Shape Coexistence and quantum phase transitions in even-even and odd-mass nuclei

Noam Gavrielov
Racah Institute of Physics, The Hebrew University, Jerusalem

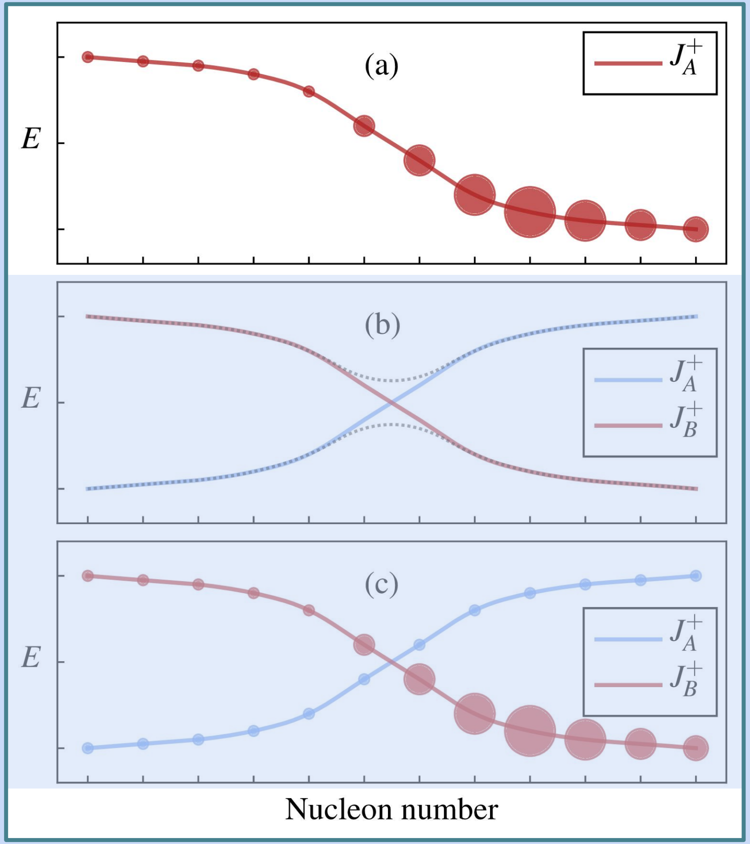
Shapes and Symmetries in Nuclei: from Experiment to Theory, Orsay 2024

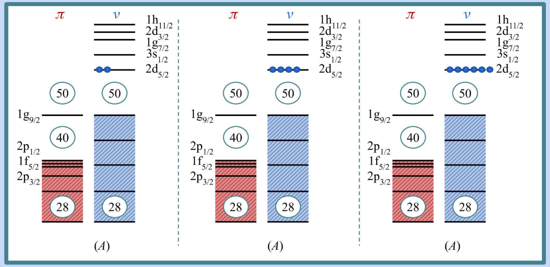
Quantum Phase Transitions in Nuclei

$$\hat{H} = (1-\xi)\hat{H}_1 + \xi\hat{H}_2$$

$$0 \leq \xi \leq 1$$

A. Dieperink, O. Scholten and F. Iachello, Phys. Rev. Lett. **44**, 1747 (1980).





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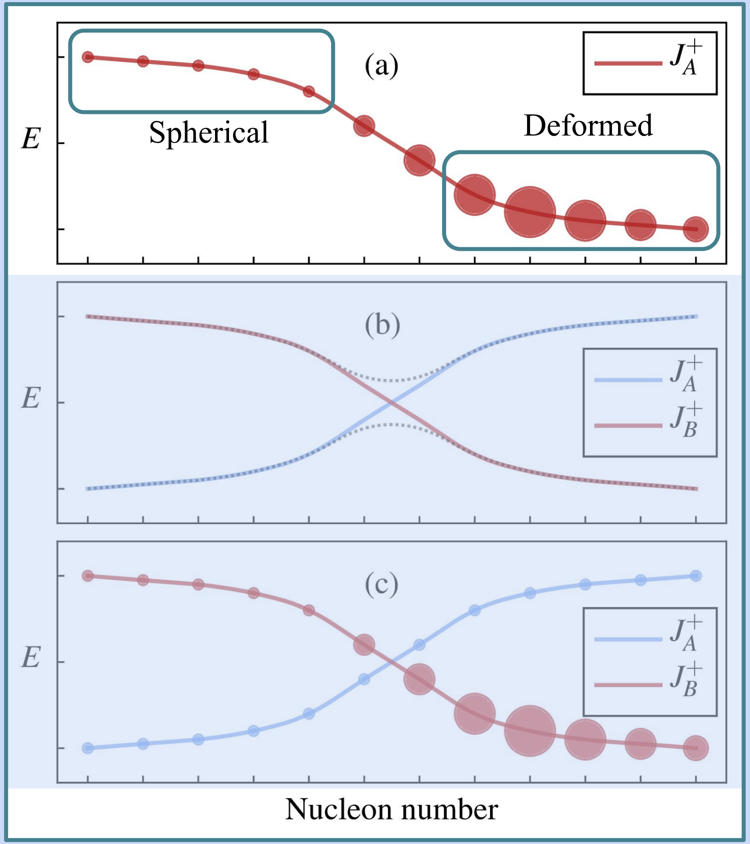

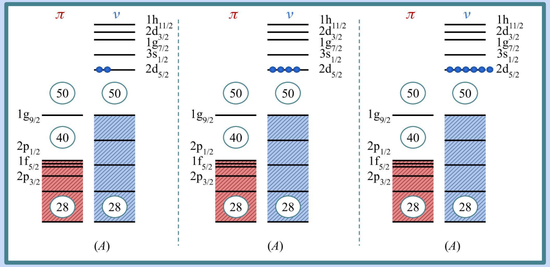

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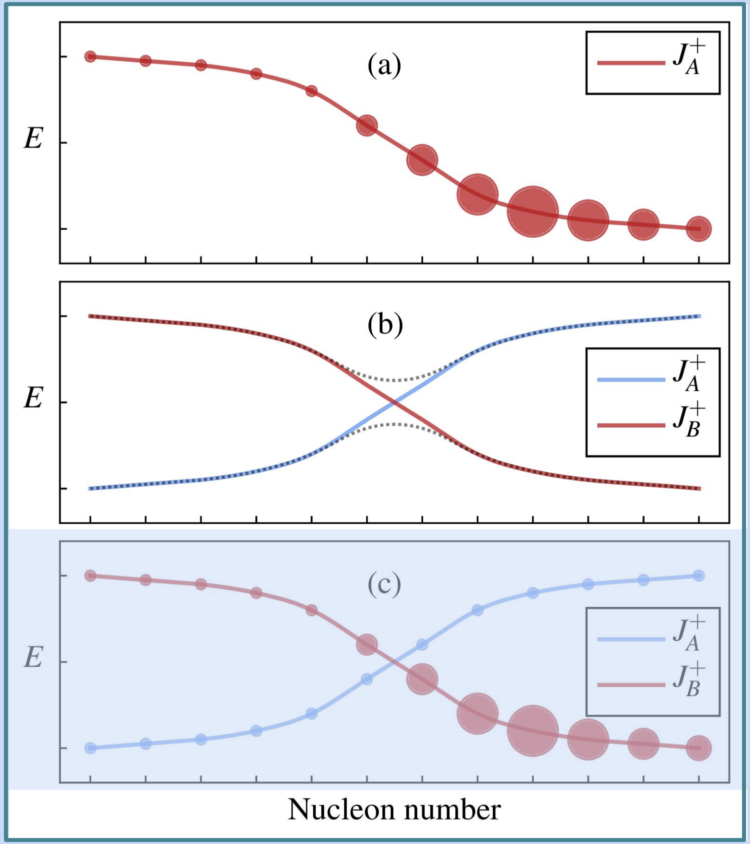
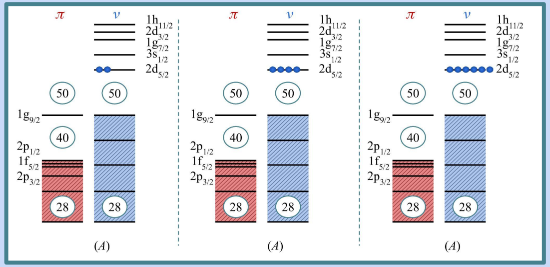
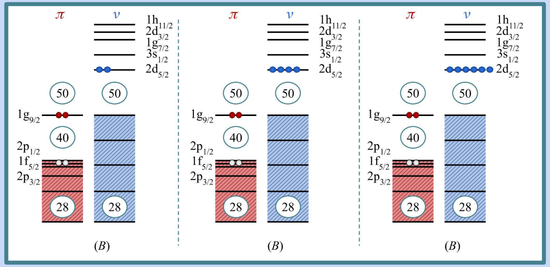
$$\hat{H} = \begin{bmatrix} \hat{H}_A(\xi_A) & \hat{W}(\omega) \\ \hat{W}(\omega) & \hat{H}_B(\xi_B) \end{bmatrix}$$

A. Frank, P. Van Isacker and F. Iachello, Phys. Rev. C **73**, 061302R (2006).







Intertwined Quantum Phase Transitions (IQPT)

$$\hat{H} = (1-\xi)\hat{H}_1 + \xi\hat{H}_2$$

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A. Dieperink, O. Scholten and F. Iachello, Phys. Rev. Lett. **44**, 1747 (1980).

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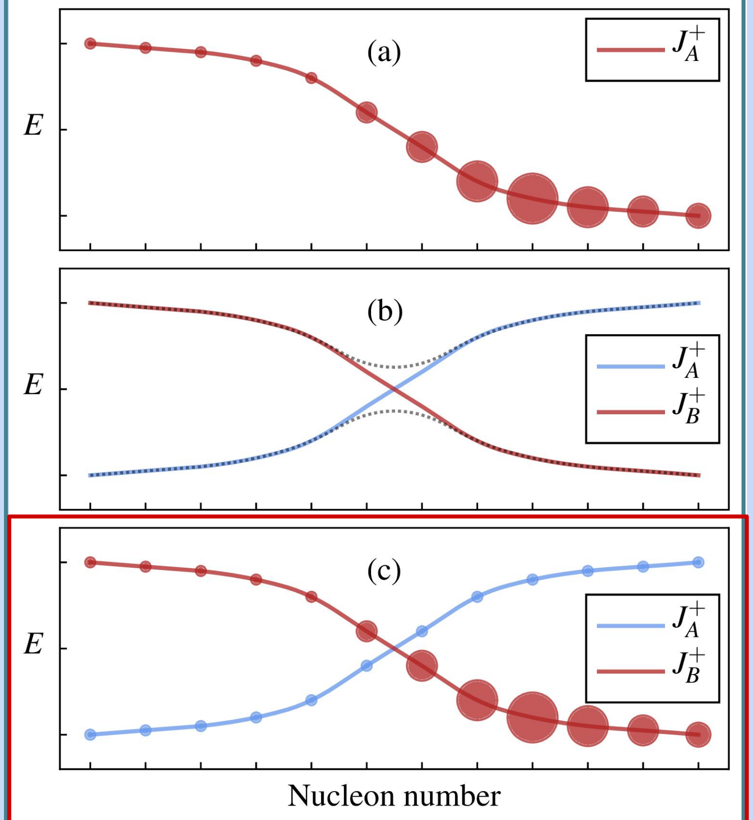
A. Frank, P. Van Isacker and F. Iachello, Phys. Rev. C **73**, 061302R (2006).



IQPT:
shape evolution
+
configuration crossing

even-even Zr isotopes

N. Gavrielov, A. Leviatan and F. Iachello,
- Phys. Rev. C **99**, 064324 (2019).
- Phys. Scr. **95**, 024001 (2020).
- Phys. Rev. C **105**, 014305 (2022).

V. Karayonchev *et al.*, N. Gavrielov,
- Phys. Rev. C **102**, 064314 (2020).

Even-even nuclei: ${}_{40}Zr$ isotopes

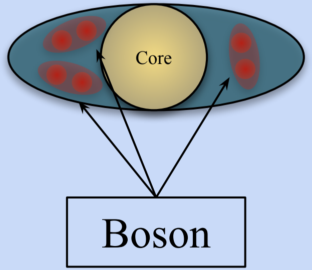
The Interacting Boson Model (IBM)

F. Iachello and A. Arima, The Interacting Boson Model (Cambridge University Press, Cambridge) 1987.

- *Low lying collective states can be described by* ***bosonic degrees of freedom***.
- *Bosons* ≈ ***correlated valence nucleon pairs***
  (no distinction between proton or neutron bosons).

- $s^\dagger$ $(L^p = 0^+)$, $d^\dagger_\mu$ $(\mu = 0, \pm 1, \pm 2; L^p = 2^+)$. $\qquad b^\dagger_\alpha \in \{s^\dagger, d^\dagger\}$

- Hamiltonian: $H = \sum_{\alpha\beta} \varepsilon_{\alpha\beta} G_{\alpha\beta} + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} G_{\alpha\beta} G_{\gamma\delta} + \ldots$ $\qquad (G_{\alpha\beta} = b^\dagger_\alpha b_\beta)$



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THE HEBREW UNIVERSITY OF JERUSALEM

- *Low lying collective states can be described by* ***bosonic degrees of freedom***.
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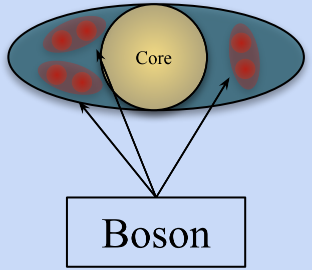
- Hamiltonian: $H = \sum_{\alpha\beta} \varepsilon_{\alpha\beta} G_{\alpha\beta} + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} G_{\alpha\beta} G_{\gamma\delta} + \ldots$ $\qquad (G_{\alpha\beta} = b^\dagger_\alpha b_\beta)$
  Hermitian, (boson) number conserving, rotational invariant (good SO(3) symmetry).



Dynamical Symmetry

F. Iachello and A. Arima, The Interacting Boson Model (Cambridge University Press, Cambridge) 1987.

$$[N_b] \uparrow U(6)$$

Algebra

$$|\, n_d \quad \tau \quad n_\Delta \quad L \,\rangle$$

$$U(6) \supset U(5) \supset SO(5) \supset SO(3)$$

Spherical vibrator

$$|\, (\lambda, \mu) \quad K \quad L \,\rangle$$

$$U(6) \supset SU(3) \supset SO(3)$$

Axially deformed rotor

$$U(6) \supset SO(6) \supset SO(5) \supset SO(3)$$

$$|\, \sigma \quad \tau \quad n_\Delta \quad L \,\rangle$$

γ-unstable deformed rotor

Dynamical Symmetry

F. Iachello and A. Arima, The Interacting Boson Model (Cambridge University Press, Cambridge) 1987.

Number of valence pairs: $[N_b]$

$U(6)$ Algebra

$$\supset U(5) \supset SO(5) \supset SO(3) \quad |\, n_d \;\; \tau \;\; n_\Delta \;\; L \,\rangle$$

Spherical vibrator

$$\supset SU(3) \supset SO(3) \quad |\, (\lambda, \mu) \;\; K \;\; L \,\rangle$$

Axially deformed rotor

$$\supset SO(6) \supset SO(5) \supset SO(3) \quad |\, \sigma \;\; \tau \;\; n_\Delta \;\; L \,\rangle$$

γ-unstable deformed rotor

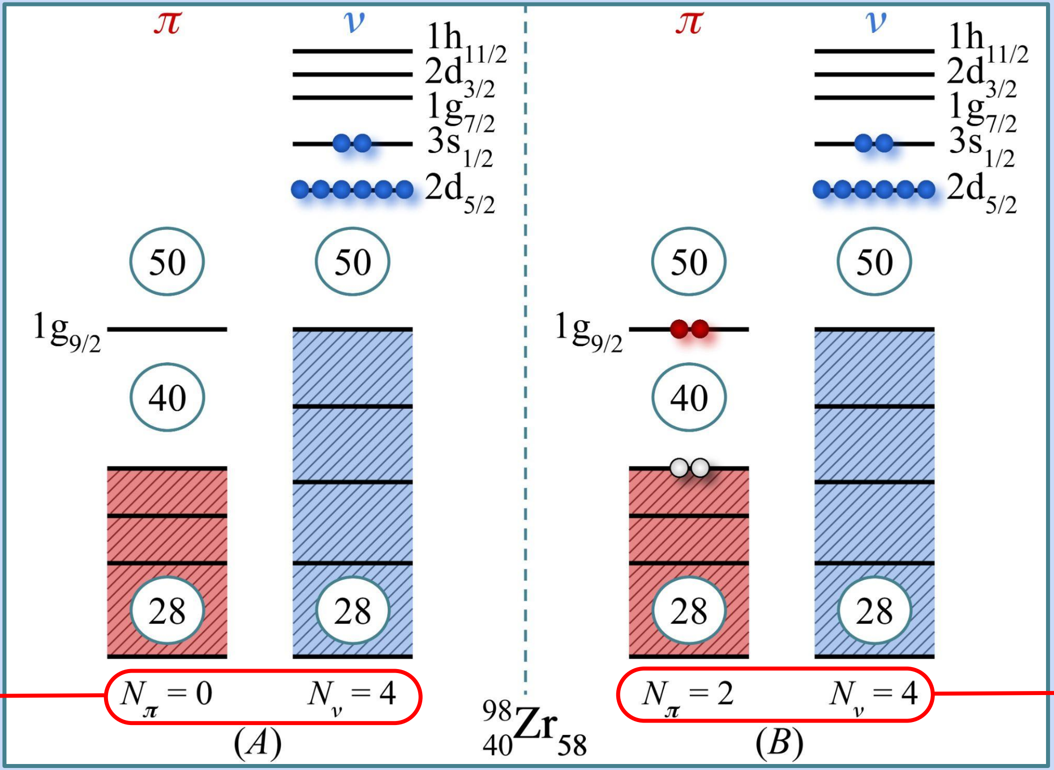
Introduction

Boson counting: ^{98}Zr

$^{90}_{40}Zr_{50}$ core

0p-0h protons

2p-2h protons



$N_b = N_\pi + N_\nu = 4$ — $N_\pi = 0$, $N_\nu = 4$

$N_\pi = 2$, $N_\nu = 4$ — $N_b = N_\pi + N_\nu = 6$

Wave function structure and spherical occupation

even-even Zr

$$|\boldsymbol{\psi}; L\rangle = a\,|\boldsymbol{\psi}_{\mathrm{A}}; N_{\mathrm{A}}, L\rangle + b\,|\boldsymbol{\psi}_{\mathrm{B}}; N_{\mathrm{B}}, L\rangle\,; \;\; a^2 + b^2 = 1$$

N. Gavrielov, Physica Scripta, **99**, 075310 (2024)

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$$|\boldsymbol{\psi}; L\rangle = a\,|\boldsymbol{\psi}_{\mathrm{A}}; N_{\mathrm{A}}, L\rangle + b\,|\boldsymbol{\psi}_{\mathrm{B}}; N_{\mathrm{B}}, L\rangle\,; \quad a^2 + b^2 = 1$$

