New Physics event re-weighting – general conditions

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- One of the main purpose of High Energy accelerator experiments is to provide confronation of theory and measurements in ever new realms.
- Any new agreement extend theory applicability domain, any discrepancy may hint to unexplained phenomena. That means needs of better calculations or new deeper theory.
- Because easy things for present day acclerators are already completed. One has to search for small variation over large Standard Model distributions.
- My talk concentrate on two classes of solutions for New Physics event weight calculation :

(i) Precision Monte Carlo and calculation from its internal variables

(ii) Calculation from information stored in production data files

- **Two examples:**

  (i) `TauSpinner` for weight embedding of anomalous dipole moment into $pp \to \tau^+\tau^- X$ simulation samples.

  (ii) Algorithm and results of anomalous moment weights, calculated simultaneously with run of `KKMC` Monte Carlo for $e^+e^- \to l^+l^-(n\gamma)$ at low energies and now also for FCC center of mass energies.

  (iii) Results from e-Print: 2307.03526 Sw. Banerjee, A.Yu. Korchin,, E. Richter-Was, Z. Was, *Electron-positron, parton-parton and photon-photon production of τ-lepton pairs: anomalous magnetic and electric dipole moments spin effects*, will be shown by Alex.

  (iv) NOTE: weights – ratios of matrix element squared, calculated in well defined points of phase space, but of distinct physics assumptions.

  (v) My talk is about general principles...

Add extra interactions:

To start: M^{SM} and M^{SM+NP} are needed.

- Really OK, for **anomalous magnetic/electric dipole** moments implementation in $e^+e^- \to \tau^+\tau^-(n\gamma)$ process (τ decays included).
- NP distinct interaction scale than (independent from) bremsstrahlung/jet emissions.
- Seem trivial, but one has to keep in mind practical details.
- I will say little about reliability proofs, even though they are essential.
- Important is to preserve precision for SM (interfering-) part of the process!
- Check if factorization properties for NP match with what is in SM. Precision requirements for New Physics implementation are not high. Use of interpolated Improved Born in presence of hard bremsstrahlung required difficult validations.
- For New Physics weights simplified kinematic is used.

Add extra interactions:

Formalism for $\tau^+\tau^-$: phase space $\times$ M.E. squared

- Because narrow τ width (τ propagator works as Dirac δ), cross-section for $f\bar{f} \to \tau^+\tau^- Y; \tau^+ \to X^+\bar{\nu}; \tau^- \to \nu\nu$ reads (norm. const. dropped):

$$d\sigma = \sum_{spin} |\mathcal{M}|^2 d\Omega = \sum_{spin} |\mathcal{M}|^2 d\Omega_{prod}\ d\Omega_{\tau^+}\ d\Omega_{\tau^-}$$

$$\mathcal{M} = \sum_{\lambda_1\lambda_2=1}^{2} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\ \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2}$$

- Pauli matrices orthogonality $\delta^{\lambda'}_{\lambda}\delta^{\bar{\lambda}'}_{\bar{\lambda}} = \sum_\mu \sigma^{\mu}_{\lambda\bar{\lambda}}\sigma^{\lambda'\bar{\lambda}'}_{\mu}$ completes condition for production/decay separation with τ spin states.
- **core formula of spin algorithms,** wt **is product of density matrices of production and decays**, $0 < wt < 4$, $< wt >= 1$ useful properties.

$$d\sigma = \Big(\sum_{spin} |\mathcal{M}^{prod}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^+}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^-}|^2\Big) wt\ d\Omega_{prod}\ d\Omega_{\tau^+}\ d\Omega_{\tau^-}$$

Add extra interactions:

Reference frames of host program

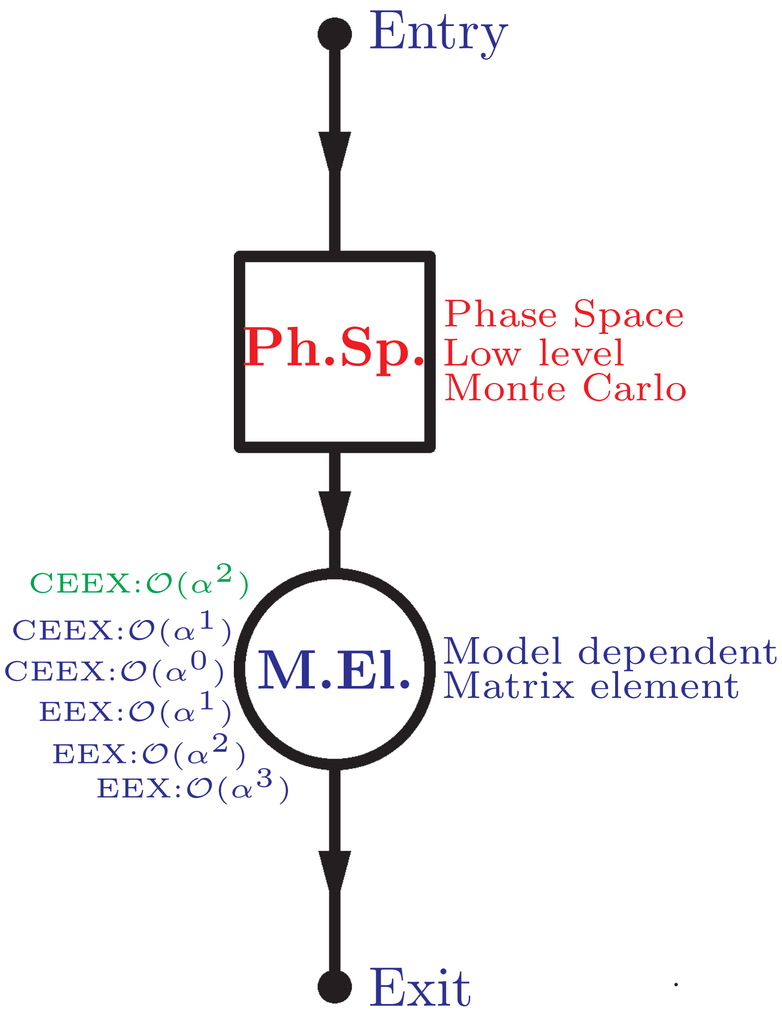
- **Host program frames convenient but not essential:** better precision, no need to worry about bremsstrahlung impact etc. Use of internal program variables helps too.
- Profit of well established routines relating τ-lepton rest frame and laboratory frame.
- That looks (and formally is) trivial, but it saves lots of worries how frames are affected in presence of bremsstrahlung photons (jets etc).
- New Physics module may have its own simpler solutions relating to lab frame.
- **On the other hand**, this prevents re-use of events for distinct models

Event record information only

- `TauSpinner` use simplified frames everywhere. It is for pp, not only kinematic is approximated, but on-flight hard process is re-evaluated for every event. Sum over all possible parton level processes is performed.
- Choice of these frames carefully prepared. Final solution was established when CP-sensitive observables were studied: Eur.Phys.J.C 79 (2019) 2, 91
- Re-use of the same event sample, with distinct New Physics models.

