Axial asymmetry of the nuclear shape and its impact on the features of observed rotational bands

Elena A. Lawrie,
iThemba LABS, Cape Town, South Africa

SSNET'24

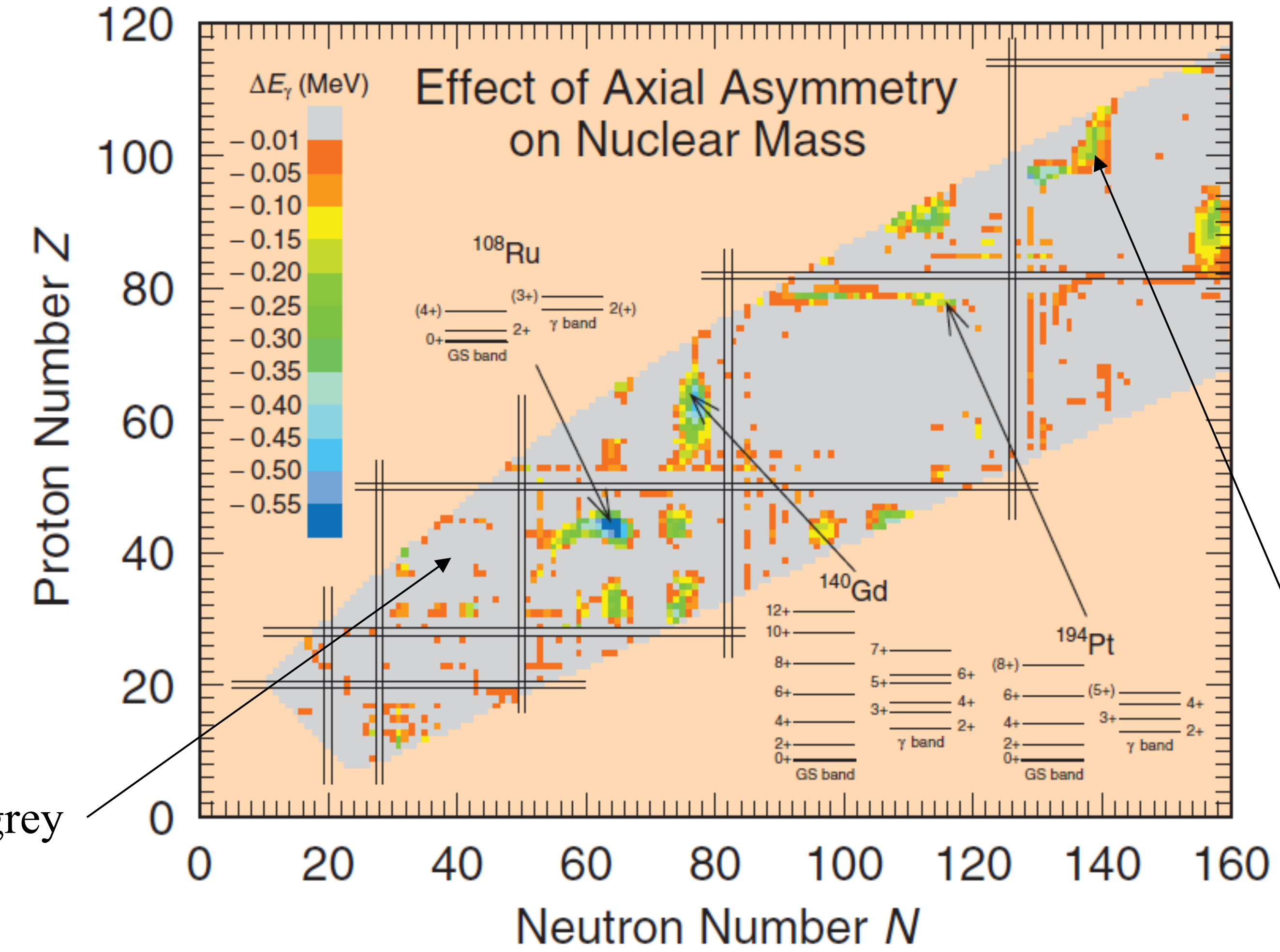
Outline

- How to deduce γ in simple, easy, (practically) model-independent way?
  - For even-even rotating nuclei
    - Davydov-Filippov model applied to 2^+ states becomes assumptions-free
    - use ratios of E2 matrix elements, $R_{22/02}$ and/or $R_{22\gamma/22}$ to deduce γ (including prolate-like or oblate-like)
  - For even-even rotating nuclei
    - Analysis of the Gamma-ray energies vs Spin plots for the ground-state band indicate nuclear shapes (axially symmetric, γ-rigid, γ-soft)

- Wobbling: vibrational and rotational excitations in even-even and odd-mass triaxial nuclei
  - Definitions and conflicting terminology
  - How to distinguish between vibrational and rotational excitations

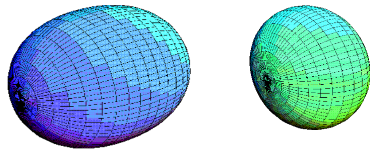
Global calculations on axial asymmetry of nuclei

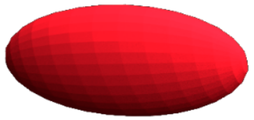
ΔE_γ - calculated energy difference assuming triaxial and axially symmetric nuclear shape at ground state



axially symmetric, grey

triaxial (like kiwi fruit), color





Moller et al., PRL 97, 162502 (2006)

How to measure triaxial deformation in a model-independent approach?

Kumar-Cline sum rules analysis

- rotational invariants – the same in the intrinsic and laboratory frame – $<Q^2>$, $<Q^3\cos(3\gamma)>$, $<Q^4>$…

$$\langle \hat{Q}^2 \rangle = \sqrt{5}\langle [\hat{E2}\times\hat{E2}]_0\rangle s,$$

$$\langle [\hat{E2}\times\hat{E2}]_0\rangle s = \frac{(-1)^{2I_s}}{\sqrt{(2I_s+1)}}\sum_t M_{st}M_{ts}\begin{Bmatrix}2 & 2 & 0\\ I_s & I_s & I_t\end{Bmatrix},$$

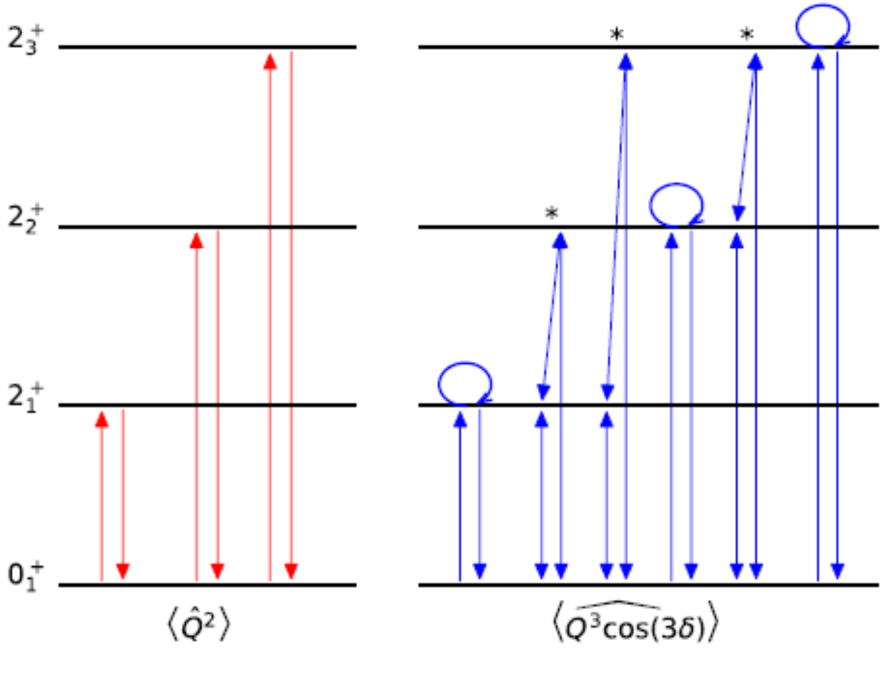
$$\langle Q^3\widehat{\cos(3\delta)}\rangle = -\frac{\sqrt{35}}{\sqrt{2}}\langle\{[\hat{E2}\times\hat{E2}]_2\times\hat{E2}\}_0\rangle s,$$

$$\langle\{[\hat{E2}\times\hat{E2}]_2\times\hat{E2}\}_0\rangle s = \frac{(-1)^{2I_s}}{2I_s+1}\sum_{tu} M_{su}M_{ut}M_{ts}\begin{Bmatrix}2 & 2 & 2\\ I_s & I_t & I_u\end{Bmatrix}.$$

M_{st} usually measured in Coulex

Need a large number of M_{st} (at least 4 for second-term sums)

not many nuclei with sufficient info on M_{st}

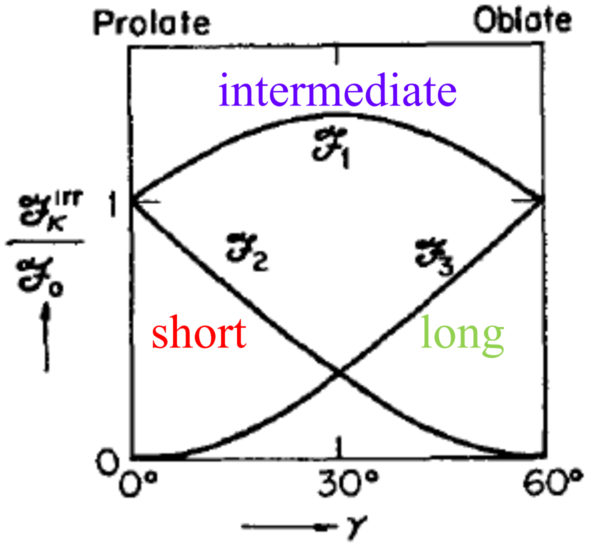


J. Henderson, PRC 102, 054306 (2020)

Even-even rotating nuclei with quadrupole shapes

Empirical evidence for MOI from measured energies and electric quadrupole matrix elements for 12 even-even rotating nuclei with $R_{4/2} > 2.7$

- Davydov-Filippov model, assumptions:
  - Spin dependence of the MoI - $\mathfrak{I}_0(I)$ is constant
  - Dependence of MoI with respect to γ - irrotational-flow model

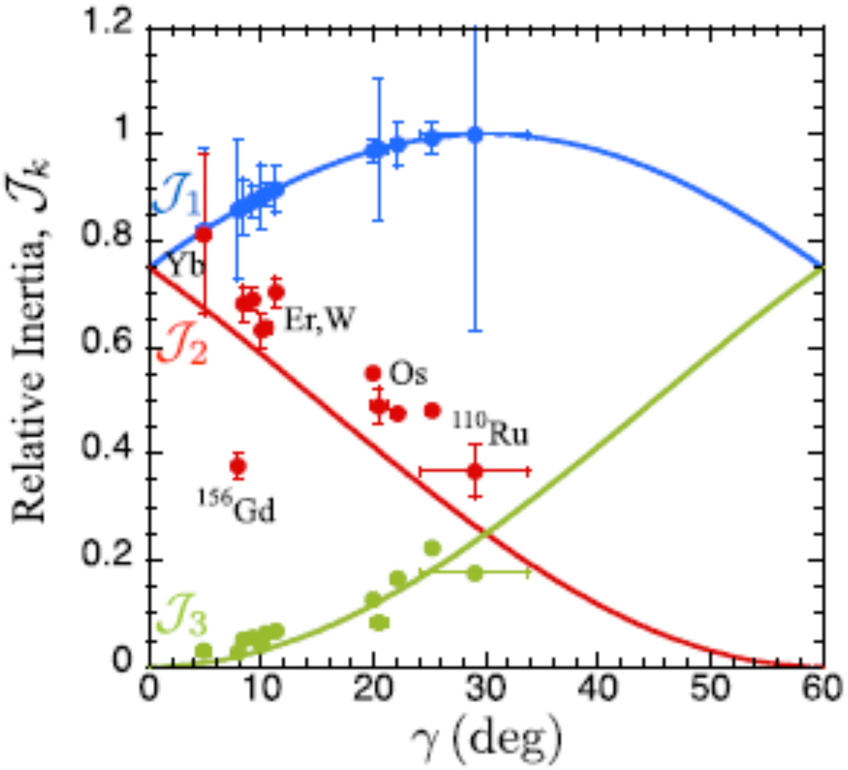


$$\mathcal{H} = \frac{\hbar^2}{2\mathfrak{I}_1}\hat{I}_1^2 + \frac{\hbar^2}{2\mathfrak{I}_2}\hat{I}_2^2 + \frac{\hbar^2}{2\mathfrak{I}_3}\hat{I}_3^2,$$

$$\mathfrak{I}_k(\gamma) = \mathfrak{I}_0 \sin^2\left(\gamma - k\frac{2\pi}{3}\right),$$

for $\gamma = 30^o$

$\mathfrak{I}_1 = 4\,\mathfrak{I}_2 = 4\,\mathfrak{I}_3$



- Generalized TR model, applied for the 2^+ and 2^+_γ states
  - Spin dependence of the MoI - redundant
  - Asymmetry of MoI described with a new parameter Γ independent of γ
    - ✓ the model becomes assumptions free for even-even rotating nuclei
    - ✓ deduced triaxial deformation γ_{TR} for **26** even-even nuclei with $R_{4/2} > 2.4$
    - ✓ needs 4 matrix elements and 2 excitation energies

J.M. Allmond, J.L.Wood, Physics Letters B 767 (2017) 226–231

J.M. Allmond, CWAN'23 conference, for $R_{4/2} > 2.4$

Measure γ for even-even rotating nuclei
in an assumptions-free way

using 2 E2 matrix elements

Even-even rotating nuclei

We have expanded the generalized TR approach

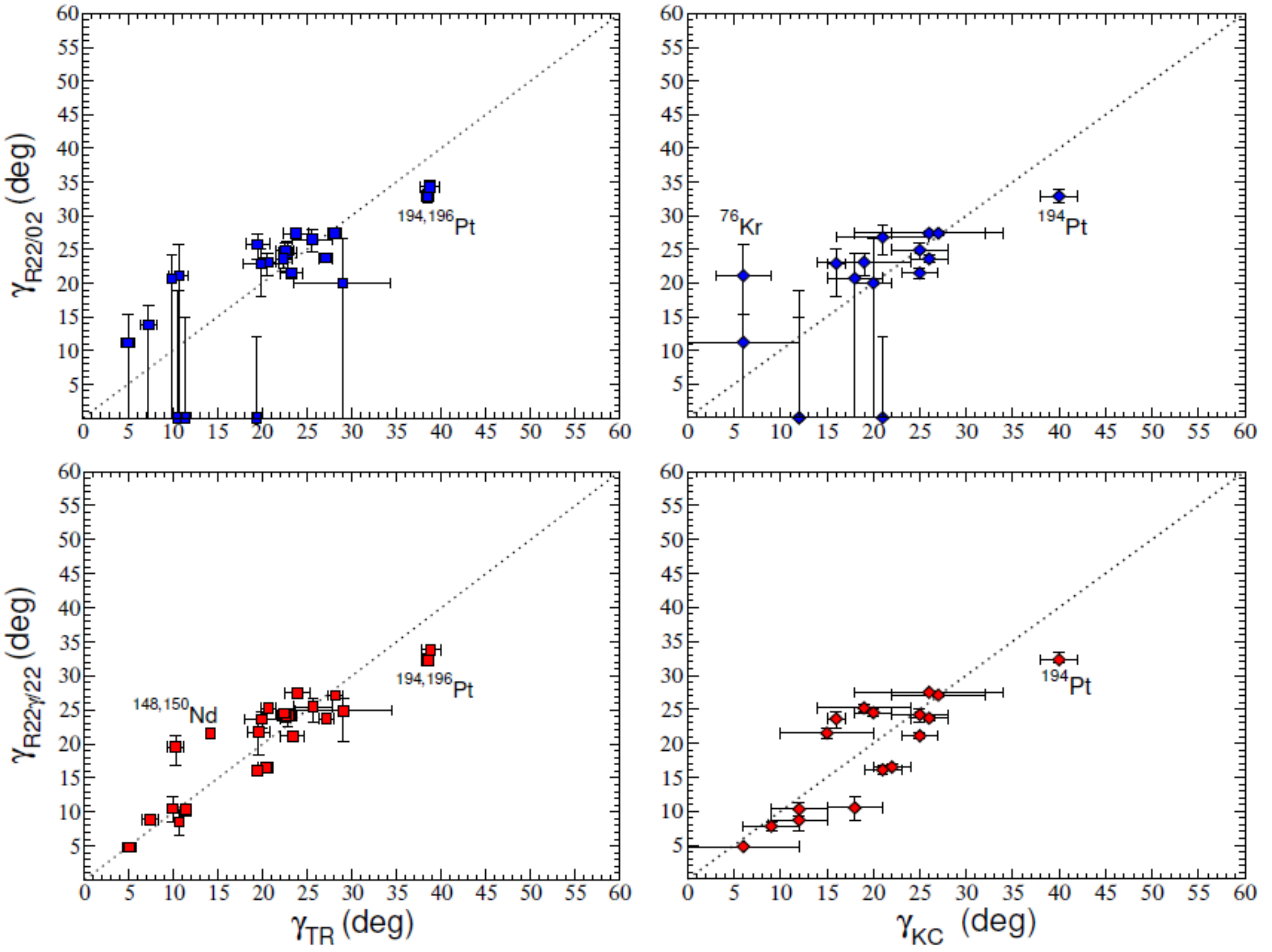
- by adopting the irrotational-flow dependence of MoI not as a model assumption but as empirically proven dependence
- work with 2^+ and 2^+_γ states
- introduce ratios of two E2 matrix elements within the DF equations (assumptions-free)
- need 2 matrix elements per ratio
- test the deduced γ against the γ_{KC} and γ_{TR}
- extract γ deformation based on these ratios
- for all even-even rotating nuclei where data on two matrix elements are available

$$R_{22/02} := \frac{\langle 2^+_1 \| \hat{E2} \| 2^+_1 \rangle}{\langle 0^+_1 \| \hat{E2} \| 2^+_1 \rangle}$$

$$R_{22/02}(\gamma) = -\sqrt{\frac{10}{7}}\,\frac{\cos\left(\gamma + \cos^{-1}\left(\frac{\cos(4\gamma)+2\cos(2\gamma)}{\sqrt{4\cos(6\gamma)+5}}\right)\right)}{\cos\left(\gamma - \frac{1}{2}\cos^{-1}\left(\frac{\cos(4\gamma)+2\cos(2\gamma)}{\sqrt{8\cos^2(3\gamma)+1}}\right)\right)}.$$

$$R_{22\gamma/22} := \frac{\langle 2^+_1 \| \hat{E2} \| 2^+_\gamma \rangle}{\langle 2^+_1 \| \hat{E2} \| 2^+_1 \rangle}$$

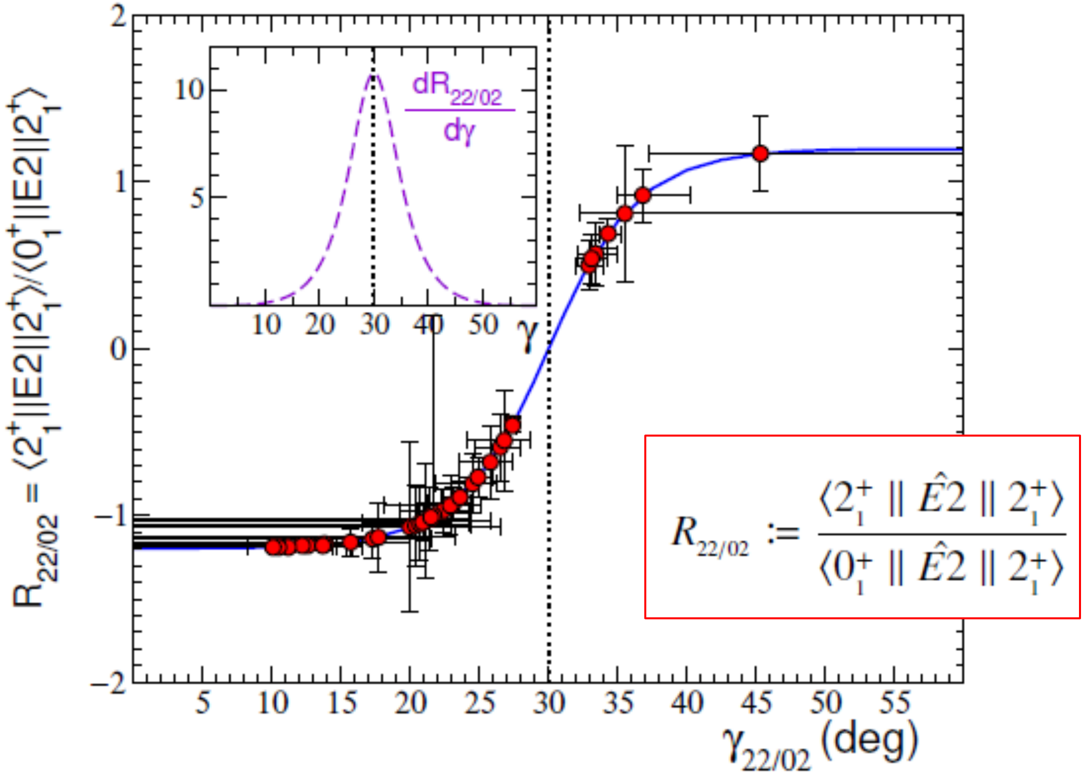
$$R_{22\gamma/22}(\gamma) = -\tan\left[\gamma + \cos^{-1}\left(\frac{\cos(4\gamma) + 2\cos(2\gamma)}{\sqrt{9 - 8\sin^2(3\gamma)}}\right)\right].$$

