User-Friendly Spectral Analysis

Third Step: Likelihood

Antoine Beauchêne - *November 14th, 2023*

LLR meeting

Steps

1. PDFs creation
    a. Background PDFs
    b. PDF for each DSNB model
2. Likelihood calculation → In progress
3. Systematics calculation → In progress
4. Fit → In progress
5. Phase combination → In progress

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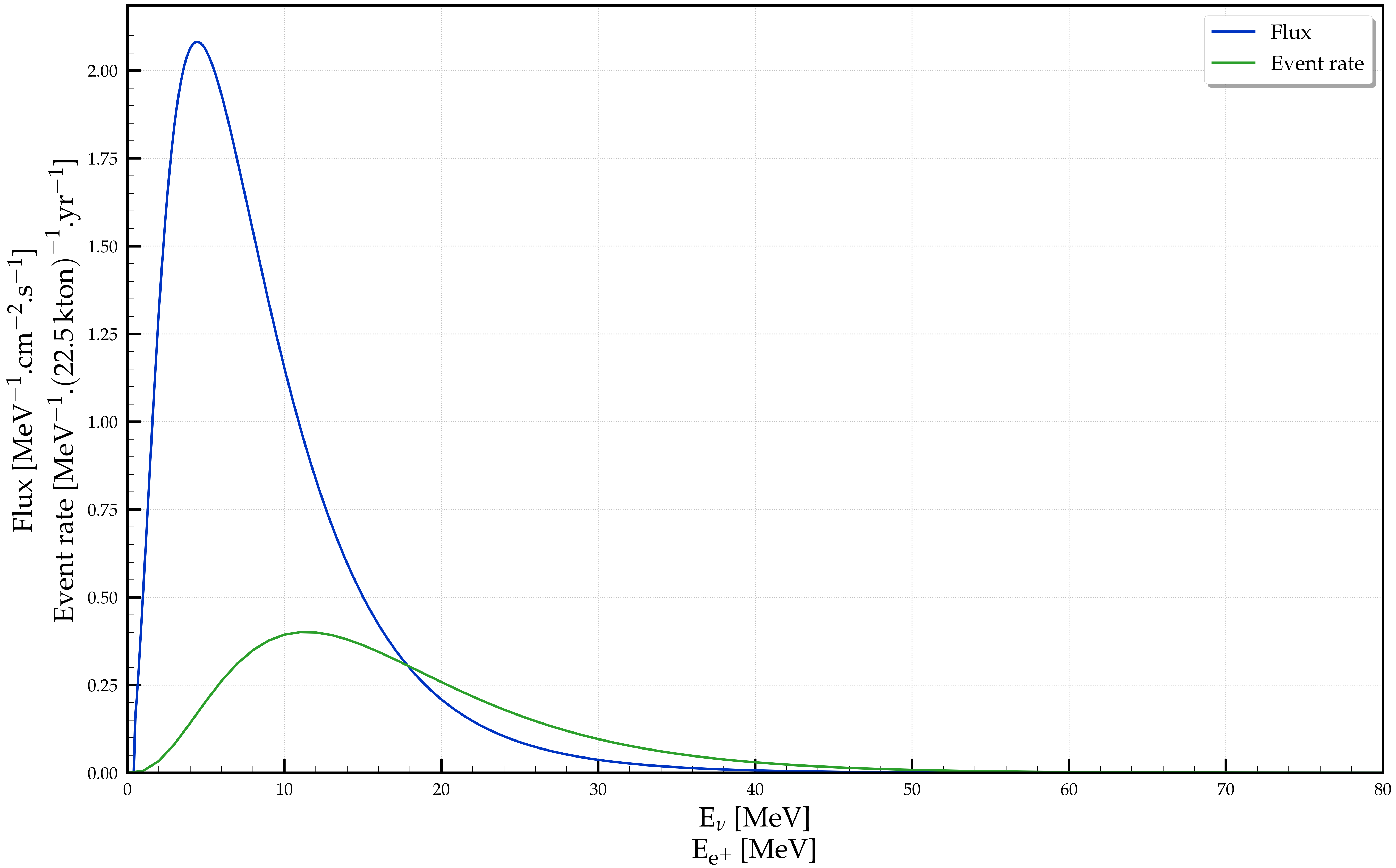
Flux & Event rate of a DSNB model

- For example: Model of S. Horiuchi (2009)

1. Extract the flux of the considered model
2. Compute the event rate:

$$\frac{dN_{\text{events}}(E_\nu)}{dE_\nu} = N_{\text{targets}}\, \Delta t_{1\text{ year}}\, \Phi(E_\nu)\, \sigma_{\text{IBD}}(E_\nu)$$

G. Ricciardi, N. Vignaroli and F. Vissani [arXiv:2206.05567v2 [hep-ph]]

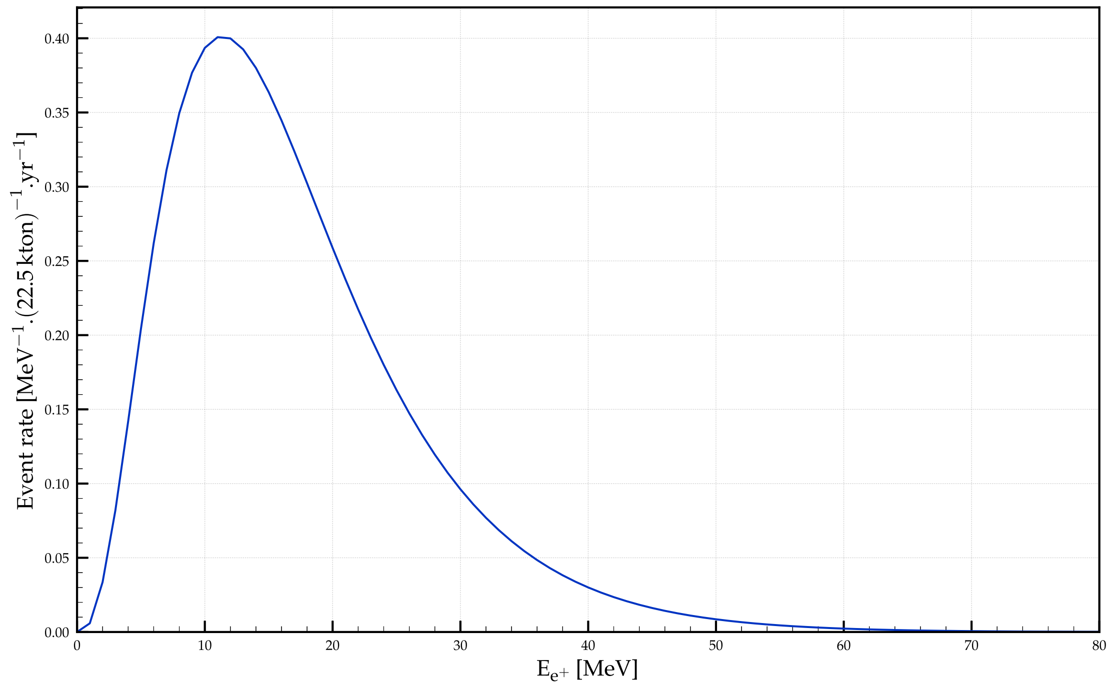


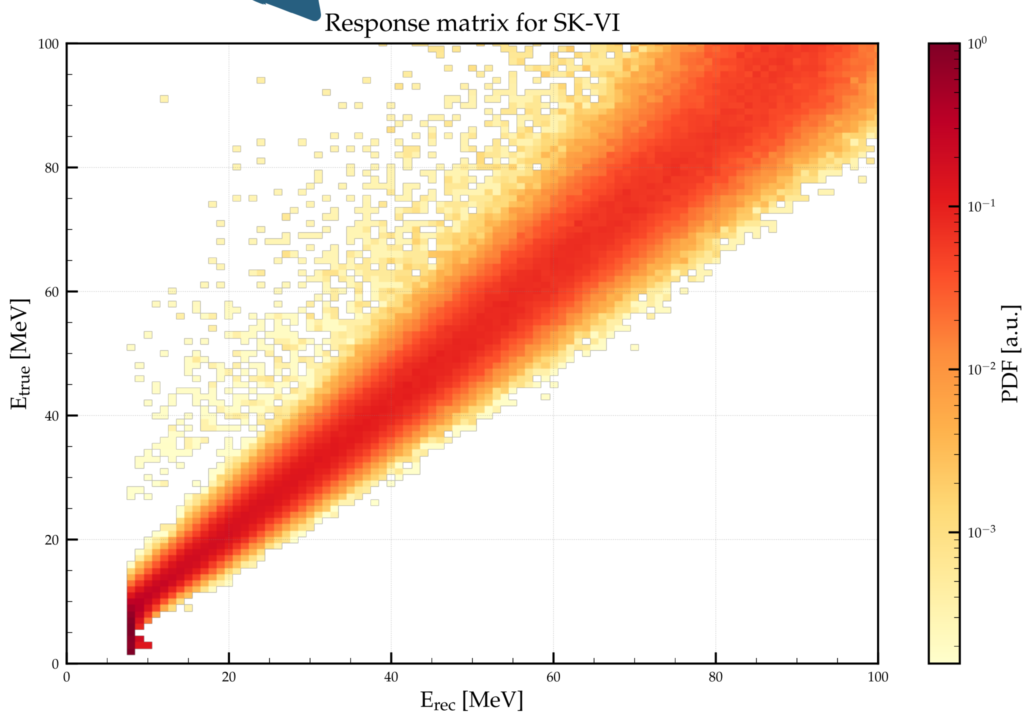
Spectral Analysis

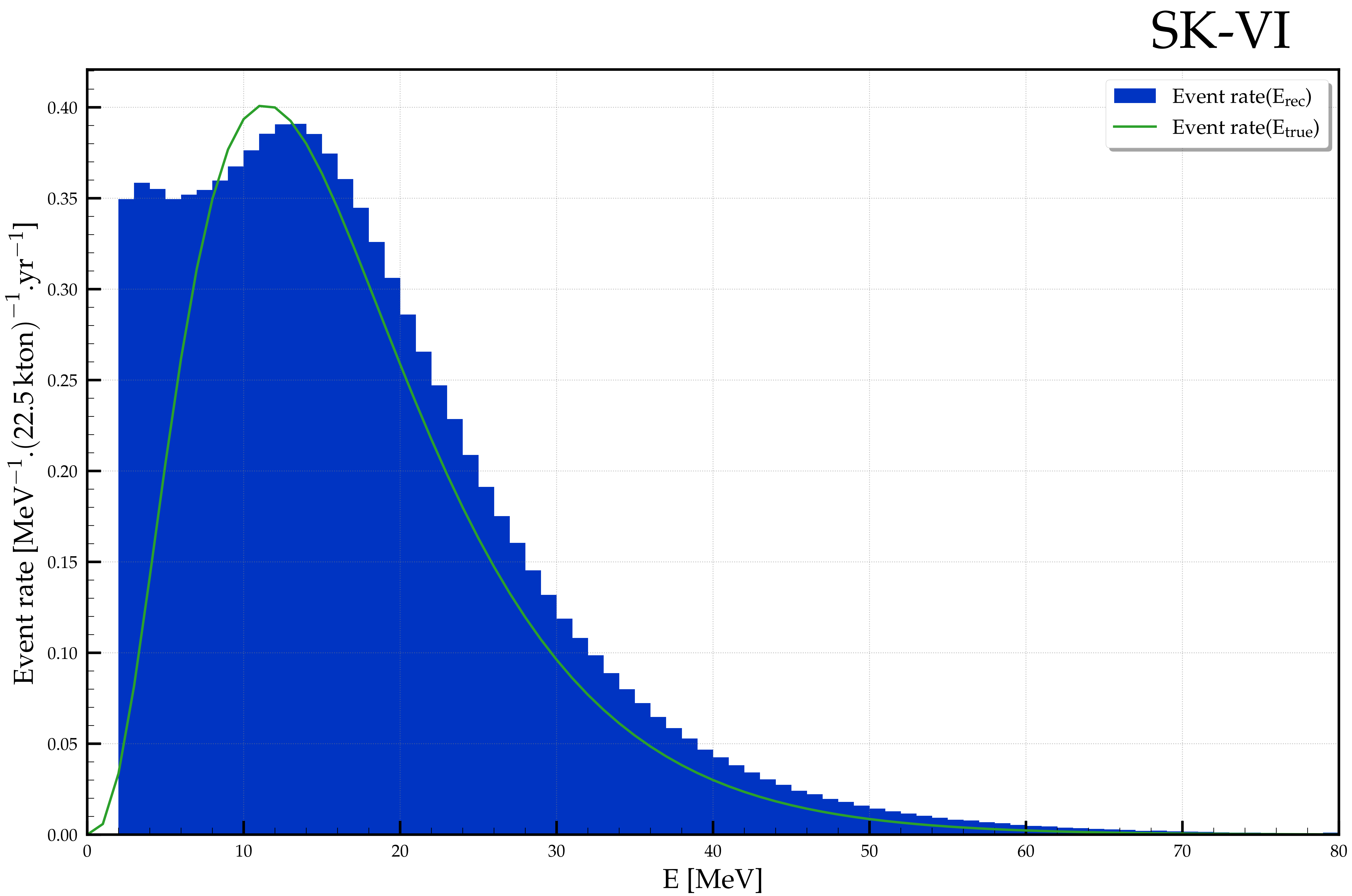
Reconstructed flux

- We have the response matrix $R_{E_{\text{true}} \to E_{\text{rec}}}$ (to convert E_{true} into E_{rec})
- Now we can create the signal PDF:

$$\Phi_{E_{\text{rec}}} = \Phi_{E_{\text{true}}} \times R_{E_{\text{true}} \to E_{\text{rec}}}$$

