Shape coexistence and superdeformation in ^{28}Si
arXiv:2404.14506 [nucl-th] (Accepted in PRC)

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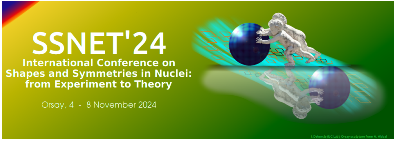

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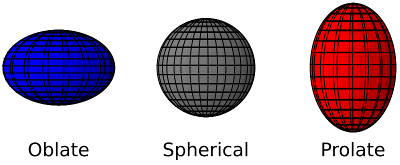
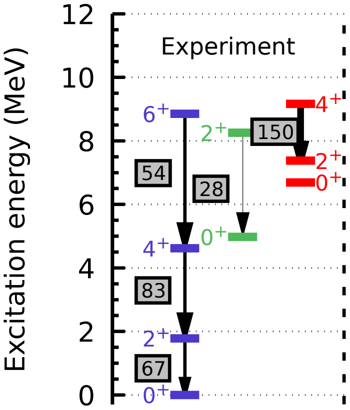
Motivation for ^{28}Si

- **Coexistence** of different collective structures:
  1. 0_1^+ (0.0 MeV): Oblate bandhead of a rotational band
  2. 0_2^+ (5.0 MeV): Vibration of the ground state
  3. 0_3^+ (6.7 MeV): Prolate bandhead of a rotational band
  4. Superdeformed rotational band? ($E \gtrsim 10$ MeV)

Taniguchi, Y., et al. Phys. Rev. C **80**, 044316 (2009)





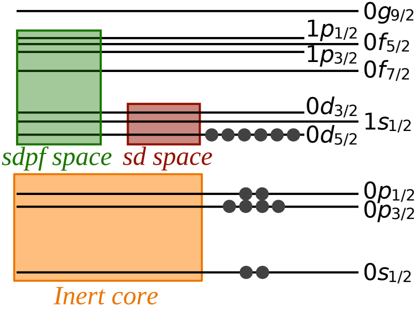
- $E \sim J(J+1)$; $J = 0^+, 2^+, 4^+ \ldots$
- Strong $B(E2)$ strengths

Theoretical framework

Schrödinger equation

$$\mathcal{H}|\Psi\rangle = E|\Psi\rangle$$

- ^{28}Si: $Z = N = 14$ Slater determinant (**spherical!**)
- **Interacting shell model**:
  $\mathcal{H}_{\mathrm{eff}} = \mathcal{H}_0 + \mathcal{H}_{\mathrm{res}}$
  $\mathcal{H}_{\mathrm{res}}$: valence space
- Slater determinant basis $\{\Phi_i\}$
- Phenomenological interactions: USDB and SDPF-NR



Valence space: $Z_v = N_v = 6$ (^{28}Si)
Inert core: ^{16}O nucleus

Caurier E., et al. Rev. Mod. Phys. **77**, 427 (2005)

Many-body methods

Exact diagonalization (ISM)

- **Most accurate solution** of $\mathcal{H}_{\text{eff}}|\Psi\rangle = E|\Psi\rangle$
- Large set of simple Slater determinants $|\Phi_i\rangle = c^{\dagger}_{i1} c^{\dagger}_{i2} \dots c^{\dagger}_{iA} |0\rangle$
- Best suited for smaller valence spaces (sd)
- Cannot explore a single degree of freedom ($Q_{\lambda\mu}$)

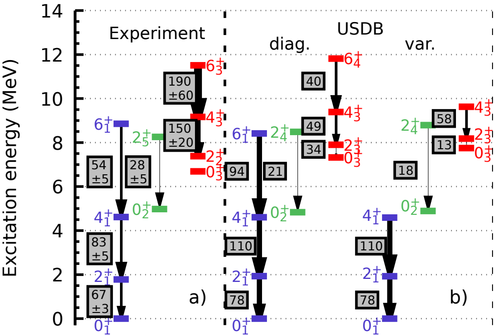
Beyond-mean-field (PGCM)

