Quantum tomography at the LHC: top quark production

Luca Mantani

In collaboration with:
R. Aoude, E. Madge, F. Maltoni
+ insights from the community

European Research Council
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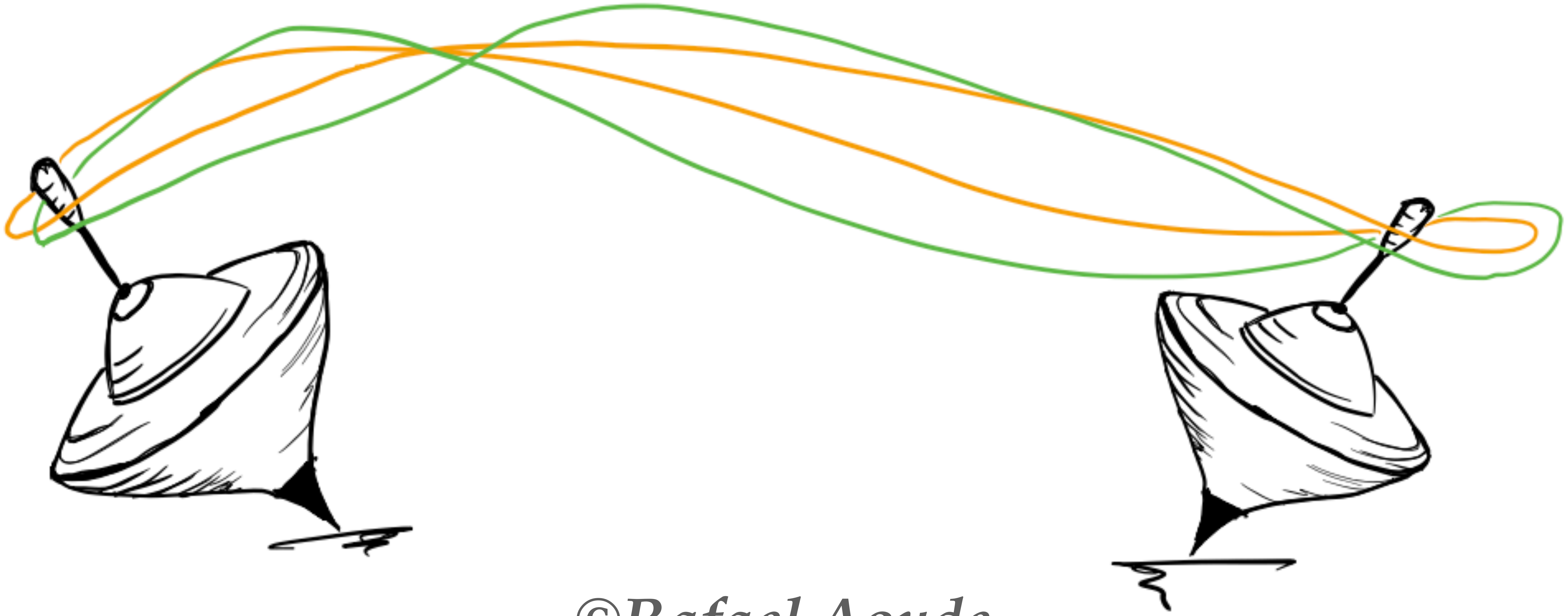
UNIVERSITY OF CAMBRIDGE

Outline

* Motivation
* Quantum Information Theory: the basics
* Why the top quark?
* Entanglement at the LHC



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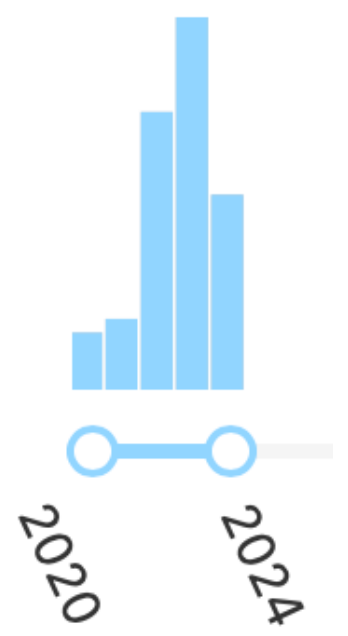
* Test Quantum Mechanics at the TeV scale
    * Quantum Information at the LHC: relativistic, fundamental particles
    * Learn from QI: fundamental interactions structure, interpretation
* Challenge: can we actually do it in a “dirty” environment?
* Rise in interest in the community: new ideas, methods, exp. strategies



Citations to the Afik & de Nova seminal paper

Quantum Information Theory

Entanglement in bipartite systems

Given a bipartite system, with Hilbert space $\mathscr{H} = \mathscr{H}_1 \otimes \mathscr{H}_2$

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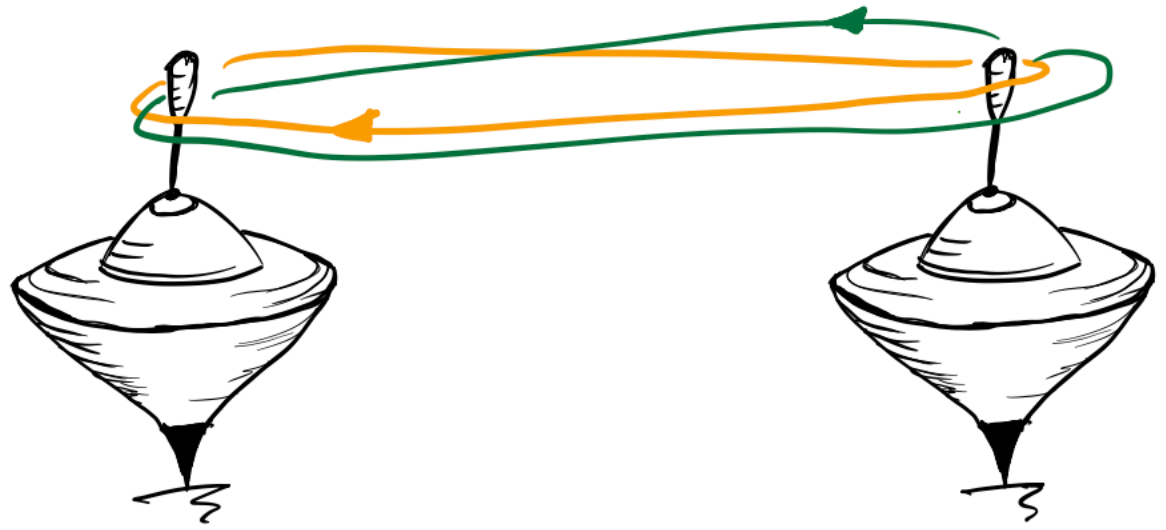
This is not always the case, e.g.: **Maximally entangled states: spin 1/2**



$$|\Phi^\pm\rangle = \frac{|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle}{\sqrt{2}} \qquad |\Psi^\pm\rangle = \frac{|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle}{\sqrt{2}}$$

Density matrix

* $\rho = \sum_k p_k \rho_k$

entangled if $\rho_k \neq \rho_1 \otimes \rho_2$

The fundamental object in QM is the density matrix *

$$\rho = \frac{1}{d}\mathbb{I} + \sum_{i=1}^{d^2-1} a_i \lambda_i$$

One particle of spin s:

d=2s+1

Generalised Gell-Mann matrix

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Two particles, each of spin s:

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The parameters completely characterise the quantum spin state of the system

How do we build the density matrix?

$$R^I_{\alpha_1\alpha_2,\beta_1\beta_2} \equiv \frac{1}{N_a N_b} \sum_{\substack{\text{colors} \\ \text{a,b spins}}} \mathcal{M}^*_{\alpha_2\beta_2}\,\mathcal{M}_{\alpha_1\beta_1}$$

Sum over initial state only

Matrix-element $\mathcal{M}_{\alpha\beta} \equiv \langle t(k_1,\alpha)\bar{t}(k_2,\beta)|\mathcal{T}|a(p_1)b(p_2)\rangle$

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$$R = \tilde{A}\,\mathbb{I}\otimes\mathbb{I} + \sum_{i=1}^{d^2-1} \tilde{a}_i\,\lambda_i\otimes\mathbb{I} + \sum_{j=1}^{d^2-1} \tilde{b}_j\,\mathbb{I}\otimes\lambda_j + \sum_{i=1}^{d^2-1}\sum_{j=1}^{d^2-1} \tilde{c}_{ij}\,\lambda_i\otimes\lambda_j \quad \Rightarrow \quad \rho = \frac{R}{tr(R)}$$

Quantum observables

Concurrence

$$\mathcal{C}(\rho) = \inf\left[\sum_i p_i c(|\psi_i\rangle)\right]$$

Entangled if > 0

2-qubits: $$C[\rho] \equiv \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

with λ_i eigenvalues of $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$

$\tilde{\rho} = (\sigma_2 \otimes \sigma_2)\,\rho^*\,(\sigma_2 \otimes \sigma_2)$

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Bell inequality

$$\langle \mathcal{B} \rangle_{\max} = \max_{U,V} \left(\mathrm{Tr} \left(\rho\, (U^\dagger \otimes V^\dagger)\, \mathcal{B}\, (U \otimes V) \right) \right) \geq 2$$

$$\mathcal{B} = -\frac{2}{\sqrt{3}} (S_x \otimes S_x + S_y \otimes S_y) + \lambda_4 \otimes \lambda_4 + \lambda_5 \otimes \lambda_5$$

The ideal candidate: the top quark

* *Aoude et al. 2307.09675*
Ashby-Pickering et al. 2209.13990
Fabbrichesi et al. 2302.00683

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Weak bosons (*) and top quarks are the ideal candidates:
EW interactions allow for spin reconstruction from decay (no hadronisation)

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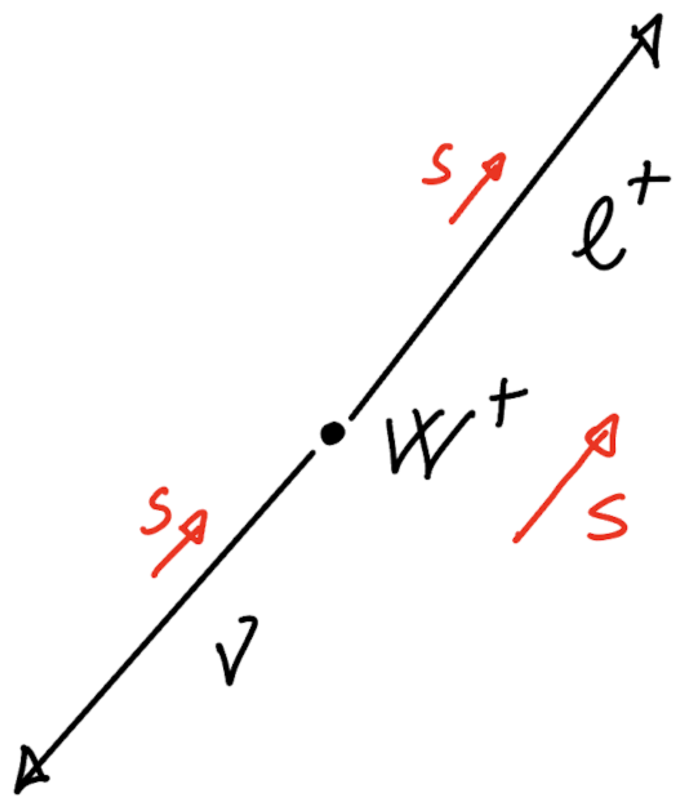
W decay:
lepton decays along W spin



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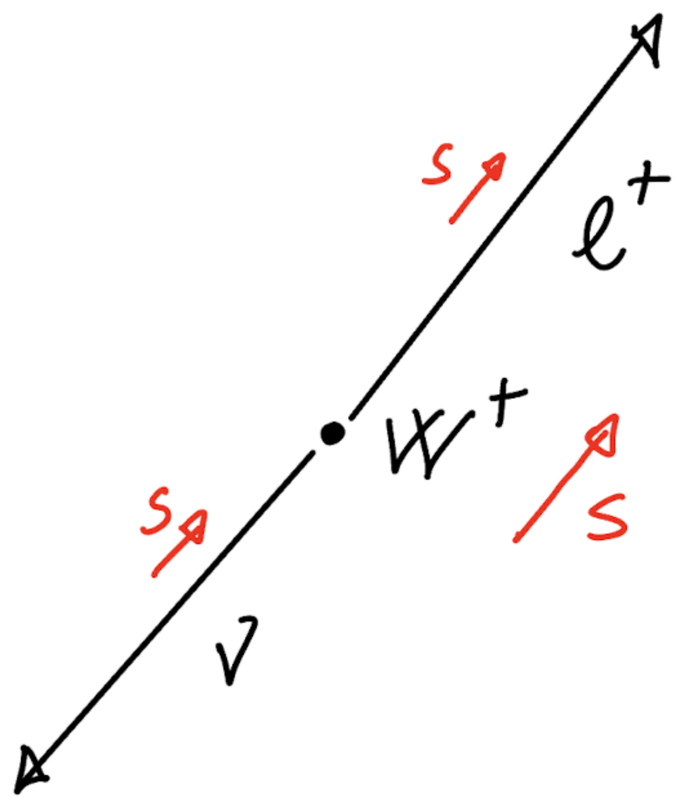

Top decay:
lepton decay correlated with top spin

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\phi} = \frac{1+\cos\phi}{2}$$

ϕ angle between lepton and spin

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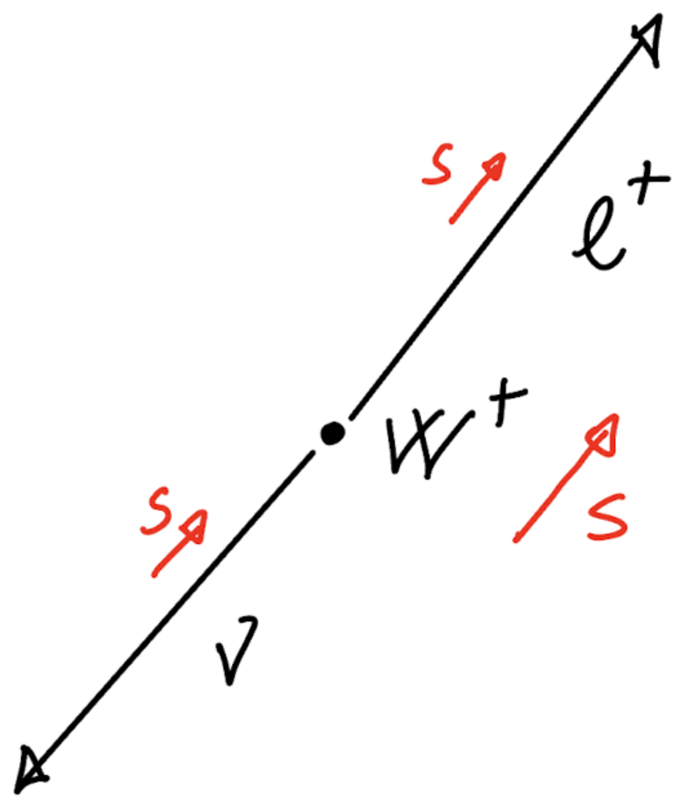