Normal — Intruder

N. Gavrielov, Physica Scripta, **99**, 075310 (2024)

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$$|\boldsymbol{\psi}; L\rangle = a\,|\boldsymbol{\psi}_{\mathrm{A}}; N_{\mathrm{A}}, L\rangle + b\,|\boldsymbol{\psi}_{\mathrm{B}}; N_{\mathrm{B}}, L\rangle\,;\;\; a^2 + b^2 = 1$$

$$N_{\mathrm{A}} = N_b; \qquad N_{\mathrm{B}} = N_b + 2$$

$$|\boldsymbol{\psi}_i; N_i, L\rangle = \sum_{n_d,\boldsymbol{\tau},n_\Delta} C^{(N,L)}_{n_d,\boldsymbol{\tau},n_\Delta}\,|N_i, n_d, \boldsymbol{\tau}, n_\Delta, L\rangle$$

U(5) basis

N. Gavrielov, Physica Scripta, **99**, 075310 (2024)

Wave function structure and spherical occupation

even-even Zr

$$|\psi; L\rangle = a\,|\psi_A; N_A, L\rangle + b\,|\psi_B; N_B, L\rangle; \quad a^2 + b^2 = 1$$

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$$|\psi_i; N_i, L\rangle = \sum_{n_d, \boldsymbol{\tau}, n_\Delta} C^{(N,L)}_{n_d, \boldsymbol{\tau}, n_\Delta} |N_i, n_d, \boldsymbol{\tau}, n_\Delta, L\rangle \quad \rightarrow P^{(N_i, L)}_{n_d} = \sum_{\boldsymbol{\tau}, n_\Delta} [C^{(N,L)}_{n_d, \boldsymbol{\tau}, n_\Delta}]^2 \quad n_d \text{ occupation}$$

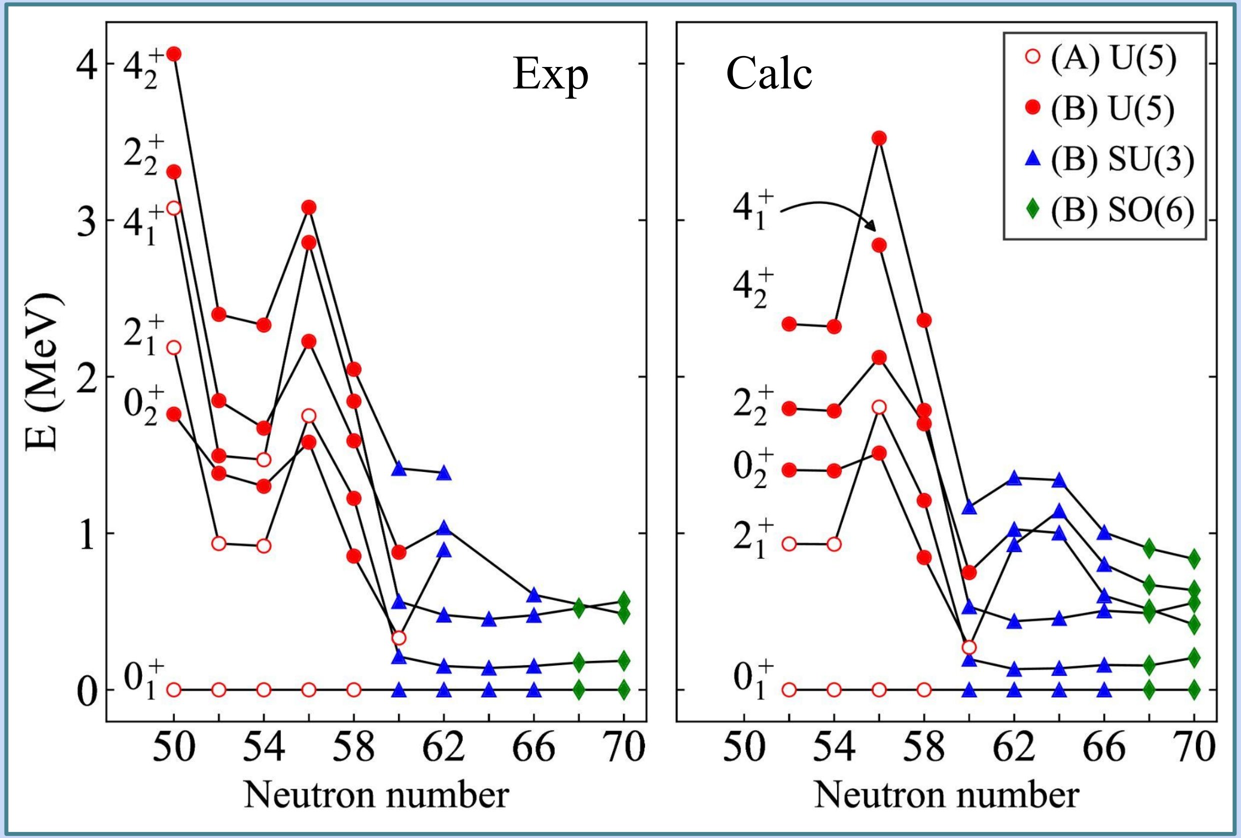
$$\rightarrow P^{(N_i, L)} = \sum_{n_d} P^{(N,L)}_{n_d} \qquad N_b \text{ occupation}$$

i = A, B

N. Gavrielov, Physica Scripta, **99**, 075310 (2024)

Results

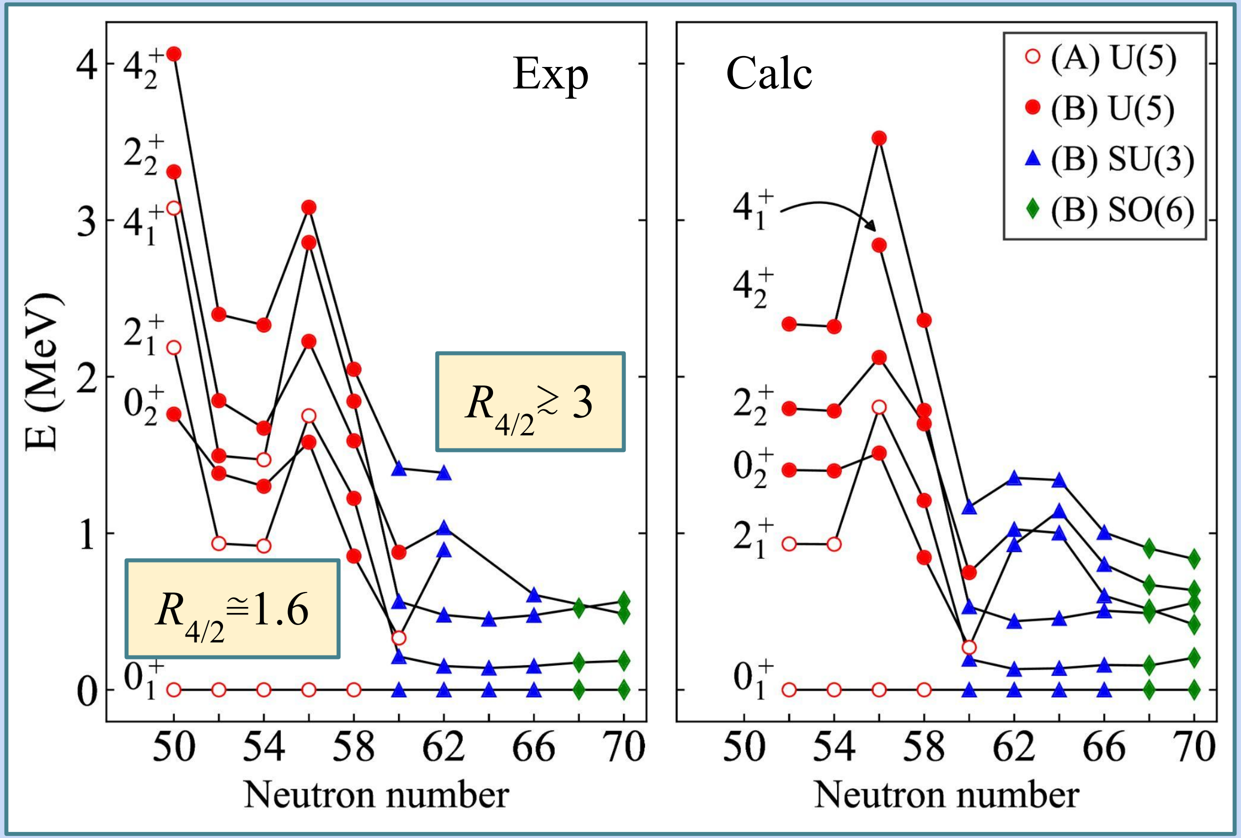
Energy levels



Results

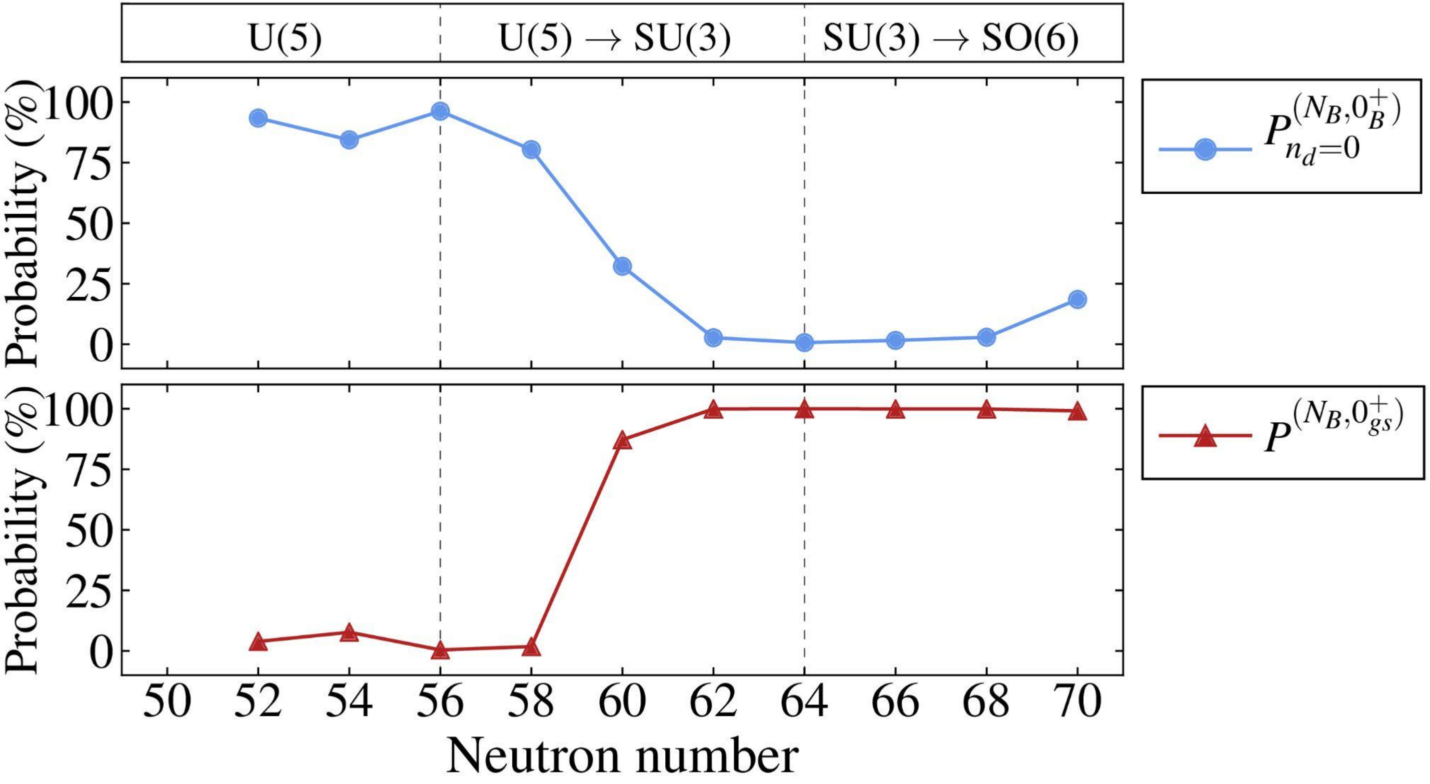
Energy levels

$R_{4/2} = E(4_1^+)/E(2_1^+)$



Results: quantum phase transitions

Evolution of shape and configuration



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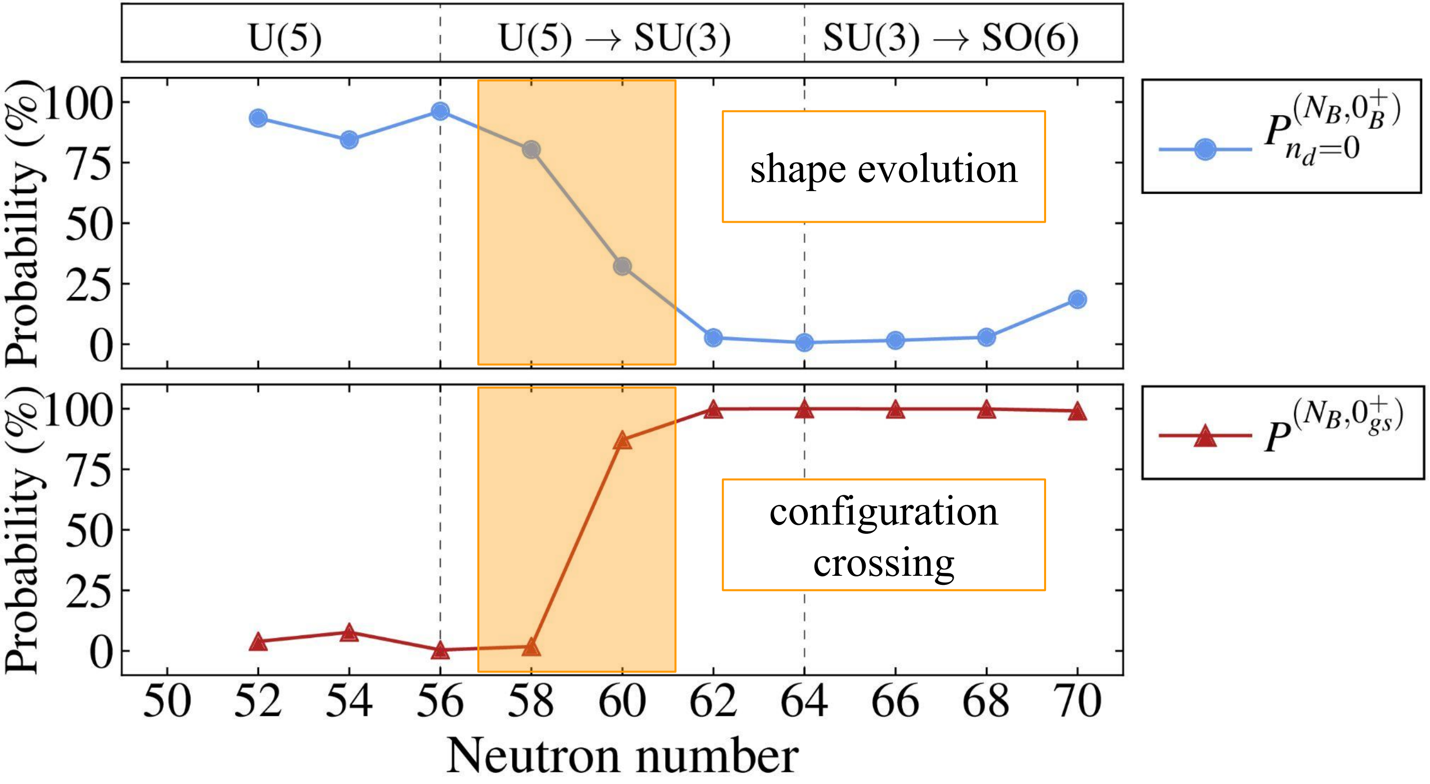


Odd-mass nuclei: $_{40}Zr$ isotopes

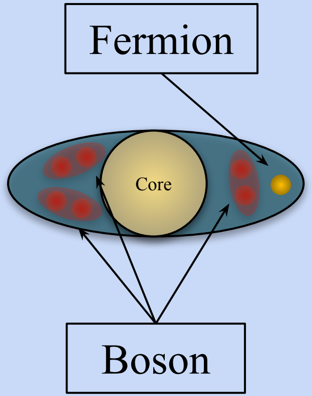
Interacting Boson Model Configuration Mixing

Hamiltonian

$$\hat{H} = \hat{H}_B$$

IBM-CM
Hamiltonian

$$\hat{H}_B = \begin{bmatrix} \hat{H}^{(1)}(\xi_1) & \hat{W}(\omega) \\ \hat{W}(\omega) & \hat{H}^{(2)}(\xi_2) \end{bmatrix}$$



Interacting Boson-Fermion Model Configuration Mixing

Hamiltonian

$$\hat{H} = \hat{H}_B + \hat{H}_F + V_{BF}$$

IBM-CM Hamiltonian

$$\hat{H}_B = \begin{bmatrix} \hat{H}^{(1)}(\xi_1) & \hat{W}(\omega) \\ \hat{W}(\omega) & \hat{H}^{(2)}(\xi_2) \end{bmatrix}$$

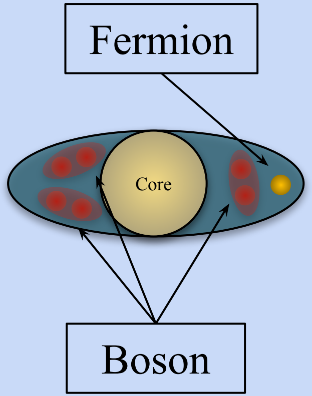
Fermion part

$$\hat{H}_F = \begin{bmatrix} \varepsilon_j \hat{n}_j & 0 \\ 0 & \varepsilon_j \hat{n}_j \end{bmatrix}$$

Bose-Fermi interaction

$\omega_j = \omega$, for all j

$$V_{BF} = \begin{bmatrix} V_{BF}{}^{(1)}(A^{(1)},\Gamma^{(1)},\Lambda^{(1)}) & 0 \\ 0 & V_{BF}{}^{(2)}(A^{(2)},\Gamma^{(2)},\Lambda^{(2)}) \end{bmatrix}$$



N. Gavrielov, Phys. Rev. C **108**, 014320 (2023)

Wave function structure and spherical occupation

odd-mass nuclei

$$|\boldsymbol{\psi}; J\rangle = \underbrace{\sum_{\alpha, L_b, j} \mathrm{C}^{(N,J)}_{\alpha, L_b, j} |\boldsymbol{\psi}_{\mathrm{A}}; N_{\mathrm{A}}, \alpha, L_b, j; J\rangle}_{\text{Normal}} + \underbrace{\sum_{\alpha, L_b, j} \mathrm{C}^{(N+2,J)}_{\alpha, L_b, j} |\boldsymbol{\psi}_{\mathrm{B}}; N_{\mathrm{B}}, \alpha, L_b, j; J\rangle}_{\text{Intruder}}$$