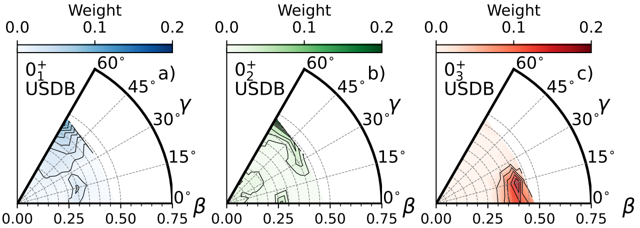
- **Approximate solution** to $\mathcal{H}_{\text{eff}}|\Psi\rangle = E|\Psi\rangle$
- Smaller set of more complex wavefunctions (HFB) $\beta^{\dagger}_k = \sum_l (U_{lk} c^{\dagger}_l + V_{lk} c_l)$
- Alternative for large valence spaces ($sdpf$)
- Exploration of relevant degrees of freedom ($Q_{\lambda\mu}$)

Oblate rotational band: **well described**, slightly more deformed

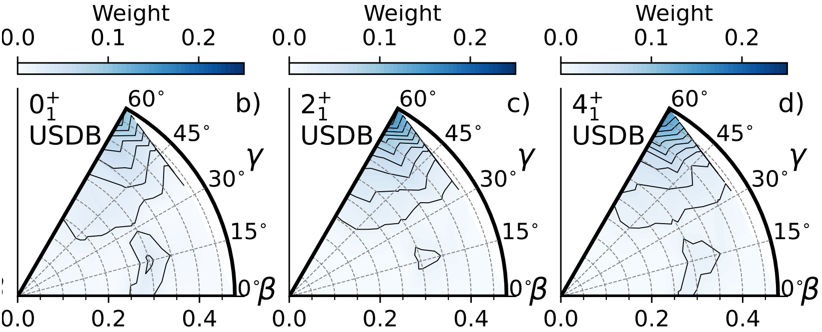
Vibrational band based on the ground state is also **well described**

Prolate rotational band has too weak $B(E2)$





Collective wavefunctions: weight of each wf in the mixed state

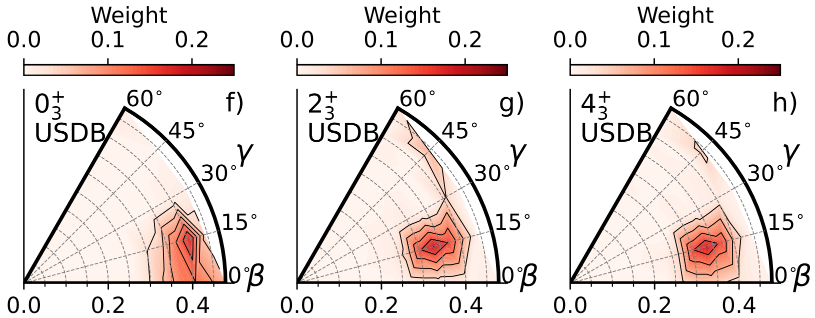


- Shared intrinsic deformation ($\beta \approx -0.45$)
- Rotational band behaviour:
  - $E \sim J(J+1)$ and **strong** $B(E2)$: 67 ± 3 vs 78 $e^2\text{fm}^4$

D. Frycz, J. Menéndez, A. Rios, B. Bally, T. R. Rodríguez and A. M. Romero, [arXiv:2404.14506 [nucl-th]]

Prolate rotational band (USDB)

Collective wavefunctions: weight of each w.f. in the mixed state



- Different deformation patterns
- Reduced deformation for $J = 2$ and $J = 4$ $(\beta \approx 0.35)$
- **Weak** $B(E2)$ transition strengths: 150 ± 20 vs 50 $e^2\text{fm}^4$

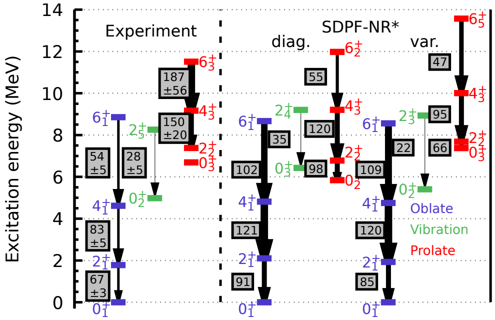
D. Frycz, J. Menéndez, A. Rios, B. Bally, T. R. Rodríguez and A. M. Romero, [arXiv:2404.14506 [nucl-th]]

Adjusted SDPF-NR† interaction to reproduce ^{28}Si shell gap

Additional deformation from *pf*-shell particles:

- Slight gain for oblate and vibration
- Significant gain in prolate deformation
- 1 particle in *pf*-shell (38% of *sdpf* 2p-2h)

†S. Nummela Phys. Rev. C **63**, 044316 (2001)



Experiment vs Theory (SDPF-NR*):

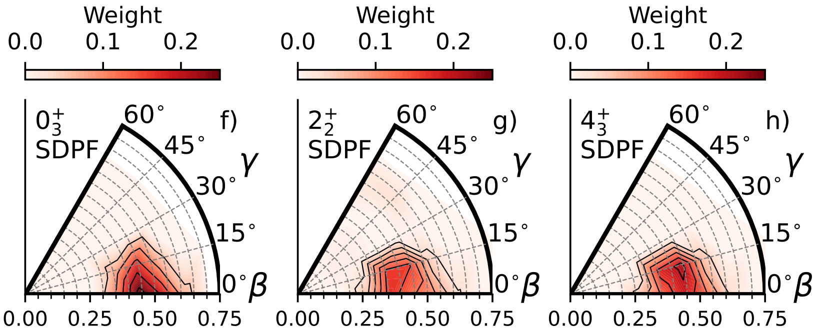
$$B(E2, 2^+_{obl} \to 0^+_{obl}) = 67 \pm 3 \text{ vs } 91 \; e^2\text{fm}^4$$

$$B(E2, 2^+_{vib} \to 0^+_{vib}) = 28 \pm 5 \text{ vs } 35 \; e^2\text{fm}^4$$

$$B(E2, 4^+_{pro} \to 2^+_{pro}) = 150 \pm 20 \text{ vs } 120 \; e^2\text{fm}^4$$

sdpf valence space

- The SDPF-NR* interaction (*sdpf* space) **naturally** reproduces the prolate rotational band.
- Enhanced collectivity from the *pf* shell
- **1 particle** in *pf* shell: 38% of *sdpf* 2p-2h contribution



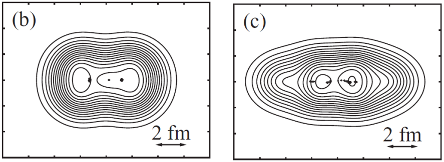
- Enhanced $B(E2, 4^+_{\text{pro}} \to 2^+_{\text{pro}}) = 150 \pm 20$ vs $120\ e^2\text{fm}^4$

Superdeformation

Superdeformed (SD) band

predicted with:

- Deformation: $\beta \approx 1$
- **4p-4h** into *pf* shell
- $\sim$ 13 MeV bandhead

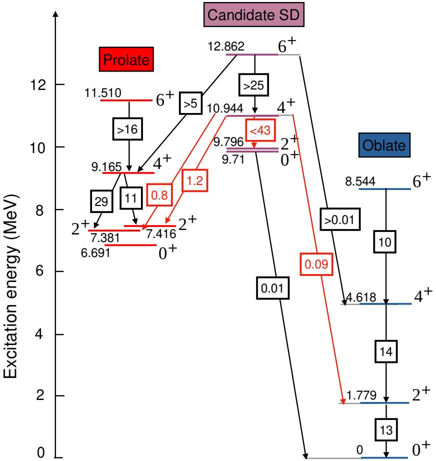


Taniguchi, Y., et al. Physical Review C, 2009. **80**, 044316

Experimental attempts

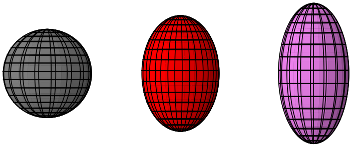
- $B(E2, 4^+ \to 2^+) \leq 217 e\mathrm{fm}^2$
- Not found: $\beta_{exp} \leq 0.6$

  Morris, L. et al. Phys. Rev. C **104**, 054323 (2021)

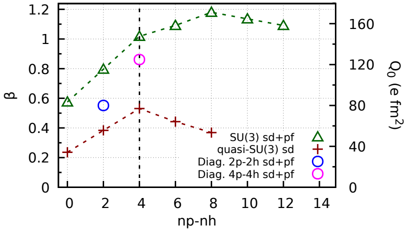


Analytical SU(3) models:

- *sd*-shell ($\beta \leq 0.5$)
- *sdpf* space ($\beta \geq 0.5$)
  SD for $\geq$4p-4h ($\beta \approx 0.8$)

