

Shape coexistence and superdeformation in ^{28}Si arXiv:2404.14506 [nucl-th] (Accepted in PRC)

Dorian Frycz

Universitat de Barcelona & Institut de Ciències del Cosmos

Collaborators: J. Menéndez, A. Rios, B. Bally,
T. R. Rodríguez & A. M. Romero

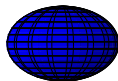


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Motivation for ^{28}Si

- **Coexistence** of different collective structures:
 1. 0_1^+ (0.0 MeV): **Oblate** bandhead of a rotational band
 2. 0_2^+ (5.0 MeV): **Vibration** of the ground state
 3. 0_3^+ (6.7 MeV): **Prolate** bandhead of a rotational band
 4. **Superdeformed** rotational band? ($E \gtrsim 10$ MeV)

Taniguchi, Y., et al. Phys. Rev. C **80**, 044316 (2009)



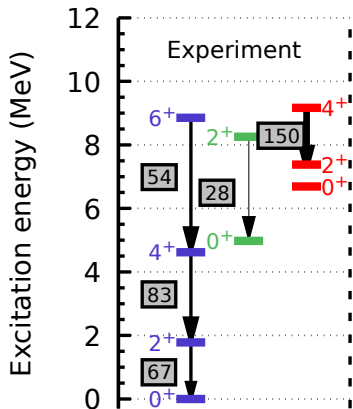
Oblate



Spherical



Prolate



- $E \sim J(J+1)$; $J = 0^+, 2^+, 4^+ \dots$
- Strong $B(E2)$ strengths

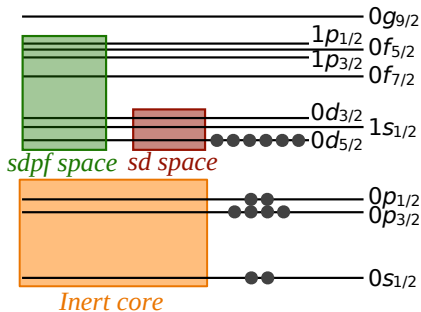
Theoretical framework

Schrödinger equation

$$\mathcal{H}|\Psi\rangle = E|\Psi\rangle$$

- ^{28}Si : $Z = N = 14$ Slater determinant (**spherical!**)
- **Interacting shell model**:
 $\mathcal{H}_{\text{eff}} = \mathcal{H}_0 + \mathcal{H}_{\text{res}}$
 \mathcal{H}_{res} : valence space
- Slater determinant basis $\{\Phi_i\}$
- Phenomenological interactions:
USDB and **SDPF-NR**

Caurier E., et al. Rev. Mod. Phys. 77, 427 (2005)



Valence space: $Z_v = N_v = 6$ (^{28}Si)
Inert core: ^{16}O nucleus

Exact diagonalization (ISM)

- **Most accurate solution** of $\mathcal{H}_{\text{eff}}|\Psi\rangle = E|\Psi\rangle$
- **Large set** of **simple Slater determinants**
 $|\Phi_i\rangle = c_{i1}^\dagger c_{i2}^\dagger \dots c_{iA}^\dagger |0\rangle$
- Best suited for **smaller valence spaces** (sd)
- Cannot explore a single **degree of freedom** ($Q_{\lambda\mu}$)

Beyond-mean-field (PGCM)

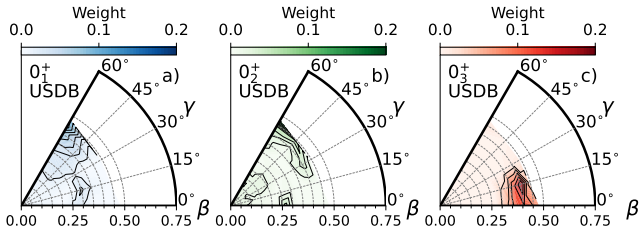
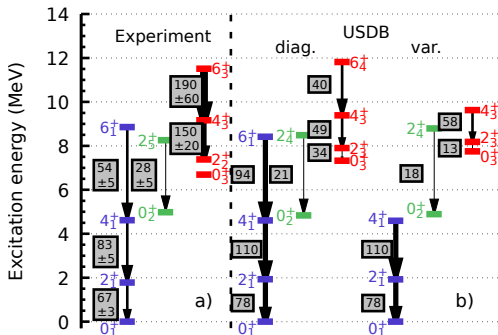
- **Approximate solution** to $\mathcal{H}_{\text{eff}}|\Psi\rangle = E|\Psi\rangle$
- **Smaller set** of more **complex wavefunctions** (HFB)
 $\beta_k^\dagger = \sum_l (U_{lk} c_l^\dagger + V_{lk} c_l)$
- Alternative for **large valence spaces** ($sdpf$)
- Exploration of relevant **degrees of freedom** ($Q_{\lambda\mu}$)