N. Gavrielov, Physica Scripta, **99**, 075310 (2024)

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$$|\boldsymbol{\psi}; J\rangle = \sum_{\alpha,\, L_b,\, j} \mathrm{C}^{(N,J)}_{\alpha,\, L_b,\, j}\, |\boldsymbol{\psi}_{\mathrm{A}}; N_{\mathrm{A}},\, \alpha,\, L_b, j;\, J\rangle + \sum_{\alpha,\, L_b,\, j} \mathrm{C}^{(N+2,J)}_{\alpha,\, L_b,\, j} |\boldsymbol{\psi}_{\mathrm{B}}; N_{\mathrm{B}},\, \alpha,\, L_b, j;\, J\rangle$$

$$N_{\mathrm{A}} = N_b; \qquad N_{\mathrm{B}} = N_b + 2$$

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N. Gavrielov, Physica Scripta, **99**, 075310 (2024)

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$$\rightarrow P^{(N_{\mathrm{i}},J)} = \sum_{n_d, \boldsymbol{\tau}, n_\Delta, L} [C^{(N_{\mathrm{i}},J)}_{n_d, \boldsymbol{\tau}, n_\Delta, j, L}]^2 \quad j \text{ occupation}$$

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N. Gavrielov, Physica Scripta, **99**, 075310 (2024)

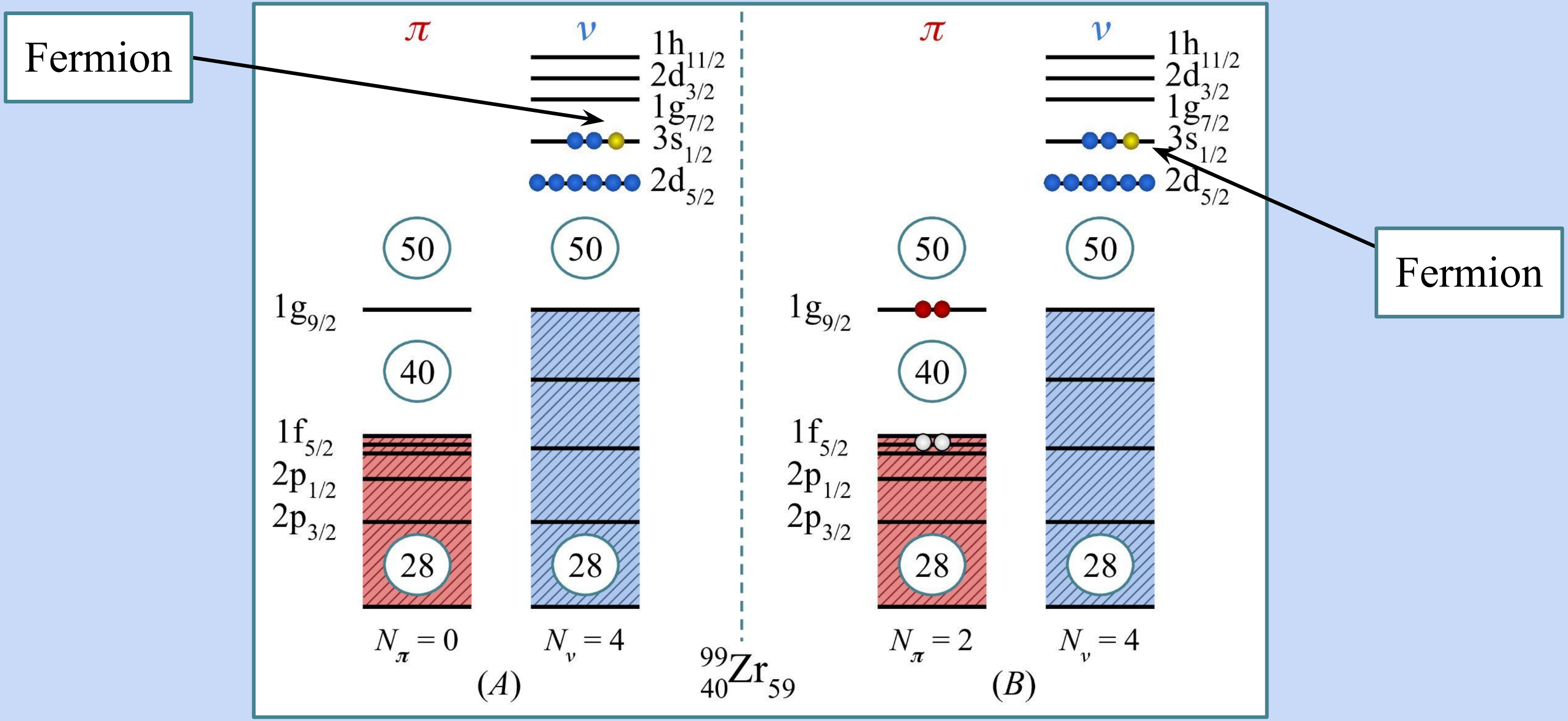
Introduction

Boson-Fermion counting: ^{99}Zr

$^{98}_{40}Zr_{58}$ boson core

0p-0h protons

2p-2h protons



Introduction

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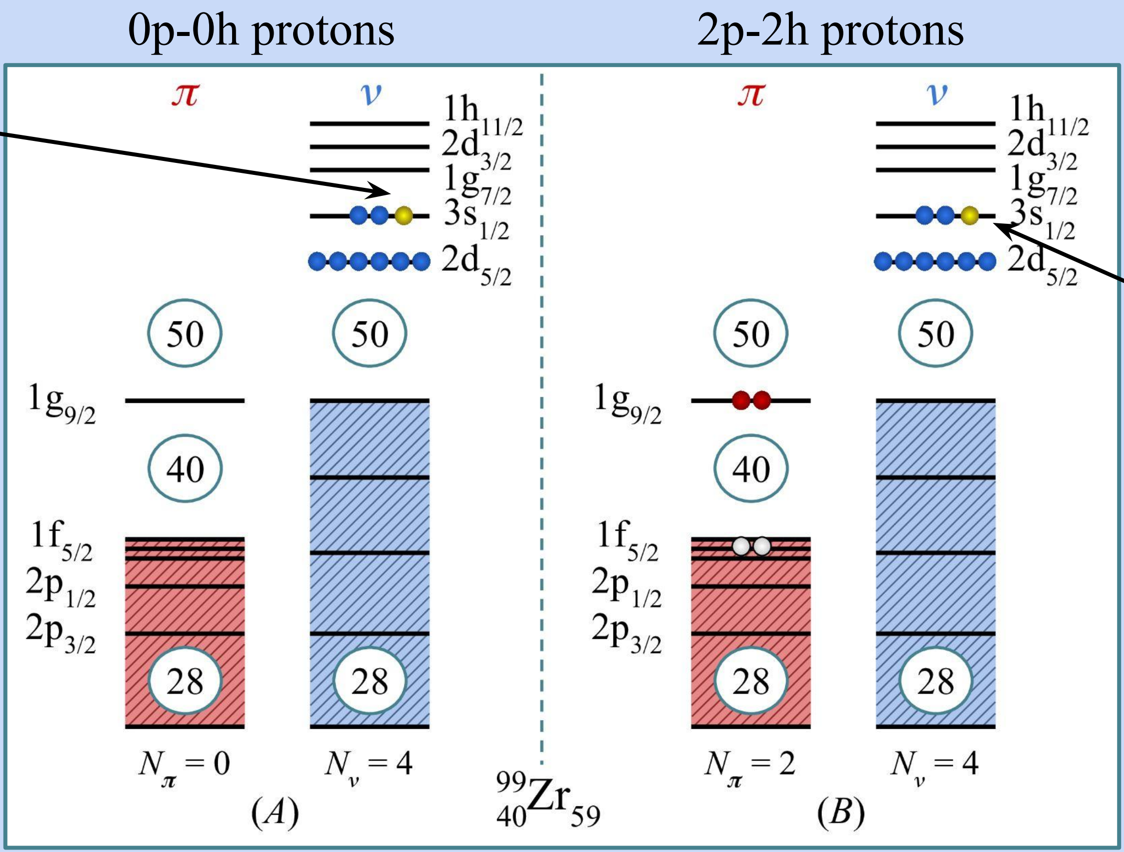
Fermion

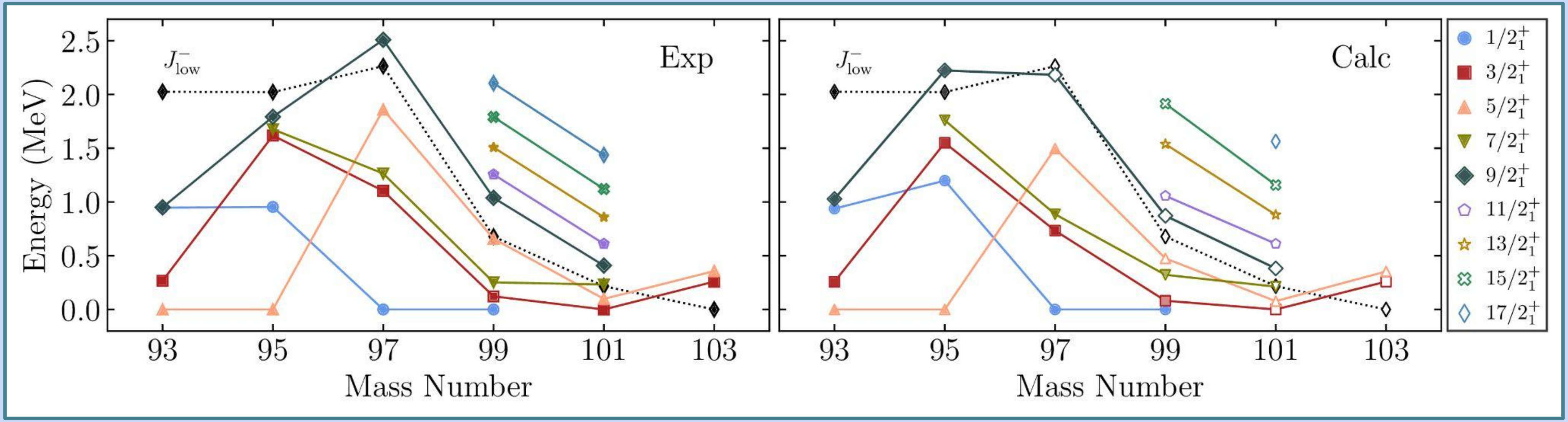


Fermion

Changing Fermi Level!

Results

Energy levels



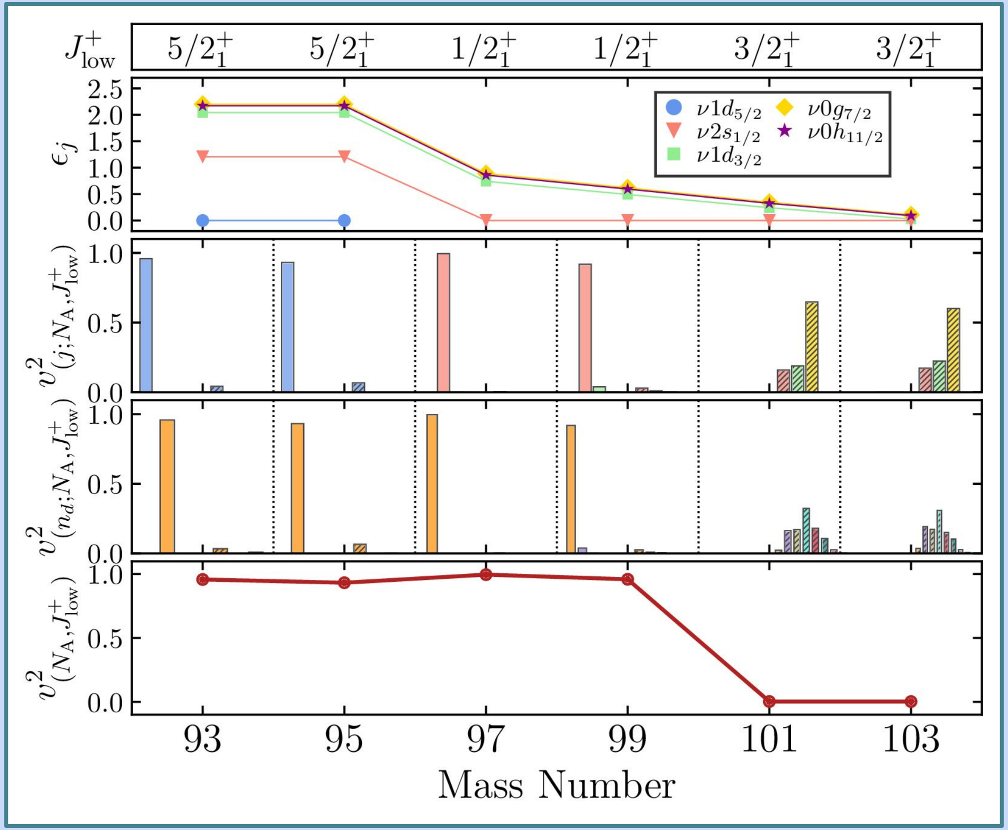
N. Gavrielov arXiv:2409.00967

Results

Evolution of occupation probabilities

Evolutions:

- single quasi-particle energies:
- j occupation (orbital):
- n_d occupation (deformation):
- N_b occupation (configuration):



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^{99}Zr debate

What is the nature of the isomeric $7/2^+$ state?

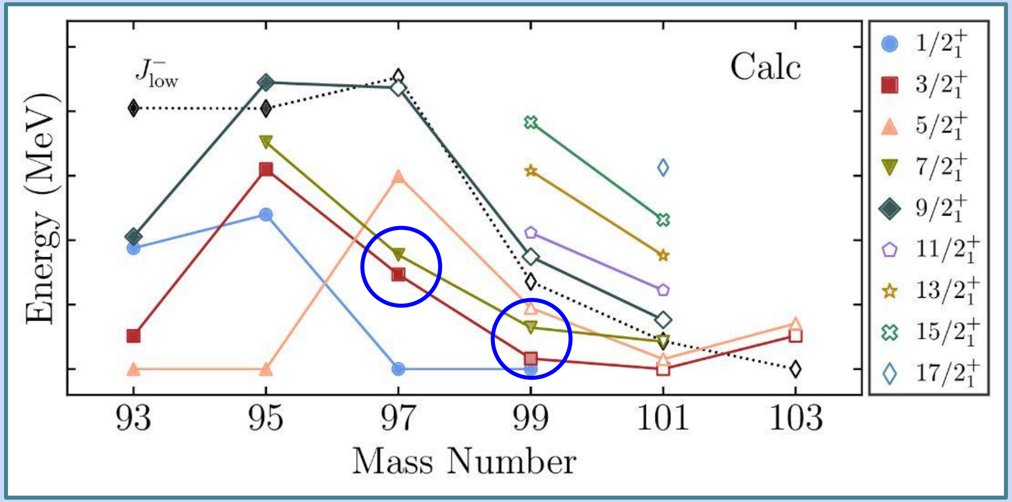
PHYSICAL REVIEW LETTERS **124,** 112501 (2020)

g Factor of the ^{99}Zr $(7/2^+)$ Isomer: Monopole Evolution in the Shape-Coexisting Region

F. Boulay,[1,2,3] G. S. Simpson,[4] Y. Ichikawa,[2] S. Kisyov,[5] D. Bucurescu,[5] A. Takamine,[2] D. S. Ahn,[2] K. Asahi,[2,6] H. Baba,[2] D. L. Balabanski,[2,7] T. Egami,[2,8] T. Fujita,[2,9] N. Fukuda,[2] C. Funayama,[2,6] T. Furukawa,[2,10] G. Georgiev,[11] A. Gladkov,[2,12] M. Hass,[13] K. Imamura,[2,14] N. Inabe,[2] Y. Ishibashi,[2,15] T. Kawaguchi,[2,8] T. Kawamura,[9] W. Kim,[12] Y. Kobayashi,[16] S. Kojima,[2,6] A. Kusoglu,[11,17] R. Lozeva,[11] S. Momiyama,[18] I. Mukul,[13] M. Niikura,[18] H. Nishibata,[2,9] T. Nishizaka,[2,8] A. Odahara,[9] Y. Ohtomo,[2,6] D. Ralet,[11] T. Sato,[2,6] Y. Shimizu,[2] T. Sumikama,[2] H. Suzuki,[2] H. Takeda,[2] L. C. Tao,[2,19] Y. Togano,[6] D. Tominaga,[2,8] H. Ueno,[2] H. Yamazaki,[2] X. F. Yang,[20] and J. M. Daugas[1,2]

^{99}Zr debate

What is the nature of the isomeric $7/2^+$ state?



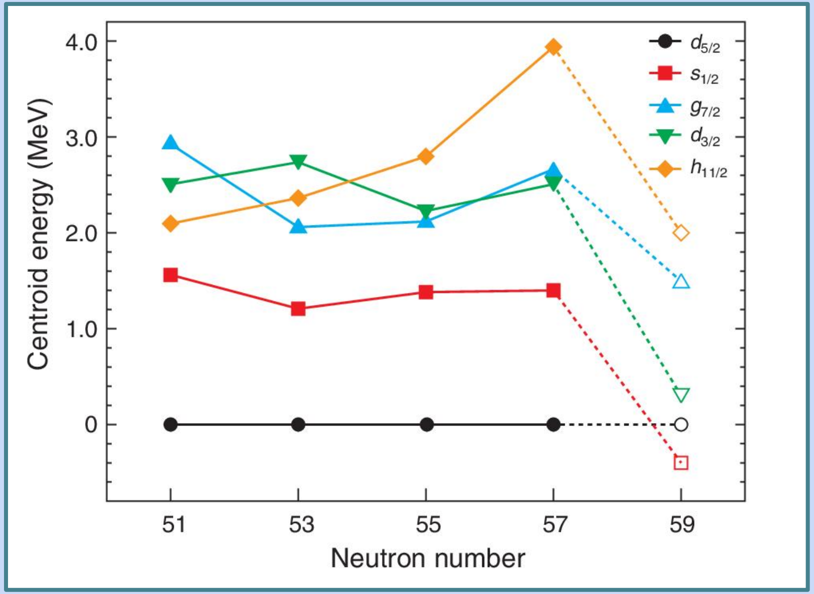
$\mu(^{97}\text{Zr}) = 1.365\ \mu_N$; $7/2^+$ is a $\nu g_{7/2}$ excitation.

$\mu(^{99}\text{Zr}) = 2.31\ \mu_N$; $7/2^+$ is … ?

^{99}Zr debate

What is the nature of the isomeric $7/2^+$ state?