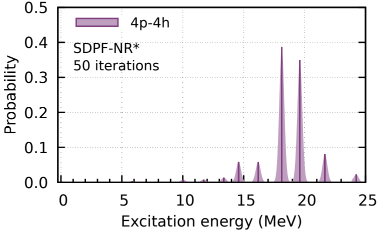


Spherical Normal Superdeformed



Lanczos strength function:
Decomposition of a fixed 4p-4h configuration into the fully mixed states of the Hamiltonian:

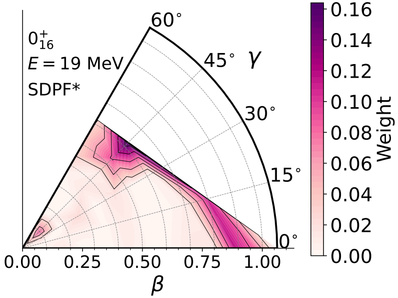
$$|0^+_{np\text{-}nh}\rangle = \frac{1}{N}\sum_\sigma S(\sigma)|0^+_\sigma\rangle$$



Energies: 4p-4h at **18-20 MeV**

- Full *sdpf* space
- Superdeformed state ($\beta \geq 0.6$)
- $\sim$ **3 particles** into the *pf* shell
- Energy: $E \approx 19$ MeV
- In agreement with the shell model calculation

PGCM calculation in *sdpf* space with SDPF-NR* interaction:



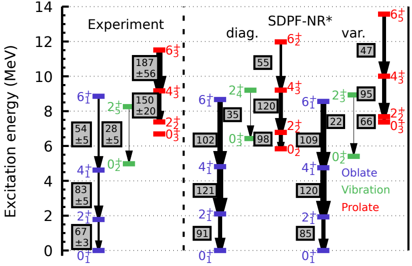
Conclusions

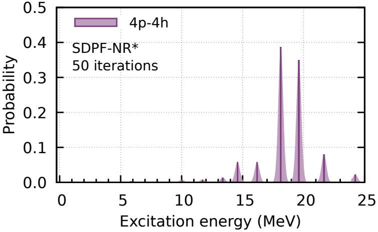
Shape coexistence of **structures** within the *sd* shell or *sdpf*

- Exact **diagonalization** and **variational** method (PGCM)
- USDB interaction describes oblate states.
- SDPF-NR* is needed for prolate band (1 particle in *pf* shell)

Superdeformed structures appear at 18 – 20 MeV (SDPF-NR*)

Article: D. Frycz, J. Menéndez, A. Rios, B. Bally, T. R. Rodríguez and A. M. Romero, [arXiv:2404.14506 [nucl-th]]





THANK YOU!

Shape coexistence of **structures** within the *sd* shell or *sdpf*

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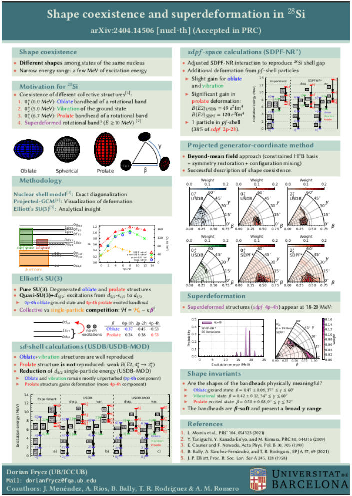


Outlook

Ab initio **interaction**:

- Valence space in-medium renormalization grop

  Stroberg, S. Ragnar, et al. Ann. Rev. Nucl. Part. Sci. **69**, 307 (2019)

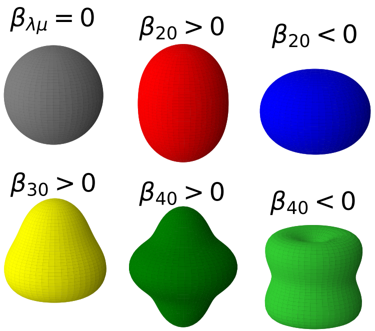
Shape coexistence and SD:

- $N = Z$: ^{32}S, ^{24}Mg...
- Neutron-rich: $^{30-42}$Si
- E0 transitions

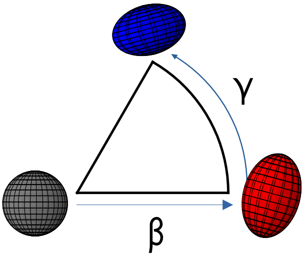
Multipole deformations:

- $\beta_4(^{28}\text{Si}) = 0.03 \pm 0.01$

Y. K. Gupta et al., Phys. Lett. B **845**, 138120 (2023)



- Quadrupole-constrained HFB basis $|\phi(q)\rangle$:
  $\mathcal{H}'_{\text{eff}} = \mathcal{H}_{\text{eff}} - \lambda \sum_\mu Q_{2,\mu}$
- (β, γ) parameters:

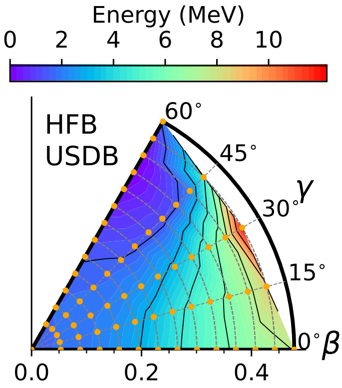


- **TAURUS**$_{vap}$ code

B. Bally, et al. Eur. Phys. J. A **57**, 69 (2021)

$Z_v = N_v = 6$ USDB interaction†

†W. A. Richter, et al. Phys. Rev. C **78**, 064302 (2008)



Oblate minimum ($\beta \approx -0.4$)

Ground state (Beyond mean field)

Mean field:

- Quadrupole-constrained HFB basis $|\phi(q)\rangle$:
  $\mathcal{H}'_{\text{eff}} = \mathcal{H}_{\text{eff}} - \lambda \sum_\mu Q_{2,\mu}$

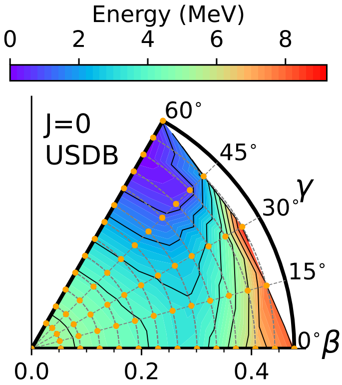
Beyond mean field:

- **Symmetry restoration**
  $|\phi^{NZJ}\rangle = P^N P^Z P^J_{MK} |\phi\rangle$
- **Configuration mixing** of projected states (PGCM)
  $|\Psi_{\text{GCM}}\rangle = \sum_q f_q |\phi^{NZJ}(q)\rangle$
  GCM: generator coordinate method
- **TAURUS**$_{pav,mix}$ codes

B. Bally, et al. Eur. Phys. J. A **60**, 62 (2024)

$Z_v = N_v = 6$ USDB interaction†

†W. A. Richter, et al. Phys. Rev. C **78**, 064302 (2008)

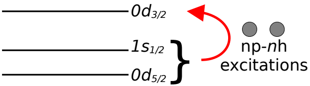


Oblate minimum ($\beta \approx -0.4$)

Modification of the interaction

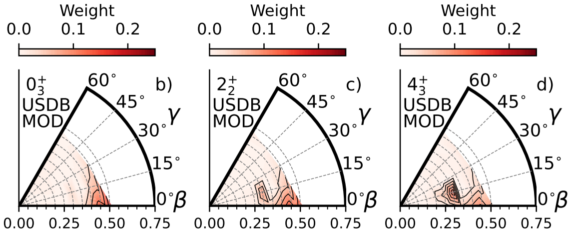
Energy competition in quasi-SU(3): $\mathcal{H} = \mathcal{H}_0 - \kappa\beta^2$

- Gains energy with deformation but looses with excitations
- Prolate band: excitations from $d_{5/2} + s_{1/2}$ to $d_{3/2}$