Spectrum of ^{28}Si (USDB)

Oblate rotational band:
well described, slightly
more deformed

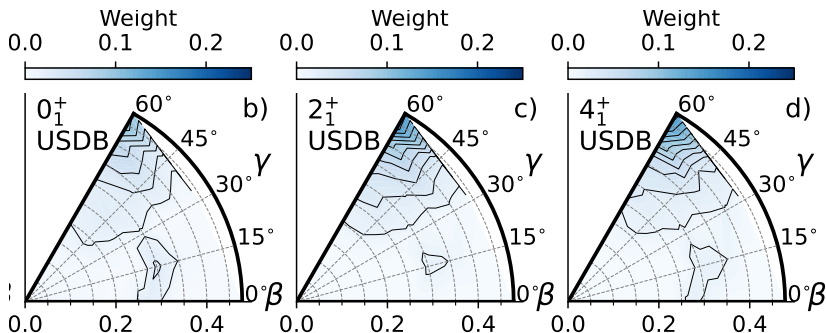
Vibrational band based on
the ground state is also
well described

Prolate rotational band
has too **weak** $B(E2)$



Oblate rotational band (USDB)

Collective wavefunctions: weight of each wf in the mixed state



- Shared intrinsic deformation ($\beta \approx -0.45$)

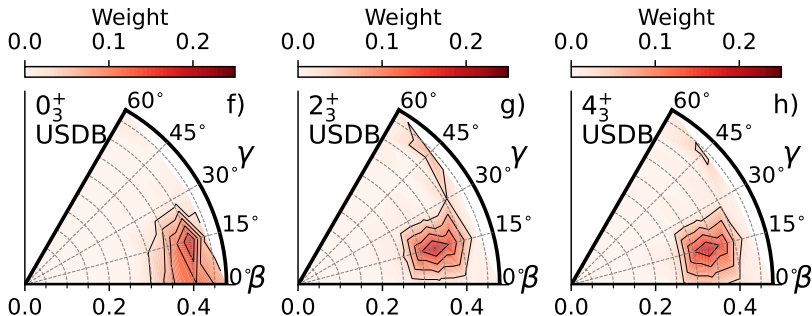
- Rotational band behaviour:

- $E \sim J(J+1)$ and **strong** $B(E2)$: 67 ± 3 vs $78 e^2\text{fm}^4$

D. Frycz, J. Menéndez, A. Rios, B. Bally, T. R. Rodríguez and A. M. Romero, [arXiv:2404.14506 [nucl-th]]

Prolate rotational band (USDB)

Collective wavefunctions: weight of each w.f. in the mixed state



- Different deformation patterns
- Reduced deformation for $J = 2$ and $J = 4$ ($\beta \approx 0.35$)
- Weak $B(E2)$ transition strengths: 150 ± 20 vs $50 e^2\text{fm}^4$

D. Frycz, J. Menéndez, A. Rios, B. Bally, T. R. Rodríguez and A. M. Romero, [arXiv:2404.14506 [nucl-th]]

Spectrum of ^{28}Si (SDPF-NR*)

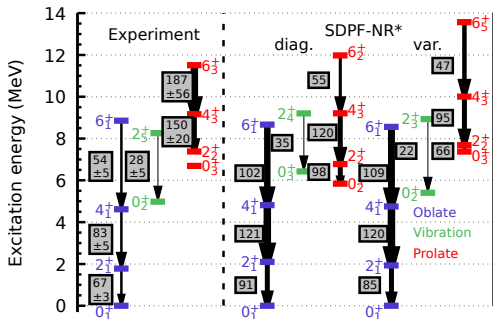
Adjusted SDPF-NR † interaction to reproduce ^{28}Si shell gap