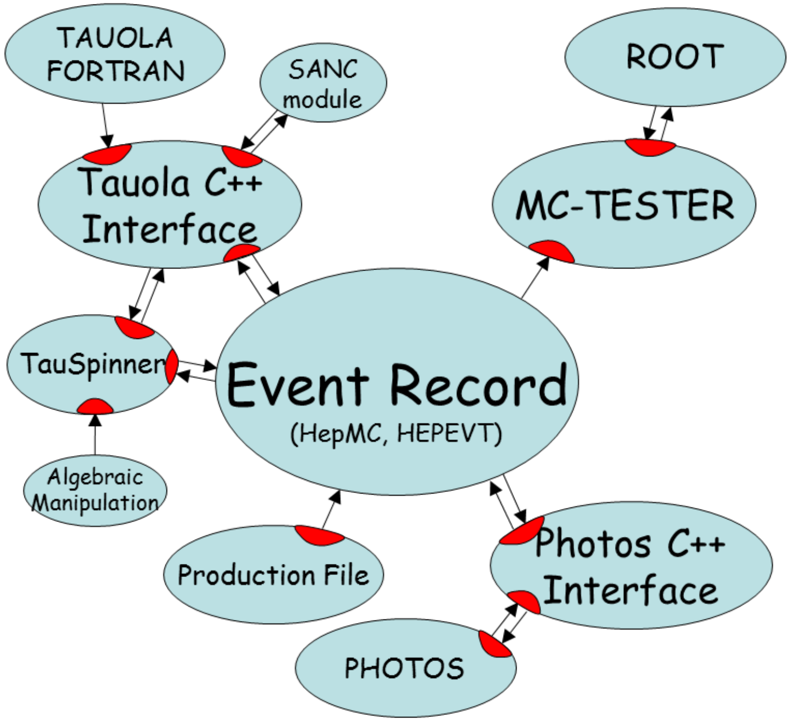
Run simultaneously with host program

KKMC basis: "matrix element × full and exact phase space" it is NOT shower-like algorithm !



- Phase-space Monte Carlo simulator is a module producing "raw events" (including importance sampling for possible intermediate resonances/singularities)
- Library of Matrix Elements; input for "model weight"; independent module
- This was used already for LEP precision Monte Carlos, like KKMC. Now it is used for Belle (FCC ...) collaboration for τ lepton pair production with decays and multi-photon radiation.
- Correlated samples techniques. Lots of technicalities collected in Phys. Rev. D41 (1990) 1425.
- Solutions useful for New Physics event weights!

TauSpinner phase space point from Event Record



● **What is advantageous?** Simultaneous event generation and weight calculation? Re-use the same (stored) events with detector response for many New Physics models?

TauSpinner (re-use) requires:

● Good control of theory on user side.

● Good understanding of tools on user side.

● Challenge: checks (often adjustments) on event record contents. **Beware:** frequent new classes of event record content abuses.

Simultaneous event generation (eg. with `KKMC`) and weight calculation:

● easier to control and to assure precision for New Physics.

● Convenient for authors, less so for users in fits, evaluation of detection ambiguities.

TauSpinner phase space point from Event Record

$$
\begin{aligned}
d\sigma_{Born} & (x_1, x_2, \hat{s}, \cos\theta) = \sum_{q_f, \bar{q}_f} \\
& [\quad f^{q_f}(x_1, ...) f^{\bar{q}_f}(x_2, ...) d\sigma^{q_f \bar{q}_f}_{Born}(\hat{s}, \cos\theta) \\
& + \quad f^{\bar{q}_f}(x_1, ...) f^{q_f}(x_2, ...) d\sigma^{q_f \bar{q}_f}_{Born}(\hat{s}, -\cos\theta)],
\end{aligned} \tag{1}
$$

where x_1, x_2 denote fractions of incoming protons momenta carried by the corresponding parton, $\hat{s} = x_1\, x_2\, s$ and $f/\bar{f}$ denotes parton (quark-/anti-quark) density functions. We assume that kinematics is reconstructed from four-momenta of the outgoing leptons.

$$
x_{1,2} = \frac{1}{2}\Big(\pm \frac{p_z^{ll}}{E} + \sqrt{(\frac{p_z^{ll}}{E})^2 + \frac{m_{ll}^2}{E^2}} \Big), \tag{2}
$$

where E denotes energy of the proton beam and $p_z^{\ell\ell}$ denotes z-axis momentum of outgoing lepton pair in the laboratory frame and m_{ll} lepton pair virtuality. Note that this formula can be used, as approximation, for the events with hard jets too.