IBFM (single configuration): $7/2^+$ is $\nu d_{5/2}$ excitation



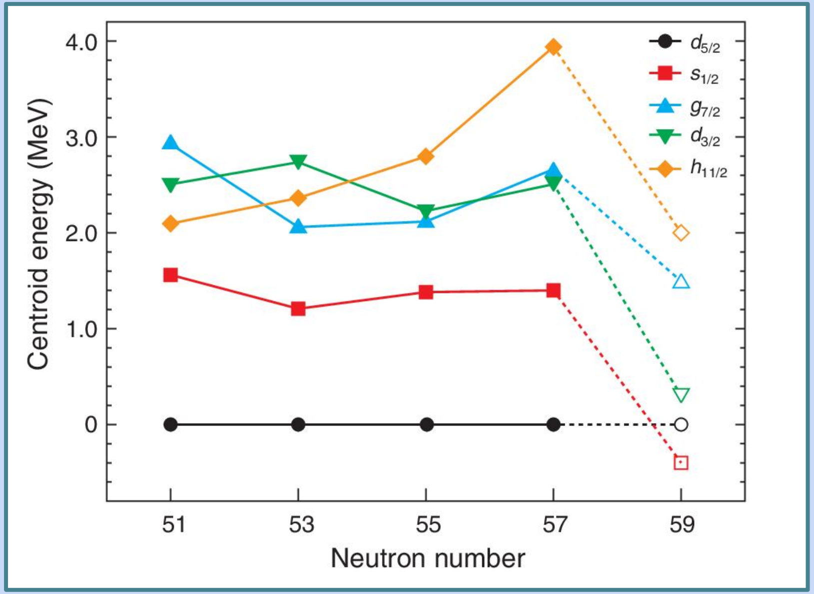
F. Boulay *et al.*, Phys. Rev. Lett. **124**, 112501 (2020)

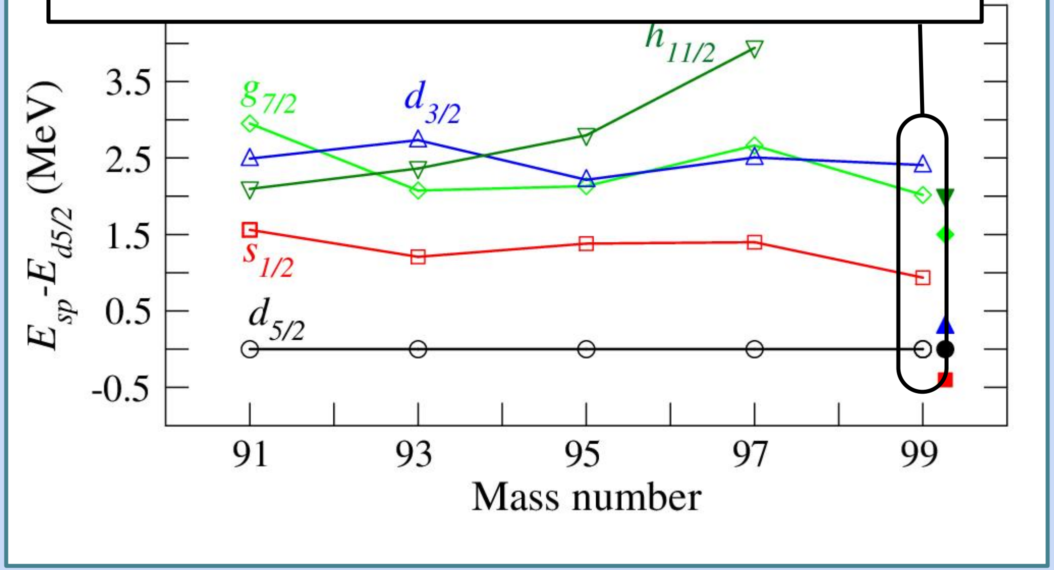
^{99}Zr debate

What is the nature of the isomeric $7/2^+$ state?

IBFM (single configuration): $7/2^+$ is $\nu d_{5/2}$ exc...

S. Brant, V. Paar, A. Wolf, Phys. Rev. C **58**, 1349 (1998)



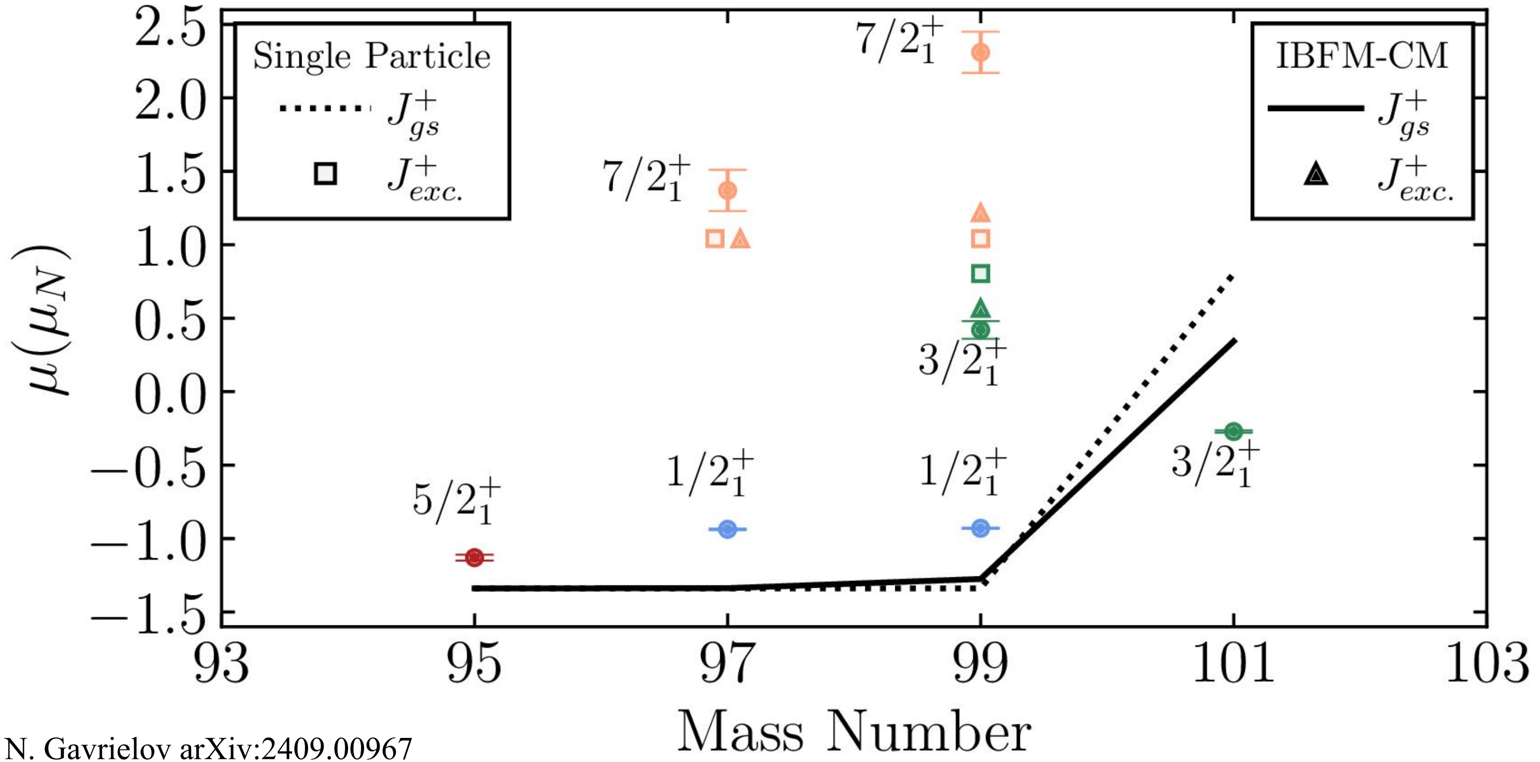


F. Boulay *et al.*, Phys. Rev. Lett. **124**, 112501 (2020)

P. E. Garrett, Phys. Rev. Lett. **127**, 169201 (2021)

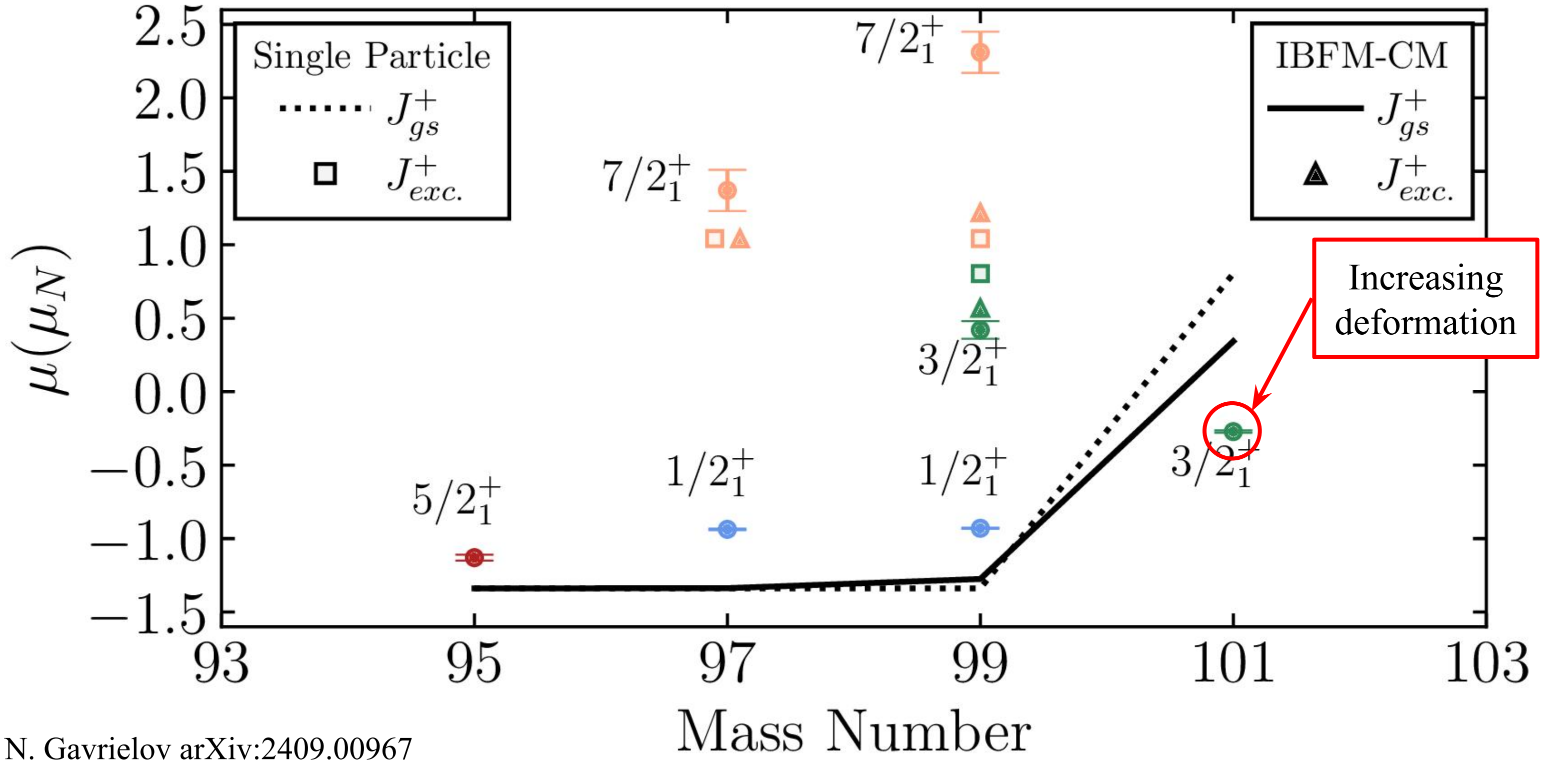
Results

Magnetic Moments



N. Gavrielov arXiv:2409.00967

Results
Magnetic Moments



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Results

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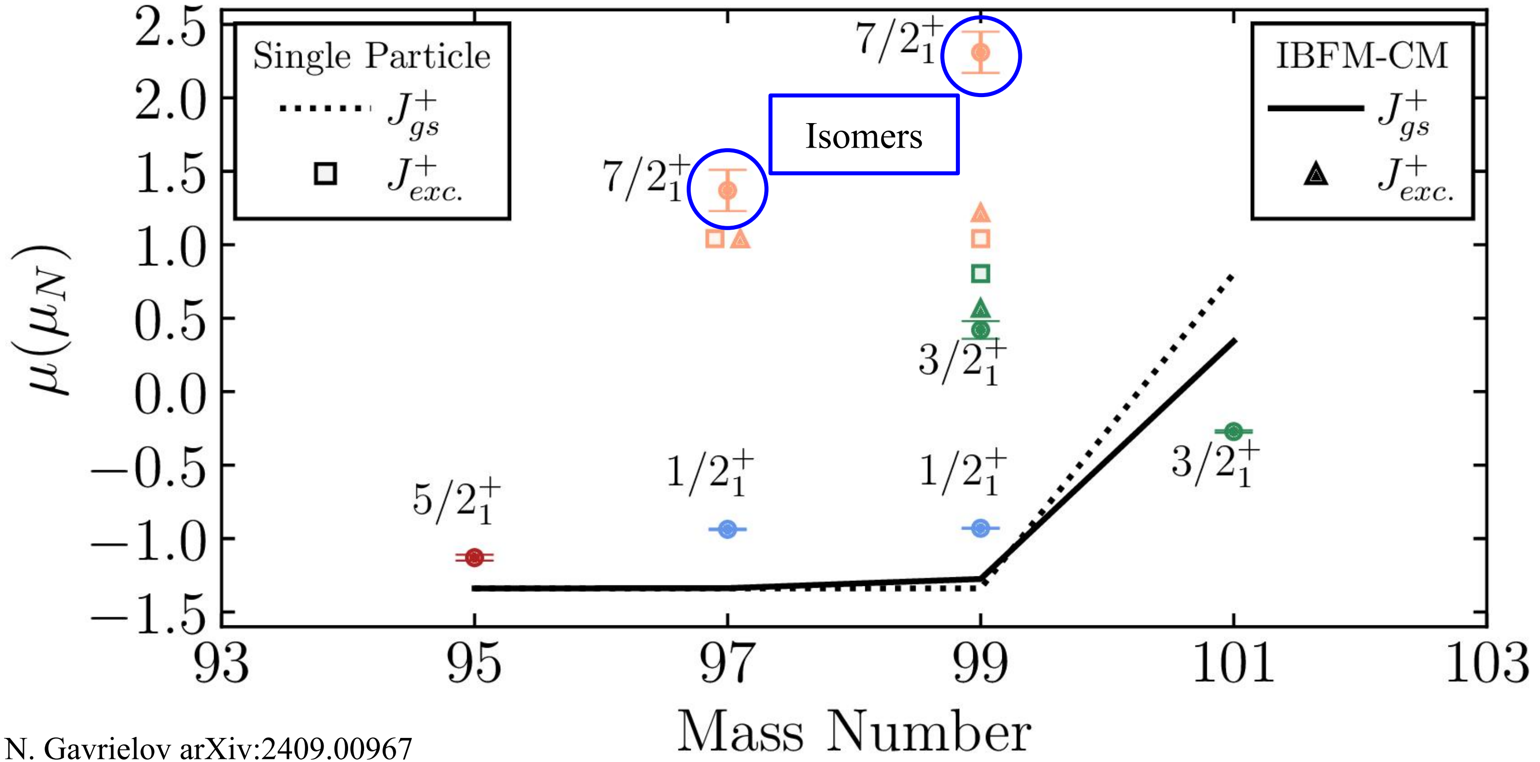

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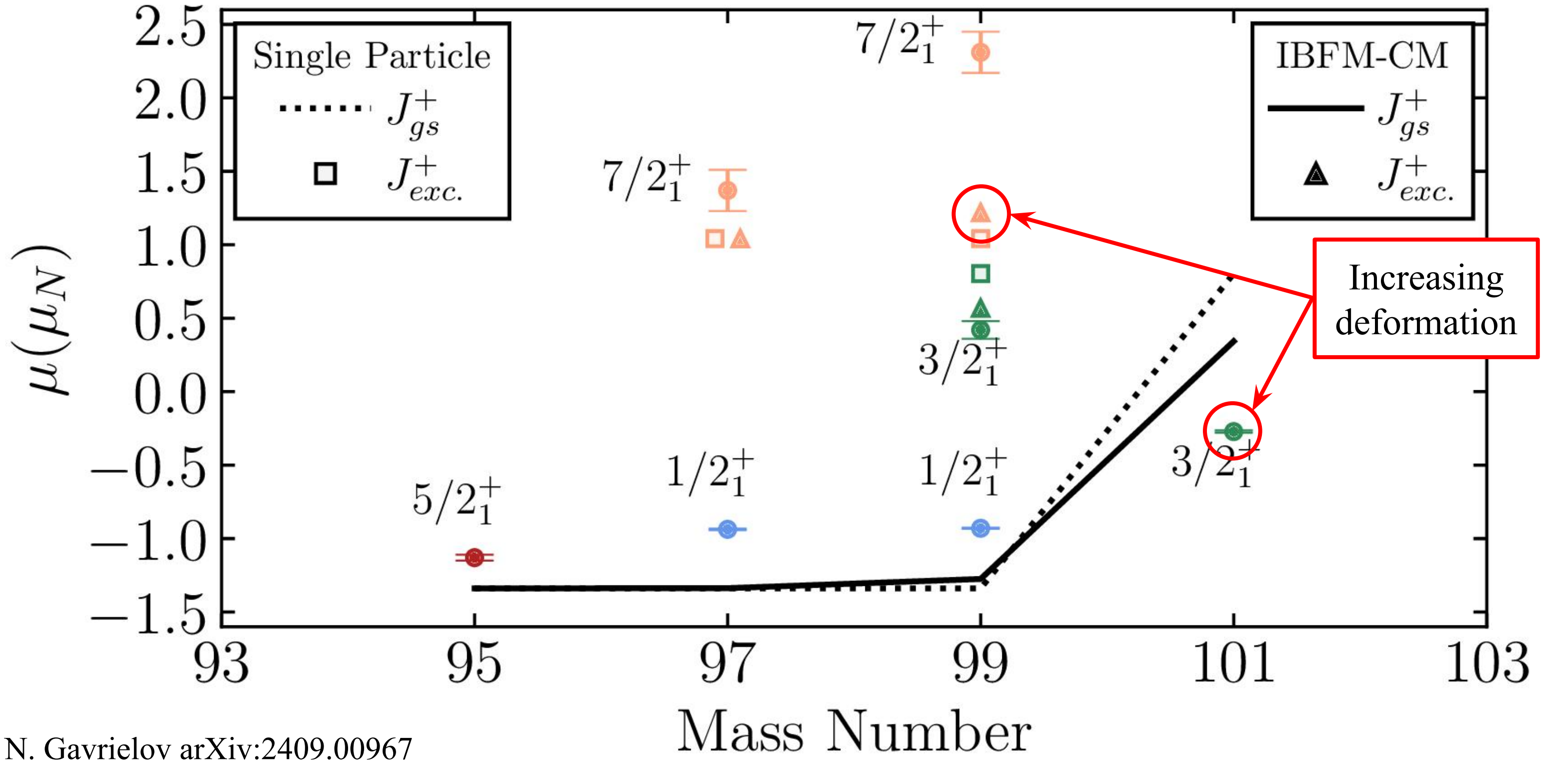