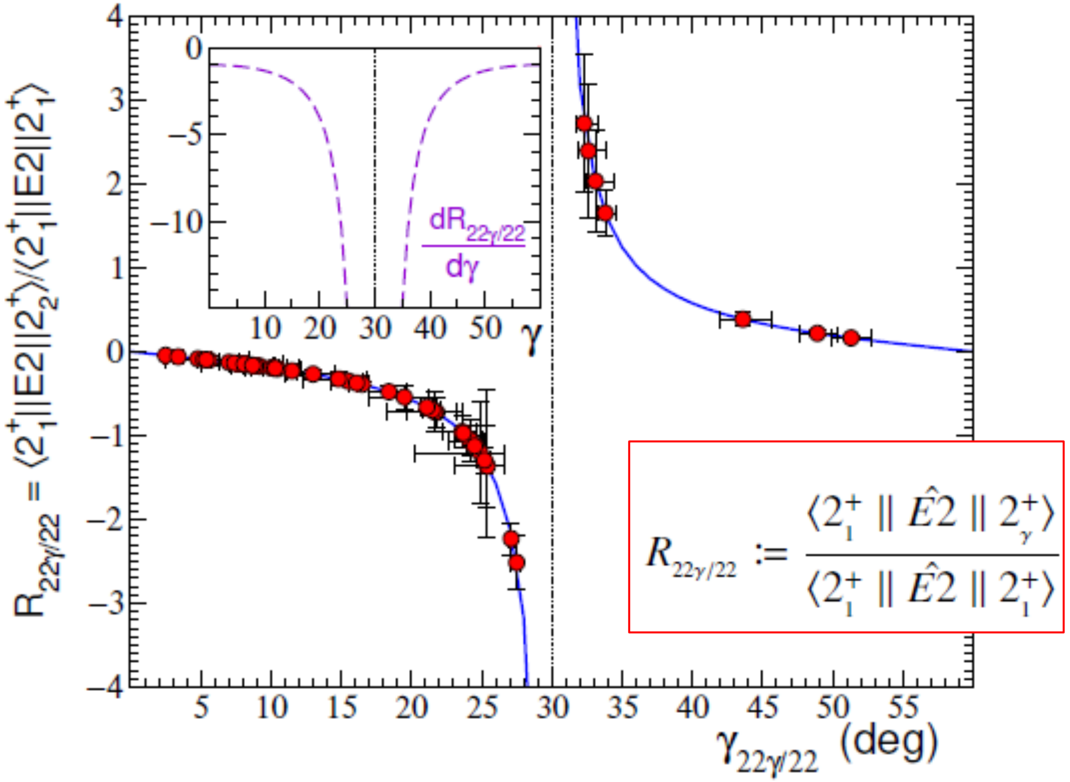


Even-even rotating nuclei,

- **$R_{4/2} > 2.4$,**
- **data on two matrix elements available for more than 60 even-even nuclei**
- **deduced triaxiality in an assumptions-free approach**
- **distinguishes between prolate-like and oblate-like shapes**

E.A. Lawrie, J.N. Orte, submitted





✓ sensitive for $20^o < \gamma < 40^o$

✓ does not require knowledge of γ band

sensitive in the full range $0^o < \gamma < 60^o$

Measure γ for even-even rotating nuclei in an assumptions free way

using γ-ray energies in the gs band

Even-even rotating nuclei

$$\mathcal{H} = \frac{\hbar^2}{2\mathfrak{I}_1}\hat{I}_1^2 + \frac{\hbar^2}{2\mathfrak{I}_2}\hat{I}_2^2 + \frac{\hbar^2}{2\mathfrak{I}_3}\hat{I}_3^2,$$

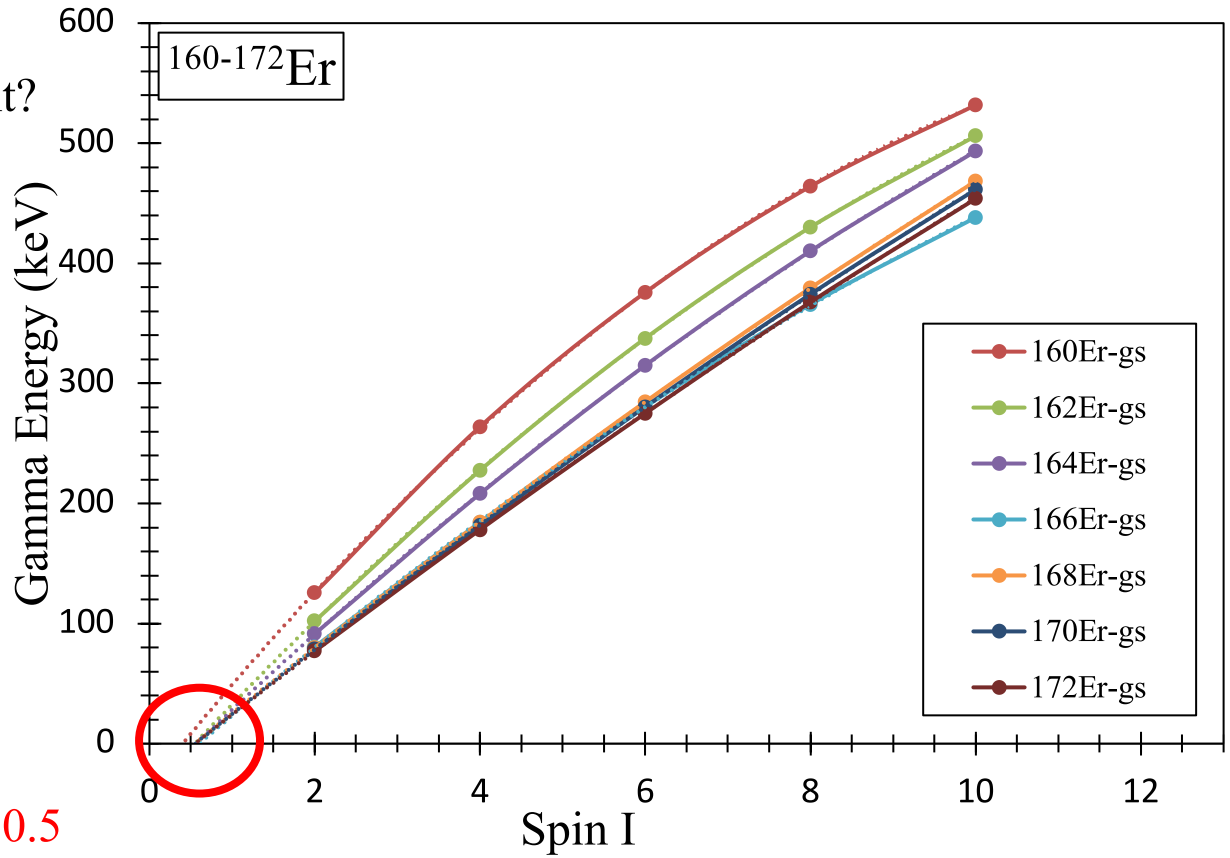
- Davydov-Filippov model assumptions:
  - Spin dependence of the MoI

    is it possible to consider states with I > 2 while keeping $\mathfrak{I}$(I) dependence irrelevant?

- axially-symmetric shape

$$E(I) = \frac{\hbar^2}{2\mathfrak{I}} I(I+1)$$

$$E_\gamma(I) = \frac{\hbar^2}{2\mathfrak{I}} [4I - 2]$$



- the shape of $E_\gamma(I)$ reflects the dependence of $\mathfrak{I}$(I)
- if $\mathfrak{I}$ is constant - E_γ vs I is linear
- the crossing point with x-axis, where $E_\gamma(I)$=0, is at $I_c = 0.5$

While the gs bands may have different MoI, the crossing point is specific for axially symmetric nuclei, $I_c = 0.5$

Even-even rotating nuclei

- **stable triaxial shape**

$$E(I) = \frac{\hbar^2}{2\mathfrak{J}_1} I_1^2 + \frac{\hbar^2}{2\mathfrak{J}_2} I_2^2 + \frac{\hbar^2}{2\mathfrak{J}_3} I_3^2$$

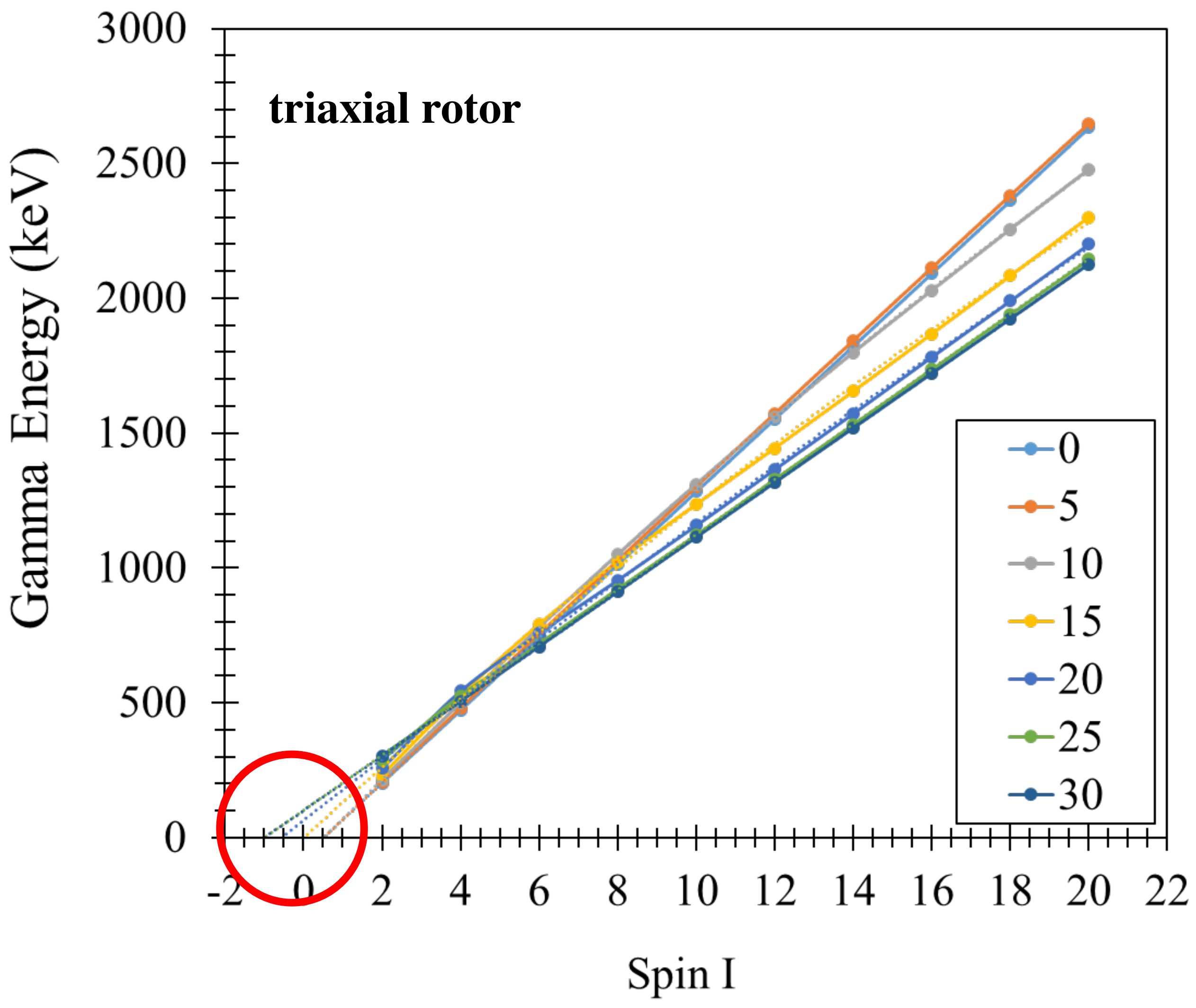
no analytical solutions, except for γ = 30º
(irrotational-flow MoI dependence with γ)

$$E_\gamma(I, \gamma = 30^o) = \frac{\hbar^2}{2\mathfrak{J}} [4I + 4]$$

- $E_\gamma(I, \gamma = 30^o)$ crosses the x-axis is $I_c = -1$

- **DF calculations yield** $0^o < \gamma < 30^o$ $-1 < I_c < +0.5$

$0^o < \gamma < 10^o$	$I_c \sim +0.5$
$\gamma = 15^o$	$I_c \sim 0$
$\gamma = 20^o$	$I_c \sim -0.5$
$25^o < \gamma < 30^o$	$I_c \sim -1$



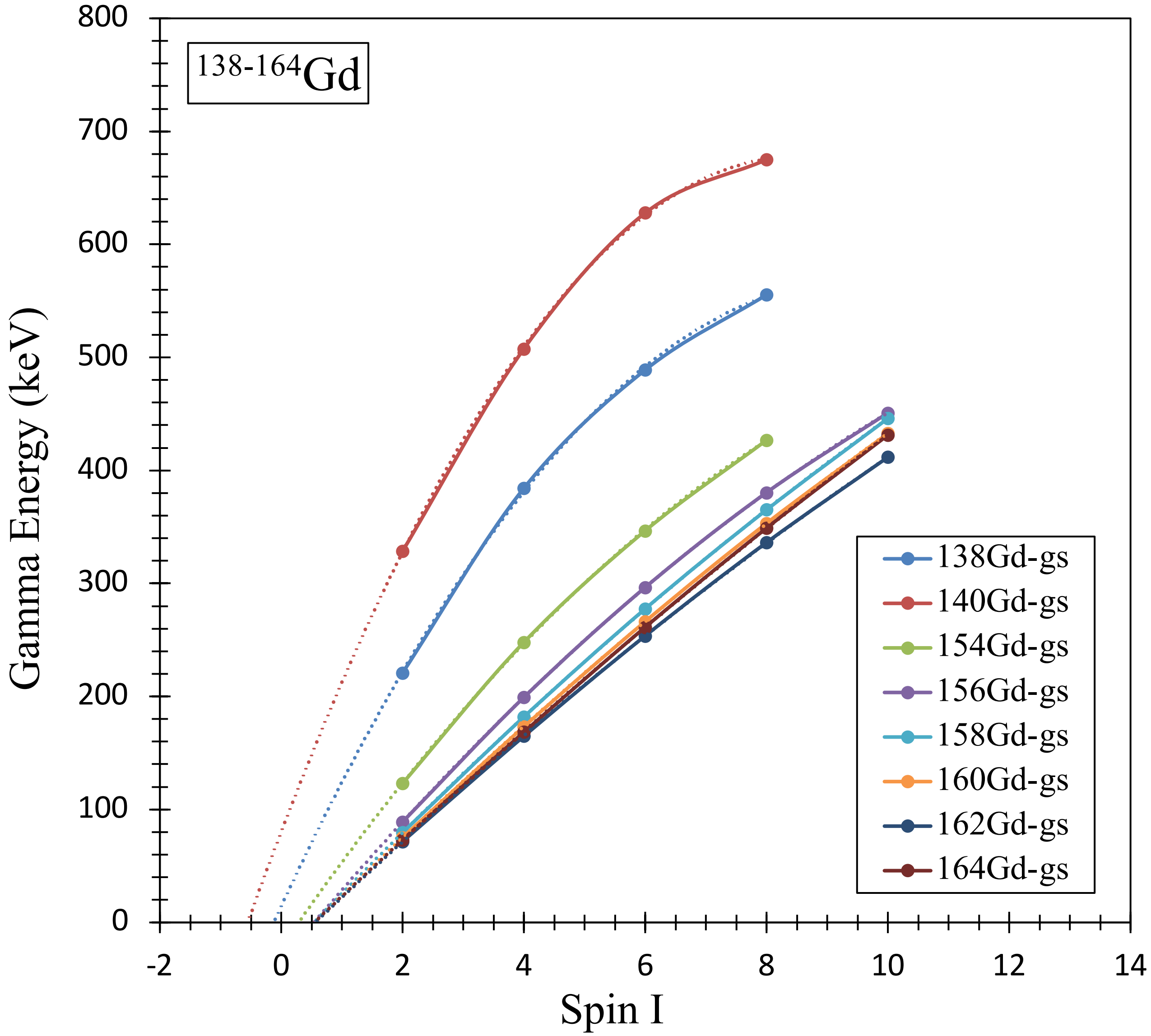
While the gs bands may have different MoI, the crossing point is specific for rigid triaxial nuclei $-1 < I_c < +0.5$

Even-even rotating nuclei

Example:
according to the global calculations
nuclei near ^{140}Gd are triaxial

Results:

$^{156\text{-}164}$Gd, $I_c = +0.5$, $\gamma < 10^o$
^{154}Gd, $I_c = +0.3$, $\gamma \sim 13^o$
^{138}Gd, $I_c = -0.1$, $\gamma \sim 16^o$
^{140}Gd, $I_c = -0.5$, $\gamma \sim 20^o$



The lighter Gd isotopes develop axial asymmetry

Even-even rotating nuclei

- ❑ γ-vibrations (Wilets-Jean)

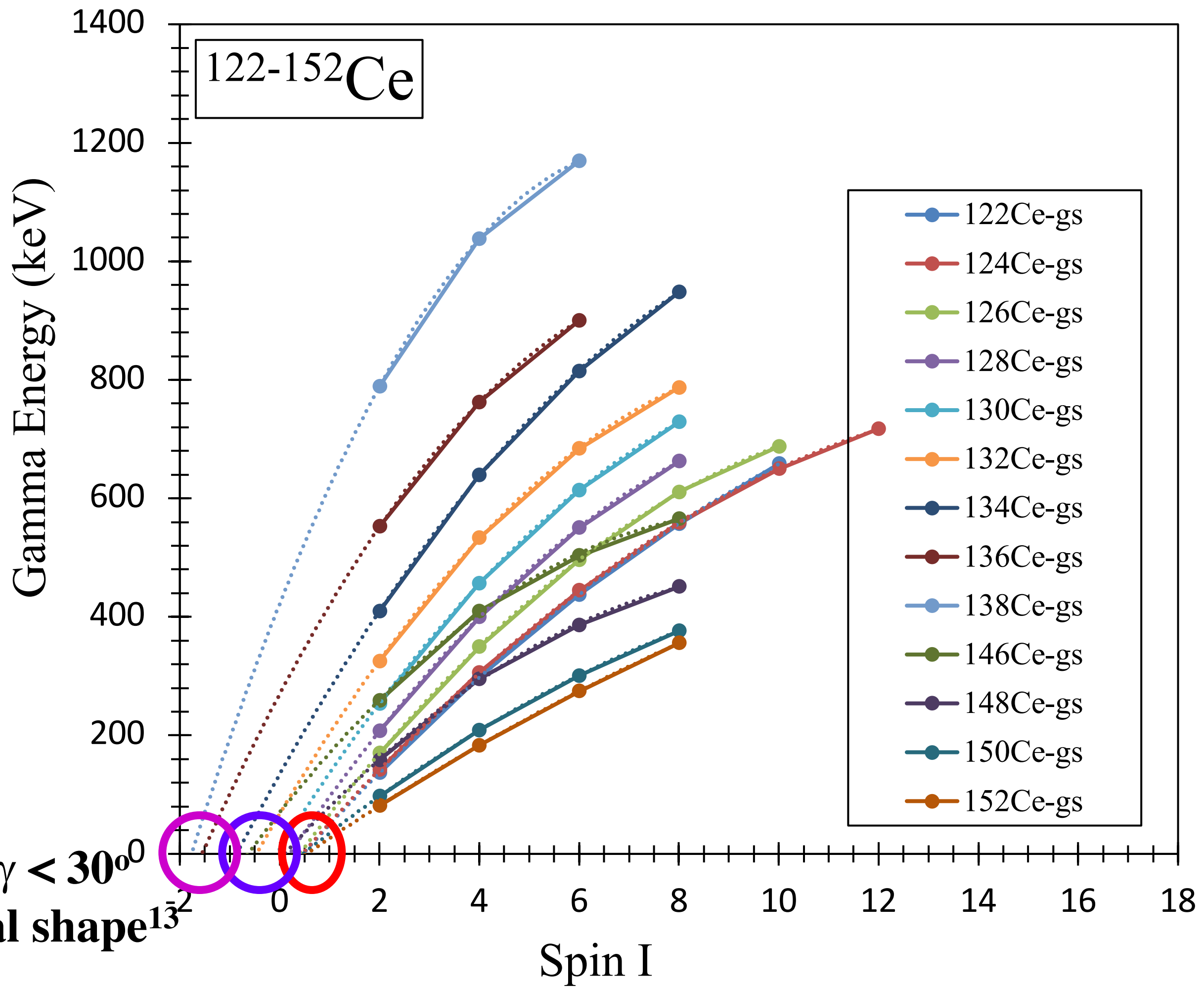
$$E(I) = A\,I(I+6)$$

$$E_\gamma(I) = A\,[4I+8]$$

  - ➢ $E_\gamma(I)$ crosses the x-axis is $I_c = -2$

- ❑ γ-vibrations around and average triaxial shape?

  - ➢ $E_\gamma(I)$ crosses the x-axis is $-2 < I_c < -1$



Example:

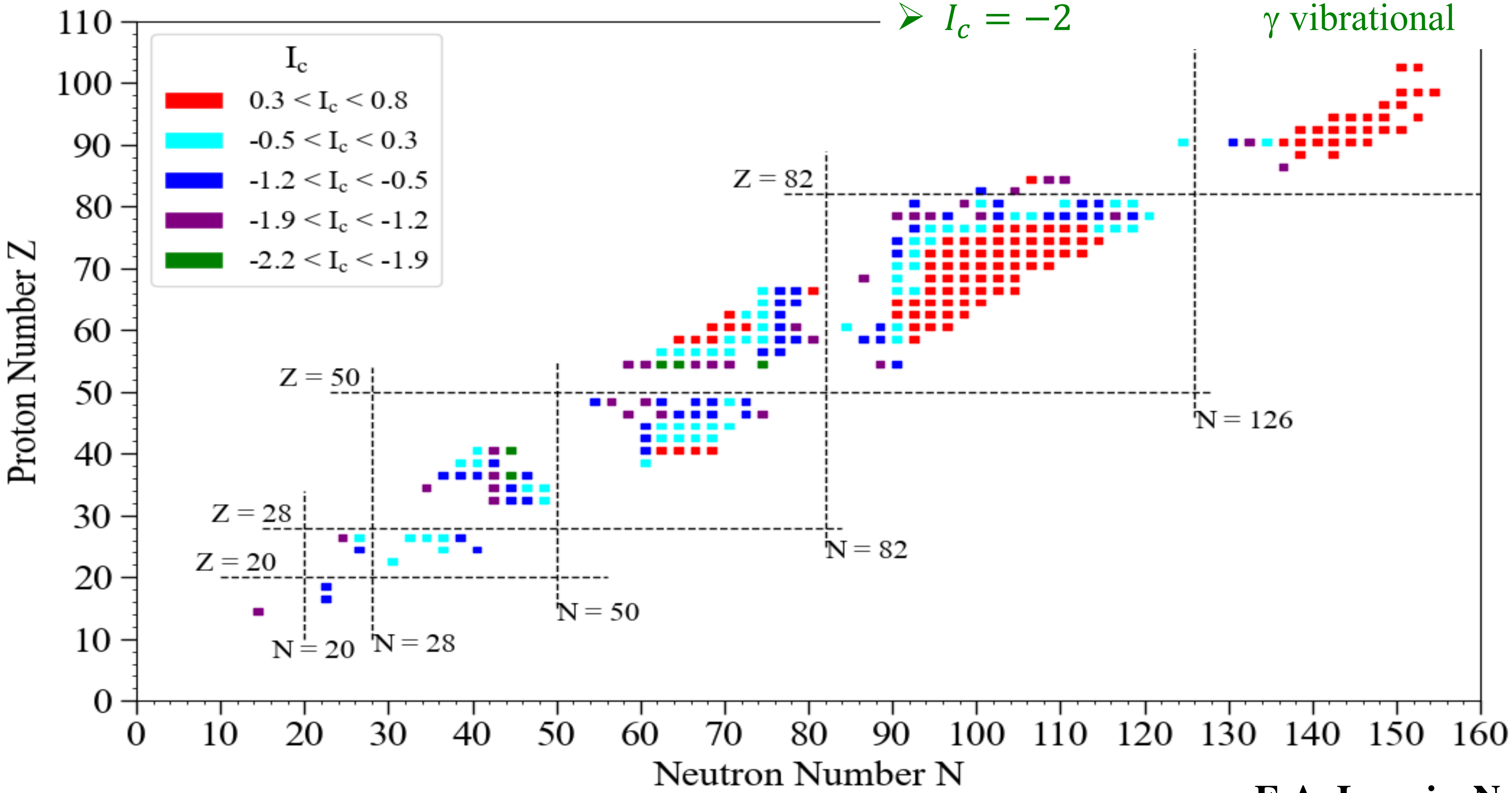
$^{122-126}$Ce →	$I_c \sim 0.5$	$0^o < \gamma < 10^o$
$^{128-130}$Ce →	$I_c \sim 0$	stable triaxial, ~15^o
$^{132-134}$Ce →	$I_c \sim -1$ to -0.5	stable triaxial, $20^o < \gamma < 30^o$
$^{136-138}$Ce →	$I_c \sim -1.5$ to -1.7	γ-soft around triaxial shape

all even-even rotating nuclei with $2.4 < R_{4/2} < 3.3$

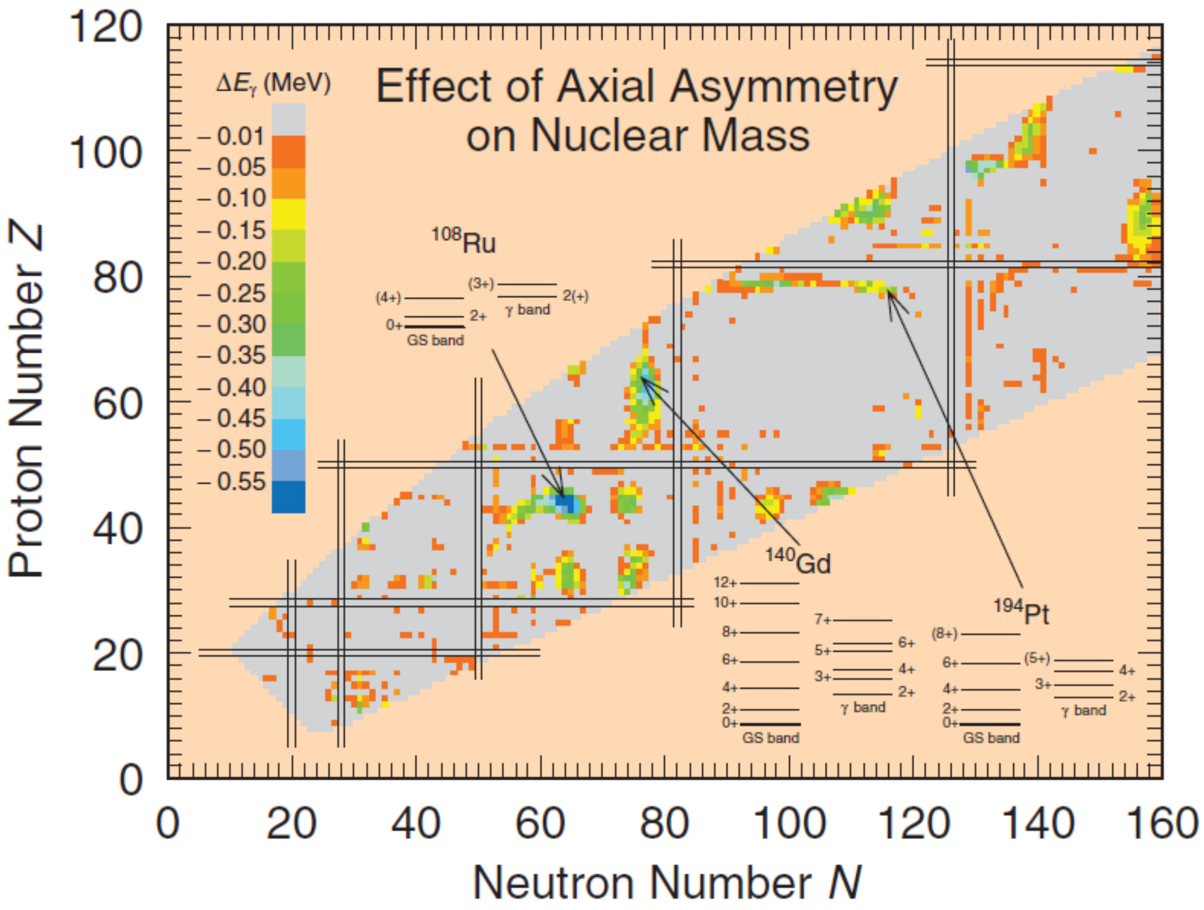
does not distinguish between prolate-like ($\gamma < 30^o$)

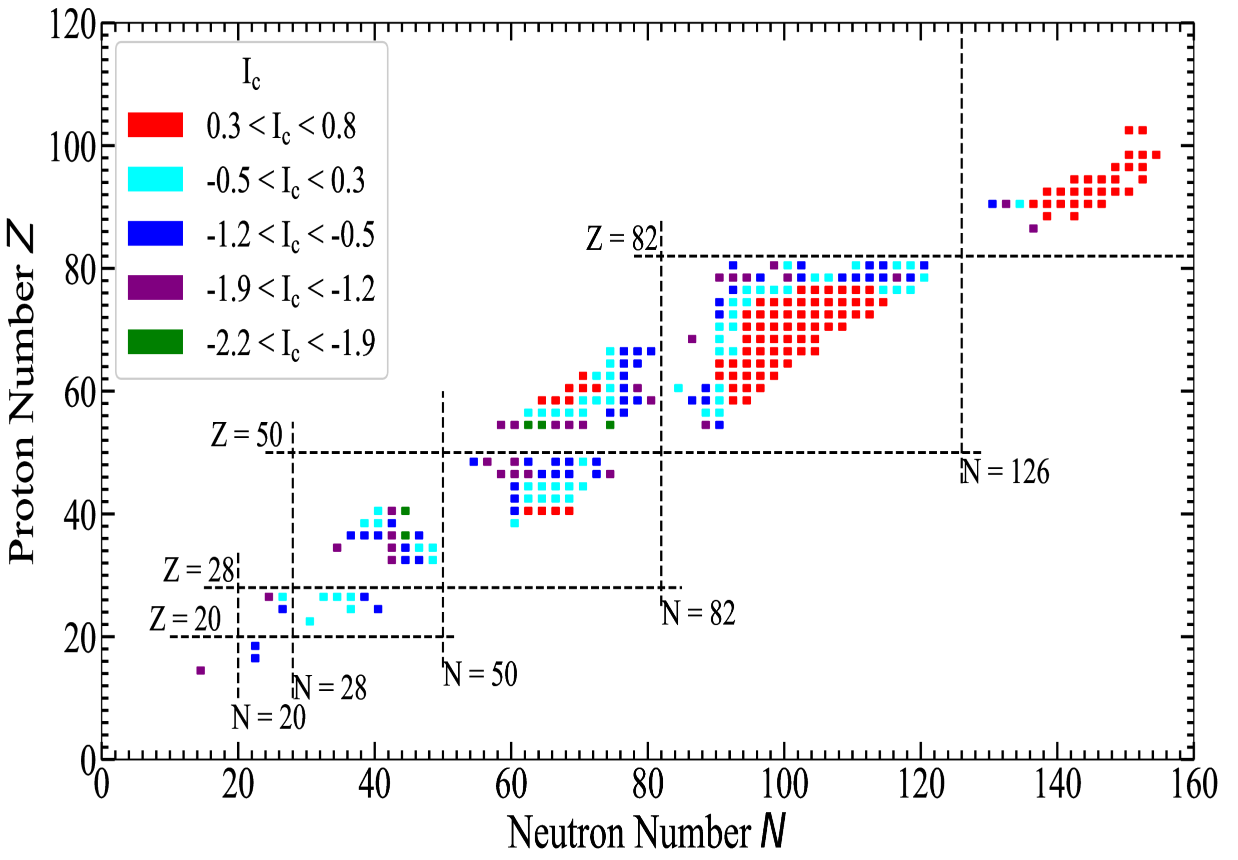
and oblate-like ($\gamma > 30^o$)

- $I_c \sim +0.5$ axially symmetric, $\gamma < 10^o$
- $-0.5 < I_c < 0.5$ stable triaxial $\gamma = 10^o - 20^o$
- $-1 < I_c < -0.5$ stable triaxial
- $-2 < I_c < -1$ γ vibrations around triaxial shape
- $I_c = -2$ γ vibrational

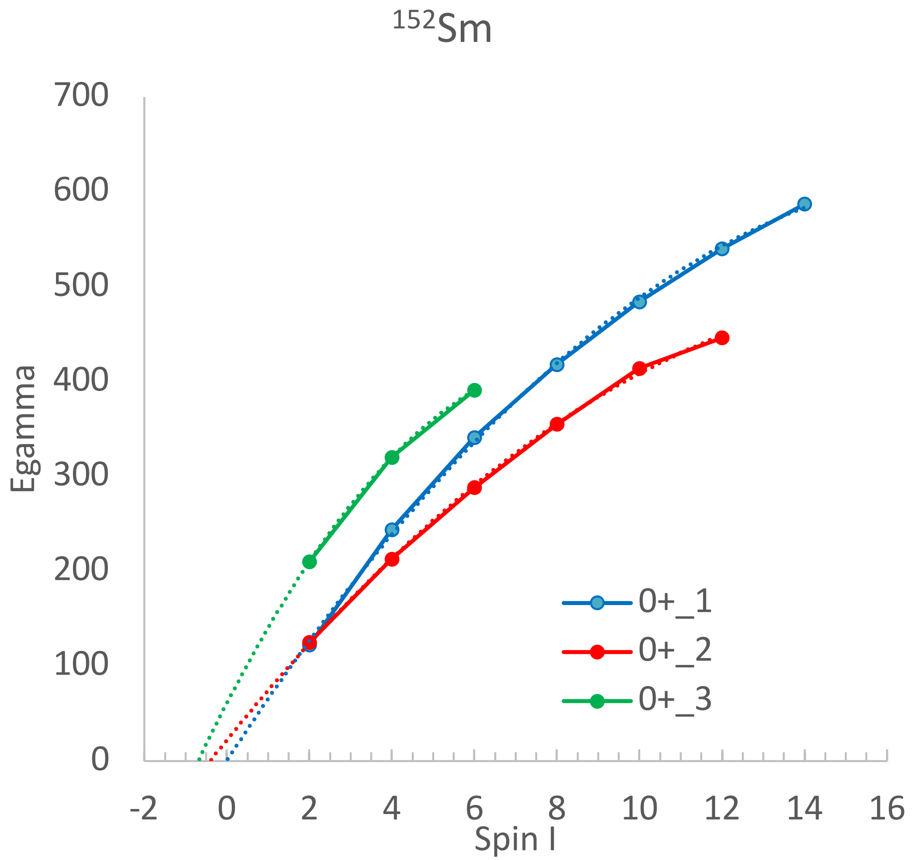


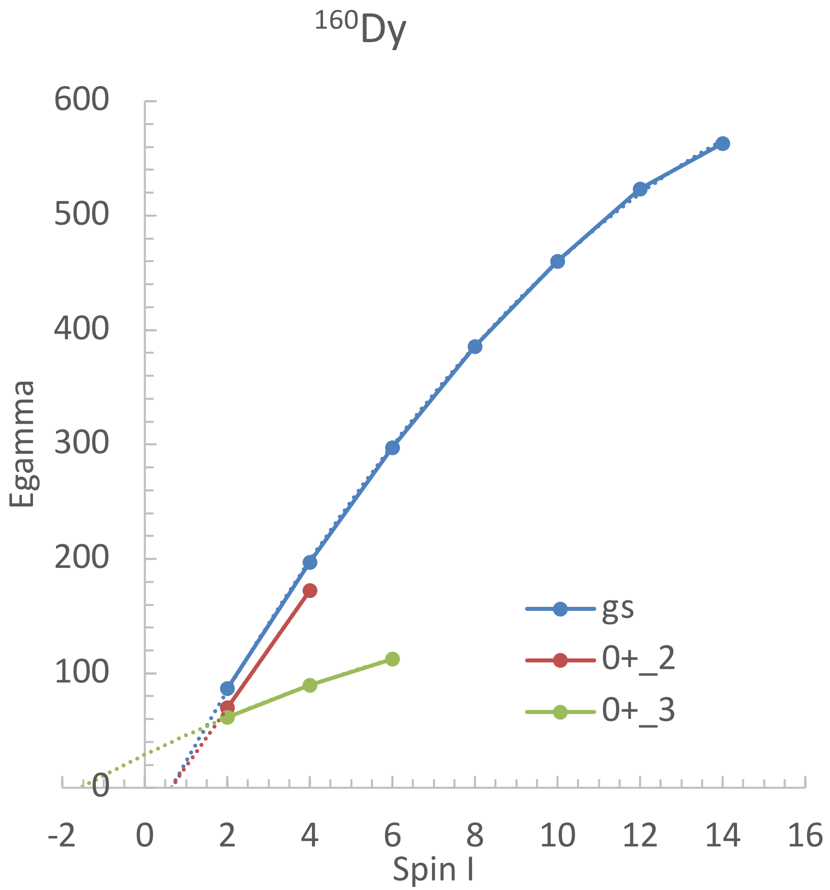
E.A. Lawrie, N. Xulu, in preparation





E.A. Lawrie, N. Xulu, in preparation


152Sm
Egamma
Spin I
0+_1
0+_2
0+_3

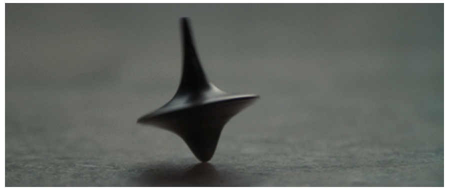

160Dy
Egamma
Spin I
gs
0+_2
0+_3

Wobbling in triaxial nuclei

harmonic vibrational
or
rotational excitation

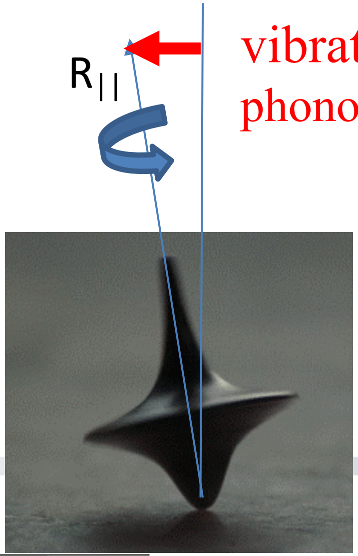
Definition of wobbling ~ 1970s

Bohr and Mottelson, Nuclear Structure



$H = A_1 I_1^2 + A_2 I_2^2 + A_3 I_3^2 \approx A_1 I^2 + \hbar\omega\,(n+1/2)$ at high spins

wobbling → harmonic vibrational excitation
→ n - number of wobbling phonons
→ quantization in excitation energy,
→ quantization B(E2; n→ n -1) ….



vibrational phonon

190 文 ROTATIONAL SPECTRA *Ch. 4*

4-5e States with Large *I*

Simple and illuminating solutions for the asymmetric rotor can be obtained for the high angular momentum states in the yrast region. In the classical theory of the asymmetric rotor, the motion reduces to a simple rotation without precession of the axes, if the angular momentum is along the axis corresponding to the largest or smallest moment of inertia. Correspondingly, in the quantal theory, the states of smallest (or largest) energy for given *I* acquire a simple structure in the limit of large *I* (Golden and Bragg, 1949).

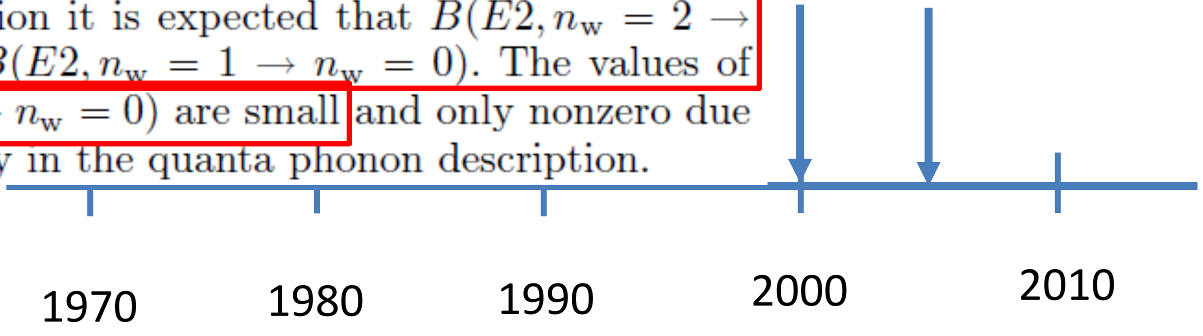
wobbling → triaxial-rotor model at high spins

B&M definition wobbling → harmonic vibrational excitation

wobbling → TSD bands in the odd-mass Lu isotopes

a sequence of wobbling bands described by the energy, $E_R(I, n_w) = I(I+1)/2\mathcal{J}_x + \hbar\omega_w(n_w + 1/2)$, where n_w is the wobbling phonon number and the wobbling fre-

The most crucial information from the particle-rotor calculations of ref. [25] is contained in the size of the electromagnetic transition matrix elements. In particular, the values of $B(E2, n_w = 1 \rightarrow n_w = 0)$ are around 22–30% of the $B(E2, n_w = 1 \rightarrow n_w = 1)$ values for the collective in-band transitions in the spin-range covered by the experiment. Furthermore, with a wobbling phonon description it is expected that $B(E2, n_w = 2 \rightarrow n_w = 1) \sim 2 \cdot B(E2, n_w = 1 \rightarrow n_w = 0)$. The values of $B(E2, n_w = 2 \rightarrow n_w = 0)$ are small and only nonzero due to anharmonicity in the quanta phonon description.



Eur. Phys. J. A **20**, 183–188 (2004)
DOI 10.1140/epja/i2002-10349-4

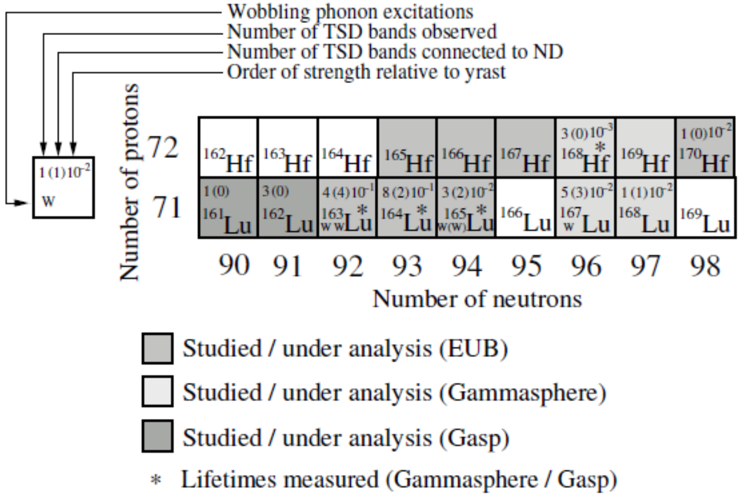
THE EUROPEAN PHYSICAL JOURNAL A

Wobbling phonon excitations in strongly deformed triaxial nuclei

G.B. Hagemann[a]

The Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

Received: 30 October 2002 /
Published online: 23 March 2004 – © Società Italiana di Fisica / Springer-Verlag 2004



F&D definition of wobbling → rotational excitation

$H = A_1 R_1^2 + A_2 R_2^2 + A_3 R_3^2$ ~~$\approx A_1 I^2 + \hbar\omega\,(n+1/2)$ at high spins~~

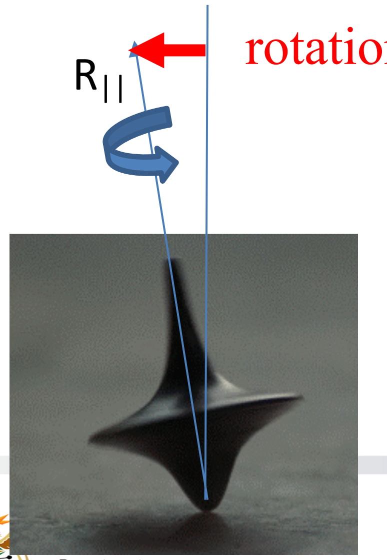
wobbling → triaxial rotor model at low spins
→ rotation excitation
→ harmonic vibrational features fall away
→ large B(E2; n → n-1)

since 2014 many wobbling bands proposed in
A=100, 130, 160, 190 mass regions

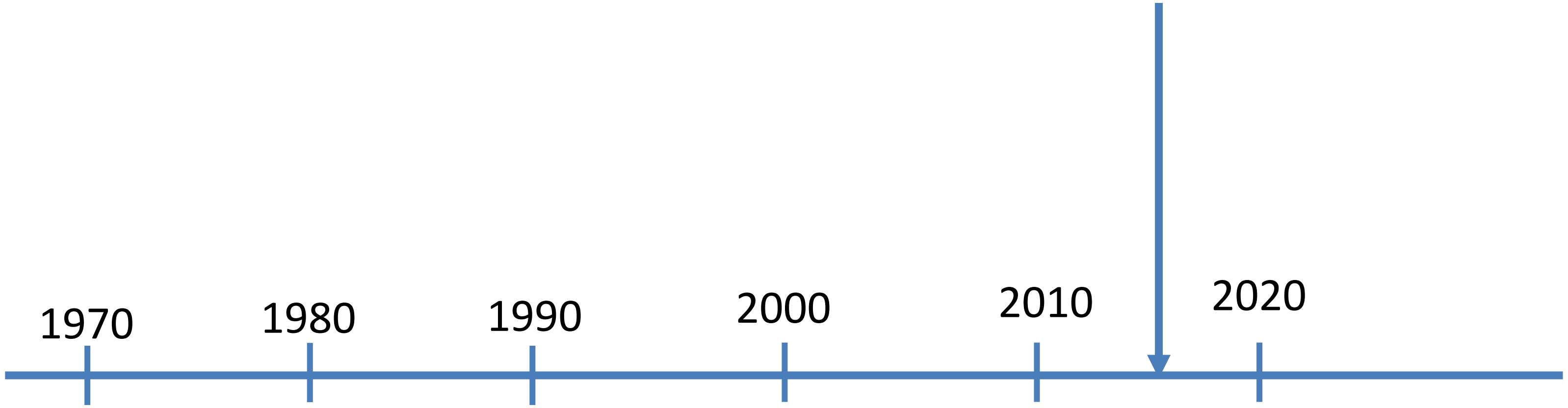
PHYSICAL REVIEW C **89**, 014322 (2014)

Transverse wobbling: A collective mode in odd-*A* triaxial nuclei

S. Frauendorf[1,*] and F. Dönau[2,†]
[1]*Department of Physics, University of Notre Dame, South Bend, Indiana 46556, USA*
[2]*Institut für Strahlenphysik, Helmholtz-Zentrum Dresden-Rossendorf, 01314 Dresden, Germany*
(Received 7 October 2013; revised manuscript received 17 November 2013; published 27 January 2014)



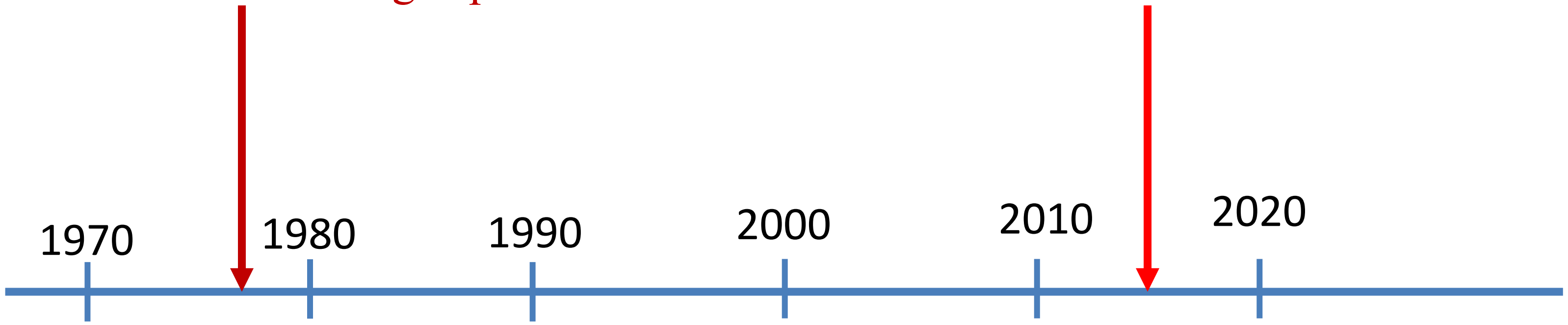
rotational



Two conflicting definitions

B&M definition wobbling → harmonic vibrational excitation triaxial rotor at high spin

F&D definition of wobbling → rotational excitation triaxial rotor at low spin



- conflict in the definitions
- many triaxial-rotor studies, which were published between 1980s and 2015 as rotational bands, would now qualify as wobbling bands
  - all γ-bands in even-even nuclei calculated with the triaxial rotor model (which were never adopted as wobbling within the B&M definition) would now, with the F&D definition, qualify as wobbling bands
  - similarly rotational bands in triaxial odd-mass nuclei that were not considered as wobbling within B&M definition, would with the F&D definition qualify as wobbling

what is core difference between the two definitions

→ B&M: wobbling is a excitation of harmonic vibrational nature
→ F&D: wobbling is a excitation of rotational nature

How to distinguish between vibrational and rotational excitations?