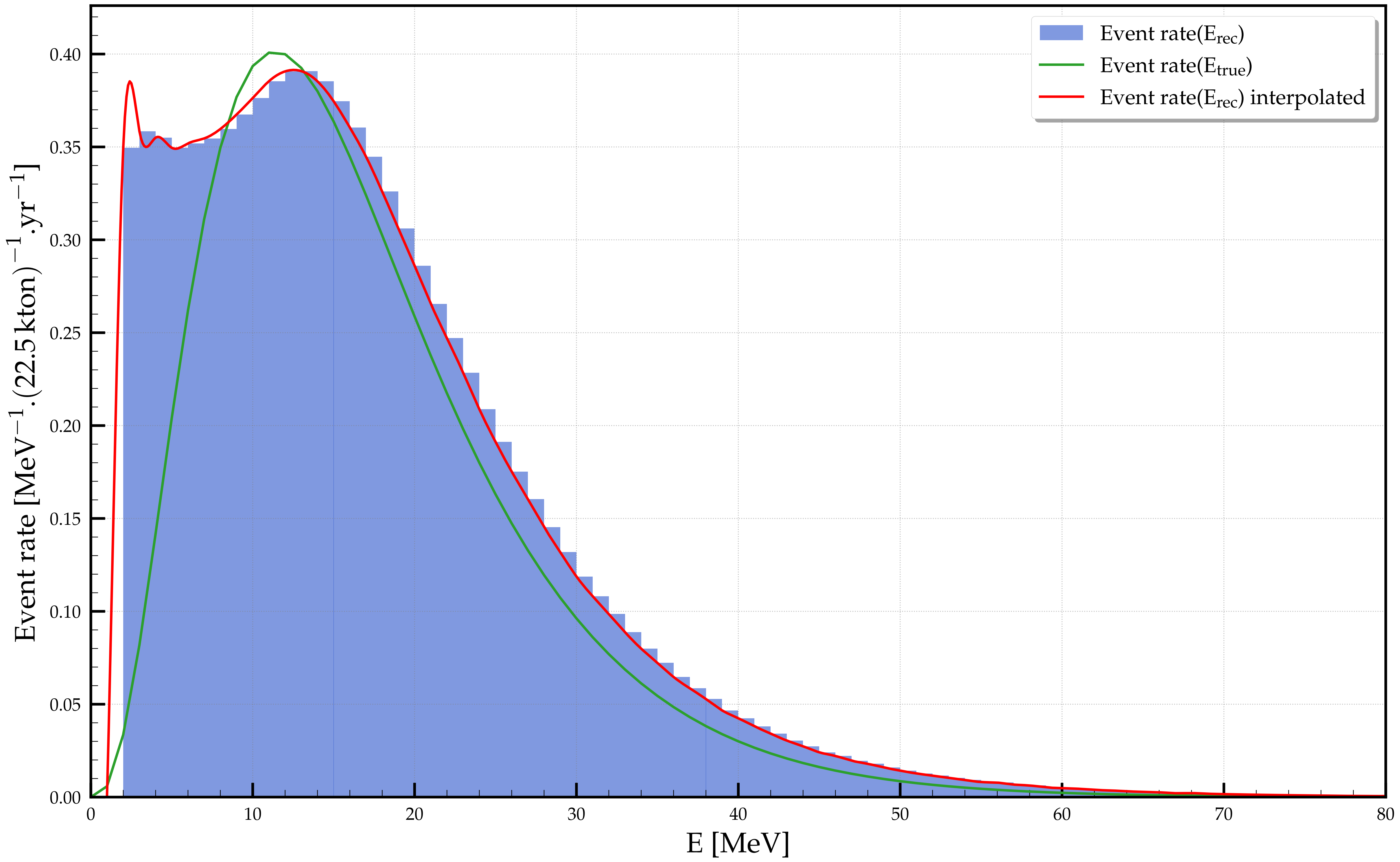


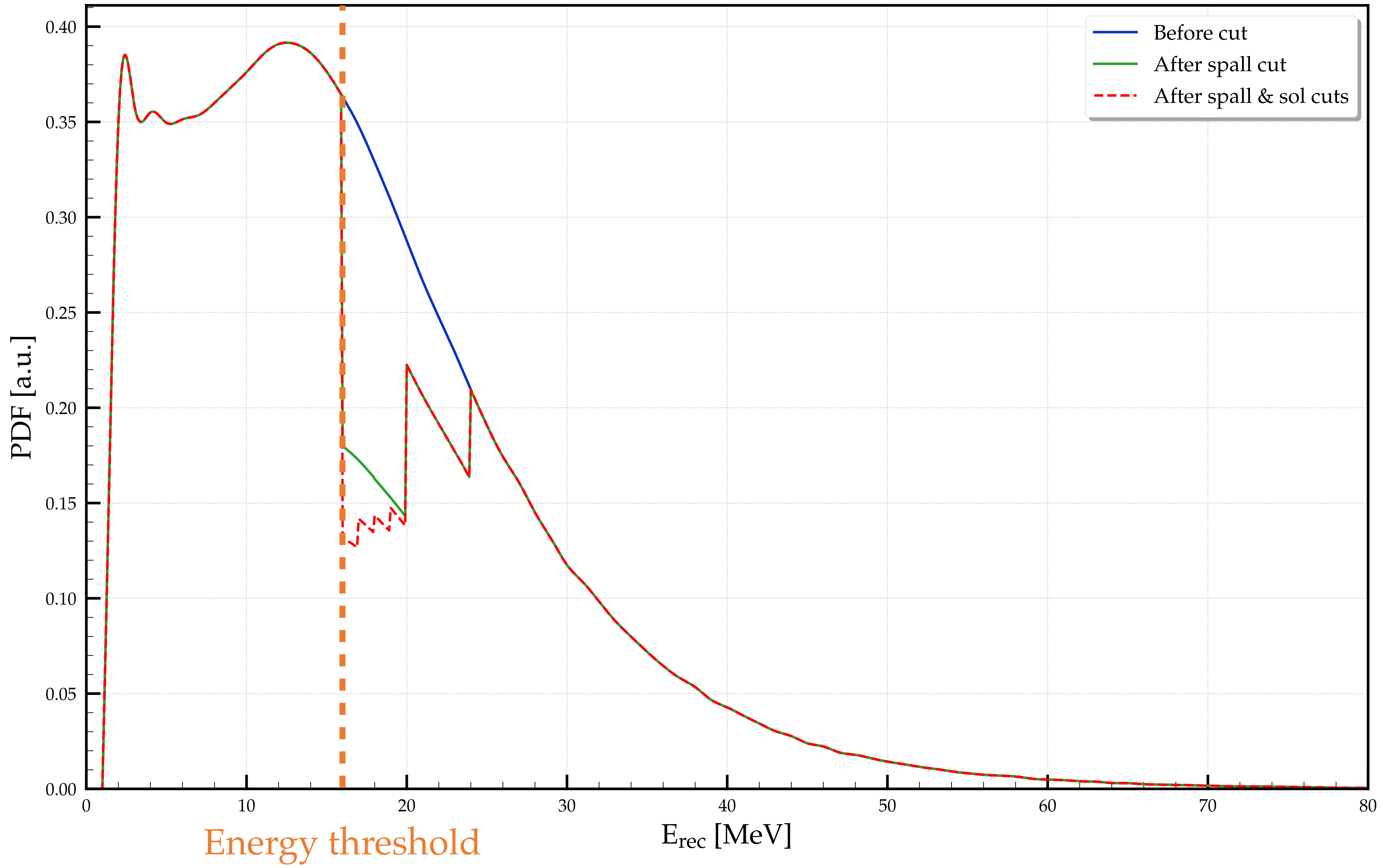




B-spline interpolation & cuts

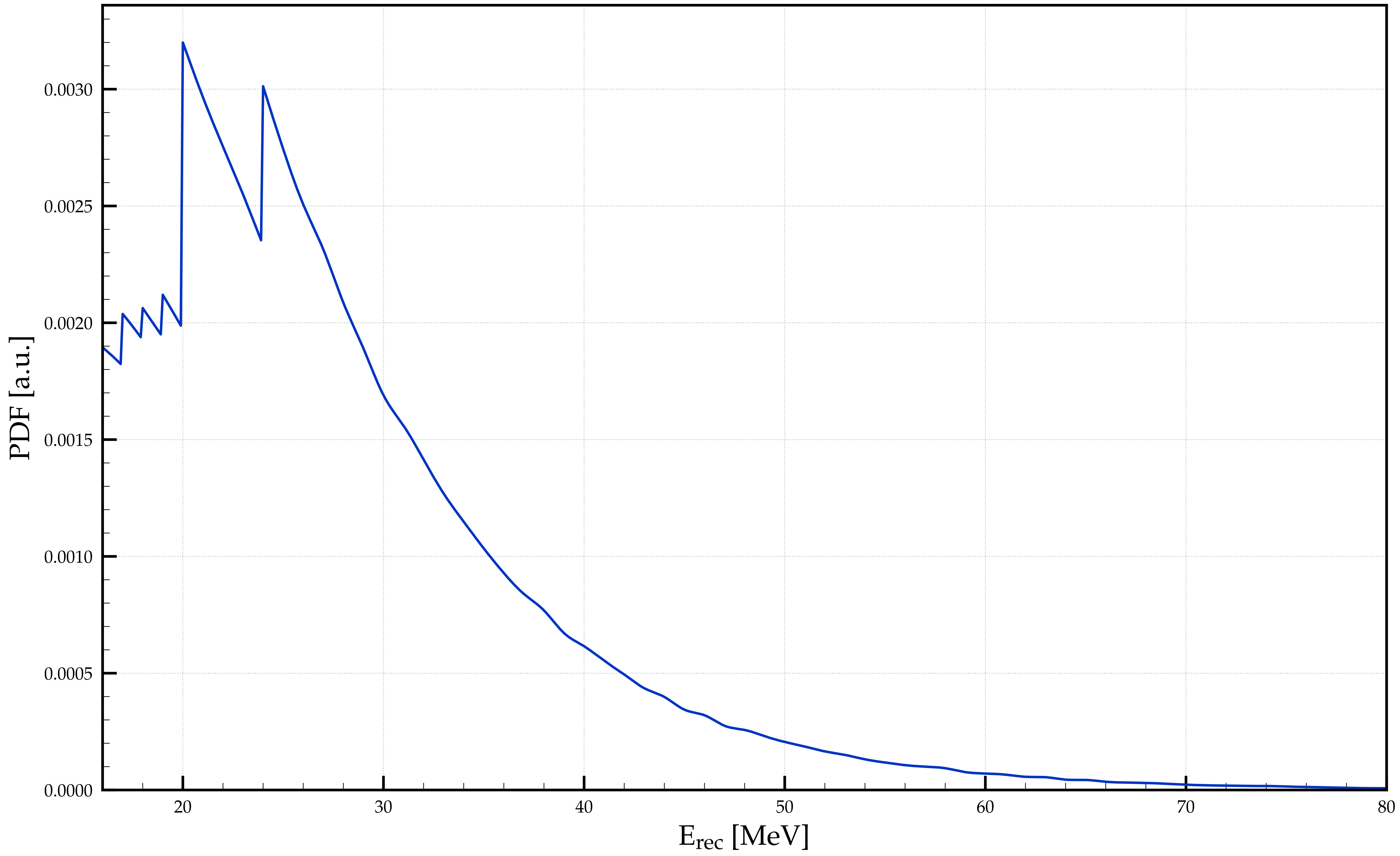
- Method to create smooth curves from control points using special functions called **B-splines**





Normalize PDFs

- Energy threshold (17.5 MeV for SK-II, 16 MeV otherwise) + *l1* normalization



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Likelihood without systematics

Simultaneously on the 6 regions

- We fit the number of observed events N_j that maximizes the following likelihood:

$$\mathscr{L}(\vec{E}\,|\,N_s, \vec{N}_b) = e^{-\sum_{j\in s+b} N_j} \prod_{i=1}^{N_{\text{data}}} \sum_{j\in s+b} N_j \,\text{PDF}_j\,(E^i\,|\,\theta_C^i, N^i_{\text{tagged } n})$$

- Minimization of:

$$-2\ln\left(\mathscr{L}(\vec{E}\,|\,N_s, \vec{N}_b)\right) = 2\cdot\left(\sum_{j\in s+b} N_j - \sum_{i=1}^{N_{\text{data}}} \ln\left(\sum_{j\in s+b} N_j \,\text{PDF}_j\,(E^i\,|\,\theta_C^i, N^i_{\text{tagged } n})\right)\right)$$

Minimization methods (tested with SK-VI)

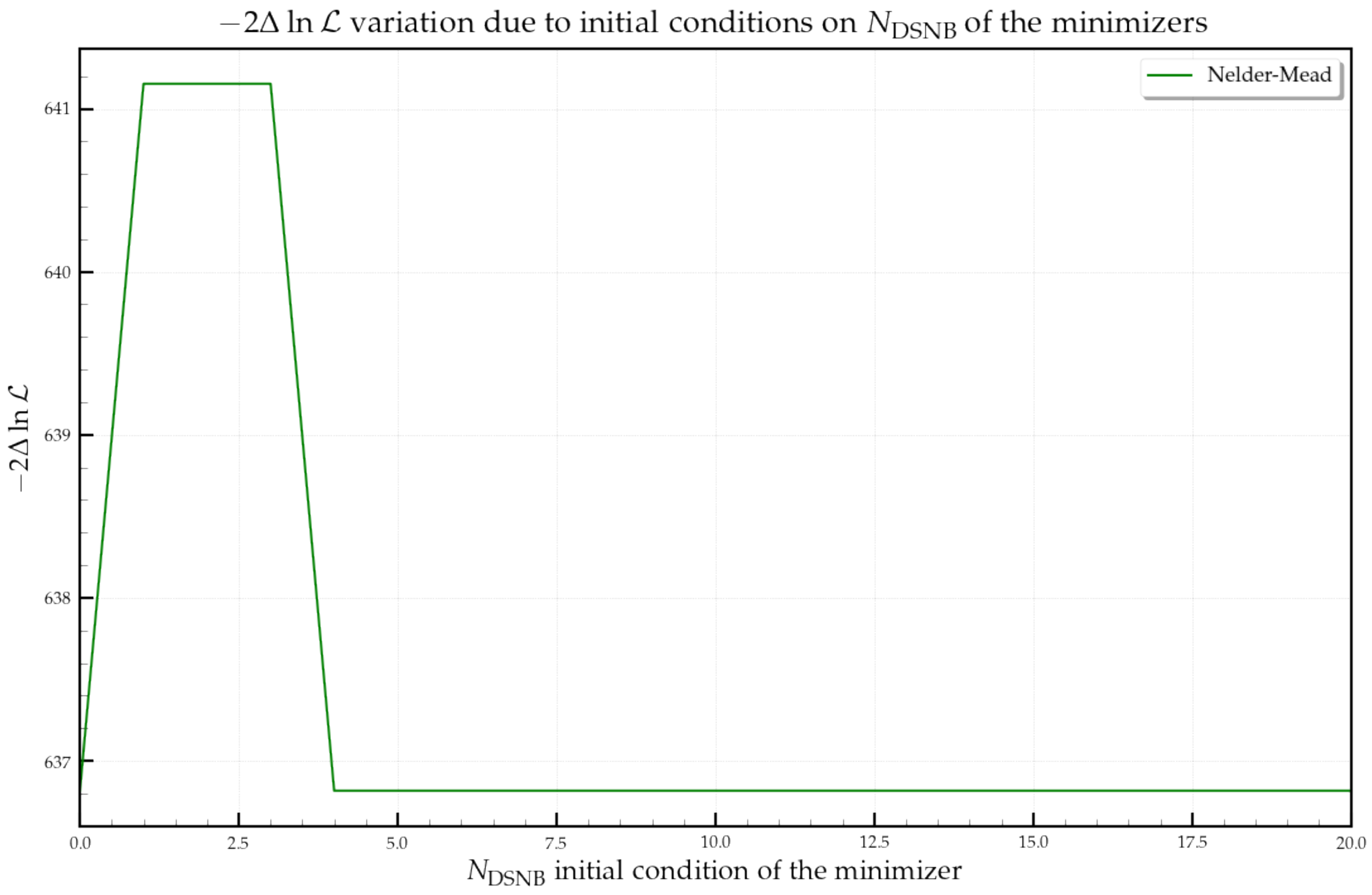
Method	Duration (s)	$-2\ln(\mathscr{L})$	N_{DSNB}	$N_{\nu_e\,\text{CC}}$	$N_{\text{Decay}\,e^-}$	N_{NCQE}	$N_{\mu/\pi}$	$N_{\text{Spallation}}$
Nelder-Mead	36.46	636.82	11.37	17.84	28.07	13.84	3.43	4.45
Powell	21.68	636.82	11.32	17.86	28.08	13.84	3.43	4.46
CG	30.91	636.82	11.37	17.84	28.07	13.84	3.43	4.45
BFGS	13.85	636.82	11.37 ± 3.94	17.84 ± 2.33	28.07 ± 3.90	13.84 ± 2.91	3.43 ± 1.69	4.45 ± 1.61
Newton-CG	80.12	636.82	11.15	17.93	27.96	13.73	3.44	4.40
L-BFGS-B	6.54	636.82	11.37 ± 4.18	17.84 ± 5.43	28.07 ± 6.09	13.84 ± 1.35	3.43 ± 1.42	4.45 ± 1.74
TNC	44.03	636.82	11.44	17.86	27.89	13.74	3.46	4.45
COBYLA	22.55	636.82	11.37	17.84	28.07	13.84	3.43	4.45
SLSQP	11.08	636.82	11.37	17.84	28.07	13.84	3.43	4.45
trust-constr	40.33	636.82	11.37	17.84	28.07	13.84	3.43	4.45
dogleg	364.26	636.82	11.36	17.84	28.06	13.84	3.43	4.45
trust-ncg	28.53	636.82	11.36	17.84	28.06	13.84	3.43	4.45
trust-exact	364.98	636.82	11.36	17.84	28.06	13.84	3.43	4.45
trust-krylov	15.33	636.82	11.36	17.84	28.07	13.84	3.43	4.45

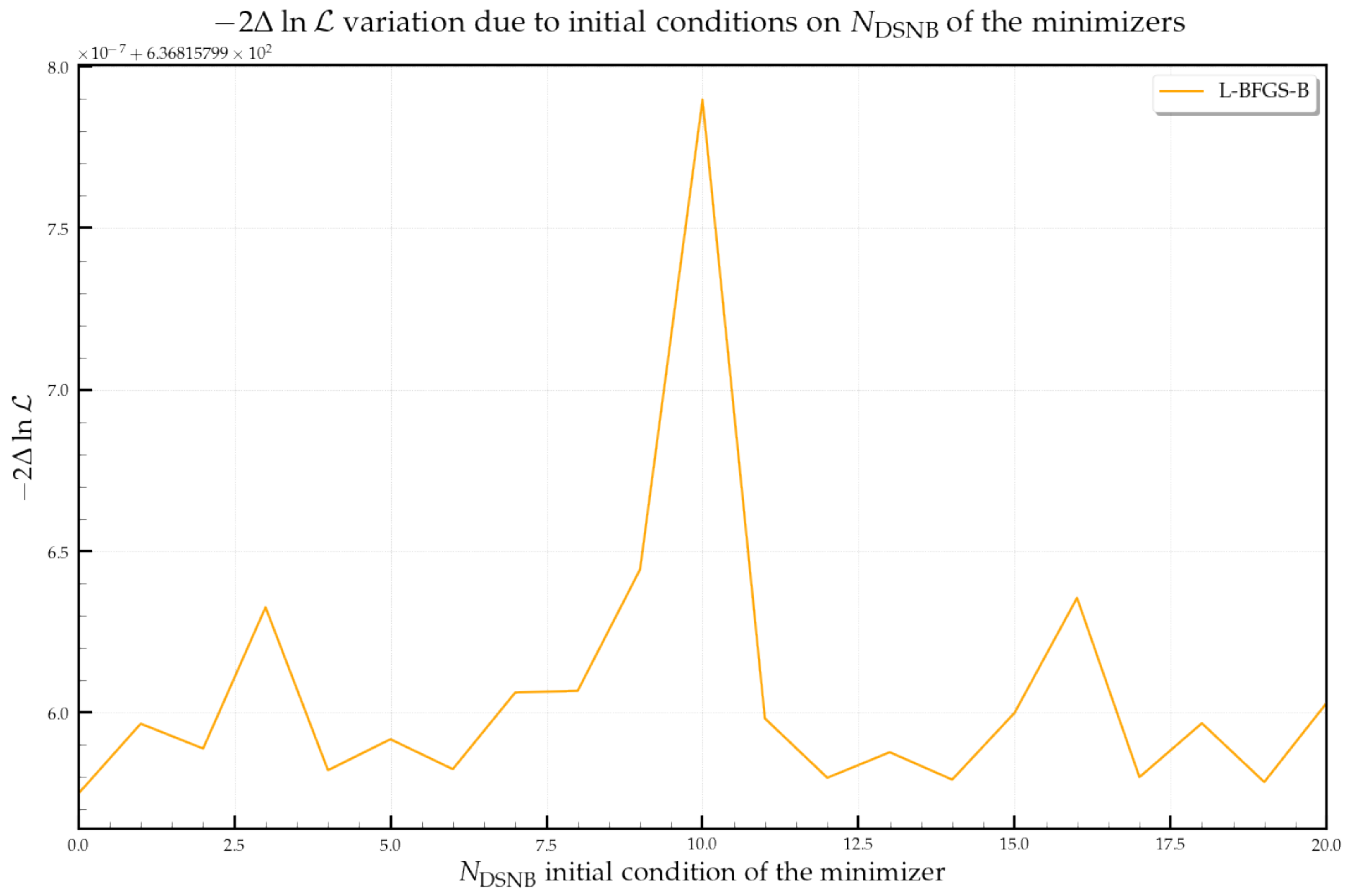
Comparison (on SK-VI)

	Nelder-Mead	BFGS	L-BFGS-B
Number of evaluations of the objective functions	629	238	119
Number of iterations	398	32	15
Number of evaluations of the Jacobian of the	None	34	17
Sensitivity to initial conditions	**Sensitive**	**Not sensitive**	**Not sensitive**
Errors	**Not provided**	**Provided**	**Provided**

(Inverse Hessian matrix = covariance matrix)

Comparison (on SK-VI)





Comparison with previous analysis (on SK-VI without systematics)

Analysis	Method	$-2\ln(\mathscr{L})$	N_{DSNB}	$N_{\nu_e\text{CC}}$	$N_{\text{Decay }e^-}$	N_{NCQE}	$N_{\mu/\pi}$	$N_{\text{Spallation}}$	$\sum N_j$
Old	Nelder-Mead	290.34	7.35	18.58	31.32	13.57	3.49	4.70	79.01
New	Nelder-Mead	636.82	11.37	17.84	28.07	13.84	3.43	4.45	79.00
New	BFGS	636.82	11.37 ± 3.94	17.84 ± 2.33	28.07 ± 3.90	13.84 ± 2.91	3.43 ± 1.69	4.45 ± 1.61	79.00 ± 7.08
New	L-BFGS-B	636.82	11.37 ± 4.18	17.84 ± 5.43	28.07 ± 6.09	13.84 ± 1.35	3.43 ± 1.42	4.45 ± 1.74	79.00 ± 9.54

SK-VI parameter estimations

7.35	18.58	31.32	13.57	3.49	4.70

+ New → $-2\ln(\mathscr{L}) = 637.36$

Comparison with previous analysis (on SK-I, II, III, IV & VI without systematics)

