Chiral three-body force and spin-orbit splitting in nuclei

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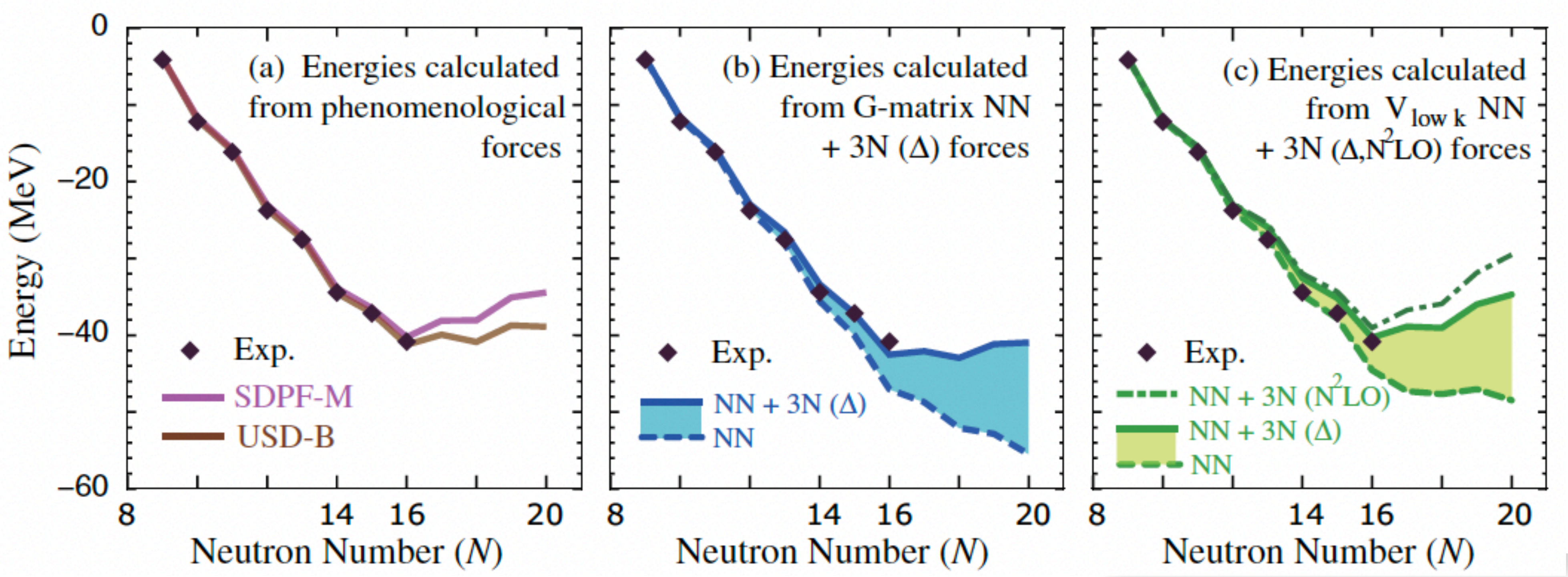
Uncovering the mechanism of chiral three-nucleon force in driving spin-orbit splitting

Physics Letters B 855, 138839 (2024)

First study explicitly including the 3NF in the derivation of the effective SM Hamiltonian

With an *NN* force-only, g.s. energies do not stop to decrease putting the dripline at N = 20, while 3N contributions correct the g.s behavior bringing a significant raise from N = 16 to 18 and then **provide the correct location of the drip line**

Ground-state energies of oxygen isotopes measured from ^{16}O



T. Otsuka et al., PRL 105, 032501 (2010)

3NF & Monopole SM effective interactions

A. Poves, A. Zuker, Phys. Rep. 70 (1981) 235
E. Caurier et al., PRC C 50 (1994) 225
G.. Martínez-Pinedo et al., PRC 55, 1871997
A. Poves et al., NPA 694, 157 (2001)
E. Caurier et al., Rev. Mod. Phys. 77 (2005) 427
S. M. Lenzi et al., PRC 82 (2010) 054301

- Only the monopole component of H_{eff} from realistic NN potential should be modified to obtain results of a quality comparable with that provided by phenomenological

Monopole component

$$H_{\text{mon}} = \sum_{a\tau} \epsilon_{a\tau} N_{a\tau} + \frac{1}{2} \sum_{ab\tau\tau'} \frac{\bar{V}_{ab}^{\tau\tau'} N_{a\tau}(N_{b\tau'} - \delta_{ab}\delta_{\tau\tau'})}{\sum_J \hat{J}^2}$$

Centroid

$$\bar{V}_{ab}^{\tau\tau'} = \frac{\sum_J \hat{J} \langle a\tau b\tau'; J \mid V_{\text{eff}} \mid a\tau b\tau'; J\rangle}{\sum_J \hat{J}}$$

⇒

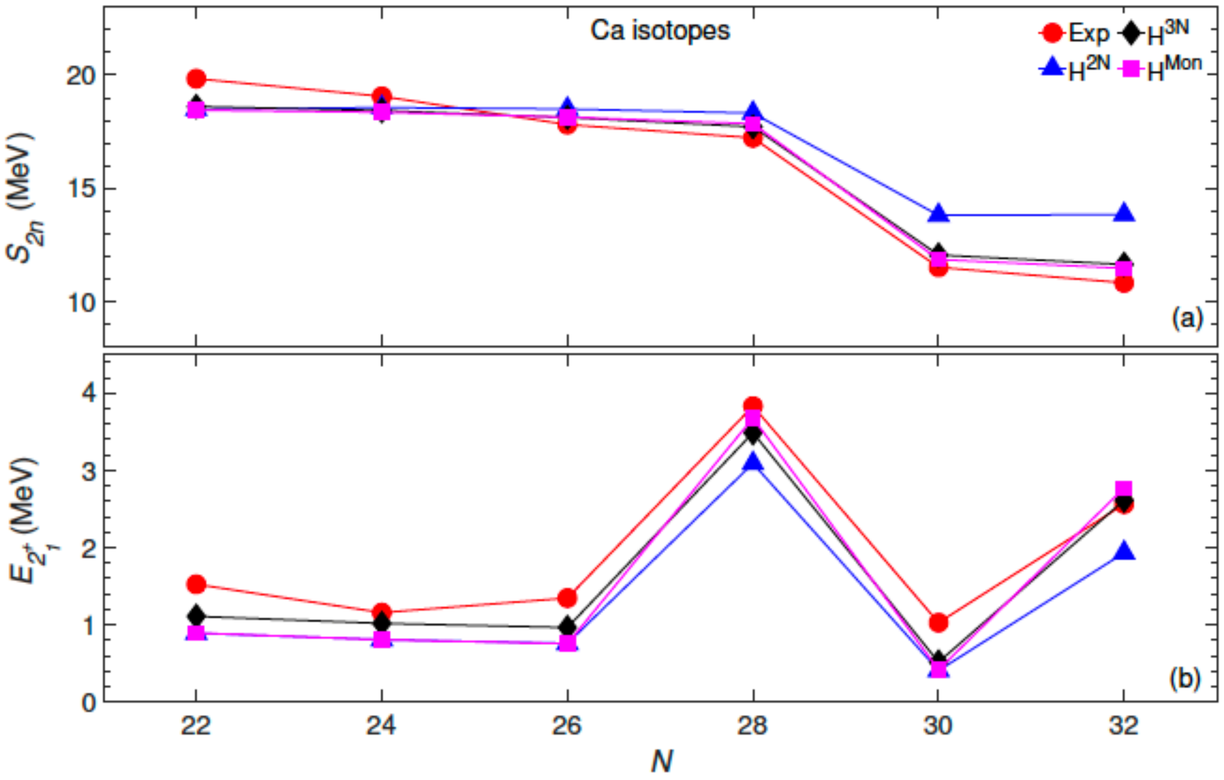
Responsible for the evolution of the SP energies

$$\text{ESPE}(a\tau) = \epsilon_{a\tau} + \sum_{b\tau'} \bar{V}_{ab}^{\tau\tau'} n_b^{\tau'}$$