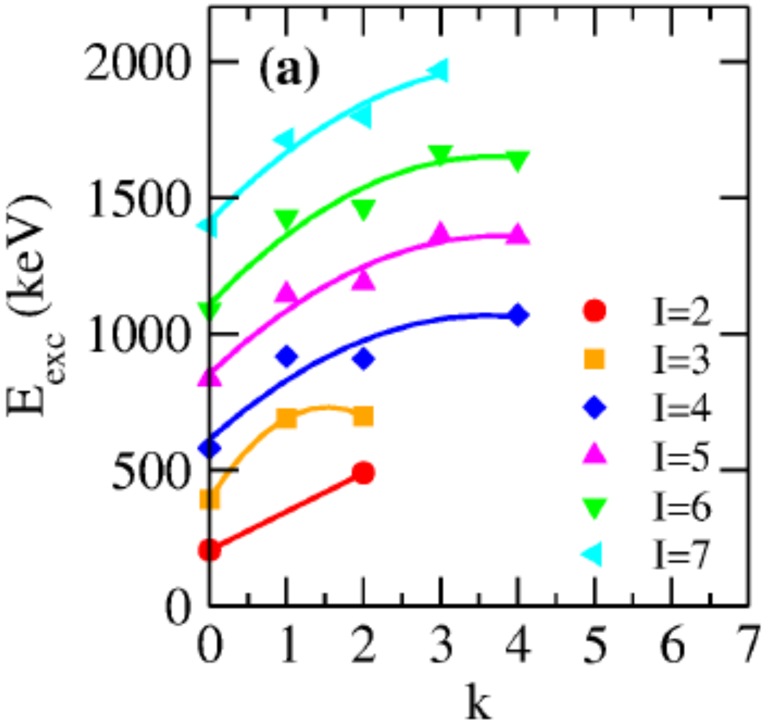
$\gamma = 30^o$

item	for	property	vibrational	rotational
1	ee, lc, tc	quantum number k	n	m
2	ee, lc, tc	$E_{exc}(I = const)$	$\propto n$	$\propto m^2$
3	ee	$E(2_g^+) + E(2_\gamma^+)$	$> E(3_\gamma^+)$	$= E(3_\gamma^+)$
4	ee, lc	E_{rel}	const with n	decreasing with m
5	ee, lc	$B(E2)_{intra}$	const with n	decreasing with m
6	ee, lc	$B(E2)_{intra}$	const with I	increasing with I
7	ee, lc, tc	$B(E2; k, I \to k-1, I-1)$	$\propto n$	not proportional to m *
8	ee, lc	$B(E2; k, I \to k-1, I-1)$	$\propto 1/I$	not proportional to $1/I$ *
9	ee, lc, tc	$B(E2)_{inter}$ for $\Delta n > 1$	0	> 0 (allowed)#
10	ee	$R_{2\gamma 2g} = \frac{B(E2;2_\gamma^+ \to 2_g^+)}{B(E2;2_g^+ \to 0_g^+)}$	0	1.4
11	ee	$R_{3\gamma 2\gamma} = \frac{B(E2;3_\gamma^+ \to 2_\gamma^+)}{B(E2;2_g^+ \to 0_g^+)}$	1	1.8
12	tc	E_{rel}	const with n	increasing with m &
13	tc	$E_{rel}(I)$	decreasing for $I < I_{max}$	decreasing for $I < I_c$ &
14	tc	$B(E2)_{intra}$	const with n	not const with m
15	tc	$B(E2)_{intra}$	const with I	not const with I

Examples on distinguishing between vibrational and rotational nature

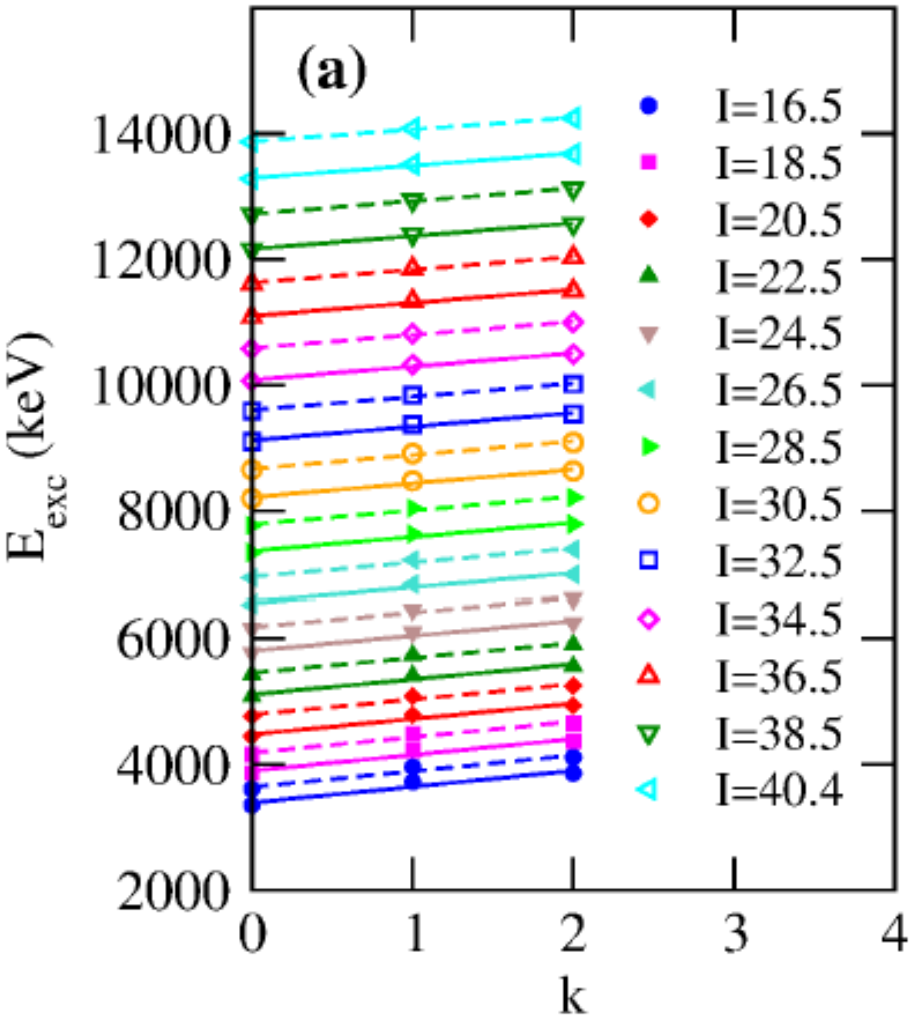
$E_{exc}(I) \propto n$ for vibrational and $\propto m^2$ for rotational type of excitation

^{192}Os



$E_{exc}(I) \propto k^2$ – rotational excitation

^{163}Lu



$E_{exc}(I) \propto k$ – vibrational excitation

what is core difference between the two definitions

→ B&M: wobbling is a excitation of harmonic vibrational nature

→ F&D: wobbling is a excitation of rotational nature

How to distinguish between vibrational and rotational excitations?

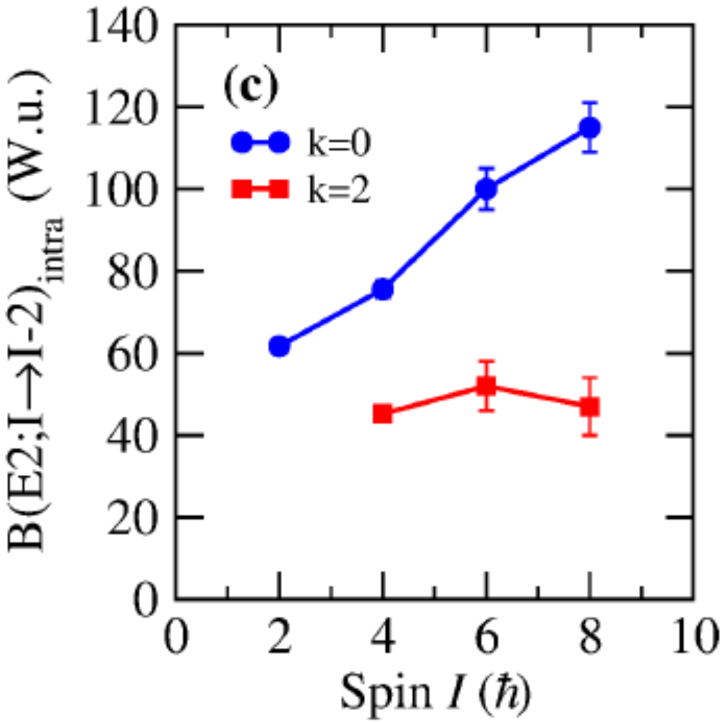
$\gamma = 30^o$

item	for	property	vibrational	rotational
1	ee, lc, tc	quantum number k	n	m
2	ee, lc, tc	$E_{exc}(I = const)$	$\propto n$	$\propto m^2$
3	ee	$E(2_g^+) + E(2_\gamma^+)$	$> E(3_\gamma^+)$	$= E(3_\gamma^+)$
4	ee, lc	E_{rel}	const with n	decreasing with m
5	ee, lc	$B(E2)_{intra}$	const with n	decreasing with m
6	ee, lc	$B(E2)_{intra}$	const with I	increasing with I
7	ee, lc, tc	$B(E2;k,I \to k-1,I-1)$	$\propto n$	not proportional to m *
8	ee, lc	$B(E2;k,I \to k-1,I-1)$	$\propto 1/I$	not proportional to $1/I$ *
9	ee, lc, tc	$B(E2)_{inter}$ for $\Delta n > 1$	0	> 0 (allowed)#
10	ee	$R_{2\gamma 2g} = \frac{B(E2;2_\gamma^+ \to 2_g^+)}{B(E2;2_g^+ \to 0_g^+)}$	0	1.4
11	ee	$R_{3\gamma 2\gamma} = \frac{B(E2;3_\gamma^+ \to 2_\gamma^+)}{B(E2;2_g^+ \to 0_g^+)}$	1	1.8
12	tc	E_{rel}	const with n	increasing with m &
13	tc	$E_{rel}(I)$	decreasing for $I < I_{max}$	decreasing for $I < I_c$ &
14	tc	$B(E2)_{intra}$	const with n	not const with m
15	tc	$B(E2)_{intra}$	const with I	not const with I

Examples on distinguishing between vibrational and rotational nature

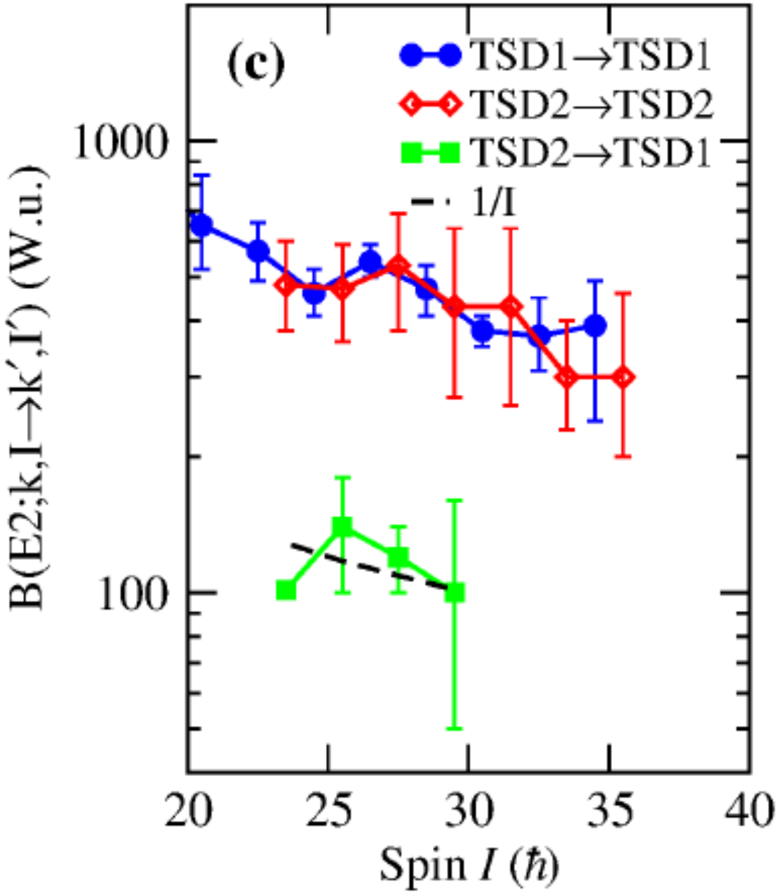
- $B(E2)_{intra}(I)$ should be constant with I for vibrational and increasing with I for rotational
- $B(E2)_{inter}(I \rightarrow I-1)$ should be ~1/I for vibrational and not so for rotational

^{192}Os



$B(E2)_{intra}(k=0) > B(E2)_{intra}(k=0) \rightarrow$ rotational

^{163}Lu



$B(E2)_{intra}(TSD1) = B(E2)_{intra}(TSD2) \rightarrow$ vibrational

$B(E2)_{intra}(TSD2 \rightarrow TSD1) \sim 1/I \rightarrow$ vibrational test whether the excitation is caused by vibrational phonon or by rotation…

$\gamma = 30^o$

item	for	property	vibrational	rotational
1	ee, lc, tc	quantum number k	n	m
2	ee, lc, tc	$E_{exc}(I = const)$	$\propto n$	$\propto m^2$
3	ee	$E(2_g^+) + E(2_\gamma^+)$	$> E(3_\gamma^+)$	$= E(3_\gamma^+)$
4	ee, lc	E_{rel}	const with n	decreasing with m
5	ee, lc	$B(E2)_{intra}$	const with n	decreasing with m
6	ee, lc	$B(E2)_{intra}$	const with I	increasing with I
7	ee, lc, tc	$B(E2; k, I \to k-1, I-1)$	$\propto n$	not proportional to m *
8	ee, lc	$B(E2; k, I \to k-1, I-1)$	$\propto 1/I$	not proportional to $1/I$ *
9	ee, lc, tc	$B(E2)_{inter}$ for $\Delta n > 1$	0	> 0 (allowed)#
10	ee	$R_{2\gamma 2g} = \frac{B(E2; 2_\gamma^+ \to 2_g^+)}{B(E2; 2_g^+ \to 0_g^+)}$	0	1.4
11	ee	$R_{3\gamma 2\gamma} = \frac{B(E2; 3_\gamma^+ \to 2_\gamma^+)}{B(E2; 2_g^+ \to 0_g^+)}$	1	1.8
12	tc	E_{rel}	const with n	increasing with m &
13	tc	$E_{rel}(I)$	decreasing for $I < I_{max}$	decreasing for $I < I_c$ &
14	tc	$B(E2)_{intra}$	const with n	not const with m
15	tc	$B(E2)_{intra}$	const with I	not const with I

E.A. Lawrie, Chirality and Wobbling; edited by C.M. Petrache, Edited by Taylor & Francis Group

Summary

- How to measure triaxiality in a (practically) model-independent way?

  - Davydov-Filippov model becomes assumptions-free for even-even rotating nuclei
    (if applied to 2^+ states only as the spin dependence of MoI becomes redundant)
    - ✓ using the ratio of two E2 matrix elements for the 2^+_1 state we can deduce γ
    - ✓ using the ratio of two E2 matrix elements only for the 2^+_1 and 2^+_γ state we can deduce γ
    - ✓ available data allows to extract γ for more than 60 even-even rotating nuclei

E. A. Lawrie, J.N. Orte, submitted

  - the spin dependence of MoI becomes irrelevant when studying I_c of the $E_\gamma(I)$ plots for even-even nuclei
    - ✓ based on the crossing I_c, one can deduce whether the nuclear shape is axially symmetric, rigid triaxial, or γ-soft around average triaxial shape, or γ-vibrational (Wilets-Jean)
    - ✓ systematic study of all even-even rotating nuclei with known rotational bands was compared with global calculations for axially asymmetric nuclei

E. A. Lawrie, N. Xulu, in preparation

- Conflicting definitions of wobbling

  (B&M wobbling is a harmonic vibrational excitation; F&D wobbling is a rotational excitation)
  - It is proposed to investigate further the nature of the experimentally observed bands, is it vibrational or rotational
  - A number of criteria to distinguish between vibrational and rotational nature were defined and applied

E.A. Lawrie, chapter in the book
Chirality and Wobbling; edited by C.M. Petrache

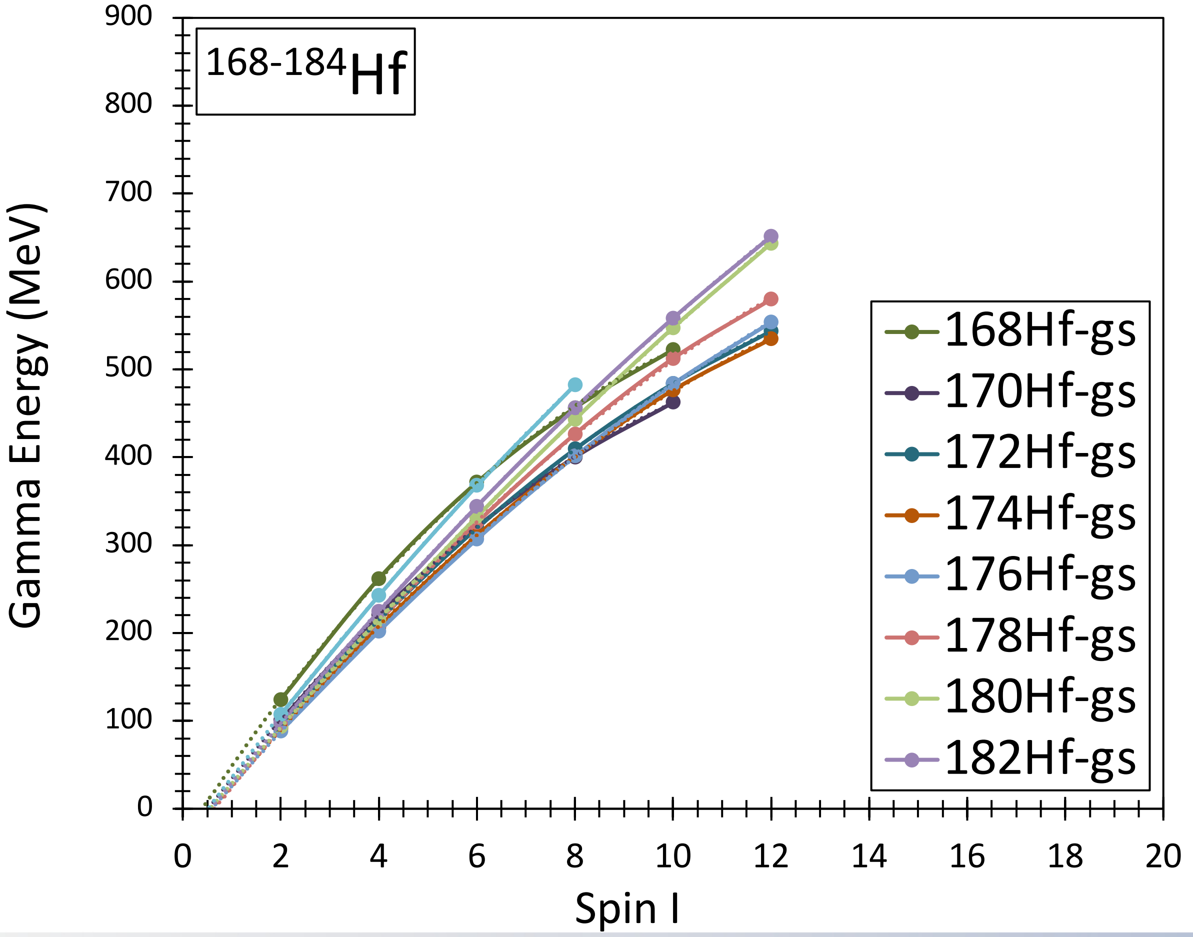
Thank you for your attention

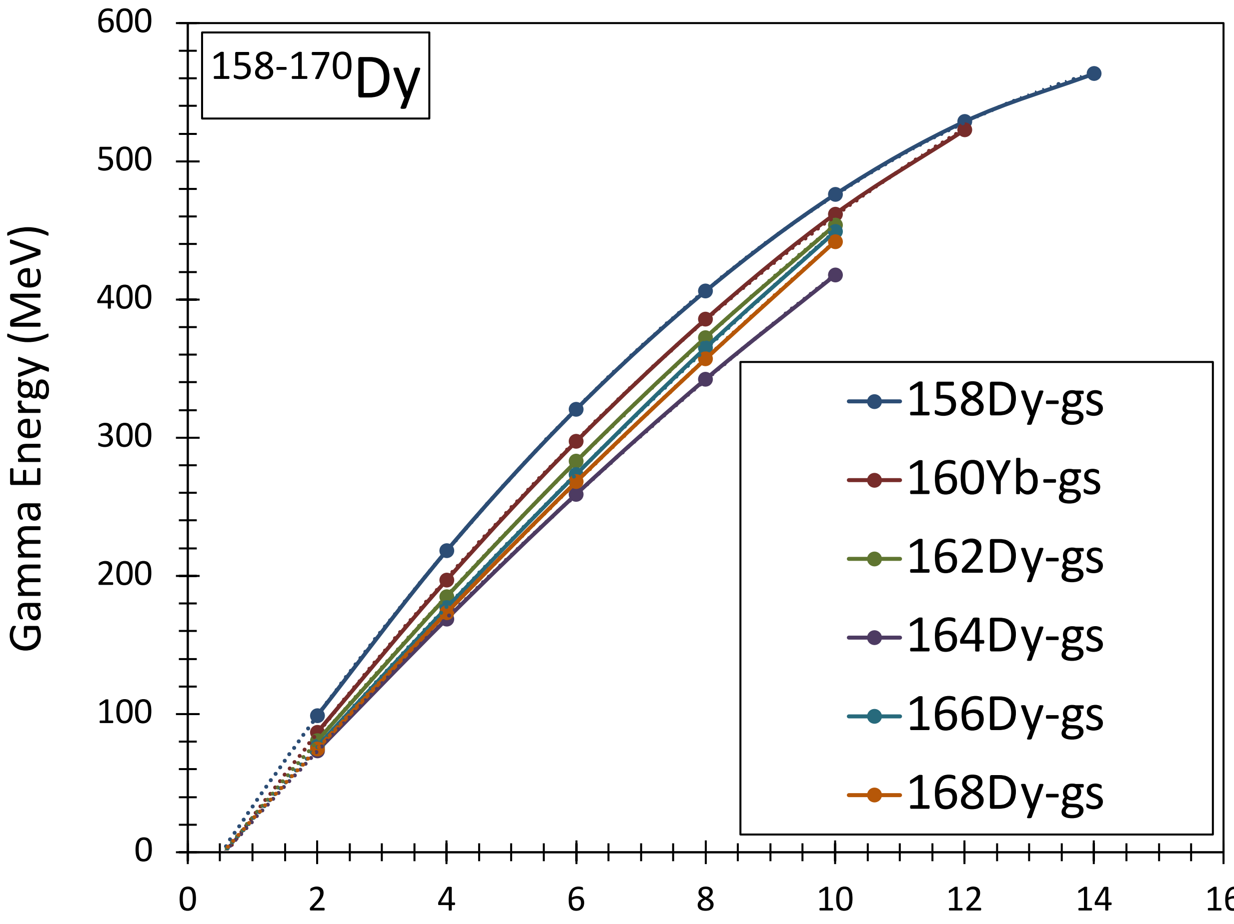
This work is based on the research supported in part by the National Research Foundation of South Africa

