Clustering and collective dynamics in atomic nuclei

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Florida State University

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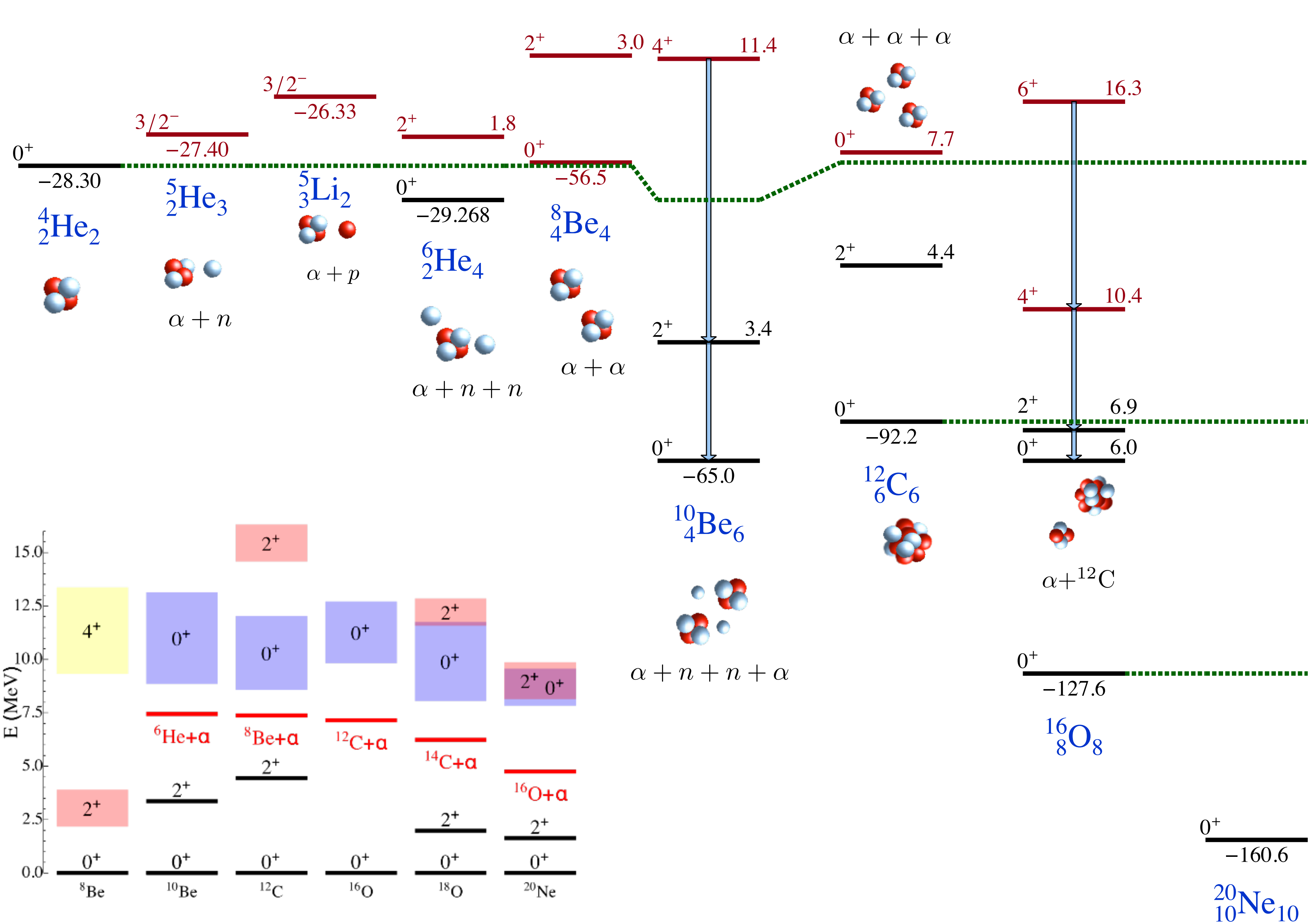
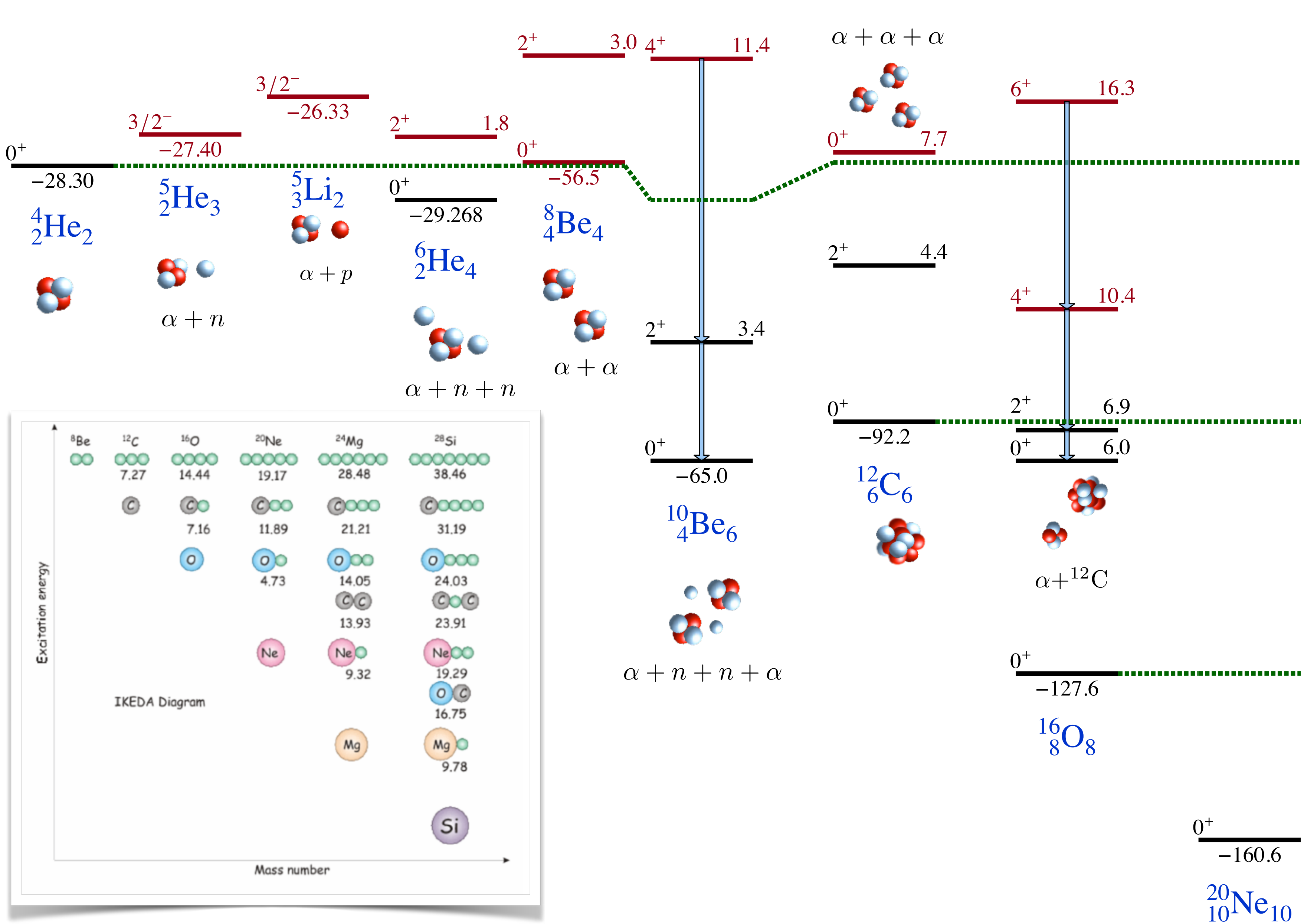
Clustering in light nuclei



Clustering in light nuclei



New look into clustering

- Fundamental questions of current interest
  - Nature and emergence of cluster degrees of freedom
  - Role of clustering in nuclear structure: interplay with shapes and rotational dynamics
  - Threshold phenomena and decay processes: superradiance, particle decay, and electromagnetic transitions
  - Cluster transfer reactions
  - Broad, overlapping alpha resonances.
- Revisiting classical ideas with advanced tools and techniques.

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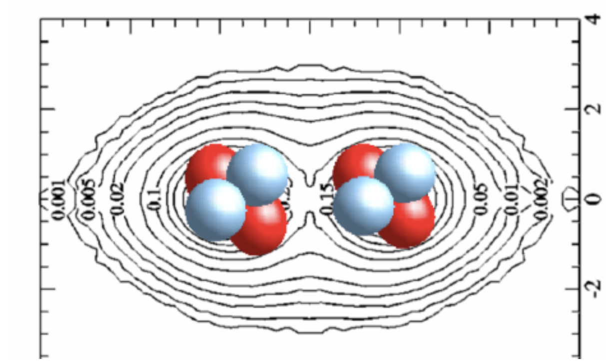
GFMC nuclear matter density distribution for ^{8}Be g.s.