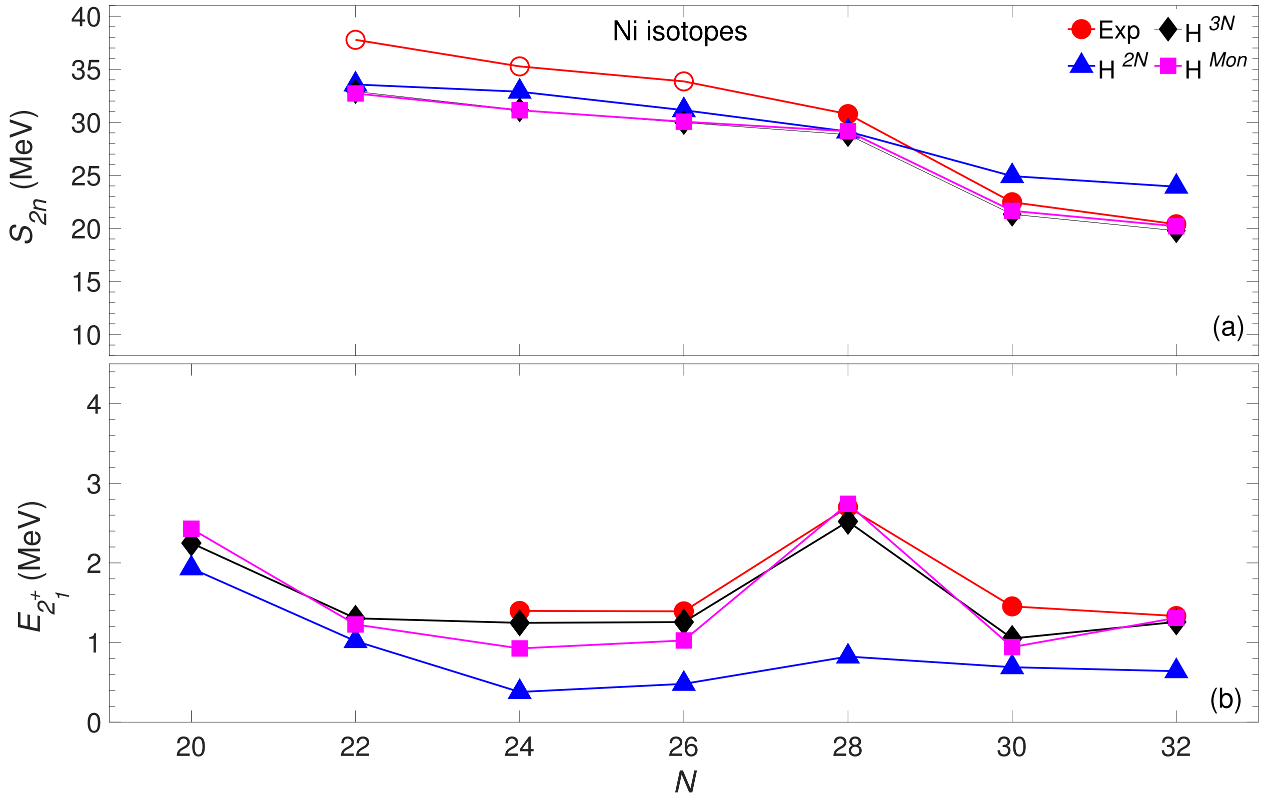
- Deficiencies in centroids are related to the bad saturation and shell formation properties of the NN interaction and can be traced back to the lack of 3N forces

An example from the fp shell: Ca and Ni isotopes

Z. Ma et al., PRC 100. 034324 (2019)
L. Coraggio et al., Prog.Part.Nucl.Phys. 134, 104079 (2024)



- H^{2N} 2NF
- H^{3N} 2NF
- H^{Mon} monopole component of H^{3N} + multipole components of H^{2N}



3NF contribution to the monopole component → substantial expansion of the $0f_{7/2}$- $0f_{5/2}$ SO splitting at N = 28 (~2-3 MeV)

What is the mechanism through which the 3NF influences the shell structure evolution and in particular the spin-orbit splitting?

Nuclear potential → NN and 3N forces in terms of chiral EFT → tensorial decomposition of the 3NF and evaluation of the relative weight of the components

Realistic Shell Model → $NN + 3N$ → many-body perturbation theory → shell-model effetive Hamiltonian: H_{eff}

p shell nuclei → ^{4}He core; proton and neutron model space : $0p_{3/2}$ $0p_{1/2}$