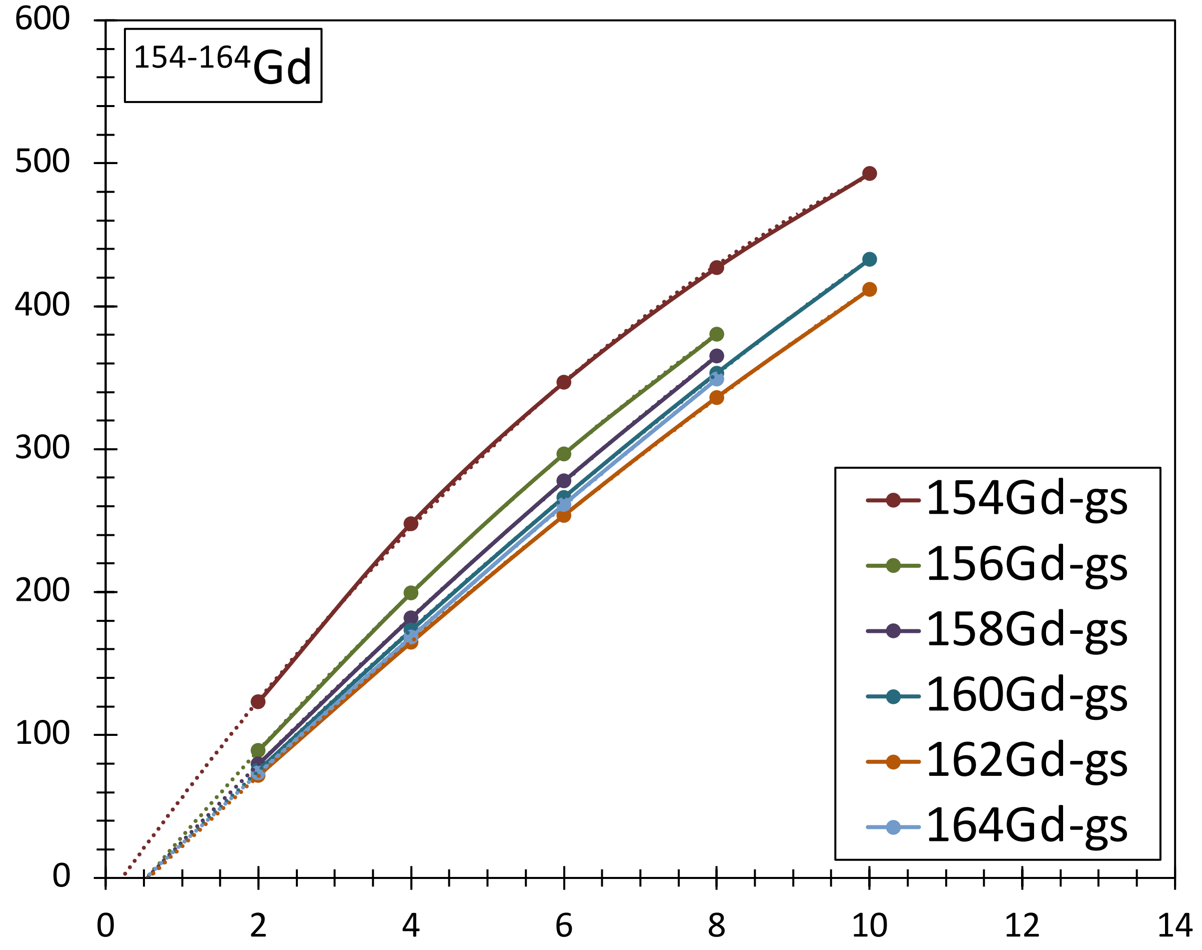
γ deduced by the ratios of matrix elements

Nucleus	$\langle 2_1^+ \Vert \hat{E}2 \Vert 2_1^+ \rangle$	$\langle 0_1^+ \Vert \hat{E}2 \Vert 2_1^+ \rangle$	$\langle 2_1^+ \Vert \hat{E}2 \Vert 2_\gamma^+ \rangle$	$\gamma_{R22/02}$	$\gamma_{R22\gamma/22}$	γ_{TR}	γ_{KC}
^{12}C	0.125(24)[a]	0.063(2)		oblate			
^{20}Ne	-0.303(40)	0.182(4)	0.052(3)	prolate	$9.2^{+1.1}_{-1.2}$		
^{22}Ne	-0.284(16)[b]	0.152(1)	0.043(17)	prolate	$8.1^{+2.7}_{-3.1}$		
^{22}Mg	-0.57(57)[b]	0.184(43)		$0^{+30.5}_{-0}$			
^{24}Mg	-0.237(26)[c]	0.209(2)	0.083(3)	$17.3^{+4.3}_{-17.3}$	$15.5^{+1.1}_{-1.2}$		
^{28}Si	0.211(40)	0.181(2)		$45.3^{+14.7}_{-8.0}$			
^{50}Cr	-0.475(92)	0.324(5)		$0^{+13.1}_{-0}$			
^{56}Fe	-0.303(40)	0.313(3)	0.145(9)	$22.4^{+1.8}_{-3.2}$	$18.4^{+1.2}_{-1.4}$		
^{58}Fe	-0.356(66)	0.349(9)	0.258(39)	$21.4^{+3.0}_{-21.4}$	$21.8^{+1.4}_{-2.1}$		
^{62}Fe	-0.11(53)[d]	0.319(97)		$28.2^{+31.8}_{-28.2}$			
^{74}Ge	-0.251(26)	0.553(14)	0.630(44)	$27.4^{+0.3}_{-0.3}$	$27.5^{+0.3}_{-0.4}$	23.8(14)	26(8)
^{76}Ge	-0.240(20)[e]	0.526(20)[e]	0.535(7)[e]	$27.3^{+0.3}_{-0.3}$	$27.1^{+0.2}_{-0.2}$	28.1(8)	27(5)
^{80}Ge	-0.61(41)[f]	0.408(10)[f]	< |0.8|[f]	$0^{+27.2}_{-0}$	<25.3 or >34.7		
^{78}Se	-0.34(12)	0.586(10)	0.469(19)	$26.5^{+1.4}_{-1.8}$	$25.4^{+1.2}_{-2.3}$	25.6(22)	
^{80}Se	-0.409(92)	0.502(8)	0.435(12)	$24.5^{+1.7}_{-2.7}$	$24.2^{+1.0}_{-1.6}$	22.8(10)	
^{82}Se	-0.290(92)	0.428(12)	0.208(25)	$25.8^{+1.6}_{-2.3}$	$21.7^{+2.0}_{-3.4}$	19.5(13)	
^{76}Kr	-0.9(3)[g]	0.871(15)	0.09(4)[g,p]	$21.1^{+4.7}_{-21.1}$	$5.6^{+2.8}_{-3.1}$	10.7(1.1)	6(3)
^{78}Kr	-0.80(4)[h]	0.796(10)	0.26(6)[h]	$21.7^{+1.0}_{-1.2}$	$14.8^{+2.0}_{-2.5}$		
^{98}Sr	-0.63(32)[i]	1.14(20)		$26.8^{+1.9}_{-2.7}$			
^{104}Ru	-0.71(11)[j]	0.917(25)[j]	0.75(4)[j]	$24.9^{+1.1}_{-1.4}$	$24.2^{+0.8}_{-1.1}$	22.6(10)	25(3)
^{110}Ru	-1.10(52)[k]	1.022(37)[k]	1.32(25)[k]	$20.0^{+6.6}_{-20.0}$	$24.9^{+1.7}_{-4.6}$	29.0(54)	
^{106}Pd	-0.72(7)[l]	0.812(10)	0.810(37)[s]	$23.6^{+1.0}_{-1.3}$	$24.5^{+0.5}_{-0.6}$	22.4(9)	20(2)
^{108}Pd	-0.810(90)[l]	0.874(11)	1.049(44)	$23.1^{+1.3}_{-1.9}$	$25.2^{+0.5}_{-0.6}$	20.6(9)	19(5)
^{110}Pd	-0.87(17)[m]	0.930(12)	0.830(28)	$22.9^{+2.2}_{-4.8}$	$23.6^{+1.0}_{-1.4}$	19.9(20)	16(1)
^{130}Ba	-1.35(20)	1.067(22)		$0^{+19.9}_{-0}$			
^{148}Nd	-1.93(18)	1.157(13)	1.342(17)	prolate	$21.5^{+0.6}_{-0.7}$	14.1(3)	15(5)
^{150}Nd	-2.64(66)	1.645(9)	1.427(9)	prolate	$19.5^{+1.8}_{-2.6}$	10.2(9)	
^{152}Sm	-2.198(21)	1.860(1)	0.422(29)	$12.6^{+1.8}_{-4.3}$	$10^{+0.6}_{-0.6}$		
^{154}Sm	-2.467(53)	2.084(11)	0.108(8)	$12.2^{+3.6}_{-12.2}$	$2.5^{+0.2}_{-0.2}$		
^{154}Gd	-2.401(53)	1.968(4)	0.549(22)[s]	$0^{+7.9}_{-0}$	$11.5^{+0.4}_{-0.4}$		
^{156}Gd	-2.546(53)	2.168(25)	0.425(7)	$13.8^{+2.8}_{-13.8}$	$8.9^{+0.2}_{-0.2}$	7.3(9)	
^{158}Gd	-2.652(53)	2.256(24)	0.390(23)	$13.7^{+2.8}_{-13.7}$	$7.9^{+0.4}_{-0.4}$		
^{160}Gd	-2.744(53)	2.277(3)	0.166(16)	$0^{+12.5}_{-0}$	$3.43^{+0.3}_{-0.4}$		
^{160}Dy	-2.38(53)[p]	2.247(9)	0.468(17)	$20.4^{+4.0}_{-20.4}$	$10.2^{+1.8}_{-2.0}$		
^{164}Dy	-2.74(20)	2.370(14)	0.444(18)	$15.7^{+4.2}_{-15.7}$	$8.6^{+0.6}_{-0.6}$		
^{166}Er	-2.51(53)	2.397(19)	0.510(16)	$20.7^{+3.6}_{-20.7}$	$10.5^{+1.7}_{-1.9}$	9.9(5)	18(3)
^{168}Er	-3.25(25)[n]	2.43(7)[n]	0.47(2)[n]	prolate	$7.8^{+0.6}_{-0.6}$	8.2(3)	9(3)
^{170}Er	-2.51(27)	2.416(14)	> 0.385	$20.9^{+2.1}_{-4.3}$	> 8.3		
^{170}Yb	-2.876(40)	2.392(15)	0.366(38)[r]	0^{+12}_{-0}	$7.0^{+0.7}_{-0.7}$		
^{172}Yb	-2.929(53)	2.468(30)	0.250(6)	$11.2^{+4.2}_{-11.2}$	$4.8^{+0.1}_{-0.1}$	5.0(7)	6 (6)
^{174}Yb	-2.876(66)	2.419 (33)	0.269(27)[r]	$10.5^{+5.3}_{-10.5}$	$5.2^{+0.5}_{-0.5}$		
^{176}Yb	-3.008(79)	2.278(20)	0.289(19)	prolate	$5.4^{+0.4}_{-0.4}$		
^{176}Hf	-2.771(26)	2.328(37)	0.387(36)[r]	$10.1^{+4.6}_{-10.1}$	$7.6^{+0.6}_{-0.6}$		
^{178}Hf	-2.665(26)	2.176(145)	0.362(12)	$0^{+16.9}_{-0}$	$7.4^{+0.2}_{-0.2}$		

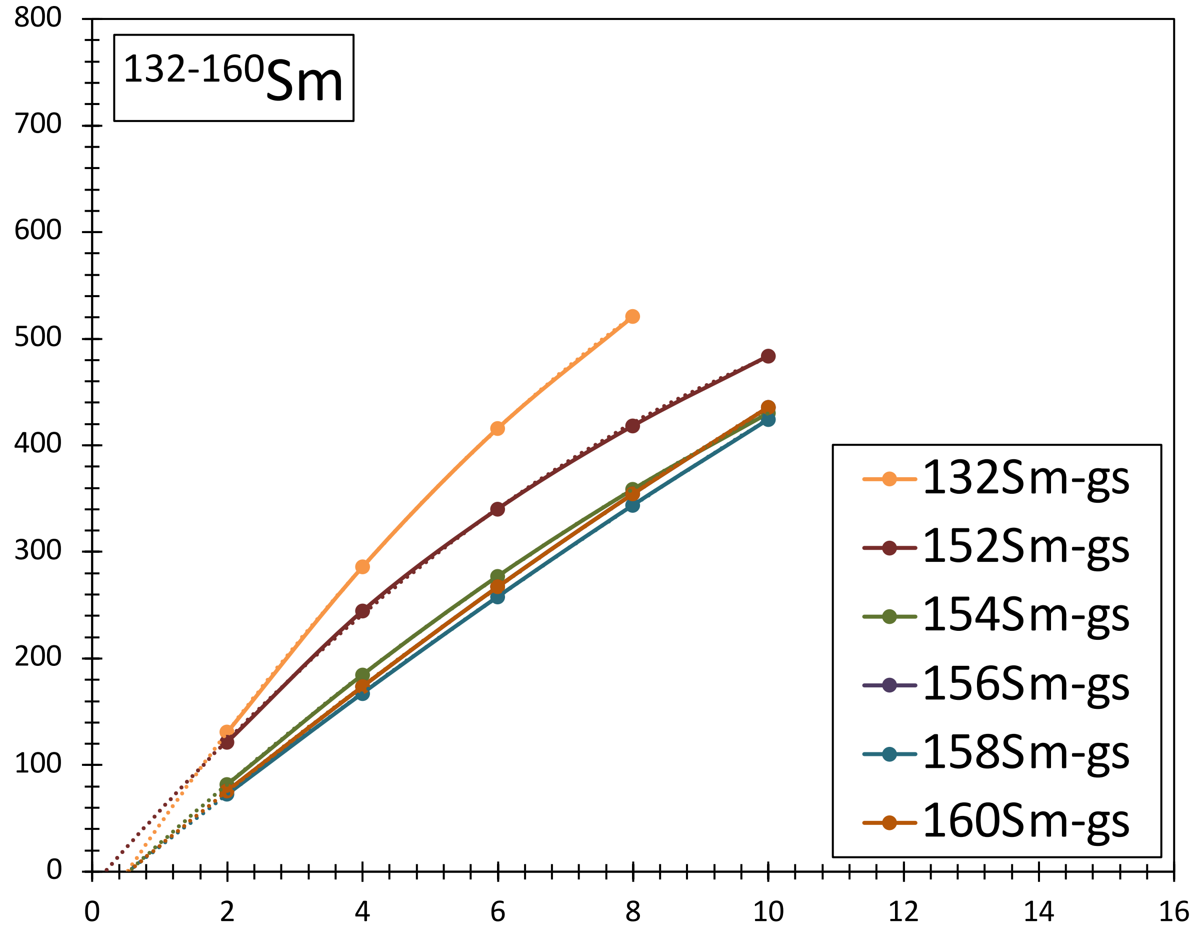
Nucleus	$\langle 2_1^+ \Vert \hat{E}2 \Vert 2_1^+ \rangle$	$\langle 0_1^+ \Vert \hat{E}2 \Vert 2_1^+ \rangle$	$\langle 2_1^+ \Vert \hat{E}2 \Vert 2_\gamma^+ \rangle$	$\gamma_{R22/02}$	$\gamma_{R22\gamma/22}$	γ_{TR}	γ_{KC}
^{180}Hf	-2.639(26)	2.156(1)	0.396(23)	prolate	$8.1^{+0.4}_{-0.4}$		
^{180}W	-2.77(53)	2.037(34)		0^{+19}_{-0}			
^{182}W	-2.77(53)	2.031(10)	0.454(6)[s]	$0^{+18.8}_{-0}$	$8.7^{+1.4}_{-1.5}$	10.6(2)	12(3)
^{184}W	-2.51(27)	1.925(9)	0.497(7)	0^{+15}_{-0}	$10.3^{+0.9}_{-0.9}$	11.4(3)	12(3)
^{186}W	-2.11(40)	1.871(10)	0.564(20)	$17.7^{+5.5}_{-17.7}$	$13.0^{+1.5}_{-2.0}$		
^{184}Os	-3.6(16)	1.793(22)		$0^{+18.9}_{-0}$			
^{186}Os	-2.151(53)	1.750(21)	0.835(32)	prolate	$16.5^{+0.4}_{-0.4}$	20.3(8)	22(2)
^{188}Os	-1.926(53)	1.581(11)	0.720(40)	$0^{+12.1}_{-0}$	$16.1^{+0.6}_{-0.6}$	19.4(5)	21(2)
^{190}Os	-1.557(40)	1.534(29)	1.028(54)	$21.5^{+0.7}_{-0.8}$	$21.1^{+0.4}_{-0.5}$	23.3(13)	25(2)
^{192}Os	-1.267(40)	1.425(35)	1.230(35)	$23.6^{+0.5}_{-0.5}$	$23.7^{+0.2}_{-0.3}$	27.1(8)	26(2)
^{192}Pt	0.79(27)	1.393(23)	1.894(61)	$33.4^{+1.6}_{-1.3}$	$32.6^{+1.3}_{-0.7}$		
^{194}Pt	0.63(0.19)	1.277(27)	1.72(12)[s]	$32.9^{+1.1}_{-0.9}$	$32.3^{+1.0}_{-0.5}$	38.5(7)	40(2)
^{196}Pt	0.82(0.11)	1.184(29)	1.35(15)[s]	$34.3^{+0.9}_{-0.7}$	$33.8^{+0.8}_{-0.5}$	38.8(11)	
^{198}Pt	0.55(16)	1.035(24)	1.13(0.11)	$33.1^{+1.2}_{-1.0}$	$33.1^{+1.3}_{-0.7}$		
^{198}Hg	0.90(0.16)	0.980(4)	0.147(9)	$36.8^{+3.4}_{-1.9}$	$51.3^{+1.5}_{-1.4}$		
^{200}Hg	1.27(0.15)	0.925(15)	0.276(31)	oblate	$48.9^{+1.5}_{-1.3}$		
^{202}Hg	1.15(0.18)	0.784(13)	0.444(59)	oblate	$43.6^{+2.1}_{-1.7}$		
^{204}Hg	0.53(27)	0.651(16)		$35.5^{+24.5}_{-3.2}$			


168-184Hf
Gamma Energy (MeV)
Spin I
168Hf-gs
170Hf-gs
172Hf-gs
174Hf-gs
176Hf-gs
178Hf-gs
180Hf-gs
182Hf-gs
900
800
700
600
500
400
300
200
100
0
0
2
4
6
8
10
12
14
16
18
20

