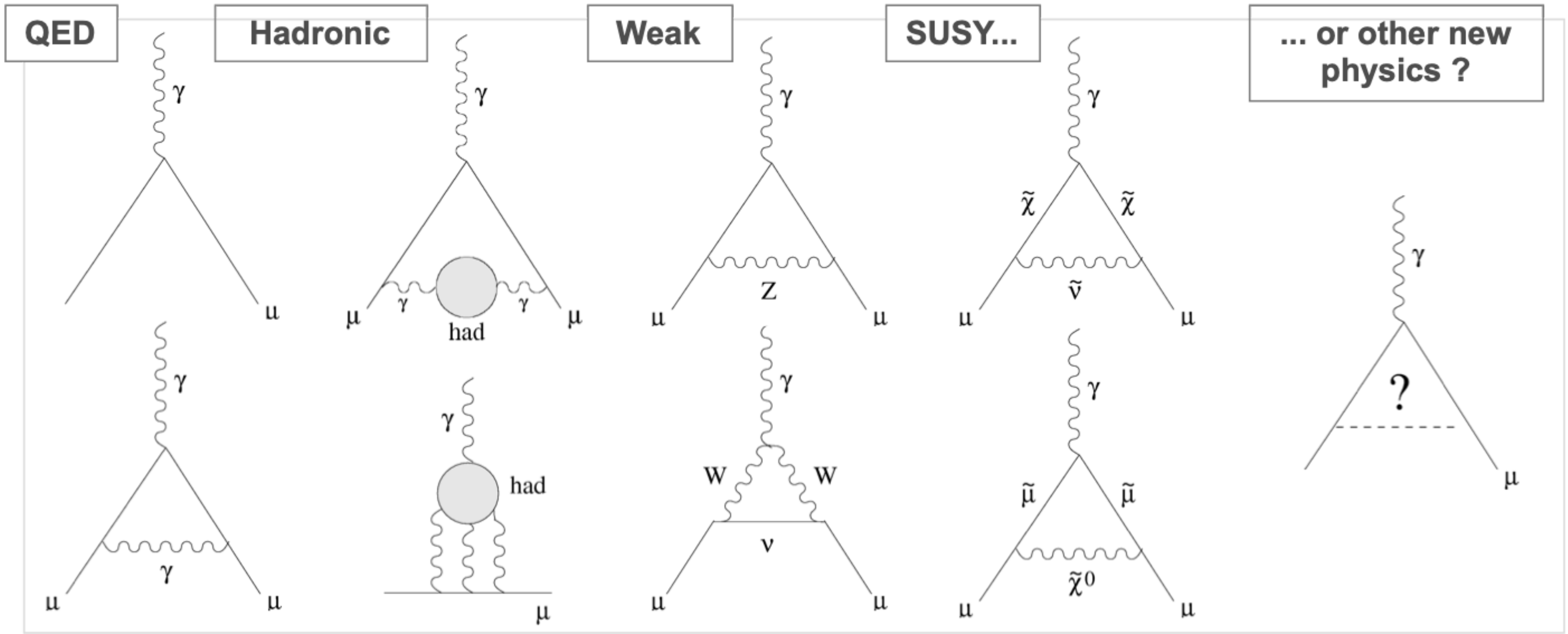


Theoretical prediction for a_μ

$$a_\mu^{\mathrm{th}} = a_\mu^{\mathrm{SM}} + a_\mu^{\mathrm{BSM}}$$

$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{had}} + a_\mu^{\mathrm{Weak}}$$

$$a_\mu^{\mathrm{BSM}}$$



Theoretical prediction for a_μ : QED

Known to 5 loops, good convergence, diagrams with internal electron loops enhanced:

$$a_\mu^{\mathrm{QED}} = \frac{\alpha}{2\pi} + A_2\left(\frac{\alpha}{\pi}\right)^2 + A_3\left(\frac{\alpha}{\pi}\right)^3 + A_4\left(\frac{\alpha}{\pi}\right)^4 + A_5\left(\frac{\alpha}{\pi}\right)^5$$

A_2 A_3 known analytically, A_4 A_5 obtained with Monte Carlo techniques, partially checked analytically for A_4

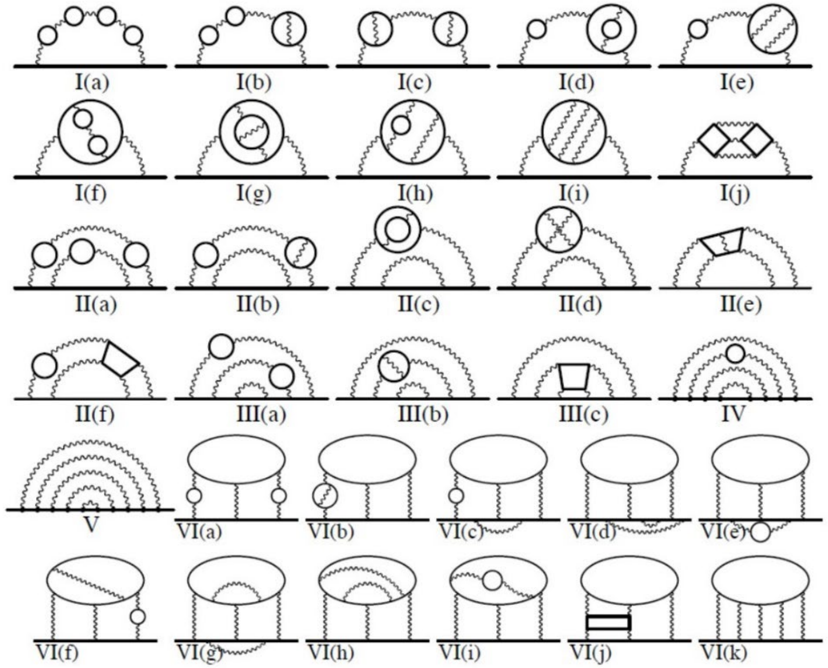
Aoyama, Hayakawa, Kinoshita, Nio (2012-2019)

α = 137.035 999 046 (27) from Cs recoil (Mueller et al., Berkeley, 2018)

α = 137.035 999 206 (8) from Rb recoil (Morel et al., LKB Paris, 2020)

$$\begin{array}{lrll}
a_\mu^{\mathrm{QED}} = & 116\,140\,973.321\;(23) & & \alpha \\
+ & 413\,217.626\;\;(7) & & \alpha^2 \\
+ & 30\,141.902\;(33) & (\times 10^{-11}) & \alpha^3 \\
+ & 381.004\;(17) & & \alpha^4 \\
+ & 5.078\;\;(6) & & \alpha^5 \\
= & 116\,584\,718.931\;(104) & &
\end{array}$$

12672 diagrams



uncertainty dominated by estimate on α^6 term

Theoretical prediction for a_μ : EW, hadronic light-by-light

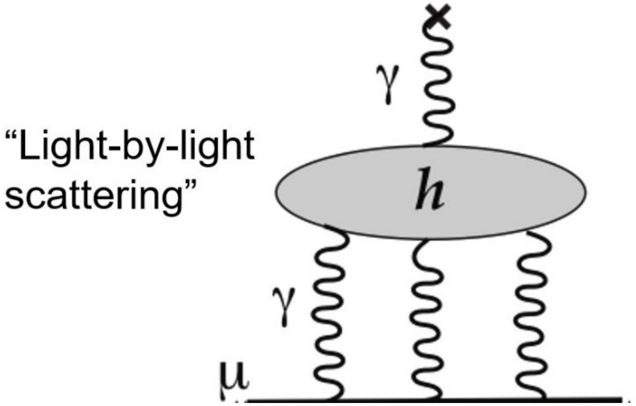
- EW: one-loop + two-loop involving W, Z bosons (little sensitivity to Higgs boson mass)

$$a_\mu^{EW} = 153.6\,(1.0) \times 10^{-11}$$

shows level of sensitivity of a_μ to physics at large mass scales ~ O(0.1 TeV)

Precision at low energies ⇔ high energy frontier

- Hadronic light-by-light: α^3 contribution not computable by analytical QCD; so far only estimated by phenomenological models using intermediate particles; new approach partly using experimental data (2017); also first results from QCD lattice simulations (2019)



small contribution

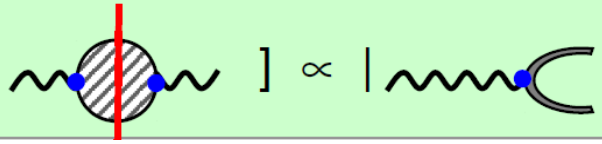
$$a_\mu^{HLbL} = 94\,(19) \times 10^{-11}$$

Theoretical prediction for a_μ : Hadronic Vacuum Polarization

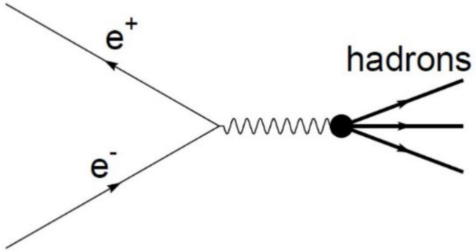
Dominant uncertainty for the theoretical prediction from HVP part which cannot be calculated from QCD (low mass scale), but one can use experimental data on $e^+ e^- \to$ hadrons cross section

Born: $\sigma^{(0)}(s) = \sigma(s)\left(\alpha / \alpha(s)\right)^2$

$$12\pi\,\mathrm{Im}\Pi_\gamma(s) = \frac{\sigma^0\,[e^+e^- \to \text{hadrons}\,(\gamma_{FSR})]}{\sigma_{pt}} \equiv R(s)$$

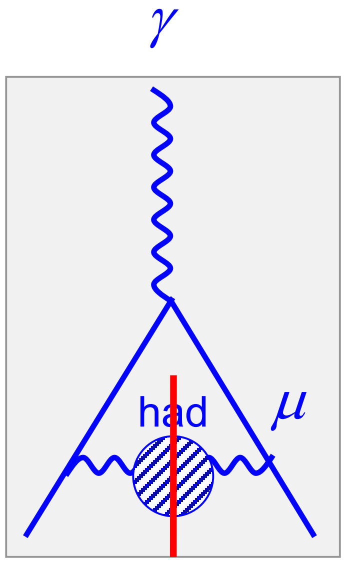


Im[ ] ∝ | hadrons |²

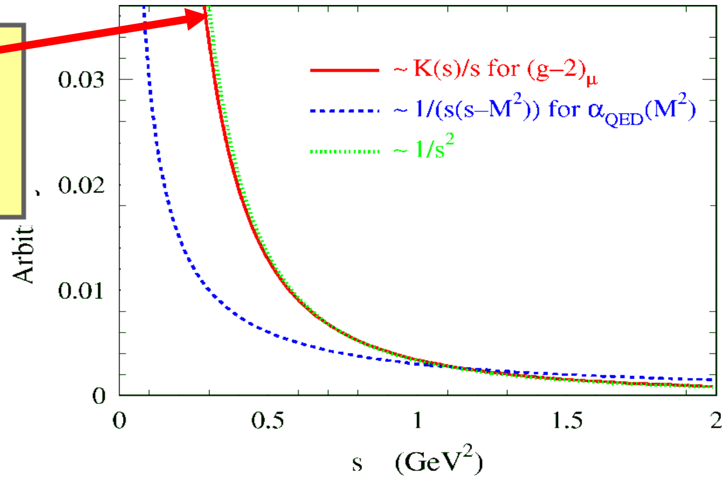


- unitarity
- analyticity

⇒ dispersion relation



$$a_\mu^{\text{had}} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} ds\,\frac{K(s)}{s}\,R(s)$$



Bouchiat-Michel (1961)
Brodsky-de Rafael (1968)

Precise $\sigma(e^+e^- \to$hadrons) measurements at low energy are necessary

Hadronic Vacuum Polarization (DHMZ group)

- HVP has been for long and still now the largest contribution to the uncertainty of the a_μ prediction in the SM
- Limited by the accuracy of e+e- experimental data
- DHMZ group (MD, Andreas Hoecker, Bogdan Malaescu, Zhiqing Zhang) involved since 1997 (τ data ALEPH)
- Result used as reference for the Brookhaven experiment: comparison revealed a deficit in the prediction at ~ 2-3 σ level, hence our motivation to continue this effort toward a more precise prediction

- Main contributions to data treatment
  - Compilation of existing data for e+e- annihilation to obtain R as a sum of exclusive processes
  - Robust combination techniques taking into account all correlated uncertainties as function of energy, between exclusive channels, and between experiments
  - Correct for unmeasured processes using isospin constraints
  - Determine energy regions where perturbative QCD calculations are safe (experience with τ physics)

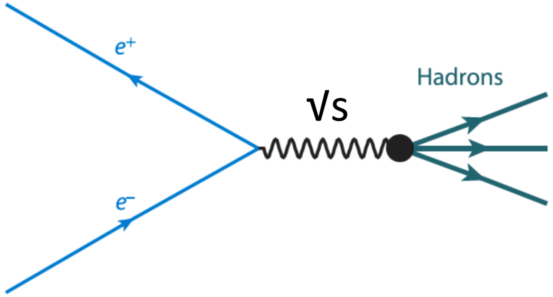
- Launched a dedicated program of e+e- cross section measurements using the BABAR detector (SLAC) to get more precise data (2001-2014) with the new Initial State Radiation (ISR) method. A new phase is still underway.