Analysis	Method	Duration
Old	Nelder-Mead	13min 31.08s
New	Nelder-Mead	8min 40.08s
New	BFGS	7min 43.73s
New	L-BFGS-B	1min 50.88s

Spectral Analysis

Results for every phase (without systematics)

Should be equal to N_{data}

SK phase	Method	$-2\ln(\mathscr{L})$	N_{DSNB}	$N_{\nu_e\,\text{CC}}$	$N_{\text{Decay}\,e^-}$	N_{NCQE}	$N_{\mu/\pi}$	$N_{\text{Spallation}}$	$\sum N_j$
I	Nelder-Mead	23.24	0.00	85.63	147.11	47.15	62.56	0.54	343.00
	BFGS	1440.82	-2.07 ± 5.82	70.94 ± 11.73	145.94 ± 4.68	48.37 ± 9.36	54.33 ± 3.31	1.48 ± 1.51	319.00 ± 17.15
	L-BFGS-B	1440.87	0.00 ± 6.80	71.12 ± 7.39	143.99 ± 10.62	48.25 ± 5.66	54.34 ± 3.04	1.30 ± 1.49	319.00 ± 16.04
II	Nelder-Mead	300.39	0.00	43.92	76.44	19.72	42.17	1.75	184.00
	BFGS	992.08	2.65 ± 2.98	33.09 ± 9.84	74.17 ± 10.88	21.04 ± 5.48	30.93 ± 6.33	2.11 ± 1.22	164.00 ± 17.19
	L-BFGS-B	992.08	2.65 ± 6.97	33.09 ± 5.97	74.17 ± 8.23	21.04 ± 3.70	30.93 ± 4.87	2.11 ± 1.58	164.00 ± 13.85
III	Nelder-Mead	284.63	4.76	45.01	50.49	10.75	24.43	1.54	137.00
	BFGS	776.22	5.75 ± 7.29	25.86 ± 4.61	52.94 ± 18.47	11.78 ± 3.25	19.97 ± 4.13	1.70 ± 1.20	118.00 ± 21.09
	L-BFGS-B	776.22	5.75 ± 5.55	25.86 ± 3.63	52.93 ± 6.84	11.78 ± 2.63	19.97 ± 3.50	1.70 ± 1.01	118.00 ± 10.53
IV	Nelder-Mead	124.39	26.21	109.72	158.75	87.57	28.19	5.61	416.05
	BFGS	2032.83	11.54 ± 20.82	106.49 ± 2.47	173.21 ± 14.84	88.37 ± 6.90	28.49 ± 2.72	7.90 ± 2.55	416.00 ± 26.86
	L-BFGS-B	2032.83	11.54 ± 7.58	106.49 ± 9.24	173.20 ± 11.65	88.38 ± 7.74	28.49 ± 4.93	7.90 ± 3.08	416.01 ± 19.29
VI	Nelder-Mead	290.34	7.35	18.58	31.32	13.57	3.49	4.70	79.01
	BFGS	636.82	11.37 ± 3.94	17.84 ± 2.33	28.07 ± 3.90	13.84 ± 2.91	3.43 ± 1.69	4.45 ± 1.61	79.00 ± 7.08
	L-BFGS-B	636.82	11.37 ± 4.18	17.84 ± 5.43	28.07 ± 6.09	13.84 ± 1.35	3.43 ± 1.42	4.45 ± 1.74	79.00 ± 9.54

Data

SK phase	Data Alberto	My data
I	343	319
II	184	164
III	137	118
IV	416	416
VI	79	79

Data

SK phase	Data Alberto	My data
I	343	319
II	184	164
III	137	118
IV	416	416
VI	79	79

or 17.5 MeV for SK-II

- Why these differences?
  - Alberto performed the fit in [16, 90] MeV range
  - He showed the results on the [16, 80] MeV range
- I will compare both and take the best energy range

Results for every phase (without systematics)

SK phase	Energies	$-2\ln(\mathscr{L})$	N_{DSNB}	$N_{\nu_e\text{CC}}$	$N_{\text{Decay } e^-}$	N_{NCQE}	$N_{\mu/\pi}$	$N_{\text{Spallation}}$	$\sum N_j$
I	[16, 90] MeV	23.24	0.00	85.63	147.11	47.15	62.56	0.54	343.00
I	[16, 90] MeV	1440.82	0.00 ± 6.38	84.93 ± 7.64	146.78 ± 9.38	48.90 ± 7.43	61.10 ± 7.08	1.28 ± 1.42	342.99 ± 17.16
I	[16, 80] MeV	1440.87	0.00 ± 6.80	71.12 ± 7.39	143.99 ± 10.62	48.25 ± 5.66	54.34 ± 3.04	1.30 ± 1.49	319.00 ± 16.04
II	Nelder-Mead	300.39	0.00	43.92	76.44	19.72	42.17	1.75	184.00
II	[17.5, 90] MeV	1131.91	2.67 ± 5.61	43.58 ± 5.34	73.59 ± 8.43	20.78 ± 3.69	41.26 ± 5.77	2.11 ± 1.38	184.00 ± 13.41
II	[17.5, 80] MeV	992.08	2.65 ± 6.97	33.09 ± 5.97	74.17 ± 8.23	21.04 ± 3.70	30.93 ± 4.87	2.11 ± 1.58	164.00 ± 13.85
III	Nelder-Mead	284.63	4.76	45.01	50.49	10.75	24.43	1.54	137.00
III	[16, 90] MeV	776.22	6.30 ± 4.96	44.77 ± 5.43	48.94 ± 6.74	11.47 ± 2.28	23.86 ± 3.71	1.67 ± 1.10	137.00 ± 10.94
III	[16, 80] MeV	776.22	5.75 ± 5.55	25.86 ± 3.63	52.93 ± 6.84	11.78 ± 2.63	19.97 ± 3.50	1.70 ± 1.01	118.00 ± 10.53
IV	Nelder-Mead	124.39	26.21	109.72	158.75	87.57	28.19	5.61	416.05
IV	[16, 90] MeV	2081.02	9.87 ± 6.82	90.59 ± 8.30	189.91 ± 8.27	89.46 ± 7.28	28.25 ± 4.58	7.91 ± 4.25	416.00 ± 16.60
IV	[16, 80] MeV	2032.83	11.54 ± 7.58	106.49 ± 9.24	173.20 ± 11.65	88.38 ± 7.74	28.49 ± 4.93	7.90 ± 3.08	416.01 ± 19.29
VI	Nelder-Mead	290.34	7.35	18.58	31.32	13.57	3.49	4.70	79.01
VI	[16, 90] MeV	643.66	10.69 ± 3.68	13.09 ± 4.15	33.02 ± 5.85	14.36 ± 1.33	3.33 ± 1.56	4.52 ± 1.65	79.00 ± 8.48
VI	[16, 80] MeV	636.82	11.37 ± 4.18	17.84 ± 5.43	28.07 ± 6.09	13.84 ± 1.35	3.43 ± 1.42	4.45 ± 1.74	79.00 ± 9.54

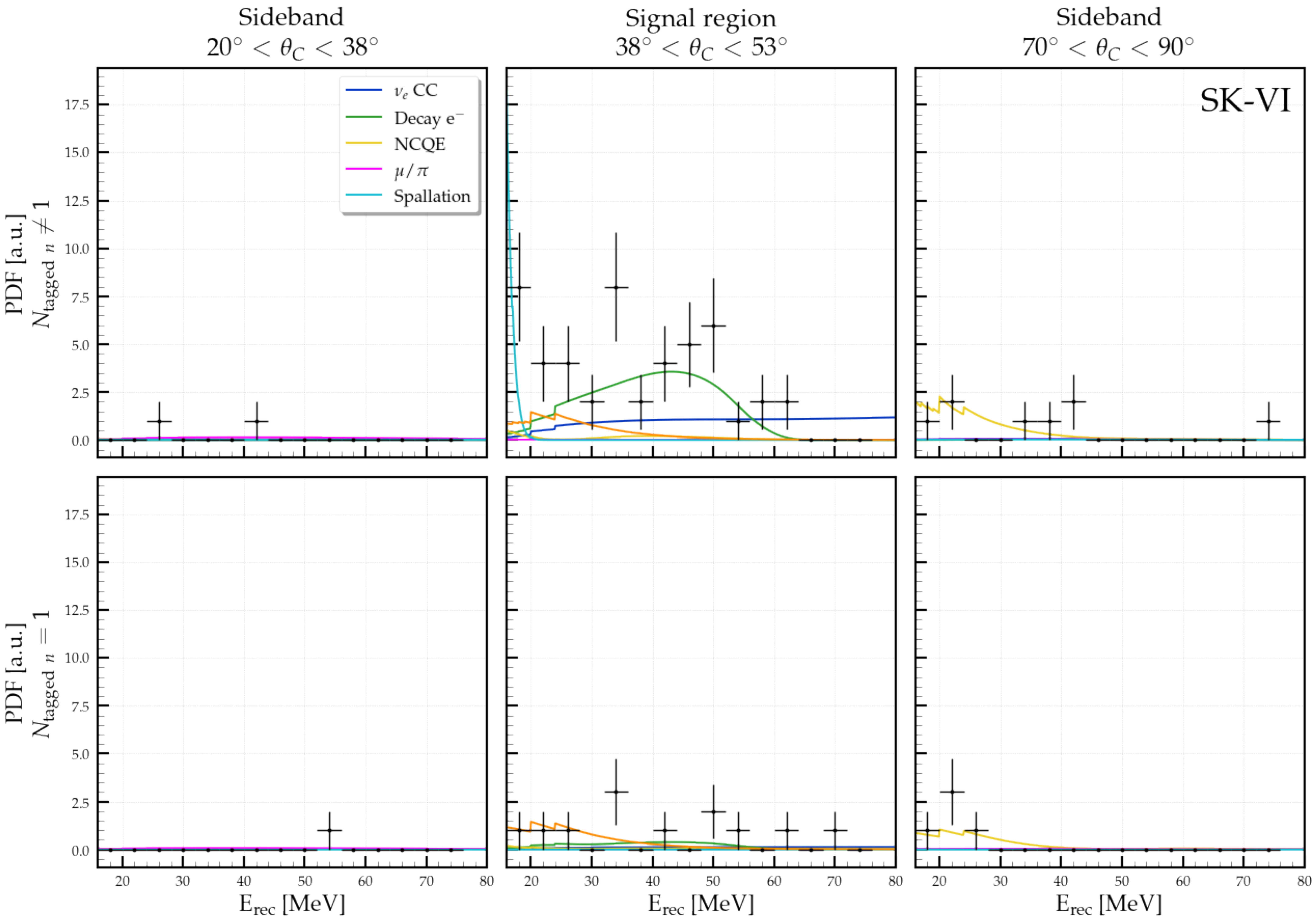
Profile likelihood ratio

- $\underline{H_0}$: Backgrounds only
- $\underline{H_1}$: Backgrounds + DSNB
- **<u>Neyman–Pearson lemma</u>**: Likelihood ratio test is the best test for H_0 against H_1

$$\frac{\mathscr{L}(\vec{E}\,|\,H_0)}{\mathscr{L}(\vec{E}\,|\,H_1)} \longrightarrow \boxed{\lambda(N_s = 0) = \frac{\mathscr{L}(\vec{E}\,|\,N_s = 0, \hat{\hat{\vec{N}}}_b)}{\mathscr{L}(\vec{E}\,|\,\hat{N}_s, \hat{\vec{N}}_b)}}$$

- $\hat{N}_s$ and $\hat{\vec{N}}_b$ are the best fit values to data sample
- $\hat{\hat{\vec{N}}}_b$ the best fit values for $N_s = 0$
- $t_0 = -2\ln(\lambda(0)) \longrightarrow Z_0 = \sqrt{t_0}$

Result for SK-VI without systematics



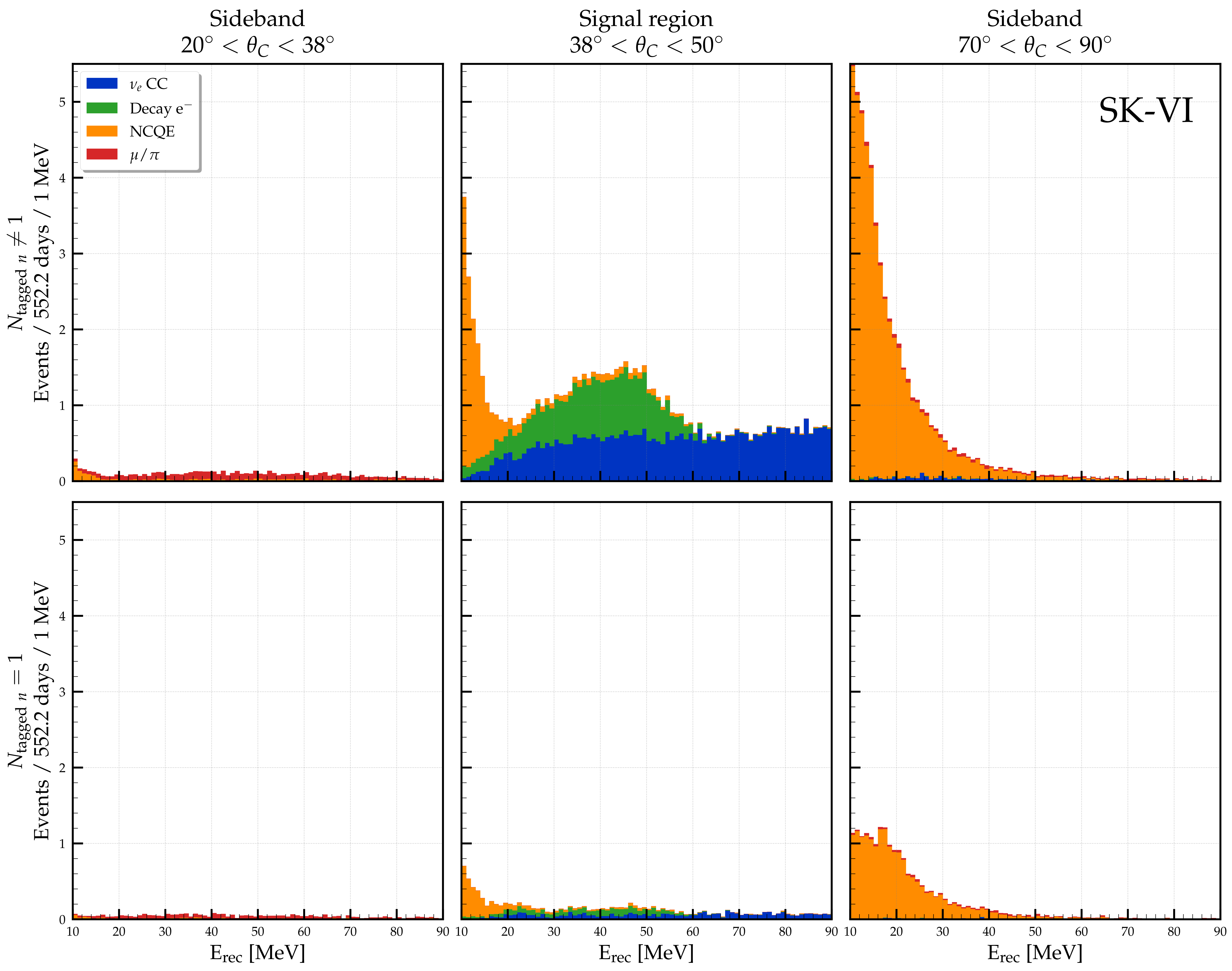
Backup

Defined regions

θ_C / $N_{\text{tagged } n}$	**[20°, 38°]**	**[38°, 53°]**	**[70°, 90°]**
≠ 1	μ/π	**DSNB** + final-state e	NCQE
= 1	μ/π	**DSNB** + final-state e	NCQE