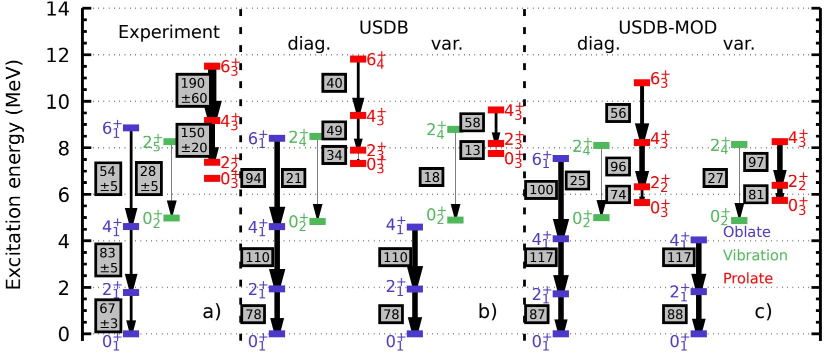


β	0p-0h	2p-2h	4p-4h
Oblate	-0.37	-0.45	-0.53
Prolate	0.24	0.38	0.53

A. P. Zuker, et al. Phys. Rev. C **92**, 024320 (2015)



USDB-MOD
Single particle energy $d_{3/2}$ from:
5 MeV → 3.5 MeV



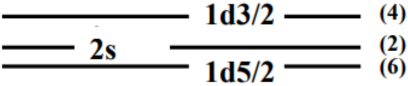
Stronger $B(E2, 4^+_{pro} \to 2^+_{pro}) = 150 \pm 20$ vs 96 e^2fm^4

Oblate and vibrational bands remain unperturbed (0p-0h)

D. Frycz, J. Menéndez, A. Rios, B. Bally, T. R. Rodríguez and A. M. Romero, [arXiv:2404.14506 [nucl-th]]

SU(3) model

- **Quadrupole interactions**: realistic Hamiltonians
- Restriction to a major **shell** (Fermi surface)

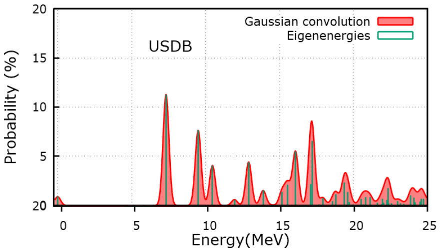


- Energy **competition**: $\mathcal{H} = \mathcal{H}_0 - \kappa Q_0^2$
  - Correlation energy decreases as Q_0^2
  - Single particle energy increases with promoted particles (from $d_{5/2}$ to $s_{1/2}$ or $d_{3/2}$)
- Intrinsic quadrupole moment Q_0:
  - Spherical: $Q_0 = 0$
  - Prolate: $Q_0 > 0$
  - Oblate: $Q_0 < 0$

Elliott, J. P. Proc R Soc Lon Ser-A, 1958. **245**, 128.

Modification of the interaction

- The prolate 4p-4h is **lost** in configuration mixing
- $|4p4h\rangle = \sum_i c_i |\Psi\rangle_{i,\text{full sd}} \rightarrow$
- USDB: prolate band only has 10% of $|4p4h\rangle$



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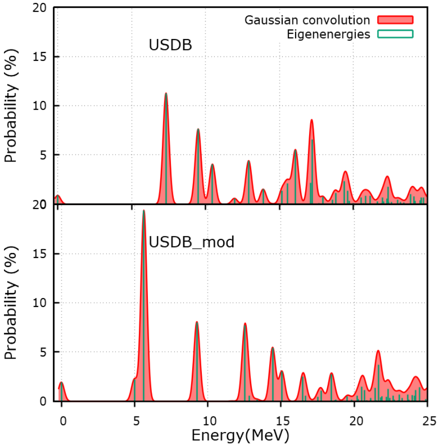
Too high single-particle energies

Gap between $d_{5/2}+s_{1/2}$ and $d_{3/2}$

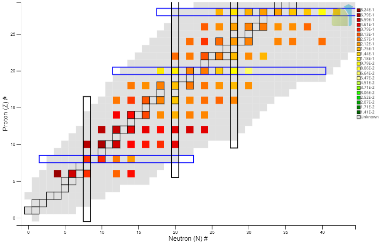
5 MeV $\rightarrow$ 3.6 MeV

- 4p-4h concentrated in 0_3^+: now goes up to 20%

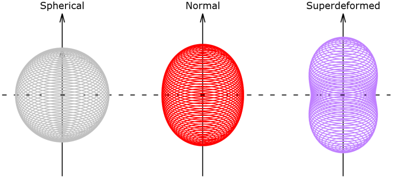


Objectives

- Why are some nuclei **deformed**?
  - Magic nuclei are spherical
  - Most nuclei are deformed
- What kind of deformation?
  - **Quadrupole** deformations
- Will they be prolate or oblate?
  - Axial symmetry
- Can different shapes **coexist**?
  - Spherical, prolate and oblate
- How much deformation?
  - Normal deformation (3:2 ratio)
  - Superdeformation (2:1 ratio)