- Same data and techniques used to study the running of α (energy) from $\alpha(0)$ to $\alpha(M_Z)$ $\Rightarrow$ prediction for M_{Higgs}

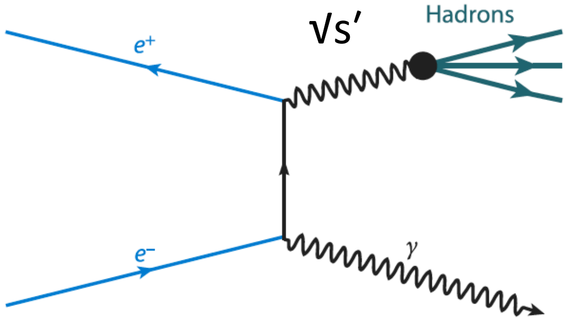
- Double role as phenomenologists and experimenters

Measurements of $\sigma(e^+e^- \to \text{hadrons})$

1. The scan method: e.g. CMD-2/3, SND at Novosibirsk
   - ➢ Advantages:
     - ➢ Well defined √s
     - ➢ Good energy resolution ~10^{-3}√s
   - ➢ Disadvantages:
     - ➢ Energy gap between two scans
     - ➢ Low luminosity at low energies
     - ➢ Limited √s range of a given experiment

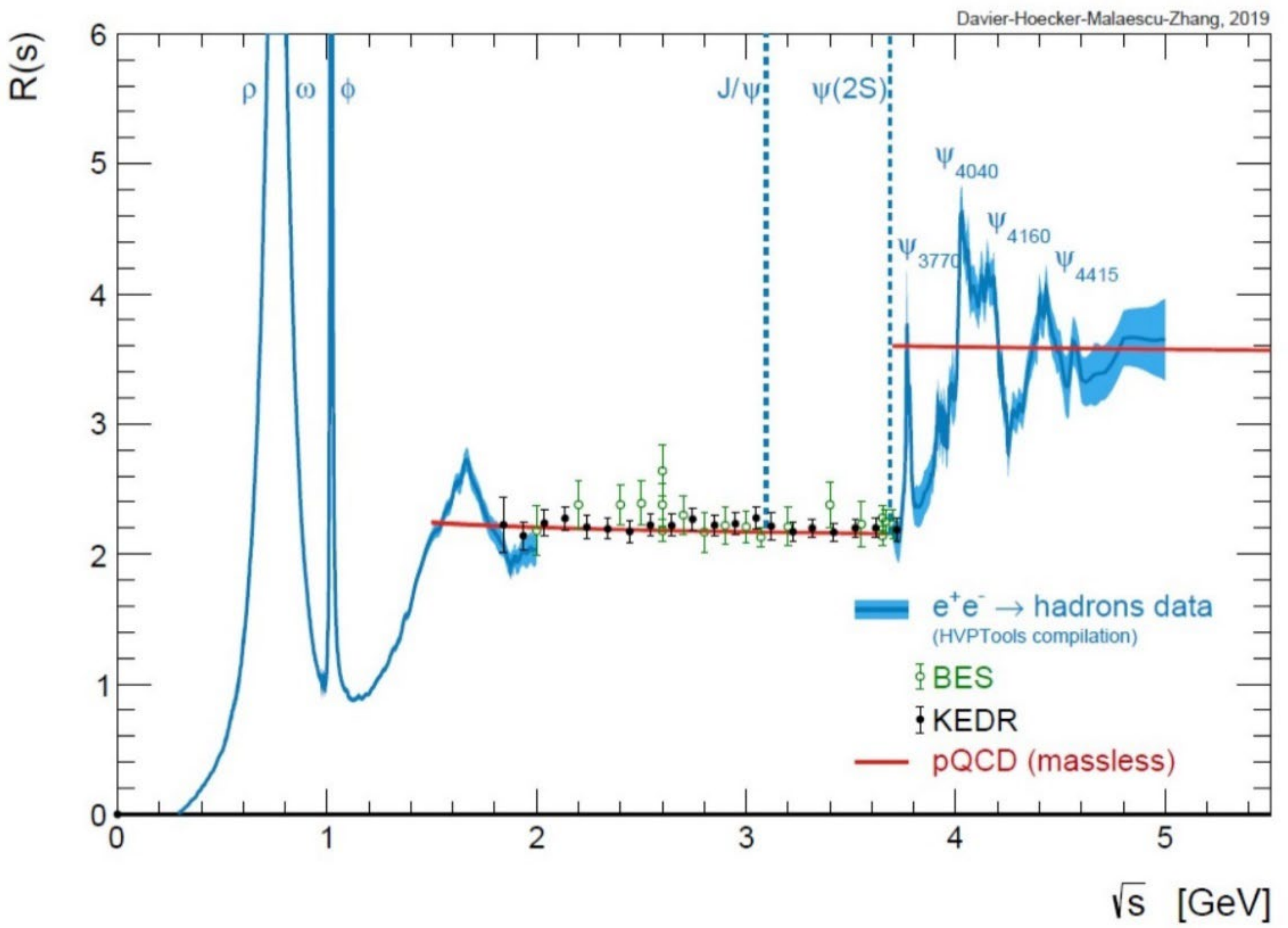


2. The ISR approach: e.g. BaBar, KLOE, BES, CLEOc
   - ➢ Advantages:
     - ➢ Continuous cross section measurement over a broad energy range down to threshold
     - ➢ large acceptance for hadrons if ISR detected at large angle
     - ➢ $\sigma(e^+e^- \to \text{hadrons})$ may be measured over $\sigma(e^+e^- \to \mu^+\mu^-)$ thus reducing some syst uncertainties
   - ➢ Disadvantages:
     - ➢ Requires high luminosity to compensate higher order in α



$s'=(1-x)/s$
$x=2E_\gamma/\sqrt{s}$

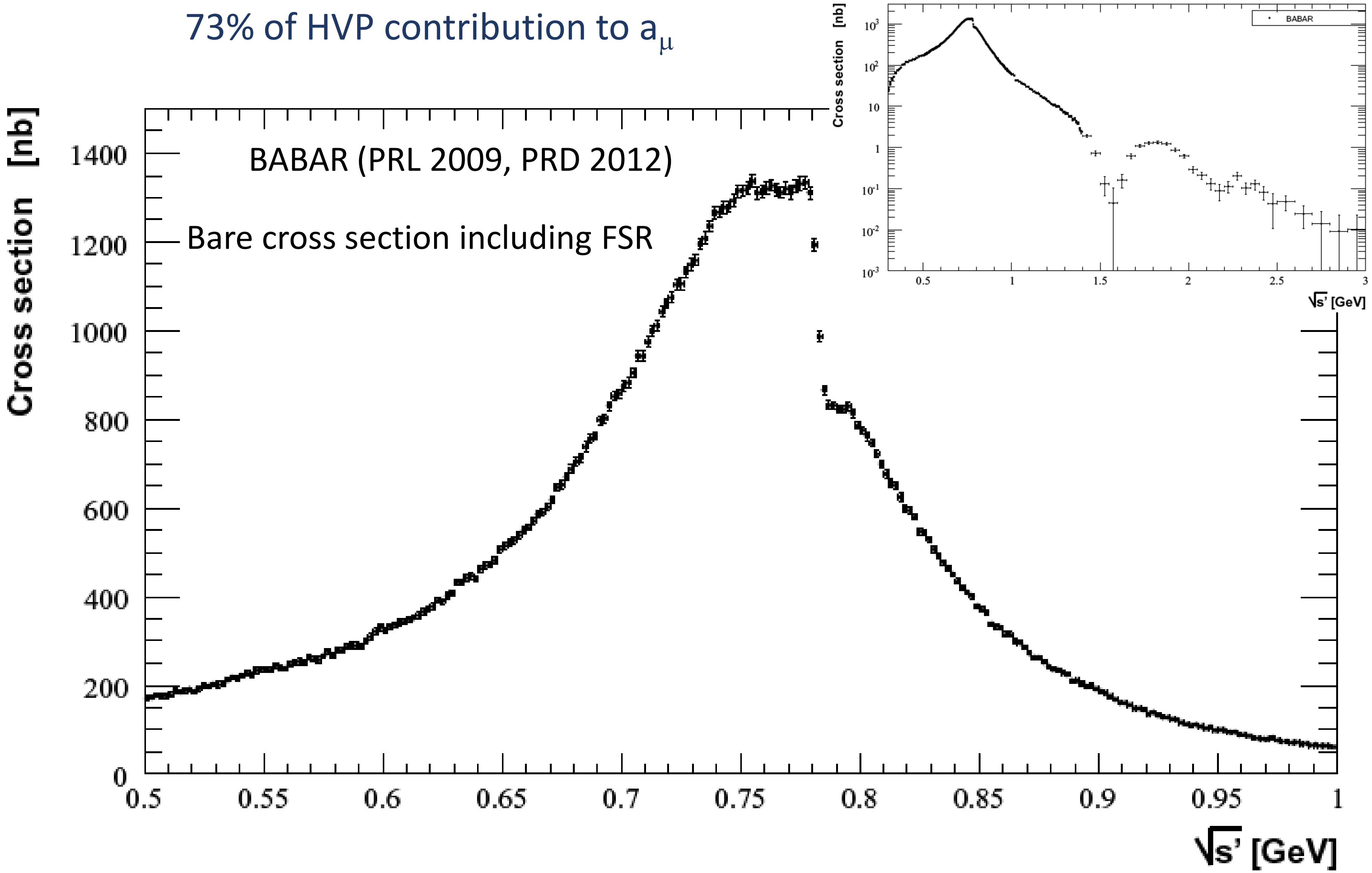
Different energy regions for R(s)


Davier-Hoecker-Malaescu-Zhang, 2019

- [$\pi^0\gamma$ threshold-1.8GeV]
  - sum about 22→40 exclusive channels
  - estimate unmeasured channels using isospin relations (now < 0.1%)
- [1.8-3.7] GeV
  - good agreement between data and pQCD calculation
    → use 4-loop pQCD
  - J/ψ, ψ(2s): Breit-Wigner integral
- [3.7-5] GeV
  use data
- >5GeV
  use 4-loop pQCD calculation