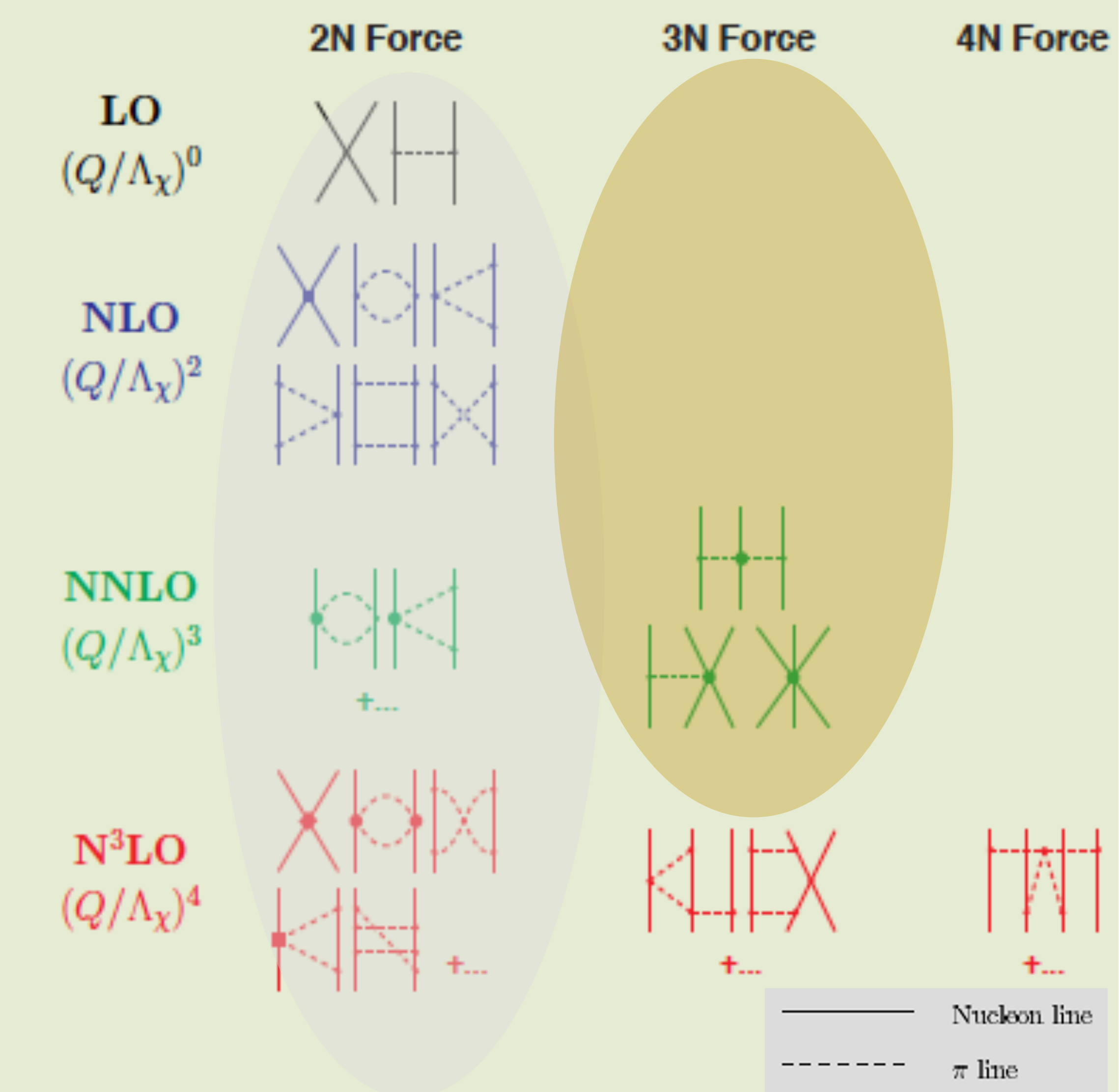
Pioneering work by Andō and Bandō [Prog.Theor. Phys. 66, (1981) 227]→ SO splitting in ^{16}O and ^{40}Ca using the rank-1 tensor component of Fujita–Miyazawa and Tucson–Melbourne 3NFs.

Chiral potentials derived in ChPT

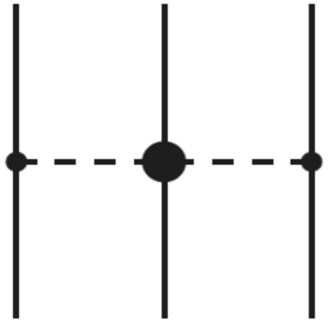
- Nucleons interact via pion exchanges and short-range contact interactions. The long-range forces are ruled by the symmetries of QCD, while short-range forces - which are not resolved - are absorbed into contacts terms proportional to low-energy constants (LEC)

- Chiral potentials are organized in a systematic low-momentum expansion, where two- and many-body forces are generated on an equal footing

- Most interaction vertices that appear in the 3N force also occur in the *NN* force → consistency requires that the parameters LECs carried by these vertices have the same values for *NN* and 3N terms



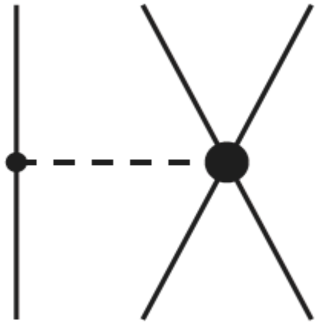
3N chiral potential at NNLO



two-pion exchange

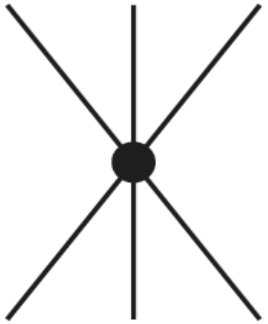
No new LEC with respect to those appearing in the expression of the N^{3}LO 2NF,

(c_1 c_3 c_4)



one-pion exchange + NN-contact interaction

New LEC parameter: c_D



pure 3N-contact interaction

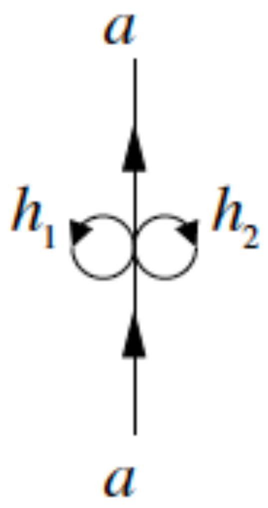
New LEC parameter: c_E

c_D and c_E may be fixed by an optimal over-all fit of the properties of light nuclei
[see for instance P. Navrátil et al., PRL 99 (2007) 042501]

Normal-ordered decomposition of the 3N component of H

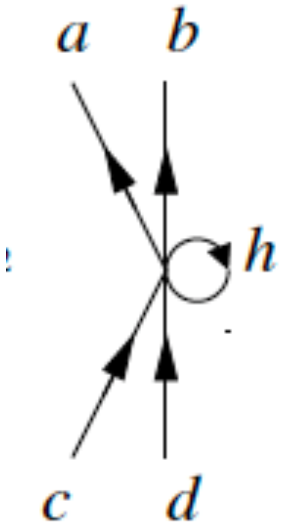
Starting from a reference state (*e.g.* g.s. represented by a Slater determinant) and using the Wick theorem, the three-body component of the nuclear Hamiltonian can be re-arranged into a sum of zero-, one-, two-, and three-body terms → only normal-ordered one- and two-body parts of 3N forces are included

normal-ordered one-body part of the 3N force



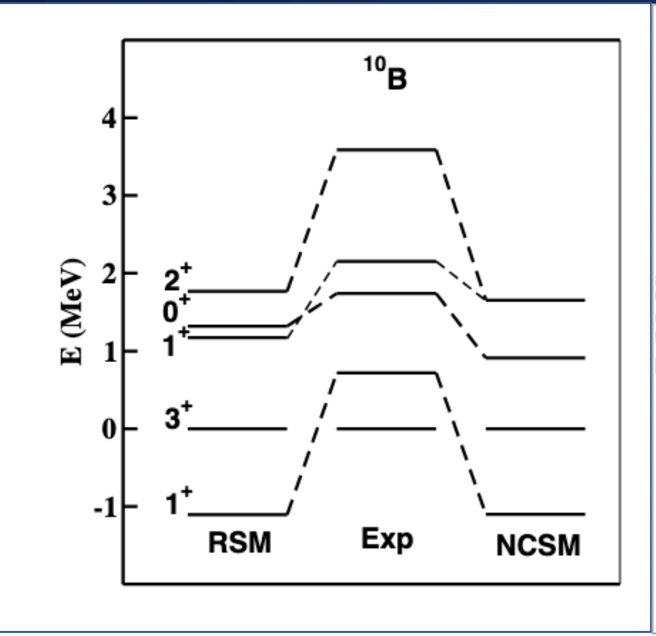
$$\epsilon_a^{(3NF)} = \sum_{\substack{h_1,h_2 \\ J_{12}J}} \frac{\hat{J}^2}{2\hat{j}_a^2} {}_A\langle (h_1,h_2), a; J_{12}J \, | V_{3N} | \, (h_1,h_2), a; J_{12}J \rangle_A$$

