Recent Progress on Halo Nuclei in Deformed Relativistic Hartree-Bogoliubov theory in continuum

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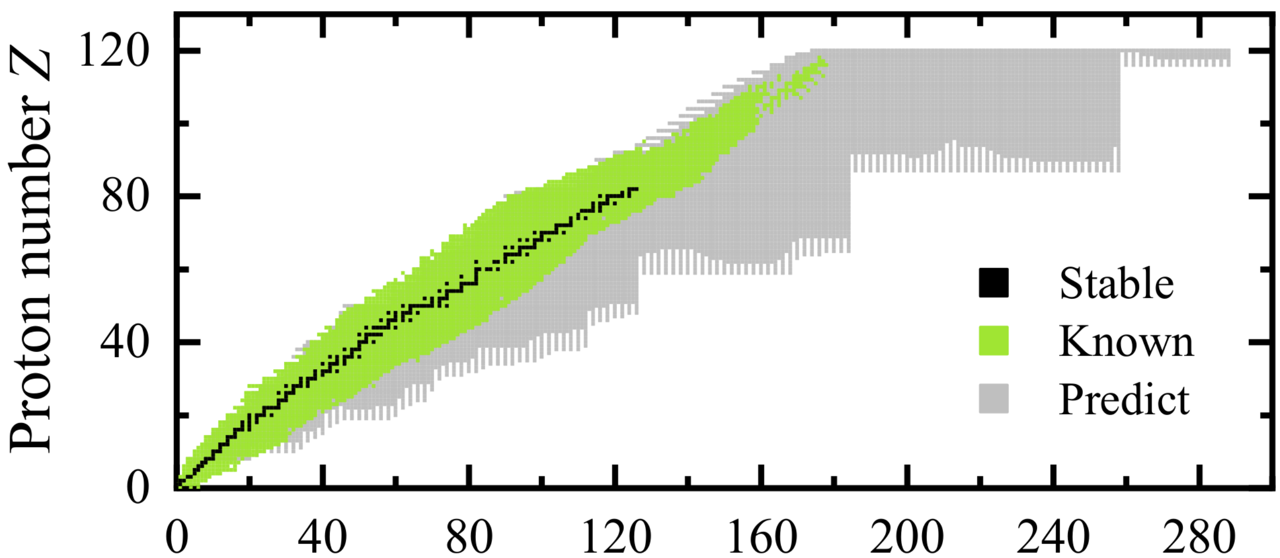
Outline

- Introduction
- Theoretical framework
- Nuclear magnetism and halo in ^{31}Ne
- Possible triaxiality and halo in ^{22}Al
- Summary

Exotic nuclei

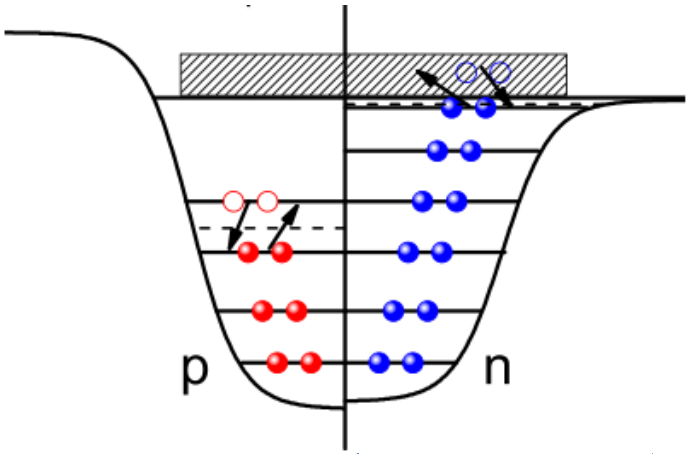
- The study of exotic nuclei far away from β-stability line is one of the most important frontiers in nuclear physics.
  - Neutron/proton halo   Tanihata *et al.*, PRL 55, 2676 (1985)
  - Change in magic number   Ozawa *et al.*, PRL 84, 5493 (2000)
  - Pygmy resonance   Adrich *et al.*, PRL 95, 132501 (2005)
  - ……