R.B. Wiringa, et al., PRC 62 (2000)

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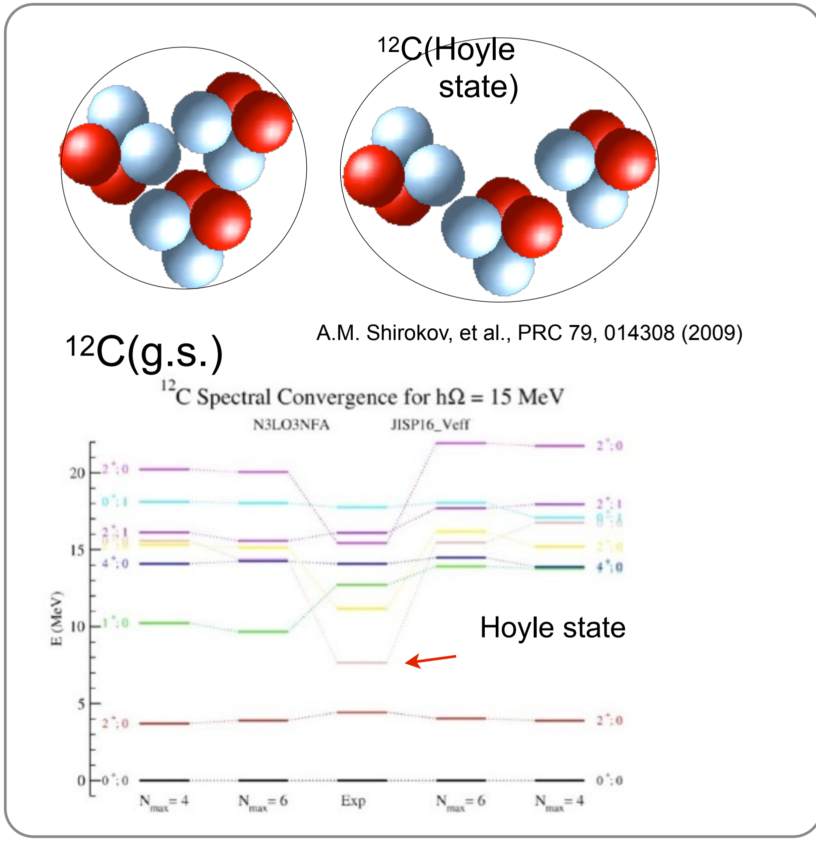
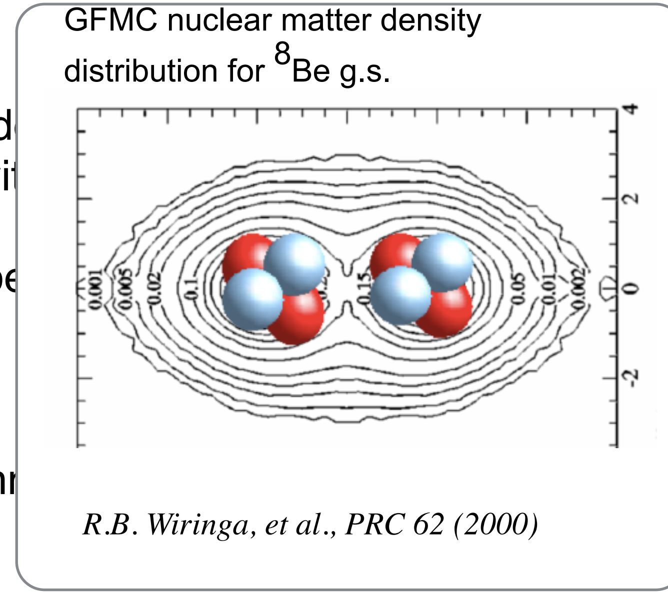


A.M. Shirokov, et al., PRC 79, 014308 (2009)



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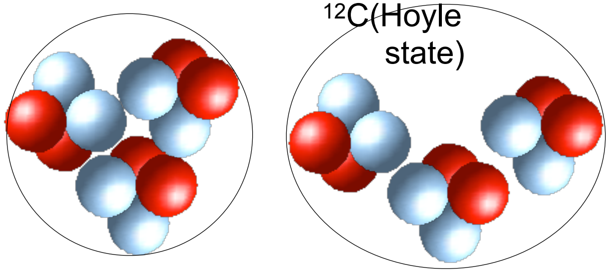

12C(g.s.)

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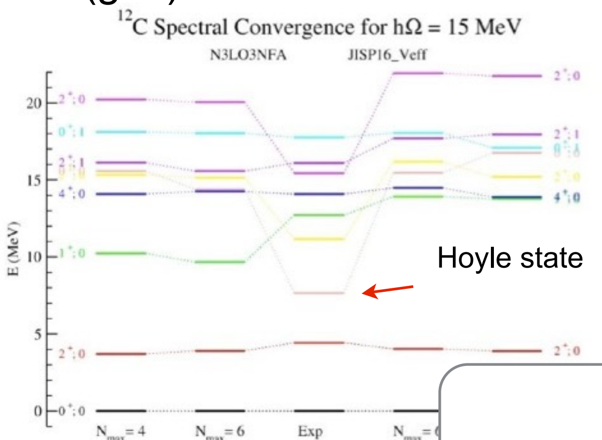

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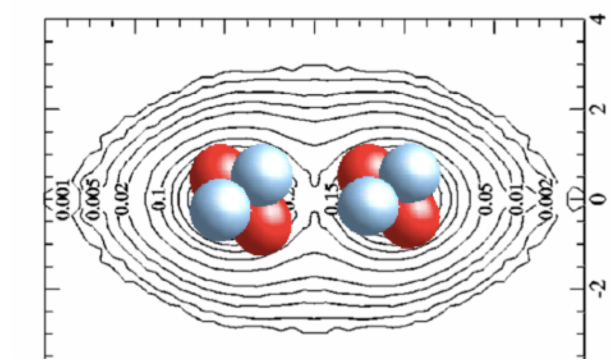
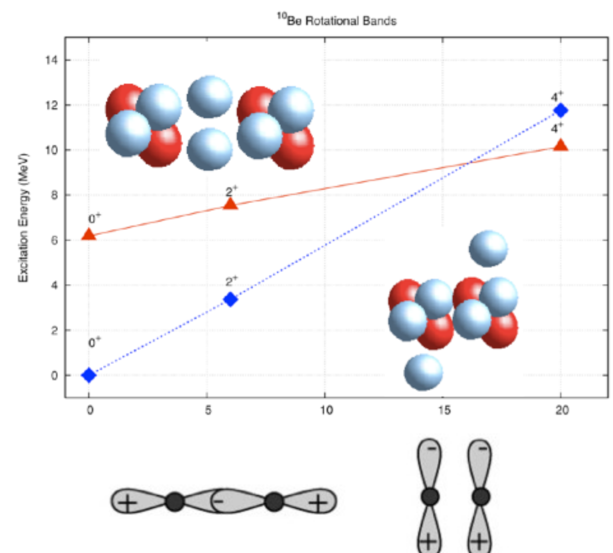


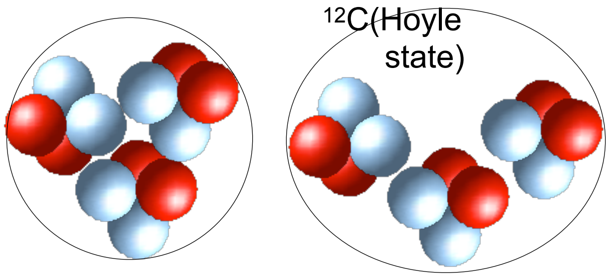
R.B. Wiringa, et al., PRC 62 (2000)

- Rotational band with high moment of inertia built on 0^+ at 6.18 MeV
- 10.15 MeV state reported to be extremely clustered [1]
- Believed to be associated with α-2n-α molecular rotational band.



[1] M. Freer, et al., PRL 96, 042501 (2006)
[2] M. Milin, et al., NPA 753, 263 (2005)
[3] N. Curtis, et al., PRC 64, 044604 (2001

Selected recent clustering studies



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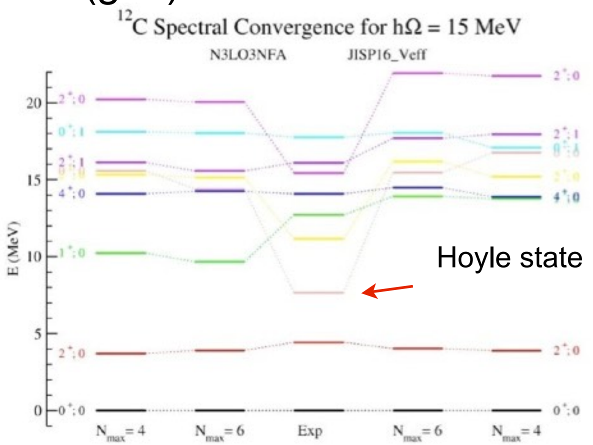

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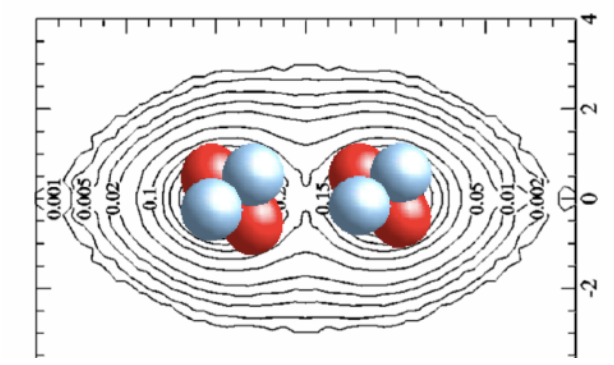