Additional deformation from pf -shell particles:

- ▶ Slight gain for **oblate** and **vibration**
- ▶ Significant gain in **prolate** deformation
- ▶ 1 particle in pf -shell (38% of *sdpf* $2p$ - $2h$)

† S. Nummela Phys. Rev. C **63**,

044316 (2001)



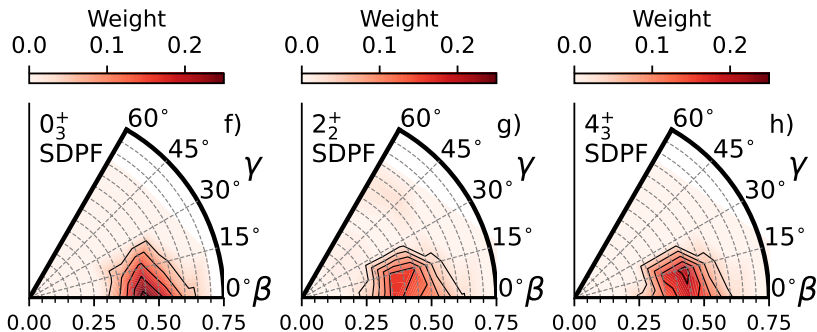
Experiment vs Theory (SDPF-NR*):

$$B(E2, 2_{obl}^+ \rightarrow 0_{obl}^+) = 67 \pm 3 \text{ vs } 91 \text{ e}^2\text{fm}^4$$

$$B(E2, 2_{vib}^+ \rightarrow 0_{vib}^+) = 28 \pm 5 \text{ vs } 35 \text{ e}^2\text{fm}^4$$

$$B(E2, 4_{pro}^+ \rightarrow 2_{pro}^+) = 150 \pm 20 \text{ vs } 120 \text{ e}^2\text{fm}^4$$

- The SDPF-NR* interaction (*sdpf* space) naturally reproduces the **prolate rotational band**.
- Enhanced collectivity from the *pf* shell
- **1 particle** in *pf* shell: 38% of *sdpf* 2p-2h contribution

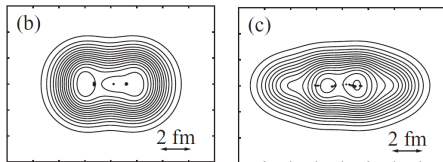


- Enhanced $B(E2, 4_{\text{pro}}^+ \rightarrow 2_{\text{pro}}^+) = 150 \pm 20$ vs $120 e^2\text{fm}^4$

Superdeformation

Superdeformed (SD) band
predicted with:

- Deformation: $\beta \approx 1$
- **4p-4h** into *pf* shell
- ~ 13 MeV bandhead

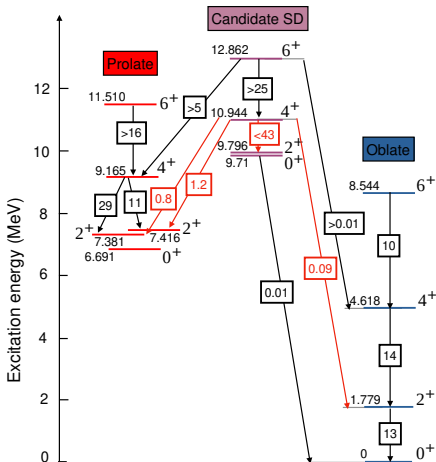


Taniguchi, Y., et al. Physical Review C, 2009. **80**, 044316

Experimental attempts

- $B(E2, 4^+ \rightarrow 2^+) \leq 217 \text{efm}^2$
- Not found: $\beta_{\text{exp}} \leq 0.6$

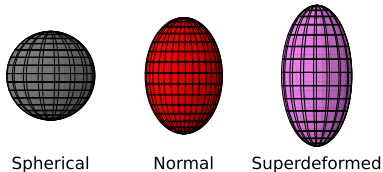
Morris, L. et al. Phys. Rev. C **104**, 054323 (2021)



Fixed np - nh configurations

Analytical SU(3) models:

- sd -shell ($\beta \leq 0.5$)
- $sdpf$ space ($\beta \geq 0.5$)
SD for $\geq 4p$ - $4h$ ($\beta \approx 0.8$)

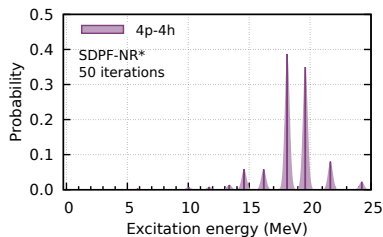
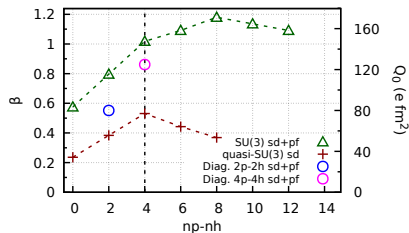


Lanczos strength function:

Decomposition of a fixed $4p$ - $4h$ configuration into the fully mixed states of the Hamiltonian:

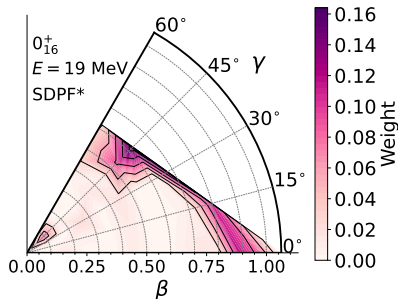
$$|0_{np-nh}^+\rangle = \frac{1}{N} \sum_{\sigma} S(\sigma) |0_{\sigma}^+\rangle$$

Energies: $4p$ - $4h$ at 18-20 MeV



- Full *sdpf* space
- Superdeformed state ($\beta \geq 0.6$)
- ~ 3 particles into the *pf* shell
- Energy: $E \approx 19$ MeV
- In agreement with the shell model calculation

PGCM calculation in *sdpf* space
with SDPF-NR* interaction:



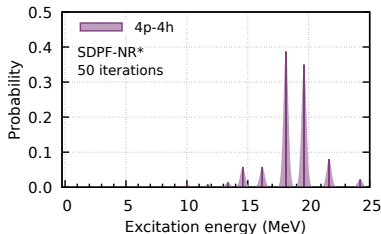
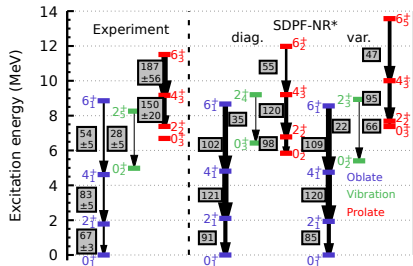
Conclusions

Shape coexistence of **structures** within the *sd* shell or *sdpf*

- Exact **diagonalization** and **variational** method (PGCM)
- **USDB** interaction describes **oblate** states.
- **SDPF-NR*** is needed for **prolate band** (1 particle in *pf* shell)

Superdeformed structures appear at 18 – 20 MeV (SDPF-NR*)

Article: D. Frycz, J. Menéndez, A. Ríos, B. Bally, T. R. Rodríguez and A. M. Romero, [arXiv:2404.14506 [nucl-th]]



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Shape coexistence and superdeformation in ^{28}Si
arXiv:2404.14506 [nucl-th] (Accepted in PRC)

Shape coexistence

- Different shapes among states of the same nucleus
- Narrow energy range: a few MeV of excitation energy

Motivation for ^{28}Si

- Coexistence of different collectives structures^{1,2}
- 1. ^{28}Si (0 MeV): Oblate bandhead of a rotational band
- 2. ^{28}Si (0.8 MeV): Vertex of the ground state
- 3. ^{28}Si (7.7 MeV): Prolate bandhead of a rotational band
- 4. Superdeformed rotational band^{3,4} (30 MeV) (1)

Methodology

Nuclear shell model^{5,6}: Exact diagonalization
Projected GCM⁷: Visualization of deformation
Elliott's SU(3)⁸: Analytical insight

Elliott's SU(3)

- Pure SU(3): Degenerated oblate and prolate structures
- Quasi-SU(3): $d_{5/2}$ excitations from $d_{5/2} \rightarrow d_{3/2}$ to $d_{5/2}$
- Specific mixed state and collective rotational band
- Collective vs single-particle competition: $\mathcal{H} = \mathcal{H}_c + \mathcal{H}_p$

sd-shell calculations (USDB/USDB-MOD)