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Z boson more complicated but doable: spin can be reco if right/left asymmetry

Spin 1/2 density matrix

The R matrix can be decomposed in the spin space

$$R = \tilde{A}\,\mathbb{1}_2 \otimes \mathbb{1}_2 + \tilde{B}_i^+ \sigma^i \otimes \mathbb{1}_2 + \tilde{B}_i^- \,\mathbb{1}_2 \otimes \sigma^i + \tilde{C}_{ij}\, \sigma^i \otimes \sigma^j$$

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Cross section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega \mathrm{d}\hat{s}} = \frac{\alpha_s^2 \beta}{\hat{s}^2} \tilde{A}\,(\hat{s}, \boldsymbol{k})$$

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Degree of top and anti-top polarisation

(zero if interactions P-invariant)

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$$\frac{d\sigma}{d\Omega d\hat{s}} = \frac{\alpha_s^2 \beta}{\hat{s}^2} \tilde{A}\left(\hat{s}, \boldsymbol{k}\right)$$

If normalised, we define the density matrix

$$\rho = \frac{\mathbb{1}_2 \otimes \mathbb{1}_2 + B_i^+ \sigma^i \otimes \mathbb{1}_2 + B_i^- \mathbb{1}_2 \otimes \sigma^i + C_{ij}\, \sigma^i \otimes \sigma^j}{4}.$$

Quantum tomography

How do we reconstruct the spin density matrix at colliders?

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For example, for the density matrix of a W boson

Ashby-Pickering et al.
2209.13990

$$\Phi_1^{P\pm} = \sqrt{2}(5\cos\theta \pm 1)\sin\theta\cos\phi$$
$$\Phi_2^{P\pm} = \sqrt{2}(5\cos\theta \pm 1)\sin\theta\sin\phi$$
$$\Phi_3^{P\pm} = \frac{1}{4}(\pm 4\cos\theta + 15\cos 2\theta + 5)$$
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$$a_j = \frac{1}{2}\int d\Omega_{\hat{\mathbf{n}}}\, p(\ell^{\pm}_{\hat{\mathbf{n}}};\rho)\,\Phi_j^{P\pm}$$

$$c_{ij} = \left(\frac{1}{2}\right)^2 \iint d\Omega_{\hat{\mathbf{n}}_1}\, d\Omega_{\hat{\mathbf{n}}_2} p\left(\ell^+_{\hat{\mathbf{n}}_1}, \ell^-_{\hat{\mathbf{n}}_2};\rho\right)\Phi_i^P\left(\hat{\mathbf{n}}_1\right)\Phi_j^P\left(\hat{\mathbf{n}}_2\right)$$

Expectation value of the Wigner P functions

Quantum tomography: top pair

Afik & De Nova
2003.02280

In the case of top pair things are simpler

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_+\mathrm{d}\Omega_-} = \frac{1 + \mathbf{B}^+ \cdot \hat{\mathbf{q}}_+ - \mathbf{B}^- \cdot \hat{\mathbf{q}}_- - \hat{\mathbf{q}}_+ \cdot \mathbf{C} \cdot \hat{\mathbf{q}}_-}{(4\pi)^2}$$

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Angle between leptons

Interestingly, at threshold, a specific angular distributions is **directly proportional to the entanglement:** entangled tops produce *small angular separation*

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$

$$D = \frac{\mathrm{tr}[\mathbf{C}]}{3} \qquad C[\rho] = \max(-1-3D, 0)/2$$

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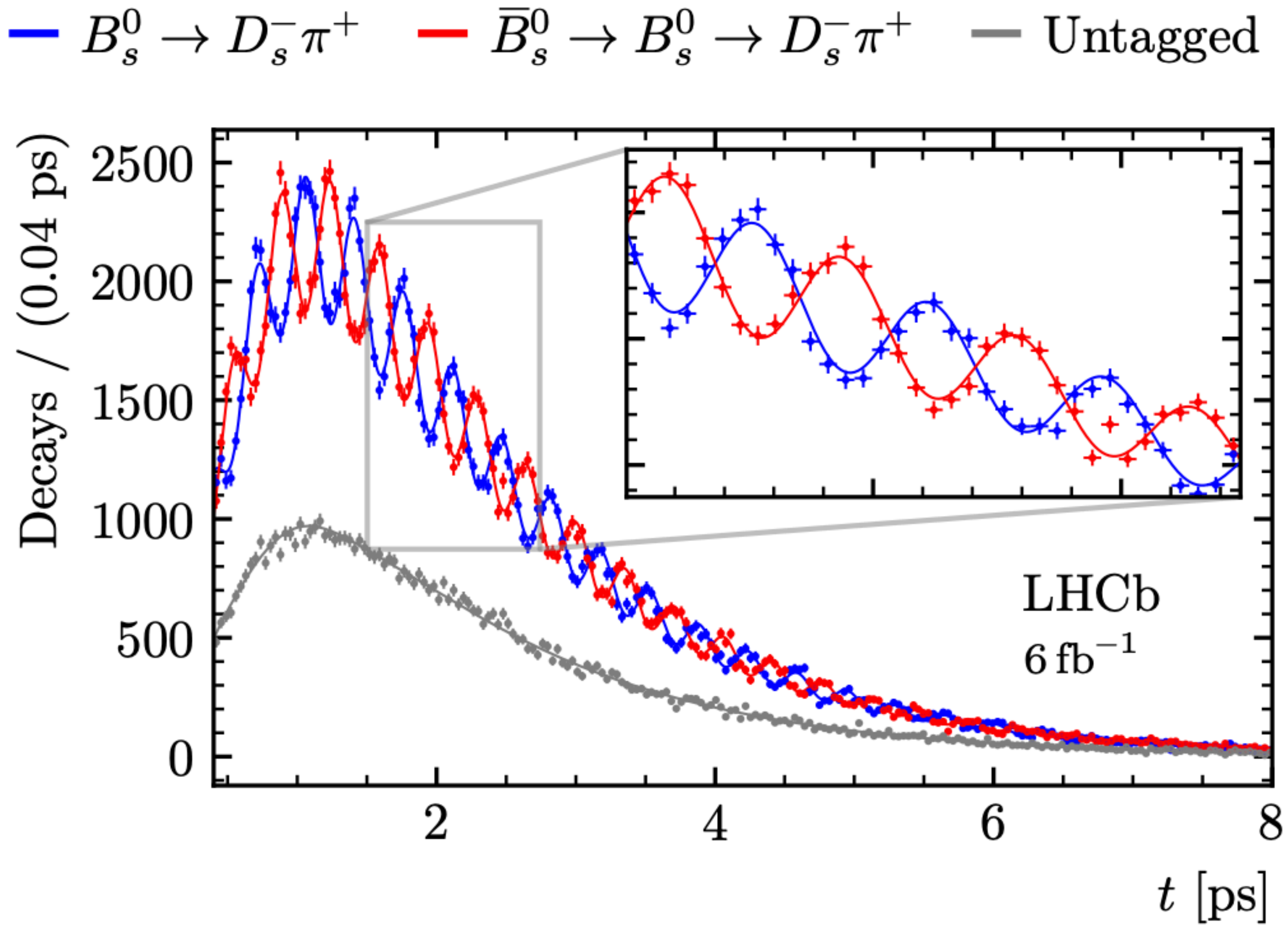
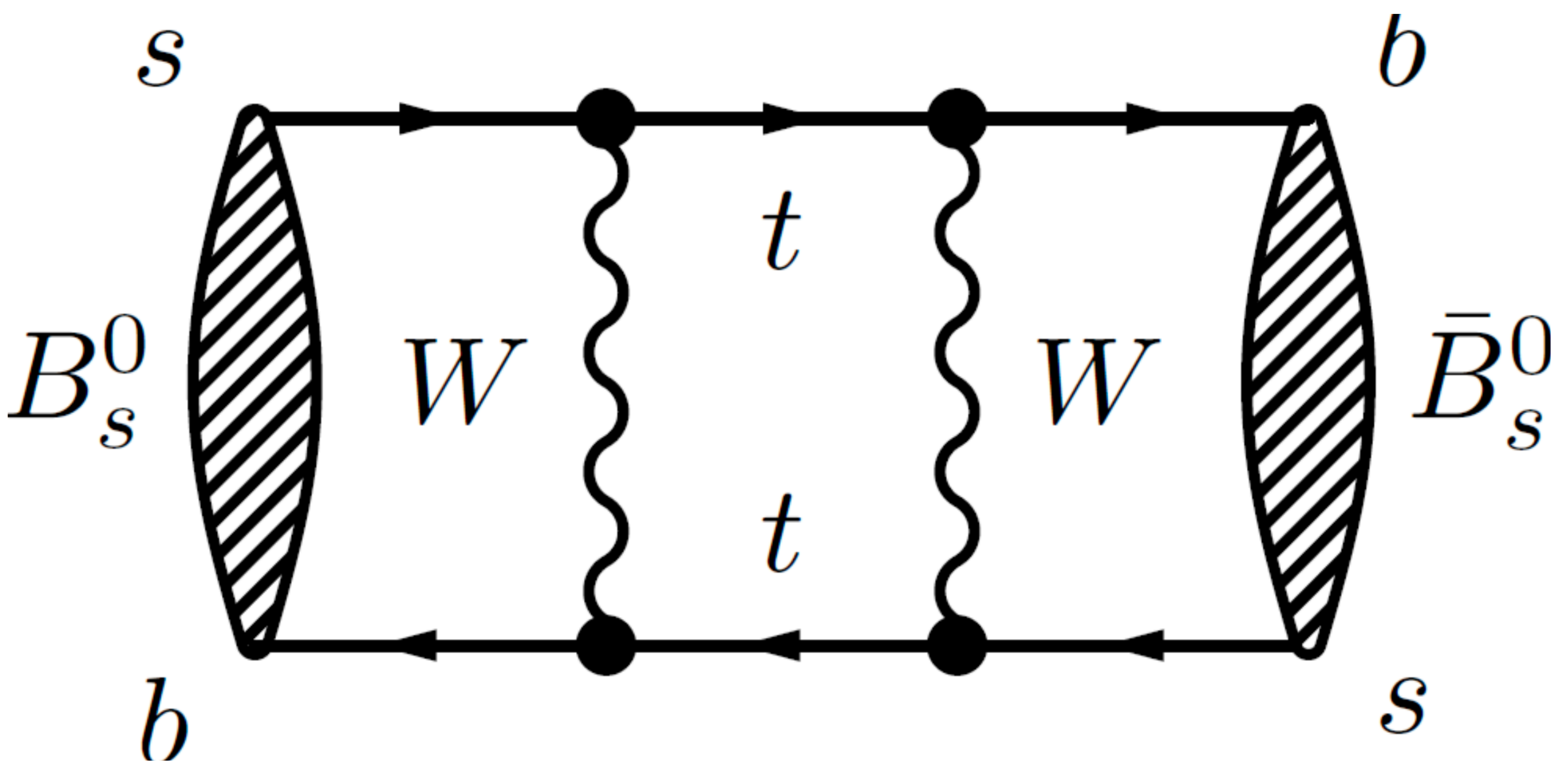
$$D < -\frac{1}{3}$$

Entanglement at the LHC

Quantum measurements at the LHC

LHCb
2104.04421





$$|B_{L,H}\rangle = p\,|B_q^0\rangle + q\,|\bar{B}_q^0\rangle$$

The R matrix at the LHC

Aoude et al.
2203.05619

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At LO in QCD

$$I = gg, q\bar{q}$$

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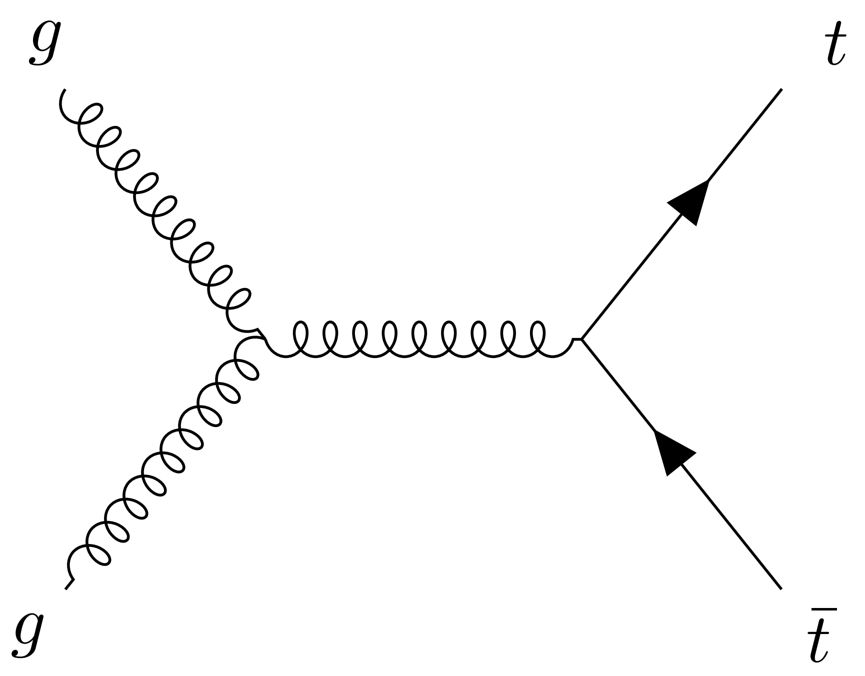
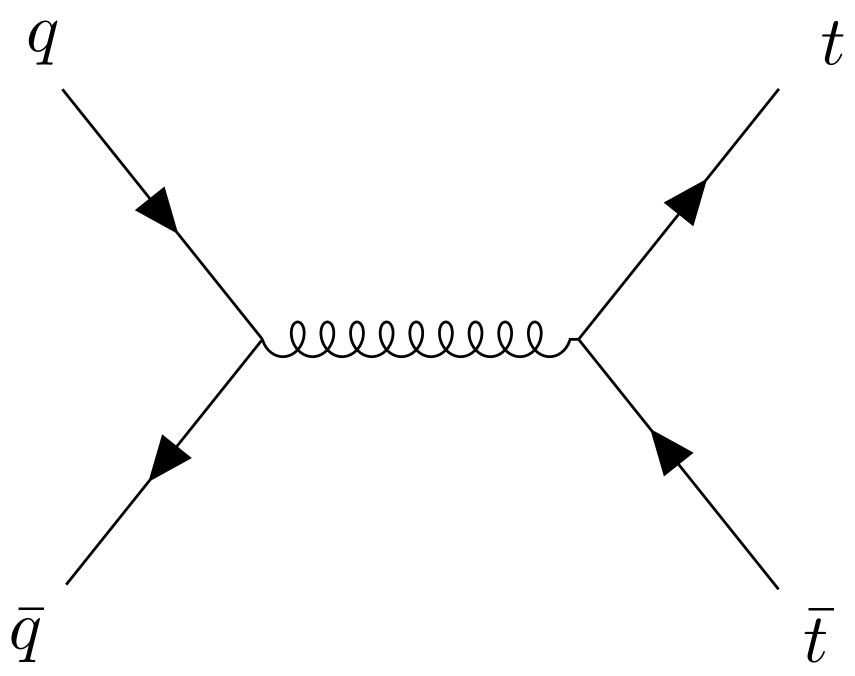
We collide protons