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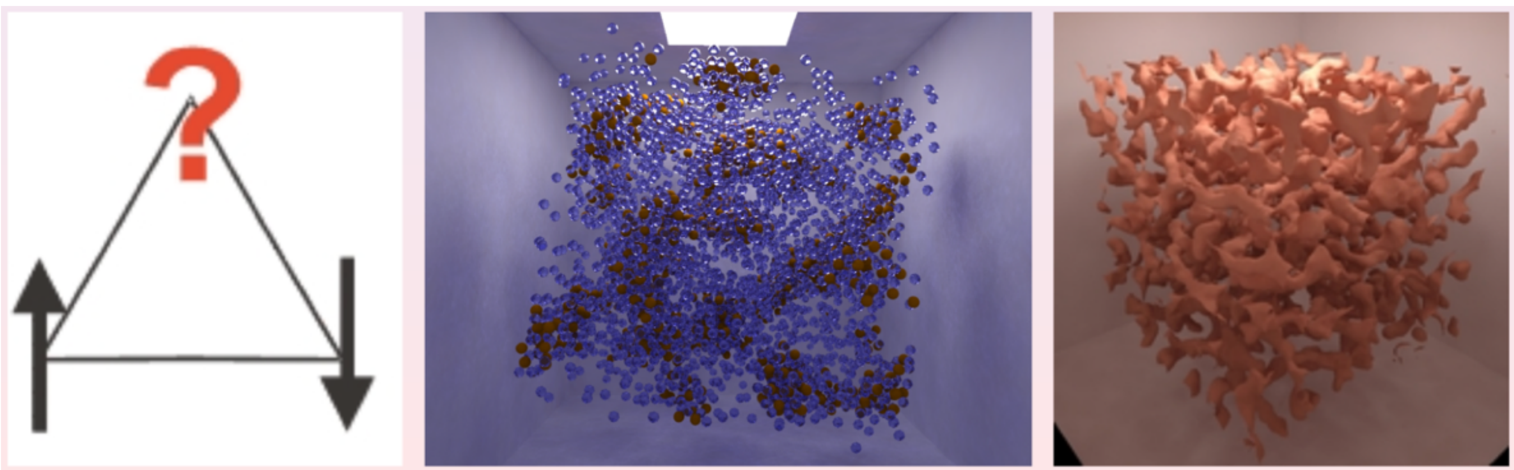
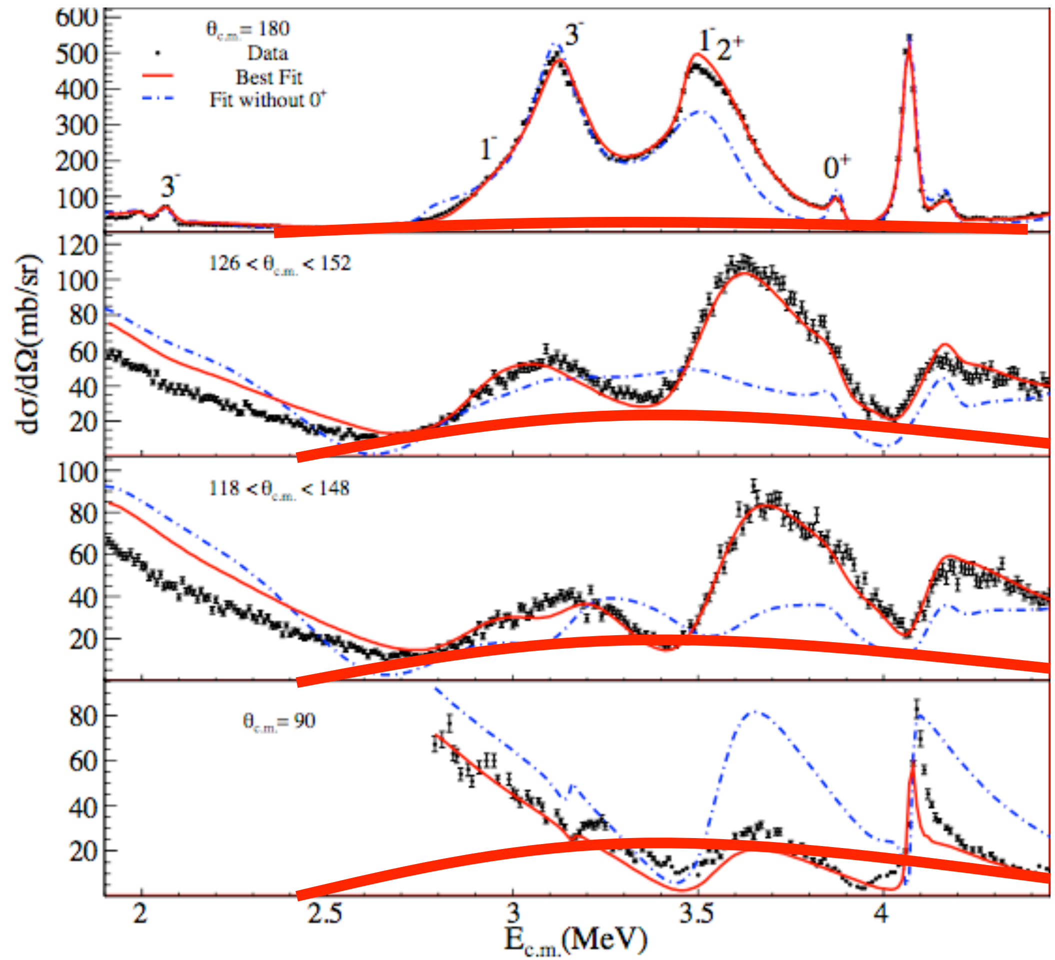


Clustering in nuclear and atomic physics: nuclear pasta

New experimental techniques

Broad 0^+ alpha state at excitation energy of 9.9 MeV $\alpha+^{14}C$



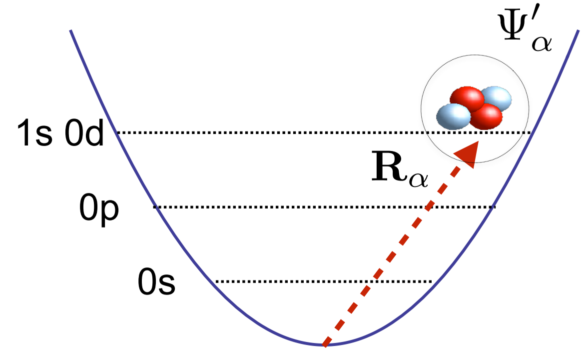
From G. Rogachev, see E.D. Johnson, et al., EPJA, 42 135 (2009)

Clustering in atomic nuclei

- Norm kernel and blocking
- Clustering in ground states
- Role of valence particles
- Emergence of cluster degrees of freedom, and entanglement
- Experimental results.
- Learning from $N \neq Z$ nuclei

Center-of-Mass boosts

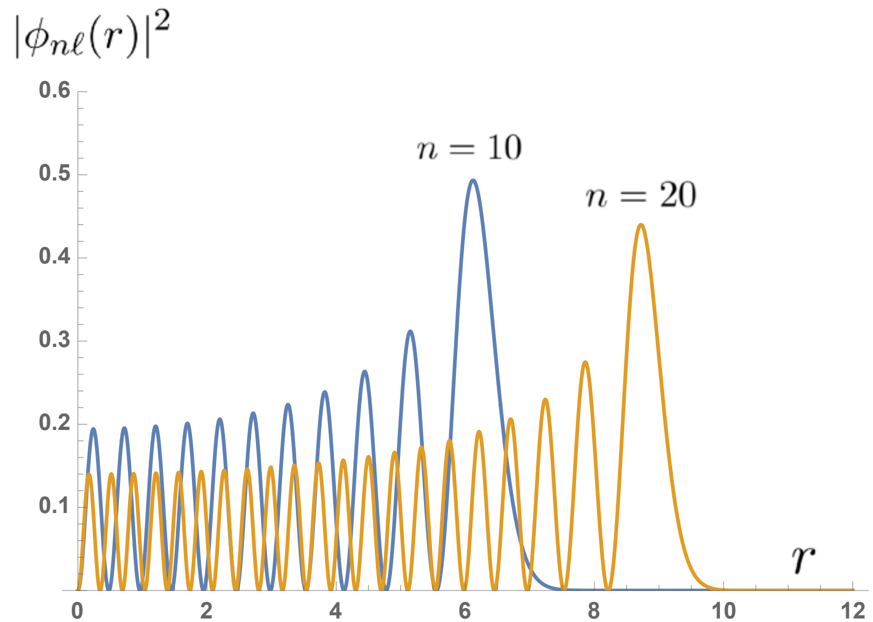
$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$



$$N = 2n + \ell$$

Control only CM quanta

$$R_\mu = \sqrt{\frac{\hbar}{2Am\omega}}(\mathcal{B}^\dagger_\mu + \mathcal{B}_\mu)$$



$\boldsymbol{\mathcal{B}}^\dagger$ and $\boldsymbol{\mathcal{B}}$ CM quanta creation and annihilation (vectors)

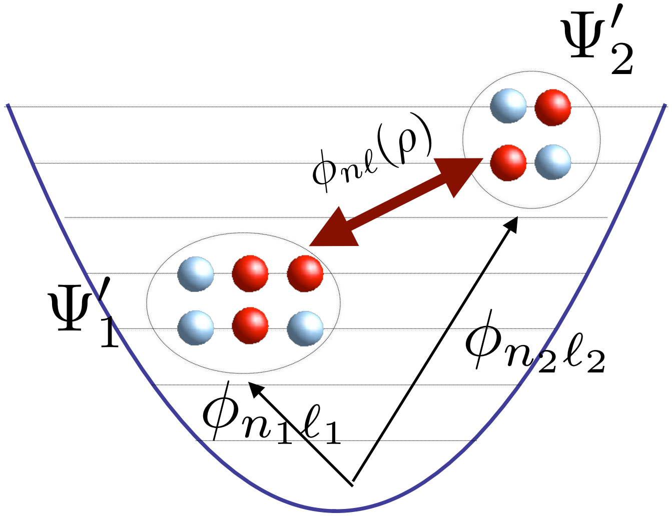
$$\Psi_{n+1\ell m} \propto \boldsymbol{\mathcal{B}}^\dagger \cdot \boldsymbol{\mathcal{B}}^\dagger \Psi_{n\ell m}$$