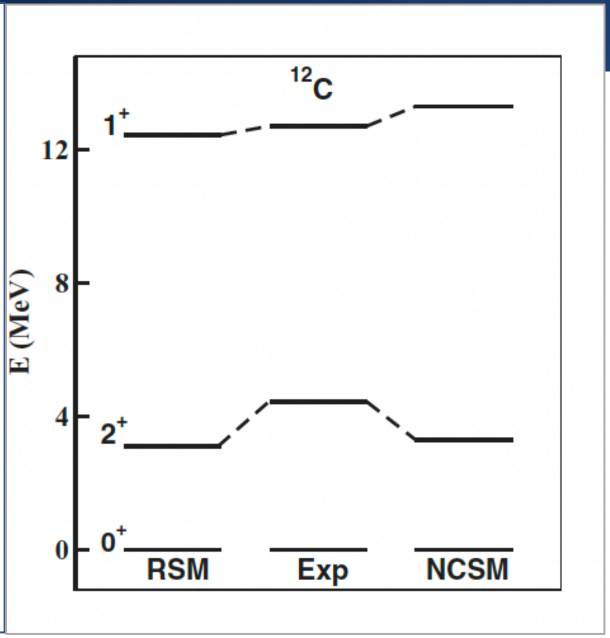
normal-ordered two-body part of the 3N force



$$V_{ab,cd,J}^{(3NF)} = \sum_{h,J'} \frac{\hat{J'}^2}{\hat{J}^2} {}_A\langle (a,b), h; JJ' \, | V_{3N} | \, (c,d), h; JJ' \rangle_A$$

p-shell nuclei: ^{10}B and ^{12}C



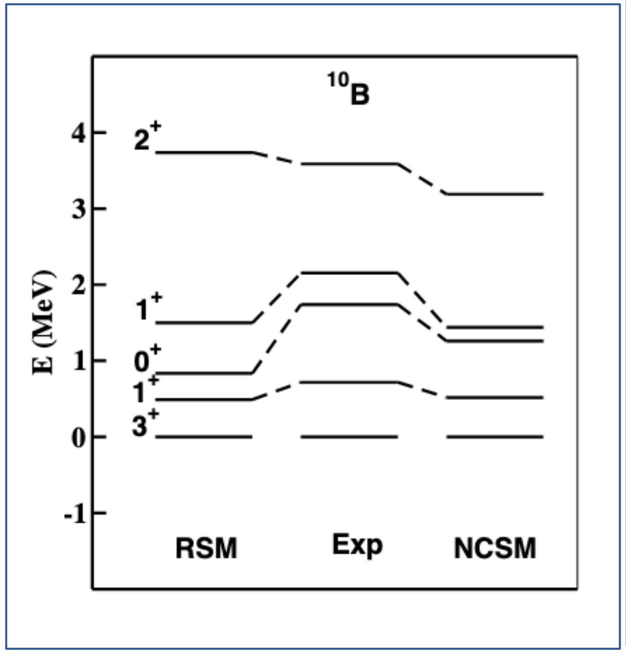


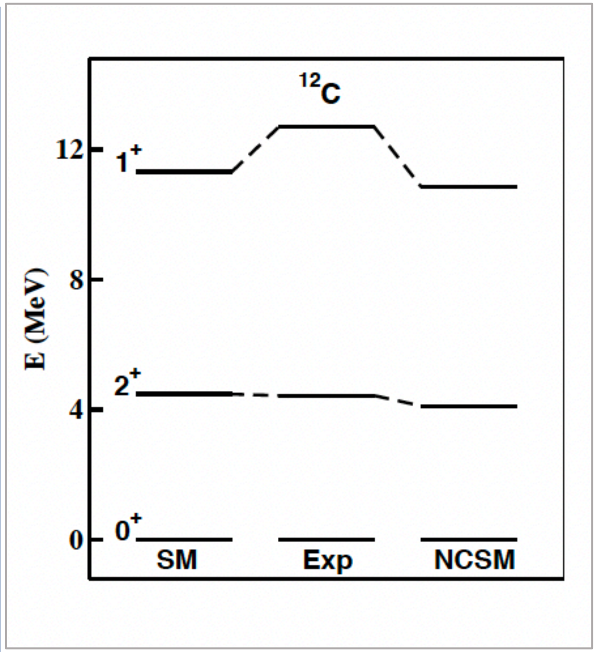
Chiral 2NF @ N^{3}LO + NO 3NF @ N^{2}LO

T. Fukui et al, PRC 98 (2018) 044305

Chiral 2NF @ N^{3}LO

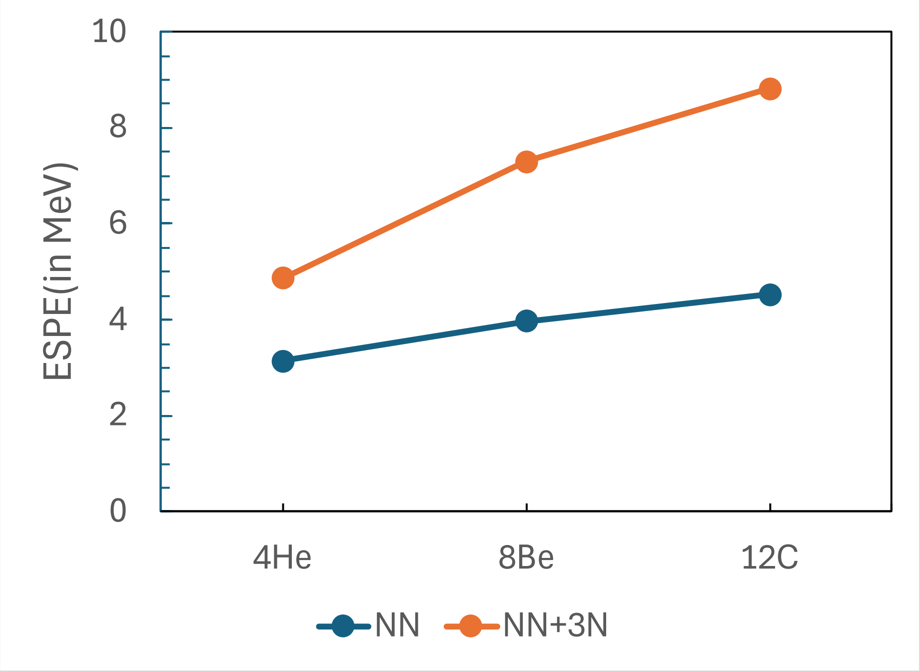
L. Coraggio et a.l, Ann. Phys. (NY) 327 (2012) 2125