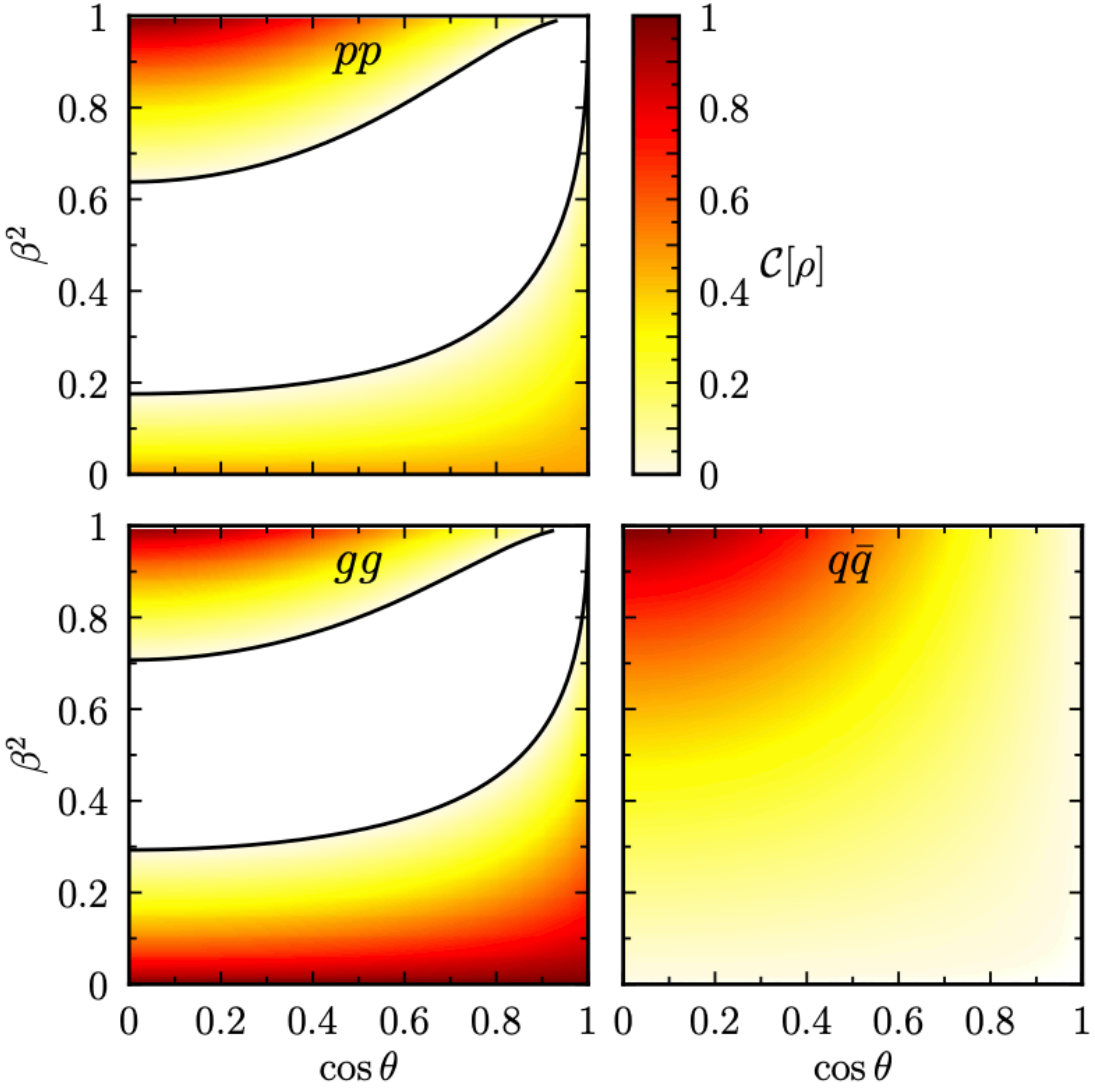
$$R(\hat{s},\boldsymbol{k}) = \sum_I L^I(\hat{s}) R^I(\hat{s},\boldsymbol{k})$$



Full density matrix is mixed state, weighted by parton luminosity

SM entanglement

Aoude et al.
2203.05619



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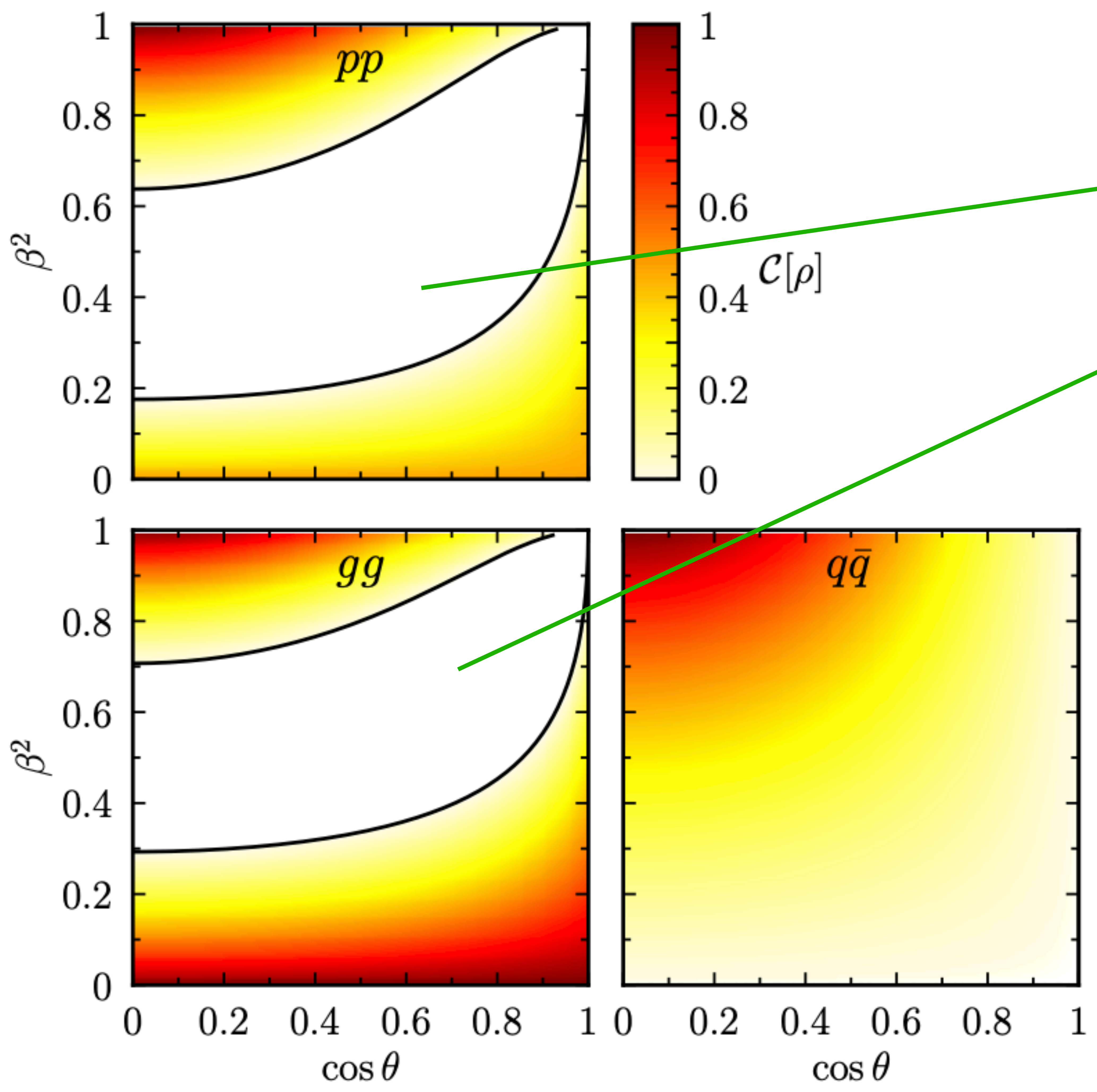

No entanglement

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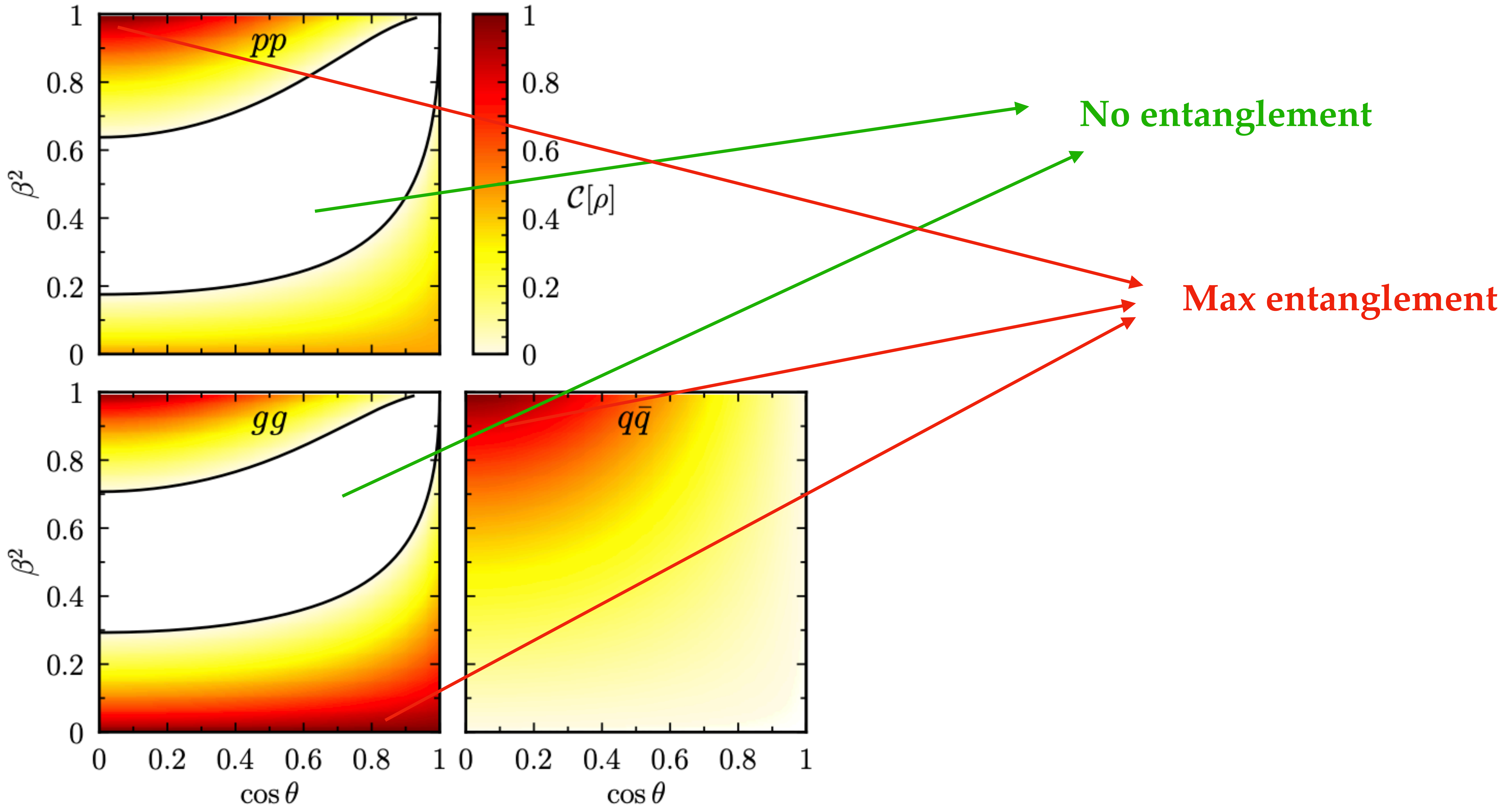
Aoude et al.
2203.05619