Neutrino background MCs

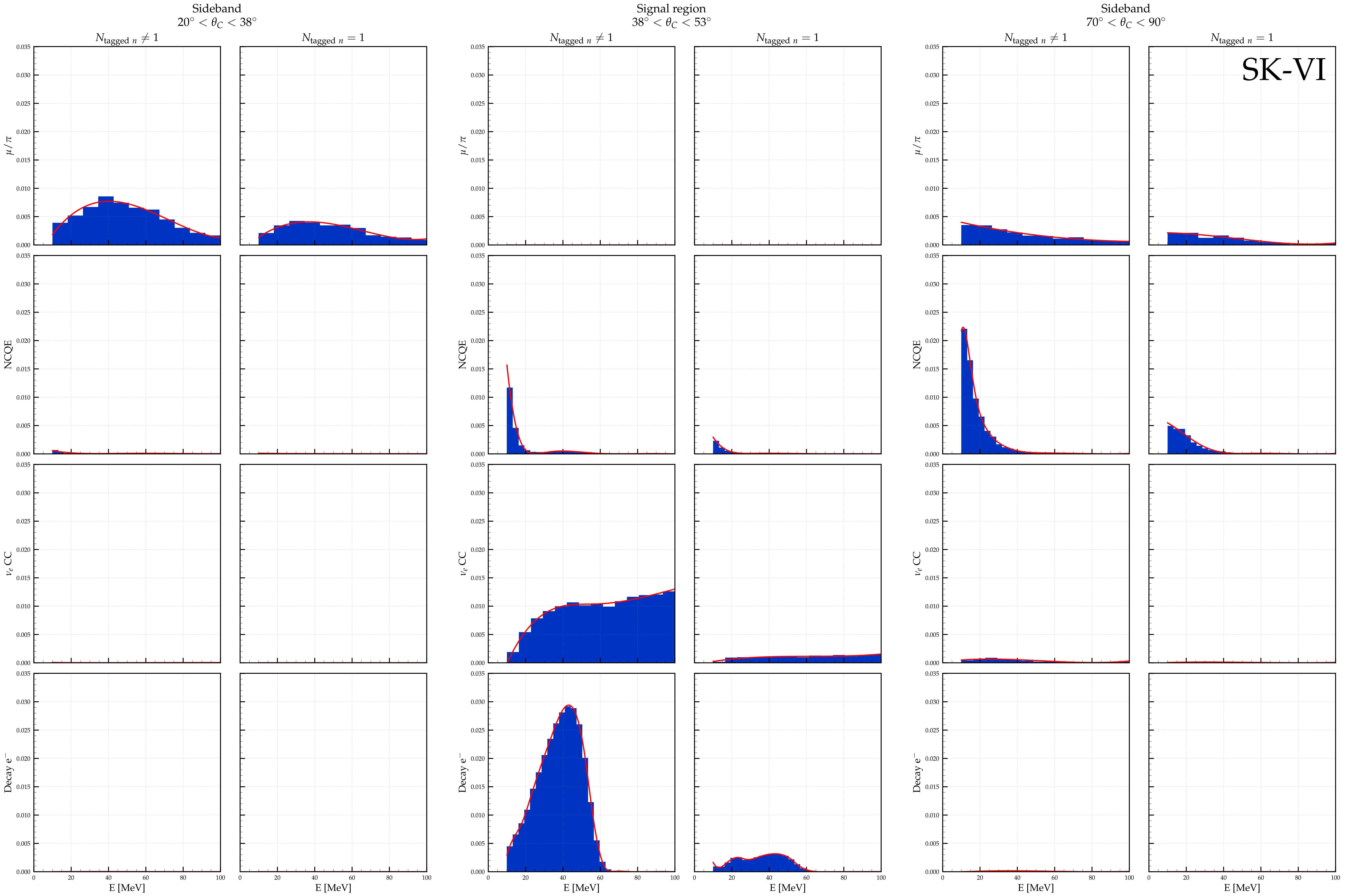
- After third reduction



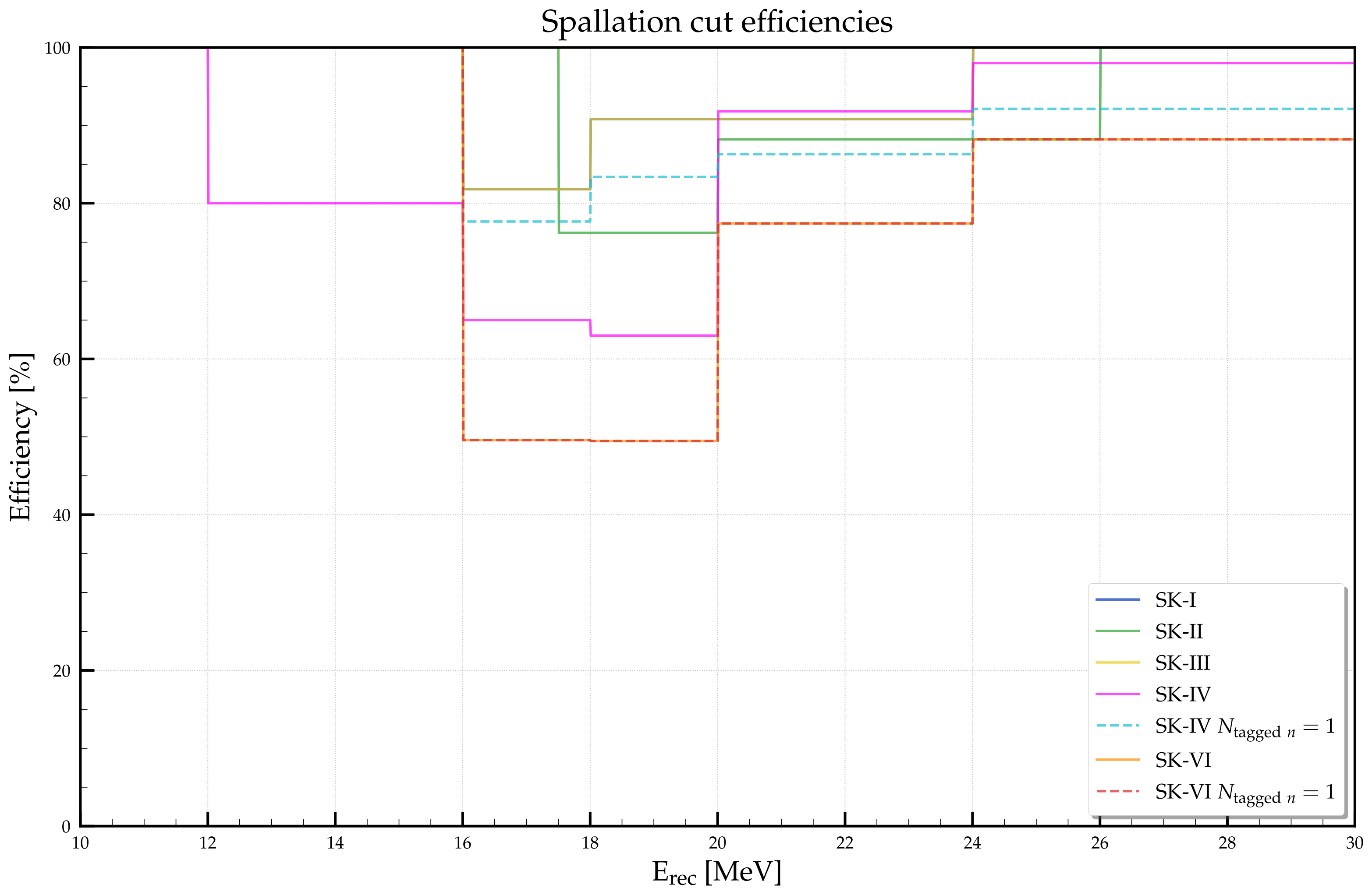
Backup

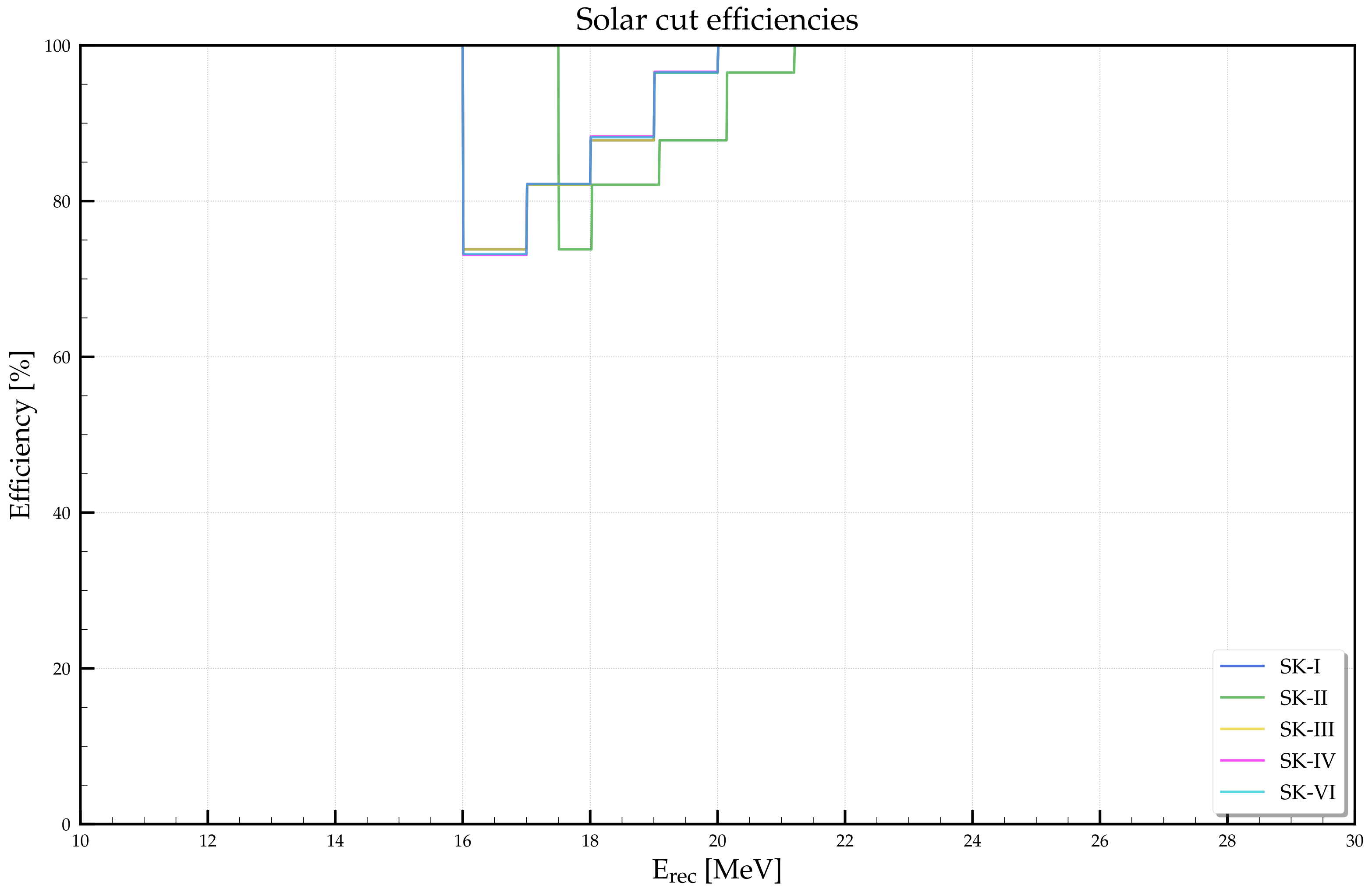
Create background PDFs

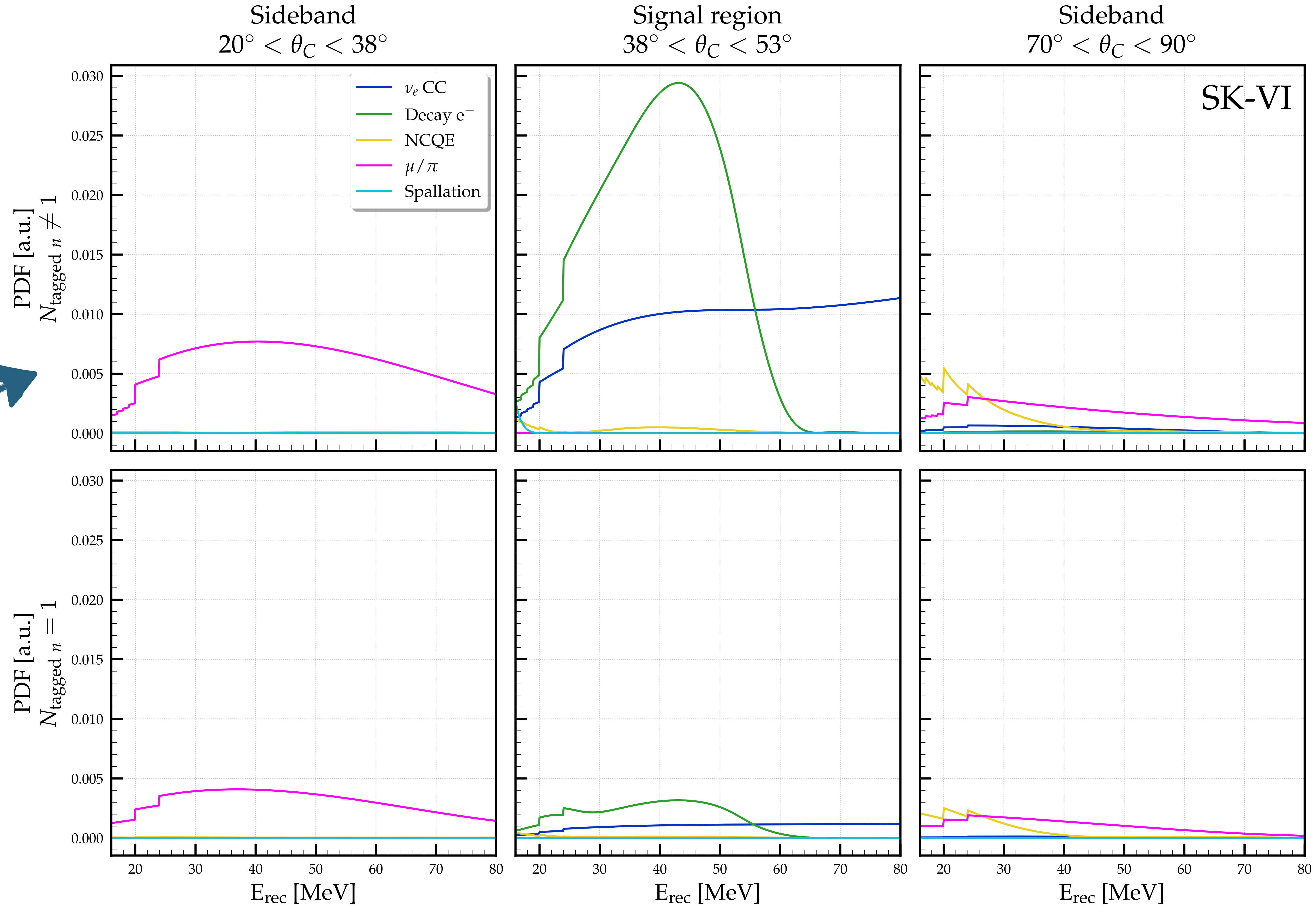
- After third reduction



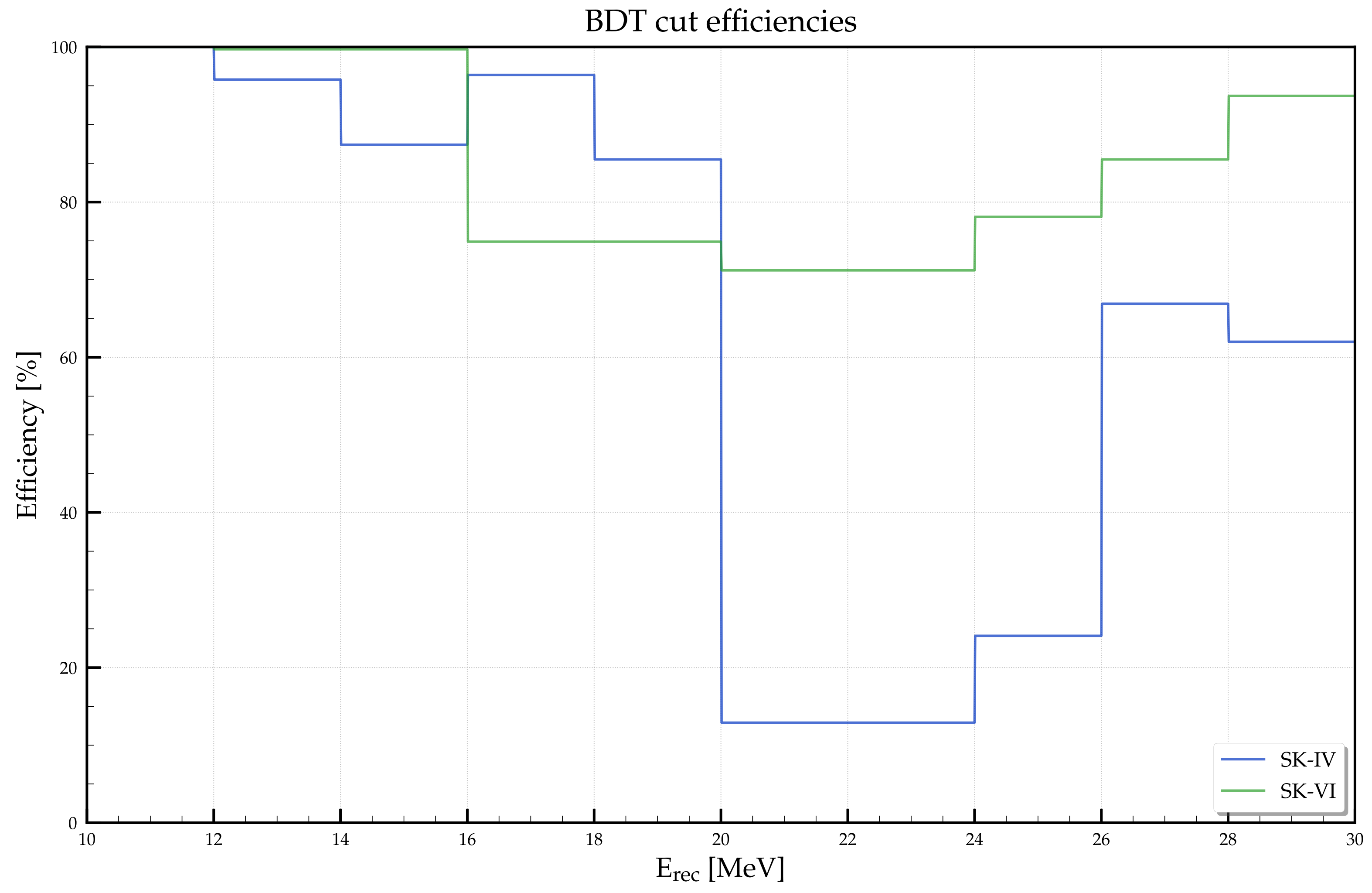
Spallation & AFT + Solar cuts

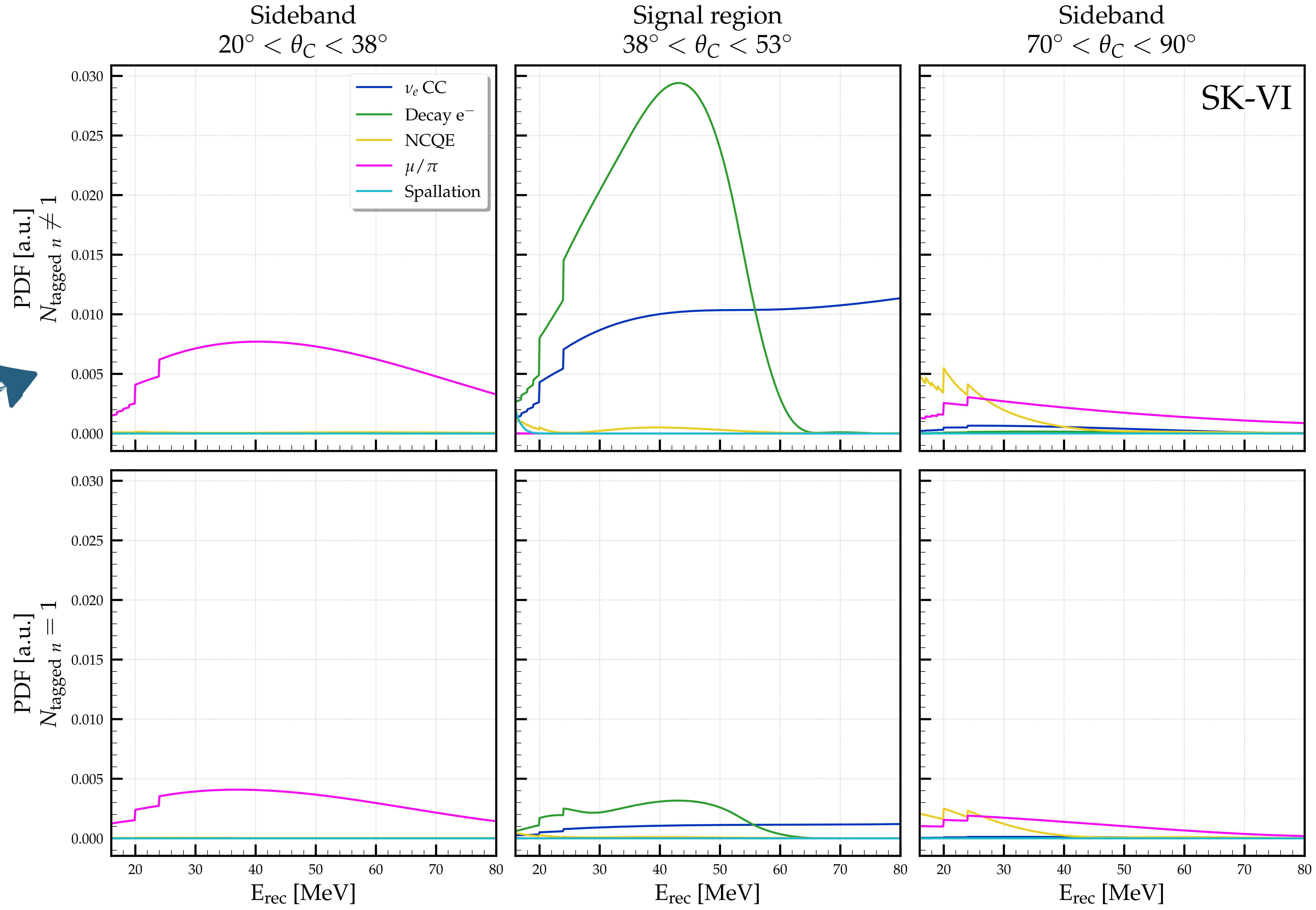