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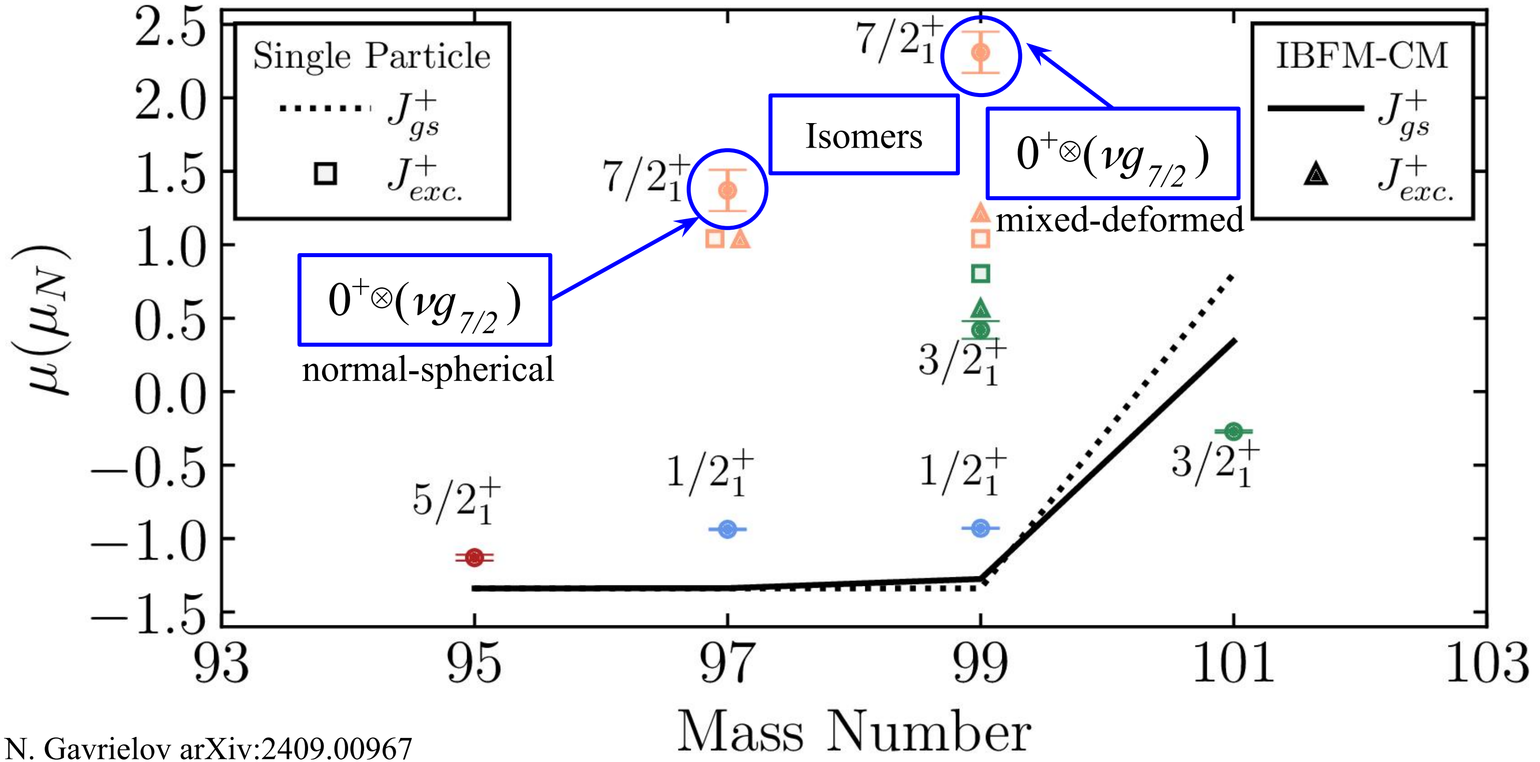

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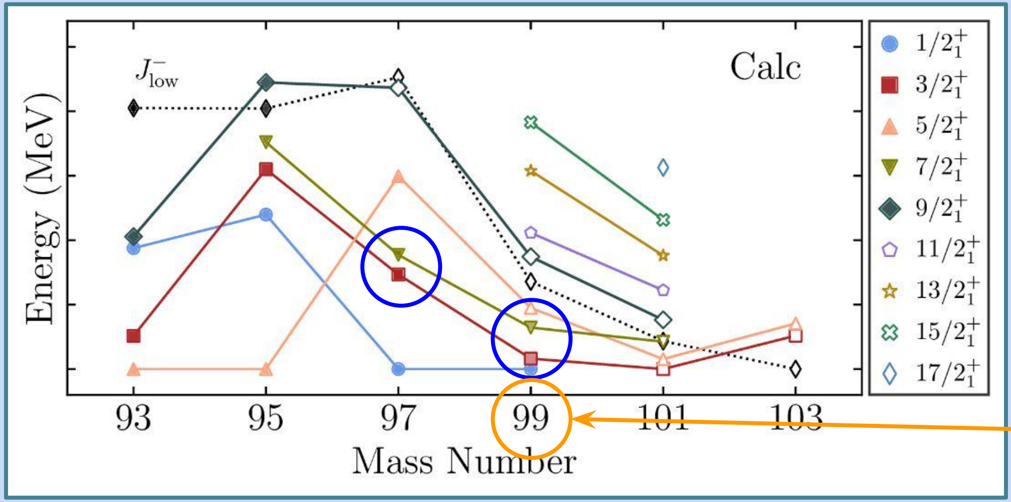
^{99}Zr debate

What is the nature of the isomeric $7/2^+$ state?



"Critical point" of

- Shape evolution
- Configuration crossing

$\mu(^{97}\text{Zr}) = 1.365\ \mu_N$; $7/2^+$ is a $\nu g_{7/2}$ excitation (normal and spherical).

$\mu(^{99}\text{Zr}) = 2.31\ \mu_N$; $7/2^+$ is a $\nu g_{7/2}$ excitation (mixed and deformed).

N. Gavrielov arXiv:2409.00967

Conclusions

- Calculation of even-even (IBM-CM) and odd-mass (IBFM-CM) Zr isotopes.
- Quantum analysis of the evolution of energy levels and other observables (two-neutron separation energies, *E2*, *E0*, isotope shift, quadrupole and magnetic moments).
- Calculated change in the configuration and symmetry content of wave functions.

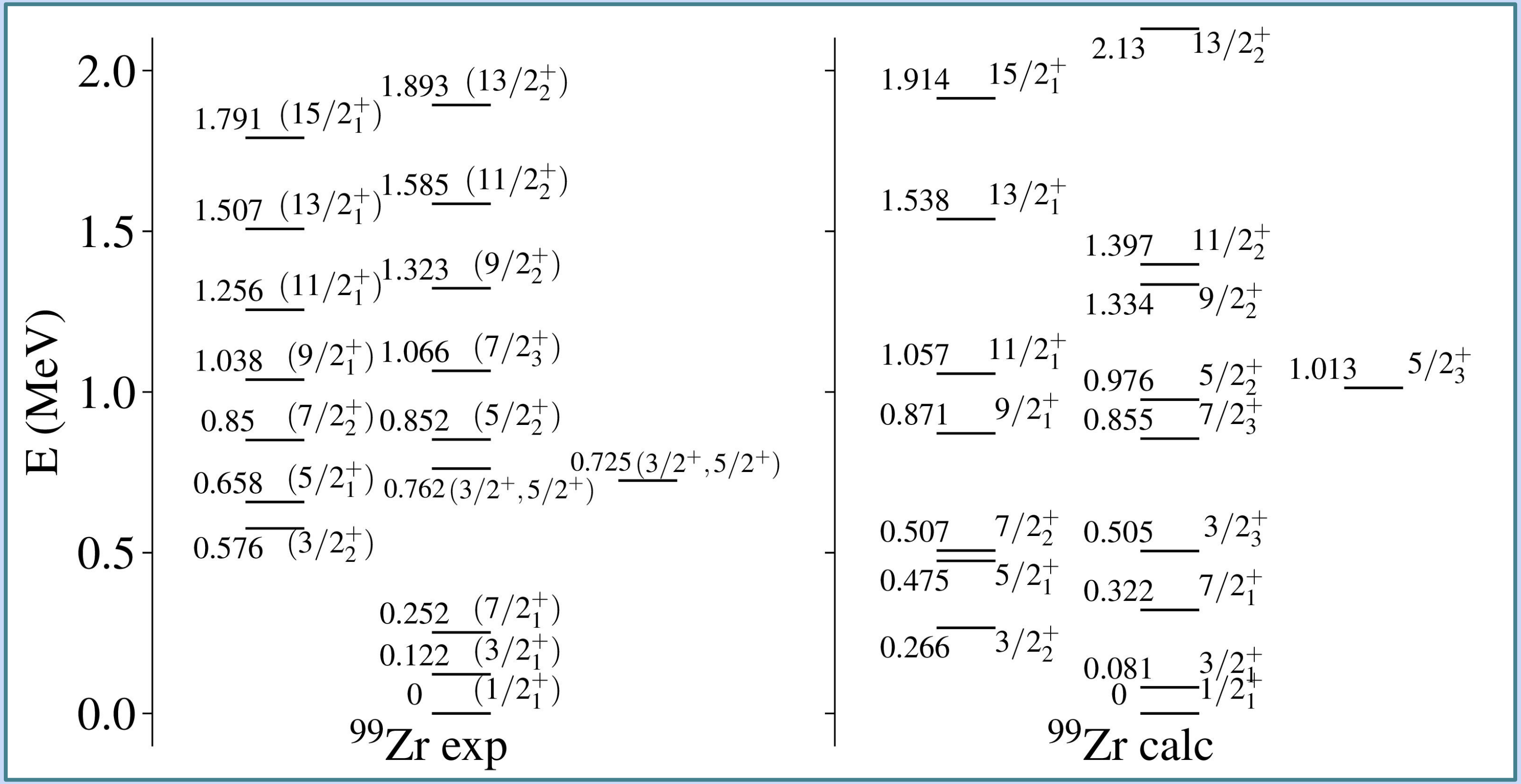
All point toward the occurrence of ***IQPTs****:*

- *Configuration crossing:* QPT between two configurations (normal and intruder).
- *Shape evolution:* QPT [spherical to deformed] of the intruder B configuration.
- *Triad of effects (odd-mass):* shape, configuration and single-quasi particle evolution.

Thank you

Appendix

^{99}Zr: critical point between ^{98}Zr and ^{100}Zr



Appendix
IBM-1-CM Hamiltonian

$$\hat{H} = \hat{H}_A^{(N)} + \hat{H}_B^{(N+2)} + \hat{W}^{(N,N+2)}$$

Normal configuration (0p-0h)*:*

$$\hat{H}_A = \varepsilon_d^{(A)} \hat{n}_d + \kappa^{(A)} Q \cdot Q$$ $[N_b]$ irrep.

Intruder configuration (2p-2h)*:*

$$\hat{H}_B = \varepsilon_d^{(B)} \hat{n}_d + \kappa^{(B)} Q \cdot Q + \kappa'^{(B)} L \cdot L + \Delta_{\mathrm{p}}$$ $[N_b+2]$ irrep.

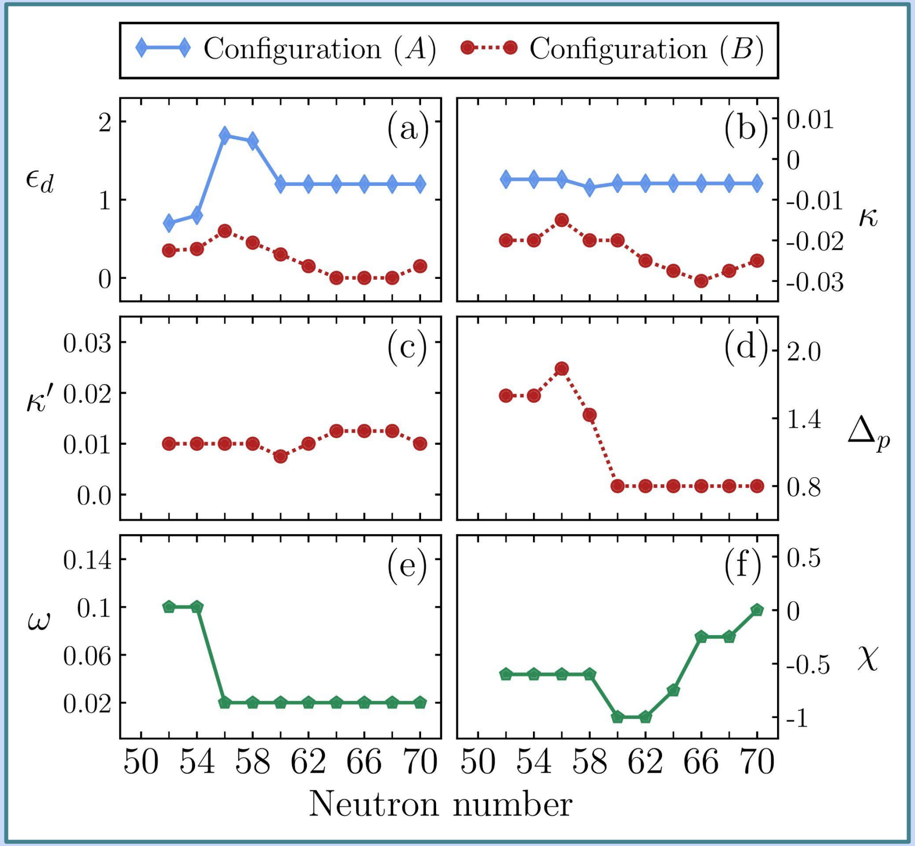
Coupling:

$$\hat{W}^{(N,N+2)} = \omega \, [(d^\dagger d^\dagger)^{(0)} + (s^\dagger)^2] + h.c.$$ $[N_b]\oplus[N_b+2]$ irrep.

Appendix
IBM-1-CM operators

- Pairing: $n_d = d^\dagger \cdot \tilde{d}$
- Quadrupole: $Q(\chi) = d^\dagger s + s^\dagger \tilde{d} + \chi(d^\dagger \times \tilde{d})^{(2)}$
- Angular momentum: $L = \sqrt{10}\,(d^\dagger \times \tilde{d})^{(1)}$

Appendix
Parameters



Appendix

Parameters

TABLE V. Parameters of the IBM-CM Hamiltonian, Eq. (14), are in MeV and χ is dimensionless. The first row of the Table lists the number of neutrons, and particle-bosons $(N, N+2)$ or hole-bosons $(\bar{N}, \bar{N}+2)$ in the (A, B) configurations.

	52(1,3)	54(2,4)	56(3,5)	58(4,6)	60(5,7)	62(6,8)	64(7,9)	66(8,10)	$68(\bar{7},\bar{9})$	$70(\bar{6},\bar{8})$
$\epsilon_d^{(A)}$	0.9	0.8	1.82	1.75	1.2	1.2	1.2	1.2	1.2	1.2
$\kappa^{(A)}$	−0.005	−0.005	−0.005	−0.007	−0.006	−0.006	−0.006	−0.006	−0.006	−0.006
$\epsilon_d^{(B)}$	0.35	0.37	0.6	0.45	0.3	0.15	0	0	0	0.15
$\kappa^{(B)}$	−0.02	−0.02	−0.015	−0.02	−0.02	−0.025	−0.0275	−0.03	−0.0275	−0.025
$\kappa'^{(B)}$	0.01	0.01	0.01	0.01	0.0075	0.01	0.0125	0.0125	0.0125	0.01
χ	−0.6	−0.6	−0.6	−0.6	−1.0	−1.0	−0.75	−0.25	−0.25	0
$\Delta_p^{(B)}$	1.6	1.6	1.84	1.43	0.8	0.8	0.8	0.8	0.8	0.8
ω	0.1	0.1	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02

Appendix

Parameters: fitting procedure

$$\epsilon_d^{(B)}(N) = \epsilon_d^{(B)}(N_0) + \frac{\partial \epsilon_d^{(B)}}{\partial N}\Big|_{N=N_0}(N - N_0) + \ldots \approx \epsilon_0 - \theta N \ ,$$

$$\kappa^{(B)}(N) = \kappa^{(B)}(N_0) + \frac{\partial \kappa^{(B)}}{\partial N}\Big|_{N=N_0}(N - N_0) + \ldots \approx \kappa_0 \ ,$$

$$\kappa'^{(B)}(N) = \kappa'^{(B)}(N_0) + \frac{\partial \kappa'^{(B)}}{\partial N}\Big|_{N=N_0}(N - N_0) + \ldots \approx \kappa'_0 \ .$$

$(\varepsilon_0, \boldsymbol{\theta}) = (1.35, 0.15)$ MeV, $\kappa^{(B)} \approx 3\kappa^{(A)}$

Appendix

Parameters: fitting procedure

TABLE VI. Experimental levels of $^{92-110}$Zr that are assigned to configuration-B and used to fit the parameters of $\hat{H}_B$ (20b). For $^{92-98}$Zr, the indicated levels correspond to calculated states dominated by U(5) components with $n_d \approx 0,\ 1,\ 2,\ 3$ within the B configuration part of the wave function $|\Psi_B; [N+2], L\rangle$, Eq. (16) (see Section V for more details).