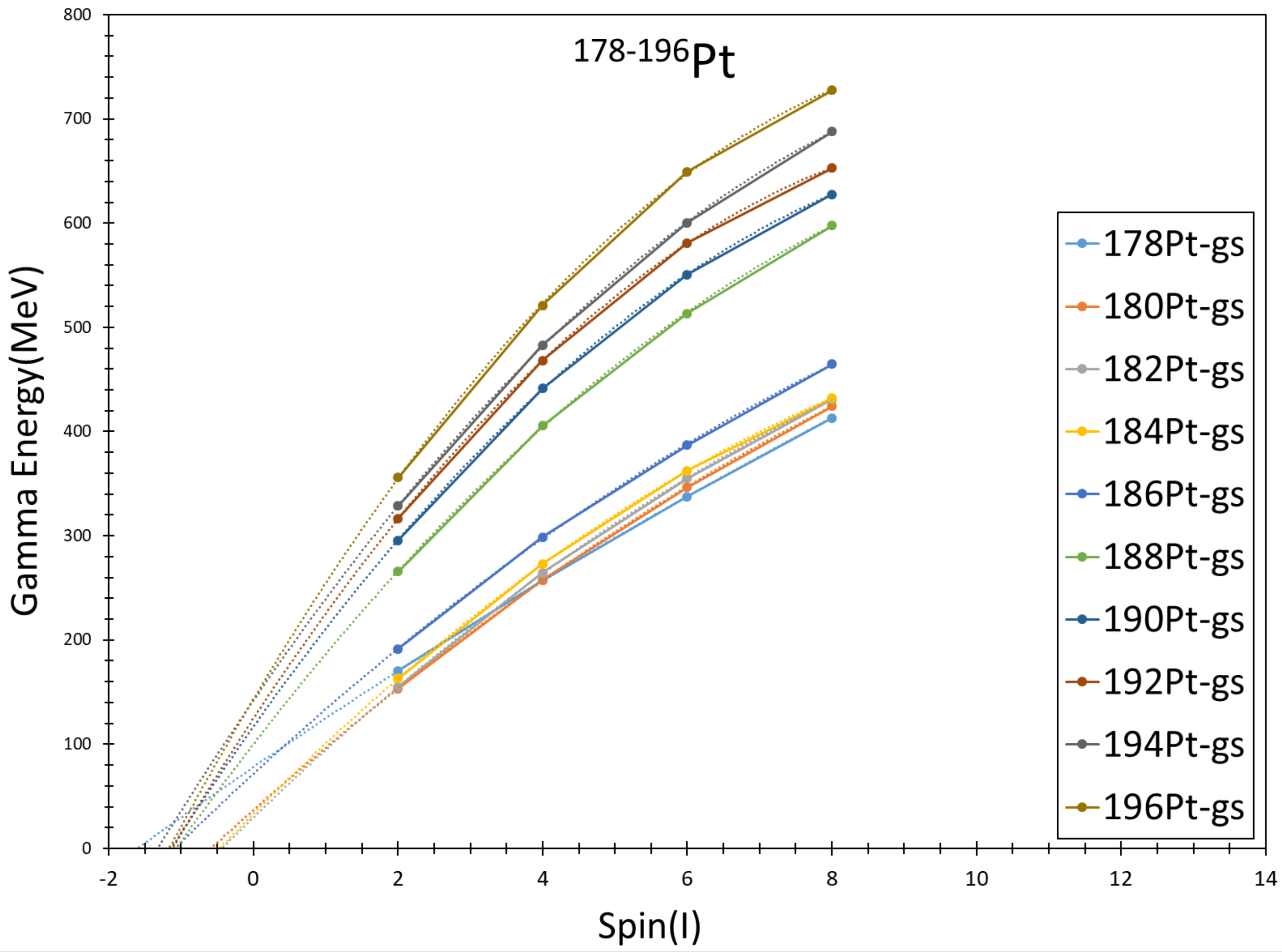

158-170Dy
Gamma Energy (MeV)
158Dy-gs
160Yb-gs
162Dy-gs
164Dy-gs
166Dy-gs
168Dy-gs
600
500
400
300
200
100
0
0
2
4
6
8
10
12
14
16



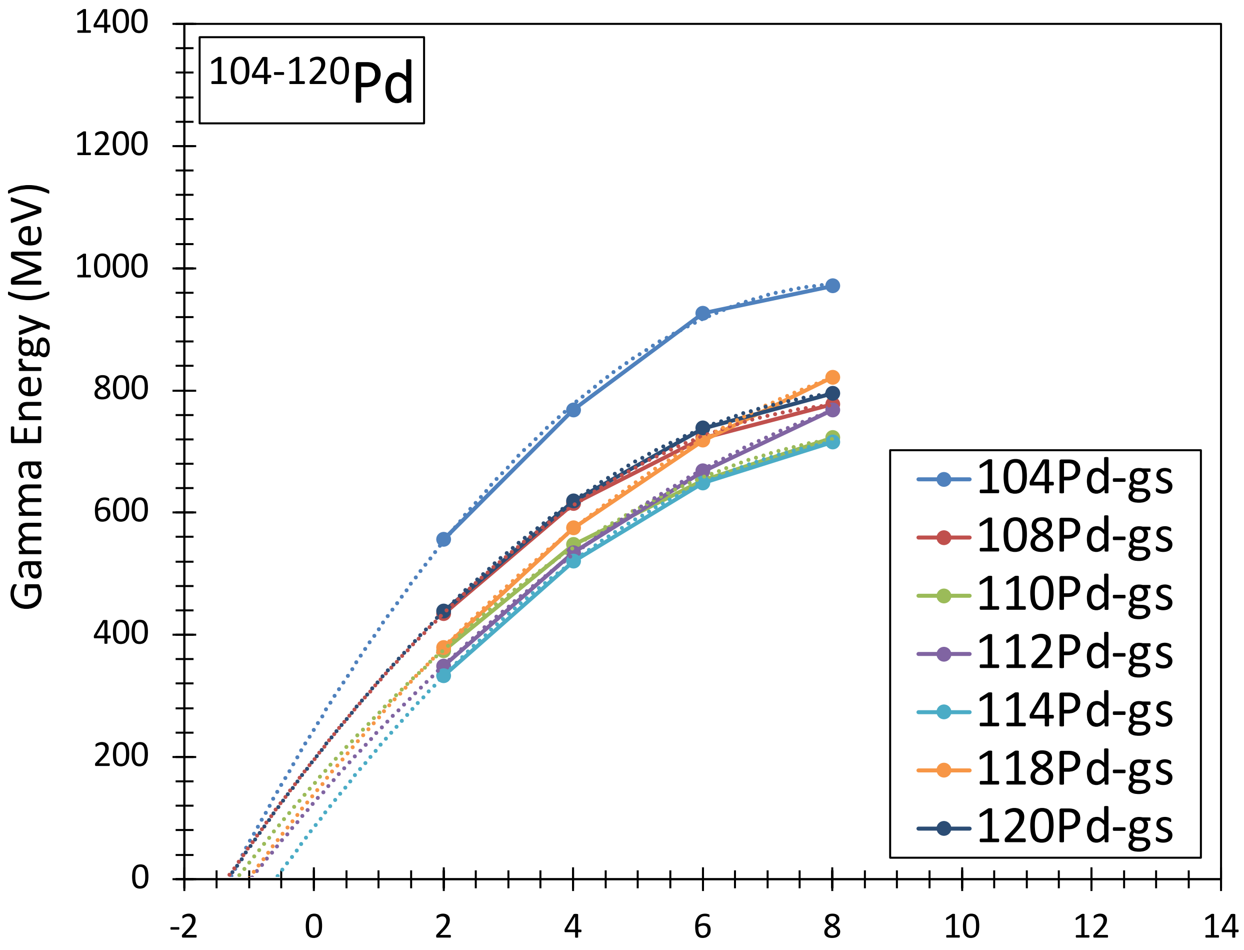
154-164Gd
154Gd-gs
156Gd-gs
158Gd-gs
160Gd-gs
162Gd-gs
164Gd-gs
0
100
200
300
400
500
600
0
2
4
6
8
10
12
14



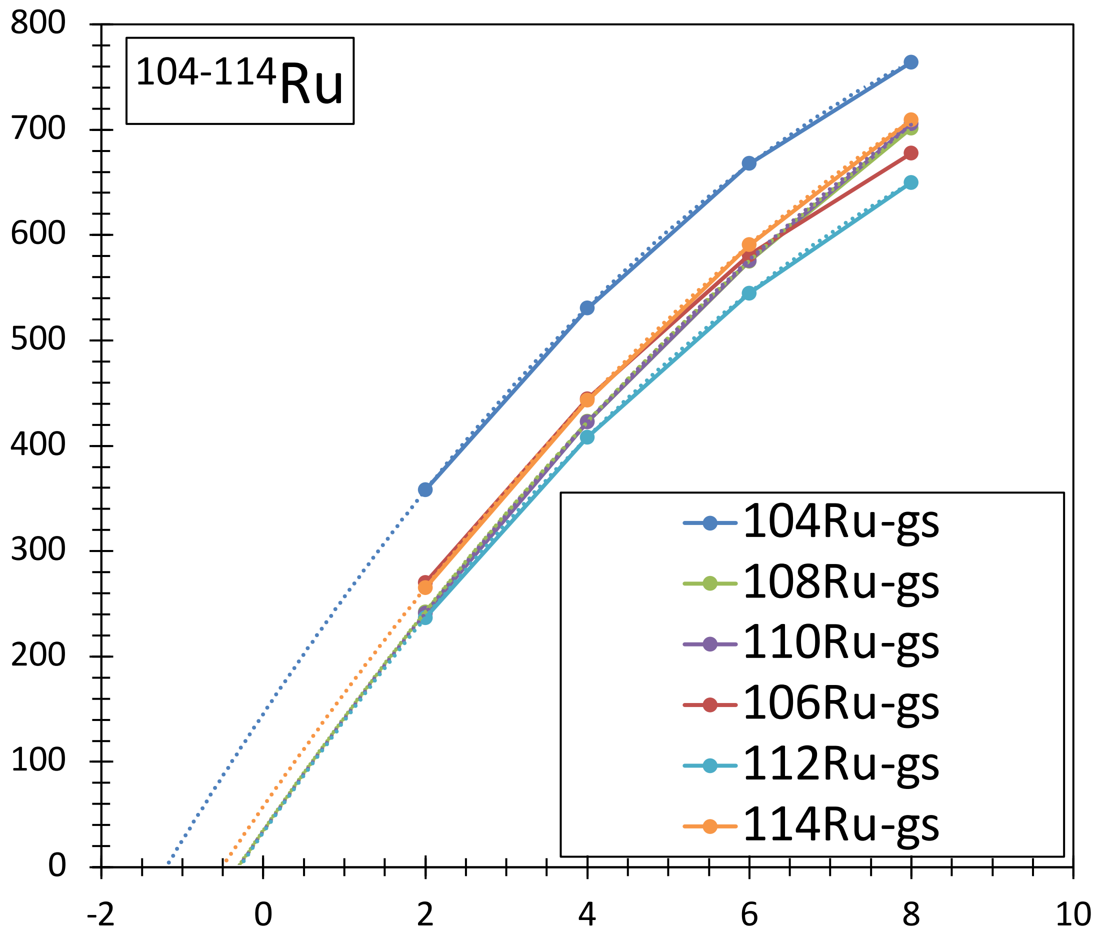
132-160Sm
132Sm-gs
152Sm-gs
154Sm-gs
156Sm-gs
158Sm-gs
160Sm-gs
0
100
200
300
400
500
600
700
800
0
2
4
6
8
10
12
14
16



178-196Pt
Gamma Energy(MeV)
Spin(I)
178Pt-gs
180Pt-gs
182Pt-gs
184Pt-gs
186Pt-gs
188Pt-gs
190Pt-gs
192Pt-gs
194Pt-gs
196Pt-gs



Gamma Energy (MeV)
104-120Pd
104Pd-gs
108Pd-gs
110Pd-gs
112Pd-gs
114Pd-gs
118Pd-gs
120Pd-gs

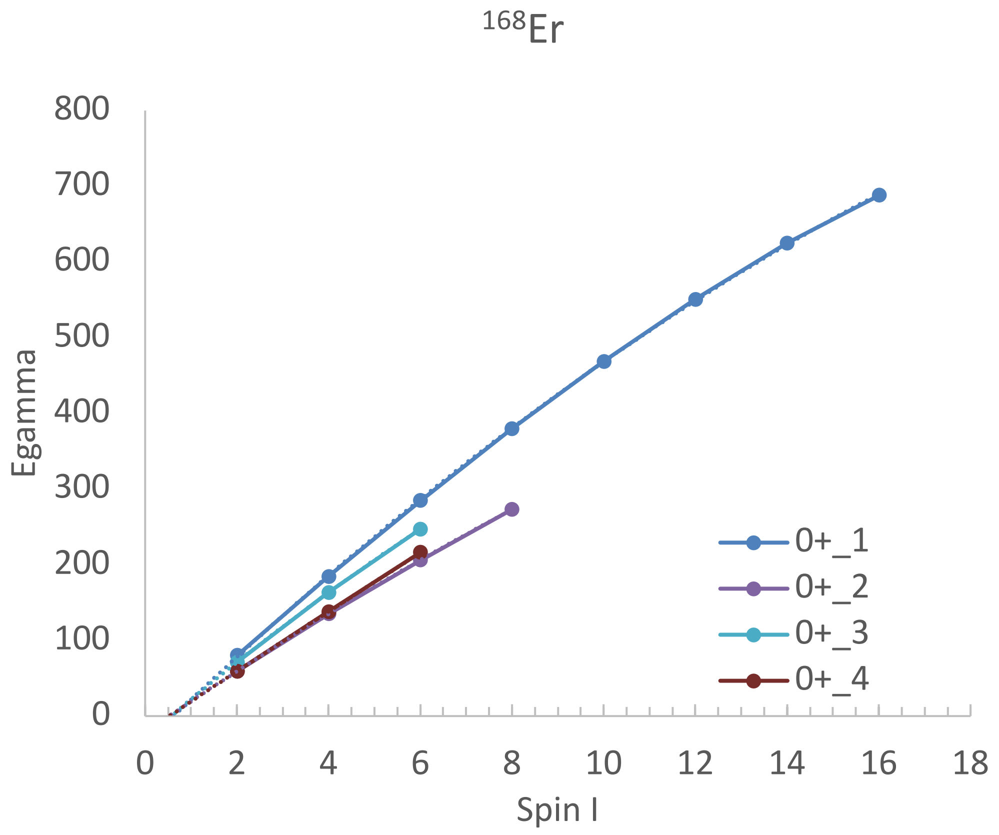


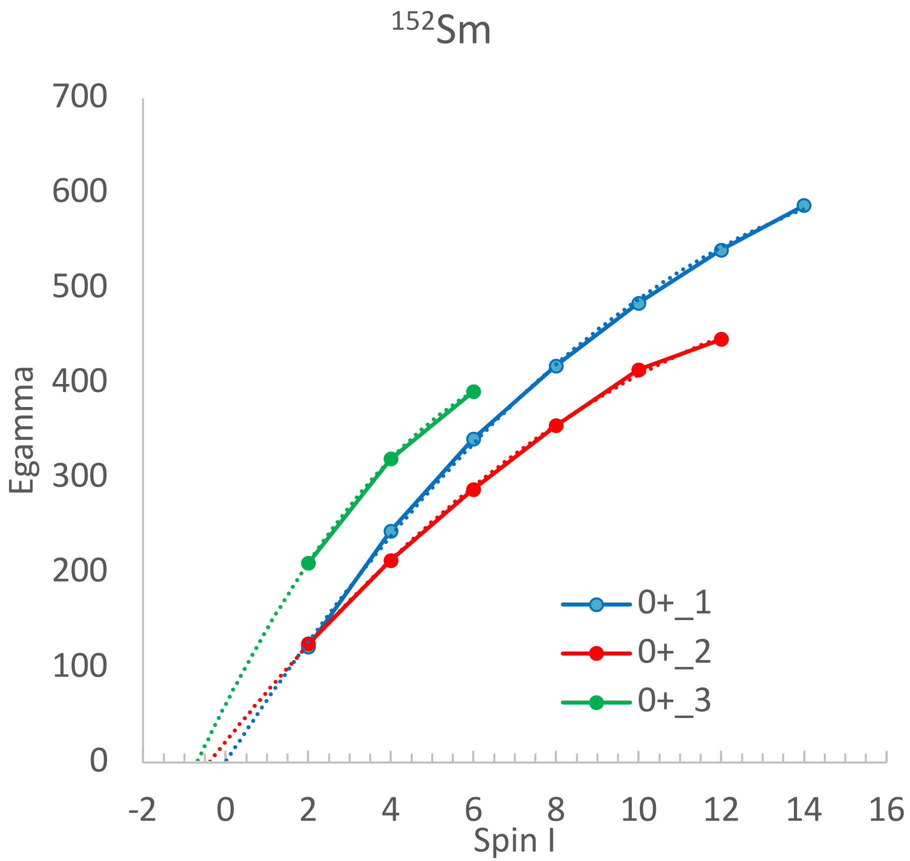
104-114Ru
104Ru-gs
108Ru-gs
110Ru-gs
106Ru-gs
112Ru-gs
114Ru-gs

shape co-existence

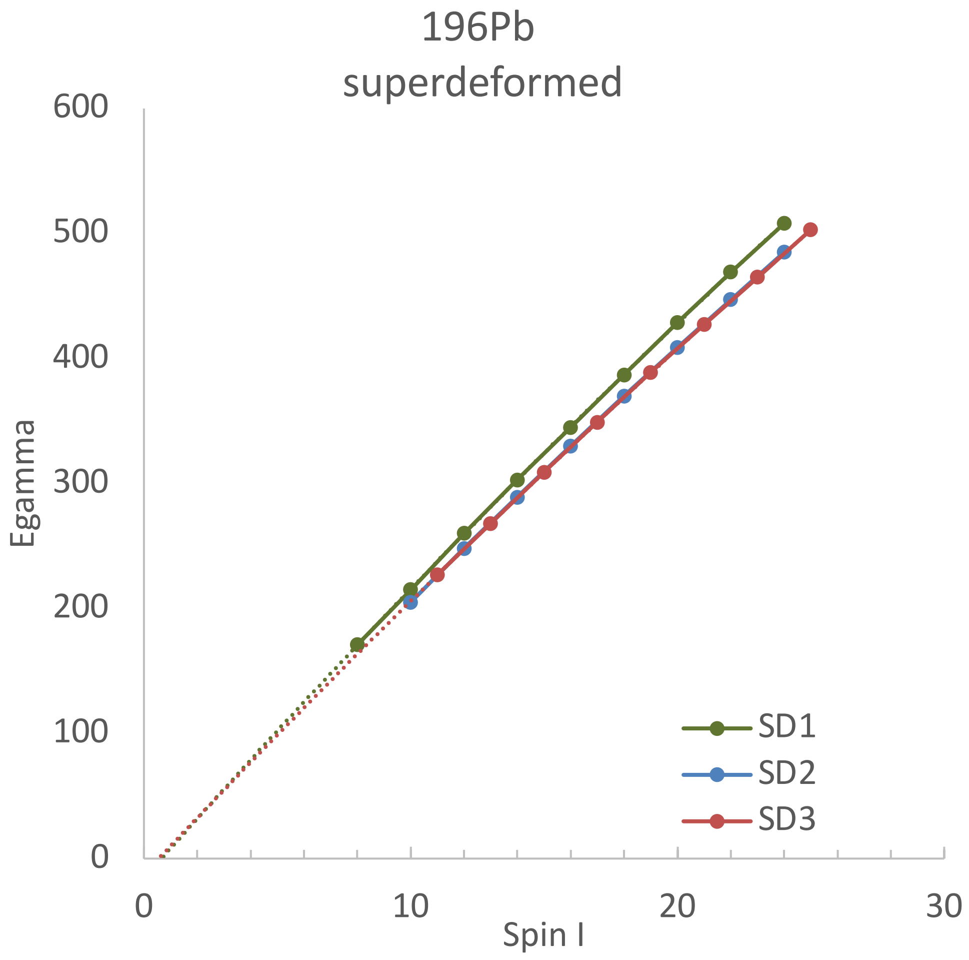
^{168}Er → all four 0^+ bands have I_c = 0.5 – axially symmetric

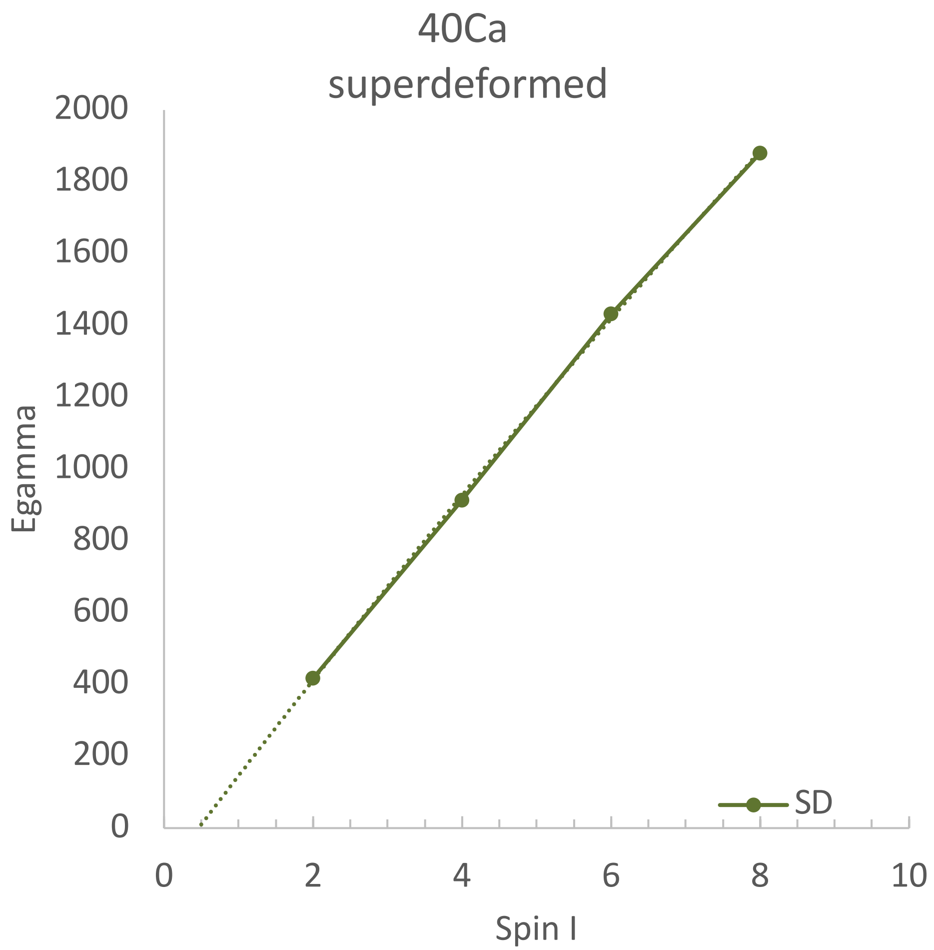
^{152}Er → the three 0^+ bands have I_c = [-0.7 – 0]