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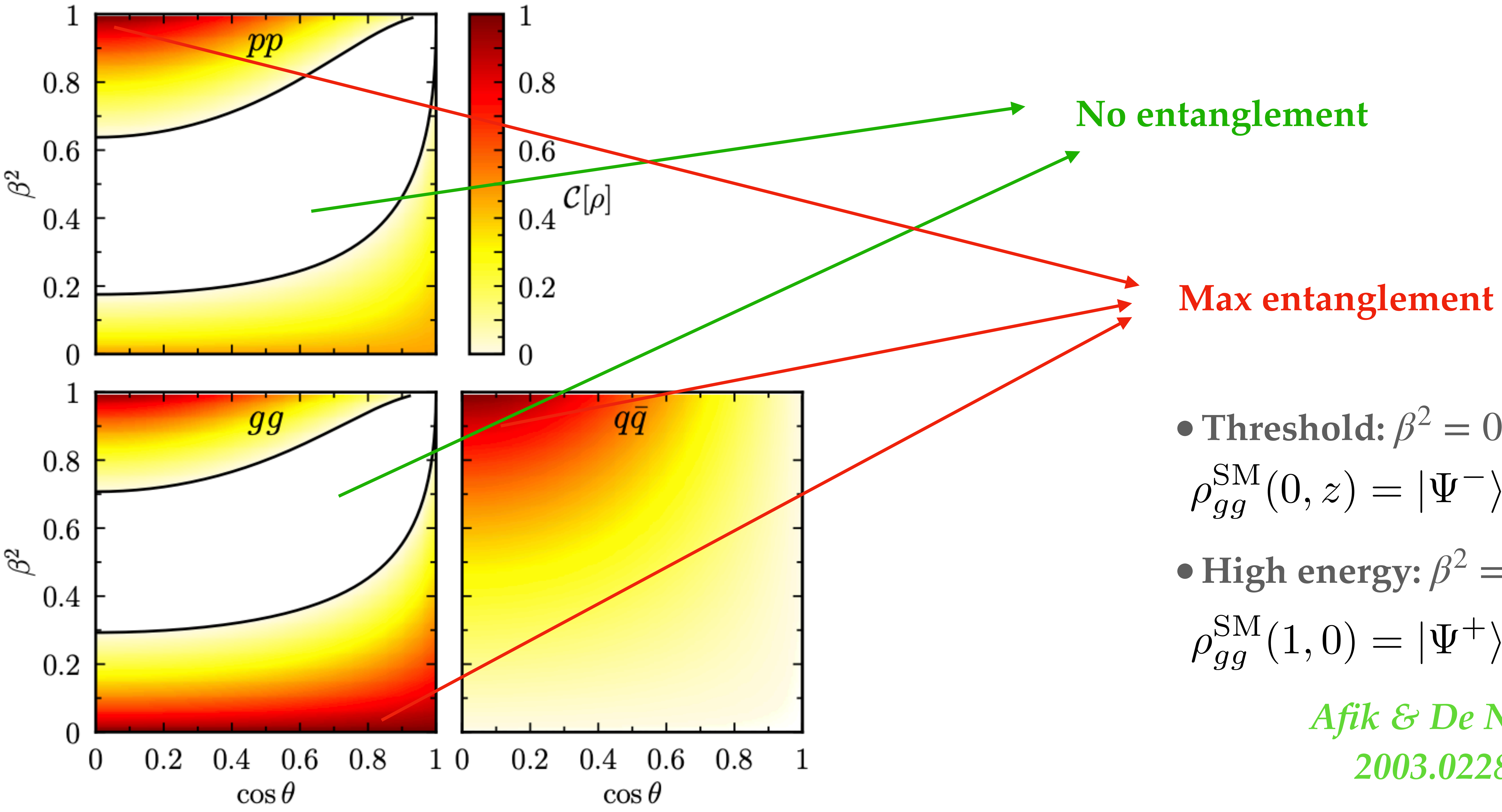

- **Threshold:** $\beta^2 = 0$

$$\rho_{gg}^{\mathrm{SM}}(0, z) = |\Psi^-\rangle_{\boldsymbol{n}}\langle\Psi^-|_{\boldsymbol{n}}$$

- **High energy:** $\beta^2 = 1\,,\cos\theta = 0$

$$\rho_{gg}^{\mathrm{SM}}(1, 0) = |\Psi^+\rangle_{\boldsymbol{n}}\langle\Psi^+|_{\boldsymbol{n}}$$

Afik & De Nova
2003.02280

Scouting for entanglement

$$\bar{R} = (4\pi)^{-1} \int \mathrm{d}\Omega \, R(\hat{s}, \boldsymbol{k}) \, ,$$

$$\delta \equiv -C_z + |2C_\perp| - 1 > 0$$

$$C[\rho] = \max(\delta/2, 0)$$

$$D = -3\langle \cos\varphi \rangle = -\frac{1+\delta}{3}$$

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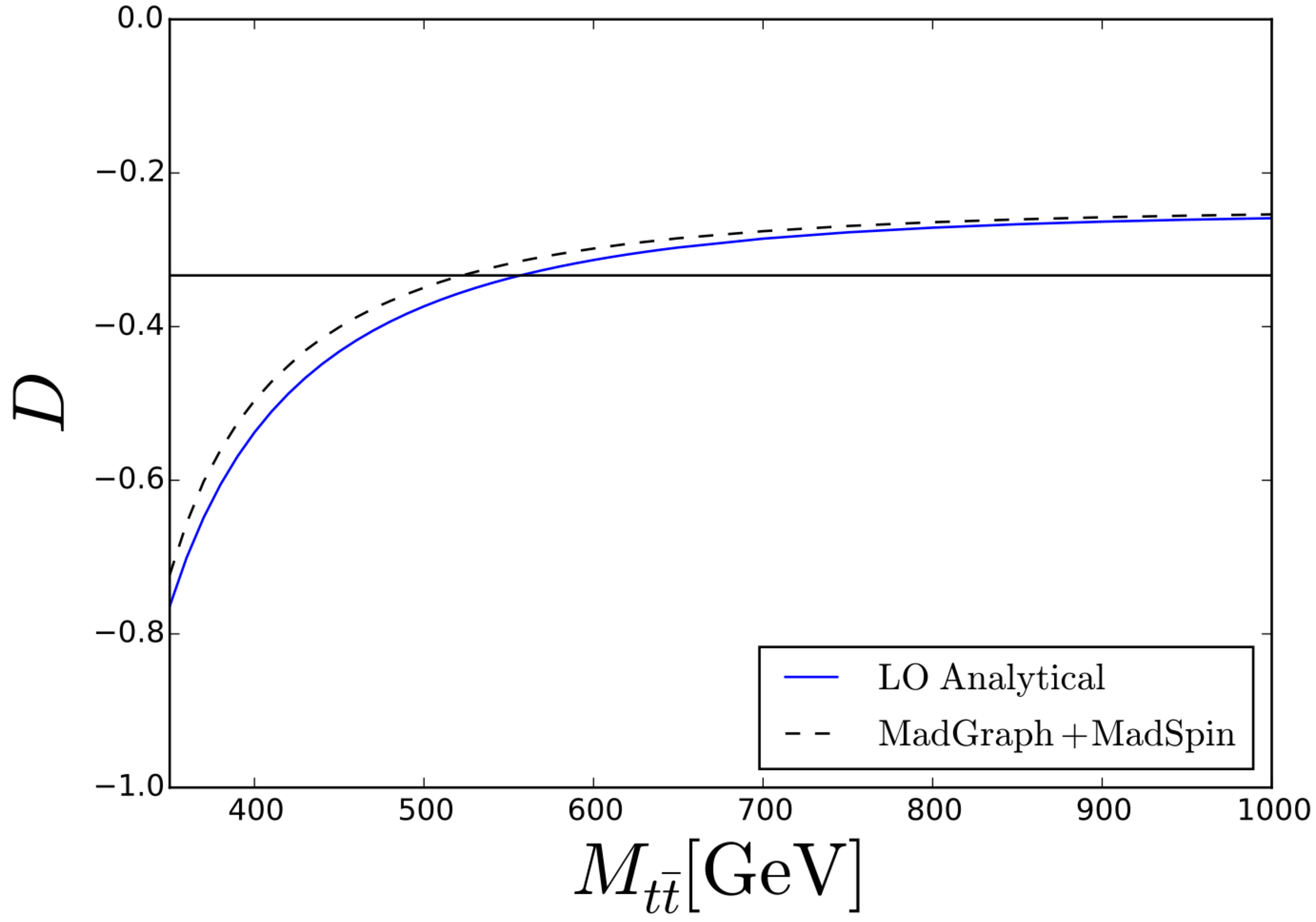

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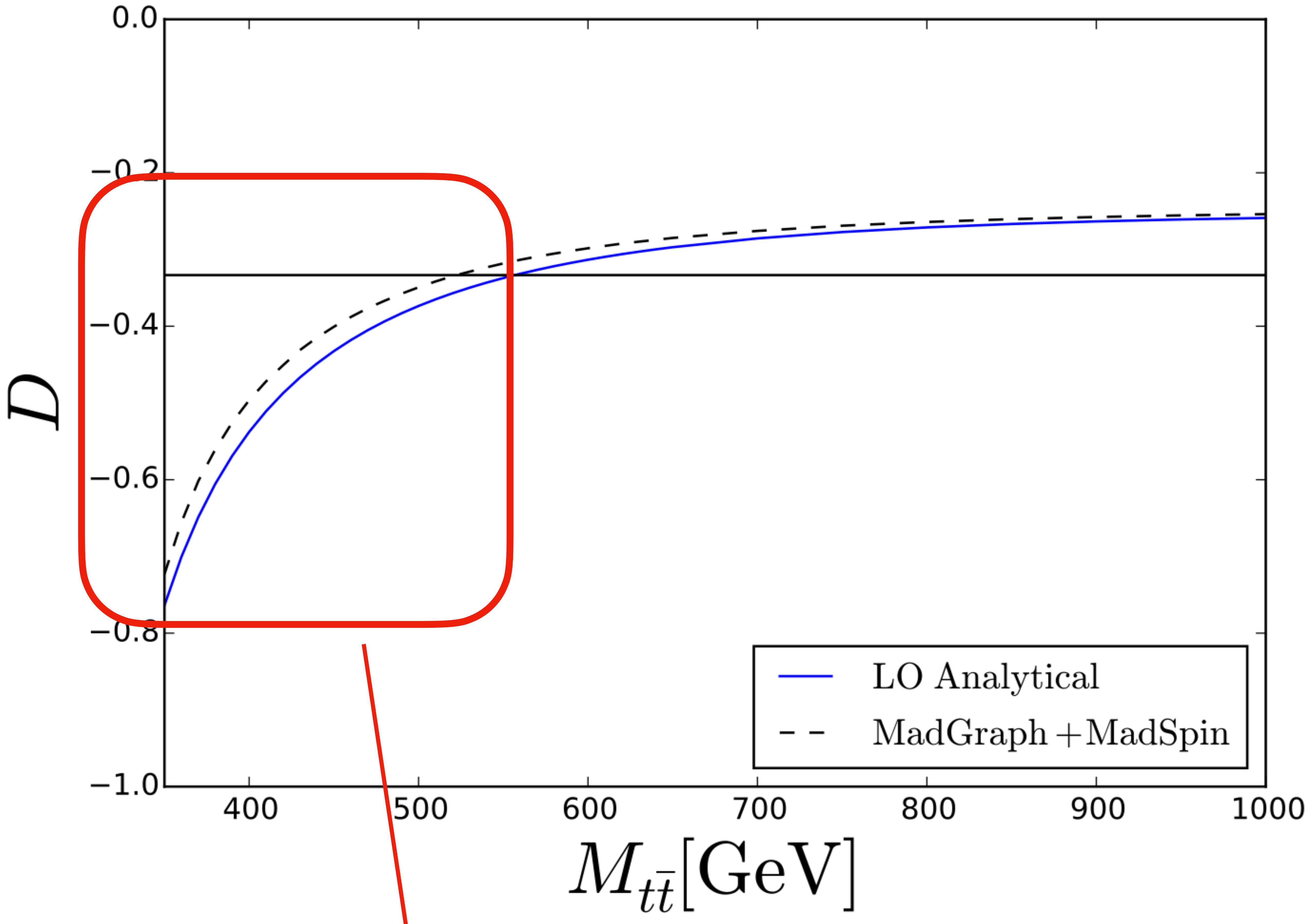
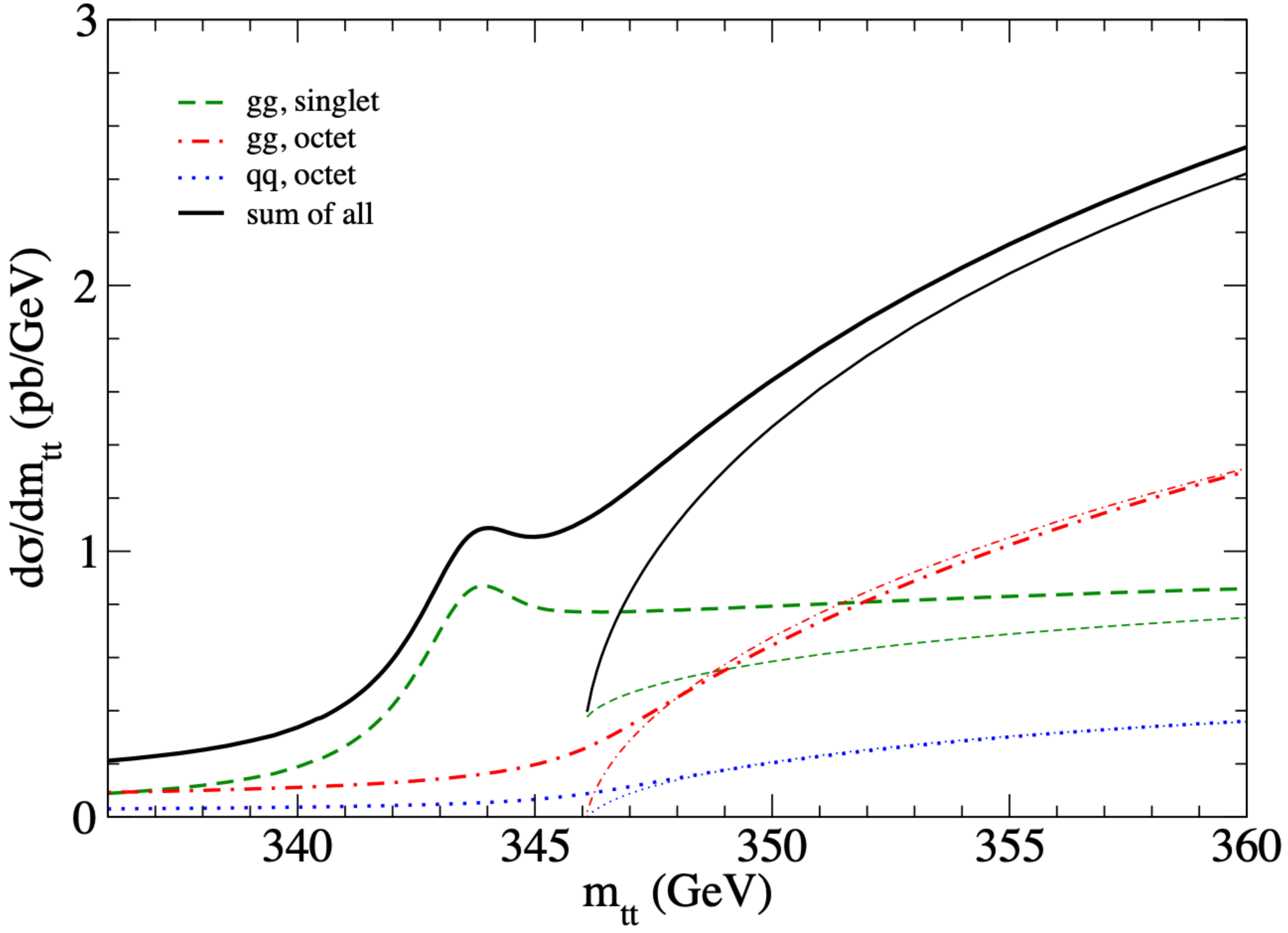