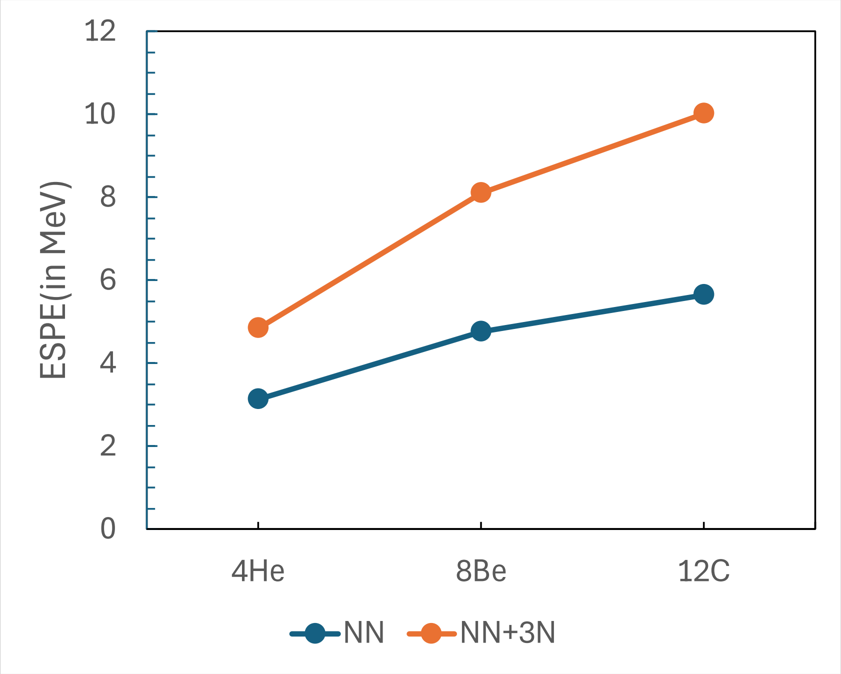


Chiral Force from D. R. Entem, R. Machleidt, PRC 66 (2002) 014002

c_D c_E from P. Navrátil et al., PRL 99 (2007)042501

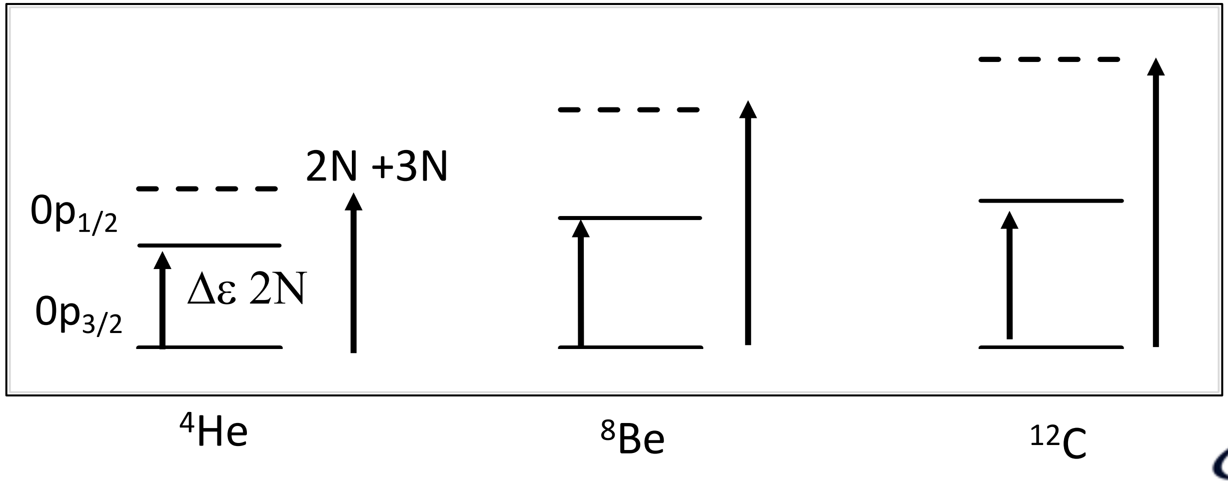


proton $\mathbf{0}\boldsymbol{p}_{1/2}$ ESPE relative to $\mathbf{0}\boldsymbol{p}_{3/2}$