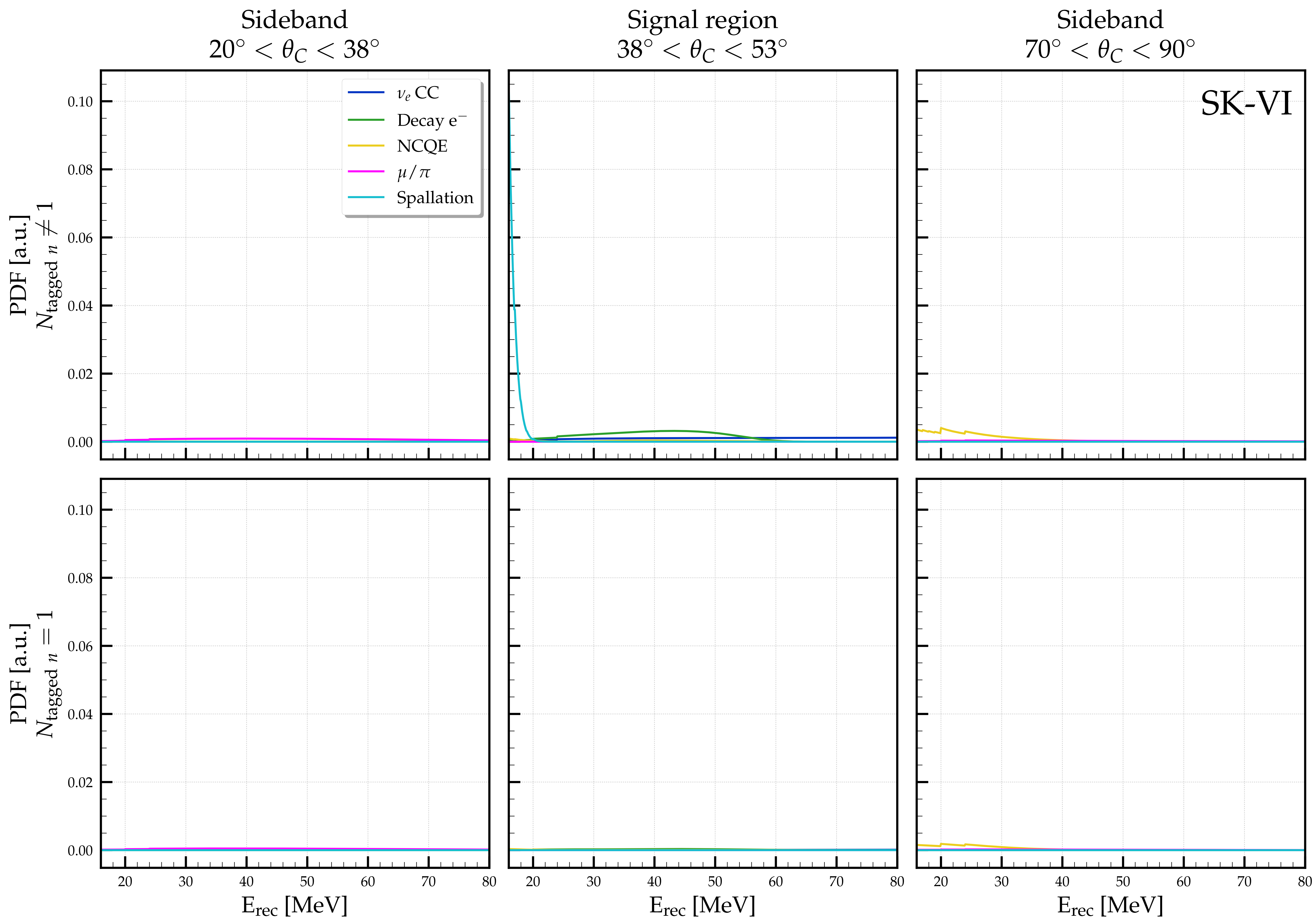
Neutron tagging cut to spallation PDF





Normalize PDFs

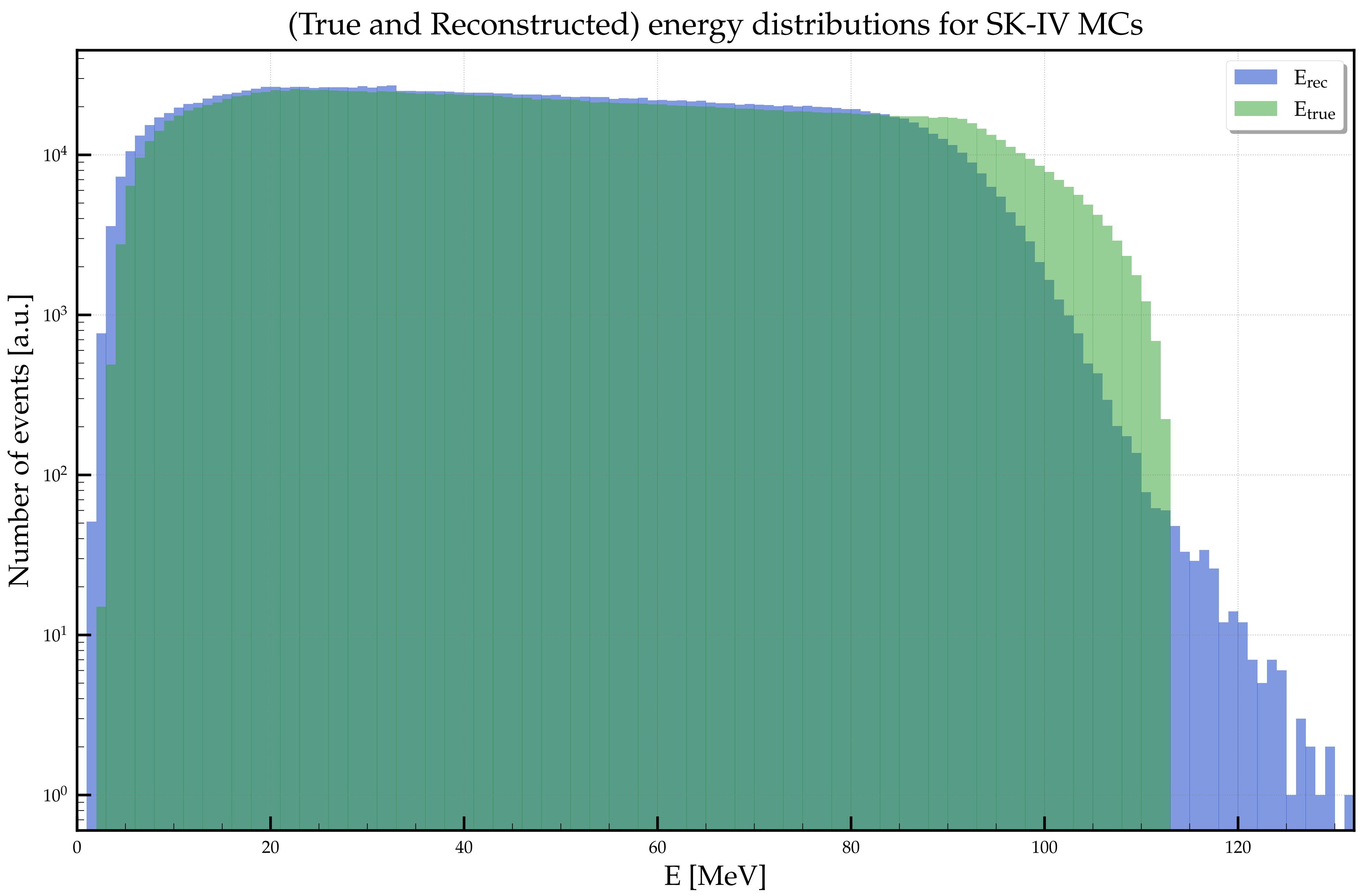
- *l1* normalization
- After this: Background PDFs ready for use in analysis

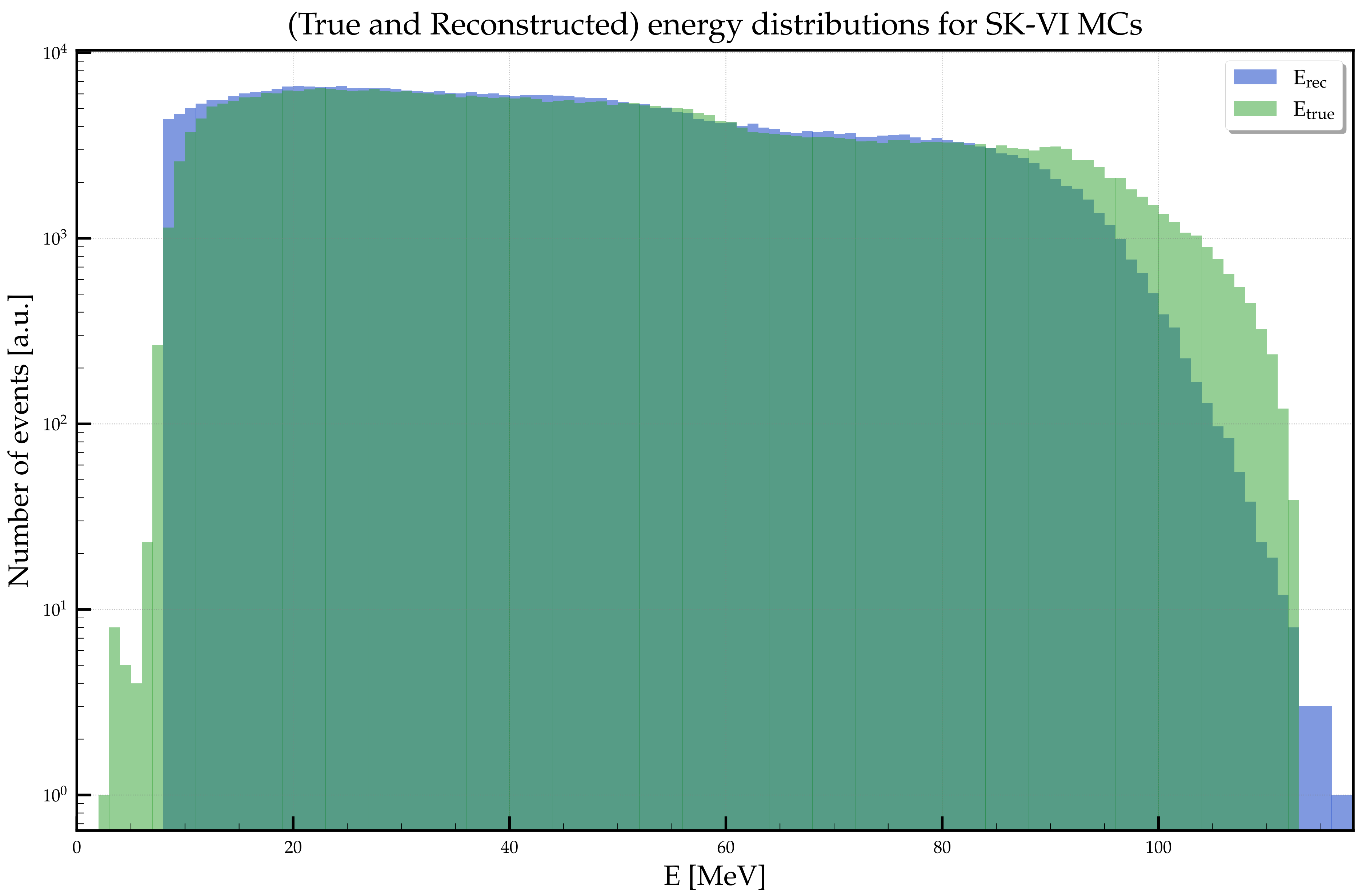


Backup

E_{true} and E_{rec} distributions

- Discarded events with E_{rec} = 9 999 MeV
- Directly applied 1st and 3rd reduction steps to SK-IV and VI MCs