Need for a narrow bin close to threshold

Open new windows

Hagiwara, Sumino & Yokoya
0804.1014



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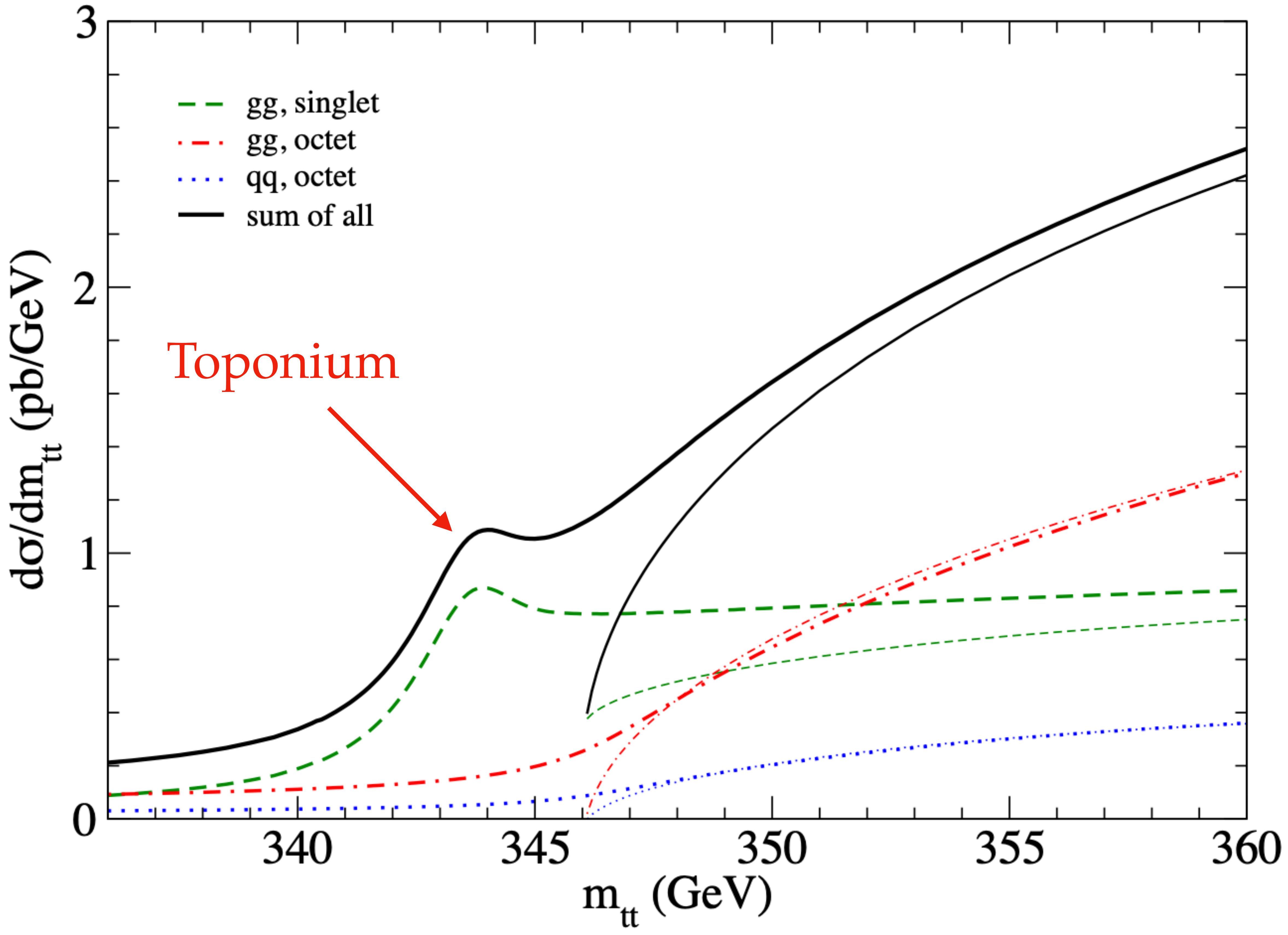
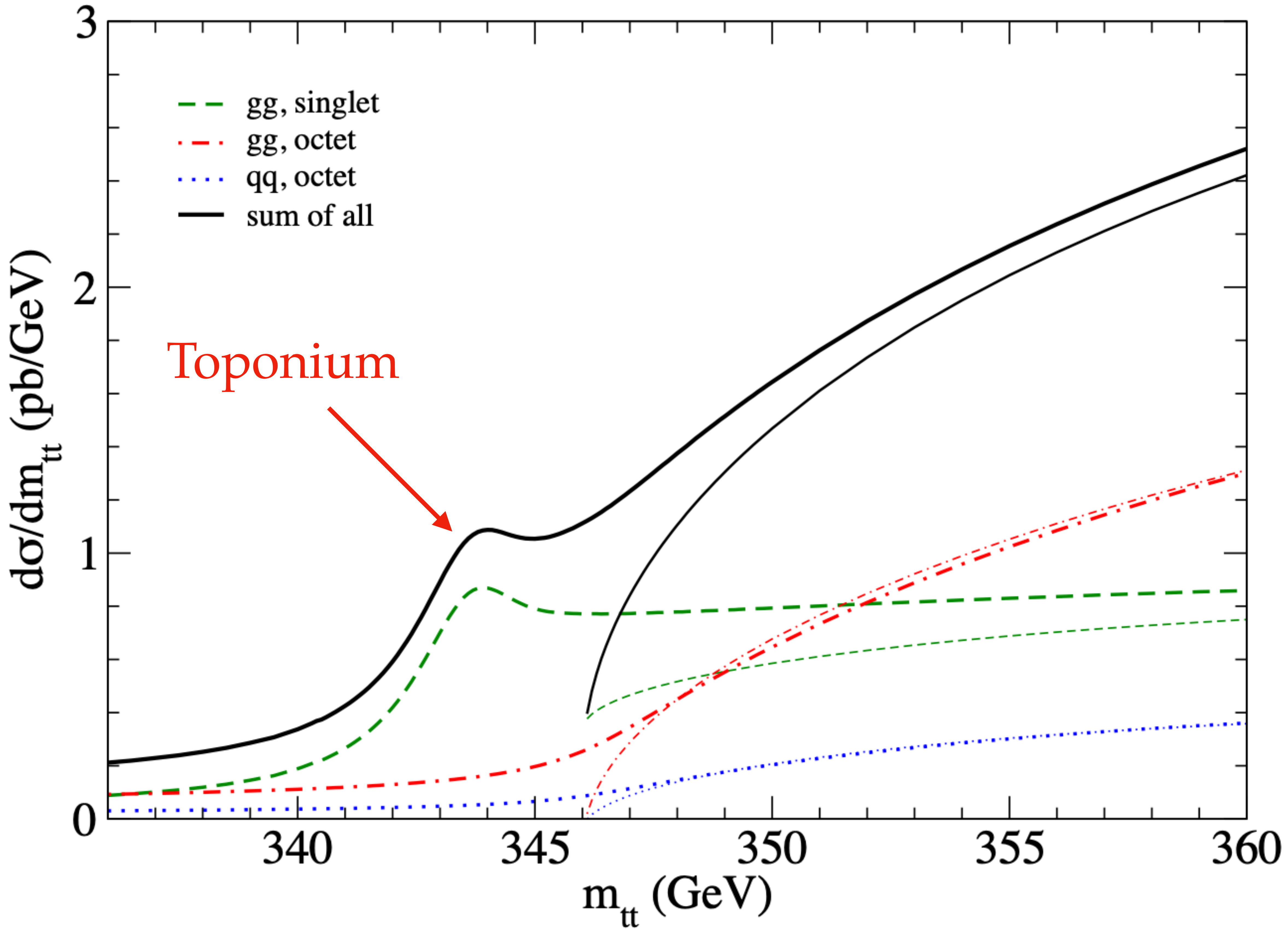
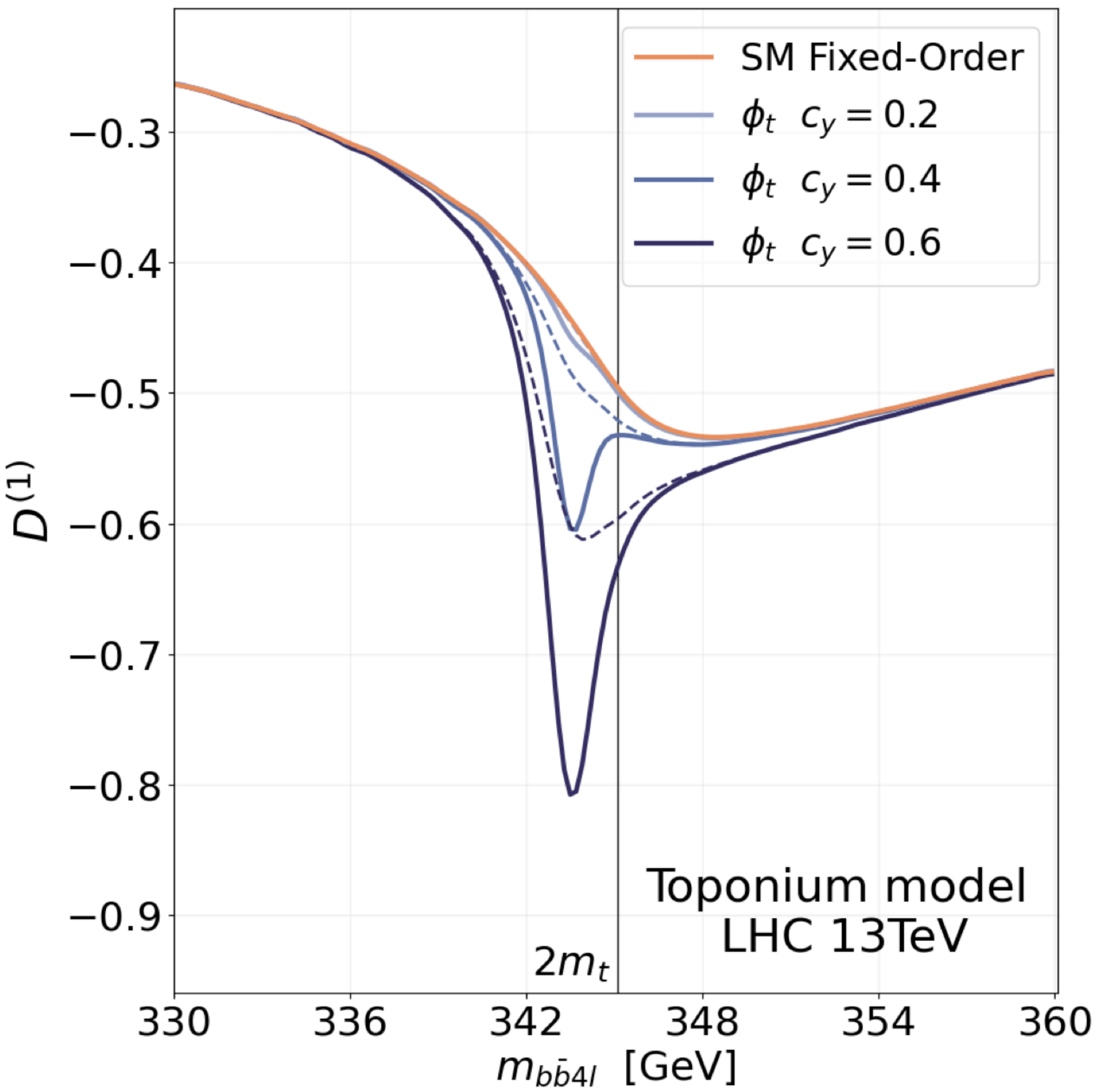


Open new windows

Hagiwara, Sumino & Yokoya
0804.1014



Maltoni et al.
2401.08751



New physics

The density matrix opens the window to new sensitivities

$$e^+e^- \to W^+W^-$$

$(\lambda_1\lambda_2\|\alpha\,\beta)$	SM	EFT Λ^{-2} : c_{WWW}
$+-00$	$-2\sqrt{2}G_F m_Z^2 \sin\theta$	-
$+--+$	$2\sqrt{2}G_F m_W^2 \sin\theta$	-
$+-+-$	$-\frac{1}{\sqrt{2}}G_F m_W^2 \sin^3\theta \csc^4(\theta/2)$	-
$+-\pm\pm$	-	$3\cdot 2^{1/4}\sqrt{G_F}m_W \sin\theta\,(4m_W^2 x^2 - m_Z^2)$
$+-0\pm$	-	$-3\cdot 2^{3/4}\sqrt{G_F}m_W^3(\pm 1+\cos\theta)\,x$
$+-\pm 0$	-	$-3\cdot 2^{3/4}\sqrt{G_F}m_W^3(\mp 1+\cos\theta)\,x$
$-+00$	$2\sqrt{2}G_F(m_Z^2 - m_W^2)\sin\theta$	-
$-+\pm\pm$	-	$6\cdot 2^{1/4}\sqrt{G_F}m_W(m_Z^2 - m_W^2)\sin\theta$

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$+-+-$	$-\frac{1}{\sqrt{2}}G_F m_W^2 \sin^3\theta \csc^4(\theta/2)$	$\longrightarrow$	-
$+-\pm\pm$	-	$\longrightarrow$	$3\cdot 2^{1/4}\sqrt{G_F}m_W \sin\theta\,(4m_W^2 x^2 - m_Z^2)$
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$-+00$	$2\sqrt{2}G_F(m_Z^2 - m_W^2)\sin\theta$	$\longrightarrow$	-
$-+\pm\pm$	-	$\longrightarrow$	$6\cdot 2^{1/4}\sqrt{G_F}m_W(m_Z^2 - m_W^2)\sin\theta$

Cross section

$$\tilde{A}(\mathcal{O}_W) \sim 0$$

New physics

$(\lambda_1\lambda_2\|\alpha\,\beta)$	SM	EFT Λ^{-2} : c_{WWW}
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$$\rho = \begin{bmatrix} \mathcal{M}_{++}\mathcal{M}^*_{++} & \mathcal{M}_{++}\mathcal{M}^*_{+-} & \cdots \\ \mathcal{M}_{+-}\mathcal{M}^*_{++} & \mathcal{M}_{+-}\mathcal{M}^*_{+-} & \cdots \\ \vdots & \ddots & \end{bmatrix}$$

New physics

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New physics

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$$\tilde{a}_1(\mathcal{O}_W) \simeq \tilde{b}_1(\mathcal{O}_W) \simeq \bar{c}_W\, 2^{5/4}\, x\ \cos^4(\theta/2)(\cos\theta + 3)\csc\theta\,,$$

Resurrected sensitivity: energy growth!