$\boldsymbol{\mathcal{B}}^\dagger \times \boldsymbol{\mathcal{B}}$ CM angular momentum operator

K Kravvaris and A. Volya, Journal of Phys, Conf. Proc. 863, 012016 (2017)

Clustering reaction basis channel

(basis states for clustering)



$$\Psi = \phi_{000}(\mathbf{R})\Psi'$$

Boost

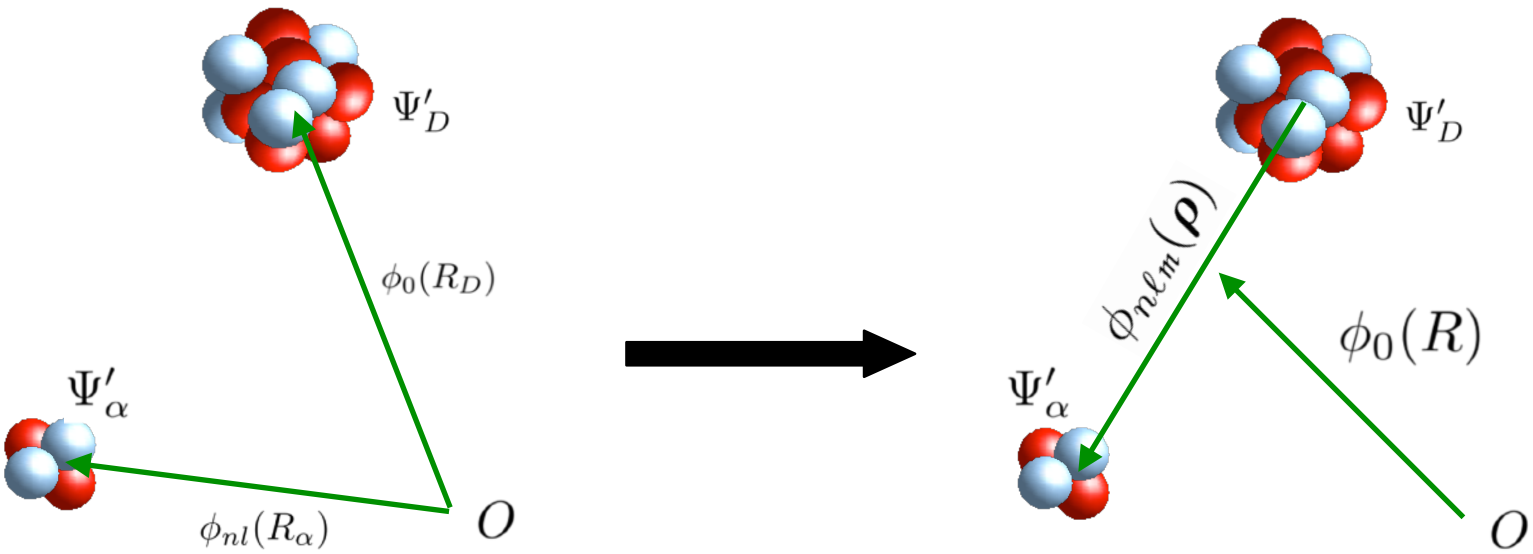
$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$

CM-Recouple

$$\Phi_{n\ell m} = \mathcal{A}\left\{\phi_{000}(\mathbf{R})\phi_{n\ell m}(\boldsymbol{\rho})\Psi'^{(1)}\Psi'^{(2)}\right\}$$

$$\Phi^{\dagger}_{n\ell} = \sum_{\substack{n_1\ell_1\\ n_2\ell_2}} \mathcal{M}^{n\ell 00;\ell}_{n_1\ell_1 n_2\ell_2}\left[\Psi^{\dagger}_{n_1\ell_1 m_1} \times \Psi^{\dagger}_{n_2\ell_2 m_2}\right]_{\ell}$$

Recoil Recoupling



- Recoupling is done with Talmi-Moshinsky brackets

$$\Phi_{n\ell m} = \mathcal{A}\left\{\phi_{000}(\mathbf{R})\phi_{n\ell m}(\boldsymbol{\rho})\Psi'^{(1)}\Psi'^{(2)}\right\}$$

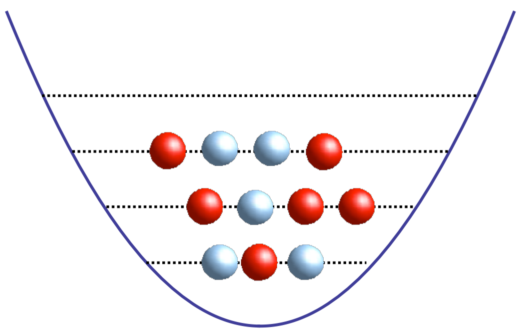
Configuration interaction approach and clustering

Traditional shell model configuration m-scheme

Cluster configuration

$$|\Psi\rangle = \Psi^\dagger|0\rangle \sim a_1^\dagger a_2^\dagger \dots a_A^\dagger|0\rangle$$

$$|\text{channel}\rangle \sim |\Phi\Psi_D\rangle \equiv \Phi^\dagger\Psi_D^\dagger|0\rangle$$



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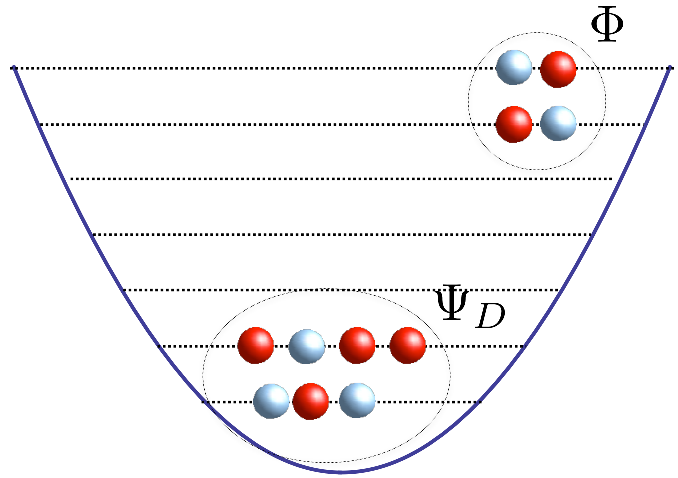
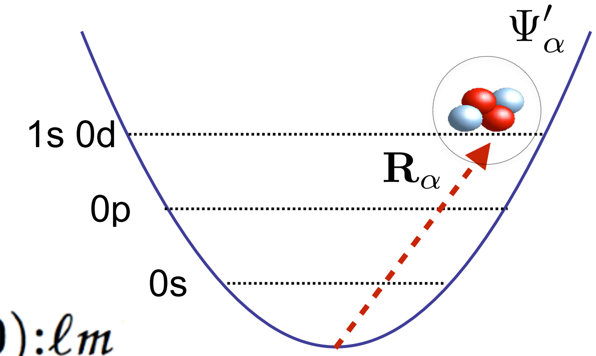


$|\Psi\rangle$ + $\Phi^\dagger|\Psi_D\rangle$

K. Kravvaris and A. Volya, Clustering in structure and reactions using configuration interaction techniques, Phys. Rev. C 100, 034321 (2019).

Center-of-Mass boosts



$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$

$$\Psi_\alpha = \phi_{n\ell m}(\mathbf{R})\,\Psi'_\alpha = \sum_\eta X^\eta_{n\ell}\,\Phi^\eta_{(n,0):\ell m}$$

Configuration	$N_{\max}=0$	$N_{\max}=4$
$(sd)^4$	0.038	0.035
$(p)(sd)^2(pf)$	0.308	0.282
$(p)^2(pf)^2$	0.103	0.094
$(p)^2(sd)(sdg)$	0.154	0.141
$(s)^2(sd)(sdgi)$	0.000	0.005
$(p)(sd)(pf)(sdg)$	0.000	0.009

Select configuration content of NCSM wave functions for ^{4}He with $\hbar\omega$ = 20 MeV boosted by 8 quanta (L = 0).

K Kravvaris and A. Volya, Journal of Phys, Conf. Proc. 863, 012016 (2017)

$$\underbrace{\phi_{n\ell m}(1)\phi_{n\ell m}(2)\phi_{n\ell m}(3)\phi_{n\ell m}(4)}_{4\times2=8\text{ quanta}} \leftrightarrow \sum_\eta X^\eta_{n'\ell}\Phi^\eta_{(8,0):\ell m} = \underbrace{\phi_{n'\ell'm'}(\mathbf{R}_\alpha)}_{8\text{ quanta}}\ \underbrace{\Psi'_\alpha}_{0\text{ quanta}}$$

m-scheme state — SU(3) symmetry state — motion of alpha

Volya and Yu. M. Tchuvil'sky, Phys. Rev. C 91, 044319 (2015).
Yu. F. Smirnov and Yu. M. Tchuvil'sky, Phys. Rev. C 15, 84 (1977).
M. Ichimura, A. Arima, E. C. Halbert, and T. Terasawa, Nucl. Phys. A 204, 225 (1973).
O. F. Nemetz, V. G. Neudatchin, A. T. Rudchik, Yu. F. Smirnov, and Yu. M. Tchuvil'sky, Nucleon Clusters in Atomic Nuclei and Multi-Nucleon Transfer Reactions (Naukova Dumka, Kiev, 1988), p. 295.