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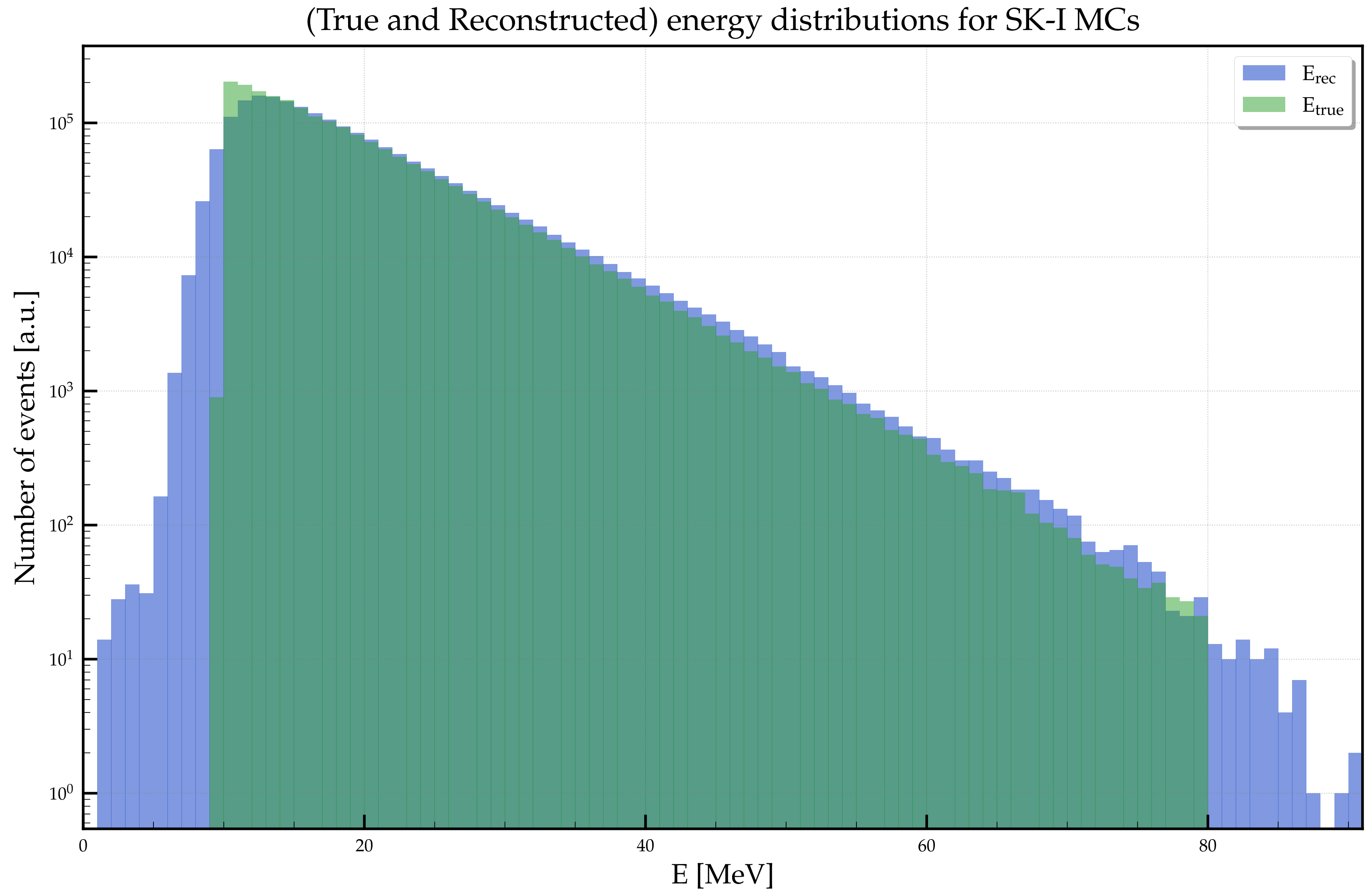
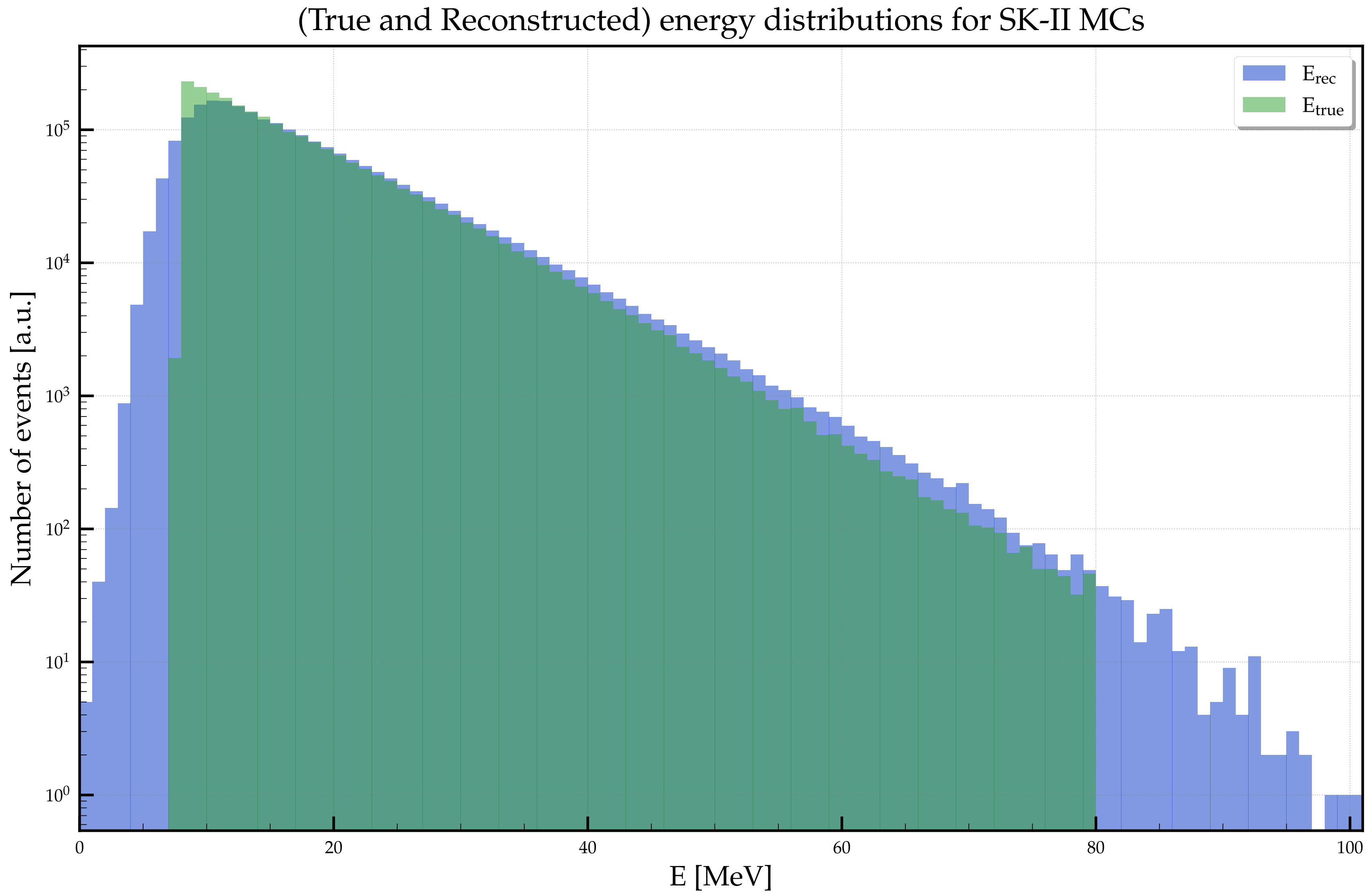
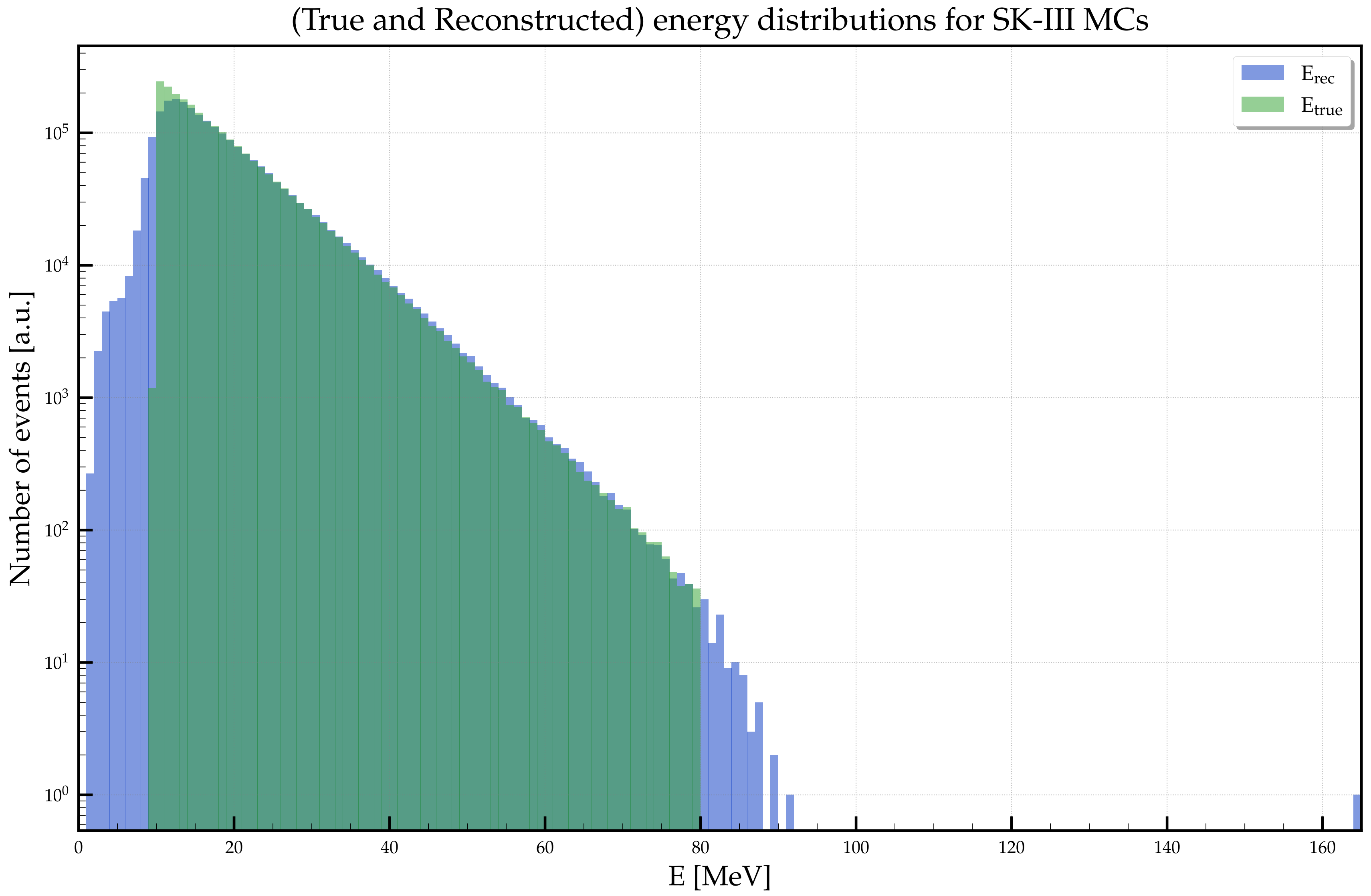