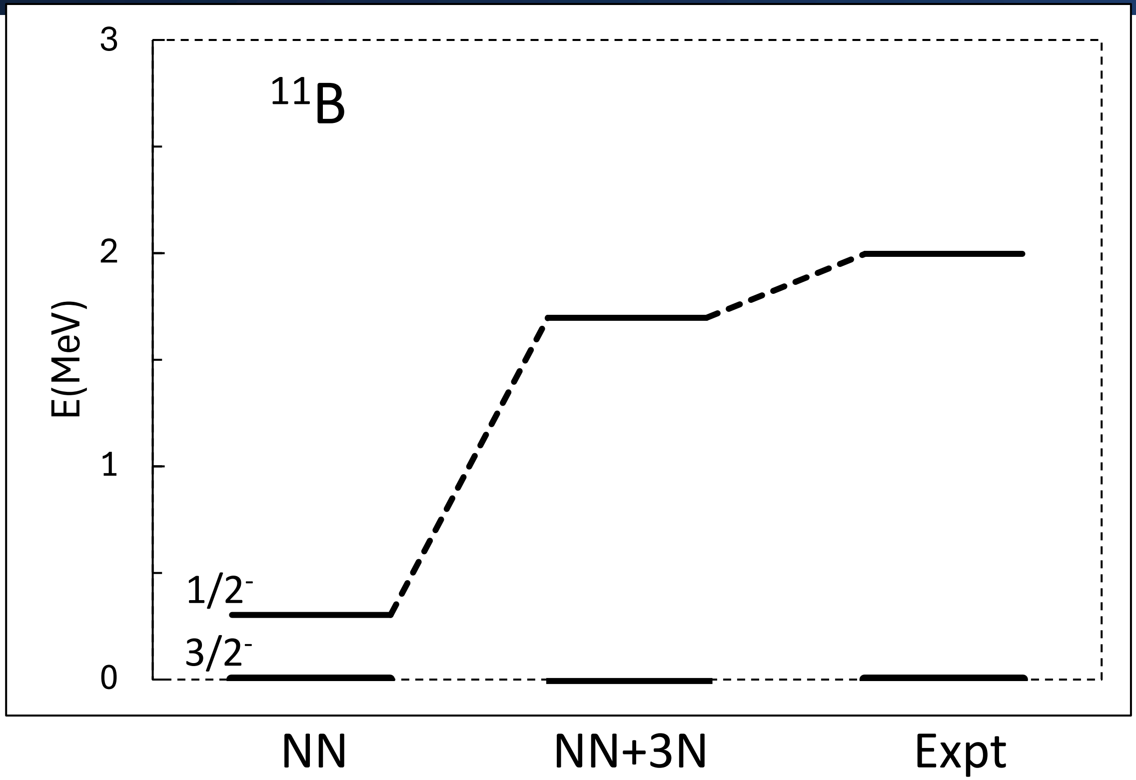


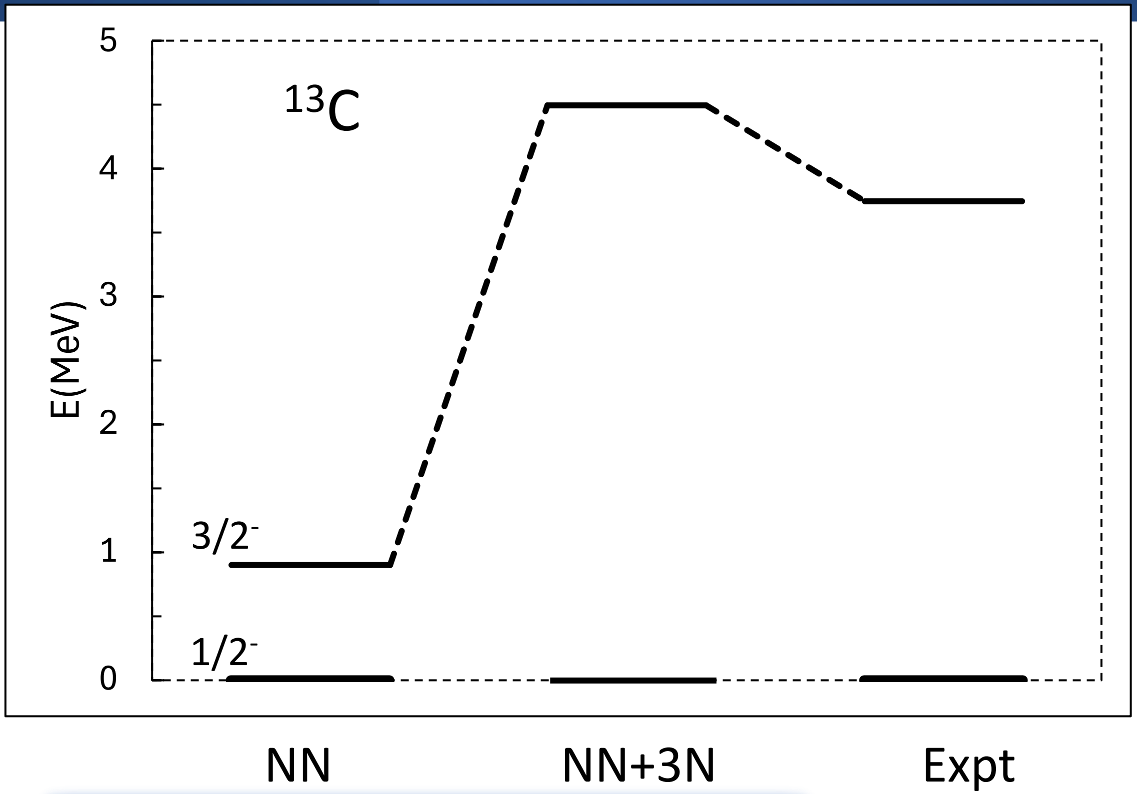
neutron $\mathbf{0}\boldsymbol{p}_{1/2}$ ESPE relative to $\mathbf{0}\boldsymbol{p}_{3/2}$

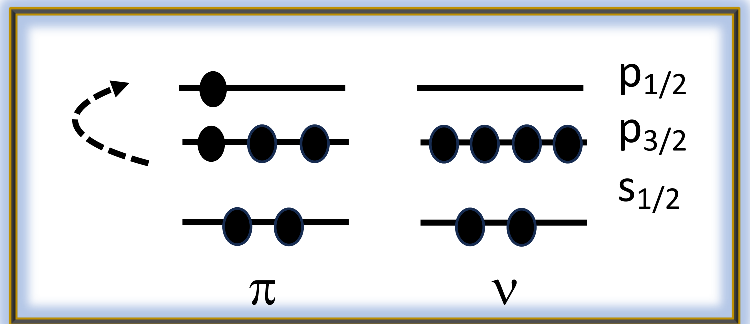
3NF is essential to enlarge the spin-orbit splitting of the 0p orbitals and ensure better closure properties → reflecting on the properties of ^{10}B and ^{12}C



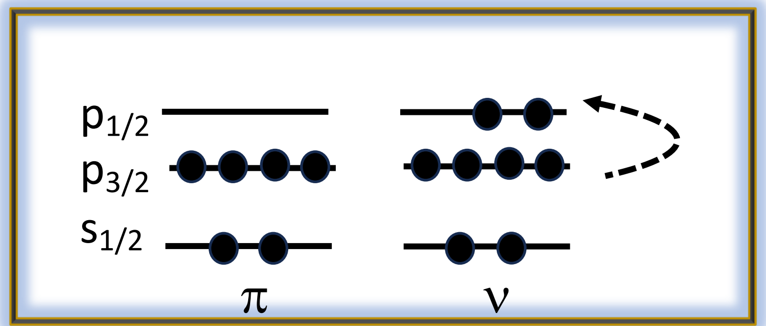
Odd *p*-shell nuclei: ^{11}B and ^{13}C







1st exc state
Ground state



What is the specific mechanism behind the increase in the SO splitting produced by the chiral 3NF? Are there specific components of the 3NF leading to this increase?

3NF can be schematically written as

T. Fukui et al., PLB 855, 138839 (2024)

$$v_{3N}^{(\alpha)} = \sum_{i \neq j \neq k} v^{(\alpha)}(\boldsymbol{\tau}_i, \boldsymbol{\tau}_j, \boldsymbol{\tau}_k) w^{(\alpha)}(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_j, \boldsymbol{\sigma}_k, \boldsymbol{q}_i, \boldsymbol{q}_j)$$

$$\alpha \in \{ct, 1\pi + ct, 2\pi\}$$

$v^{(\alpha)}$ is the isospin part and $w^{(\alpha)}$ represents the spin-momentum dependent part