The dominant channel : $e^+e^- \to \pi^+\pi^-(\gamma)$

73% of HVP contribution to a_μ

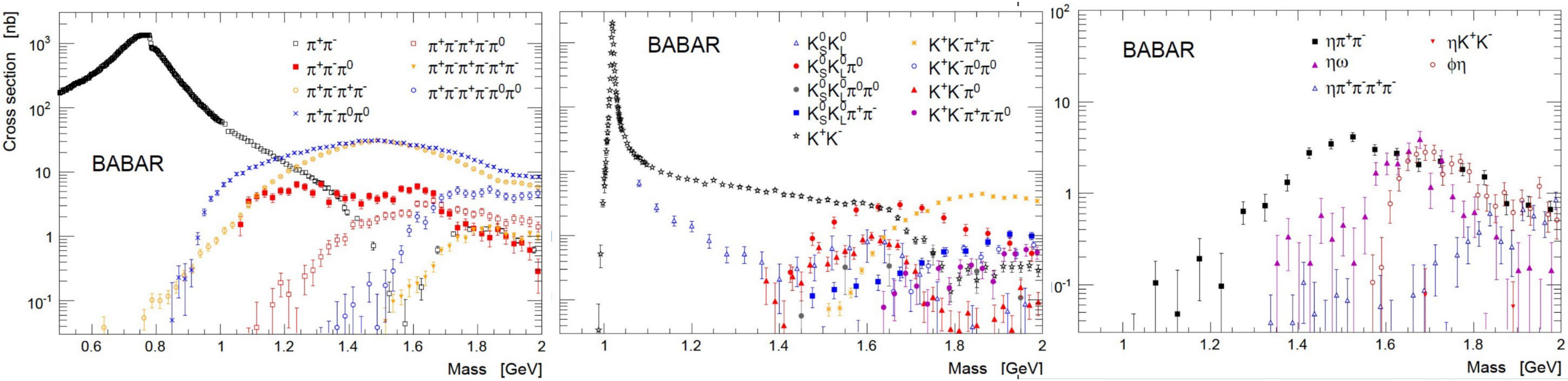

BABAR (PRL 2009, PRD 2012)

Bare cross section including FSR

BABAR: multi-hadronic channels

Besides our team for the dominant $\pi^+\pi^-$ and K^+K^- cross sections, other BABAR groups have taken the lead to measure the rest of exclusive cross sections (altogether ~ 40 processes)

$\Rightarrow$ complete and precise reconstruction of R below 2 GeV



Consistency between experimental data

- Latest dispersive evaluations rely on a rather complete set of measurements of $e^+e^- \to$ hadrons up to 6π, $\eta 4\pi$, $KK2\pi$ in all charge configurations, and a few more higher-multiplicity processes

- A significant discrepancy occurs in the $\pi^+\pi^-$ channel between the 2 most precise results (BABAR and KLOE)

- Taking into account the BABAR/KLOE disagreement in the combination, all experiments are in agreement within an enlarged combination uncertainty (0.7%), already a remarkable result given different experimental conditions: ISR (10.6 GeV BABAR, ~4 GeV BES CLEOc, 1.02 GeV KLOE), direct scan (CMD-2, SND)

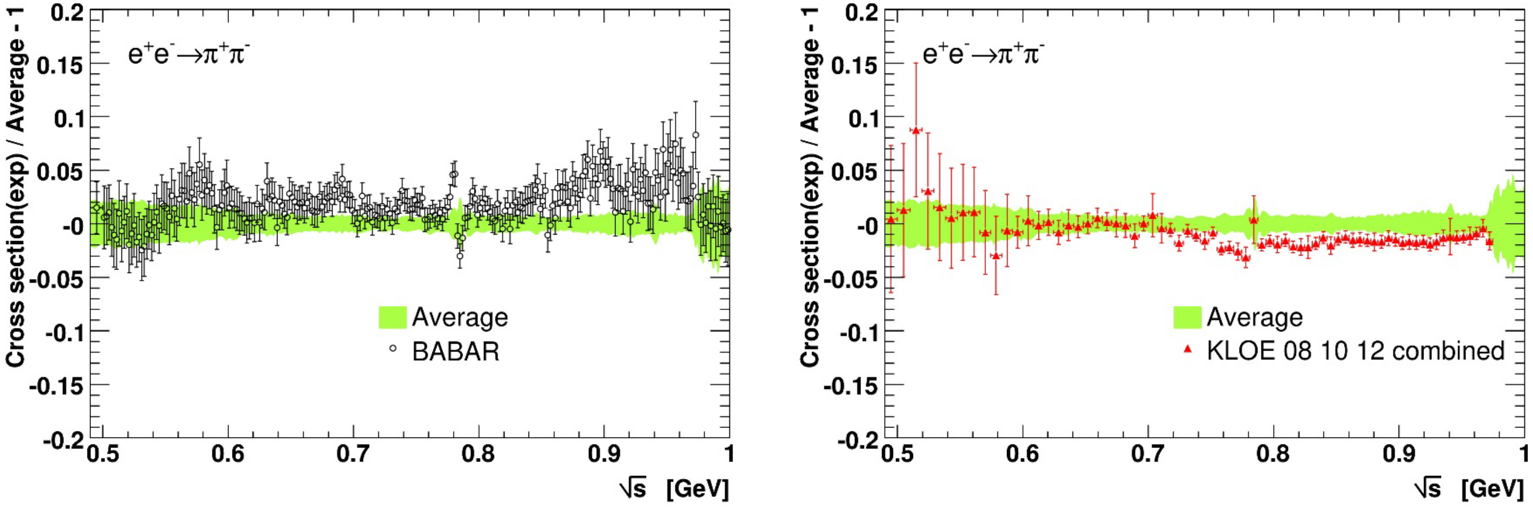
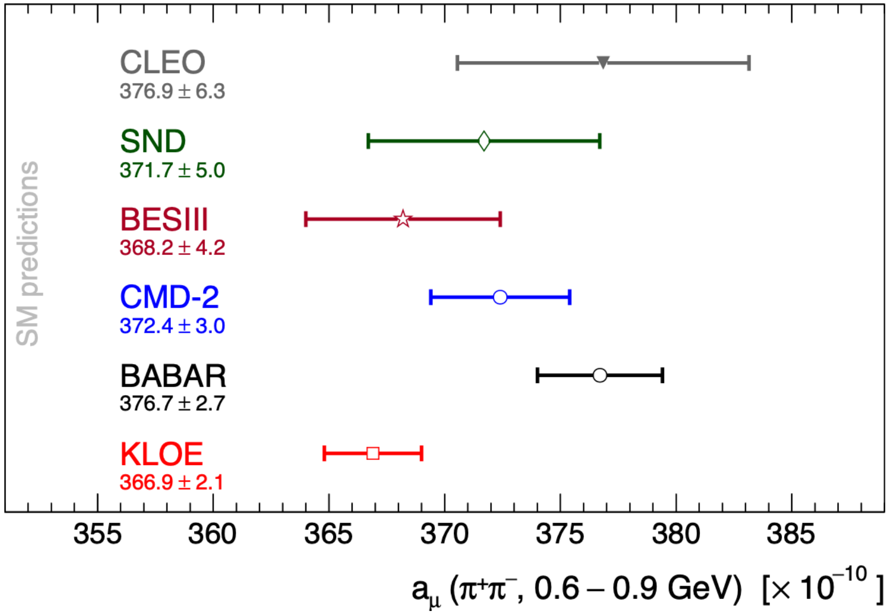
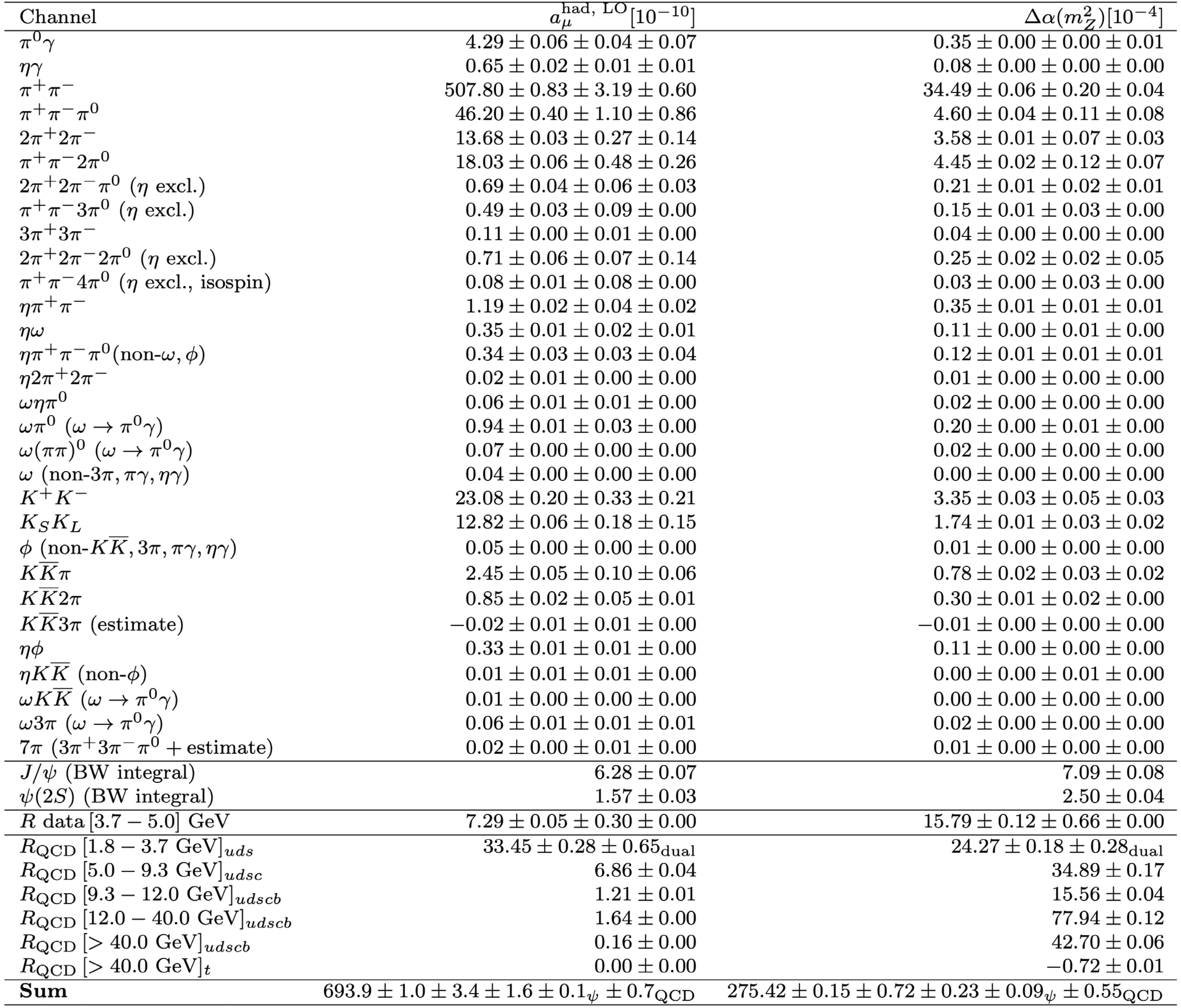


Figure from DHMZ, EPJC80 (2020) 241



- Additional systematic error added because of BABAR-KLOE difference $\Rightarrow$ degrades uncertainty by 30%

All contributions (DHMZ19)



