The high spin level scheme of ^{156}Dy and neighbouring nuclei.

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Mark Riley, John Sharpey-Schafer and John Simpson,
Phys. Rev. C110, 034313 (2024))

Outline

- The CNS, CNSB (and CNS(B)) models.
- Interpretation of the high-spin $(\pi, \alpha) = (+, 0)$ bands in ^{156}Dy.
- Coexistence at low spin for the $(\pi, \alpha) = (+, 0)$ bands in ^{156}Dy and neighbouring nuclei.
- The $\pi(h_{11/2})$ crossing in ^{156}Dy and neighbouring nuclei
- Summary and conclusions.

Cranked Nilsson-Strutinsky (CNS) -Bogoliubov (CNSB) models.

Modified oscillator or Nilsson potential:

$$V_{MO} = V(\varepsilon_2, \gamma, \varepsilon_4) - \kappa\hbar\,\overset{0}{\omega_0}\,\left(2\vec{l}_t \cdot \vec{s} + \mu\left(l_t^2 - \langle l_t^2\rangle_N\right)\right).$$

The cranking Hamiltonian is diagonalized (CNS):

$$H^{\omega}_{CNS} = H_{MO} - \omega j_x; \quad H^{\omega}_{CNS}\chi^{\omega}_{\nu} = e^{\omega}_{\nu}\chi^{\omega}_{\nu}; \quad e_{\nu} = \langle\chi^{\omega}_{\nu}\,|H_{CNS}|\,\chi^{\omega}_{\nu}\rangle$$

CNSB: $H^{\omega}_{CNSB} = H_{MO} - \omega j_x - \Delta(P^{\dagger} + P) - \lambda\hat{N}$

Total quantities:

$$E_{tot} = E_{rld} + E_{shell}(+E_{pair}); \quad E_{shell} = \sum_{occ} e_{\nu} - \left\langle \sum_{occ} e_{\nu} \right\rangle \quad I = \sum_{occ}\langle j_x\rangle$$

Rotating liquid drop energy:

$$E_{\rm rld}(Z, N, I, \varepsilon_i) = E_{\rm ld}(Z, N, \varepsilon_i) + \frac{\hbar^2 I(I+1)}{2\mathcal{J}_{\rm rig.}(Z, N, \varepsilon_i)}$$

$E_{\rm ld}$: Lublin-Strasbourg drop (LSD model) [or FRLDM]
$\mathcal{J}_{\rm rig.}$: diffuse surface: $r_0 = 1.16$ fm, $a = 0.6$ fm

T. Bengtsson and I.R., NPA 436, 14 (1986); B.G. Carlsson and I.R., PRC 74, 011302 (2006); B.G. Carlsson *et al.* PRC 78, 034316 (2008), Hai-Liang Ma, B.G. Carlsson, I.R. and H. Ryde, PRC 90, 014316 (2014).

Removal of virtual crossings - diabatic configurations.

CNS: Mesh in deformation space: $(\varepsilon_2, \gamma, \varepsilon_4)$.
Unique features to label orbitals and thus to fix conf's, e.g.:

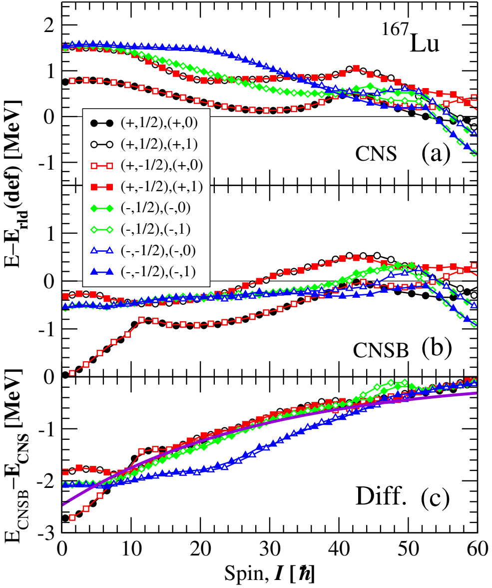
- Diagonalization in rotating harmonic oscillator basis, N_{rot}-shells treated as pure
- Labelling of high-j shells and low-j shells, respectively