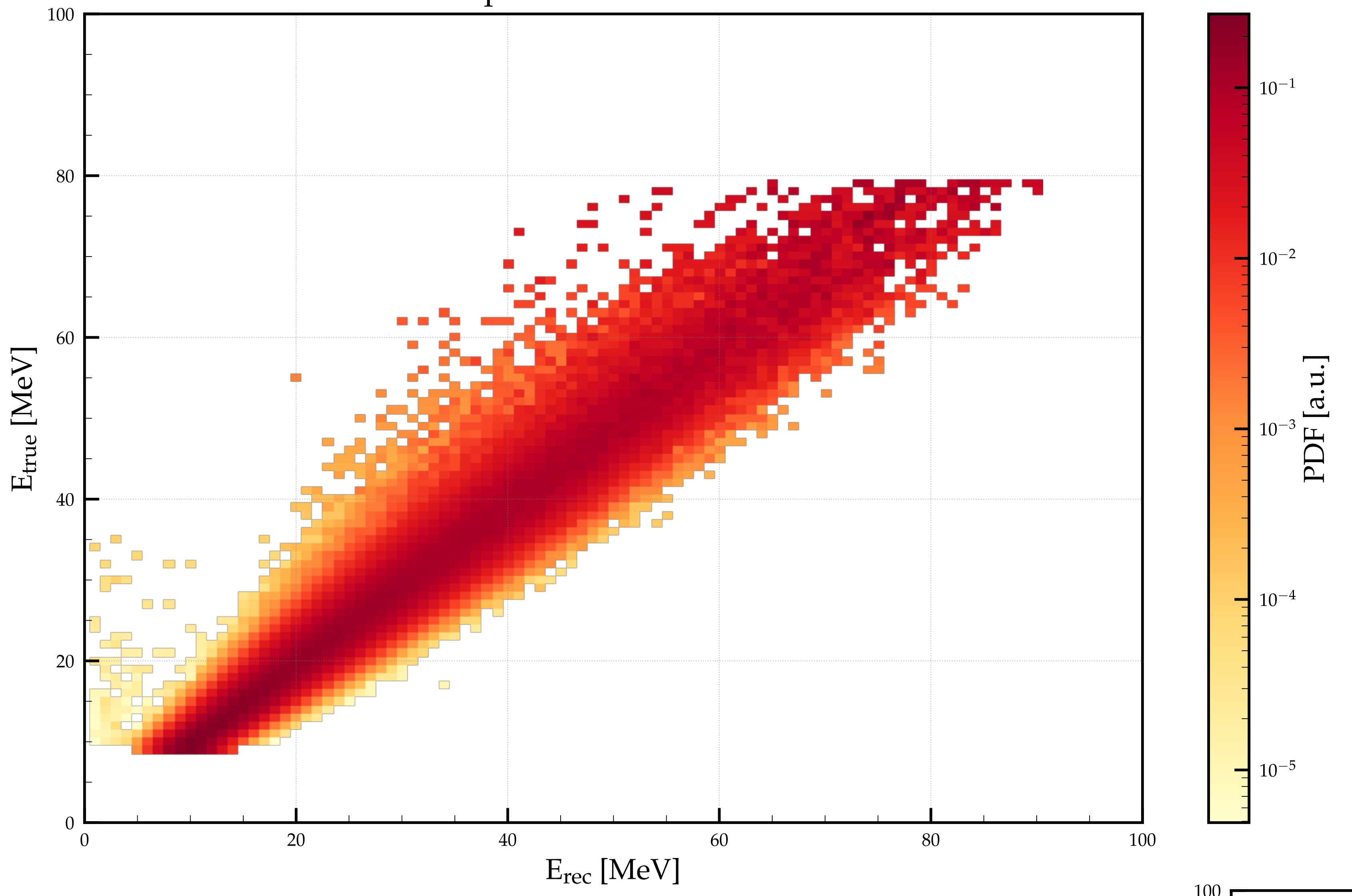


Building response matrices

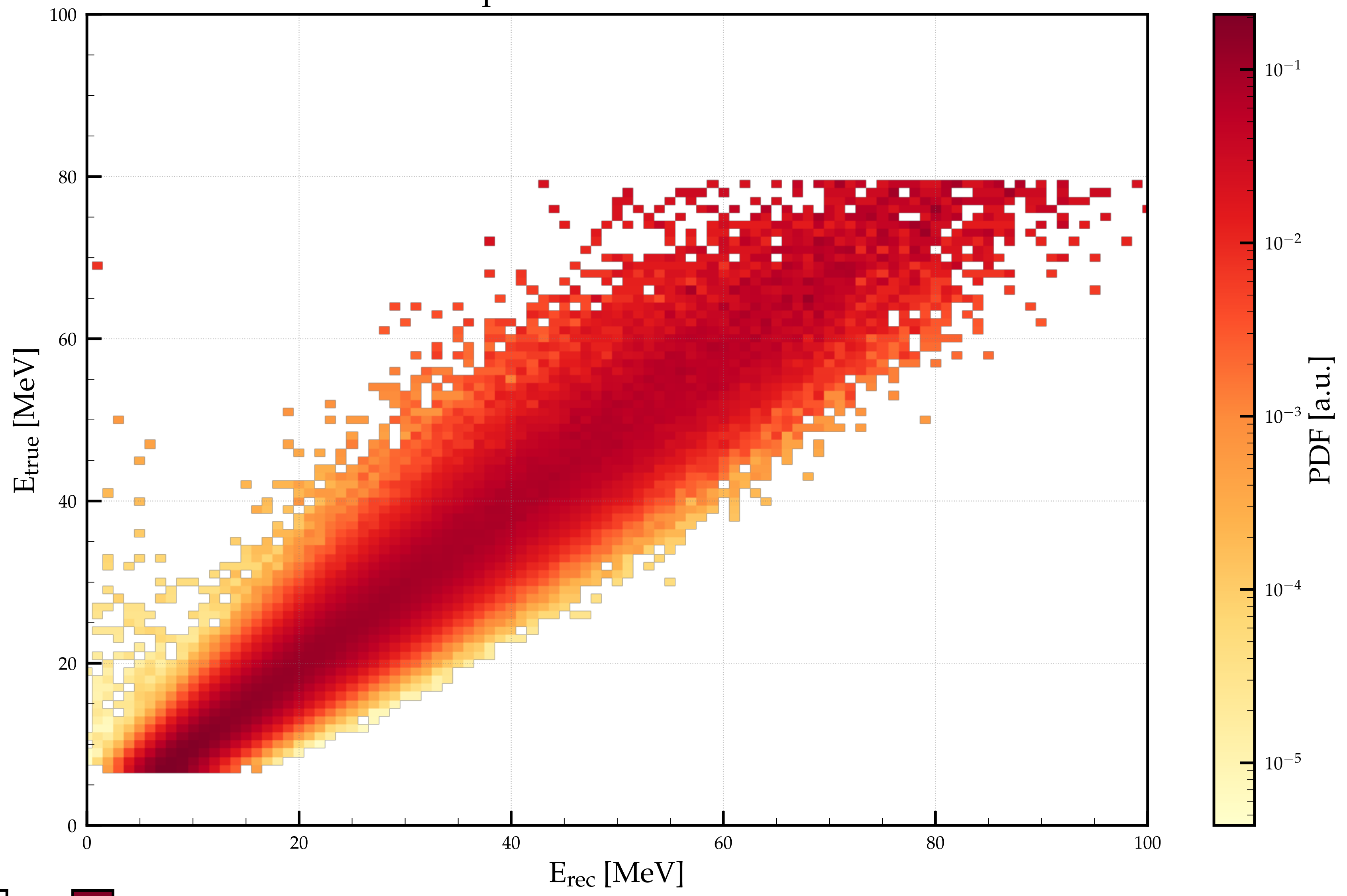
Normalized

Response matrices

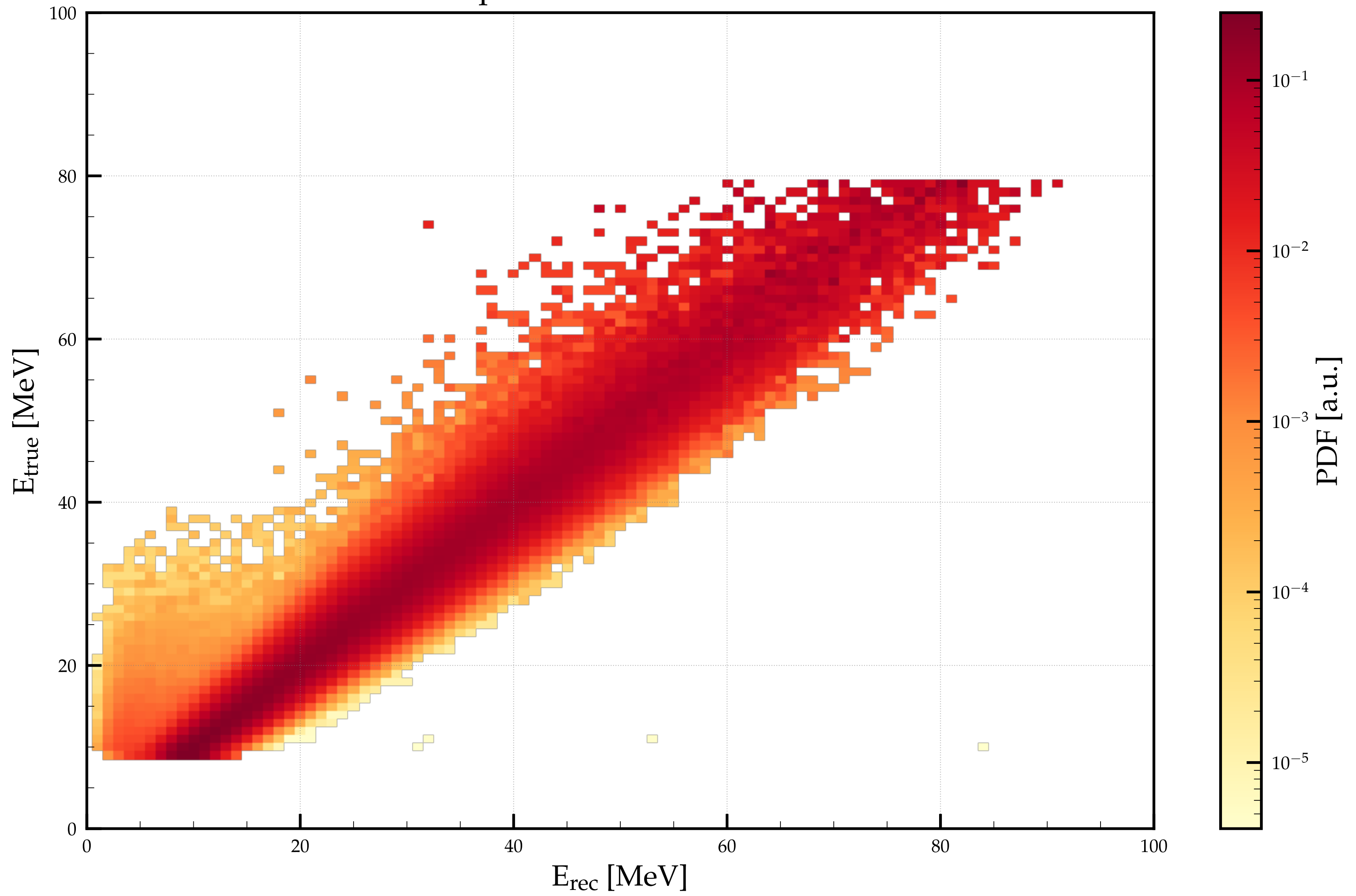
Response matrix for SK-I



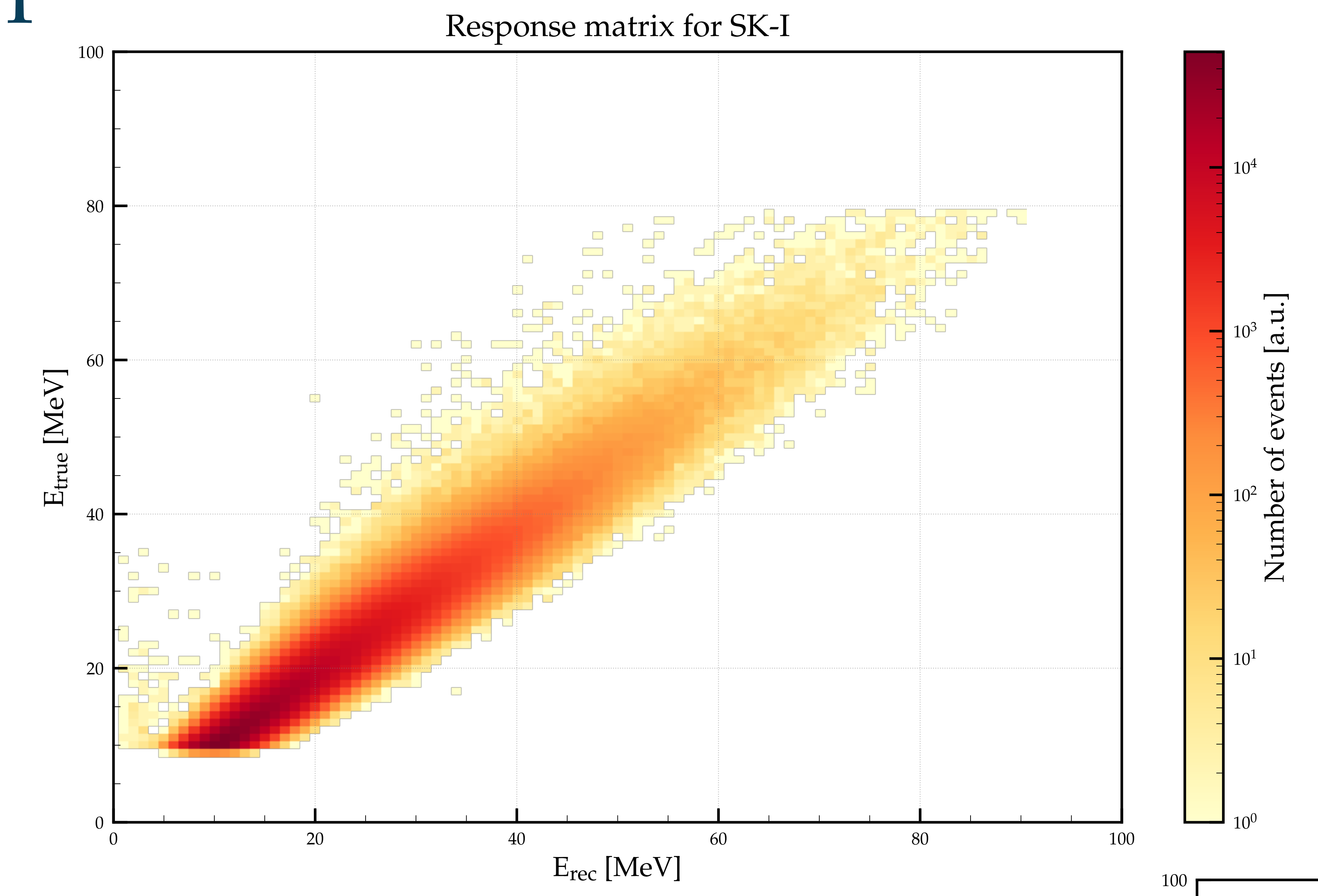
Response matrix for SK-II

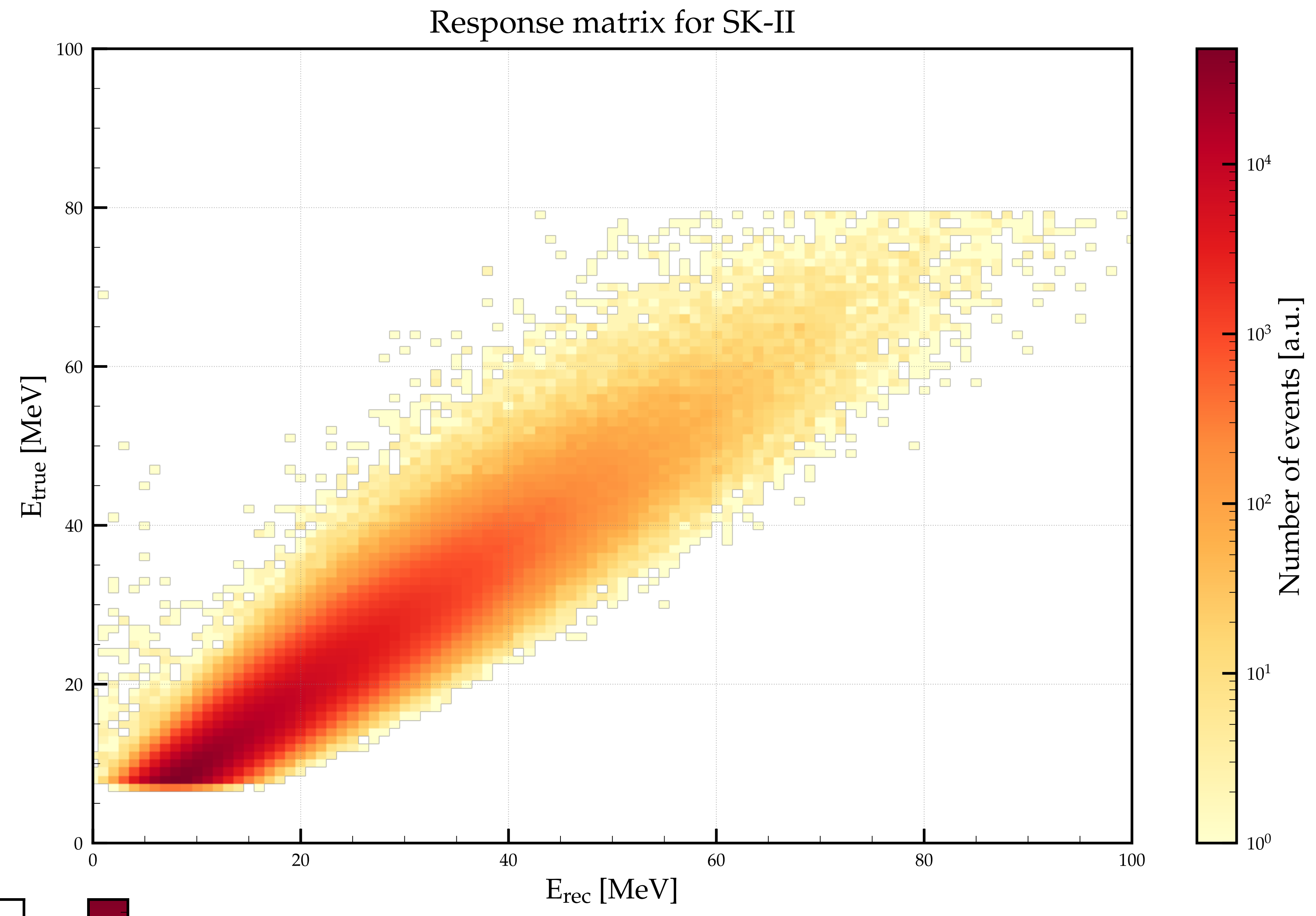


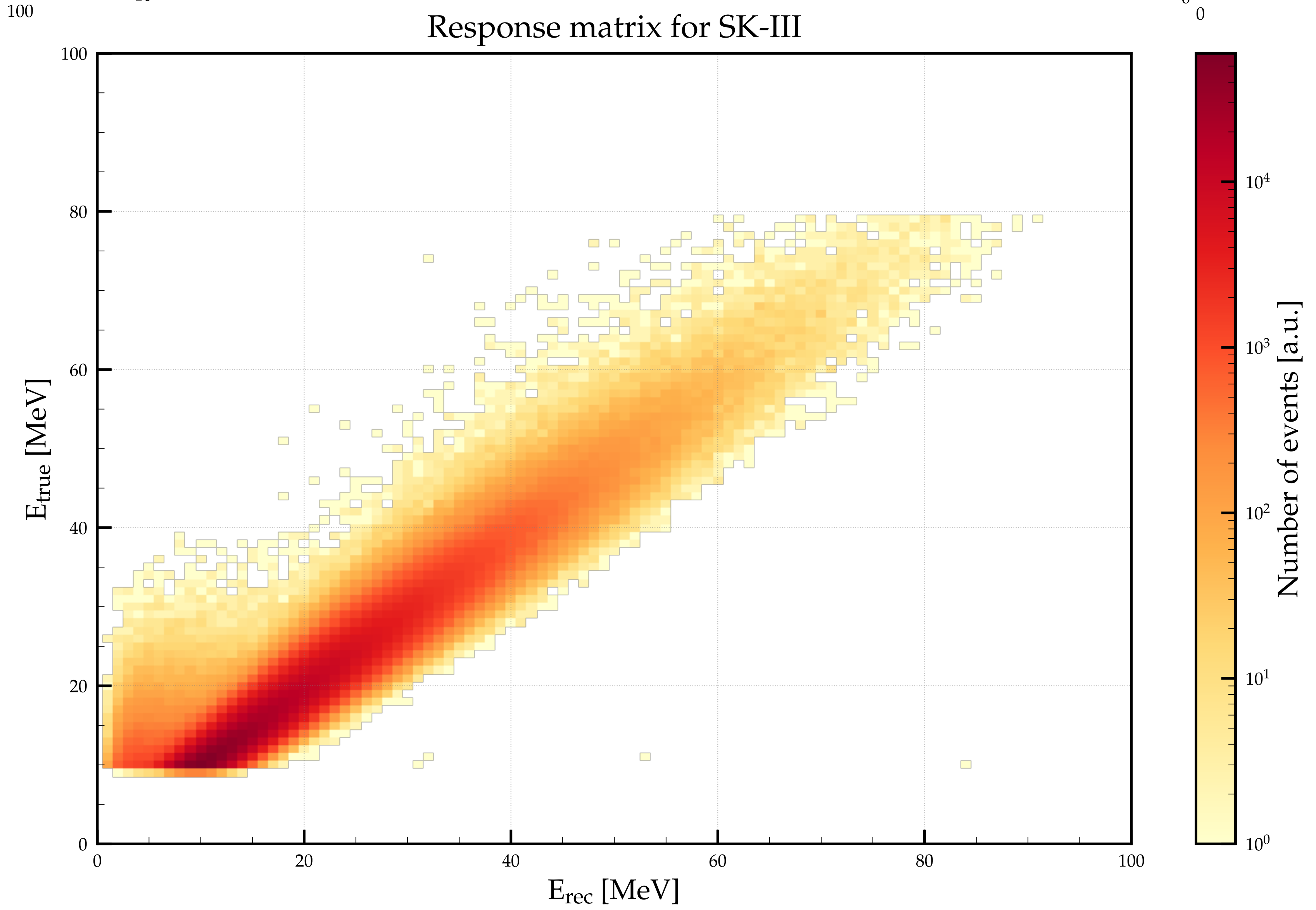
Response matrix for SK-III



Response matrices



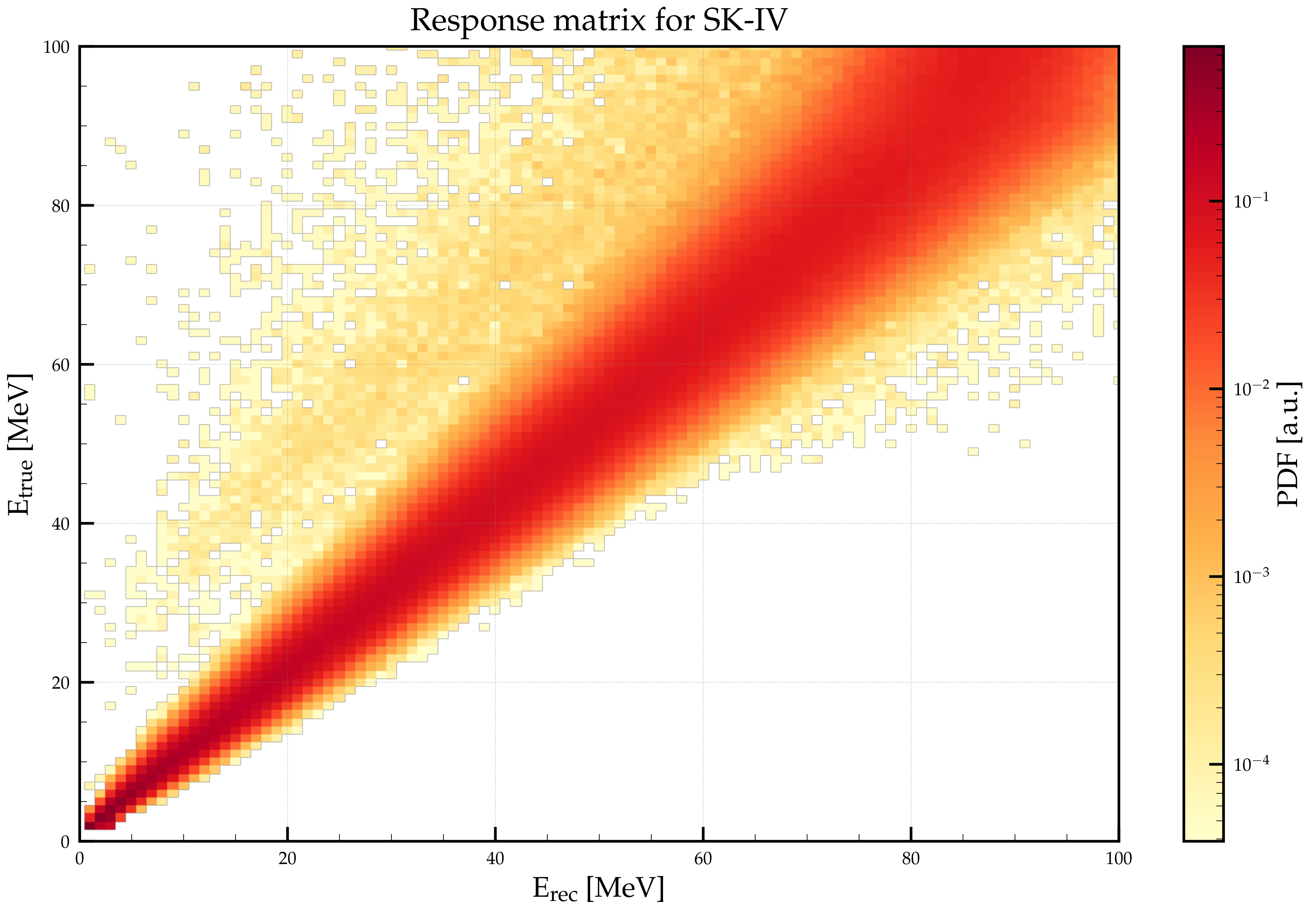


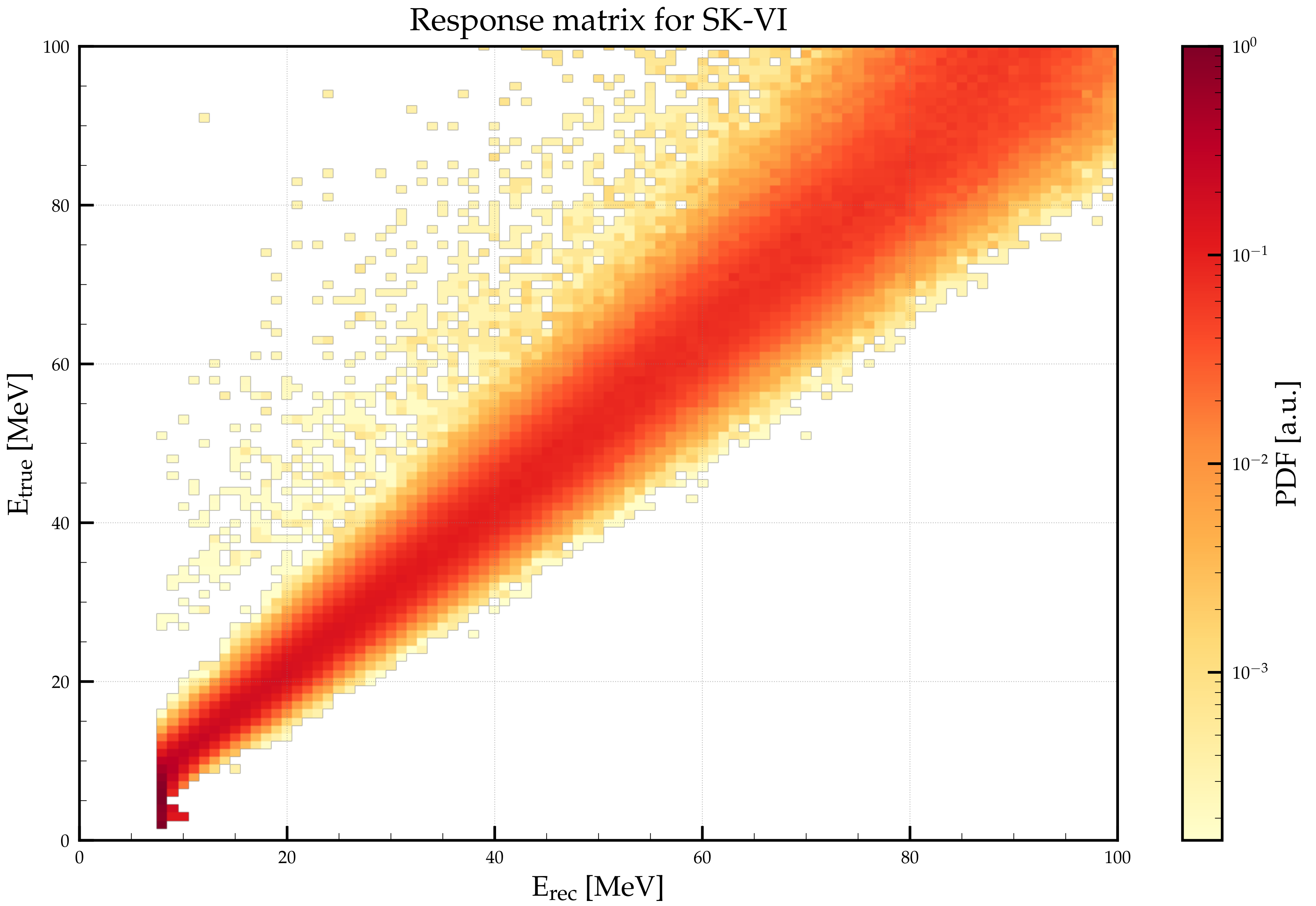


Building response matrices

Not a problem as the threshold for analysis is 16 MeV

- Linear behaviour except under $E_{rec} = 8$ MeV in SK-VI
- Why do we have this threshold at $E_{rec} = 8$ MeV with SK-VI?
  - Answer: Effect due to the normalization/Resolution effect