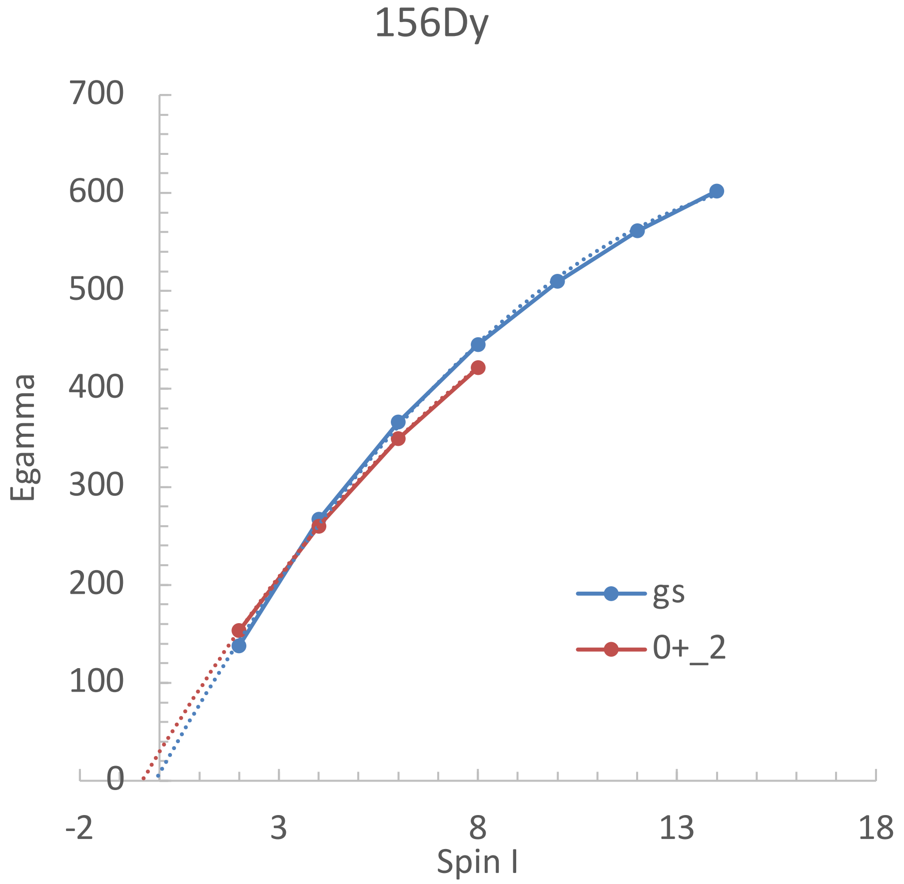


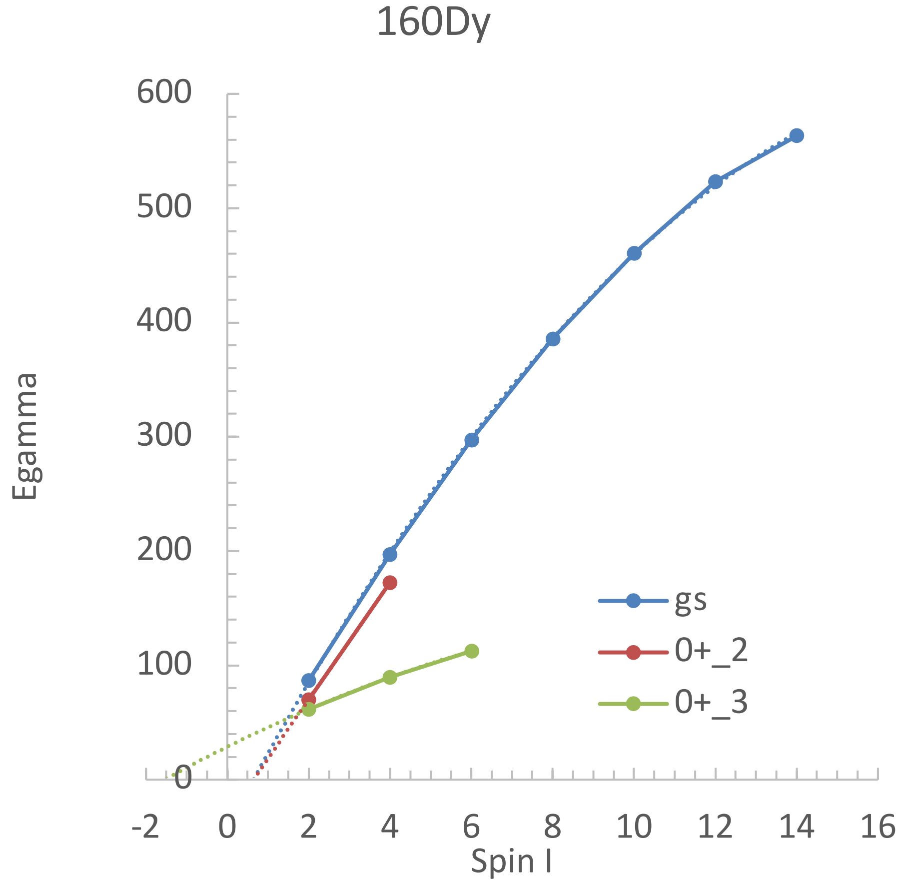


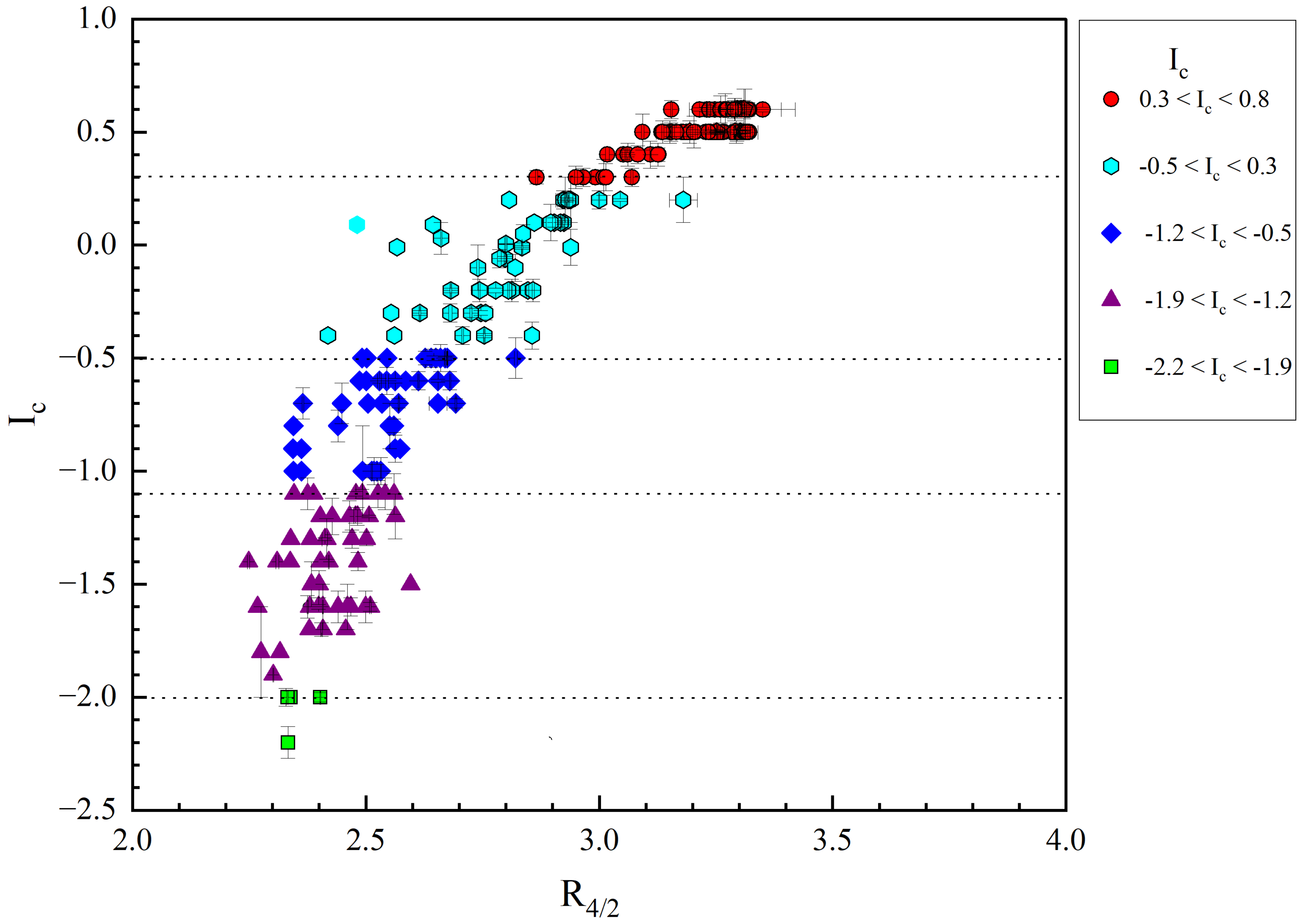
superdeformed bands in ^{196}Pb, ^{40}Ca – axially symmetric shape



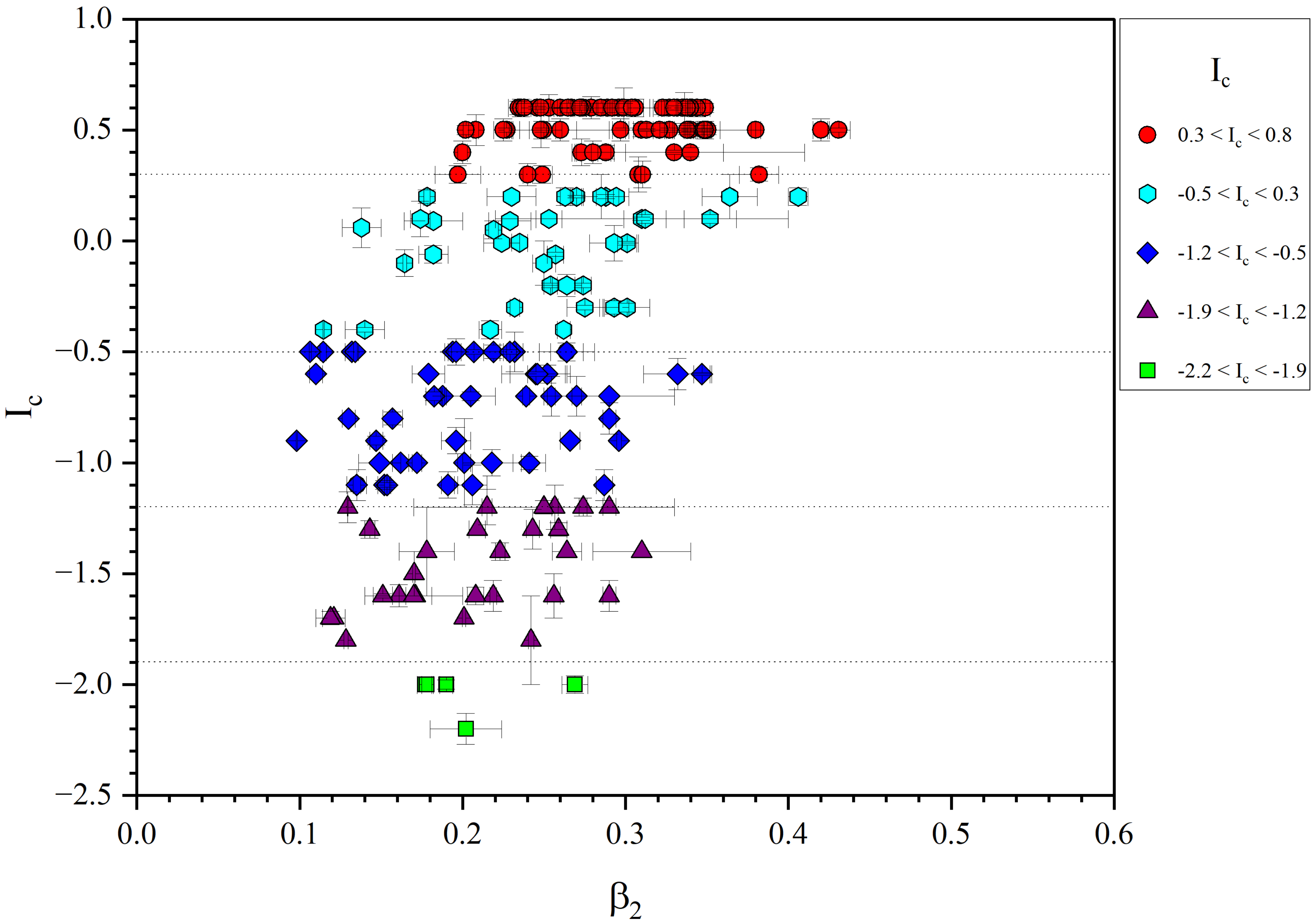



156Dy
700
600
500
400
300
200
100
0
Egamma
-2
3
8
13
18
Spin I
gs
0+_2


160Dy
600
500
400
300
200
100
0
Egamma
-2
0
2
4
6
8
10
12
14
16
Spin I
gs
0+_2
0+_3



1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
2.0
2.5
3.0
3.5
4.0
I_c
$R_{4/2}$
I_c
$0.3 < I_c < 0.8$
$-0.5 < I_c < 0.3$
$-1.2 < I_c < -0.5$
$-1.9 < I_c < -1.2$
$-2.2 < I_c < -1.9$



1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
I_c
β_2
I_c
$0.3 < I_c < 0.8$
$-0.5 < I_c < 0.3$
$-1.2 < I_c < -0.5$
$-1.9 < I_c < -1.2$
$-2.2 < I_c < -1.9$

B&M definition wobbling → harmonic vibrational excitation

wobbling – triaxial rotor model at high spin

2.N: 3.A

Nuclear Physics A297 (1978) 136–162; © North-Holland Publishing Co., Amsterdam

the states of lowest energy for angular momentum I (the yrast states) have $I \approx |I_1|$. For the first few excited states with the same angular momentum I the components of I perpendicular to the 1-axes are still small. These excited states are produced by a wobbling rotational motion of the nucleus with respect to the direction of I.

MULTIPLICITIES AND CONTINUUM γ-RAYS FOLLOWING HEAVY-ION REACTIONS

R. J. LIOTTA

Research Institute of Physics, S-10405 Stockholm 50 †

and

Institut des Sciences Nucléaires, BP 257, F-38044 Grenoble

and

R. A. SORENSEN

Research Institute of Physics, S-10405 Stockholm 50 †

and

Carnegie-Mellon University, Pittsburgh, PA, USA ††

Received 26 September 1977

The quantization of this wobbling motion gives the energy values

$$E' = \hbar\omega(n+\tfrac{1}{2}), \qquad (2.16)$$

$$B(\mathrm{E2}; n, I \rightarrow n-1, I\pm 1) = \frac{5e^2}{16\pi}\frac{n}{I}(\sqrt{3}Q'_0 x_{(\mp)} - \sqrt{2}Q'_2 x_{(\pm)})^2,$$

$$B(\mathrm{E2}; n, I \rightarrow n, I\pm 2) = \frac{5e^2}{16\pi}(Q'_2)^2,$$

$$B(\mathrm{E2}; n, I \rightarrow n+1, I\pm 1) = \frac{5e^2}{16\pi}\frac{n+1}{I}(\sqrt{3}Q'_0 x_{(\pm)} - \sqrt{2}Q'_2 x_{(\mp)})^2,$$

2010 2020

B&M definition wobbling → harmonic vibrational excitation

wobbling – cranking + RPA

I.D.2

Nuclear Physics A331 (1979) 429–463; © *North-Holland Publishing Co., Amsterdam*

NUCLEAR WOBBLING MOTION

E. R. MARSHALEK [+]

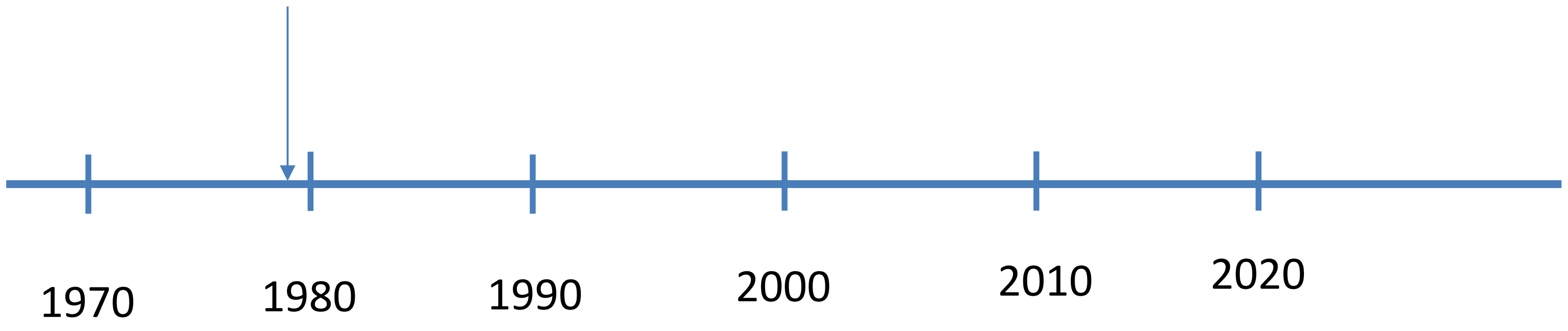
artment of Physics, University of Notre Dame, Notre Dame, Indiana 46556, USA [++]

and

Niels Bohr Institute, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

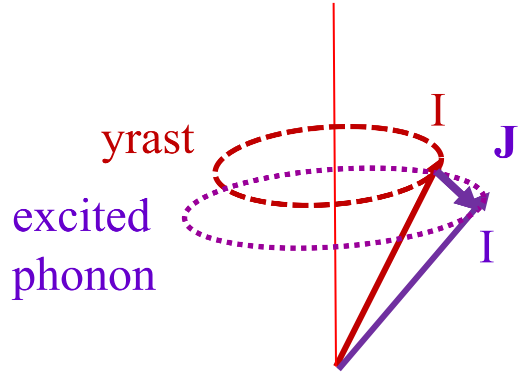
Received 7 May 1979

triaxial nucleus. This deficiency is remedied in the present paper, which presents a new way to derive the SCC+RPA, with the focus exclusively on the wobbling motion. The non-wobbling modes have been adequately treated elsewhere [4,10,11]).



What is “high spins”?

Bohr and Mottelson, Nuclear Structure

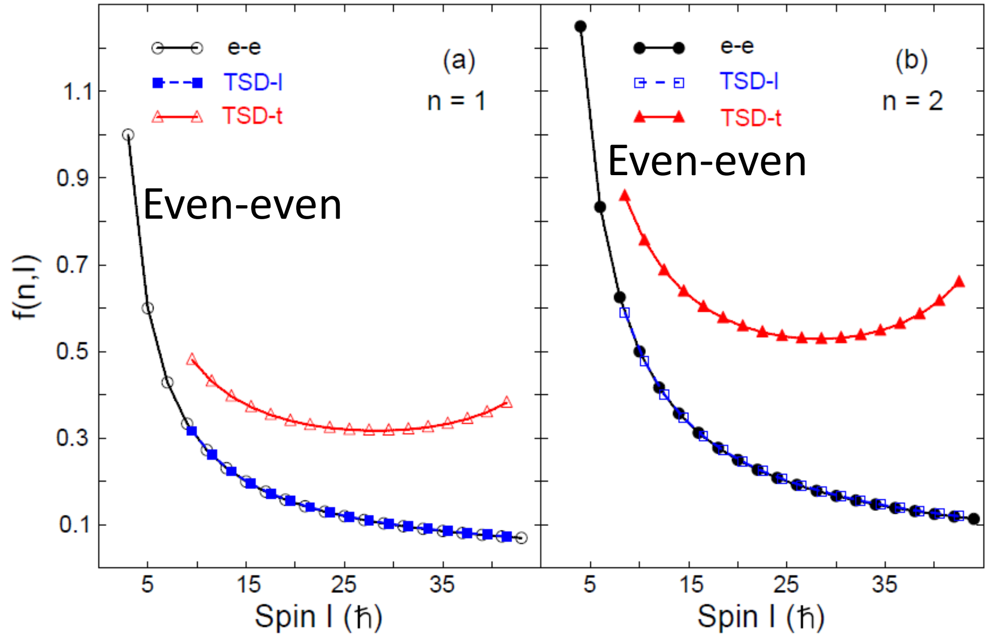


$J \rightarrow$ vibrational phonon

If the condition is satisfied

$$f(n,I) = (2n+1)\frac{(A_2+A_3-2A_1)}{2I\sqrt{(A_2-A_1)(A_3-A_1)}} << 1$$

$$A_1 = \frac{\hbar^2}{2\mathfrak{I}_1}$$



f(n, I) < 0.15 for (n = 1 , I > 20)
(n = 2 , I > 34)

PRC, 101, 034306 (2020)

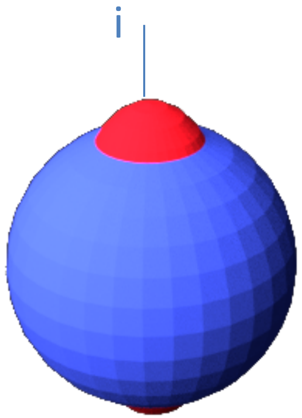
Rotation of even-even triaxial nucleus: tilted precession

$$H_R = \frac{\hbar^2}{2\mathfrak{I}_1} R_1{}^2 + \frac{\hbar^2}{2\mathfrak{I}_2} R_2{}^2 + \frac{\hbar^2}{2\mathfrak{I}_3} R_3{}^2$$

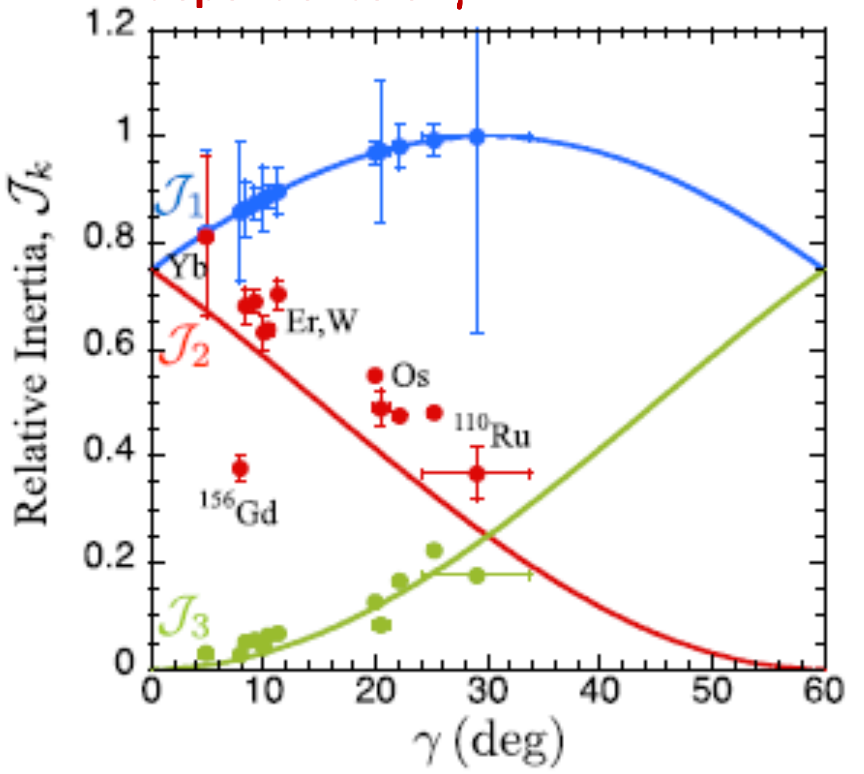
With hydrodynamical-type MoI for **γ = 30°** there is a symmetry in H because $\mathfrak{I}_2$ (short) = $\mathfrak{I}_3$ (long) = ¼ $\mathfrak{I}_1$

$$H = \frac{\hbar^2}{2\mathfrak{I}_1} R_1{}^2 + \frac{4\hbar^2}{2\mathfrak{I}_1}(R_2{}^2 + R_3{}^2) = \frac{\hbar^2}{2\mathfrak{I}_1}\{R_1{}^2 + 4[I(I+1) - R_1{}^2]\} =$$
$$= \frac{\hbar^2}{2\mathfrak{I}_1}\{4I(I+1) - 3R_1{}^2\}$$

- R_1 projection of I on the intermediate axis,
- R_1 is good q.n.
- $R_1 = I, I-1, I-2$….
- each R_1 → a rotational band



Empirical evidence for MoI from measured energies and electric quadrupole matrix elements follow **hydrodynamical MoI dependence of γ**