CNSB: Mesh in def. and pairing space: $(\varepsilon_2, \gamma, \varepsilon_4, \lambda_p, \lambda_n, \Delta_p, \Delta_n)$

- Minimization in $(\varepsilon_2, \gamma, \varepsilon_4, \lambda_p, \lambda_n, \Delta_p, \Delta_n)$ space
- Particle number projection.
- Removal of virtual interactions for fixed parameters

$\Rightarrow$ Possible to draw smooth PES's in the full (ε_2, γ)-plane.
Conf's: $(\pi, \alpha)_p(\pi, \alpha)_n$

Comparison CNS) ↔ CNSB



Analyze rotational bands: $\mathcal{J}^{(1)}$, $\mathcal{J}^{(2)}$ or I vs. ω

More illustrative:
$E - E_{rld}(def)$

Comparison between CNS and CNSB for ^{167}Lu.

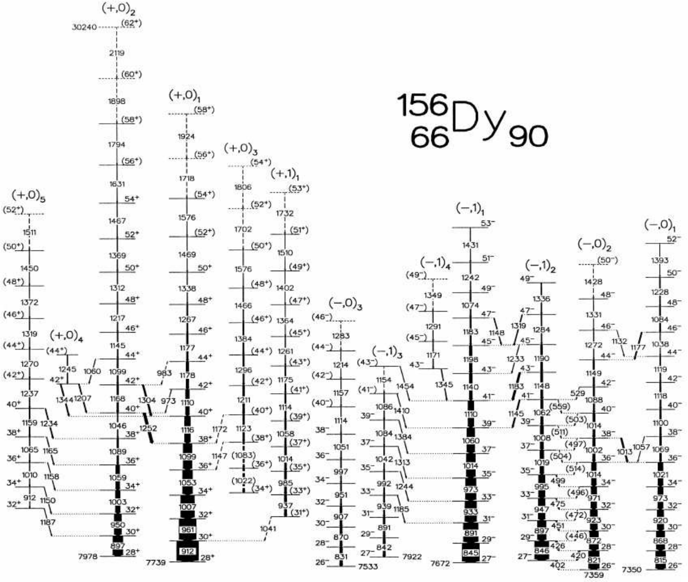
Conclusion:
Add average pairing to CNS:

$$< E_{pair} >= -2.47 \exp(-I/29)$$

CNS(B)

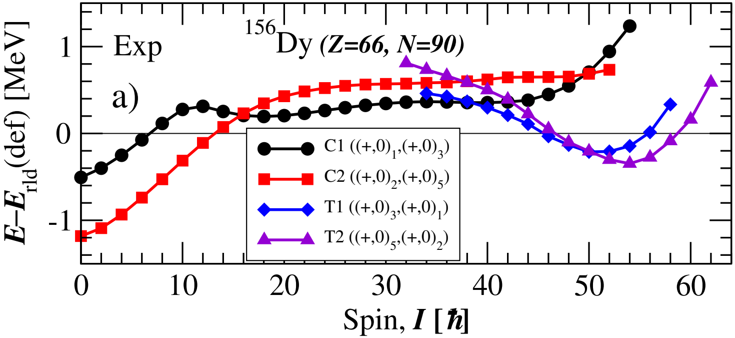
A. Kardan, I.R., B.G. Carlsson and Hai-Liang Ma, PRC 101, 014323 (2020)

High spin level scheme; ^{156}Dy



F.G. Kondev, M.A. Riley, R,V.F. Janssens, J. Simpson, A.V. Afanasjev, I.R. et al., PLB 437, 35 (1998).

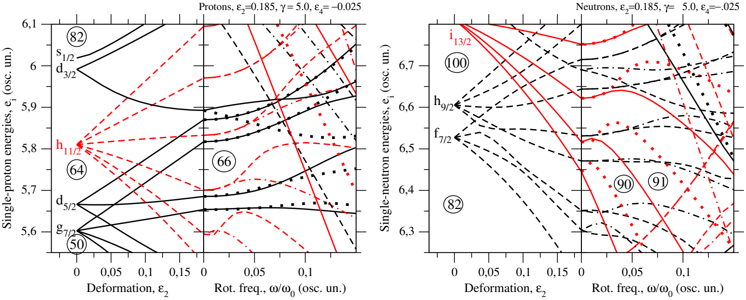
$E - E_{rld}$(def); $(+,0)$ bands, ^{156}Dy.