Channel	$a_\mu^{\text{had, LO}}[10^{-10}]$	$\Delta\alpha(m_Z^2)[10^{-4}]$
$\pi^0\gamma$	$4.29 \pm 0.06 \pm 0.04 \pm 0.07$	$0.35 \pm 0.00 \pm 0.00 \pm 0.01$
$\eta\gamma$	$0.65 \pm 0.02 \pm 0.01 \pm 0.01$	$0.08 \pm 0.00 \pm 0.00 \pm 0.00$
$\pi^+\pi^-$	$507.80 \pm 0.83 \pm 3.19 \pm 0.60$	$34.49 \pm 0.06 \pm 0.20 \pm 0.04$
$\pi^+\pi^-\pi^0$	$46.20 \pm 0.40 \pm 1.10 \pm 0.86$	$4.60 \pm 0.04 \pm 0.11 \pm 0.08$
$2\pi^+2\pi^-$	$13.68 \pm 0.03 \pm 0.27 \pm 0.14$	$3.58 \pm 0.01 \pm 0.07 \pm 0.03$
$\pi^+\pi^-2\pi^0$	$18.03 \pm 0.06 \pm 0.48 \pm 0.26$	$4.45 \pm 0.02 \pm 0.12 \pm 0.07$
$2\pi^+2\pi^-\pi^0$ (η excl.)	$0.69 \pm 0.04 \pm 0.06 \pm 0.03$	$0.21 \pm 0.01 \pm 0.02 \pm 0.01$
$\pi^+\pi^-3\pi^0$ (η excl.)	$0.49 \pm 0.03 \pm 0.09 \pm 0.00$	$0.15 \pm 0.01 \pm 0.03 \pm 0.00$
$3\pi^+3\pi^-$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ (η excl.)	$0.71 \pm 0.06 \pm 0.07 \pm 0.14$	$0.25 \pm 0.02 \pm 0.02 \pm 0.05$
$\pi^+\pi^-4\pi^0$ (η excl., isospin)	$0.08 \pm 0.01 \pm 0.08 \pm 0.00$	$0.03 \pm 0.00 \pm 0.03 \pm 0.00$
$\eta\pi^+\pi^-$	$1.19 \pm 0.02 \pm 0.04 \pm 0.02$	$0.35 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta\omega$	$0.35 \pm 0.01 \pm 0.02 \pm 0.01$	$0.11 \pm 0.00 \pm 0.01 \pm 0.00$
$\eta\pi^+\pi^-\pi^0$(non-ω,ϕ)	$0.34 \pm 0.03 \pm 0.03 \pm 0.04$	$0.12 \pm 0.01 \pm 0.01 \pm 0.01$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\eta\pi^0$	$0.06 \pm 0.01 \pm 0.01 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega\pi^0$ ($\omega \to \pi^0\gamma$)	$0.94 \pm 0.01 \pm 0.03 \pm 0.00$	$0.20 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega(\pi\pi)^0$ ($\omega \to \pi^0\gamma$)	$0.07 \pm 0.00 \pm 0.00 \pm 0.00$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
ω (non-$3\pi,\pi\gamma,\eta\gamma$)	$0.04 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
K^+K^-	$23.08 \pm 0.20 \pm 0.33 \pm 0.21$	$3.35 \pm 0.03 \pm 0.05 \pm 0.03$
K_SK_L	$12.82 \pm 0.06 \pm 0.18 \pm 0.15$	$1.74 \pm 0.01 \pm 0.03 \pm 0.02$
ϕ (non-$K\overline{K},3\pi,\pi\gamma,\eta\gamma$)	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$K\overline{K}\pi$	$2.45 \pm 0.05 \pm 0.10 \pm 0.06$	$0.78 \pm 0.02 \pm 0.03 \pm 0.02$
$K\overline{K}2\pi$	$0.85 \pm 0.02 \pm 0.05 \pm 0.01$	$0.30 \pm 0.01 \pm 0.02 \pm 0.00$
$K\overline{K}3\pi$ (estimate)	$-0.02 \pm 0.01 \pm 0.01 \pm 0.00$	$-0.01 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta\phi$	$0.33 \pm 0.01 \pm 0.01 \pm 0.00$	$0.11 \pm 0.00 \pm 0.00 \pm 0.00$
$\eta K\overline{K}$ (non-ϕ)	$0.01 \pm 0.01 \pm 0.01 \pm 0.00$	$0.00 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega K\overline{K}$ ($\omega \to \pi^0\gamma$)	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$
$\omega 3\pi$ ($\omega \to \pi^0\gamma$)	$0.06 \pm 0.01 \pm 0.01 \pm 0.01$	$0.02 \pm 0.00 \pm 0.00 \pm 0.00$
7π ($3\pi^+3\pi^-\pi^0$ + estimate)	$0.02 \pm 0.00 \pm 0.01 \pm 0.00$	$0.01 \pm 0.00 \pm 0.00 \pm 0.00$
J/ψ (BW integral)	6.28 ± 0.07	7.09 ± 0.08
$\psi(2S)$ (BW integral)	1.57 ± 0.03	2.50 ± 0.04
R data $[3.7 - 5.0]$ GeV	$7.29 \pm 0.05 \pm 0.30 \pm 0.00$	$15.79 \pm 0.12 \pm 0.66 \pm 0.00$
$R_{\text{QCD}}\,[1.8 - 3.7\text{ GeV}]_{uds}$	$33.45 \pm 0.28 \pm 0.65_{\text{dual}}$	$24.27 \pm 0.18 \pm 0.28_{\text{dual}}$
$R_{\text{QCD}}\,[5.0 - 9.3\text{ GeV}]_{udsc}$	6.86 ± 0.04	34.89 ± 0.17
$R_{\text{QCD}}\,[9.3 - 12.0\text{ GeV}]_{udscb}$	1.21 ± 0.01	15.56 ± 0.04
$R_{\text{QCD}}\,[12.0 - 40.0\text{ GeV}]_{udscb}$	1.64 ± 0.00	77.94 ± 0.12
$R_{\text{QCD}}\,[> 40.0\text{ GeV}]_{udscb}$	0.16 ± 0.00	42.70 ± 0.06
$R_{\text{QCD}}\,[> 40.0\text{ GeV}]_{t}$	0.00 ± 0.00	-0.72 ± 0.01
Sum	$693.9 \pm 1.0 \pm 3.4 \pm 1.6 \pm 0.1_\psi \pm 0.7_{\text{QCD}}$	$275.42 \pm 0.15 \pm 0.72 \pm 0.23 \pm 0.09_\psi \pm 0.55_{\text{QCD}}$

40 exclusive channels (<1.8 GeV) evaluated

Estimation for missing modes based on isospin constraints becomes negligible (0.016%)

DHMZ EPJC 80 (2020) 241

The g-2 theory initiative (2017-2020)

- By 2012, prediction using more precise e+e- data confirmed the discrepancy with the Brookhaven measurement, reaching ~ 3.5 σ

- In view of forthcoming results from the new g-2 direct experiment at Fermilab, a concerted effort was organized to try to produce the most reliable prediction ahead of time (blind to the new result)

- Organized 6 workshops followed by ~ 130 physicists (many lattice QCD theorists)

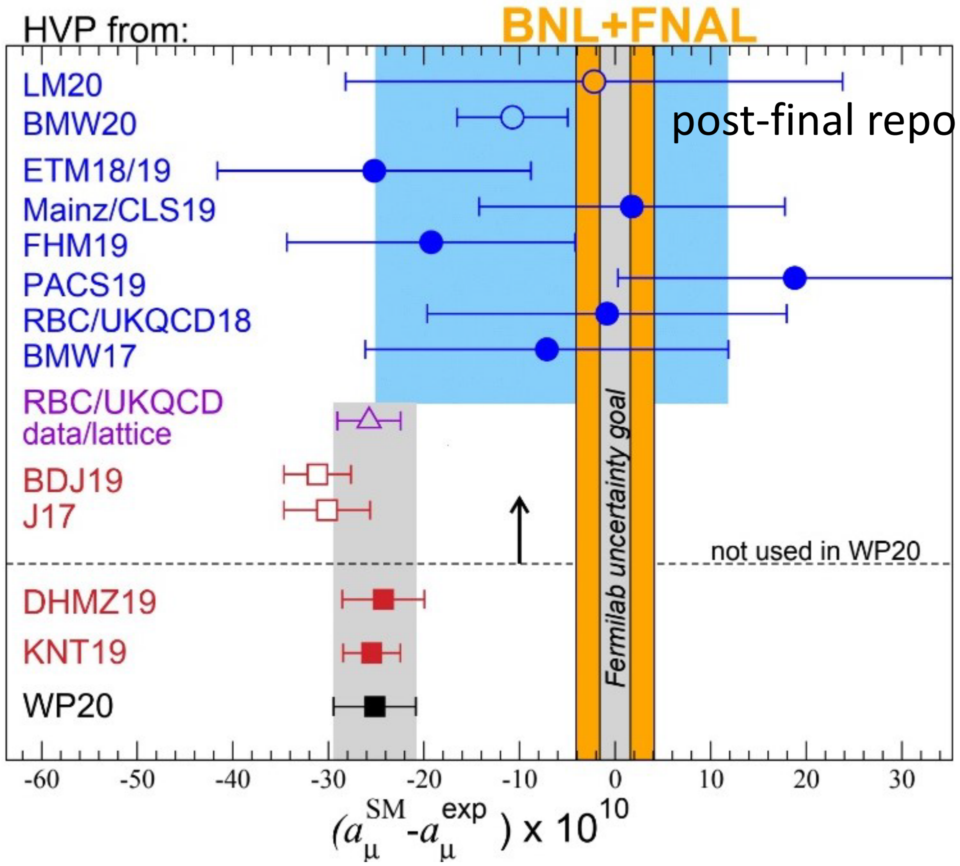
- Progress in hadronic LbL calculations with phenomenological and lattice methods, uncertainty reduced

- For HVP
  - ➢ lattice groups very active, but could not produce a reliable and competitive result
  - ➢ the dispersive approach based on data was adopted: results of 2 groups used (DHMZ and KNT) with the DHMZ conservative approach of estimating uncertainties prevailing

- Comprehensive report (166 pages) ready early 2020 and published in Physics Reports, well before the Fermilab release

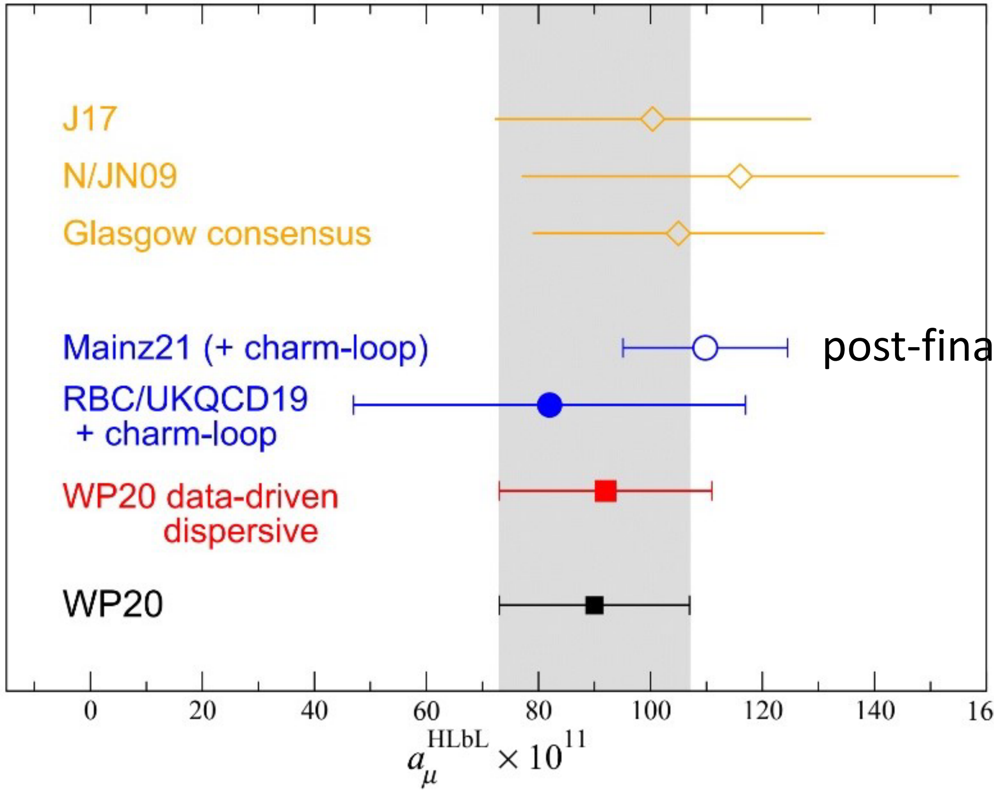
The g-2 theory initiative prediction (WP2020)

HVP



post-final report (see later)

HLbL



post-final report

The muon g-2 Fermilab run 1 result (2021)