^{92}Zr	$0_2^+,\ 2_2^+,\ (4_2^+,\ 2_3^+,\ 0_3^+),\ (6_1^+,\ 4_3^+,\ 3_1^+,\ 2_5^+)$
^{94}Zr	$0_2^+,\ 2_2^+,\ (4_2^+,\ 2_3^+),\ (6_1^+,\ 4_3^+,\ 3_1^+,\ 2_5^+)$
^{96}Zr	$0_2^+,\ 2_2^+,\ (4_1^+,\ 2_3^+,\ 0_3^+),\ (6_4^+,\ 4_3^+,\ 2_4^+,\ 0_4^+)$
^{98}Zr	$0_2^+,\ 2_1^+,\ (0_3^+,\ 2_2^+,\ 4_1^+),\ (6_1^+,\ 4_3^+,\ 3_1^+,\ 2_4^+,\ 0_4^+)$
^{100}Zr	$0_1^+,\ 2_1^+,\ 4_1^+,\ 0_3^+,\ 2_2^+,\ 6_1^+,\ 2_3^+$
^{102}Zr	$0_1^+,\ 2_1^+,\ 4_1^+,\ 0_2^+,\ 6_1^+,\ 2_2^+,\ 2_3^+,\ 3_1^+$
^{104}Zr	$0_1^+,\ 2_1^+,\ 4_1^+,\ 6_1^+$
^{106}Zr	$0_1^+,\ 2_1^+,\ 4_1^+,\ 2_2^+,\ 6_1^+$
^{108}Zr	$0_1^+,\ 2_1^+,\ 4_1^+,\ 6_1^+$
^{110}Zr	$0_1^+,\ 2_1^+,\ 4_1^+,\ 2_2^+$

Appendix

Classical analysis

$$\mathcal{E}(\beta,\gamma) = \frac{\langle [N]; \beta,\gamma | \, H \, | \, [N]; \beta,\gamma \rangle}{\langle [N]; \beta,\gamma | \, [N]; \beta,\gamma \rangle}$$

$$H = \begin{bmatrix} \hat{H}_A(\xi_A) & \hat{W}(\omega) \\ \hat{W}(\omega) & \hat{H}_B(\xi_B) \end{bmatrix} \Rightarrow \mathcal{E}(\beta,\gamma) = \begin{bmatrix} \mathcal{E}(\beta,\gamma)_{\mathrm{A}} & \Omega \\ \Omega & \mathcal{E}(\beta,\gamma)_{\mathrm{B}} \end{bmatrix} \Rightarrow \mathcal{E}_{\pm}(\beta,\gamma)$$

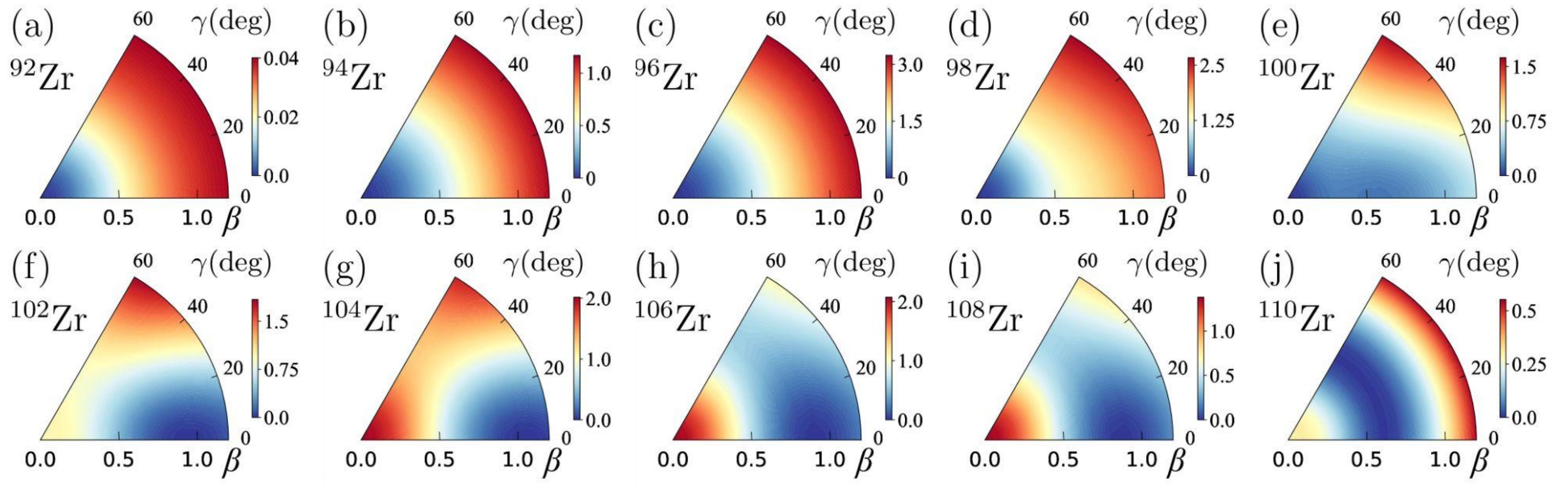
A. Frank, P. Van Isacker and C. E. Vargas, Phys. Rev. C **69**, 034323 (2004)

A. Frank, P. Van Isacker and F. Iachello, Phys. Rev. C **73**, 061302(R) (2006)

Appendix

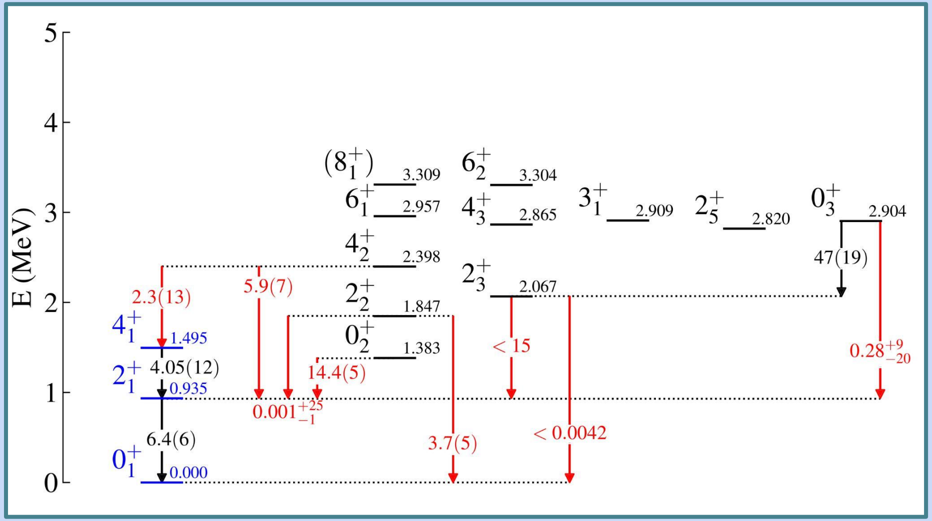
Classical analysis: Ɛ_(β,γ)

THE HEBREW UNIVERSITY OF JERUSALEM

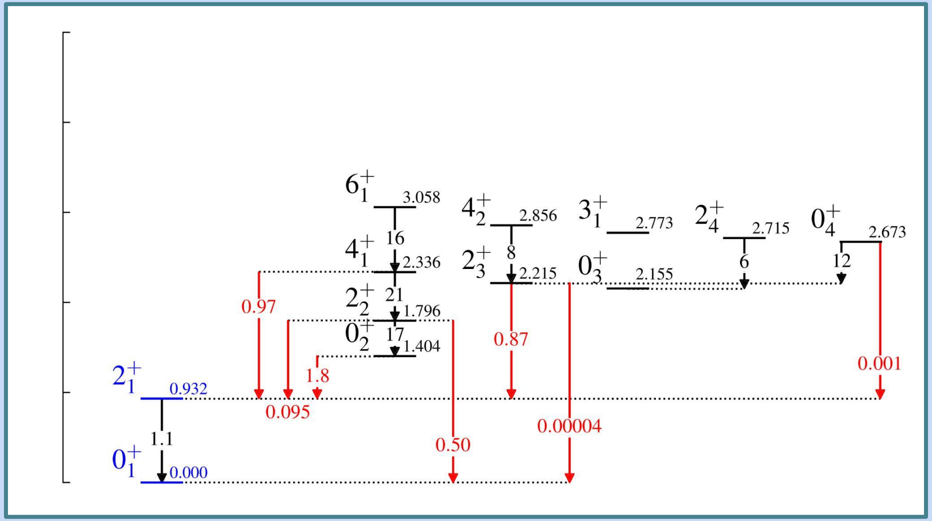


Appendix
U(5)-coexistence region

^{92}Zr exp



^{92}Zr calc

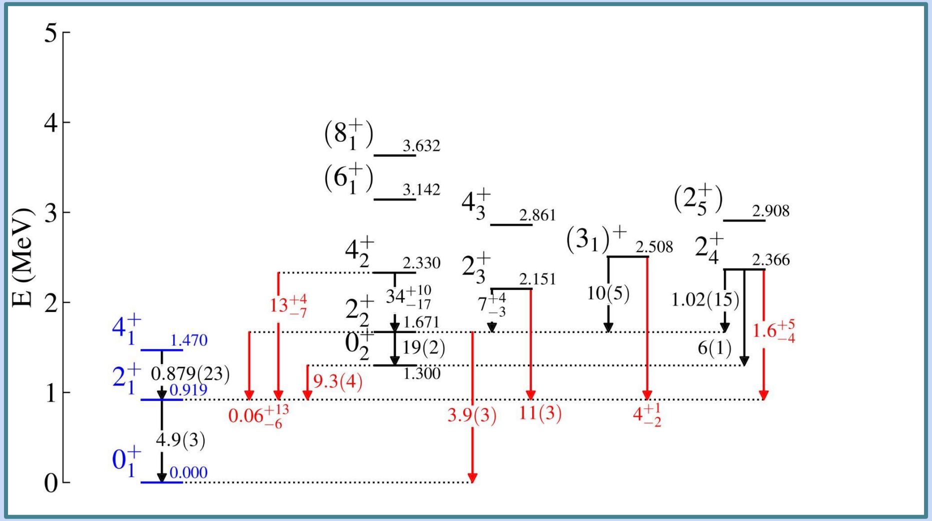


Blue: normal levels
Black: intruder levels
Arrows: *E2* transitions

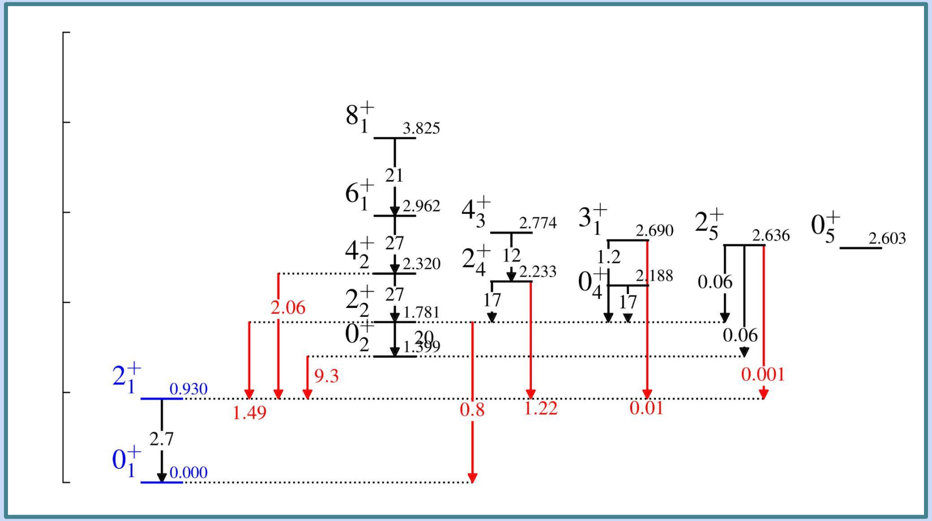
Appendix

U(5)-coexistence region

^{94}Zr exp



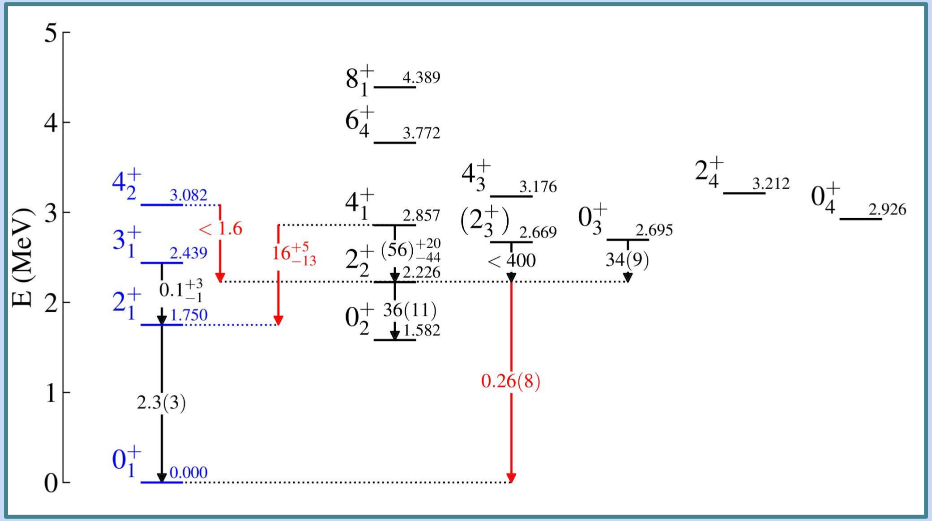
^{94}Zr calc



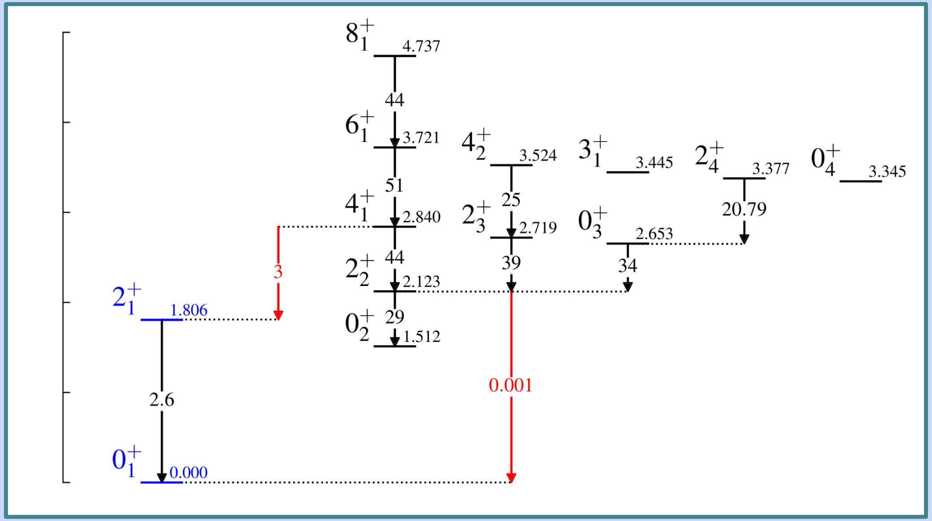
Blue: normal levels
Black: intruder levels
Arrows: *E2* transitions

Appendix
U(5)-coexistence region

^{96}Zr exp



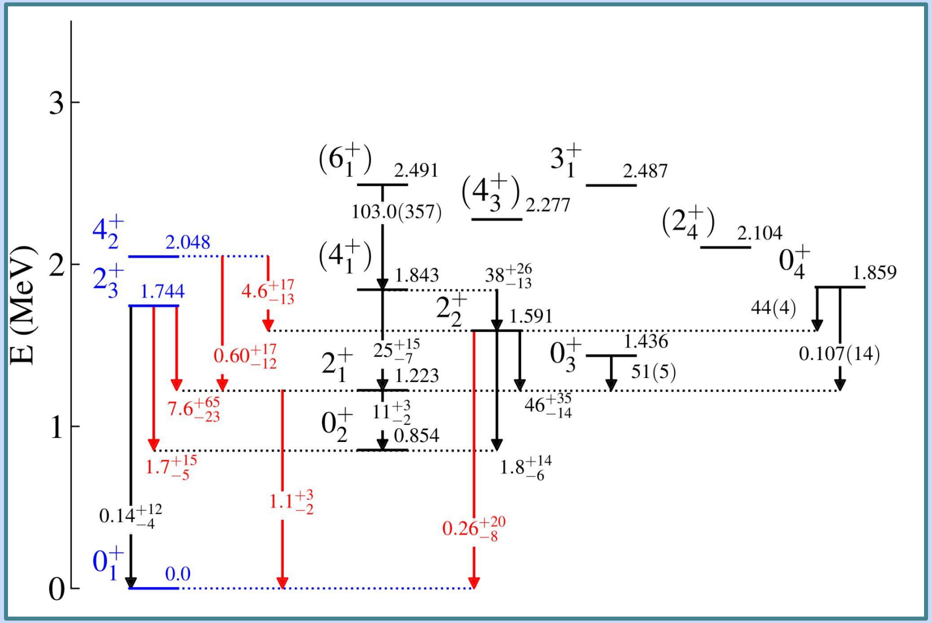
^{96}Zr calc



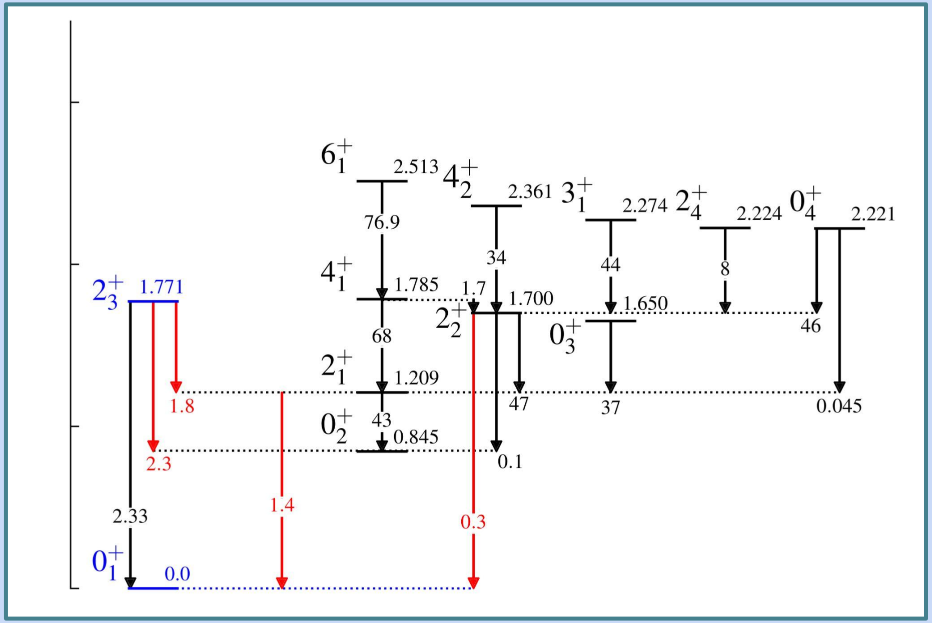
Blue: normal levels
Black: intruder levels
Arrows: *E2* transitions