Collective bands: C1, C2

Terminating bands: T1,T2

Nilsson diagrams; single particle routhians.

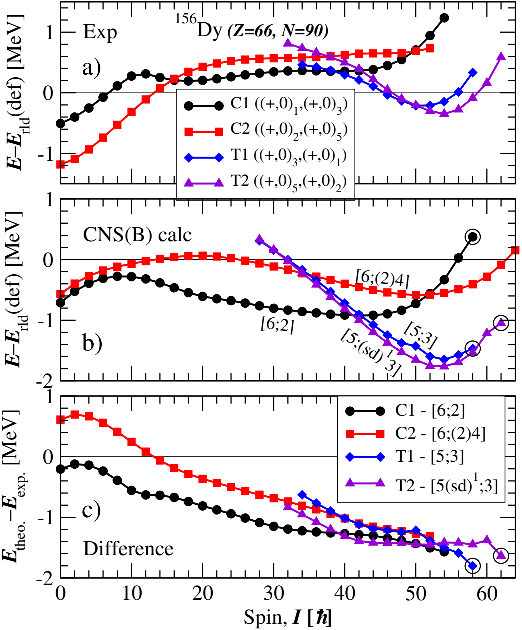


Configurations, ^{156}Dy: [# of $h_{11/2}$ protons; # of $i_{13/2}$ neutrons]

Low-energy configurations at low spin: [6;2], [6;(2)4]

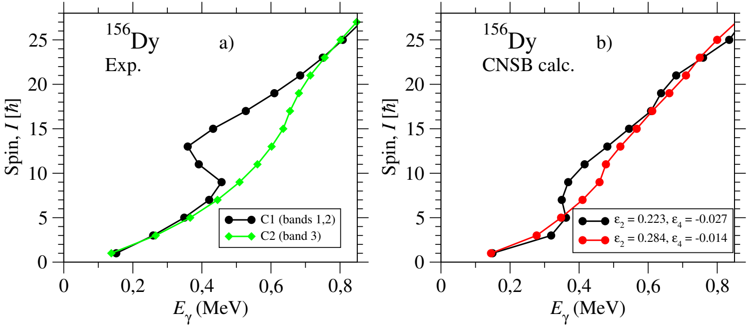
at high spin [5;3]

Comparison, exp ↔ calc; $(+,0)$ bands, ^{156}Dy.



Convincing agreement between exp. and calc. Note that $I = 58$ is the highest spin you can build in [5;3] conf. $+4\hbar$ with $(dg)^{-1}sd^{1}$ excitation.

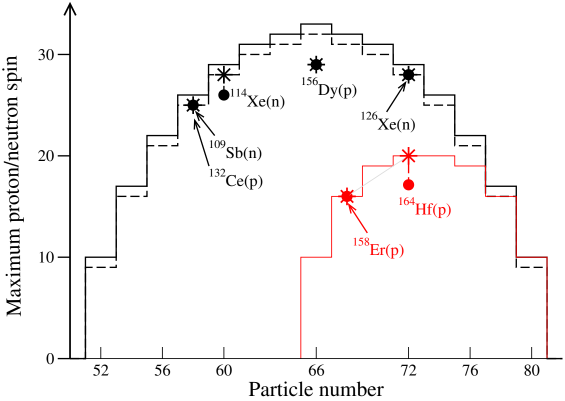
$i_{13/2}$ alignment in ^{156}Dy (backbend or upbend)



Calculations at fixed deformations:
Minima of [6;2] and [6;(2)4] conf's in CNS calculations.

Alignment in the bottom of a high-*j* shell:
F.S. Stephens, R.M. Diamond and S.G. Nilsson, Phys. Lett. 44B, 429.
S.G. Nilsson and I.R., Shapes and shells in Nuclear Structure, Cambridge Univ. Press, 1995, 2005.