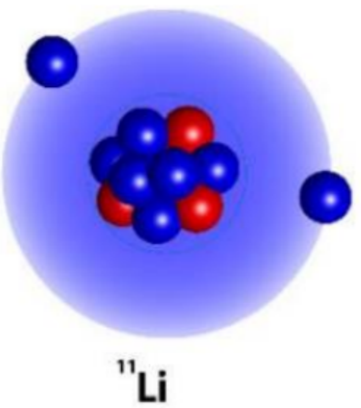
National Nuclear Data Center (NNDC)
Xia *et al.*, ADNDT 121,1(2018)

Halo and continuum

- In halo nuclei, the system is weakly bound, and the Fermi energy is close to the continuum threshold:



Meng *et al.*, PPNP 57, 470 (2006)



From X.H. Wu

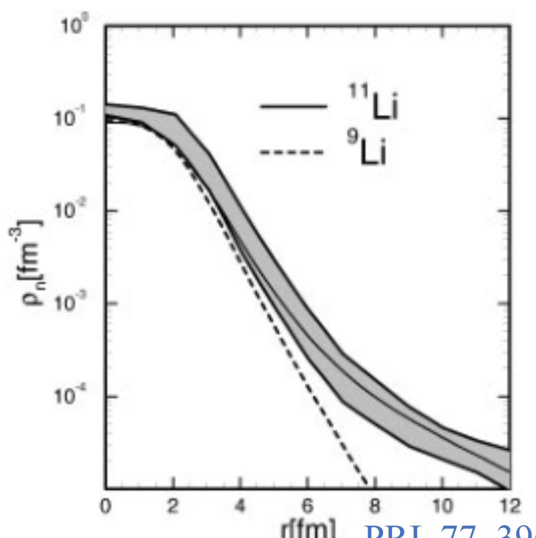
- The pairing interaction can scatter nucleons from bound states to the resonant states in the continuum.
  —— Effects of pairing correlations and continuum
- The density could become more diffuse.
  —— Asymptotic behaviors of nuclear density far away from the center.

RCHB theory

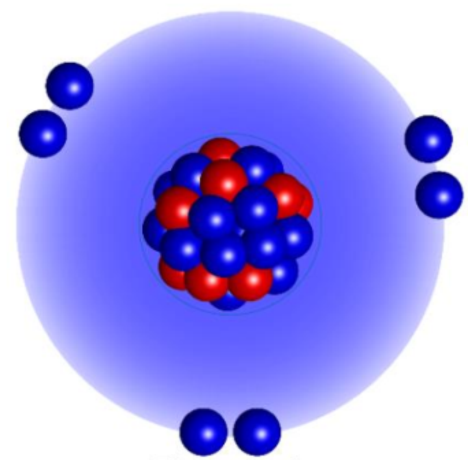
- Based on relativistic density functional theory, with pairing correlation and continuum effects properly considered, Meng and Ring developed the spherical relativistic continuum Hartree-Bogoliubov (RCHB) theory.

Meng and Ring, PRL 77, 3963 (1996)
Meng, NPA 635, 3 (1998)

- RCHB was successfully applied in many studies on exotic nuclei:
  - Reproducing and interpreting the halo in ^{11}Li Meng and Ring, PRL 77, 3963 (1996)
  - Predicting the giant halo phenomena Meng and Ring, PRL 80, 460 (1998)
  - First mass table including continuum effects Xia *et al*., ADNDT 121,1 (2018)
  - …… Meng *et al*., PPNP 57, 470 (2006)



PRL 77, 3963 (1996)



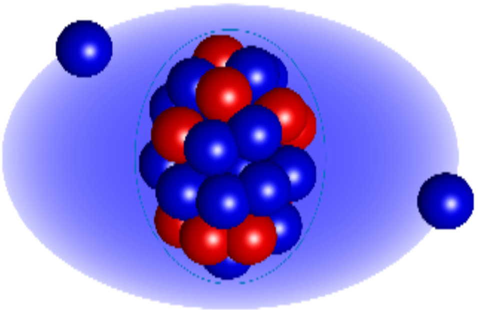
From X.H. Wu

DRHBc theory

- For axially deformed halo nuclei, Zhou *et al*. developed the deformed relativistic Hartree-Bogoliubov theory in continuum (DRHBc), with the effects of deformation, pairing correlations and continuum taken into account simultaneously. Zhou *et al*., PRC 82, 011301 (2010)

- The deformed halo nuclei $^{42,44}Mg$ were predicted by the DRHBc theory, and the shape decoupling between halo and core was revealed.