TauSpinner phase space point from Event Record

The $\cos\theta^*$ is then calculated from

$$\cos\theta_1 = \frac{\tau_x^{(1)} b_x^{(1)} + \tau_y^{(1)} b_y^{(1)} + \tau_z^{(1)} b_z^{(1)}}{|\vec{\tau}^{(1)}||\vec{b}^{(1)}|},$$
$$\cos\theta_2 = \frac{\tau_x^{(2)} b_x^{(2)} + \tau_y^{(2)} b_y^{(2)} + \tau_z^{(2)} b_z^{(2)}}{|\vec{\tau}^{(2)}||\vec{b}^{(2)}|}, \qquad (3)$$

as follows:

$$\cos\theta^* = \frac{\cos\theta_1 \sin\theta_2 + \cos\theta_2 \sin\theta_1}{\sin\theta_1 + \sin\theta_2} \qquad (4)$$

where $\vec{\tau}^{(1)}, \vec{\tau}^{(2)}$ denote 3-vectors of outgoing leptons and $\vec{b}^{(1)}, \vec{b}^{(2)}$ denote 3-vectors of incoming beams' four-momenta.

This polar angle definition, is at present the `TauSpinner` default. For tests we have used variants; *Mustraal* (Berends:1983mi) and *Collins-Soper* (Collins:1977iv) frames, which differ when high p_T jets are present.

I skip presentation of how ϕ orientation angle in (x-y) plane is established.

Spin weight, details for intution: easiest but ...

The spin weight $wt = wt_{spin}$ is dimensionless, and contains information of all spin effects transmitted from the production to the decay of τ leptons,

$$wt_{spin} = \sum_{i,j=t,x,y,z} R_{i,j} h^i_{\tau^+} h^j_{\tau^-}. \tag{5}$$

The polarimetric vector $h^i_{\tau^\pm}$ is calculated from formula

$$h^i_{\tau^\pm} = \sum_{\lambda,\bar\lambda} \sigma^i_{\lambda,\bar\lambda} \mathcal{M}^{\tau^\pm}_{\lambda} \mathcal{M}^{\tau^\pm\,\dagger}_{\bar\lambda}, \tag{6}$$

where $\sigma^i_{\lambda,\bar\lambda}$ stands for Pauli matrices. The $h^i_{\tau^\pm}$ are normalized further to set their time-like component to 1

The spin correlation matrix $R_{i,j}$ is calculated from formula

$$R_{i,j} = \sum_{\lambda_1,\bar\lambda_1 \lambda_2,\bar\lambda_2} \sigma^i_{\lambda_1,\bar\lambda_1} \sigma^j_{\lambda_2,\bar\lambda_2} \mathcal{M}^{prod}_{\lambda_1\lambda_2} \mathcal{M}^{prod\,\dagger}_{\bar\lambda_1\bar\lambda_2} \tag{7}$$

Spin weight, details for intution: easiest but ...

The $R_{i,j}$ normalized, its time-like component set to 1, namely all $R_{i,j} = \frac{R_{i,j}}{R_{t,t}}$.