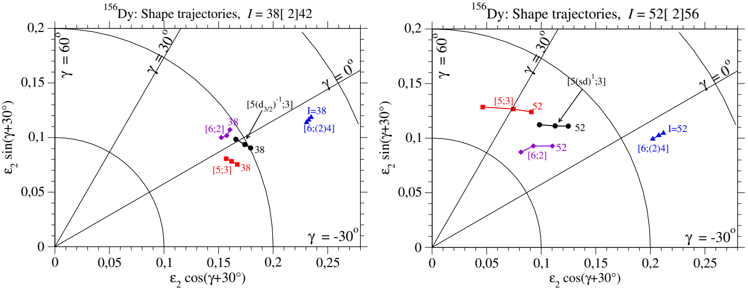
Maximum spin in shells.



Maximum spin in a half-full shell. Thus $Z = 66$ for ^{156}Dy is in the middle of the Z/N = 50-82 shell.

Max. value is $33\hbar$. In the highest spin band in ^{156}Dy, $I_p = 29$ has been observed, i.e. only $4\hbar$ below the maximum value where all 16 valence particles rotate in the same direction around the core.

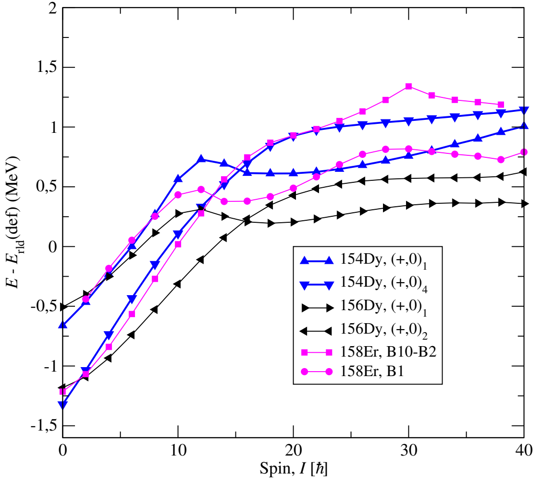
Shape coexistence, ^{156}Dy; $I = 38, 40, 42$, $I = 52, 54, 56$



Valence space
configurations: $\varepsilon_2 \approx 0.20$ for $I \approx 40$
coming close to termination for $I \approx 55$.

$(h_{11/})^{-2}$ configuration: Stays at $\varepsilon_2 > 0.20$ for spin values up to $I \approx 60$.

Similar bands in ^{154,156}Dy, ^{158}Er.



Coexistence between:

$\pi(h_{11/2})^6\nu(i_{13/2})^2$

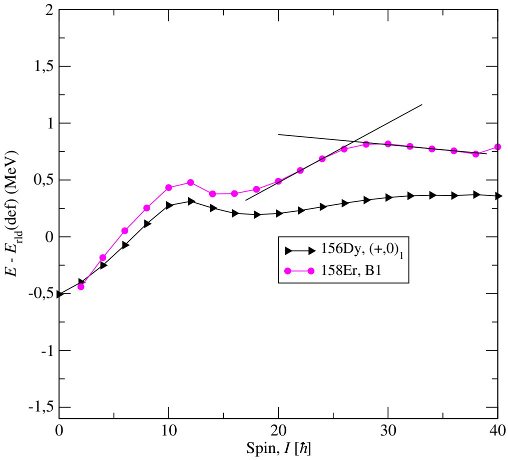
and

$\pi(h_{11/2})^6$
$\nu(h_{11/2})^{-2}(i_{13/2})^4$

configurations.

^{158}Er: J. Simpson, M.A. Riley et al., PRC 107, 054305 (2023)
^{154}Dy: W.C. Ma et al., PRC 65, 034312 (2002).