Conclusions

* Measurement of entanglement between tops is highest energy evidence ever.
* In the SM, specific spin configurations are expected, dictated by interactions.
    - High degree of entanglement present at threshold and high energy (+ Bell violation)
    - Need to design measurements in corners of phase space.
* Quantum observables probe complementary directions to the cross-section: e.g. toponium and resurrect the EFT interference.

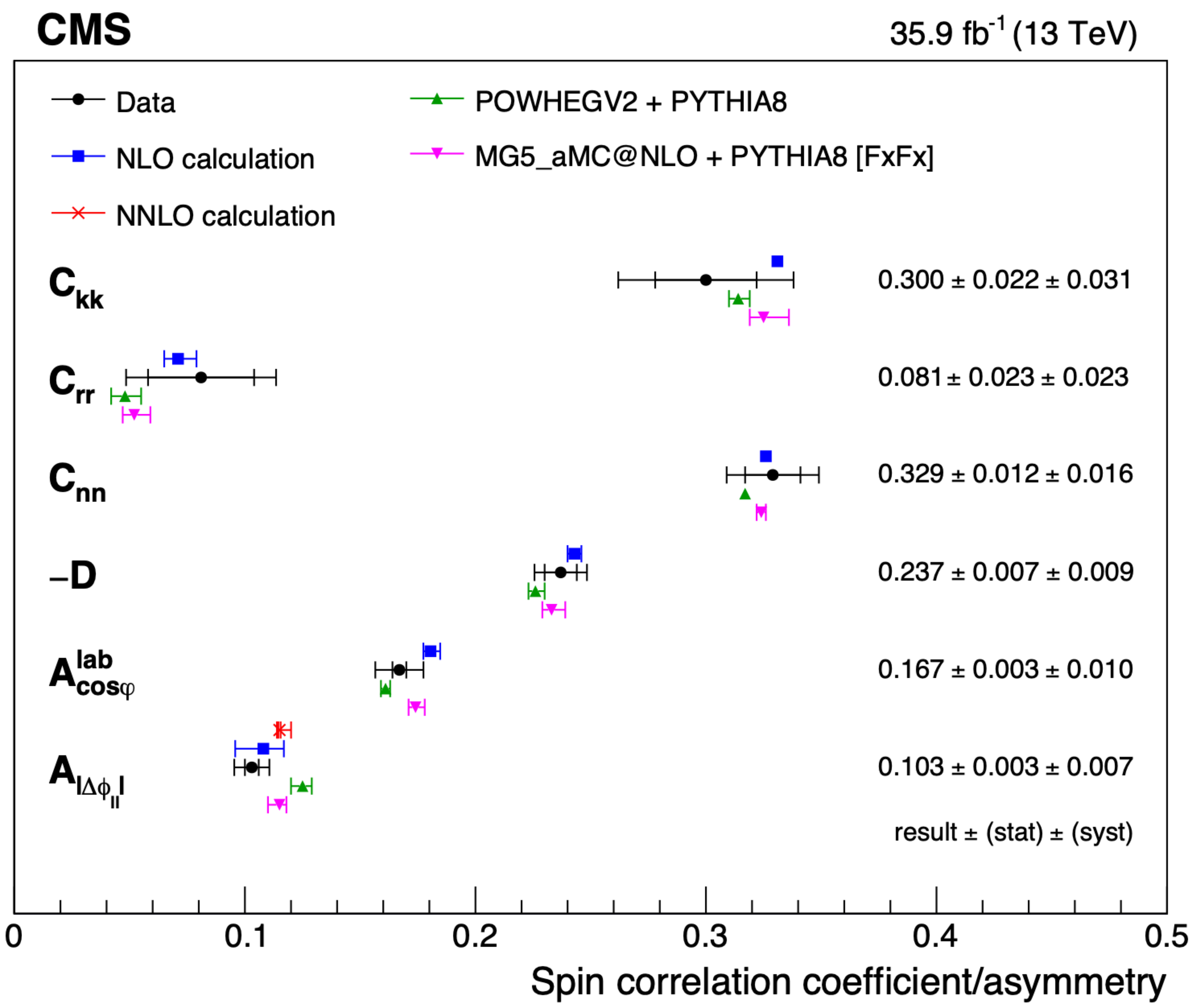

Thank you

Back-up

First workshop gathering the new community

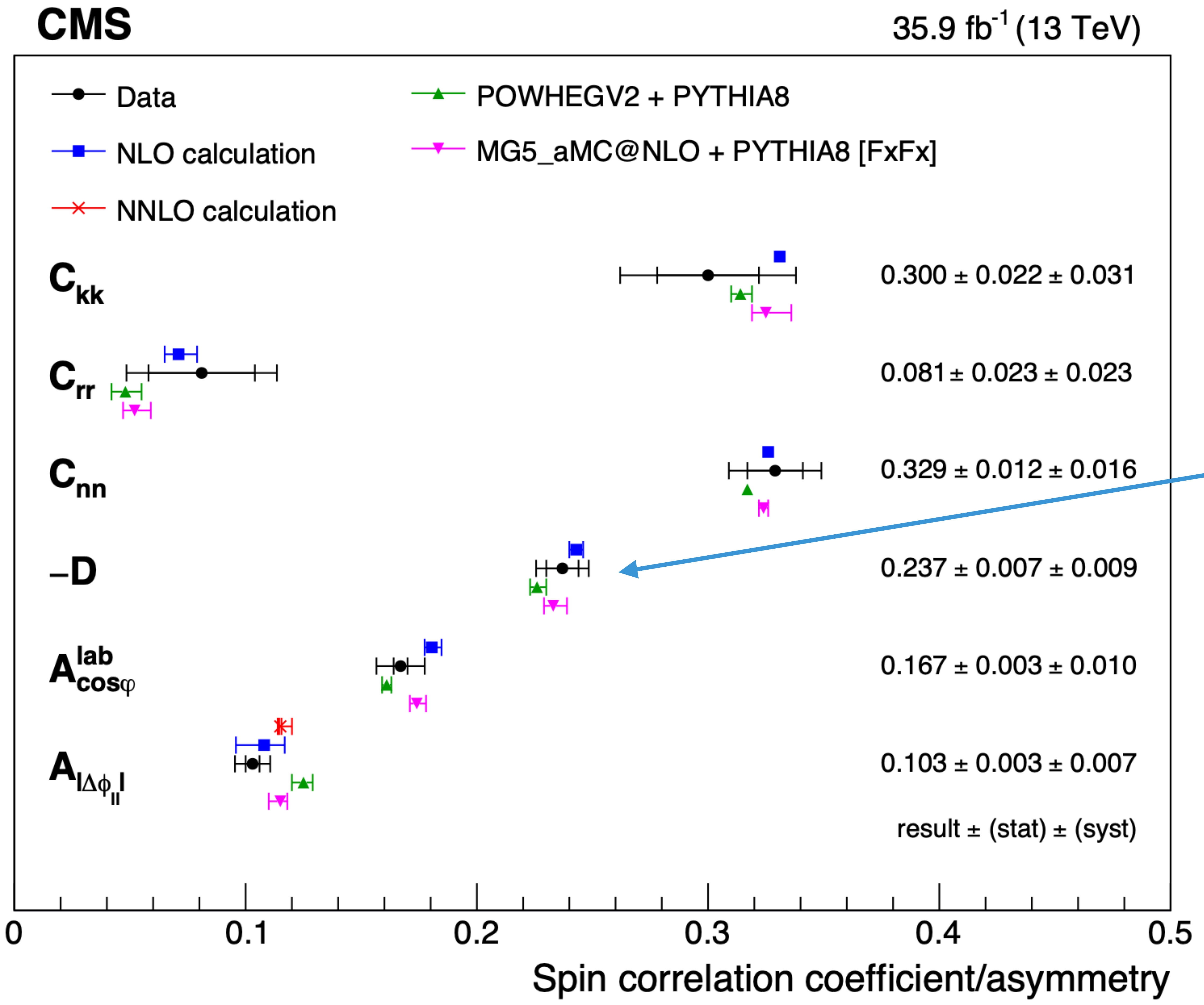
Spin correlation measurement

Inclusive measurement *CMS 1907.03729*



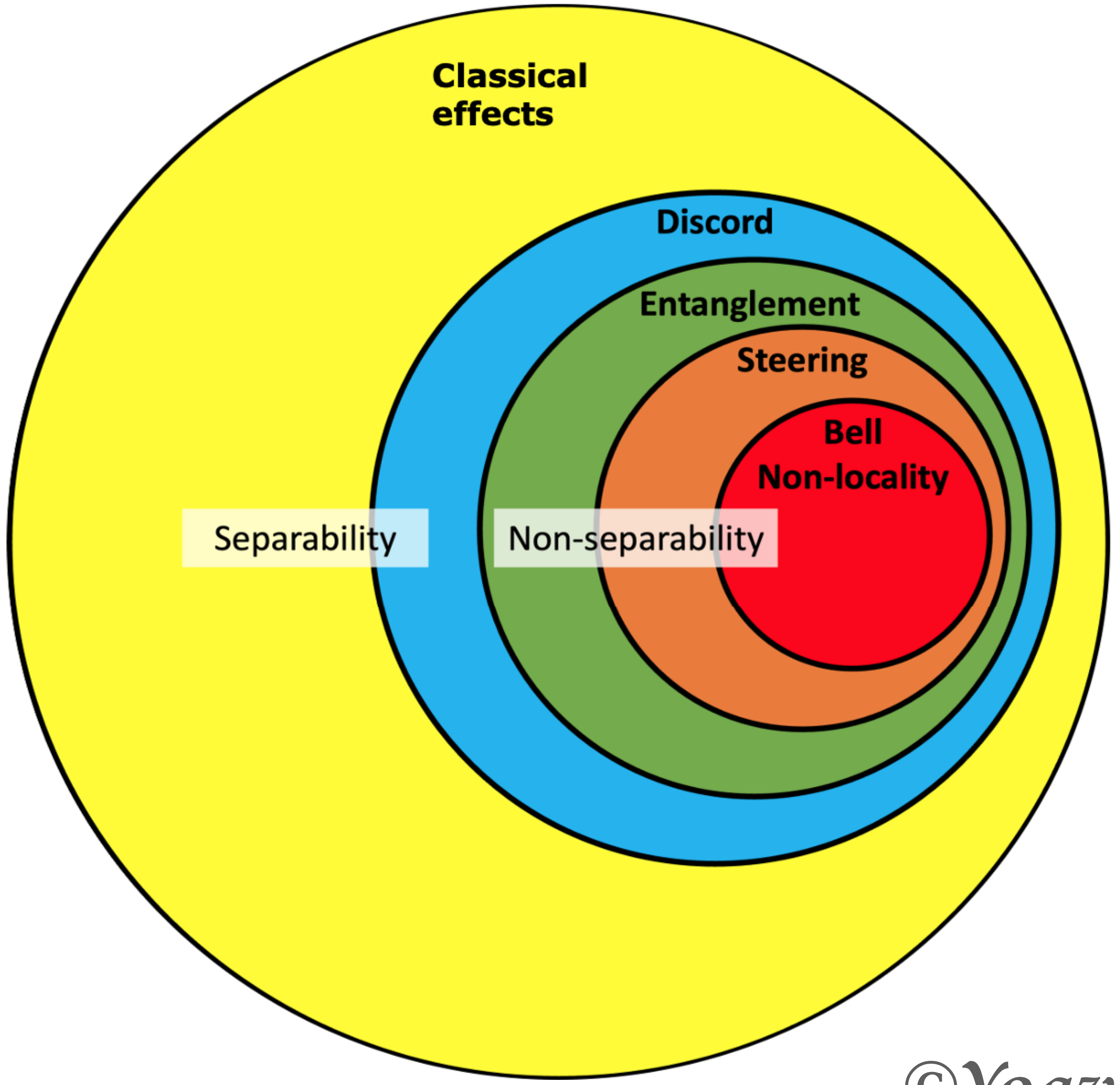
Spin correlation measurement

Inclusive measurement *CMS 1907.03729*



Not enough to see entanglement!