National nuclear data center: ENSDF. July 21, 2023



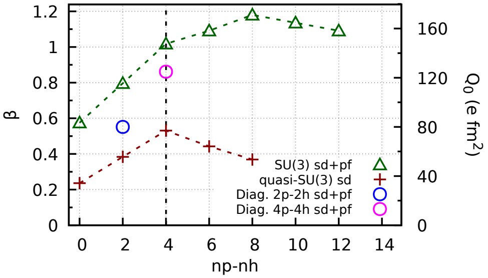
Fixed *np-nh* configurations

Analytical SU(3) models:

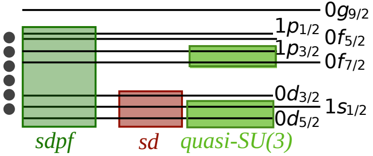
- sd-shell deformations:
  $\beta \leq 0.5$; $Q_0 \leq 80$ efm^2
  too low for SD!
- sdpf space deformations:
  $\beta \geq 0.5$; $Q_0 \geq 80$ efm^2
  SD for ≥4p-4h

Numerical calculations:

- Shell model *np-nh*:
  Similar to quasi-SU(3) sdpf

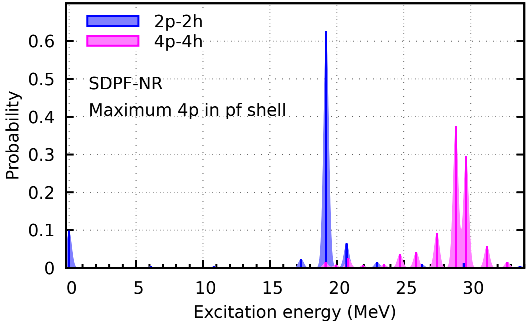


β parameters for SU(3) schemes and nuclear shell model *np-nh* configurations



Decomposition of a fixed *np-nh* configuration into the fully mixed states of the Hamitonian: $|0^+_{np\text{-}nh}\rangle_{sdpf} = \frac{1}{N}\sum_\sigma S(\sigma)|0^+_\sigma\rangle_{sdpf}$

Truncation: maximum of 4 particles into *pf* shell (**dimensions!**)



- Energies: 2p-2h at 19 MeV and 4p-4h at **30 MeV**!!!

Lanczos algorithm

- Initial state: $|1\rangle$
- Next step: $E_{12}|2\rangle = (H - E_{11})|1\rangle$
- Then: $E_{23}|3\rangle = (H - E_{22})|2\rangle - E_{12}|1\rangle$
- Generalizing: $E_{NN+1}|N+1\rangle = (H - E_{NN})|N\rangle - E_{N-1N}|N-1\rangle$

Where: $E_{NN} = \langle N|H|N\rangle$

$$\begin{pmatrix} E_{11} & E_{12} & 0 \\ E_{12} & E_{22} & E_{23} \\ 0 & E_{23} & E_{33} \end{pmatrix},$$

- Finally, diagonalize and check convergence

Projected generator coordinate method (PGCM)

Variational approach:

- Configuration mixing of Hartree-Fock-Bogoliubov (HFB) states: $|\Psi_{\rm GCM}\rangle = \sum_q f_q |\phi_{HFB}(q)\rangle$

  B. Bally, et al. Eur. Phys. J. A **60**, 62 (2024)
- Similar deformations for all interactions
- $2\nu\beta\beta$ matrix elements are larger for similar deformations

  T. R. Rodríguez and G. Martínez-Pinedo

  Phys. Rev. C **85**, 044310 (2012)

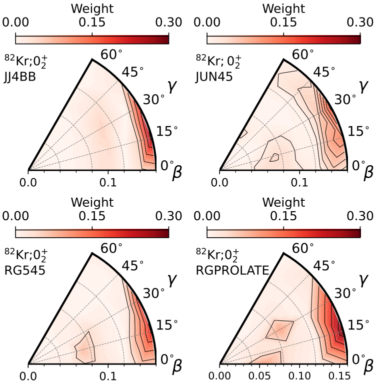


Figure: Contribution of each HFB wavefunction to fully mixed state for ^{82}Kr (0_2^+) with all interactions.