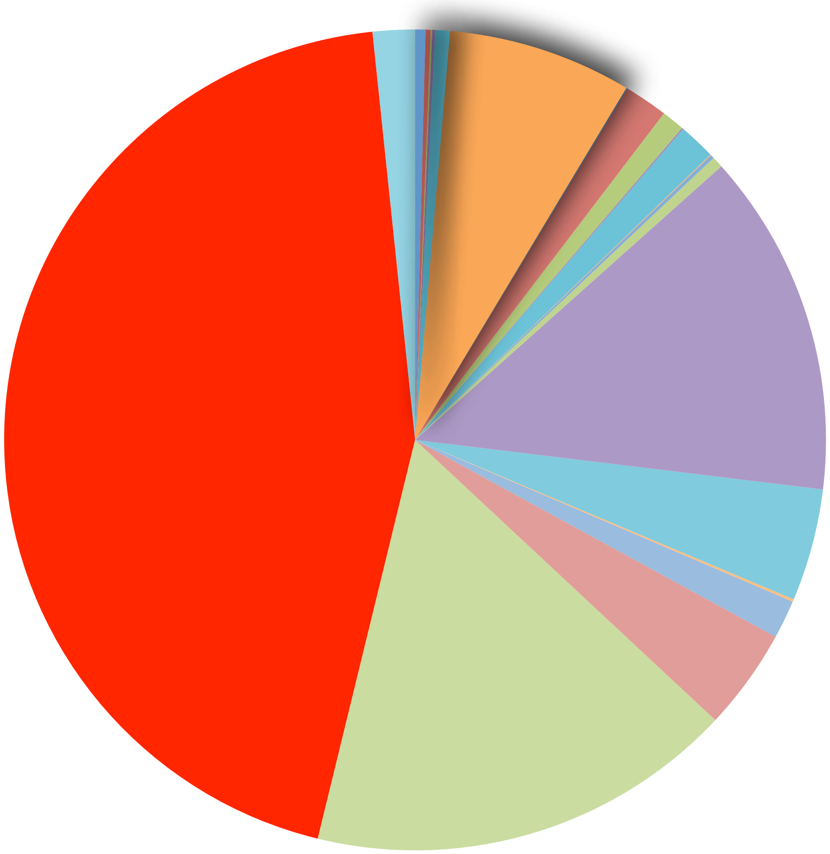
Quartet that corresponds to alpha cluster $\ell = 0$

n	X^2	(8,0)	(4,2)	(0,4)	(2,0)
4	0.02848	1.0	0.0	0.0	0.0
3	0.00697	0.561658	0.438338	0.0	0.0
2	0.00169	0.549804	0.0451847	0.3363	0.0636439
1	0.00018	0.0693304	0.735878	0.0134005	0.147418
0	0.00011	0.0693304	0.261291	0.0990471	0.0384533

$L = S = T = 0 \qquad (\lambda, \mu) = (8{,}0),\ (4{,}2),\ (0{,}4),\ \text{or}\ (2{,}0)$

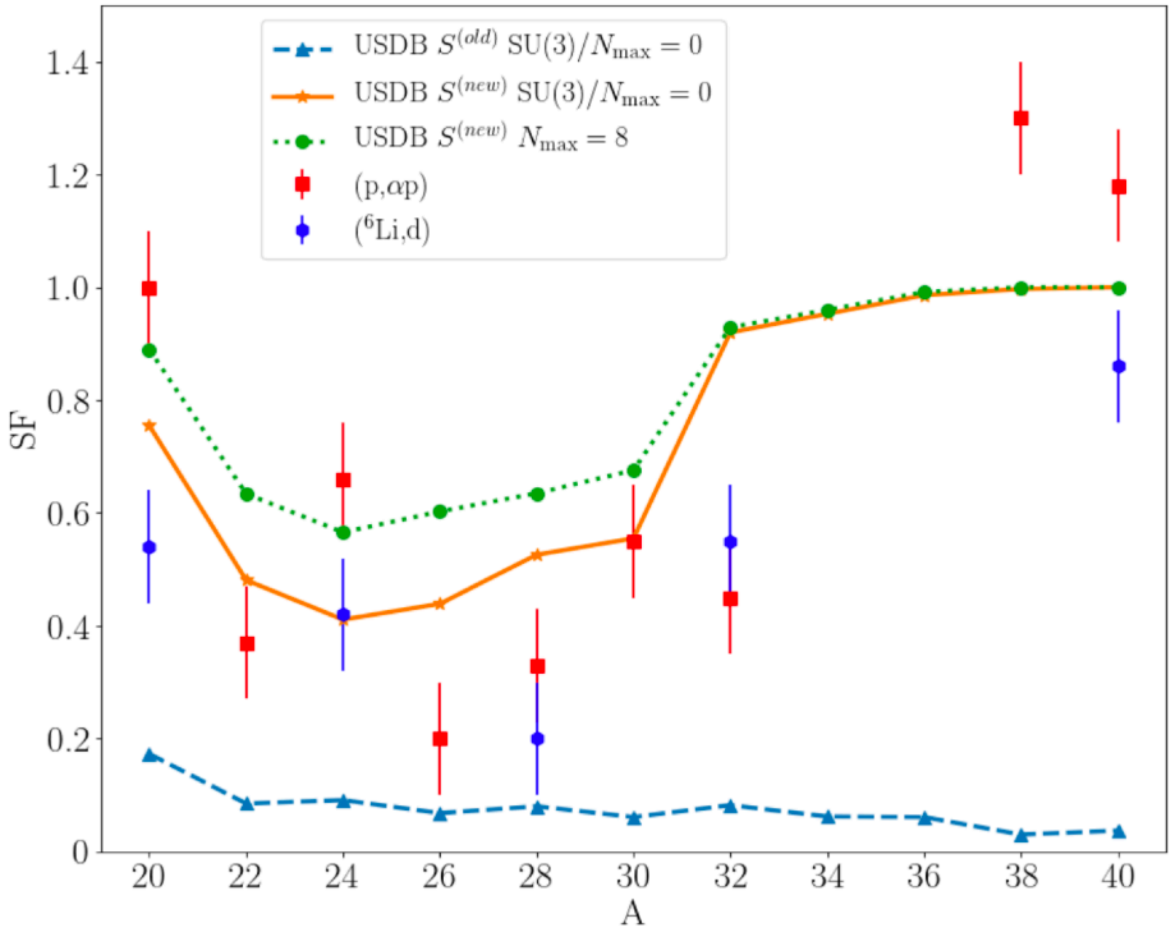
In light nuclei SU(3) symmetry plays significant role for clustering and deformation
Norm kernel and Pauli blocking plays an important role.
Effective operator for alpha cluster is significantly modified.

Classic Example: ^{24}Mg, 8 nucleons in sd-shell



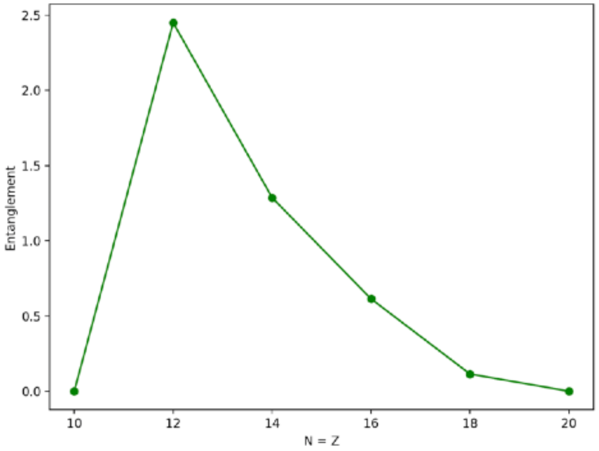
USD realistic interaction
Ground state J=T=0
breakdown in SU(3) irreps

Understanding alpha clustering in sd-shell nuclei



USDB interaction [5]
(8,0) configuration

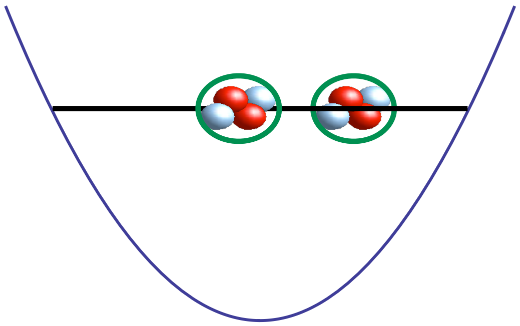
- Old SF are small
- Old SF decrease with A



[1] T. Carey, P. Roos, N. Chant, A. Nadasen, and H. L. Chen, Phys. Rev. C 23, 576(R) (1981).
[2] T. Carey, P. Roos, N. Chant, A. Nadasen, and H. L. Chen, Phys. Rev. C 29, 1273 (1984).
[3] N. Anantaraman and et al., Phys. Rev. Lett. 35, 1131 (1975).
[4] W. Chung, J. van Hienen, B. H. Wildenthal, and C. L. Bennett, Phys. Lett. B 79, 381 (1978).
[5] B. A. Brown and W. A. Richter, Phys. Rev. C 74, 034315 (2006)