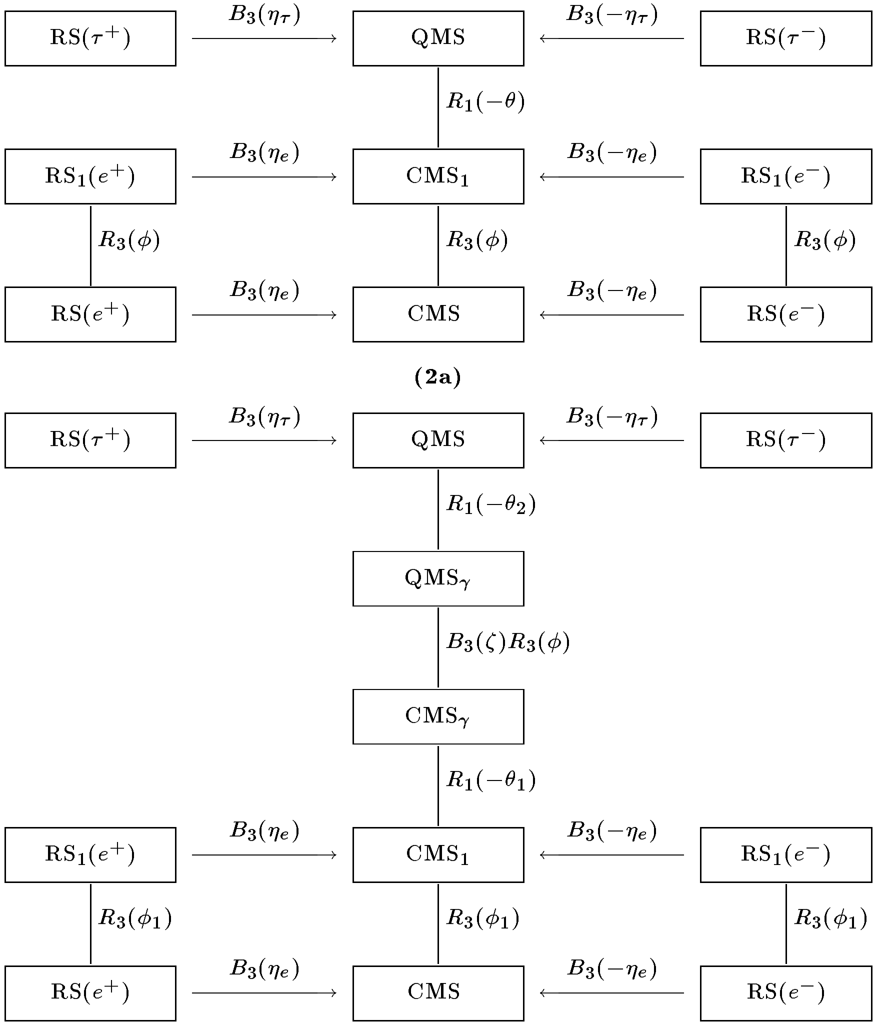
After that average over flavou is taken. No quantume entanglement between incoming parton flavours, but between τ's is taken in:

$$R_{i,j} \to \frac{\sum_{flav.} f(x_1,...)f(x_2,...)\Big(\sum_{\lambda_1,\lambda_2} |\mathcal{M}^{prod}_{parton\ level}|^2\Big) R_{i,j}}{\sum_{flav.} f(x_1,...)f(x_2,...)\Big(\sum_{\lambda_1,\lambda_2} |\mathcal{M}^{prod}_{parton\ level}|^2\Big)}. \tag{8}$$

Spin weight, details for intution: easiest but ...

Frames, boosts rotations for spin; scheme used in: `KORLAB`, `KORALZ`, `TauSpinner`.

Figure 2



$$\begin{tikzcd}
\mathrm{RS}(\tau^+) \arrow[r, "B_3(\eta_\tau)"] & \mathrm{QMS} \arrow[d, dash, "R_1(-\theta)"] & \mathrm{RS}(\tau^-) \arrow[l, "B_3(-\eta_\tau)"'] \\
\mathrm{RS}_1(e^+) \arrow[r, "B_3(\eta_e)"] \arrow[d, dash, "R_3(\phi)"] & \mathrm{CMS}_1 \arrow[d, dash, "R_3(\phi)"] & \mathrm{RS}_1(e^-) \arrow[l, "B_3(-\eta_e)"'] \arrow[d, dash, "R_3(\phi)"] \\
\mathrm{RS}(e^+) \arrow[r, "B_3(\eta_e)"] & \mathrm{CMS} & \mathrm{RS}(e^-) \arrow[l, "B_3(-\eta_e)"']
\end{tikzcd}$$