Formalism developed by Tokuro Fukui

Tensor decomposition

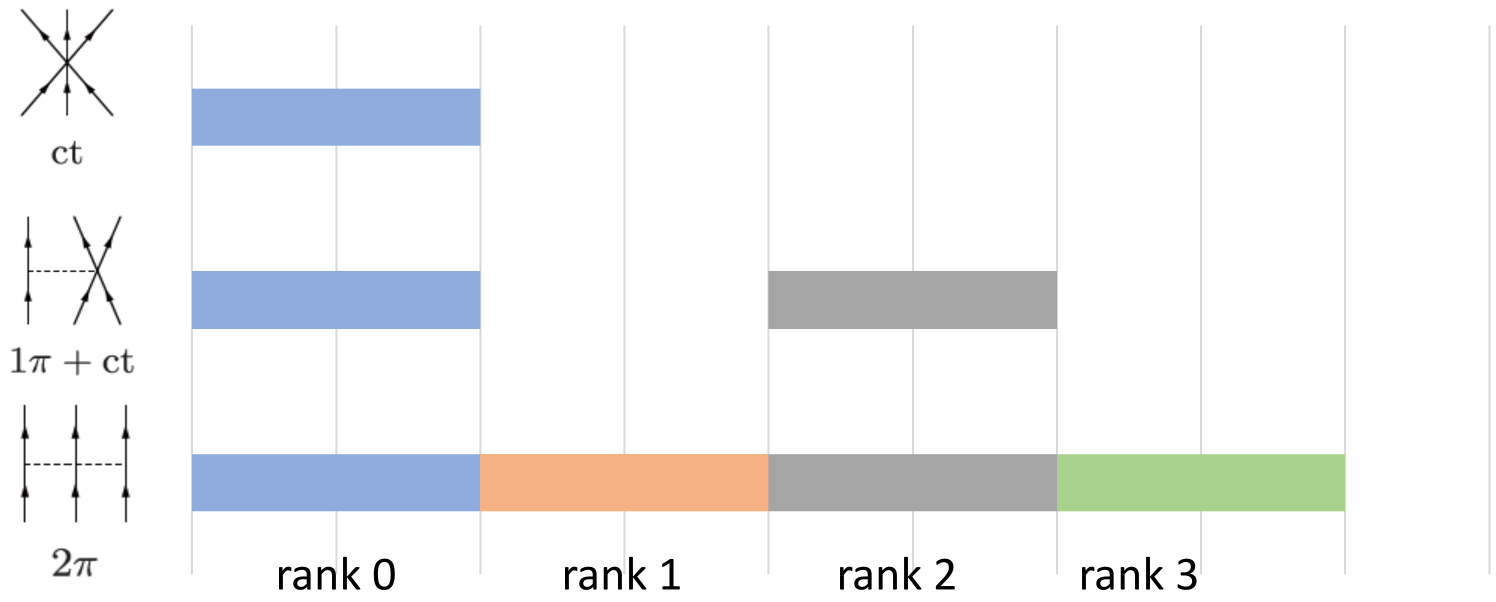
$$w^{(\alpha)}(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_j, \boldsymbol{\sigma}_k, \boldsymbol{q}_i, \boldsymbol{q}_j) = w_{\text{pro}}^{(\alpha)}(q_i, q_j) \sum_{\lambda} \mathcal{O}_{\lambda}^{(\alpha)}(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_j, \boldsymbol{\sigma}_k, \hat{\boldsymbol{q}}_i, \hat{\boldsymbol{q}}_j)$$

$$\mathcal{O}_{\lambda}^{(\alpha)}(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_j, \boldsymbol{\sigma}_k, \hat{\boldsymbol{q}}_i, \hat{\boldsymbol{q}}_j) = A_{\lambda} \left[\mathcal{M}_{\lambda}^{(\alpha)}(\boldsymbol{\sigma}_i, \boldsymbol{\sigma}_j, \boldsymbol{\sigma}_k) \otimes \mathcal{N}_{\lambda}^{(\alpha)}(\hat{\boldsymbol{q}}_i, \hat{\boldsymbol{q}}_j) \right]_{00}$$

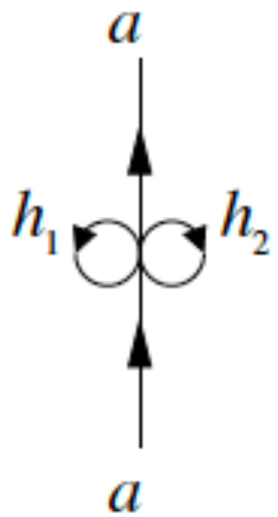
$\mathcal{O}_{\lambda}^{(\alpha)}$ is expressed as the coupling of the rank–λ spin tensor operator $\mathcal{M}_{\lambda}^{(\alpha)}$ with the corresponding rank tensor operator in the momentum space $\mathcal{N}_{\lambda}^{(\alpha)}$

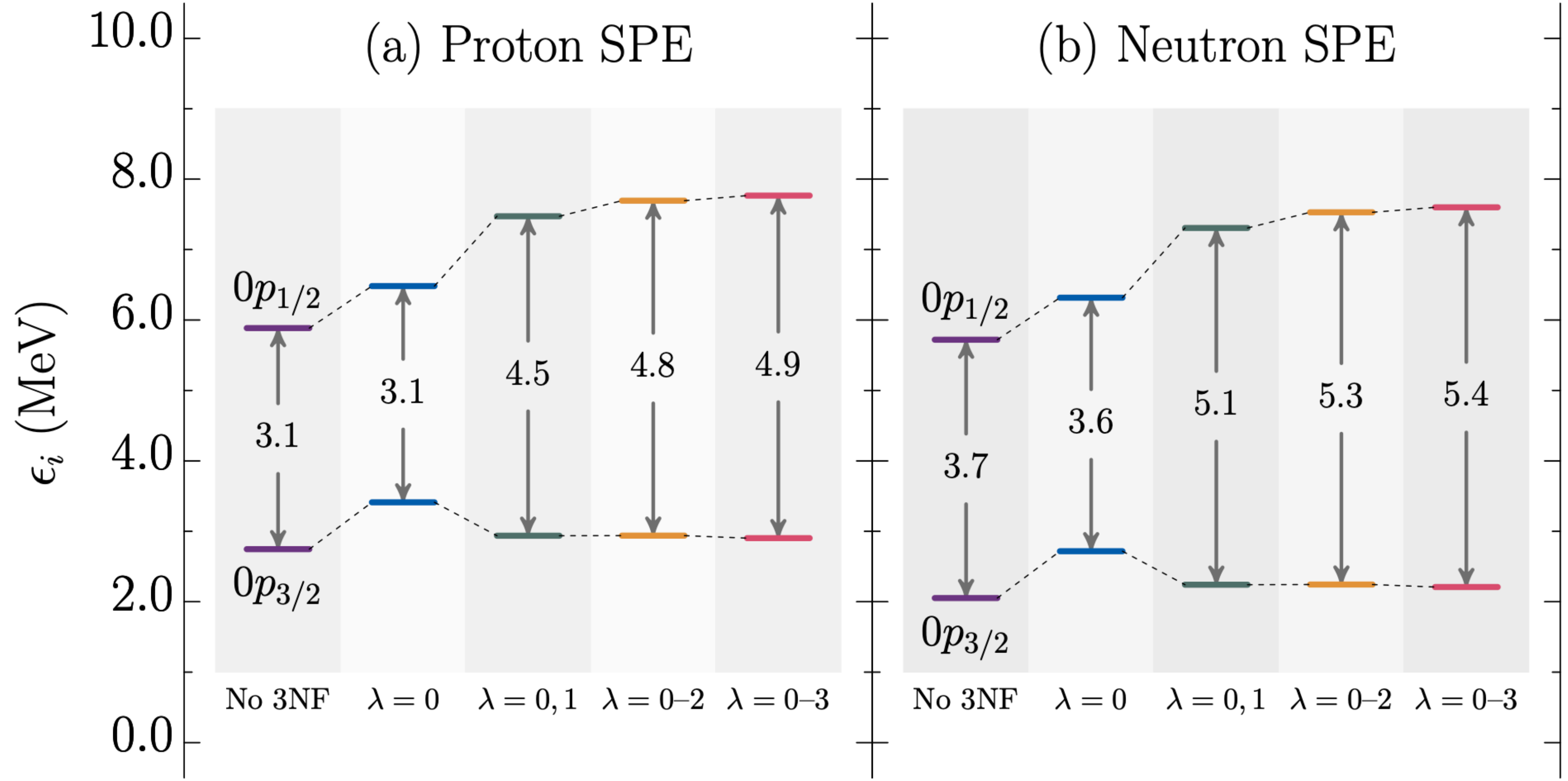
Classification of the chiral 3NF at N2LO by the number of exchanged pions and rank of the irreducible tensor

	ct	1π + ct	2π		
			c_1	c_3	c_4
λ	0	0, 2	0–2	0–2	0–3



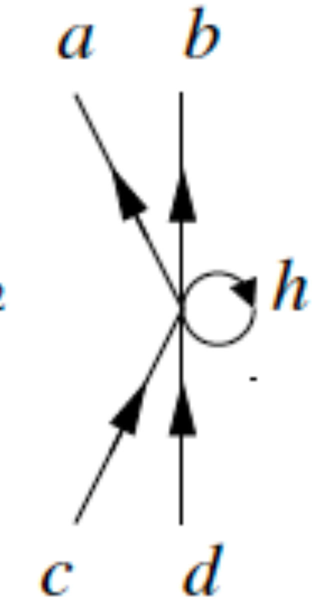
SPEs of the $0p_{3/2}$ and $0p_{1/2}$ orbitals: test of the NO1B term