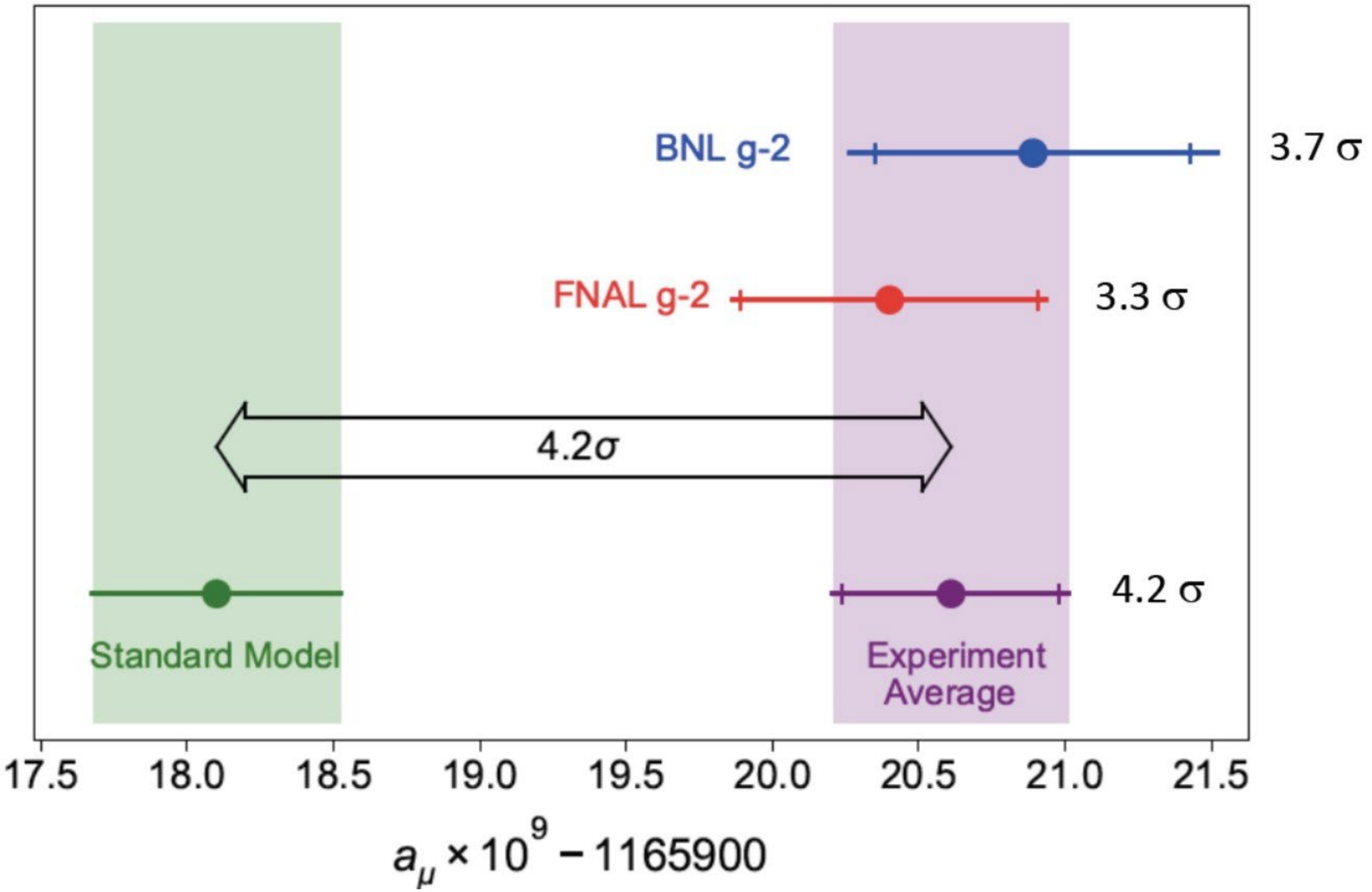
$$a_\mu(\text{Fermilab}) = 116\,592\,040\,(50)_{stat}\,(23)_{syst} \times 10^{-11}$$

- Agreement with Brookhaven value
- Precision comparable

- Excess / SM prediction increased to 4.2σ

- Caution about significance:
  - statistics-dominated measurement
  - prediction uncertainty limited by systematic effects (not Gaussian)

- Nevertheless, large discrepancy (the largest so far between measurement and SM anywhere)



New developments since 2020-1 (WP and run 1 FNAL)

- First precise HVP result using lattice QCD (BMW collaboration, 2021) in disagreement with dispersive approach

- Confrontation with other lattice groups, still going on (1) no new full result yet (2) comparison in reduced regions where uncertainties are smallest (intermediate window in Euclidean time)

- New (2023) precise measurement of $e^+ e^- \rightarrow \pi^+ \pi^-$ at Novosibirsk (CMD-3) in disagreement with all previous results (KLOE, BABAR, CMD-2….), consistent with no discrepancy with direct g-2 result

- Update (2023) from g-2 measurement in Fermilab (runs 2/3): precision increased ×2, consistent with run 1

- DMZ and BMW collaborating to localize the energy regions where differences data-driven/lattice occur

- Dedicated study by BABAR (2023) of additional radiation in ISR processes $e^+ e^- \rightarrow \mu^+ \mu^- \gamma$, $e^+ e^- \rightarrow \pi^+ \pi^- \gamma$ (LO) with one (NLO) or two (NNLO) photons $\Rightarrow$ consequences for ISR analyses such as KLOE and BESIII

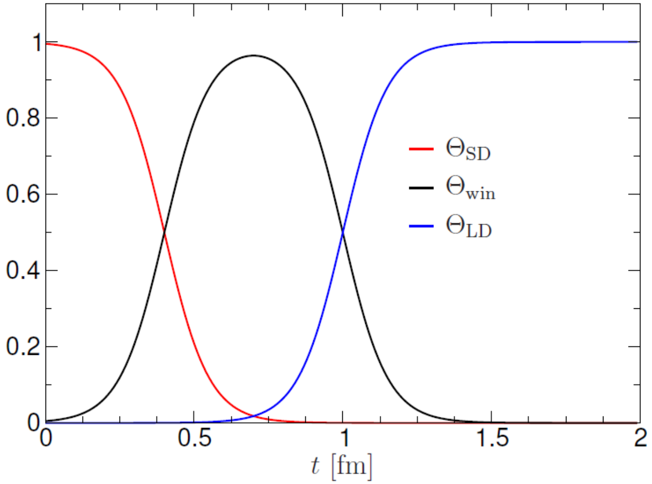
- Re-evaluation (DHLMZ, 2023) of input data in view of strong inconsistencies between experiments, reconsideration of data from hadronic τ decays (DHLMZ = DHMZ + Anne-Marie Lutz)

Purely theoretical approach: QCD on lattice

- Cannot use perturbative QCD calculations because of too-low an energy scale for a_μ (Landau pole)
- QCD on space-time lattice: simulations to compute electromagnetic-current two-point function

$$C(t) = \frac{1}{3e^2}\sum_{i=1}^{3}\int d^3x\,\langle J_i(\vec{x},t)J_i(0)\rangle \qquad C(t) = \frac{1}{24\pi^2}\int_0^\infty ds\,\sqrt{s}R(s)\,e^{-|t|\sqrt{s}}$$

$$\frac{J_\mu}{e} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c - \frac{1}{3}\bar{b}\gamma_\mu b + \frac{2}{3}\bar{t}\gamma_\mu t$$

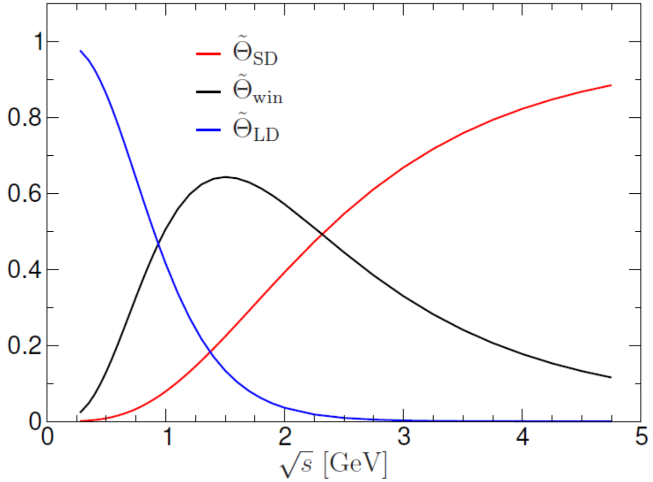


- Beyond QCD: extrapolation to real world
  lattice spacing → 0   lattice volume → ∞   π physical mass

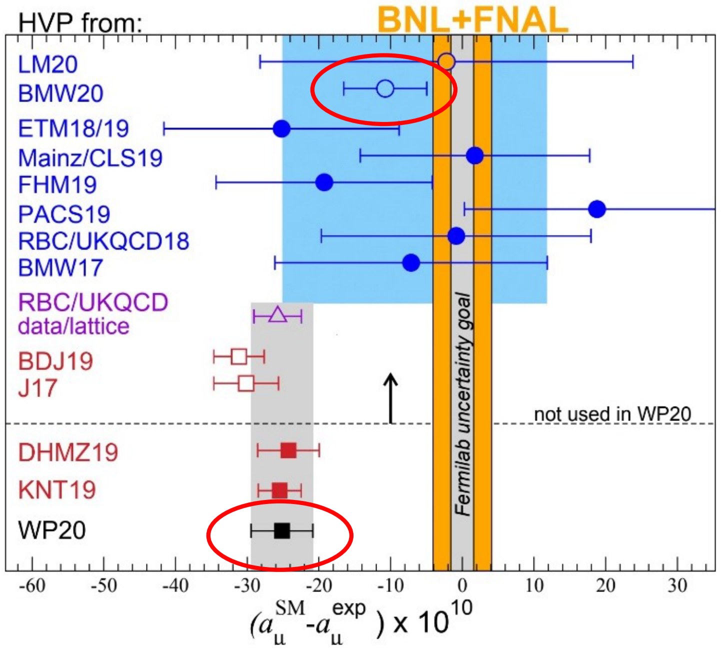
- Difficult to compare directly dispersive (timelike) and lattice (spacelike) approaches

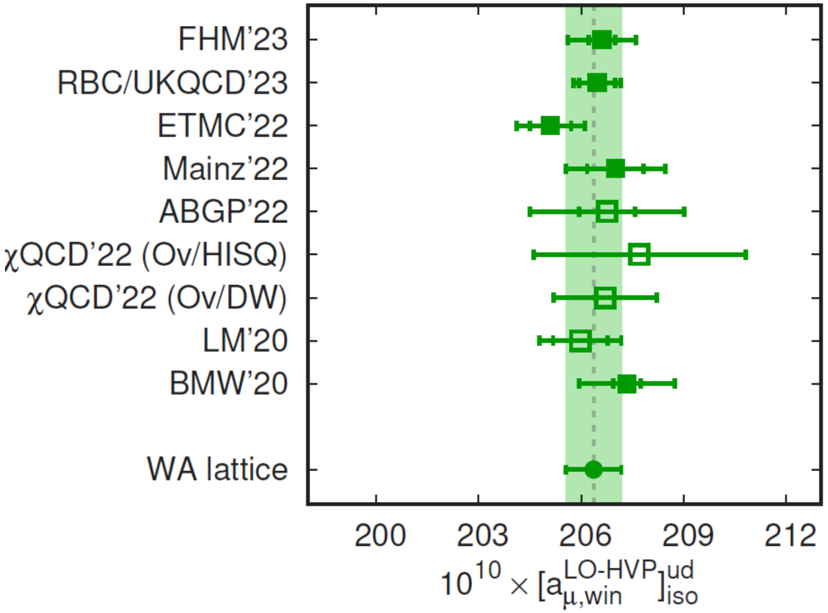
- Possibility to consider windows in lattice space

- CPU time-consuming to get statistical accuracy and study variation of lattice parameters, needs large computer resources