Hierarchy of quantumness



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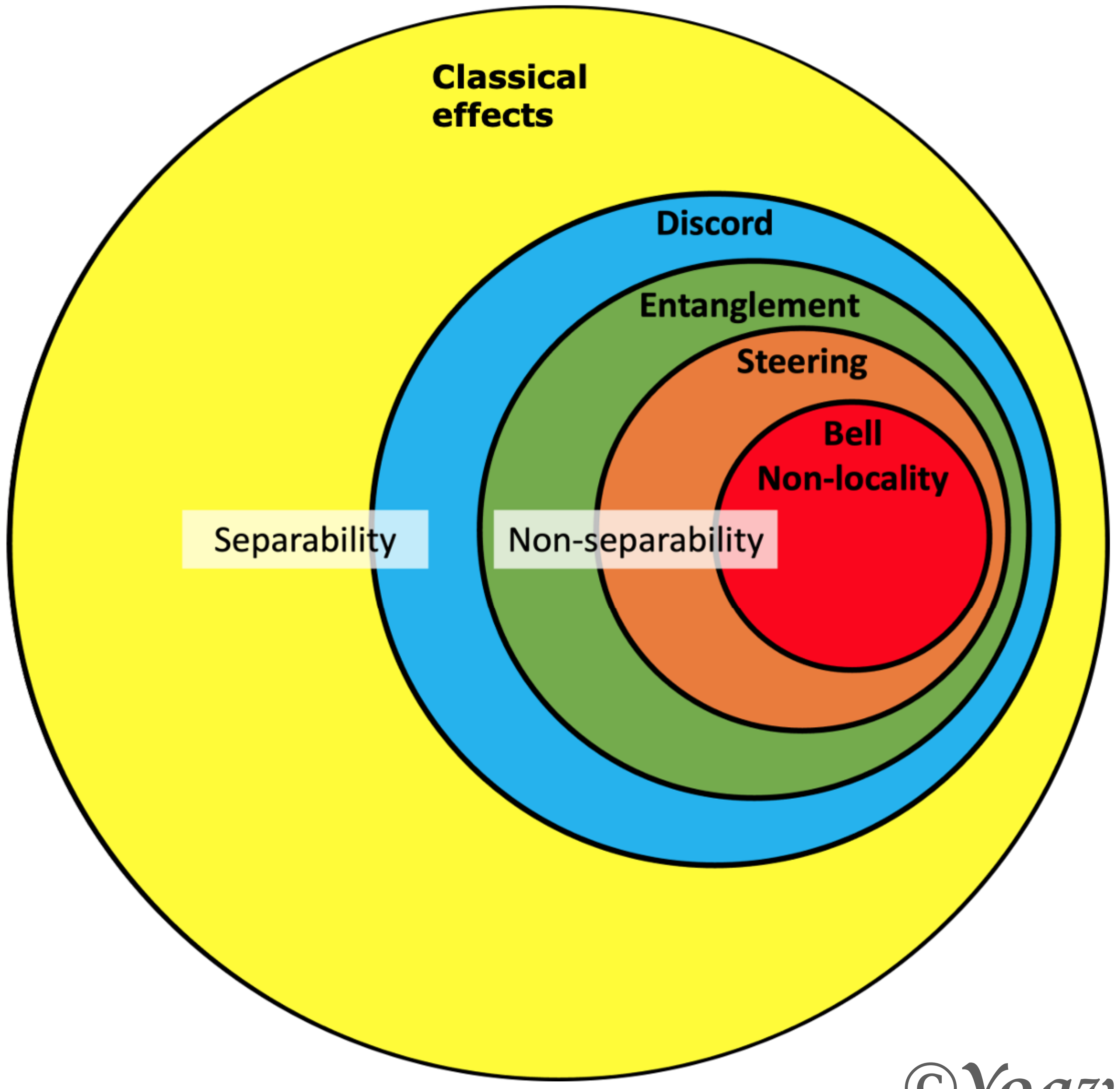
Hierarchy of quantumness



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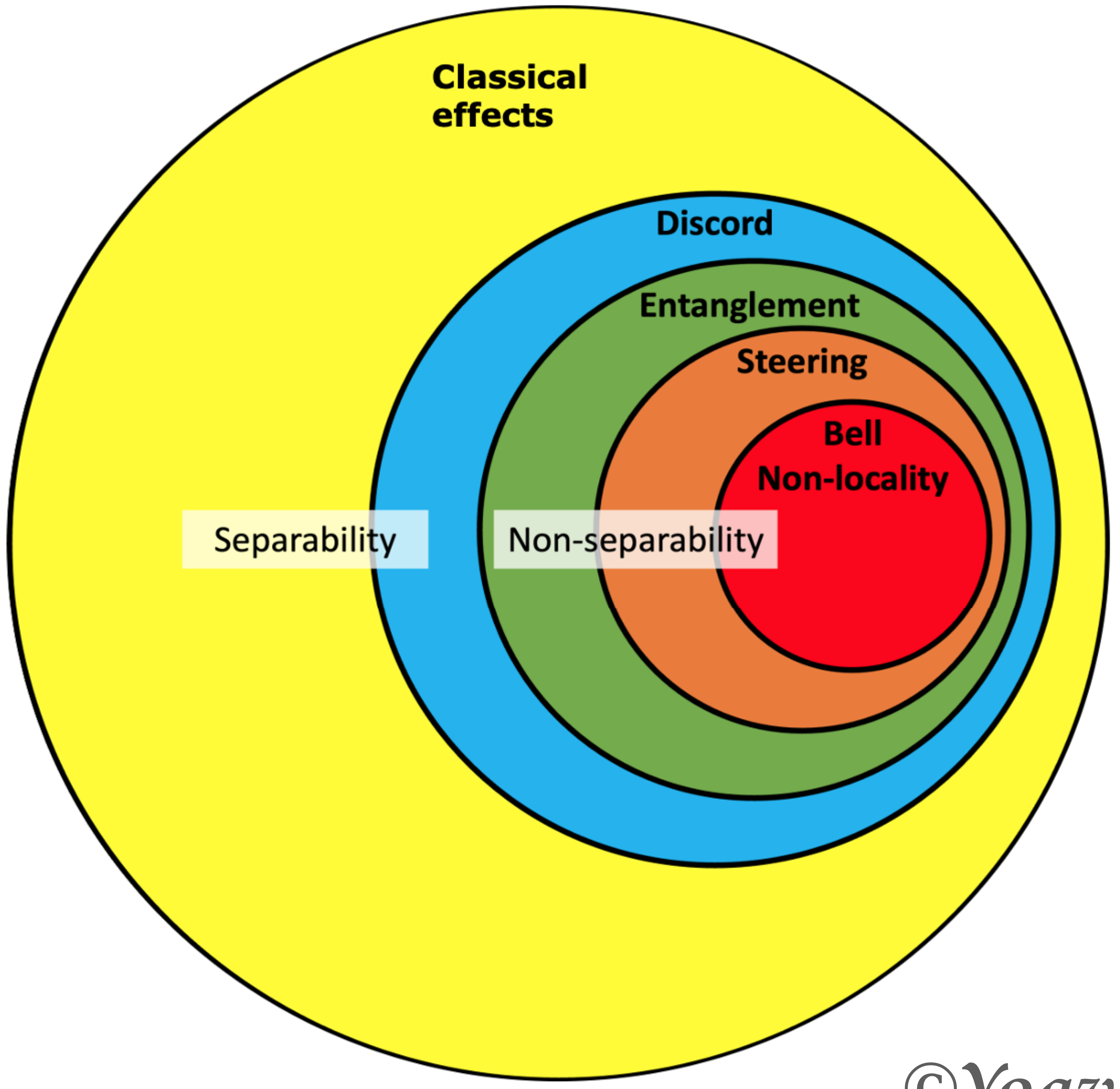
* Quantum discord: shared information.

Hierarchy of quantumness



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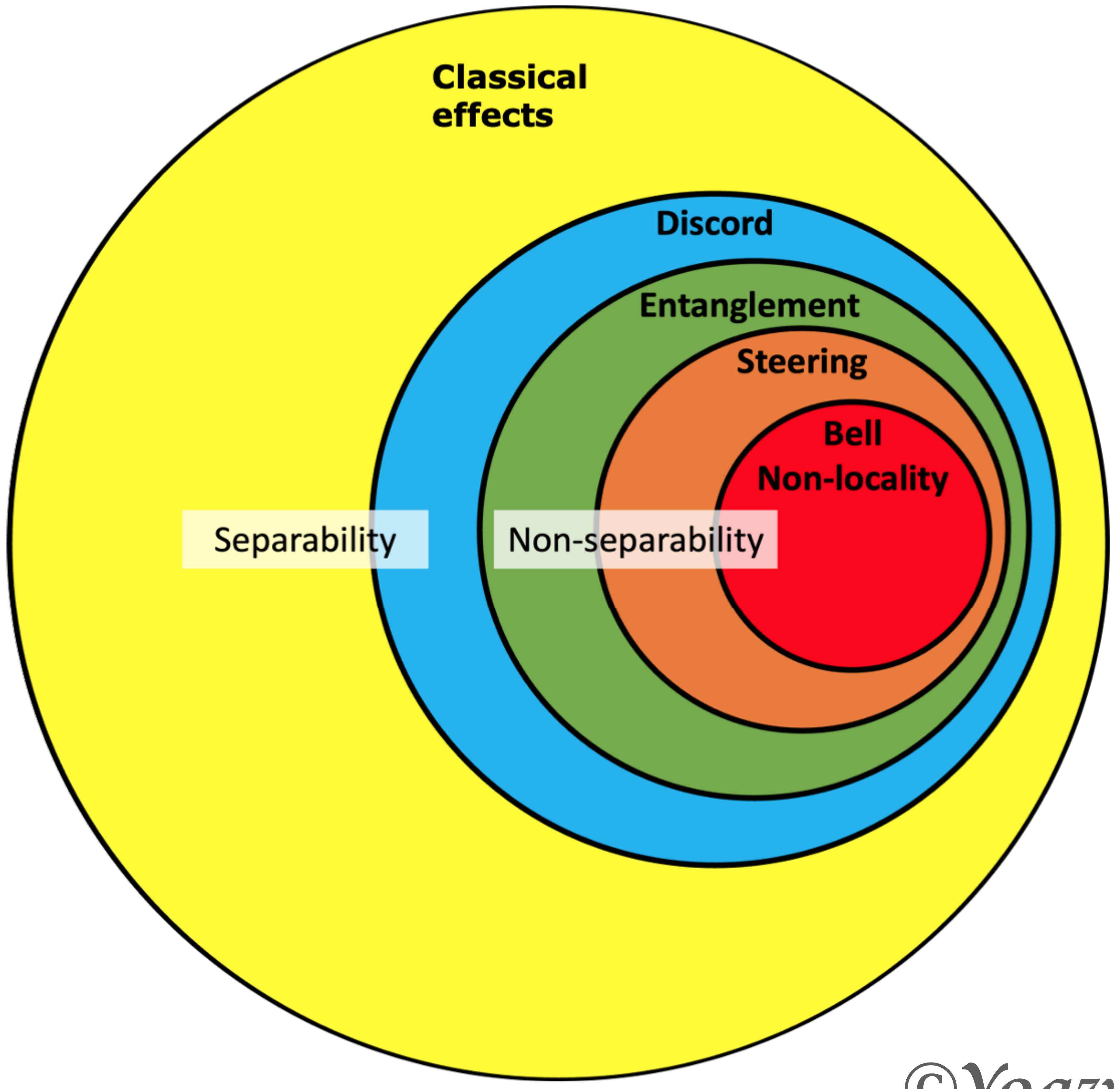
* Quantum discord: shared information.
* Entanglement: non-separability.

Hierarchy of quantumness



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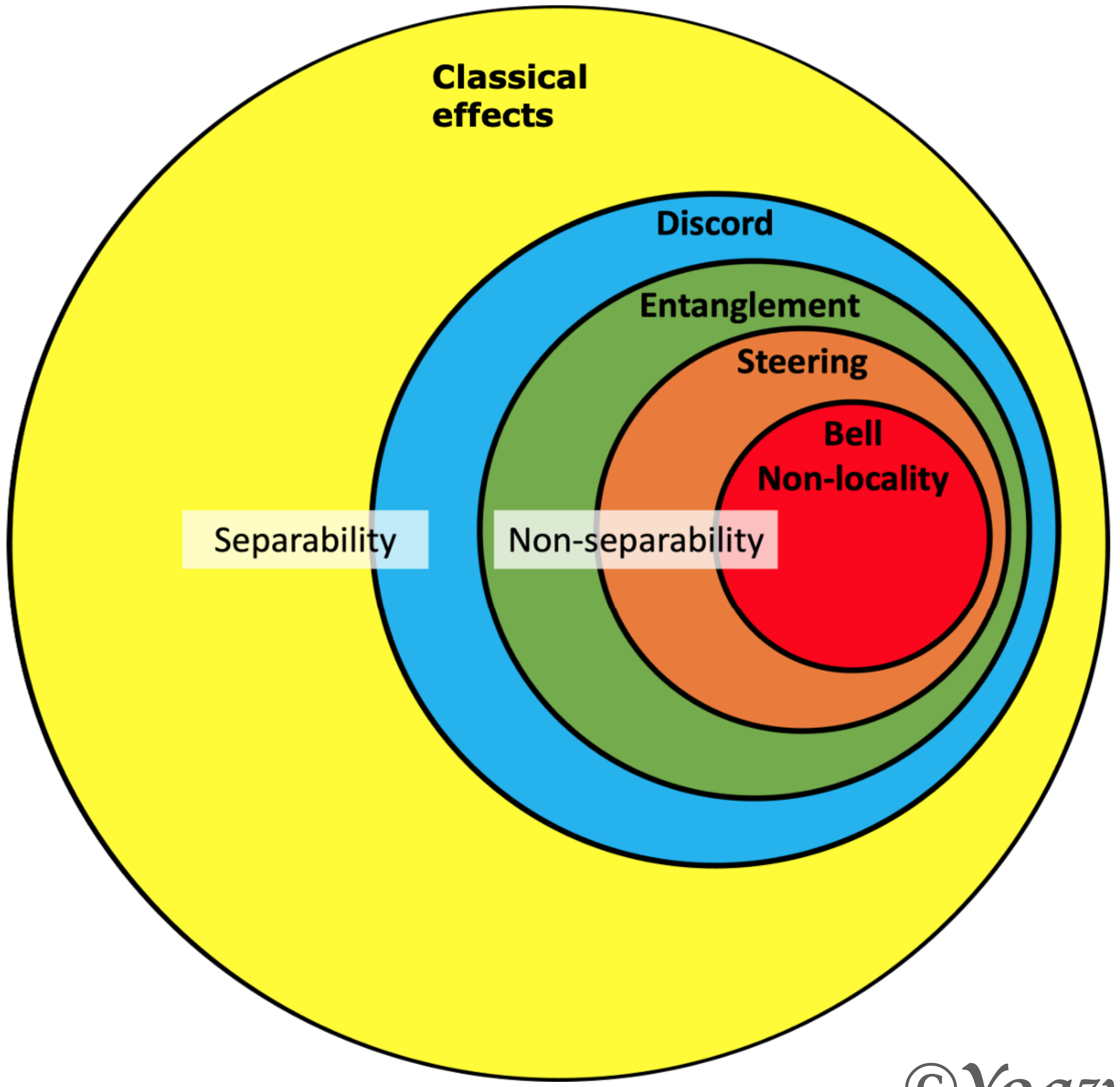
- Quantum discord: shared information.
- Entanglement: non-separability.
- Steering: "spooky action at a distance"

Hierarchy of quantumness



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* Quantum discord: shared information.
* Entanglement: non-separability.
* Steering: “spooky action at a distance”
* Bell non-locality: very strong correlations: Non local.