  Zhou *et al*., PRC 82, 011301 (2010)
  Li *et al*., PRC 85, 024312 (2012)



From X.H. Wu

- The DRHBc theory has been applied in many studies on halo nuclei.
  - C PLB 785, 530 (2018); NPA 1003, 122011 (2020)
  - B PRL 126, 082501 (2021); PRC 103, 054315 (2021)
  - Ne SCPMA 65, 262011 (2022); PLB 855, 138792 (2024)
  - Mg PLB 844, 138112 (2023); PLB 849, 138422 (2024)
  - Na PRC 107, L041303(2023)

DRHBc theory: applications

- Possible "peninsulas" in the nuclear landscape

  Zhang *et al*., PRC 104, L021301 (2021) Pan *et al*., PRC 104, 024331 (2021)
  He *et al*., CPC 45, 101001 (2021) He *et al*., PRC 110, 014301 (2024)

- Proton emission and α-decay

  Xiao *et al*., PLB 845, 138160 (2023) Choi *et al*., PRC 109, 054310 (2024)

- Angular momentum projection and rotating deformed halo nuclei

  Sun *et al*., Sci. Bull., 66 2072 (2021) Sun *et al*., PRC, 104 064319 (2021)

- Dynamical correlations with collective Hamiltonian

  Sun *et al*., CPC 46 064103 (2022) Zhang *et al*., PRC 108, 024310 (2023)