J.M. Allmond, J.L.Wood, Physics Letters B 767 (2017) 226–231

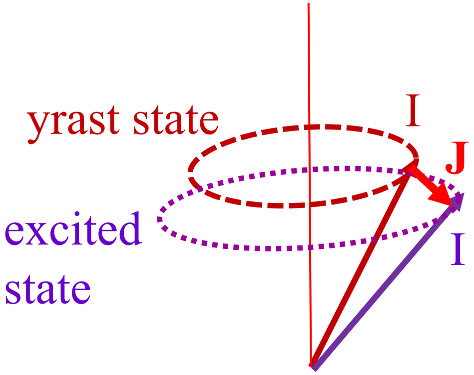
Rotation of even-even triaxial nucleus: tilted precession

$$H = \frac{\hbar^2}{2\mathfrak{J}_1}\{4I(I+1) - 3{R_1}^2\}$$

$R_1 = I$ → g.s. band

$R_1 = I - 1$ → γ band, odd spins

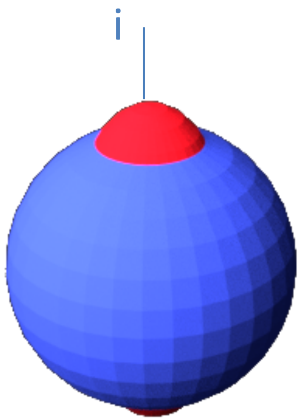
$R_1 = I - 2$ → γ band, even spins

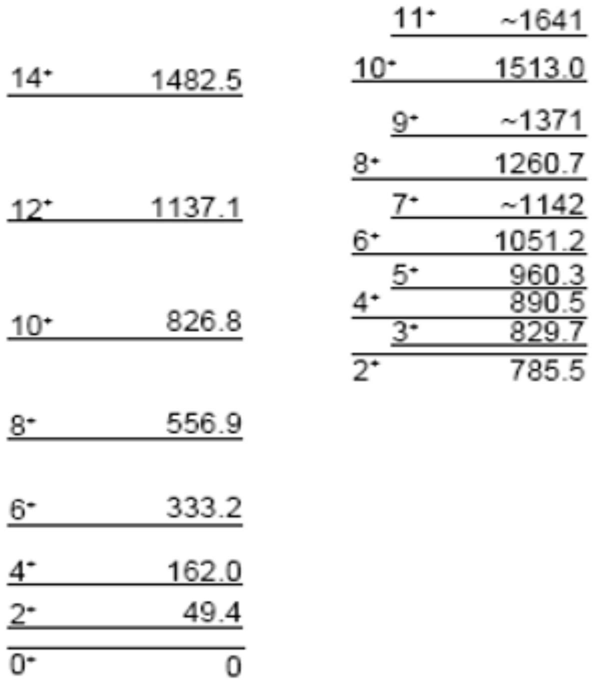


$R_1 = I, I - 1, I - 2.... = I - m$, where $m = 0,1,2,3...$

$$E = \frac{\hbar^2}{2\mathfrak{J}_1}\{I(I+4) + 3m(2I - m)\}$$

Quadratic dependence on I } rotational
Quadratic dependence on m } nature





g.s. band γ band

Wobbling due to phonon excitation

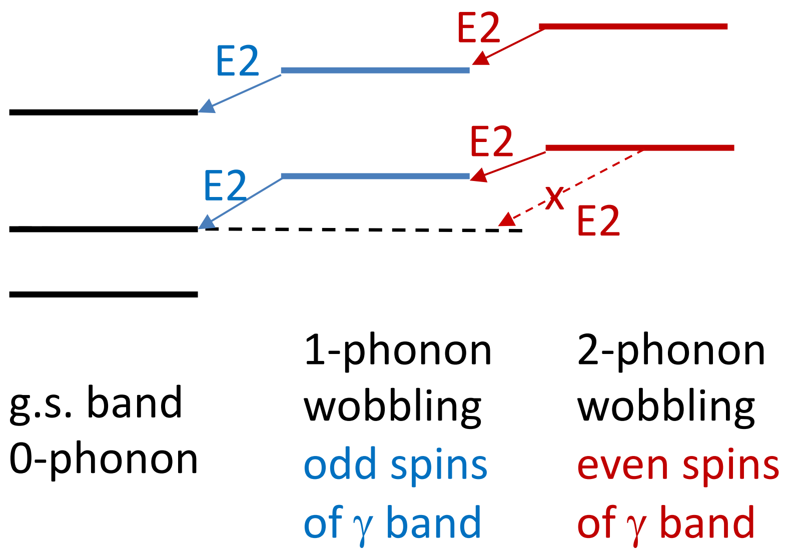
Precession in the γ band approximated at high spins with wobbling phonon

$$H = \frac{\hbar^2}{2\mathfrak{I}_1} I^2 + \hbar\omega\,(\mathbf{n}+1/2)$$

the quantization characteristics of phonon excitations:

- quantization in energy, *E(I, n) =* ***n*** *E(I, 1)*, i.e. *E(I,n=2)* **= 2** *E(I,n=1)*
- quantization in $B(E2)_{out}$, *B(E2; n → n-1) =* ***n*** *B(E2; 1 → 0),*
  eg *B(E2; n=2 → n=1)* **= 2** *B(E2; n=1 → n=0),*
- decays between the even-spin members of the γ band and the g.s. band are forbidden (simultaneous destruction of two phonons)

at high spins



Tilted precession due to rotation

Precession in the γ band at low spins (for γ = 30°)

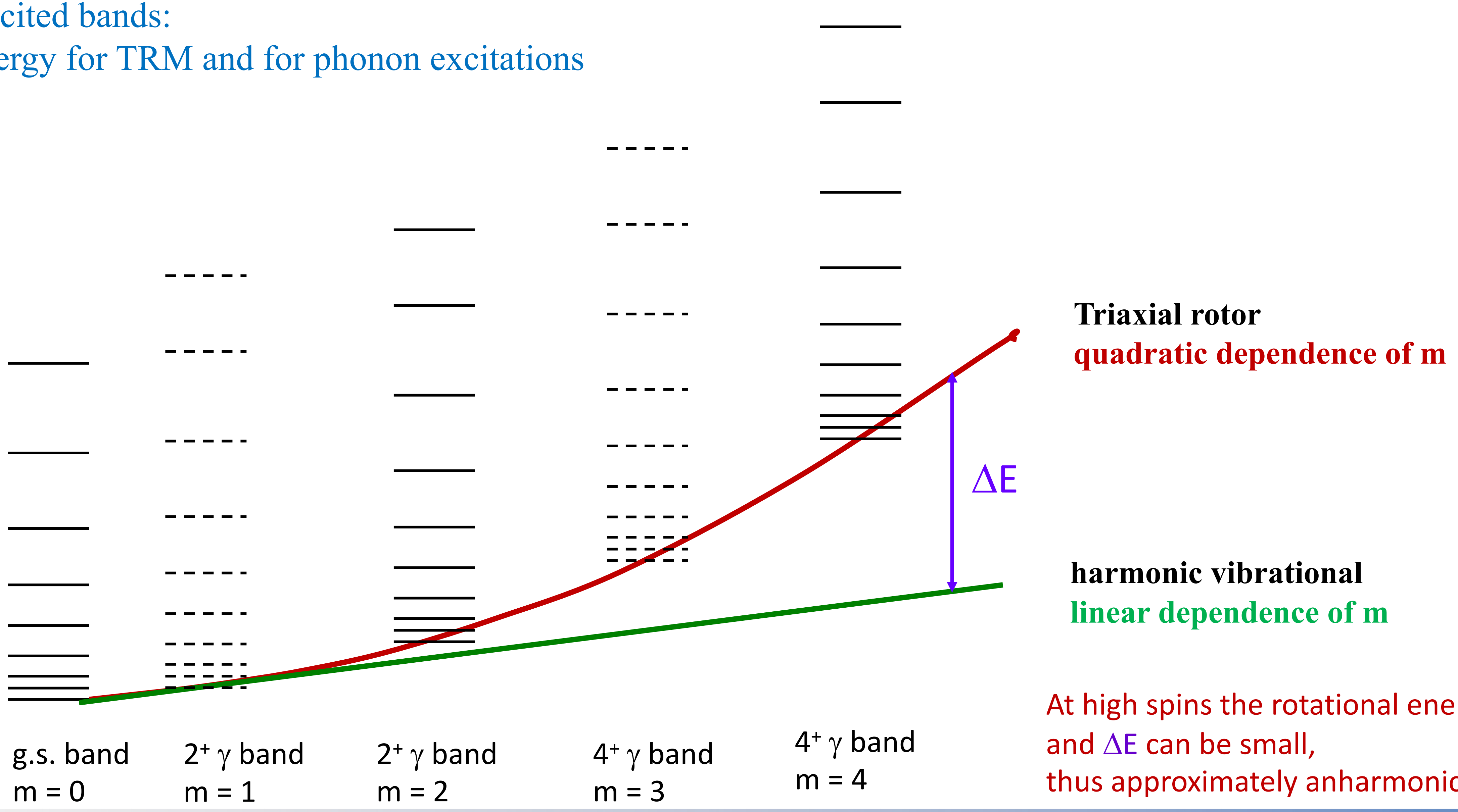
$$E = \frac{\hbar^2}{2\mathfrak{I}_1}\{I(I+4) + 3m(2I-m)\}$$

- The energy E(I, n) depends on m in quadrature, the quadratic term is small if *m << 2I*
- no quantization required in $B(E2)_{out}$,
  eg *B(E2; n=2 → n=1)* ≠ **2** *B(E2; n=1 → n=0),*
- decays between the even-spin members of the γ band the g.s. band are allowed

γ band in triaxial-rotor model is understood as precession

It looks like anharmonic wobbling at high spins

Excited bands:
energy for TRM and for phonon excitations

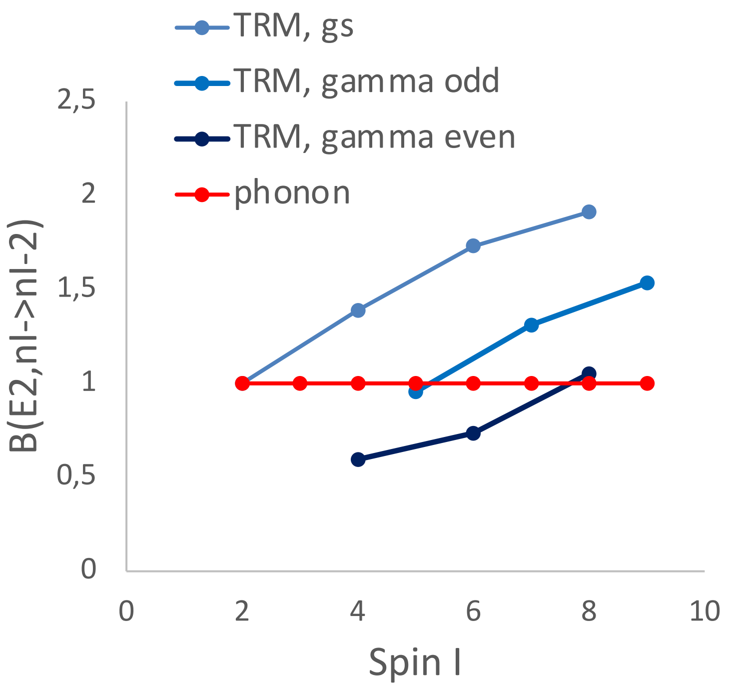


At high spins the rotational energy is large and ΔE can be small, thus approximately anharmonic vibration

B(E2) transition probabilities:
TRM and phonon excitations

Intra-band

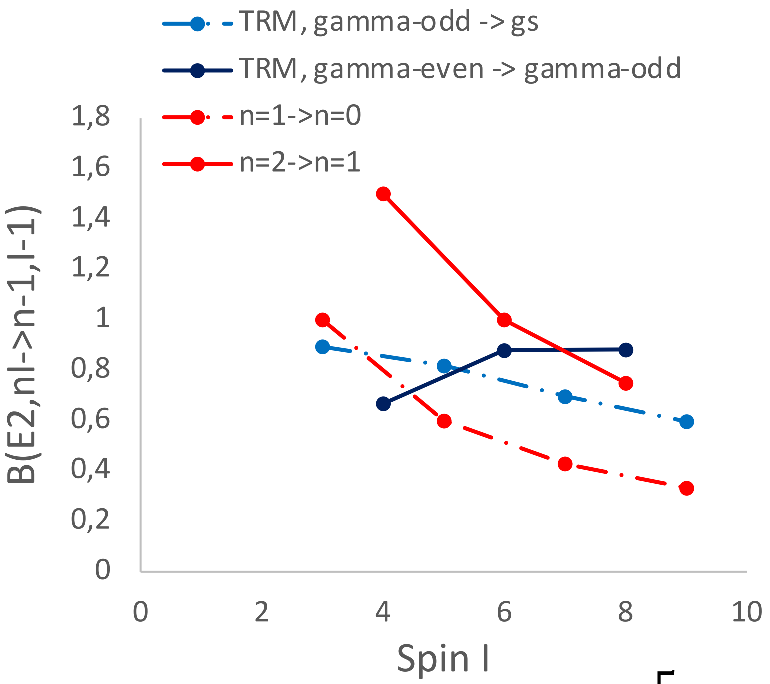
B(E2,n,I->n,I-2)



$$B(E2,n,I\rightarrow n,I\pm 2)=\frac{5}{16\pi}e^2{Q_2}^2$$

Inter-band n → n-1

B(E2,n,I->n-1,I-1)



$$B(E2,n,I\rightarrow n-1,I-1)=\frac{5}{16\pi}e^2\frac{n}{I}\left(\sqrt{3}Q_o x-\sqrt{2}Q_2 y\right)^2$$

Inter-band n → n-2

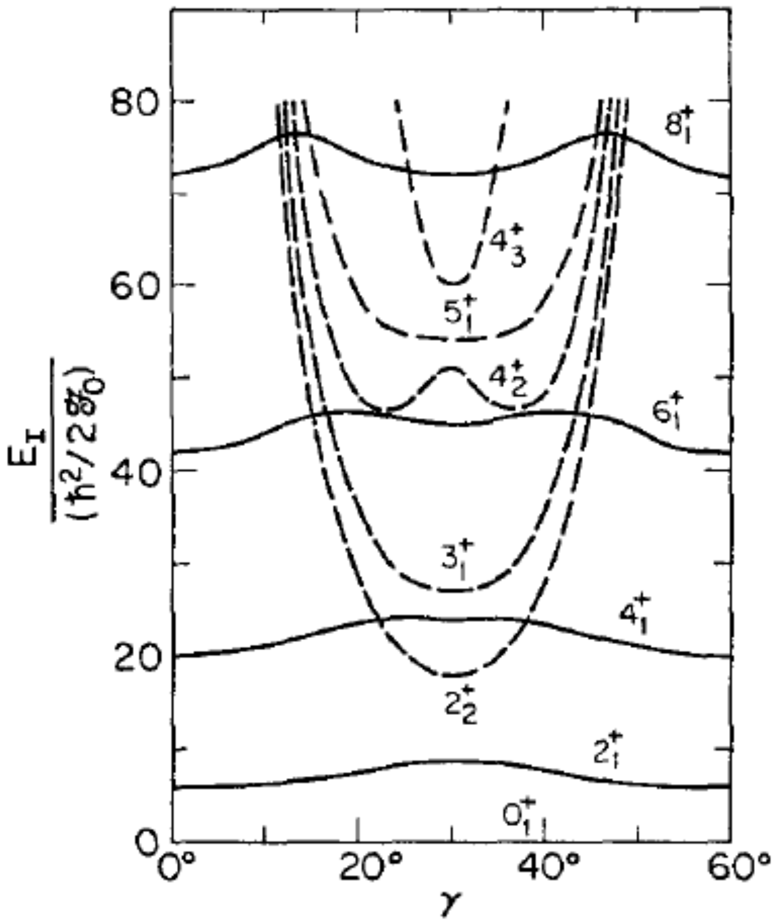
B(E2,n,I->n-2,I-2)
B(E2,n,I->n-2,I)

TRM – allowed,
$2^+_\gamma \rightarrow 2^+_{gs}$, $2^+_\gamma \rightarrow 0^+_{gs}$
$4^+_\gamma \rightarrow 4^+_{gs}$, $4^+_\gamma \rightarrow 2^+_{gs}$
$6^+_\gamma \rightarrow 6^+_{gs}$, $6^+_\gamma \rightarrow 4^+_{gs}$

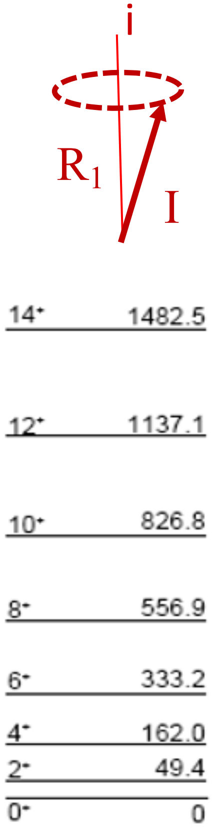
Phonon – forbidden
destruction of 2 phonons

Considerable differences in the B(E2) probabilities for tilted precession and wobbling phonons

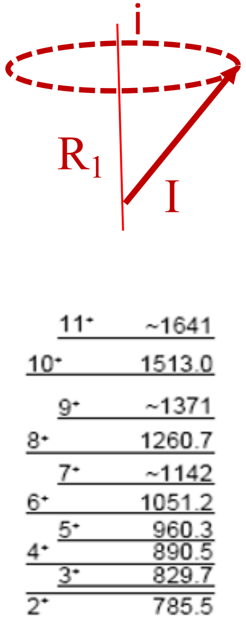
Rotation of triaxial nucleus: tilted precession



J. Meyer-ter-Vern, Nucl. Phys. A 249, 111 (1975)



g.s. band

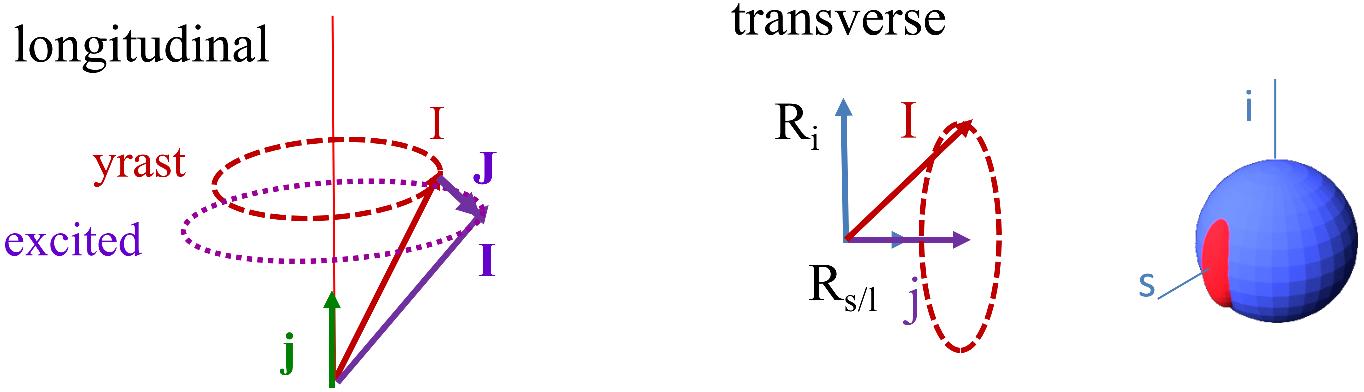


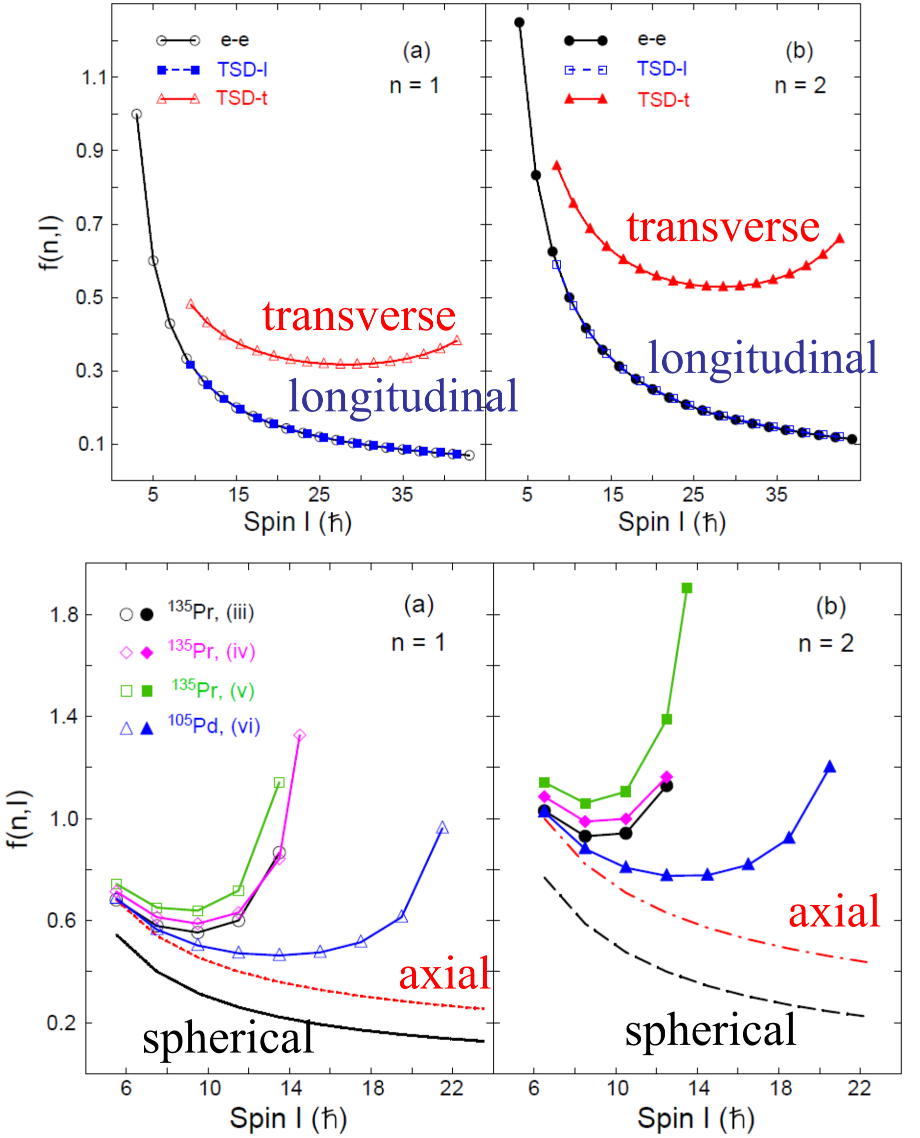
γ band

γ band is precession (rotational nature)

Since its introduction in the 1950s wobbling has been searched for in even-even nuclei for many decades.
Many γ bands were discovered at low spins, some of them have been interpreted using the triaxial-rotor model, but they were never considered as wobbling…

Precession in odd-mass nuclei





Longitudinal coupling:

→ phonon approximation is applicable at high spins;
anharmonic phonons

Transverse coupling:

→ phonon approximation is not valid at any spin

PRC, 101, 034306 (2020)