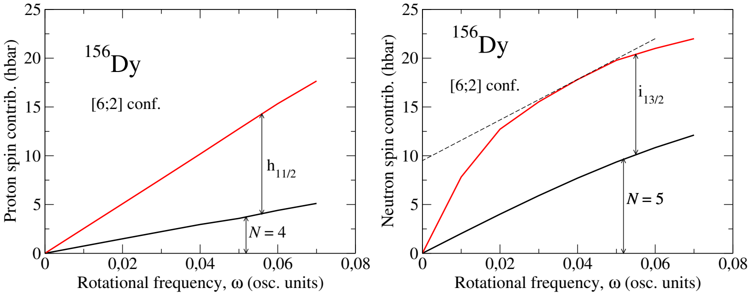
^{156}Dy, ^{158}Er: $\pi(h_{11/2})$ bandcrossing around $I = 30$?



"Accepted" explanation: Alignment of a pair of $h_{11/2}$ protons but interaction strength so large in ^{156}Dy that the alignment is smeared out over an extended spin range.

Oscillating behaviour of the yrast-yrare interaction:
R. Bengtsson, I. Hamamoto and B.R. Mottelson, PL 73B, 259 (1978)

Building of spin in the [6;2] configuration of ^{156}Dy, CNS

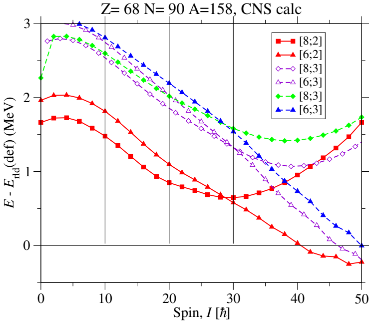


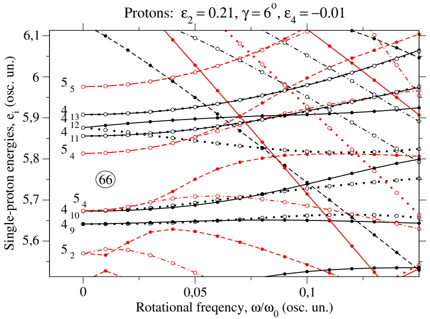
Large spin contribution from $i_{13/2}$ already at small frequencies.
Linear increase with ω from other j-shells.

$$N = 4\text{: } \pi(d_{5/2}, g_{7/2});\ N = 5\text{: } \nu(f_{7/2}, h_{9/2});$$

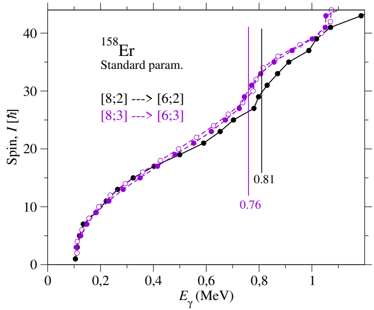
No special contribution from $h_{11/2}$ protons!

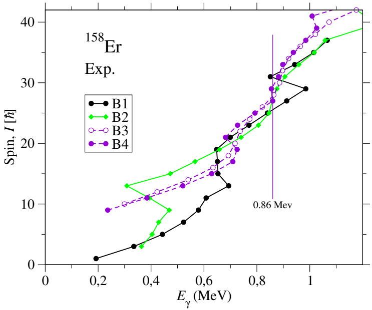
Crossing between $\pi(h_{11/2})^8$ and $\pi(h_{11/2})^8$ bands in ^{158}Er, CNS.





The $\pi(h_{11/2})^8 \rightarrow \pi(h_{11/2})^6$ transition in ^{158}Er

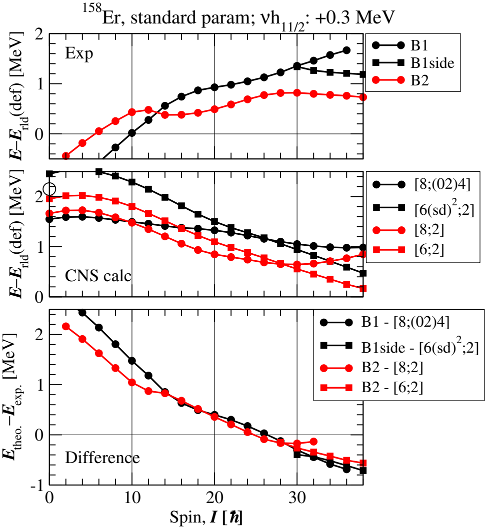




Calculation, crossing at
$E_\gamma \approx 0.78$ MeV
$\hbar\omega = 0.39$ MeV

Exp., crossing at
B2-B4: $E_\gamma \approx 0.86$ MeV
B1: Crossing looks different
$\nu(h_{11/2})^{-2} \rightarrow \pi(h_{11/2})^6$

Compare exp ↔ calc., $(+,0)$ in ^{158}Er.