The SO splitting induced by the chiral 3NF is primarily governed by its rank-1 component arising from the 2π-exchange term, suppling an attractive and repulsive contribution to the $0p_{3/2}$ and $0p_{1/2}$ energies; rank-0 component leaves the gap unchanged, while rank-2 and rank-3 components yield smaller contributions (13% and 2%)

NO2B of 3NF: monopole component & ESPE



$$\bar{V}_{ab}^{\tau\tau'} = \frac{\sum_J \hat{J} \langle a\tau b\tau'; J \mid V_{\text{eff}} \mid a\tau b\tau'; J \rangle}{\sum_J \hat{J}}$$

$$\text{ESPE}(a\tau) = \epsilon_{a\tau} + \sum_{b\tau'} \bar{V}_{ab}^{\tau\tau'} n_b^{\tau'}$$

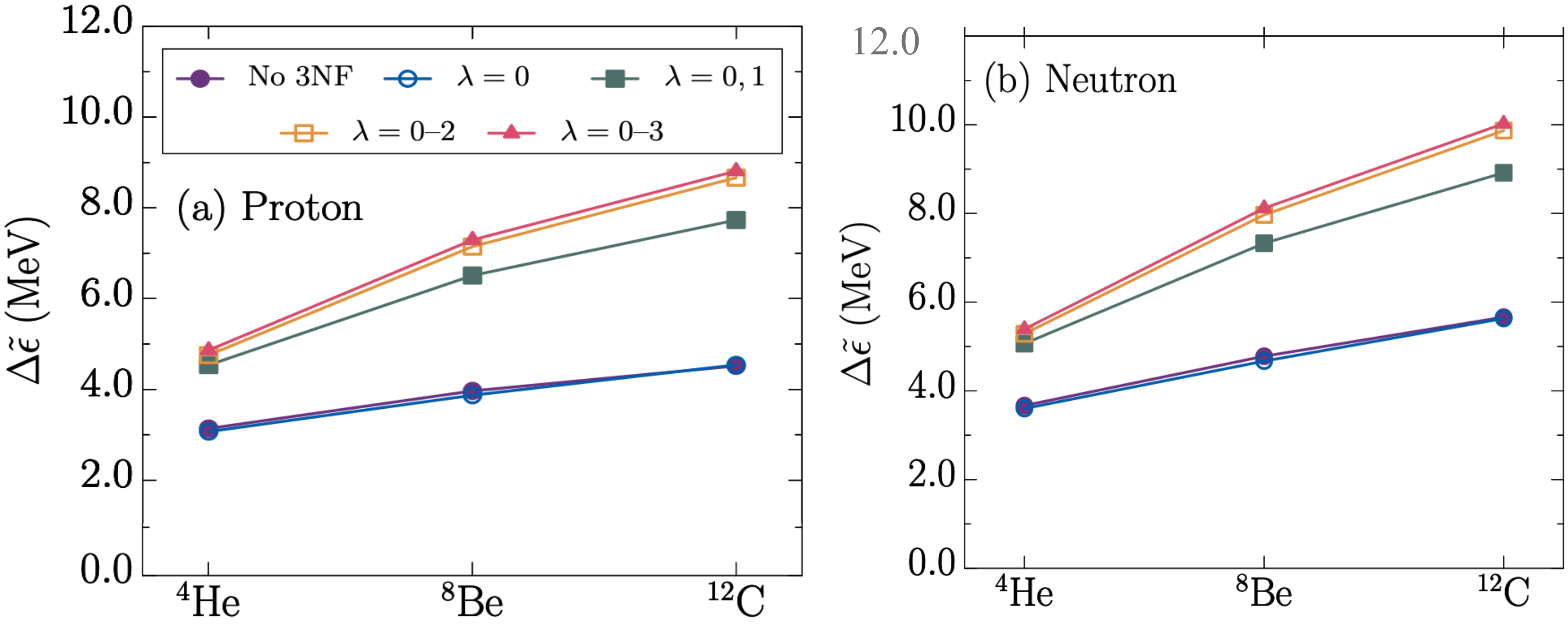
ππ/νν component

	full	rank-0	rank-1	rank-2	rank-3
$p_{3/2}\ p_{3/2}$	0.062	0.379	-0.293	-0.008	-0.017
$p_{3/2}\ p_{1/2}$	0.544	0.240	0.177	0.121	0.006
$P_{1/2}\ p_{1/2}$	1.582	0.402	0.935	0.244	0.002

πν component

	full	rank-0	rank-1	rank-2	rank-3
$p_{3/2}\ p_{3/2}$	-0.057	0.061	-0.080	-0.024	-0.013
$p_{3/2}\ p_{1/2}$	0.404	0.063	0.153	0.175	0.012
$P_{1/2}\ p_{1/2}$	0.755	0.184	0.442	0.124	0.005

$0p_{1/2}$ - $0p_{3/2}$ ESPE spacing: direct test of the NO2B term



Similarly to the SPEs, the primal impact comes from the rank-1 3NF of the 2π-exchange process, accounting for almost 75% of the gap enhancements by the whole 3NF. The effect of the rank-2 3NF (about 20%) is smaller yet appreciable.

Summary & perspectives

- 3NFs provide an overall repulsive contribution in determining the location of the neutron dripline and the evolution of the shell structure

- 3NFs affect essentially the monopole component of the shell-model Hamiltonian, leading to substantial changes in the energy spacings between SP orbitals and in particular between SO partners

- A crucial role in the SO splitting is played by the vector (rank-1) component arising from the the 2π– exchange term of the chiral 3NF → the SO splitting is not affected by the choice of the contact LECs (c_D and c_E) appearing at the level of 3NF

To strengthen our conclusions

- Extension to heavier mass regions, as sd- and fp-nuclei,

- Impact of the different rank components of 3NFs on the so-called intruder levels ($0f1p0g_{9/2}$ and $0g_{7/2}1d2s0h_{11/2}$ shells)

- Applications of the tensor decomposition of 3NFs to different approaches beyond the standard shell model, as the Gamow shell model

Collaborators

- Luigi Coraggio (UNICampania & INFN-NA)
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- Yuanzhuo Ma (South China Normal University)
- Furong Xu (Peking University)

Thanks for your attention!