SMEFT relative effects

$$\Delta = -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$$

$$\Delta_1 \equiv \Delta - \Delta_0$$

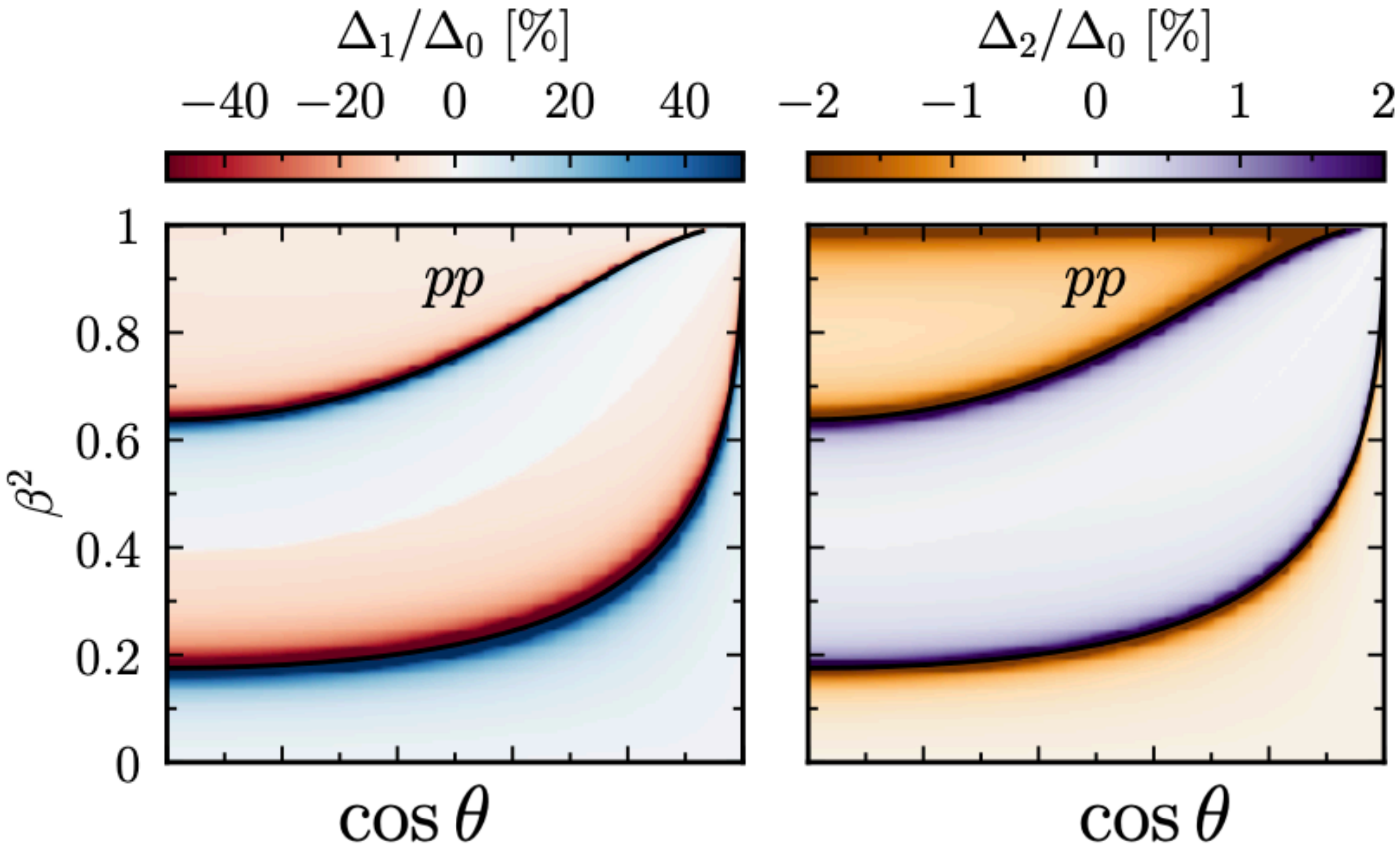
Δ computed up to $\mathcal{O}(1/\Lambda^2)$

$$\Delta_2 \equiv \Delta - \Delta_1 - \Delta_0$$

Δ computed up to $\mathcal{O}(1/\Lambda^4)$

SMEFT relative effects

$$\Delta = -C_{nn} + |C_{kk} + C_{rr}| - 1 > 0$$



$$c_{tG}/\Lambda^2 = 0.1\,\mathrm{TeV}^{-2}$$

$$\Delta_1 \equiv \Delta - \Delta_0$$

Δ computed up to $\mathcal{O}(1/\Lambda^2)$

$$\Delta_2 \equiv \Delta - \Delta_1 - \Delta_0$$

Δ computed up to $\mathcal{O}(1/\Lambda^4)$

Average concurrence

$$\bar{R} = (4\pi)^{-1} \int \mathrm{d}\Omega \, R(\hat{s}, \boldsymbol{k}) \,,$$

→

$$\delta \equiv -C_z + |2C_\perp| - 1 > 0$$

$$C[\rho] = \max(\delta/2, 0)$$

Average concurrence

$$\bar{R} = (4\pi)^{-1} \int \mathrm{d}\Omega \, R(\hat{s}, \boldsymbol{k}) \, ,$$

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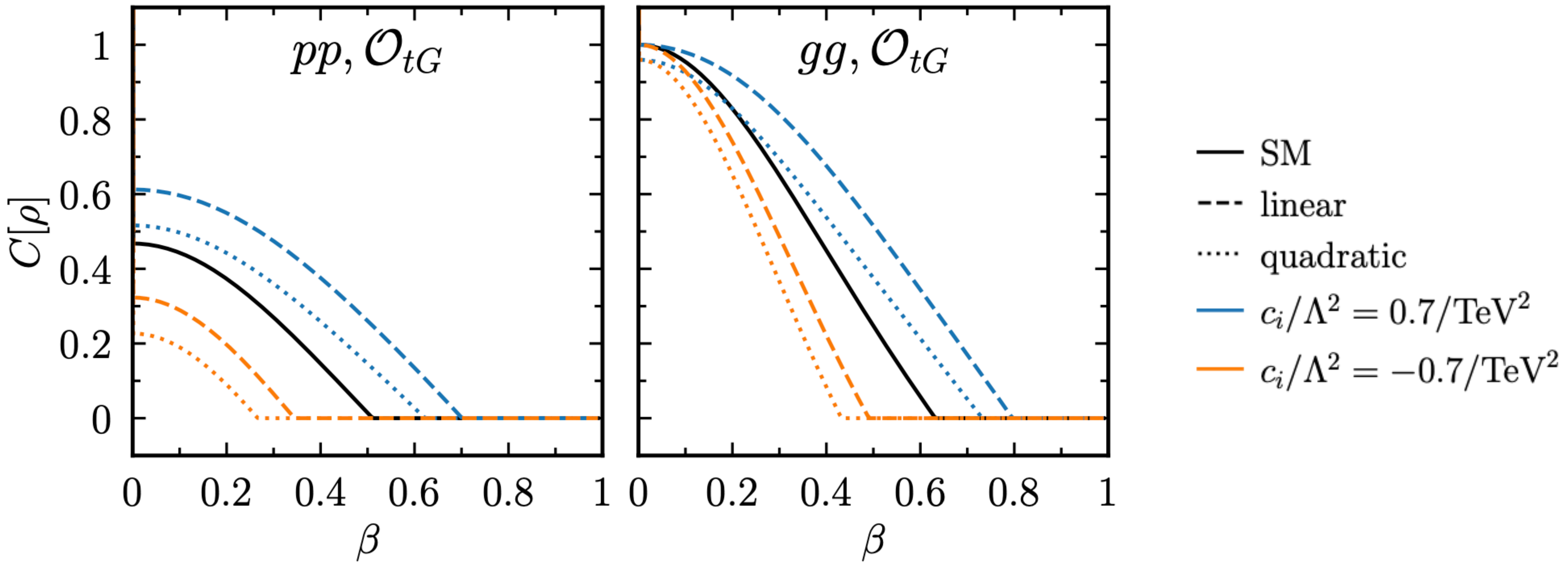

Quantum state in the EFT

gg-induced

$$\rho_{gg}^{\mathrm{EFT}}(0,z) = p_{gg}|\Psi^+\rangle_{\boldsymbol{p}}\langle\Psi^+|_{\boldsymbol{p}} + (1-p_{gg})|\Psi^-\rangle_{\boldsymbol{p}}\langle\Psi^-|_{\boldsymbol{p}}$$

$$p_{gg} = \frac{72}{7\Lambda^4} m_t^2 (3\sqrt{2} m_t\, c_G + v\, c_{tG})^2$$

Only quadratic effects!

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qq-induced

$$\rho_{q\bar{q}}^{\mathrm{EFT}}(0,z) = p_{q\bar{q}}\,|\uparrow\uparrow\rangle_{\boldsymbol{p}}\,\langle\uparrow\uparrow|_{\boldsymbol{p}} + (1-p_{q\bar{q}})\,|\downarrow\downarrow\rangle_{\boldsymbol{p}}\,\langle\downarrow\downarrow|_{\boldsymbol{p}}$$

$$p_{q\bar{q}} = \frac{1}{2} - 4\frac{c_{VA}^{(8),u}}{\Lambda^2} + \frac{8m_t^4}{\Lambda^4}\left(\frac{v\sqrt{2}}{m_t} c_{VA}^{(8),u} c_{tG} - 9 c_{VA}^{(1),u} c_{VV}^{(1),u} + 2 c_{VA}^{(8),u} c_{VV}^{(8),u}\right)$$

Quantum state in the EFT

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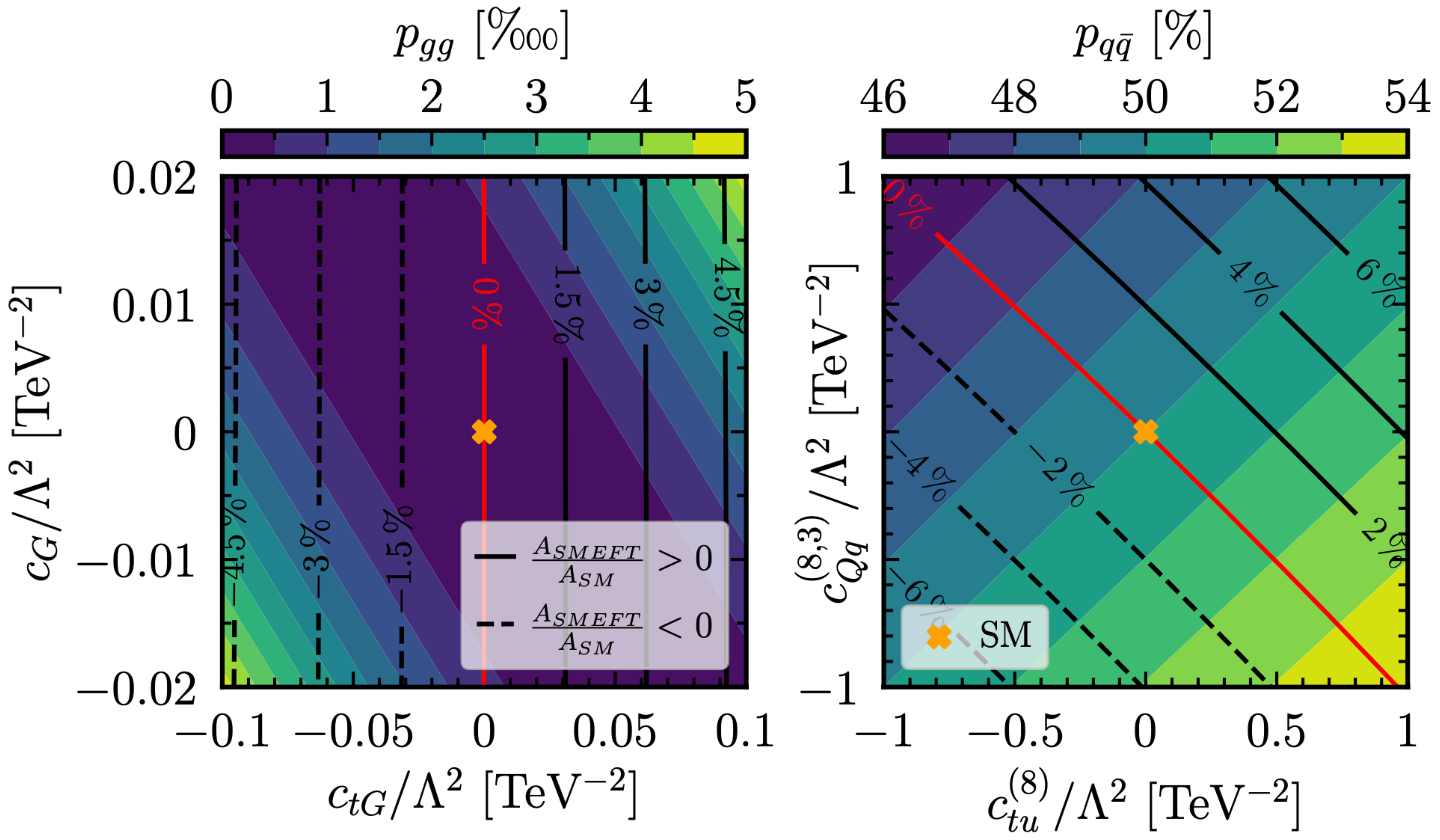

Luca Mantani