Differences well collected showing a pairing energy decreasing with spin.

Previous publications; # of protons in $h_{11/2}$.

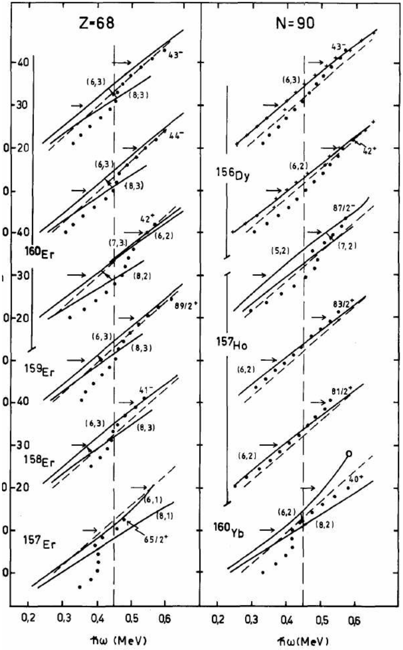
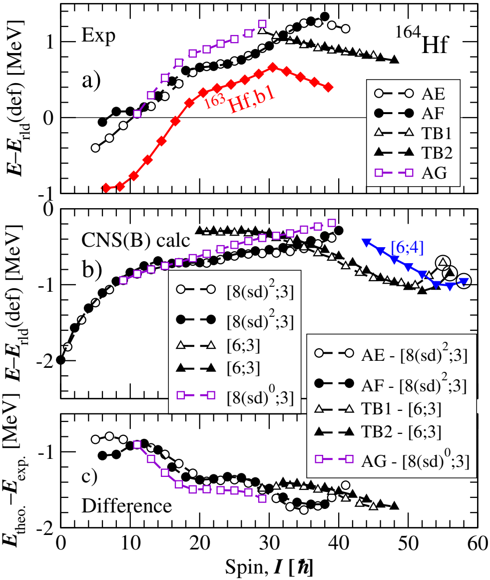


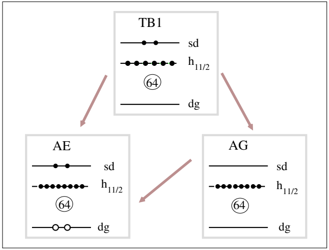
Figure from T. Bengtsson and I.R., Phys. Lett. **163B**, 31 (1985).

F. Grümmer, K.W. Schmid and A. Faessler, Nucl. Phys. A326, 1 (1979) :
It is shown that the oscillations of the yrast-yrare coupling can be explained as being due to the mixing of states with different particle numbers inside the valence shell.

$\pi(h_{11/2})^8 \rightarrow \pi(h_{11/2})^6$ transition in ^{164}Hf



J.C. Marsh et al., Phys. Rev. C88, 041306(R) (2013)



Transition from AE configuration to TB1 configuration.

Summary

- Detailed understanding of collective and terminating bands in ^{156}Dy up to the highest spin, $I \approx 60$
- The bands built on the two lowest 0^+ states in ^{154,156}Dy and ^{158}Er are assigned as collective coexisting bands for $I \approx 10 - 40$ suggesting coexistence also for lower spin values down to $I = 0$.
- Assignments at higher spin are often more reliable because pairing is negligible. These assignments can be helpful for the understanding of lower spin states.
- The band crossings at $I \approx 30$ in rare earth region can be understood in an unpaired formalism as caused by a change of the number of $h_{11/2}$ protons at the crossing.
- In the nuclei analyzed by us, except for the first $i_{13/2}$ alignment, the band-crossings can be described in an unpaired formalism (or adding an average pairing).

Thank you!