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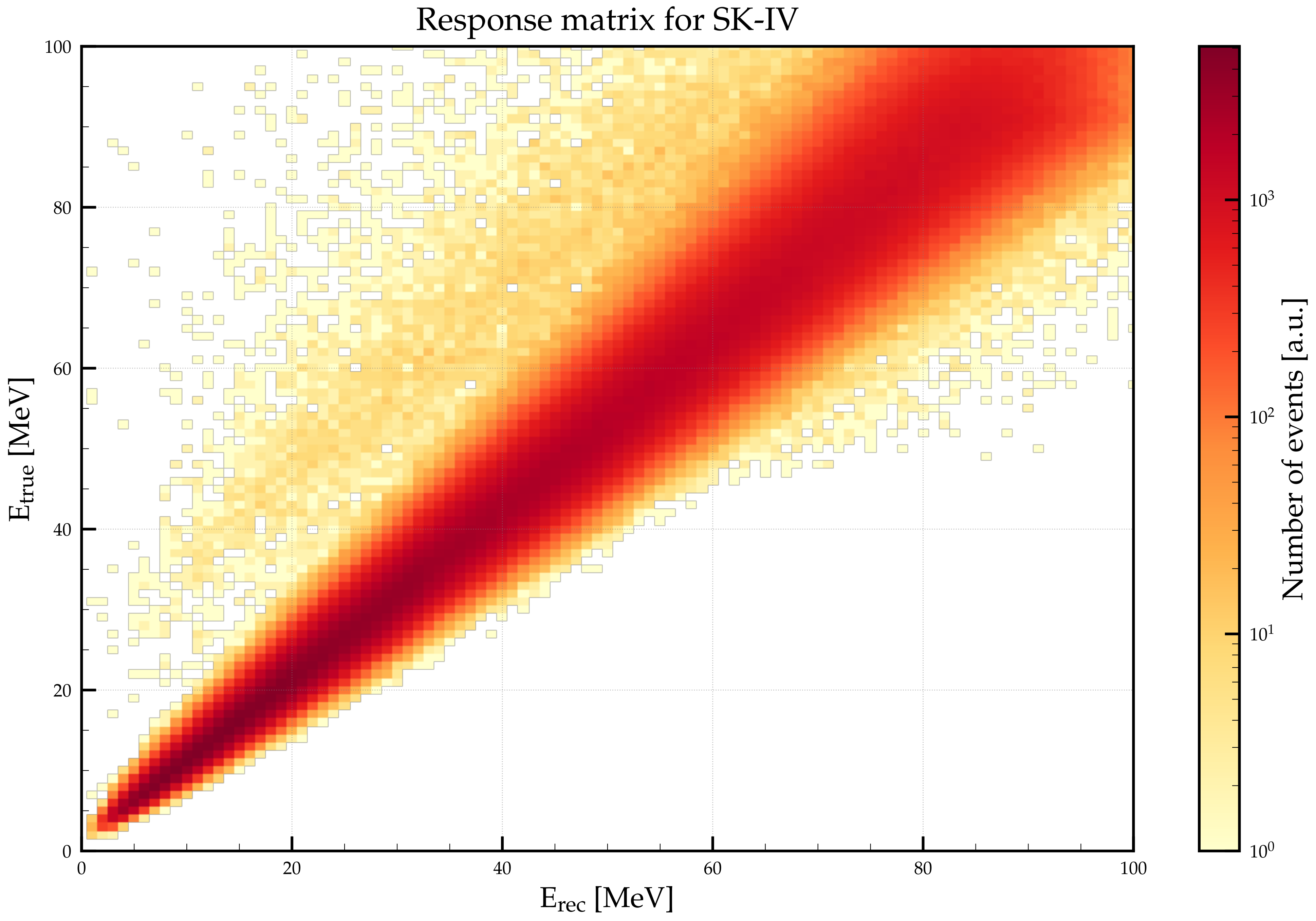
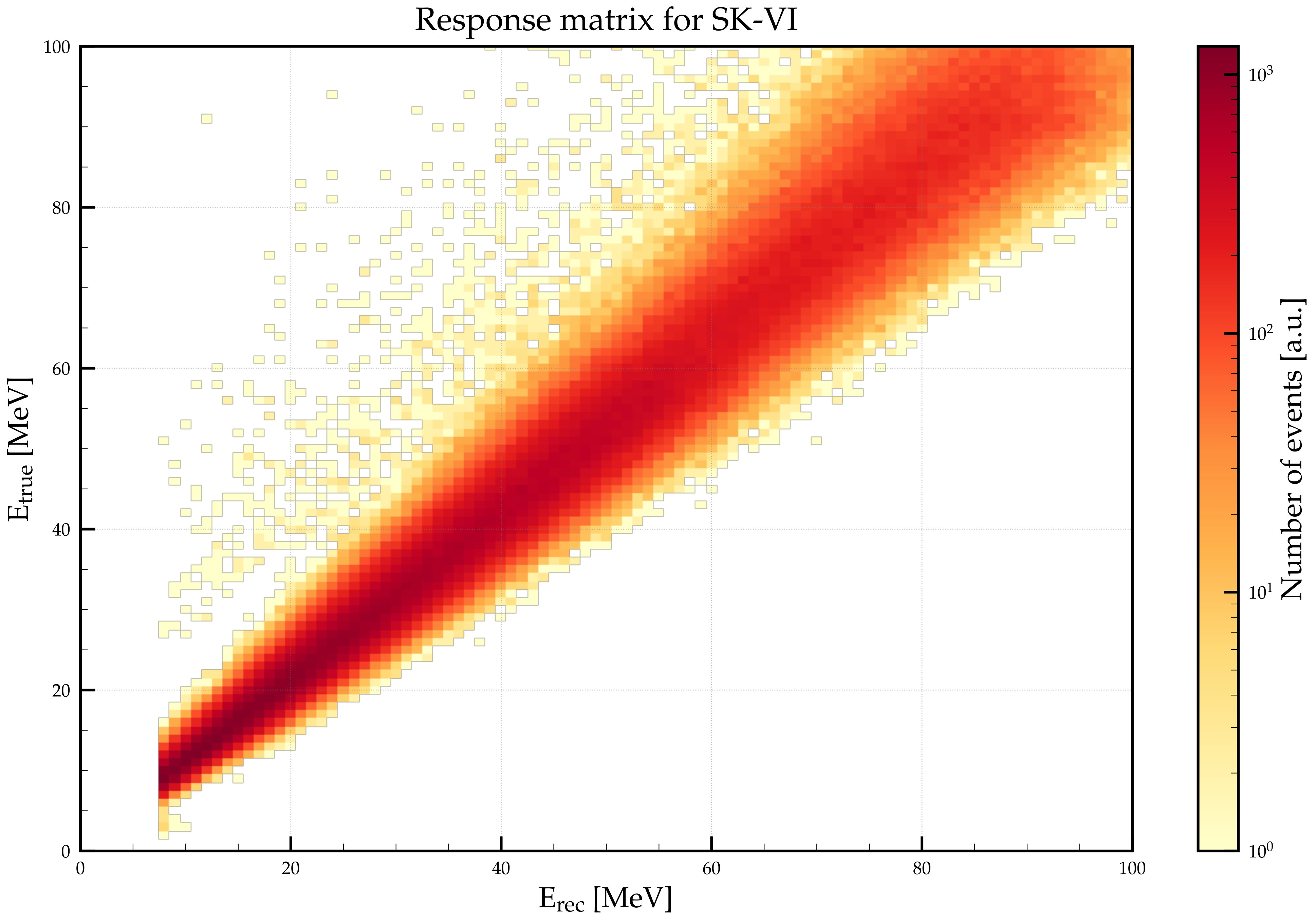
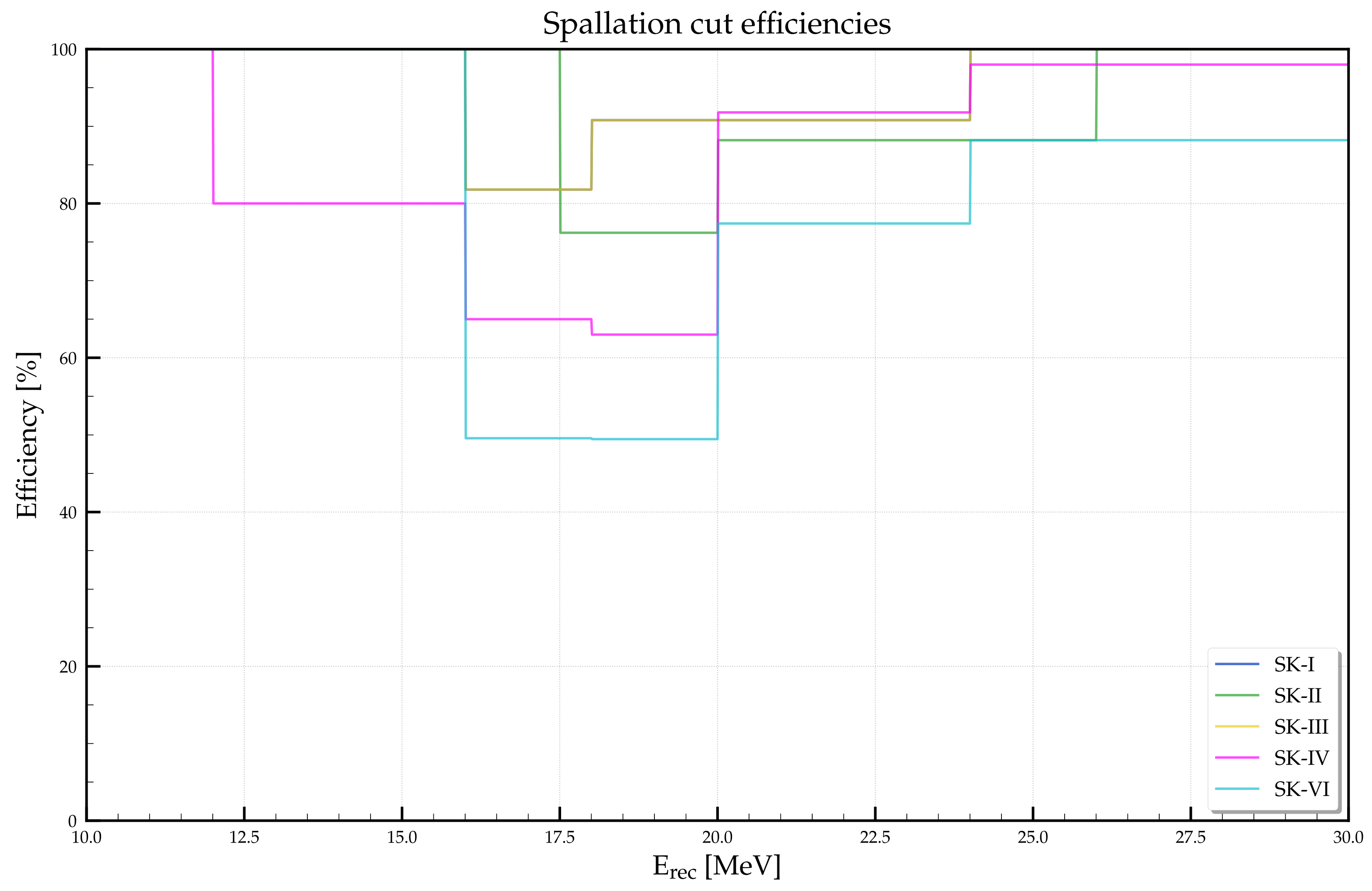
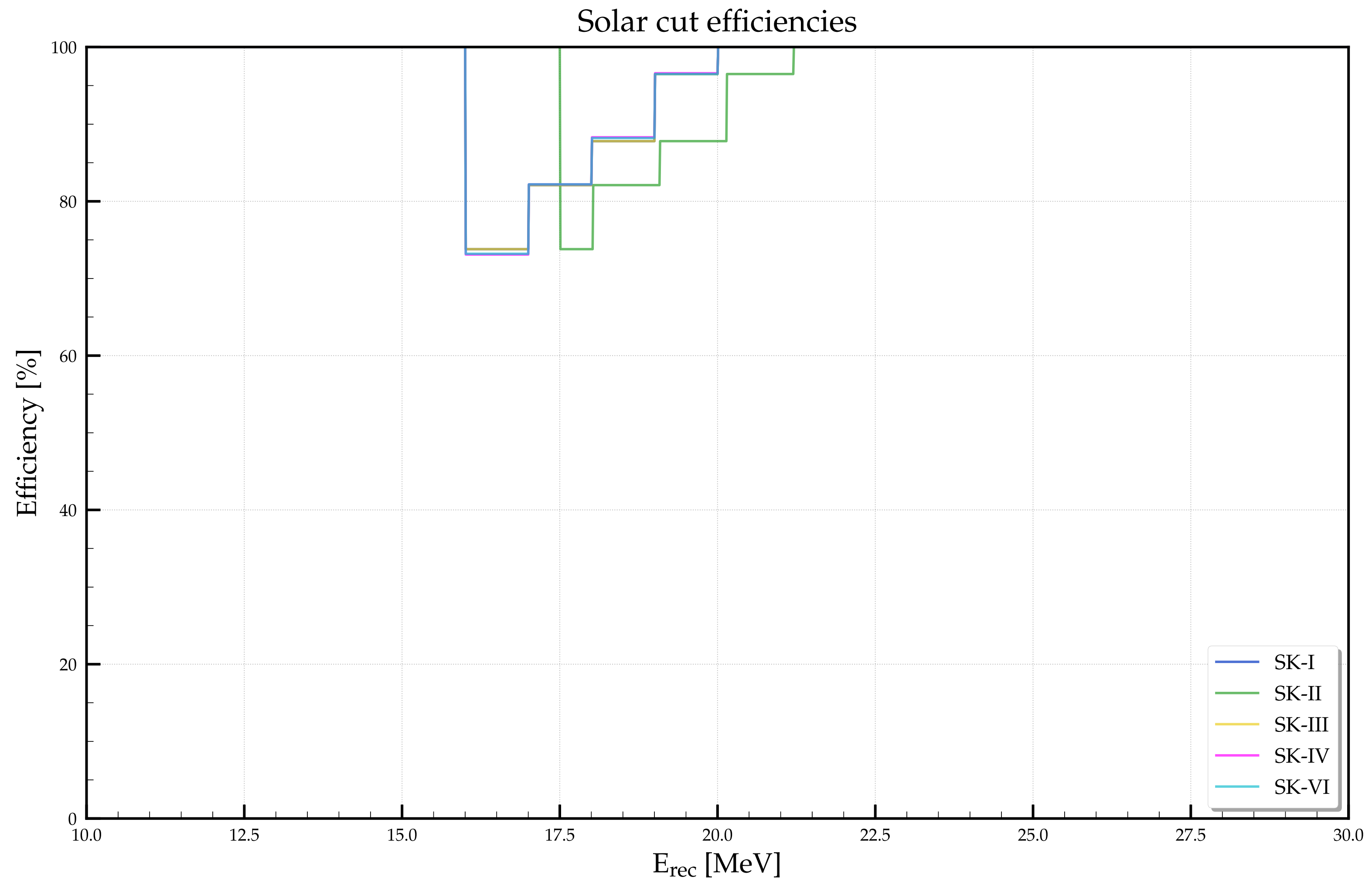
- Why do we have this threshold at E_{rec} = 8 MeV with SK-VI?
  - Answer: Effect due to the normalization/Resolution effect

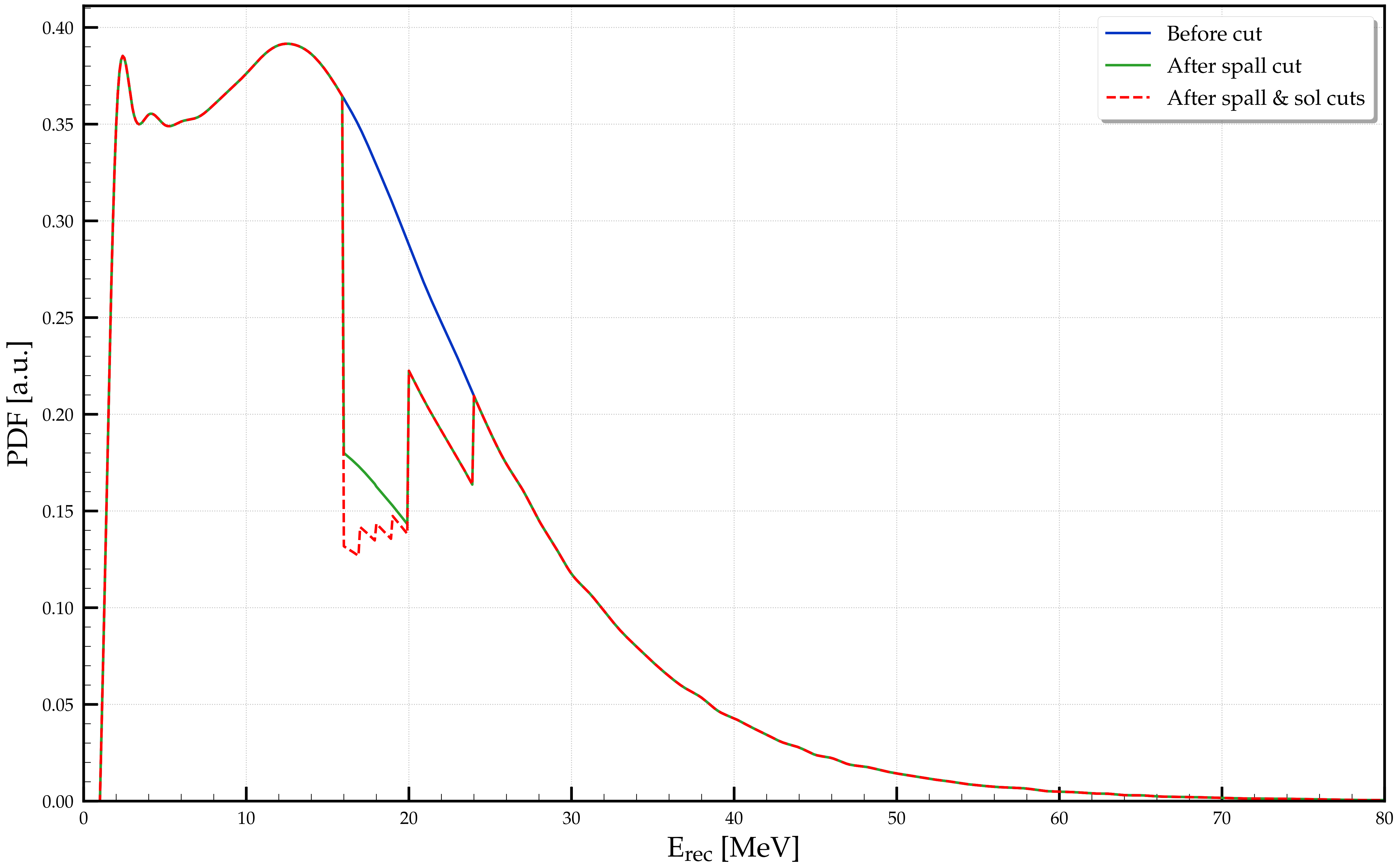




Spallation & AFT + Solar cuts







Cuts & Normalization for SK-IV

- Model of S. Horiuchi (2009)