- Oblate structure structures are well reproduced
- Prolate structure is not reproduced: weak $R(E_2)$ ($\sigma = 2$)
- Reduction of $d_{5/2}$ single-particle energy (USDB-MOD)
- Oblate and oblate non-locally separated $4p$ - 0 component
- Prolate structure gains deformation (more $4p$ component)

sdpf space calculations (SDPF-NR*)

- Adjusted SDPF-NR interaction to reproduce ^{28}Si shell gap
- Additional deformation loss *pf*-shell particles
- Right gain for oblate and oblate
- Significant gain in *prolate* deformation
- $R(E_2)_{\text{oblate}} = 49 \text{ e}^2 \text{ fm}^4$
- $R(E_2)_{\text{prolate}} = 128 \text{ e}^2 \text{ fm}^4$
- 1 particle in *pf* shell (18% of *sdpf* (2p-2h))

Projected generator-coordinate method

- Beyond-mean field approach (constrained HFB basis + symmetry restoration + configuration mixing)
- Successful description of shape coexistence

Superdeformation

- Superdeformed structures ($4p$ - 0) appear at 18-20 MeV

Shapes (anisotropy)

- Are the shapes of the bandheads physically meaningful?
- Oblate ground state: $\beta = 0.47 \pm 0.05$, $\beta \leq 0.67$
- Oblique ground state: $\beta = 0.62 \pm 0.12$, $\beta \leq 0.67$
- Prolate excited state: $\beta = 0.68 \pm 0.05$, $\beta \leq 0.7$
- The bandheads are β -soft and present a broad γ range

References

1. Morcrette et al. PRC 86, 044312 (2012)
2. Y. Tanihata, T. Kanadao, Y. Yan, and M. Kimura, PRC 45, 2447 (1992)
3. E. Casper and J. Smeekals, Acta Phys. Pol. B, 8, 703 (1998)
4. B. Bally, A. Martínez-Fernández, and T. Rodríguez, PRA 81, 014101 (2010)
5. J. B. Elliott, Proc. R. Soc. Lond. Ser. A 245, 129 (1958)

Dorian Frycz (UB/ICCUB)
Mail: dorianfrycz@upb.edu
Coauthors: J. Menéndez, A. Rios, B. Bally, T. R. Rodríguez & A. M. Romero

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Ab initio interaction:

- Valence space in-medium renormalization group

Stroberg, S. Ragnar, et al. *Ann. Rev. Nucl. Part. Sci.* **69**, 307 (2019)

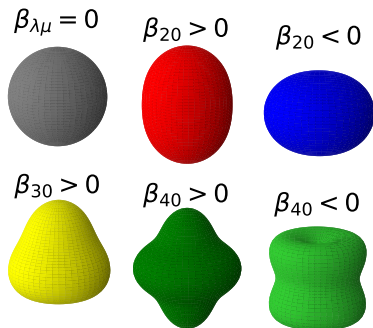
Shape coexistence and SD:

- $N = Z$: ^{32}S , ^{24}Mg ...
- Neutron-rich: $^{30-42}\text{Si}$
- E0 transitions

Multipole deformations:

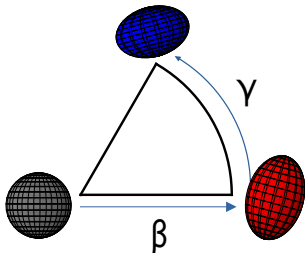
- $\beta_4(^{28}\text{Si}) = 0.03 \pm 0.01$

Y. K. Gupta et al., *Phys. Lett. B* **845**, 138120 (2023)



Ground state (Mean field)

- **Quadrupole-constrained HFB basis $|\phi(q)\rangle$:**
 $\mathcal{H}'_{\text{eff}} = \mathcal{H}_{\text{eff}} - \lambda \sum_{\mu} Q_{2,\mu}$
- (β, γ) parameters:

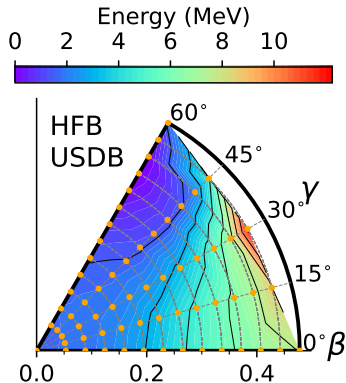


- **TAURUS_{vap}** code

B. Bally, et al. Eur. Phys. J. A **57**, 69 (2021)

$Z_V = N_V = 6$ USDB interaction[†]

[†]W. A. Richter, et al. Phys. Rev. C **78**, 064302 (2008)



Oblate minimum ($\beta \approx -0.4$)

Ground state (Beyond mean field)

Mean field:

- **Quadrupole-constrained HFB basis** $|\phi(q)\rangle$:

$$\mathcal{H}'_{\text{eff}} = \mathcal{H}_{\text{eff}} - \lambda \sum_{\mu} Q_{2,\mu}$$

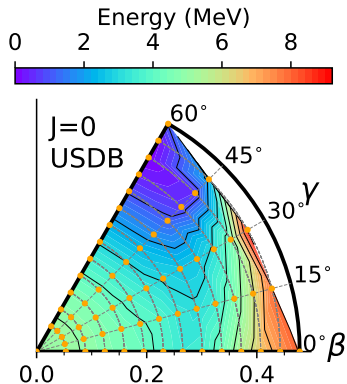
Beyond mean field:

- **Symmetry restoration**
 $|\phi^{NZJ}\rangle = P^N P^Z P^J_{MK} |\phi\rangle$
- **Configuration mixing** of projected states (PGCM)
 $|\Psi_{\text{GCM}}\rangle = \sum_q f_q |\phi^{NZJ}(q)\rangle$
GCM: generator coordinate method
- **TAURUS** $_{pav,mix}$ codes

B. Bally, et al. Eur. Phys. J. A **60**, 62 (2024)

$Z_V = N_V = 6$ USDB interaction[†]

[†]W. A. Richter, et al. Phys. Rev. C **78**, 064302 (2008)

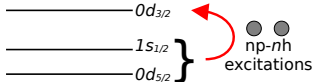


Oblate minimum ($\beta \approx 0.4$)

Modification of the interaction

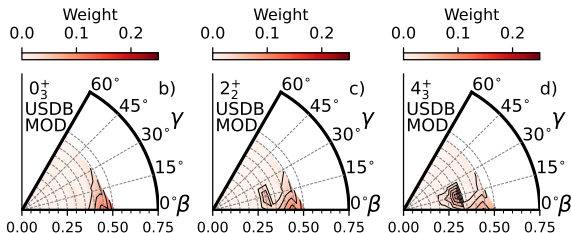
Energy competition in quasi-SU(3): $\mathcal{H} = \mathcal{H}_0 - \kappa\beta^2$

- Gains energy with **deformation** but loses with **excitations**
- **Prolate band**: excitations from $d_{5/2} + s_{1/2}$ to $d_{3/2}$



β	0p-0h	2p-2h	4p-4h
Oblate	-0.37	-0.45	-0.53
Prolate	0.24	0.38	0.53

A. P. Zuker, et al. Phys. Rev. C **92**, 024320 (2015)

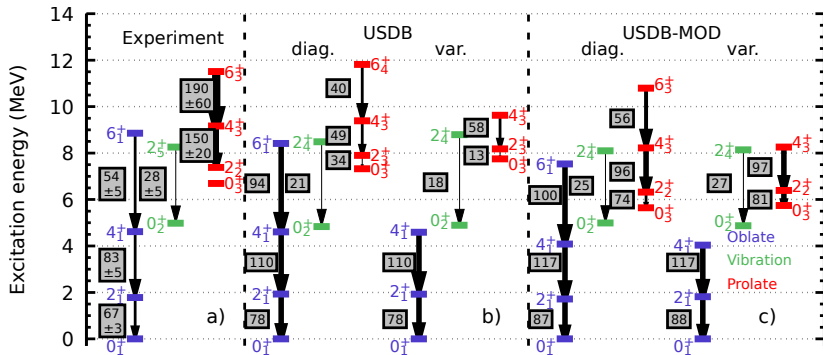


USDB-MOD

Single particle energy $d_{3/2}$ from:

5 MeV \rightarrow 3.5 MeV

Spectrum of ^{28}Si (USDB-MOD)

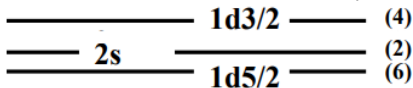


Stronger $B(E2, 4^+_{\text{pro}} \rightarrow 2^+_{\text{pro}}) = 150 \pm 20$ vs $96 e^2\text{fm}^4$

Oblate and vibrational bands remain unperturbed ($0p-0h$)

D. Frycz, J. Menéndez, A. Rios, B. Bally, T. R. Rodríguez and A. M. Romero, [arXiv:2404.14506 [nucl-th]]

- **Quadrupole interactions:** realistic Hamiltonians
- Restriction to a major **shell** (Fermi surface)

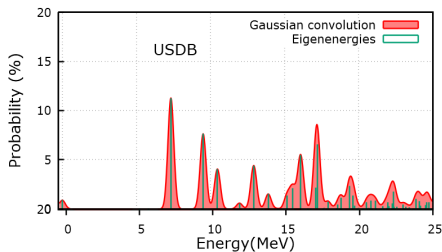


- Energy **competition**: $\mathcal{H} = \mathcal{H}_0 - \kappa Q_0^2$
 - Correlation energy **decreases** as Q_0^2
 - Single particle energy **increases** with promoted particles (from $d_{5/2}$ to $s_{1/2}$ or $d_{3/2}$)
- Intrinsic quadrupole moment Q_0 :
 - Spherical: $Q_0 = 0$
 - **Prolate**: $Q_0 > 0$
 - **Oblate**: $Q_0 < 0$

Elliott, J. P. Proc R Soc Lon Ser-A, 1958. **245**, 128.

Modification of the interaction

- The prolate 4p-4h is **lost** in configuration mixing
- $|4p4h\rangle = \sum_i c_i |\Psi\rangle_{i, \text{full sd}} \rightarrow$
- USDB: prolate band only has 10% of $|4p4h\rangle$



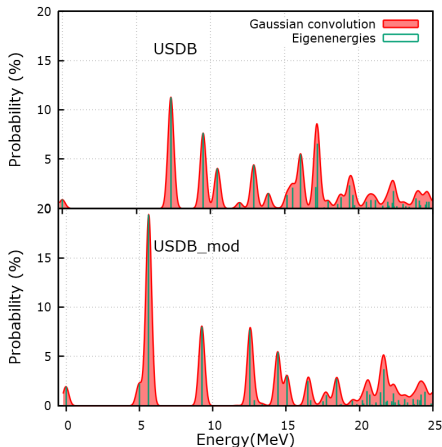
Modification of the interaction

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- $|4p4h\rangle = \sum_i c_i |\Psi\rangle_{i, \text{full sd}} \rightarrow$
- USDB: prolate band only has 10% of $|4p4h\rangle$

Too high single-particle energies

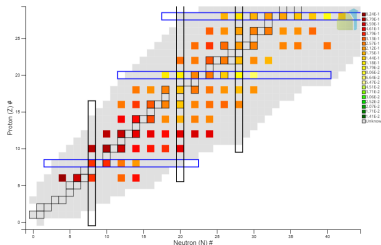
Gap between $d_{5/2}+s_{1/2}$ and $d_{3/2}$
5 MeV \rightarrow 3.6 MeV

- 4p-4h concentrated in 0_3^+ :
now goes up to 20%

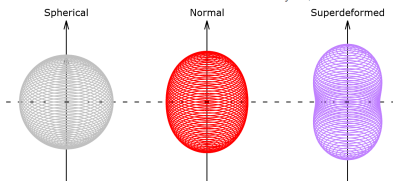


Objectives

- Why are some nuclei **deformed**?
 - Magic nuclei are spherical
 - Most nuclei are deformed
- What kind of deformation?
 - **Quadrupole** deformations
- Will they be **prolate** or **oblate**?
 - Axial symmetry
- Can different shapes **coexist**?
 - Spherical, **prolate** and **oblate**
- How much deformation?
 - Normal deformation (3:2 ratio)
 - Superdeformation (2:1 ratio)