Bosonic nature of 4-nucleon operators non-orgothogonality

If $\Phi^\dagger$ is thought of as being a boson then $\Phi\Phi^\dagger = 1 + N_b$



$$|\Psi_D\rangle = |\Phi\rangle \quad \langle\Phi_D|\hat{\Phi}\hat{\Phi}^\dagger|\Psi_D\rangle = \langle 0|\hat{\Phi}\hat{\Phi}\hat{\Phi}^\dagger\hat{\Phi}^\dagger|0\rangle = 2$$

$$L = S = T = 0$$

Φ	Ψ_P	$\left|\langle\Psi_P|\hat{\Phi}^\dagger|\Psi_D\rangle\right|^2$	$\langle 0|\hat{\Phi}\hat{\Phi}\hat{\Phi}^\dagger\hat{\Phi}^\dagger|0\rangle$
$(p)^4\,(4,0)$	$(p)^8\,(0,4)$	$1.42222^{\star}$	1.42222
$(sd)^4\,(8,0)$	$(sd)^8\,(8,4)$	0.487903	1.20213
$(fp)^4\,(12,0)$	$(fp)^8\,(16,4)$	0.292411	1.41503
$(sdg)^4\,(16,0)$	$(sdg)^8\,(24,4)$	0.209525	1.5278

* For p-shell the result is known analytically 64/45

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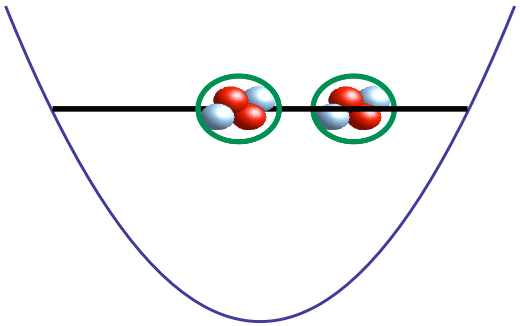

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Effective operators (alphas) are not ideal bosons

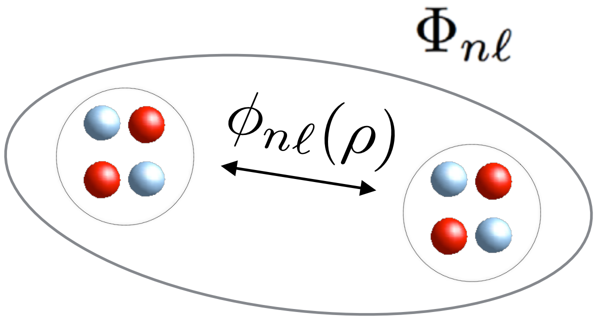
Cluster configurations are not orthogonal and not normalized

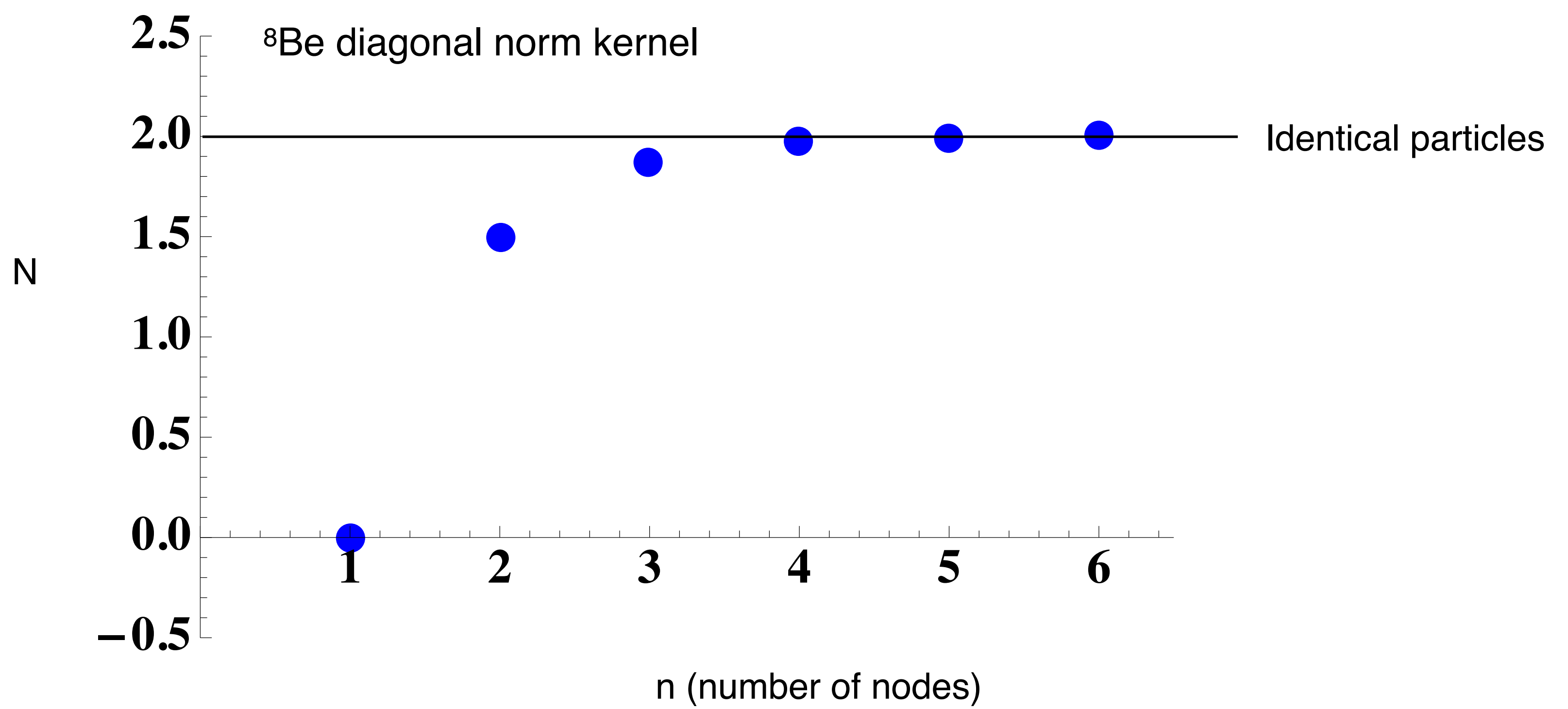
Resonating group method

$$\mathcal{F}_\ell(\rho) = \sum_n \chi_n \Phi_{n\ell}$$

$$\sum_n \mathcal{H}^{(\ell)}_{nn'} \chi_{n'} = E \sum_n \mathcal{N}^{(\ell)}_{nn'} \chi_{n'}$$

$$\mathcal{H}^{(\ell)}_{nn'} = \langle \Phi_{n\ell} | H | \Phi_{n'\ell} \rangle \qquad \mathcal{N}^{(\ell)}_{nn'} = \langle \Phi_{n\ell} | \Phi_{n'\ell} \rangle$$

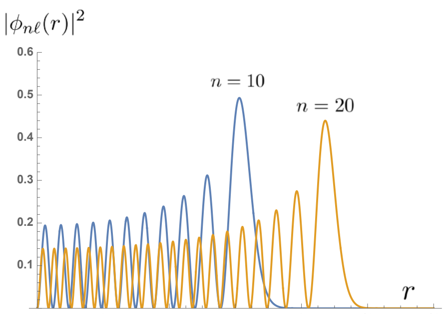




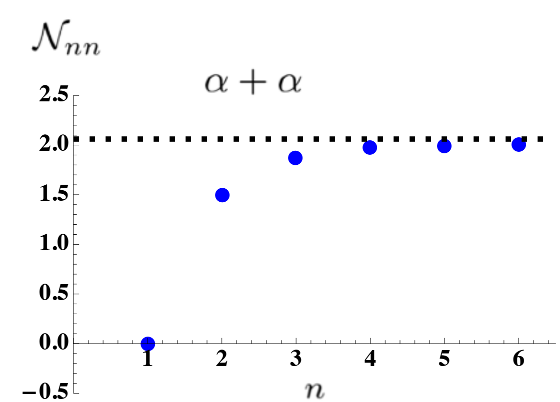
Resonating group method and reactions

$$\sum_n \mathcal{H}^{(\ell)}_{nn'}\chi_{n'} = E\sum_n \mathcal{N}^{(\ell)}_{nn'}\chi_{n'}$$

$$\begin{pmatrix} \mathcal{H}_{00} & \dots & \mathcal{H}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{H}_{n0} & \dots & \mathcal{H}_{nn} & T_{nn+1} & 0 & \vdots \\ 0 & 0 & T_{n+1n} & T_{n+1n+1} & T_{n+1n+2} & 0 \\ 0 & \dots & 0 & T_{n+2n+1} & T_{n+2n+2} & \ddots \\ 0 & \dots & \dots & 0 & \ddots & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix} = \mathrm{E} \begin{pmatrix} \mathcal{N}_{00} & \dots & \mathcal{N}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{N}_{n0} & \dots & \mathcal{N}_{nn} & 0 & 0 & \vdots \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \dots & 0 & 0 & 1 & 0 \\ 0 & \dots & \dots & 0 & 0 & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix}$$



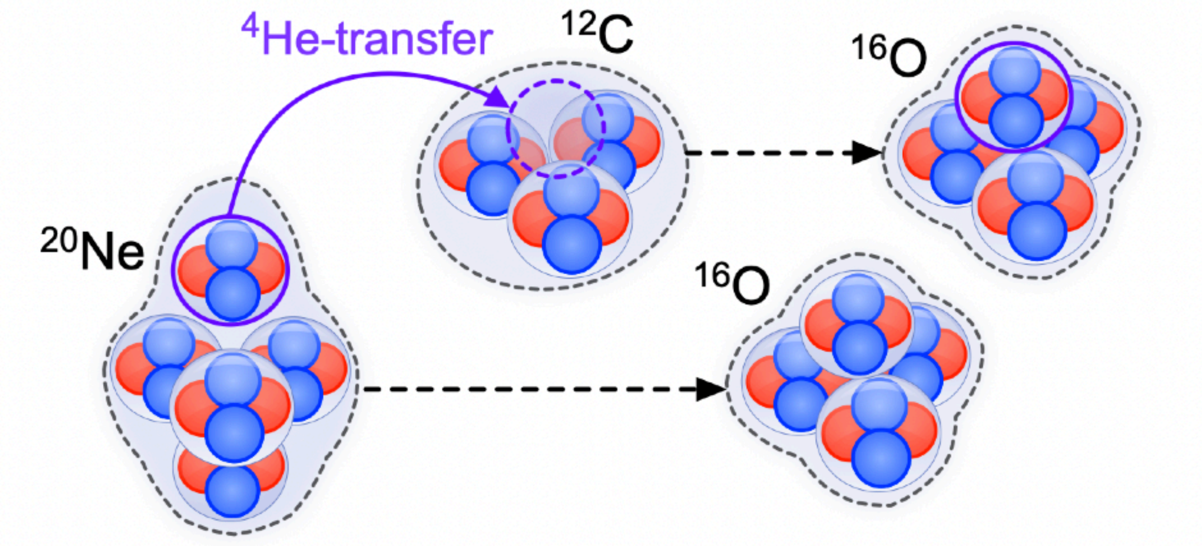
Asymptotic solution with phase shift



K. Kravvaris and A. Volya, Clustering in structure and reactions using configuration interaction techniques, Phys. Rev. C 100, 034321 (2019).