Appendix
IQPT region

^{98}Zr exp

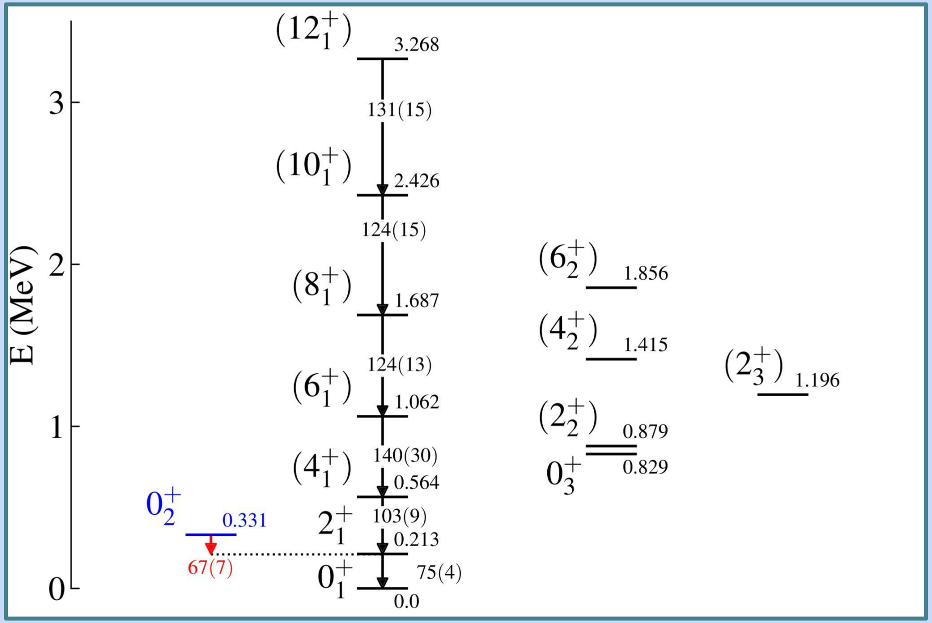


^{98}Zr calc

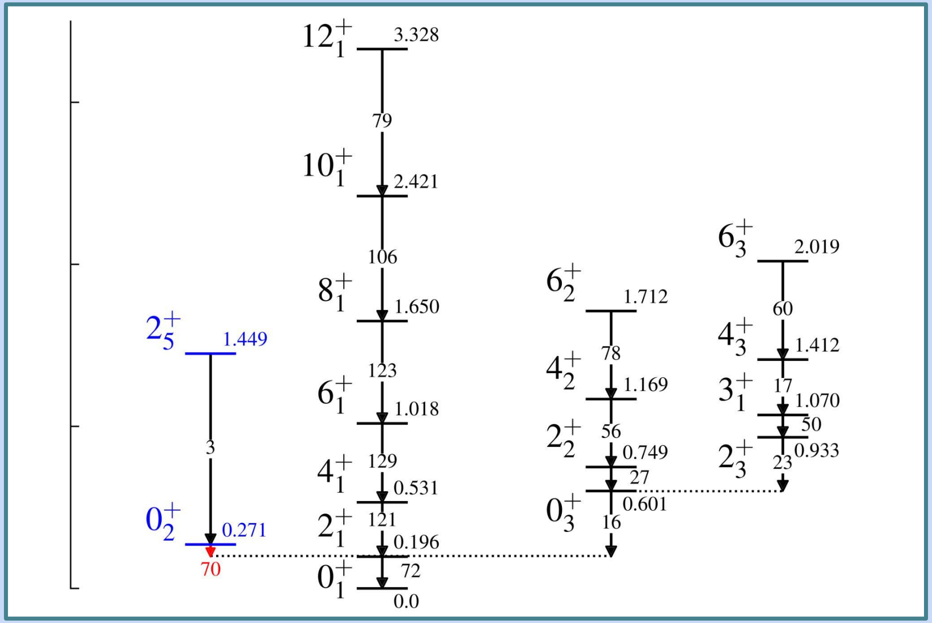


Appendix
IQPT region

^{100}Zr exp

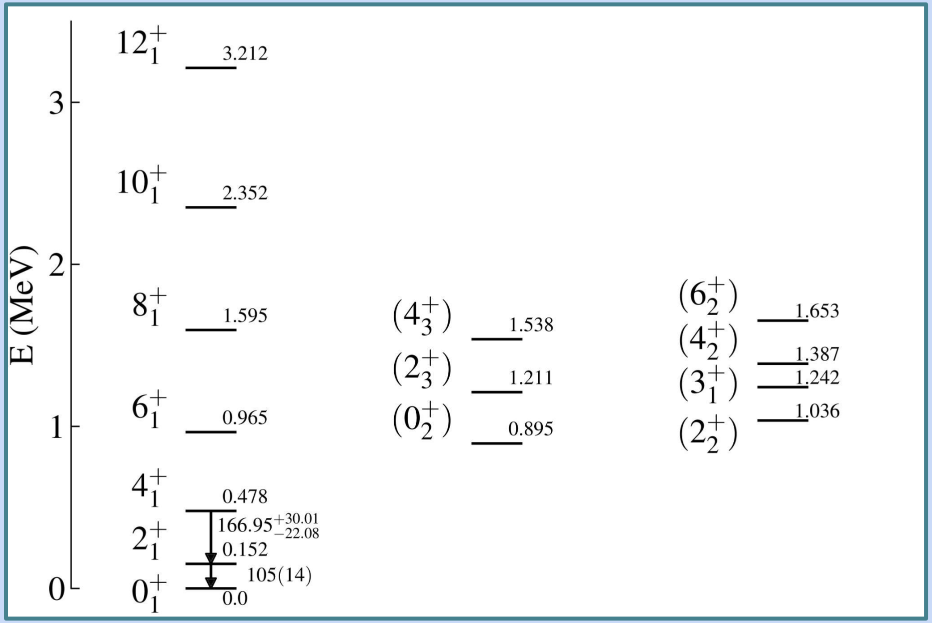


^{100}Zr calc

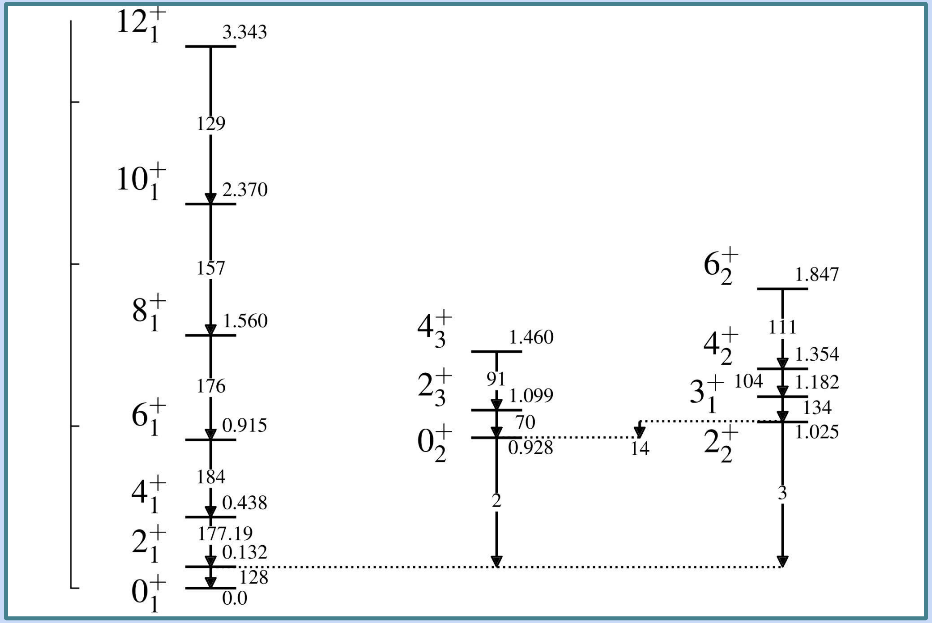


Appendix
IQPT region

^{102}Zr exp



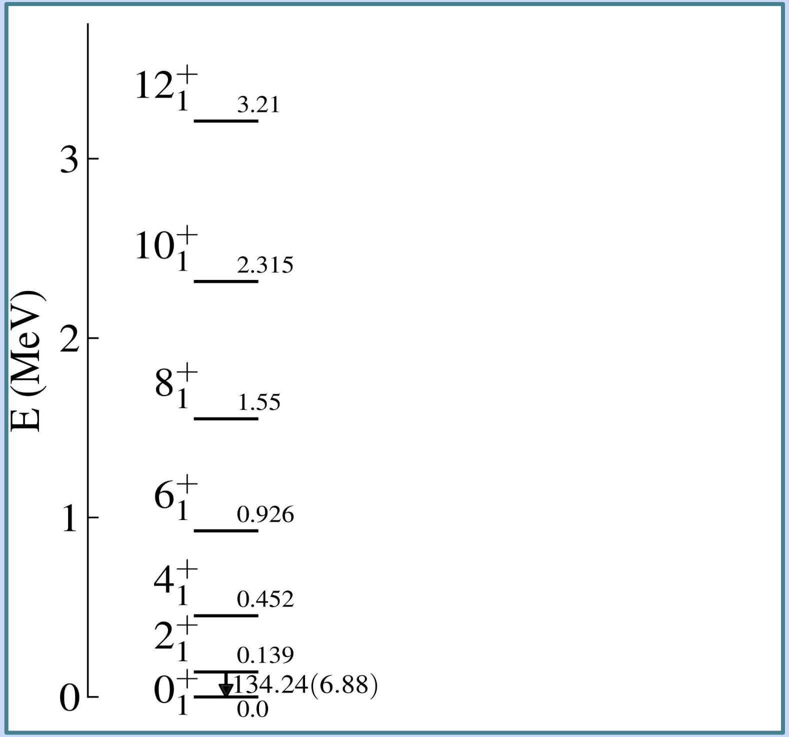
^{102}Zr calc



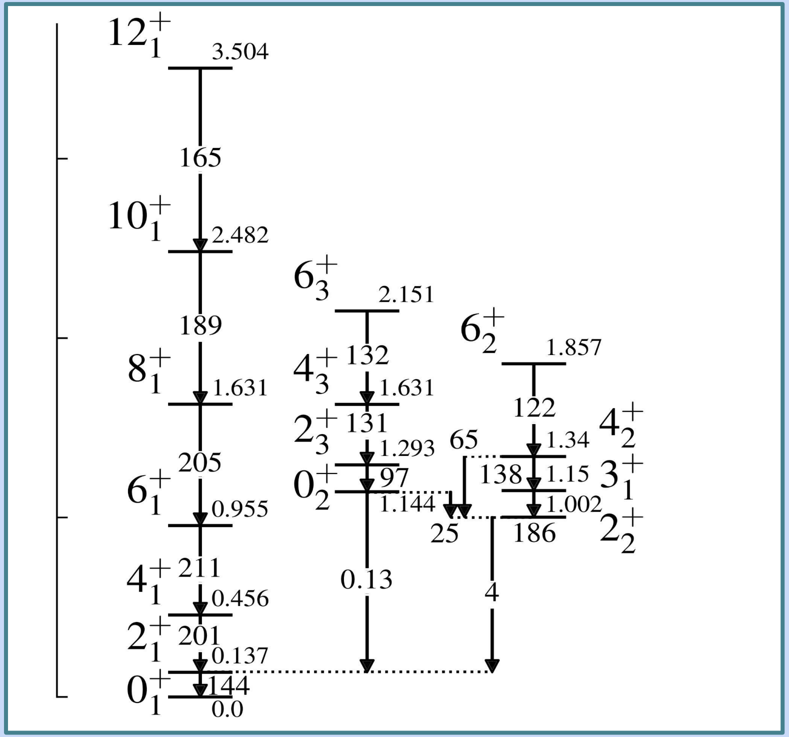
Appendix

SU(3)-SO(6) crossover

^{104}Zr exp



^{104}Zr calc

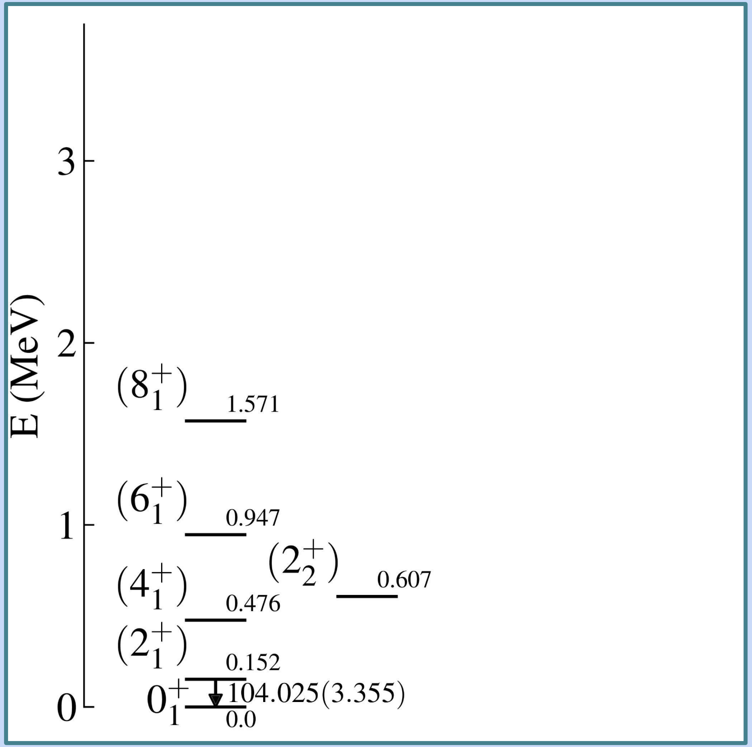


Appendix

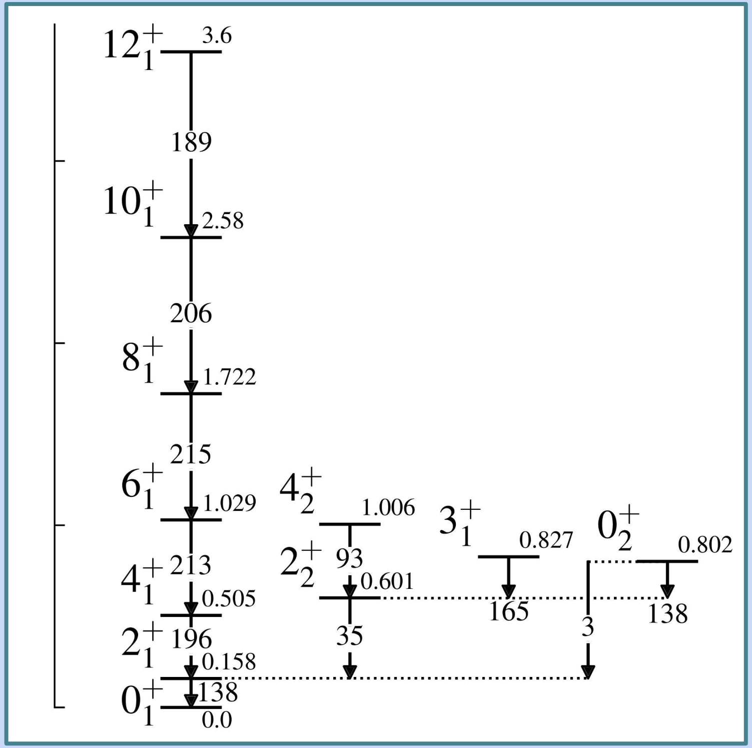
SU(3)-SO(6) crossover

THE HEBREW UNIVERSITY OF JERUSALEM

^{106}Zr exp



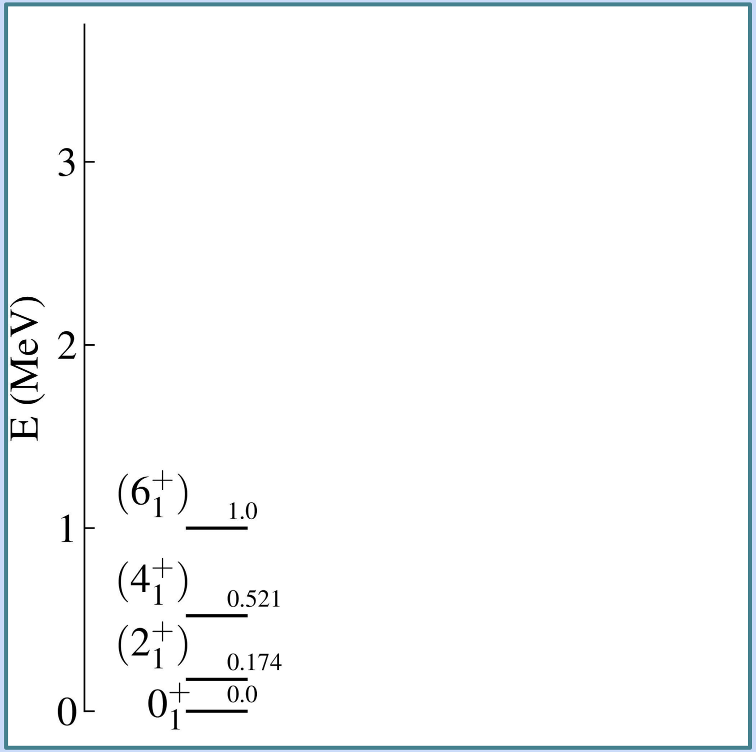
^{106}Zr calc



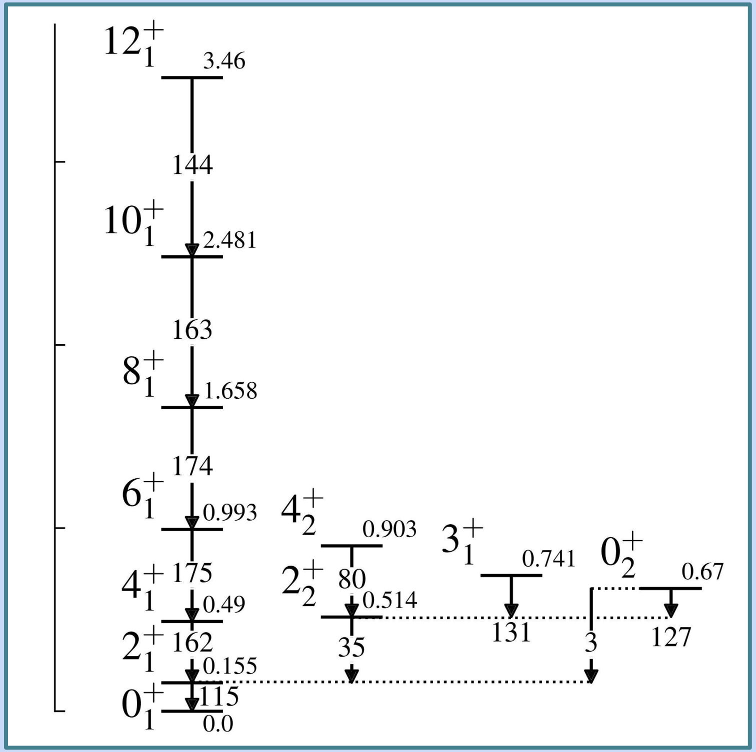
Appendix

SU(3)-SO(6) crossover

^{108}Zr exp

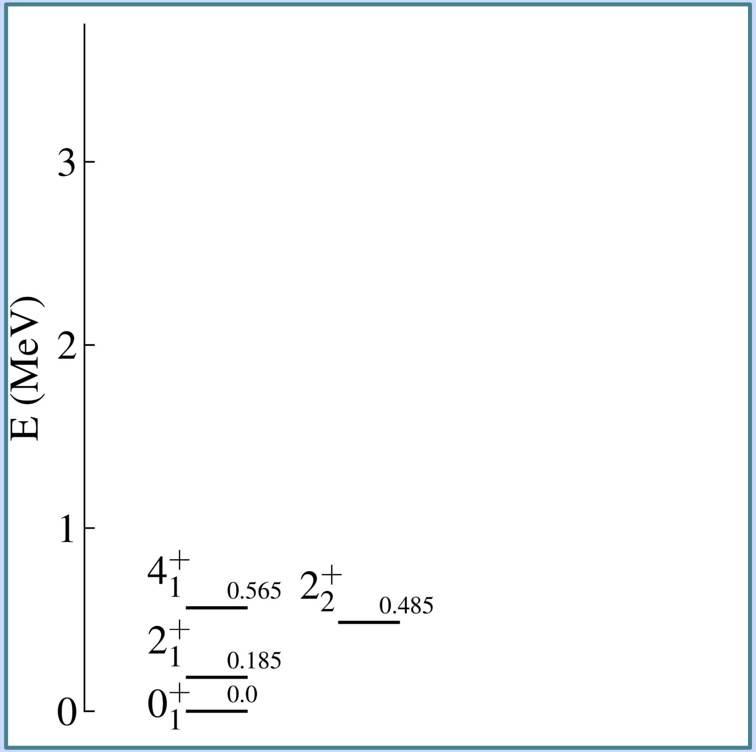


^{108}Zr calc

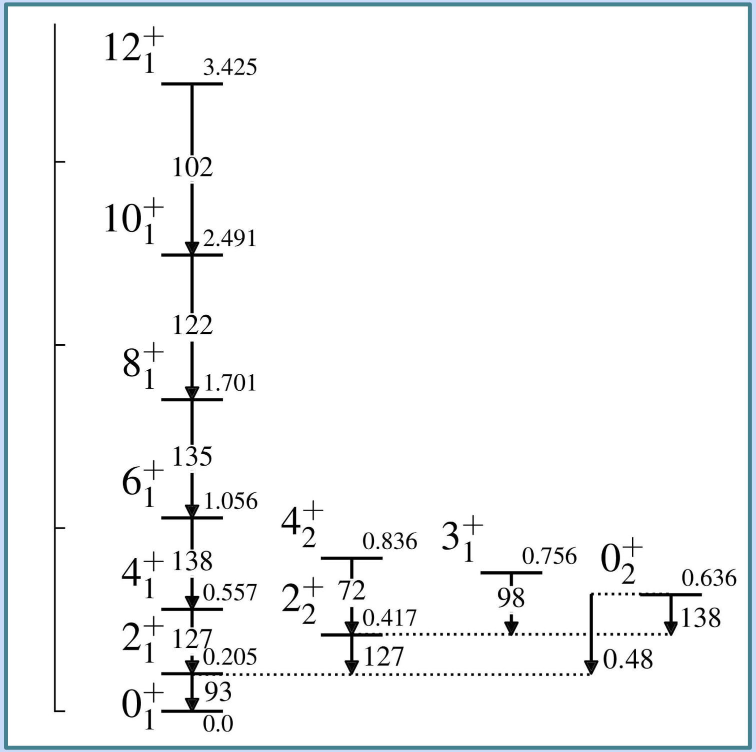