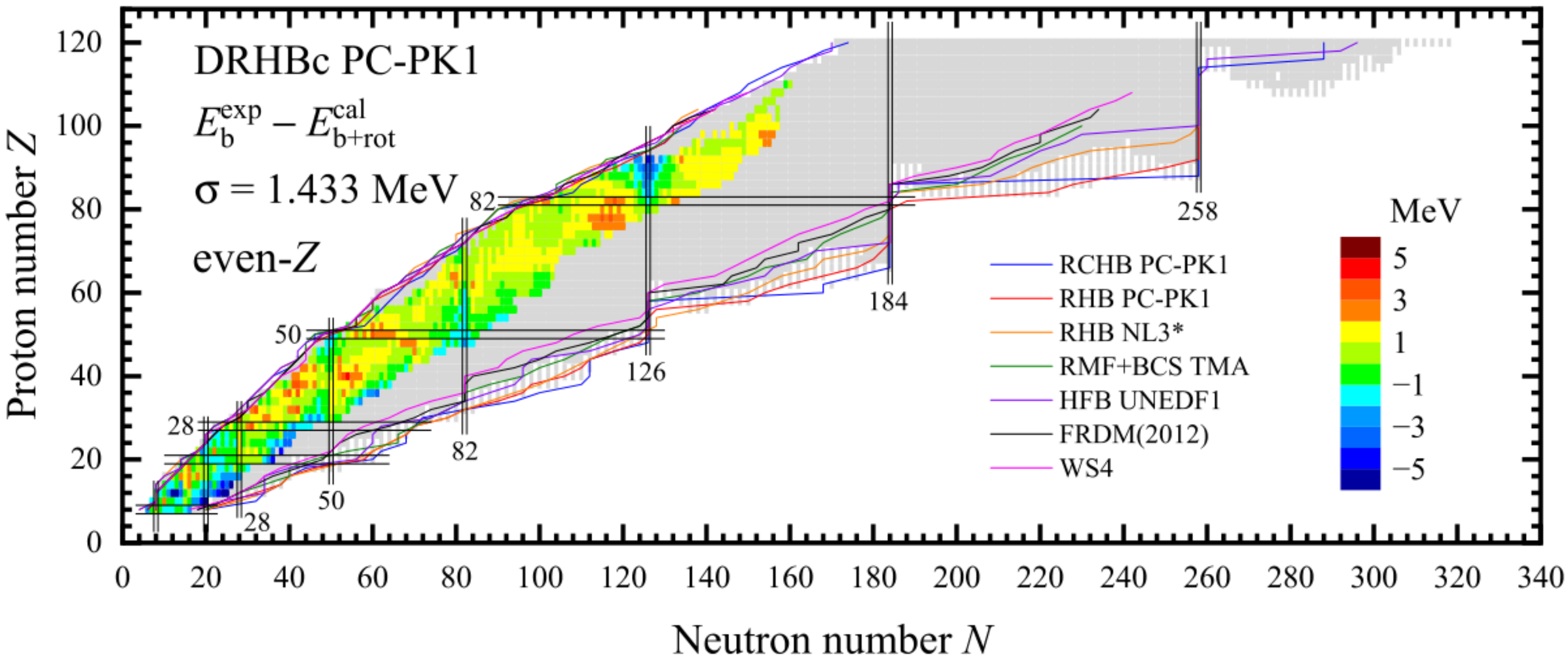
- More interesting studies ……

  Zhang *et al*., PRC 100, 034312 (2019) In *et al*., IJMPE 30, 2150009 (2021)
  Choi *et al*., PRC 105, 024306 (2022) Kim *et al*., PRC 105, 034340 (2022)
  Guo *et al*., PRC 108, 014319 (2023) Mun *et al*., PLB 847, 138298 (2023)
  Zheng *et al*., CPC 48, 014107 (2024) Mun *et al*., PRC 110, 014314 (2024)

DRHBc Mass Table

https://drhbctable.jcnp.org/

- The DRHBc mass table aims at providing a microscopic mass table including the effects of deformation and continuum.
  - ➢ The even-Z part of the DRHBc mass table has been completed.



  - ➢ 2584 even-even nuclei and 2245 even-Z odd-N ones are predicted with $8 \leq Z \leq 120$, and their ground-state properties are obtained.

Zhang *et al.*, (DRHBc Mass Table Collaboration) PRC 102, 024314 (2020)
Pan *et al.*, (DRHBc Mass Table Collaboration) PRC 106, 014316 (2022)
Zhang *et al.*, (DRHBc Mass Table Collaboration) ADNDT 144, 101488 (2022)
Guo *et al.*, (DRHBc Mass Table Collaboration) ADNDT 158, 101661 (2024)

In this work

- Recently, the DRHBc theory was extended to a version for triaxial deformation, and to a version incorporating nuclear magnetism, respectively.

Zhang, Zhang and Meng, PRC 108, L041301 (2023)
Pan, Zhang and Zhang, PLB 855, 138792 (2024)

- The effect of nuclear magnetism in the halo nucleus ^{31}Ne is investigated.

  Pan, Zhang and Zhang, PLB 855, 138792 (2024)

- The possible proton halo in ^{22}Al is examined, and the effect of triaxial deformation is studied.

  Zhang, Pan and Wang, PRC 110, 014320 (2024)

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DRHBc theory

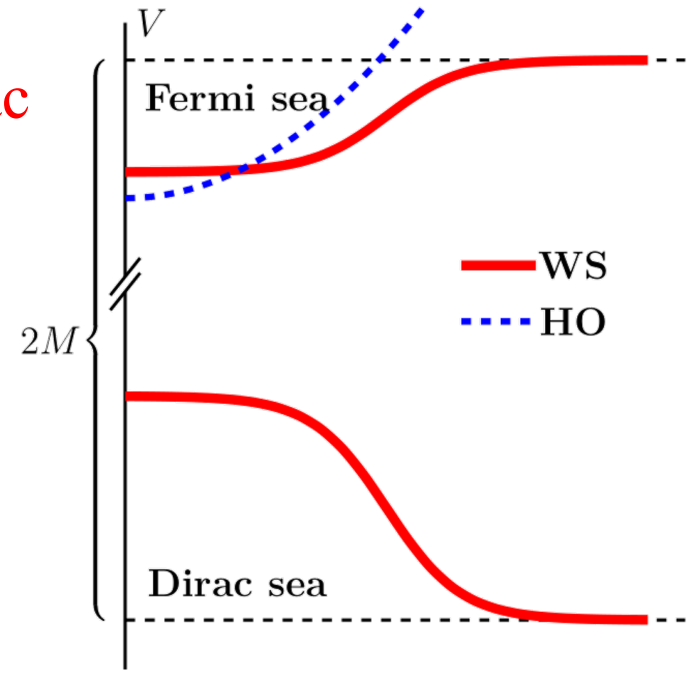
- The relativistic Hartree-Bogoliubov (RHB) equation for nucleons:

$$\begin{pmatrix} \hat{h}_D - \lambda_\tau & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h}_D^* + \lambda_\tau \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}$$

Kucharek and Ring, ZPA 339, 23 (1991)

- To properly consider continuum effect, the RHB equation is solved in a spherical Dirac Woods-Saxon (DWS) basis.

Zhou *et al*., PRC, 68, 034323 (2003)



- For the nuclei with axial deformation and spatial reflection symmetry, the densities and potentials are expanded with Legendre polynomials:

$$f(\boldsymbol{r}) = \sum_\lambda f_\lambda(r) P_\lambda(\cos\theta), \quad (\lambda = 0, 2, 4, \ldots, \lambda_{max})$$

Zhou *et al*., PRC 82, 011301 (2010)

Blocking effect and Nuclear magnetism

- For odd-A and odd-odd nuclei, the blocking effect of the unpaired nucleon should be considered:

$$(U_{k_b}, V_{k_b}) \leftrightarrow (V^*_{k_b}, U^*_{k_b}), \quad E_{k_b} \leftrightarrow -E_{k_b}$$

Ring and Schuck, The Nuclear Many-Body Problem (Springer, 1980)

- The odd nucleon breaks the time-reversal symmetry, making the system more complicated:

Koepf and Ring, NPA 511, 279 (1990)

$$h_D = \boldsymbol{\alpha} \cdot (\boldsymbol{p} - \boldsymbol{V}) + V^0 + \beta(M + S)$$

$$S(\boldsymbol{r}) = \alpha_S \rho_S + \beta_S \rho_S^2 + \gamma_S \rho_S^3 + \delta_S \Delta \rho_S,$$

$$V^0(\boldsymbol{r}) = \alpha_V j^0 + \gamma_V (j_\nu j^\nu) j^0 + \delta_V \Delta j^0 + e\frac{1-\tau_3}{2} A^0 + \alpha_{TV} j_3^0 + \delta_{TV} \Delta j_3^0$$

Time-even

$$\boldsymbol{V}(\boldsymbol{r}) = \alpha_V \boldsymbol{j} + \gamma_V (j_\nu j^\nu) \boldsymbol{j} + \delta_V \Delta \boldsymbol{j} + e\frac{1-\tau_3}{2} \boldsymbol{A} + \alpha_{TV} \boldsymbol{j}_3 + \delta_{TV} \Delta \boldsymbol{j}_3$$

Time-odd

[Nuclear magnetism]

- In the DRHBc theory, the nuclear magnetism is neglected.

Perez-Martin *et al*., PRC 78, 014304 (2008)
Li *et al*., CPL 29, 042101 (2012)

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  Pan, Zhang and Zhang, PLB 855, 138792 (2024)
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Nuclear magnetism

- The nuclear magnetism in deeply bound or rotational nuclei has been well studied within the RDFT framework:
  - Binding energy

    Rutz *et al*., NPA 634, 67 (1998); Afanasjev *et al*., PRC 81, 014309 (2010)
  - Odd-even mass difference

    Rutz *et al*., PLB 468, 1 (1999); Afanasjev *et al*., PRC 67, 024309 (2003)
  - Magnetic moment