(2a)

$$\begin{tikzcd}
\mathrm{RS}(\tau^+) \arrow[r, "B_3(\eta_\tau)"] & \mathrm{QMS} \arrow[d, dash, "R_1(-\theta_2)"] & \mathrm{RS}(\tau^-) \arrow[l, "B_3(-\eta_\tau)"'] \\
& \mathrm{QMS}_\gamma \arrow[d, dash, "B_3(\zeta)R_3(\phi)"] & \\
& \mathrm{CMS}_\gamma \arrow[d, dash, "R_1(-\theta_1)"] & \\
\mathrm{RS}_1(e^+) \arrow[r, "B_3(\eta_e)"] \arrow[d, dash, "R_3(\phi_1)"] & \mathrm{CMS}_1 \arrow[d, dash, "R_3(\phi_1)"] & \mathrm{RS}_1(e^-) \arrow[l, "B_3(-\eta_e)"'] \arrow[d, dash, "R_3(\phi_1)"] \\
\mathrm{RS}(e^+) \arrow[r, "B_3(\eta_e)"] & \mathrm{CMS} & \mathrm{RS}(e^-) \arrow[l, "B_3(-\eta_e)"']
\end{tikzcd}$$

(2b)

Spin weight, details for intution: easiest but ...

NOTE:

- In `KKMC` different solution was used for compatibility with Kleiss-Stirling spinor techniques. See Eur.Phys.J.C 22 (2001) 423 for details.

- The idea was to relate τ leptons quantization frames used in production and decay through consecutive transformation from τ rest frame to lab frame and back to τ rest frame.

- That sound complicated for simple rotation representation, but is safe and independent from number of bremsstrahlung photons.

- The $\tau^{\pm}$ decay products and its $h^i_{\tau^{\pm}}$ vector is transformed to the laboratory frame. Then $h^i_{\tau^{\pm}}$ is transformed back to τ lepton rest-frame, but this time of axes oriented as chosen in Kleiss-Stirling spinor techniques.

- We have explored partly this solution for `KKMC` anomalous moment event re-weighting.

Spin weight, details for intution: easiest but ...

Simplified kinematic for NP implementation. Cross section:

$$wt_{ME} = \Big(\sum_{spin} |\mathcal{M}^{prod\ SM+NP}|^2\Big)/\Big(\sum_{spin} |\mathcal{M}^{prod\ SM}|^2\Big)$$

Complicated is spin weight

$$wt_{spin} = \Big(\sum_{ij} R_{ij}^{SM+NP} h_+^i h_-^j\Big)/\Big(\sum_{ij} R_{ij}^{SM} h_+^i h_-^j\Big)$$

The R_{ij} depend on kinematic of τ-pair production, $h_\pm^i$ on $\tau^\pm$ decays.

Spin quantization frames orientation need care. It must be the same for production and decay.

We use `KKMC` routines to transfer $h_\pm^i$ to lab frame and another routines to transfer back to $\tau^\pm$ but oriented as in New Physics calculation.

In this way reference frames are OK and impact of photons on phase space parametrisations is under control.

Solution works for all τ decays!

Spin weight, details for intution: easiest but ...

Main message. Technical side:

- **Use of host program frames is convenient but not essential:** better precision, no need to worry about bremsstrahlung impact etc. Use of internal program variables is helpful too.
- **On the other hand**, this prevents re-use of events for distinct models
- **So far nothing new since last year slides ...**
- **NEW (details next talk):**
  For FCC (`KKMC`): extension of re-weighting algorithm to FCC center of mass energies, electroweak corrections included.
  For LHC (`TauSpinner`): $\gamma\gamma$ parton level processes added, explicit spin correlation matrix R_{ij} prepared for quark initialized processes as well.

but it all helps to develop intuition, and bring fun.

Spin weight, details for intution: easiest but ...

Thank you for listening