First precise lattice calculation: BMW 2021





- Lattice calculations in progress during g-2 Theory Initiative
- Precision not competitive with data-driven dispersive method, not included in WP2020
- Highly publicized release of BMW result (post-final WP report) synchronized with run 1 Fermilab announcement
- Confrontation with independent lattice determinations strongly desired, but no result yet for the complete calculation (3 years…)
- Important partial comparisons performed in intermediate window 0.4-1.5 fm, but it keeps only about 1/3 of HVP contribution

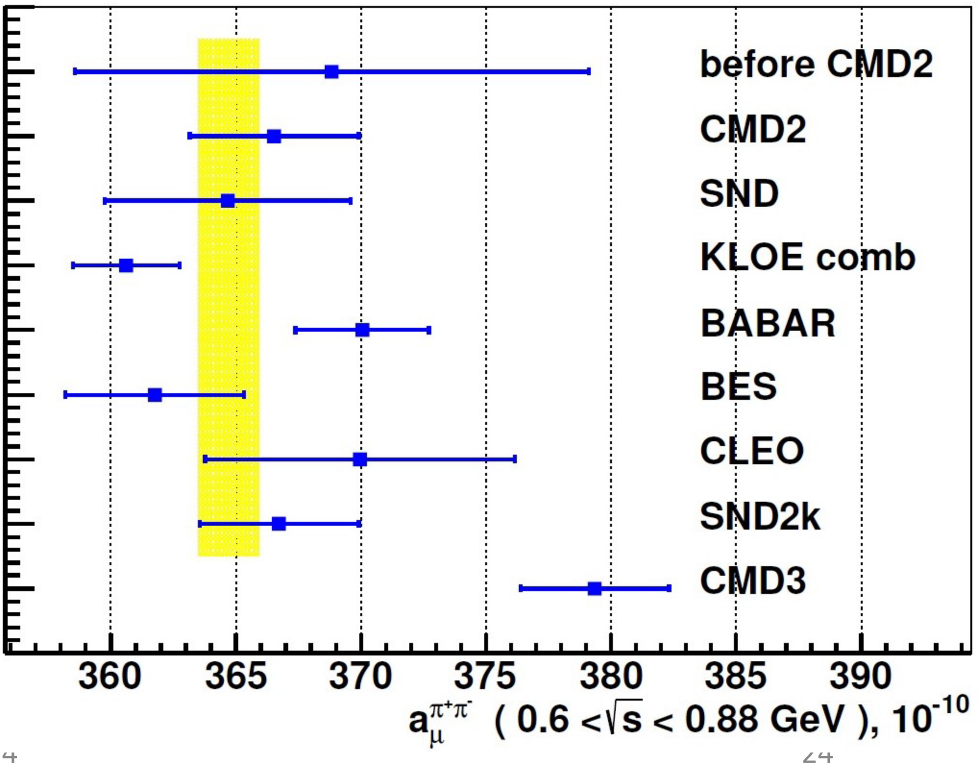
  ⇒ 4 groups in agreement with BMW

CMD-3 2023 2π result

- CMD-3 at VEPP-2000 paper arXiv:2302.08834 letter arXiv:2309.12910 both unpublished yet
- Large statistics accumulated mainly in 2017-2018 (34 M ππ, 3.7 M μμ, 4.4 M ee) allowing for detailed systematic studies
- Two independent methods for channel separation: momentum-based (better at low energy) and energy deposition in calorimeter (better at high energy): overlap of the two methods in the ρ peak region

- ππ cross section disagrees with all previous experiments whether from ISR or scan techniques
- Discrepancy with CMD-2 at VEPP-2M also using calorimetry is not understood and needs to be clarified

- Thorough review organized by the g-2 Theory Initiative
- No major issue identified significantly impacting the results
- Some questions about estimate of systematic uncertainties
- Analysis not blinded (even worse…)



The muon g-2 Fermilab runs 2-3 result (2023)

Run 1 $a_\mu(\text{Fermilab}) = 116\,592\,040\,(50)_{\text{stat}}\,(23)_{\text{syst}} \times 10^{-11}$

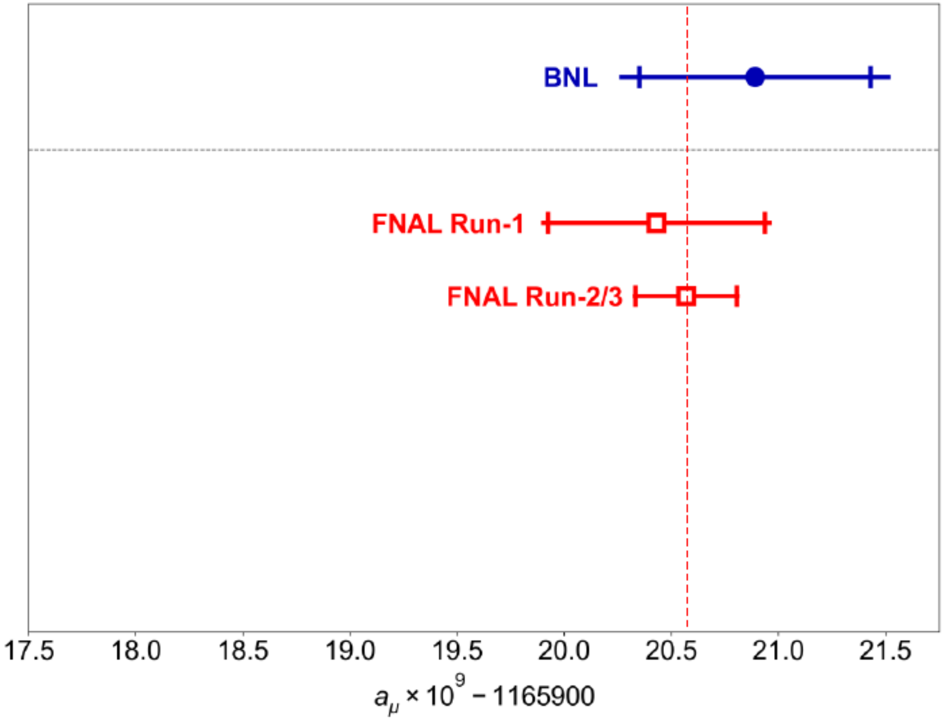
Runs 2-3 $a_\mu(\text{Fermilab}) = 116\,592\,057\,(24)_{\text{stat}}\,(8)_{\text{syst}} \times 10^{-11}$

- Agreement with Brookhaven and run 1 Fermilab values
- Precision × 2
- Systematic uncertainty below final goal

- Excess / WP2020 prediction increases to > 5σ
- Would be wonderful news if not for the confusing situation for the SM prediction

- Still more in store with results from runs 4-6 to come in 2025, another factor of 2 expected

- Clearly the burden of proof is to straighten out the discrepancies between e+e- experiments on the one hand and with lattice on the other hand



Systematic approach to compare data-driven/lattice (2023)

- Collaboration DMZ-BMW arXiv: 2308.04221
- Try to find origin of tensions between dispersive and lattice approaches
- Comparison not trivial: weighted integrals of C(t) for lattice, weighted integrals of R(s) for dispersive
- R to lattice straightforward, lattice to R inverse Laplace transform (ill-posed)

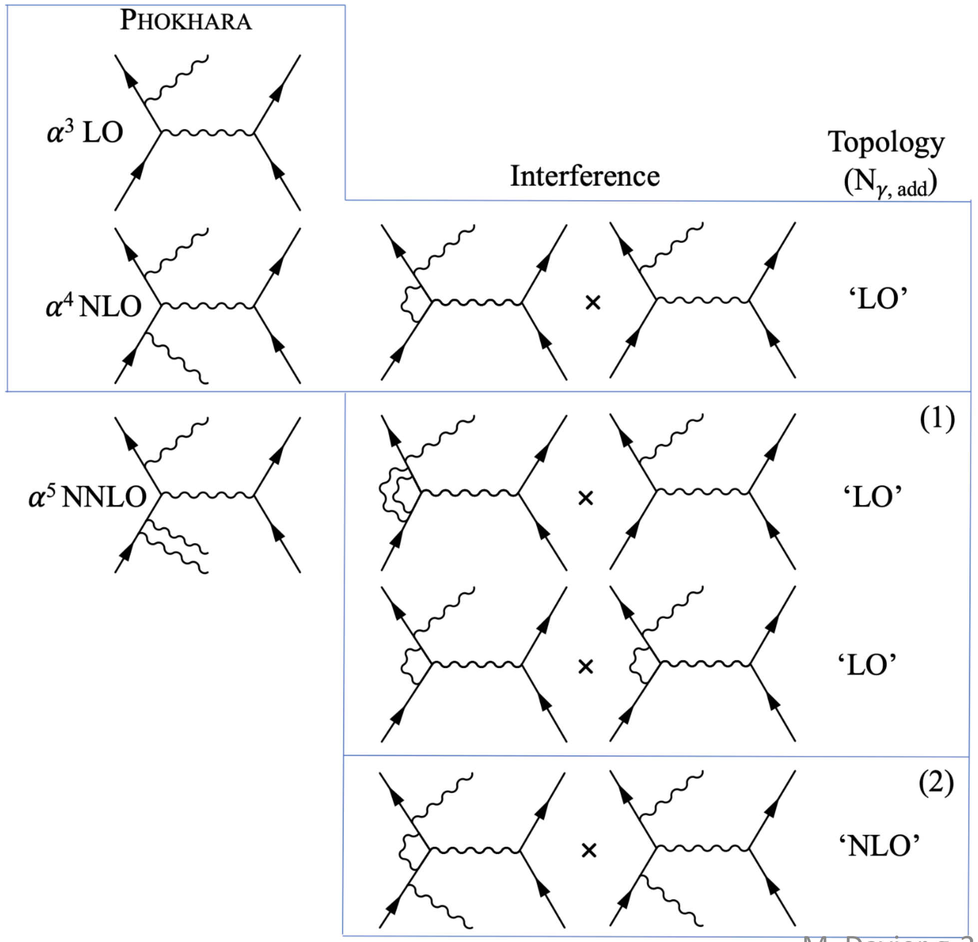
- For the moment few observables available for HVP: a_μ, $a_{\mu,\text{window}}$, $\Delta\alpha(Q^2)$, important to combine them to get more information, more moments can be considered (correlations)
- Current analysis uses full set of e+e- data as of WP2020
- Detailed work on lattice uncertainties/correlations and uncertainties on them
- Tensions expressed in t (lattice): excess mostly in 0.4-1.5 fm range, small below and above
- Tensions expressed in s (data): deficit mostly in the ρ region
- No significant impact on precision EW fits at the Z scale

- Once differences understood, same framework can be used to combine dispersive and lattice results to improve precision on HVP prediction

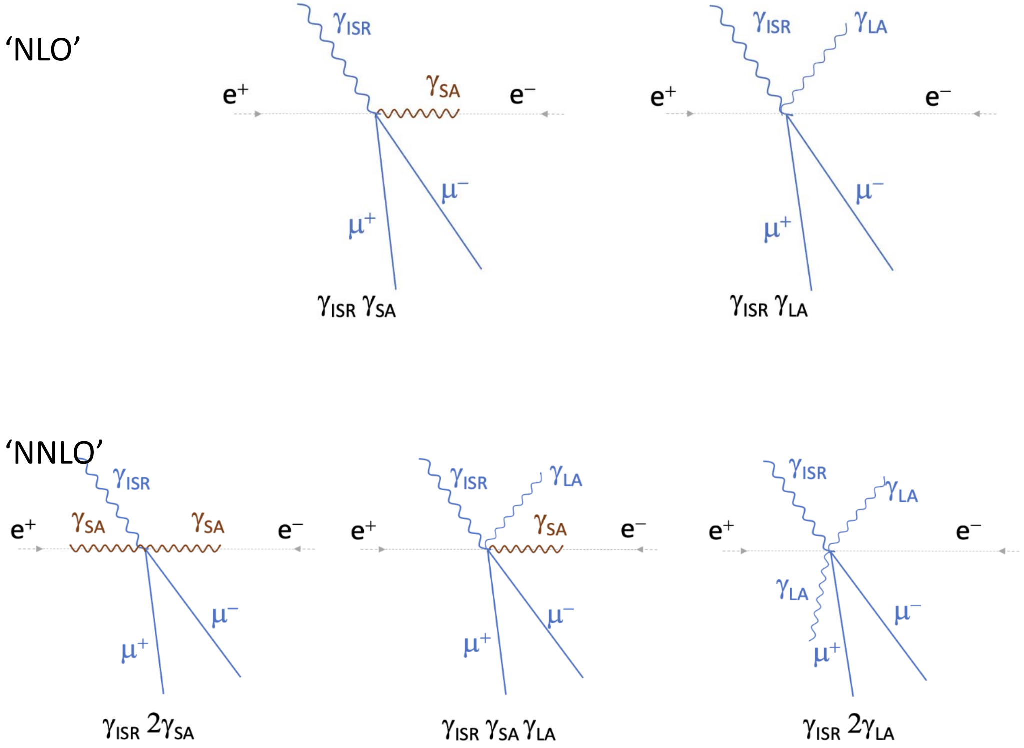
Impact of higher order radiation: unique ‘(N)NLO’ BaBar study