Appendix
SU(3)-SO(6) crossover

^{110}Zr exp

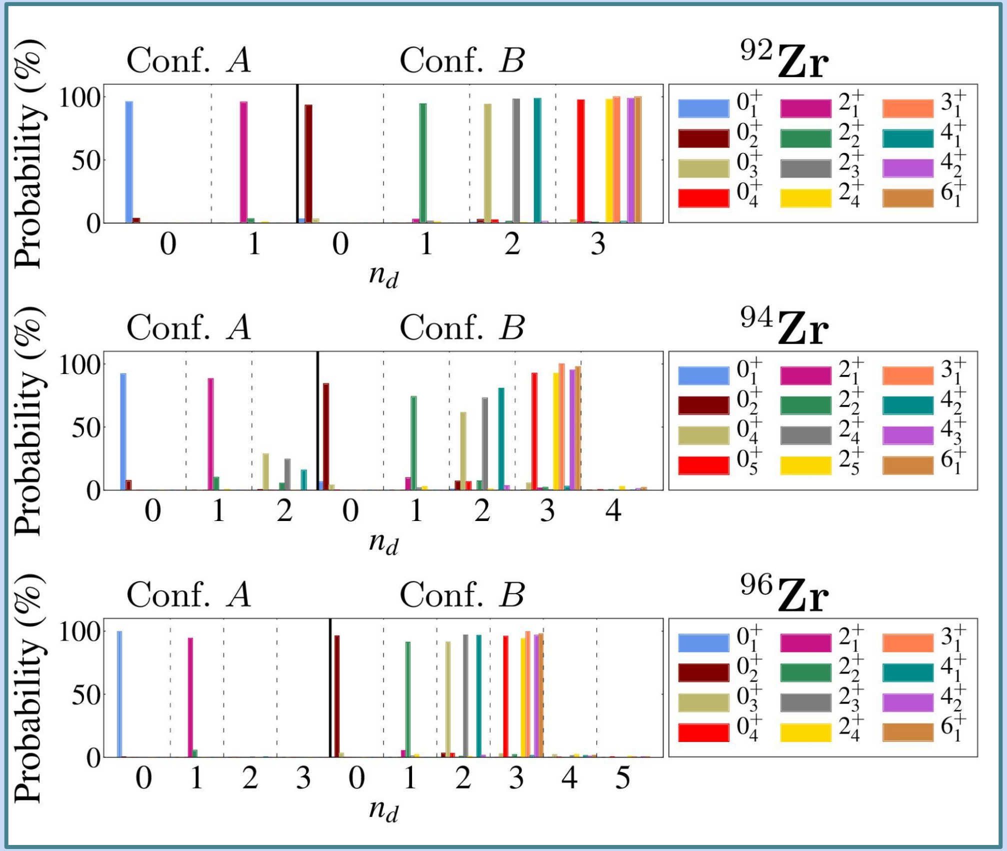


^{110}Zr calc



Appendix

Decomposition



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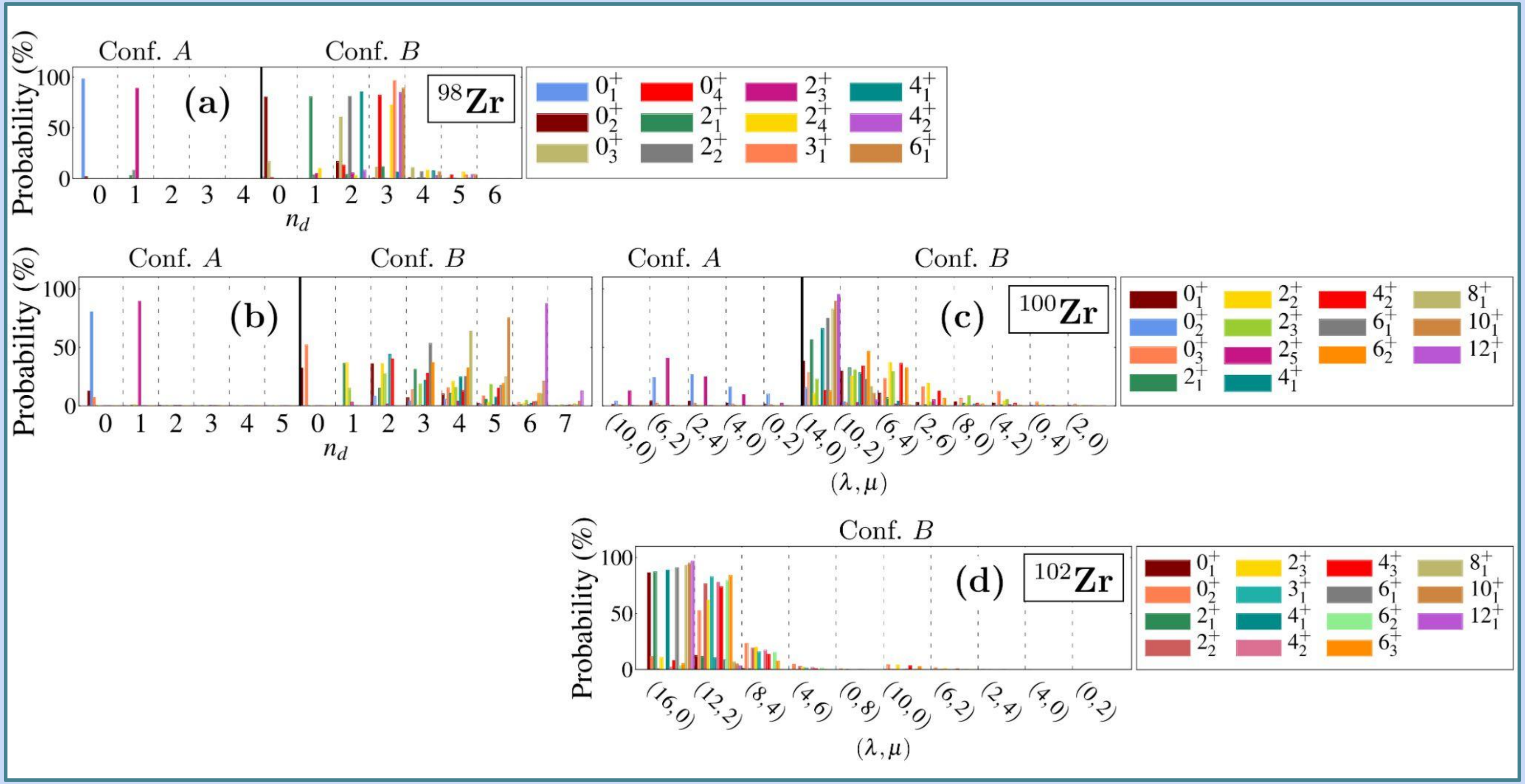
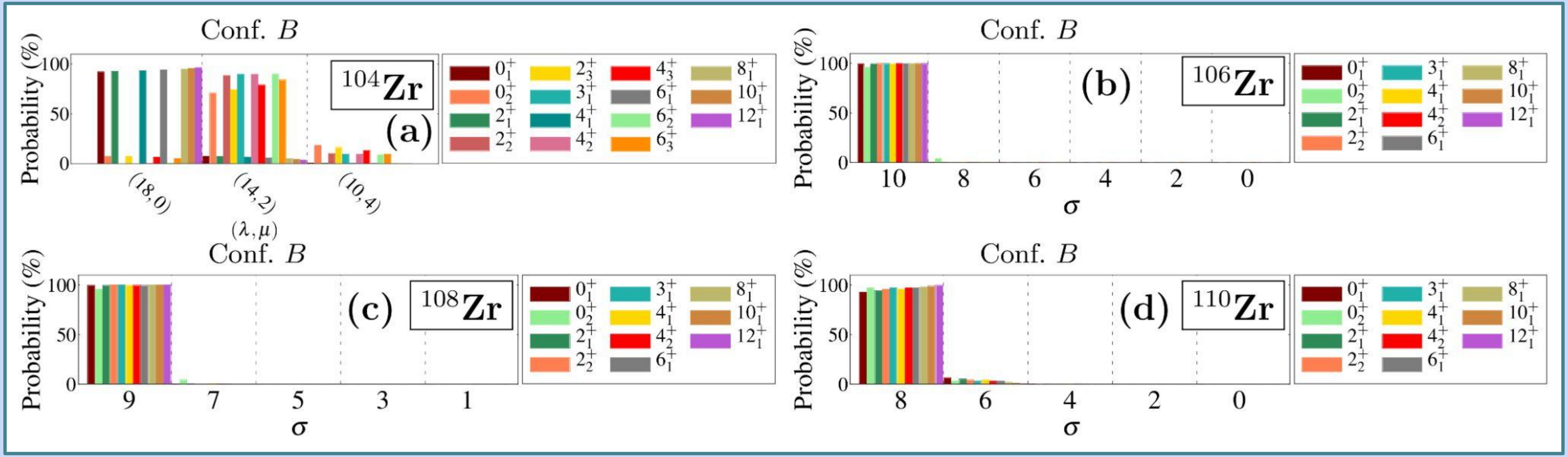
Decomposition

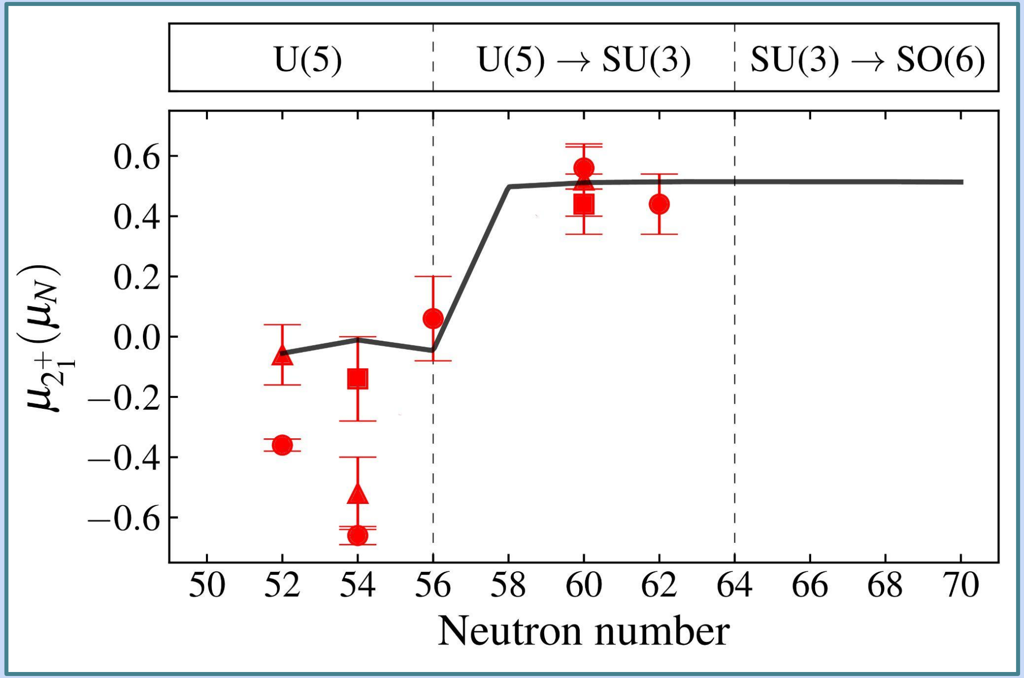


Appendix
Decomposition



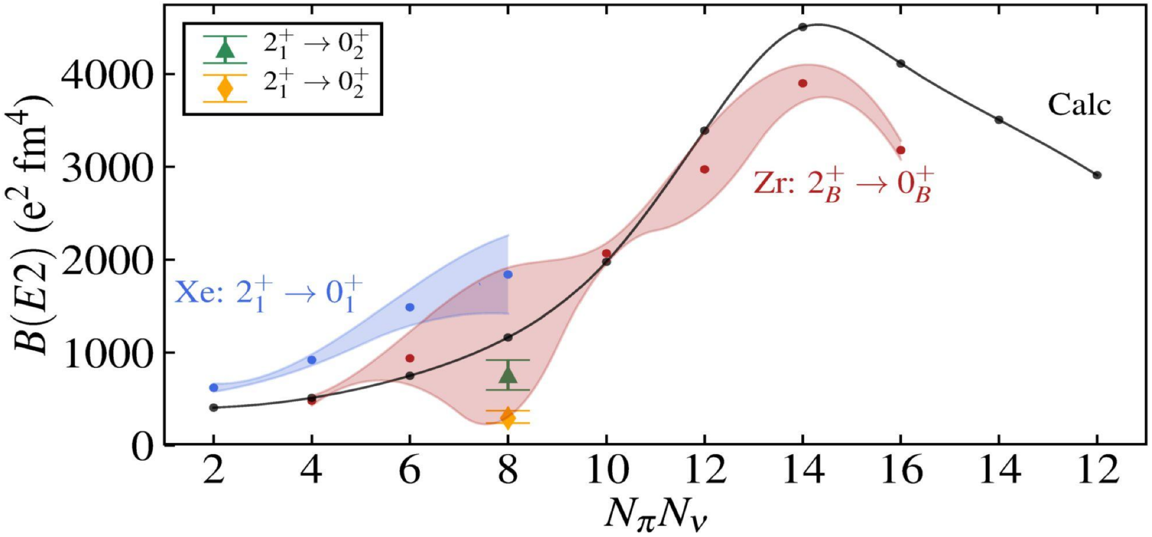
Appendix
Magnetic moments

$$T(M1) = \sqrt{(3/4\pi)}(g^{(A)} L^{(N)} + g^{(B)} L^{(N+2)})$$
$$\mu_L = (a^2 g^{(A)} + b^2 g^{(B)}) L$$



$$g^{(A)} = -\,0.04\,\mu_N$$
$$g^{(B)} = +0.2575\,\mu_N$$

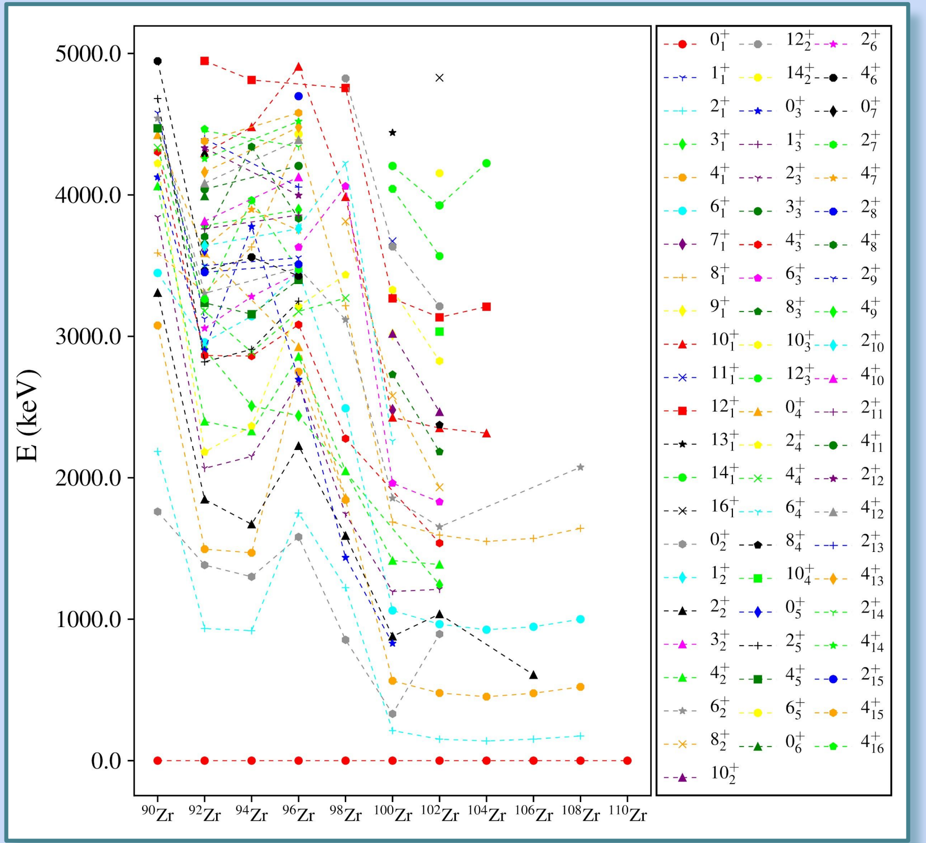
Appendix
^{98}Zr



$N_\pi N_\nu$	2	4	6	8	10	12	14	16	14	12
Zr	^{92}Zr	^{94}Zr	^{96}Zr	^{98}Zr	^{100}Zr	^{102}Zr	^{104}Zr	^{106}Zr	^{108}Zr	^{110}Zr
Xe	^{134}Xe	^{132}Xe	^{130}Xe	^{128}Xe						

Appendix

Zr experimental spectrum



Thank you