Clustering in the ground states

$$^{12}\text{C}(^{20}\text{Ne},^{16}\text{O})^{16}\text{O}$$

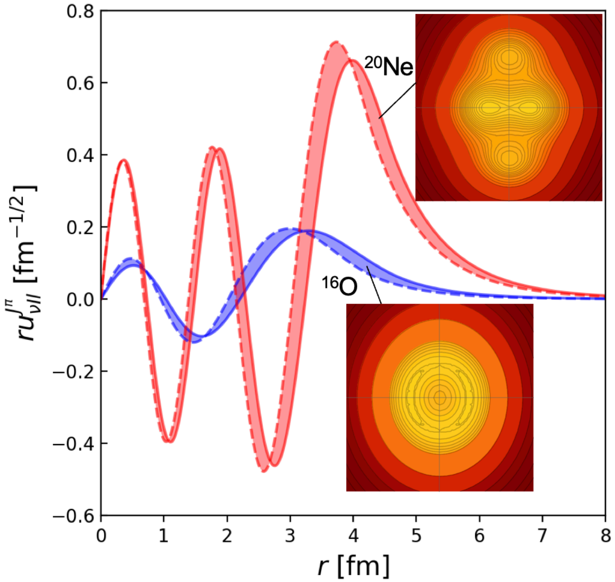


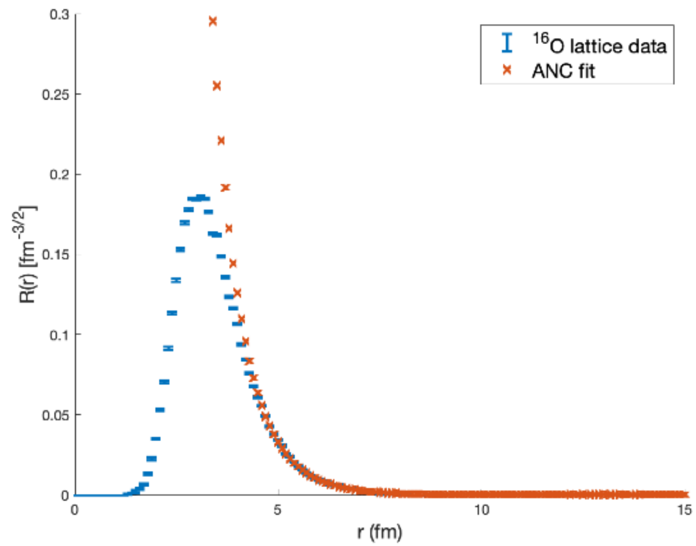
Model	C(^{16}O) ($\text{fm}^{-1/2}$)	C(^{20}Ne) ($\text{fm}^{-1/2}$)	C^2C^2 (10^{12} fm^{-2})
Experiment	Preliminary result		10(1.5)(2.0)
NLEFT	380(80)	3800(950)	2.1(1.4)
SA-NCSM	210(30)	3760(470)	0.6(0.4)
CI	520(30)	4500(500)	5.4(1.3)

E. Harris, et al. Quantifying clustering in the ground states of ^{16}O and ^{20}Ne, in preparation for publication

Clustering in the ground states

$$^{12}\mathrm{C}(^{20}\mathrm{Ne},^{16}\mathrm{O})^{16}\mathrm{O}$$





A. C. Dreyfuss et al., Phys. Rev. C 102, 044608 (2020).
K. Kravvaris and A. Volya, Phys. Rev. C 100, 034321 (2019).
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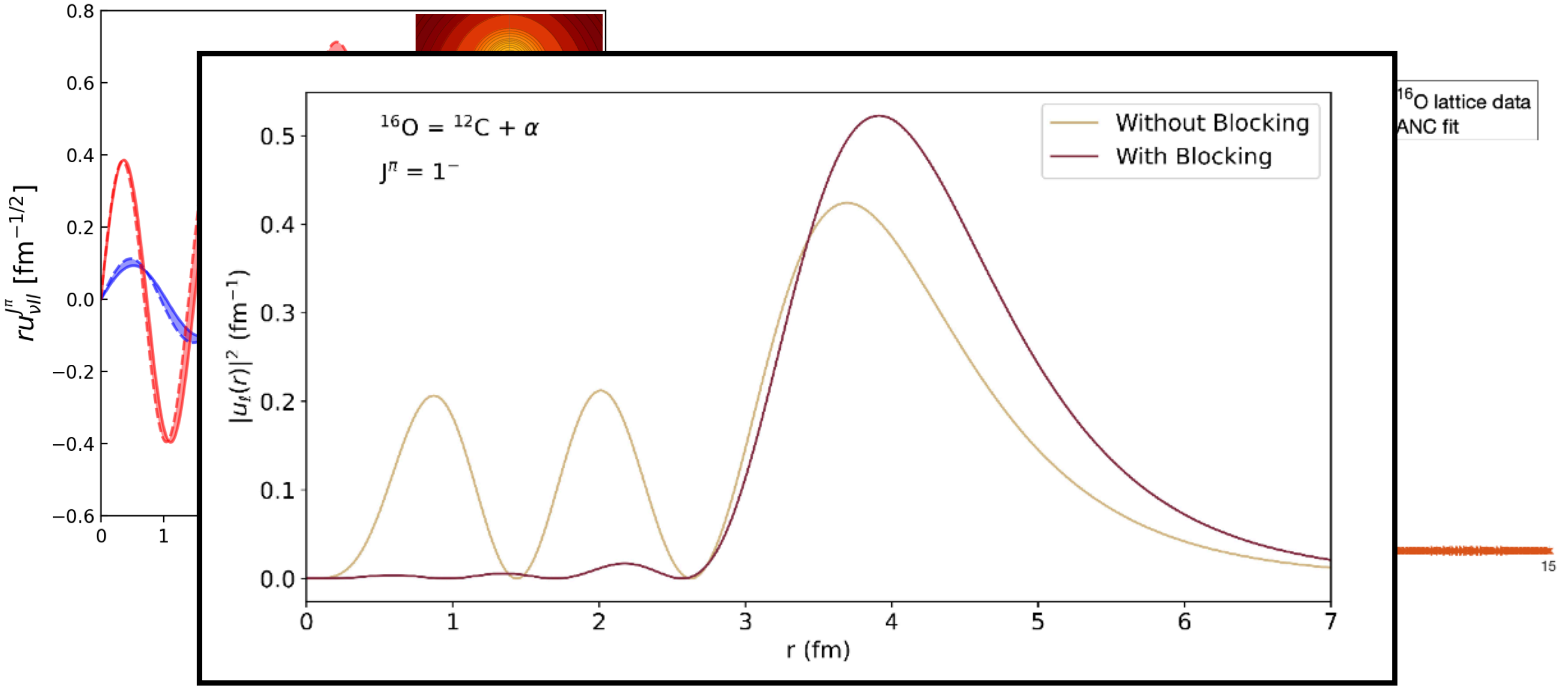
Clustering in the ground states