- Studied in-situ in BaBar data, using kinematic fits: test the most frequently used Monte Carlo generators
  - PHOKHARA: limited to NLO, but full matrix element for ISR and FSR
  - AFKQED: NLO and NNLO, with collinear approximation for additional ISR
- Large cancellations between hard emission and soft/virtual contributions

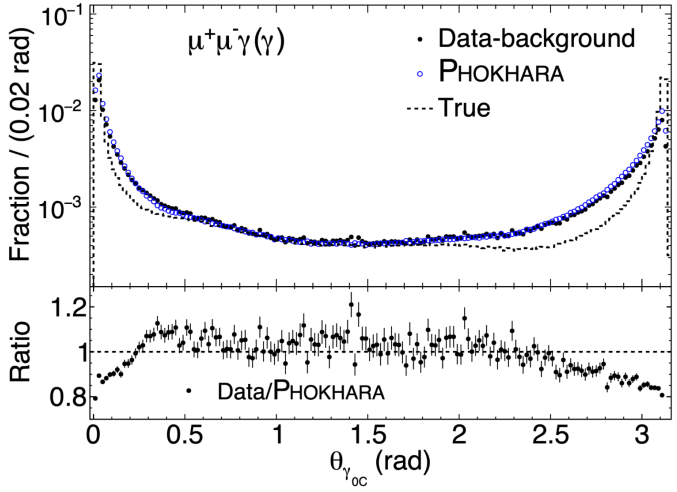
Generic ISR diagrams

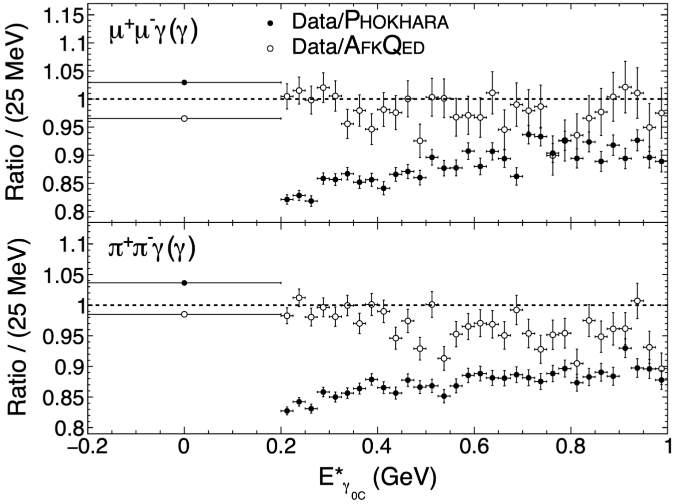


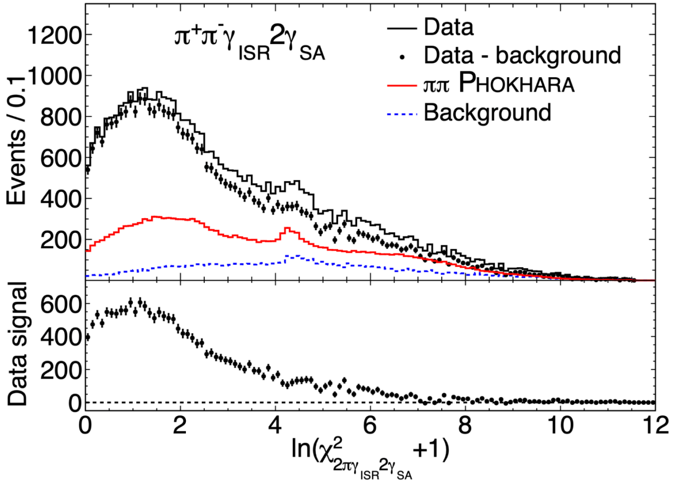
arXiv:2308.05233



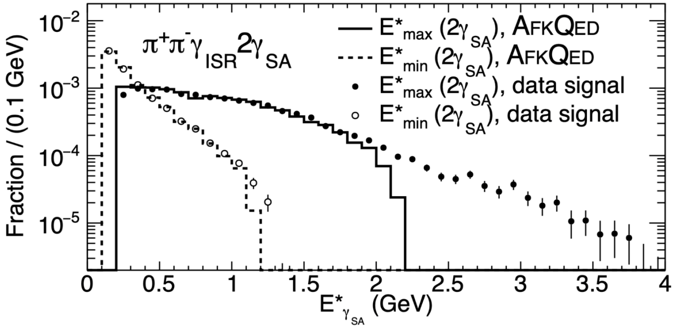
BaBar results on higher order radiation NLO and NNLO







- NLO small-angle ISR in PHOKHARA higher than in data; large-angle ratios consistent with unity
- AFKQED: reasonable description of rate and energy distributions for ‘(N)NLO’ data

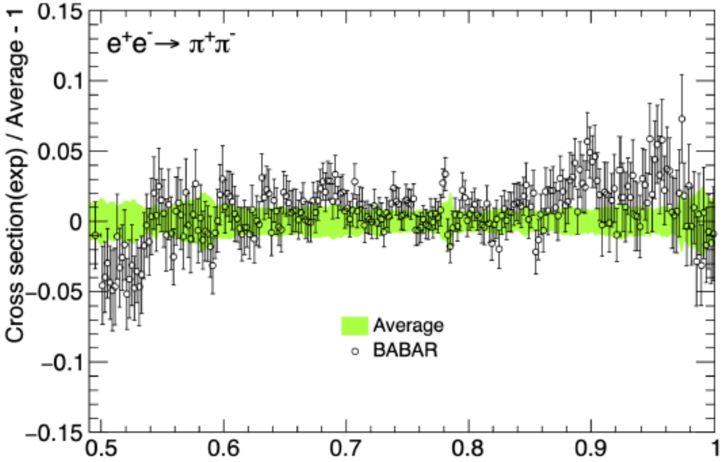


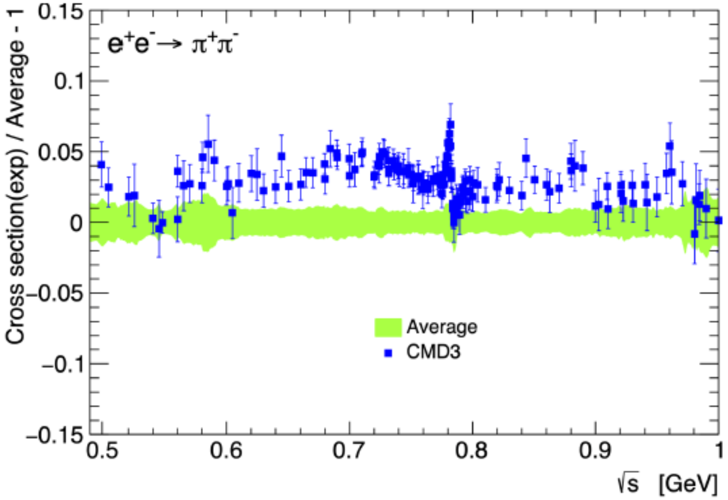
- NNLO contributions clearly observed in data (3.5% $E_\gamma > 200$ MeV)

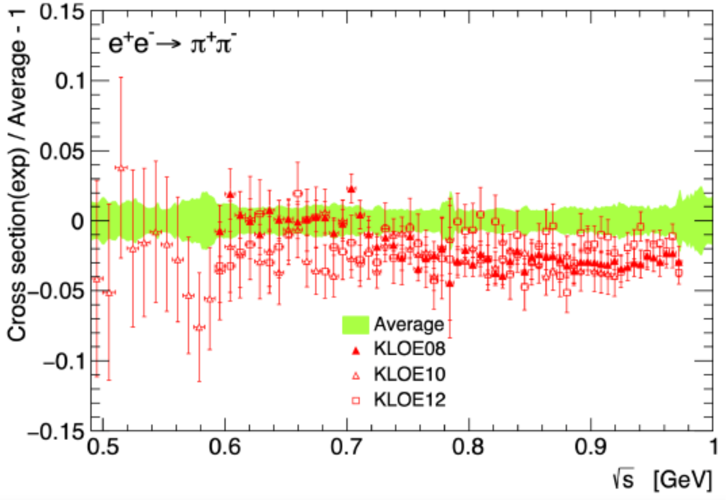
- BaBar measurements with loose selection incorporate NLO and HO radiation minimizing MC-dependence
- Other ISR measurements select ‘LO’ topology and rely on PHOKHARA for hard NLO (but not NNLO)
- Impact for KLOE and BESIII needs to be investigated (arXiv:2312.02053)

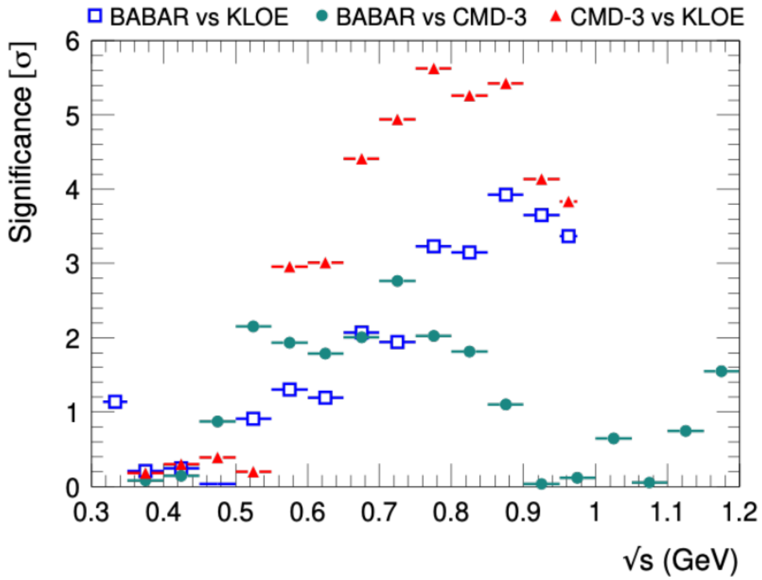
The new landscape of data dispersive (DHLMZ,2023)
(1) Tensions

- The new CMD-3/all and old BABAR/KLOE discrepancies necessitate a re-evaluation of the situation of e+e- data used in the dispersive approach
- Performed new combination of all experiments (+CMD-3, SND2020, updated BESIII) to identify the differences among the most precise experiments









- Discrepancy between CMD-3 and KLOE 5.1 σ in the 0.6-0.95 GeV ρ region
- “ “ CMD-3 and BABAR 2.2 σ “
- “ “ KLOE and BABAR 3.0 σ “

The new landscape of data dispersive (DHLMZ,2023)
(2) Impact of NLO/NNLO BABAR study