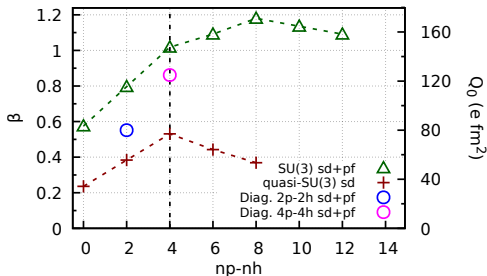
National nuclear data center: ENSDF. July 21, 2023



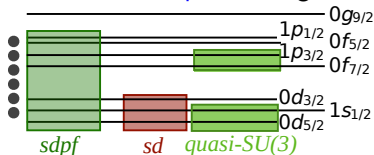
Fixed np - nh configurations

Analytical SU(3) models:

- **sd-shell** deformations:
 $\beta \leq 0.5$; $Q_0 \leq 80 \text{ efm}^2$
 too low for SD!
- **sdpf** space deformations:
 $\beta \geq 0.5$; $Q_0 \geq 80 \text{ efm}^2$
 SD for $\geq 4p$ - $4h$



β parameters for SU(3) schemes and nuclear shell model np - nh configurations



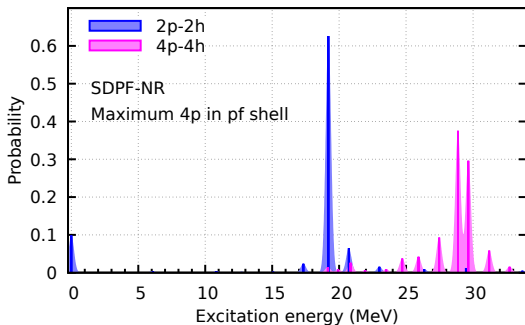
Numerical calculations:

- Shell model np - nh :
 Similar to **quasi-SU(3) sdpf**

Lanczos strength functions

Decomposition of a fixed $np-nh$ configuration into the **fully mixed states** of the Hamiltonian: $|0_{np-nh}^+\rangle_{sdpf} = \frac{1}{N} \sum_{\sigma} S(\sigma) |0_{\sigma}^+\rangle_{sdpf}$

Truncation: maximum of 4 particles into pf shell (**dimensions!**)



- Energies: 2p-2h at 19 MeV and 4p-4h at 30 MeV!!!

- Initial state: $|1\rangle$
- Next step: $E_{12}|2\rangle = (H - E_{11})|1\rangle$
- Then: $E_{23}|3\rangle = (H - E_{22})|2\rangle - E_{12}|1\rangle$
- Generalizing: $E_{NN+1}|N+1\rangle = (H - E_{NN})|N\rangle - E_{N-1N}|N-1\rangle$

Where: $E_{NN} = \langle N|H|N\rangle$

$$\begin{pmatrix} E_{11} & E_{12} & 0 \\ E_{12} & E_{22} & E_{23} \\ 0 & E_{23} & E_{33} \end{pmatrix},$$

- Finally, diagonalize and check convergence

Projected generator coordinate method (PGCM)

Variational approach:

- Configuration mixing of Hartree-Fock-Bogoliubov (HFB) states:

$$|\Psi_{\text{GCM}}\rangle = \sum_q f_q |\phi_{\text{HFB}}(q)\rangle$$

B. Bally, et al. Eur. Phys. J. A **60**, 62 (2024)

- Similar deformations for all interactions
- $2\nu\beta\beta$ matrix elements are larger for similar deformations

T. R. Rodríguez and G. Martínez-Pinedo

Phys. Rev. C **85**, 044310 (2012)

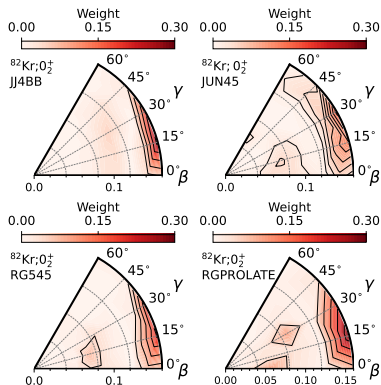


Figure: Contribution of each HFB wavefunction to fully mixed state for $^{82}\text{Kr} (0_2^+)$ with all interactions.