$$^{12}C(^{20}Ne,^{16}O)^{16}O$$



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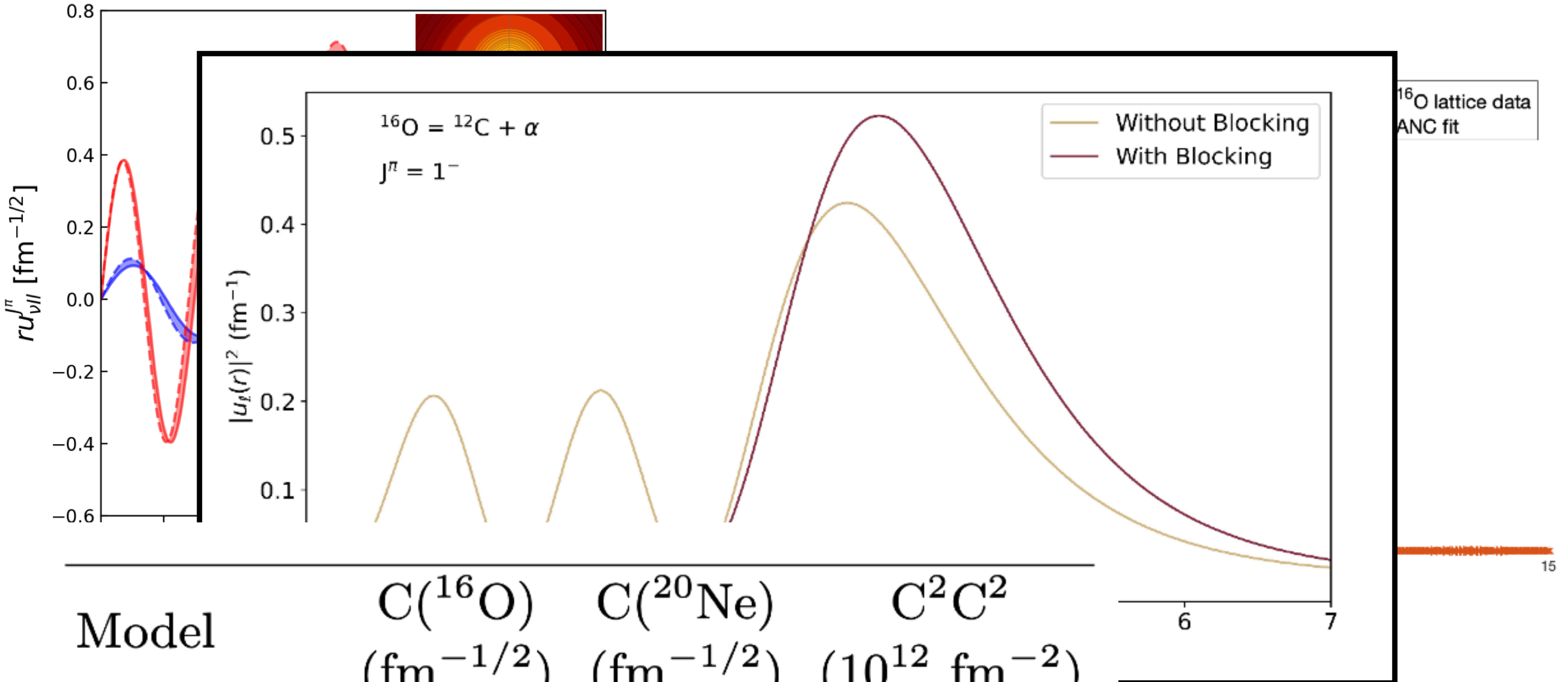
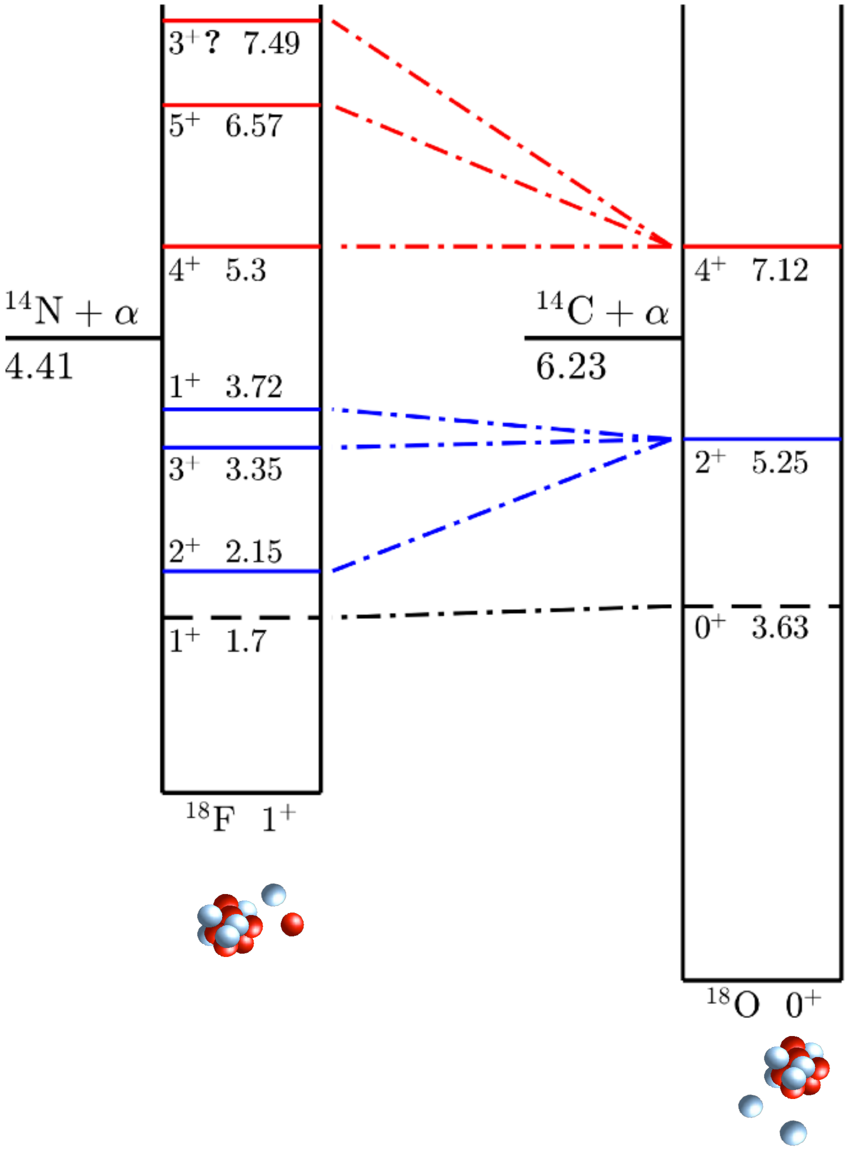


Model	C(^{16}O) (fm$^{-1/2}$)	C(^{20}Ne) (fm$^{-1/2}$)	C^{2}C^2 (10^{12} fm^{-2})
Experiment			10(1.5)(2.0)
NLEFT	380(80)	3800(950)	2.1(1.4)
SA-NCSM	210(30)	3760(470)	0.6(0.4)
CI	520(30)	4500(500)	5.4(1.3)

18F and 18O, pn and nn pairs and clustering

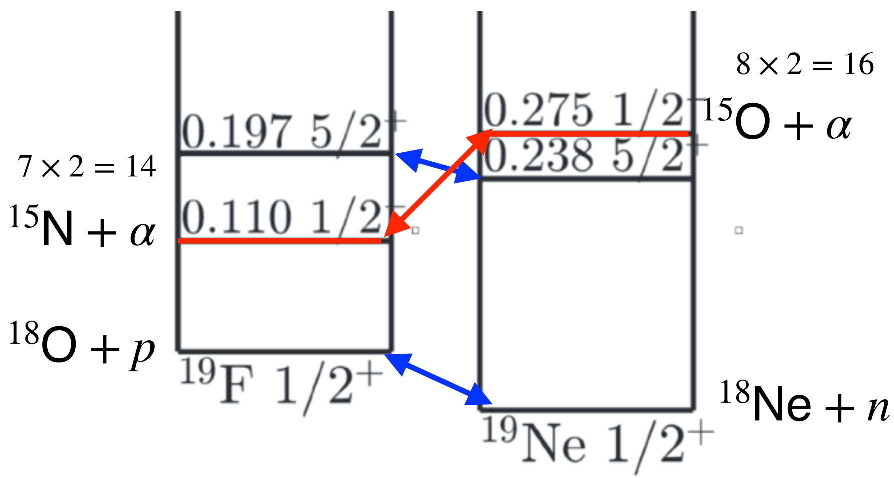


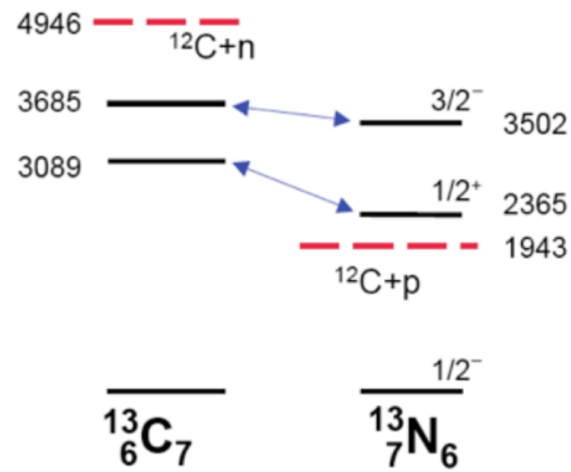
- Significant effect of nucleon pair on alpha blocking
- Low ground state alpha SF
- Clear clustering in excited states
- Core spin coupling to orbital motion.

A.K.Nurmukhanbetova, et al, EPJ in press.

Clustering, interaction with continuum

Near-threshold behavior has a significant impact on structure, reactions, and various key phenomena.





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Remarks and outlook

- Microscopic studies of clustering allow for quantitative evaluation of long standing questions related to identical particles, entanglement, and emergence of cluster degrees of freedom
- Advances in experimental techniques and studies for $N \neq Z$ nuclei provide sensitive tests for theory.

Main publications:

- E. Harris, et al. Quantifying clustering in the ground states of 16O and 20Ne
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- N. Sandulescu, M. Sambataro, and A. Volya, EPJ Web Conf. 292, 1003 (2024).
- K Kravvaris and A. Volya, Phys. Rev. Lett, 119(6), 062501 (2017); Journal of Phys 863, 012016 (2017), Phys. Rev. C 100 (2019) 034321.
- V. Z. Goldberg, et al., Phys. Rev. C 105 (2022) 014615.
- A. Volya, M. Barbui, V. Z. Goldberg, and G. V. Rogachev, Commun Phys 5, 1 (2022).
- V. Zelevinsky and A. Volya, Mesoscopic Nuclear Physics: From Nucleus to Quantum Chaos to Quantum Signal Transmission (World Scientific, 2023).

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