- BABAR only experiment measuring simultaneously LO/NLO/NNLO ISR processes $e^+ e^- \rightarrow \mu^+ \mu^- \gamma / \pi^+ \pi^- \gamma$
  - ⇒ questioned validity of NLO Phokhara + absence of NNLO
  - ⇒ BABAR measurement essentially independent of description of radiation the MC generator

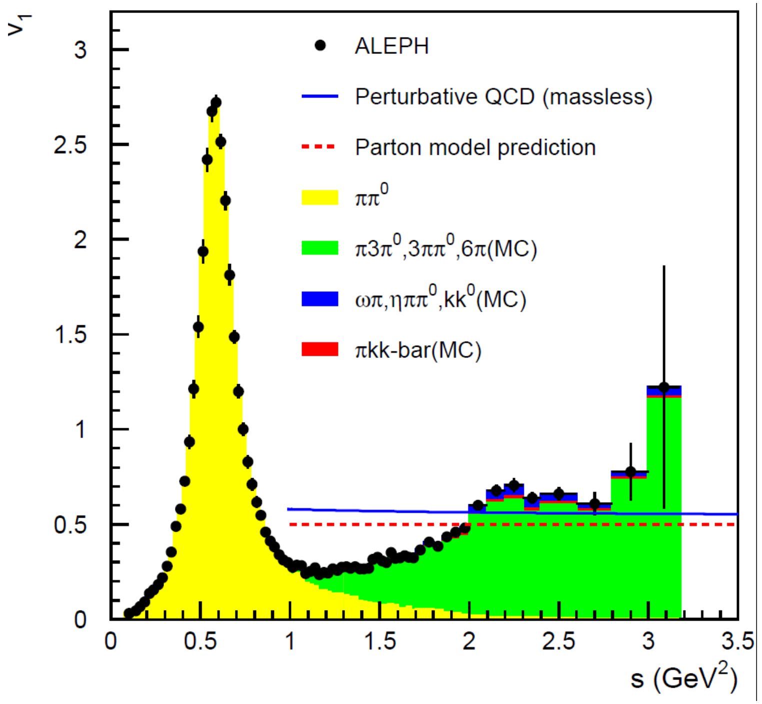
- KLOE and BESIII select LO topology and rely heavily on NLO Phokhara generator to correct for
  - ⇒ doubts expressed about validity of their approach

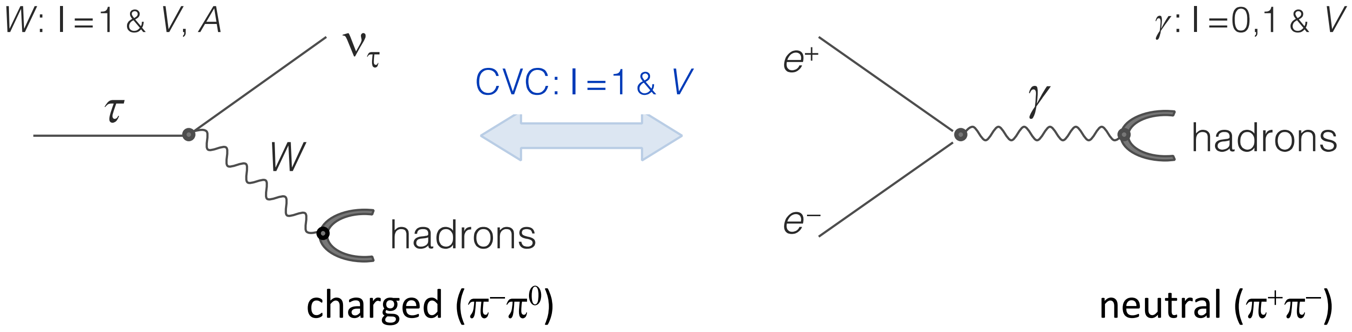
- Performed fast simulations of KLOE and BESIII experimental conditions to check impact of Phokhara shortcomings
  - ⇒ potential biases found at a level larger than quoted systematic uncertainties which could explain the BABAR/KLOE discrepancy

- More realistic tests should be performed by KLOE and BESIII collaborations themselves

The new landscape of data dispersive (DHLMZ,2023)
(3) re-consideration of τ hadronic spectral functions

- Precise measurements of τ branching fractions and hadronic spectral functions with ALEPH prompted their use for computing HVP in a_μ and $\Delta\alpha(M_Z)$ (Alemany-Davier-Hoecker, 1997)



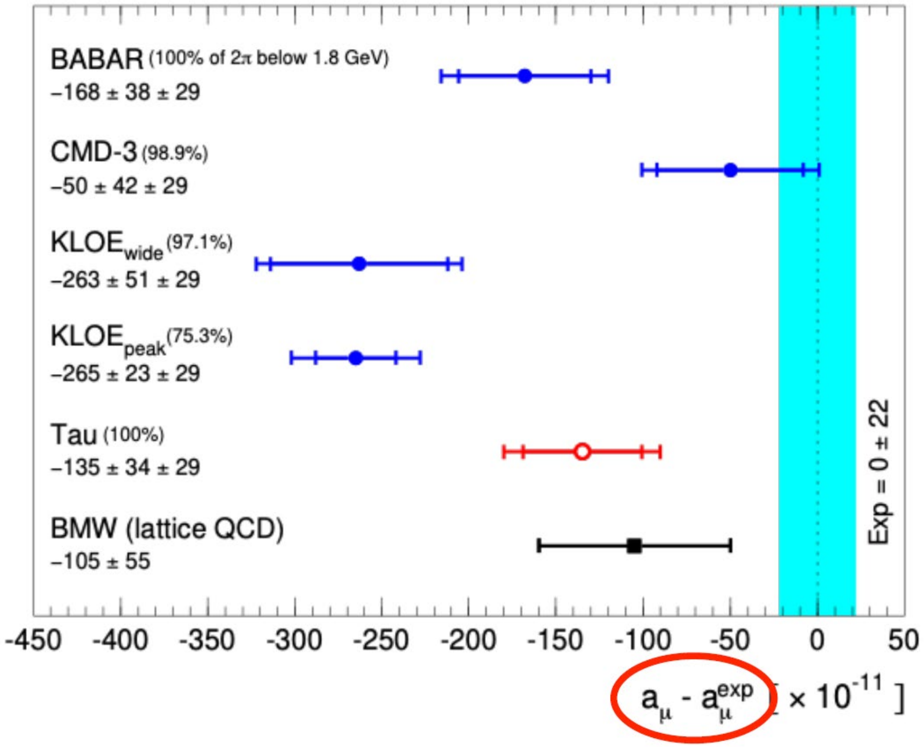


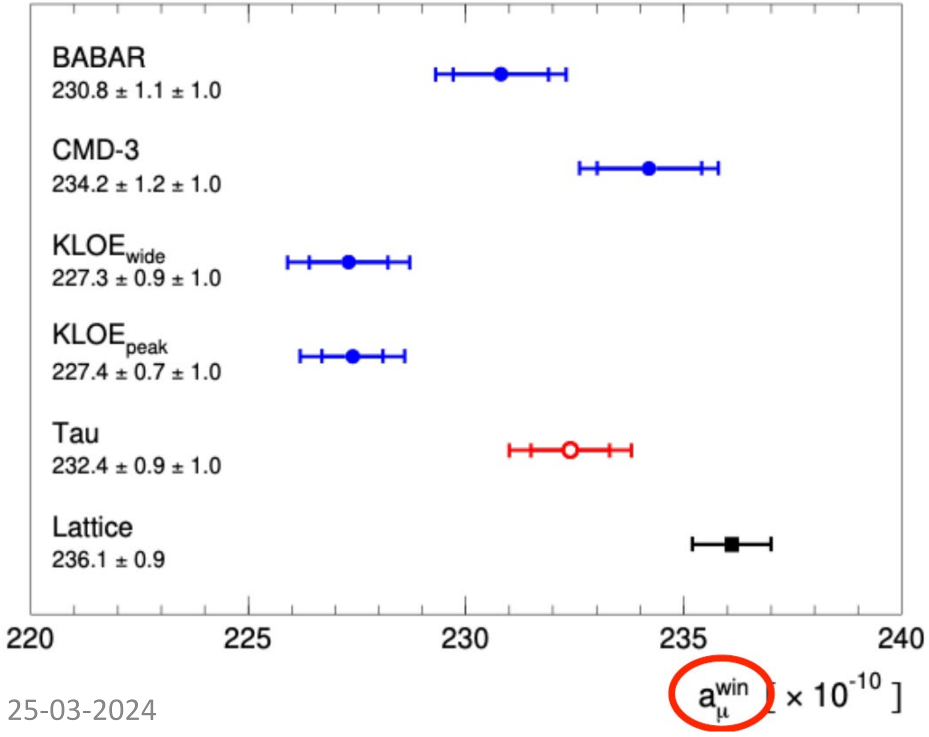
- Complementarity LEP (BR) and b/c factories (mass spectra)
- Isospin-breaking corrections identified: electroweak radiative effects, charged/neutral mass/width differences, ρ-ω interference
- Good agreement between BABAR and τ in ππ channel (2009)
- Use of τ data discontinued for WP2020 (more e+e- data, avoid IB corrections)

- In view of discrepancies between KLOE/BABAR/CMD-3 for ππ, it is advantageous to reconsider the τ approach
- Re-evaluated τ ππ contribution agrees with BABAR and CMD-3, not with KLOE

The new landscape of data dispersive (DHLMZ,2023)
(4) Comparison of the most precise results

- So far BMW result compared with the combination of all e+e- data. Since there is no agreement between them and some doubts expressed for some, it is interesting to perform the comparison experiment by experiment
- Use ππ contribution from different experiments, all other contributions from WP2020 ⇒ full a_μ values
- BABAR, CMD-3, τ in fair agreement, average 2.5σ below Fermilab, contradiction with most precise KLOE value
- BABAR/CMD-3/τ in good agreement with BMW, average of all 4 results 2.8σ below Fermilab
- Still a deviation with lattice average for the intermediate window, to be understood





Perpectives

- Short-term efforts on 3 fronts: Fermilab, lattice, new e+e- measurements

- Fermilab: runs 4-5-6 being analyzed, systematics under control, precision statistically limited
  precision ×2 /runs 1-2-3, final results expected in early 2025

- Lattice: expected results for long- and short-distance windows, full HVP, improvements for HLbL
  long-distance is the most problematic part to obtain with precision (but 57% of total contribution)

- Data-driven dispersive: several efforts in progress
  - Feedback from past KLOE analyses?
  - Feedback from CMD-2/3 problem
  - SND update with full data
  - BES III with more statistics, attention to additional radiation
  - BABAR with full data and independent method separating $\mu\mu/\pi\pi$/KK without PID,
    well advanced (DLMZ + Léonard Polat post-doc ANR au LPNHE/IJCLab)
  - Belle II for both e+e- and τ
  - New KLOE analysis of full data, additional radiation?, development of an NNLO generator
  - In the longer run: MuonE at CERN, very challenging

Summary

- The understanding of the muon g-2 has been a bit chaotic, but remains an exciting and challenging task
- Significant progress on the direct measurement at Fermilab: precision × 2 /previous BNL, goal is × 4
- A large effort was devoted to produce a reliable and conservative theoretical prediction within the Standard Model, culminating in the 2020 WP of the muon g-2 Theory Initiative
- The HVP contribution plays a very important role in the value and accuracy of the prediction
- Our DHMZ group has more than 20 years of experience using the mature dispersive approach based on data from τ decays and e+e- cross sections for which we contributed with innovative methods
- Unfortunately discrepancies among different experiments prevent us from using the full data potential
- The alternative approach using QCD on a lattice is very promising with so far only one result which disagrees with the WP2020 prediction, needing confirmation by other groups

- In the past year many developments occurred: measurement by BABAR of additional radiation up to NNLO with impact on KLOE and BES III analyses
- As a result some clarification seems to be emerging with BABAR, CMD-3, and τ driven estimates being in agreement and also with the lattice calculation, still departing from the Fermilab measurement by about 3σ, however much smaller than using the 2020 prediction.
- The coming year will continue to be exciting…