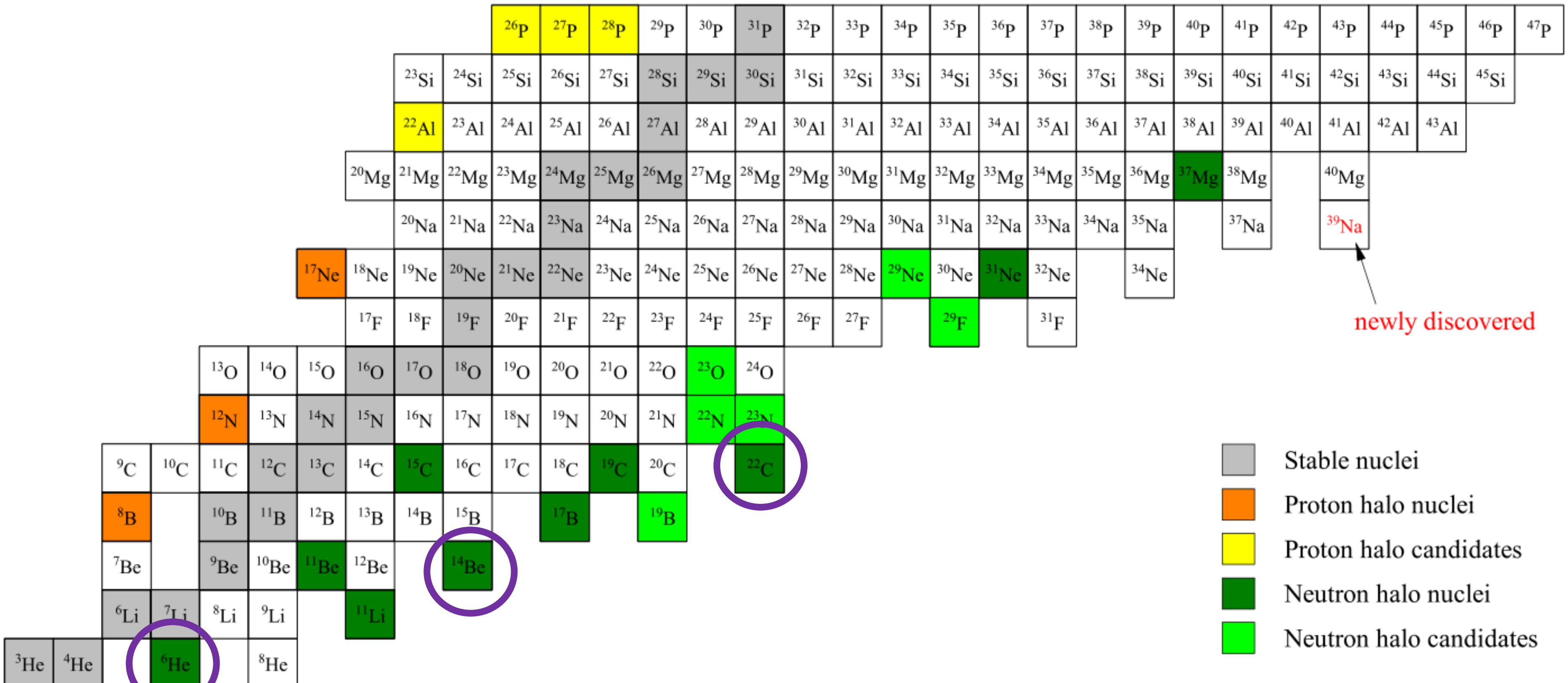
    Hofmann *et al*., PLB 214, 307 (1988); Yao *et al*., PRC 74, 024307 (2006)
  - Rotational properties

    Koepf and Ring, NPA 493, 61 (1989); Koepf and Ring, NPA 511, 279 (1990)
  - …
- For the exotic nuclei near the drip line, the nuclear magnetism may influence the exotic structures, potentially altering the drip-line location.

  Afanasjev *et al*., PRC 81, 014309 (2010); Kasuya *et al*., PTEP 2021, 013D01 (2020)

Nuclear magnetism in halo nuclei

- In most suggested halo nuclei or candidates from experiments, the neutron or proton number is odd.



Zhang *et al*., PRC 107, L041303 (2023)

- To explore the nuclear magnetism in halo nuclei, the time-odd DRHBc (TODRHBc) theory was developed.

Neutron halo nucleus ^{31}Ne

- A halo structure has been experimentally suggested in ^{31}Ne, and measurements show $J^{\pi} = 3/2^{-}$, and $S_n = 0.15^{+0.16}_{-0.10}$ MeV.

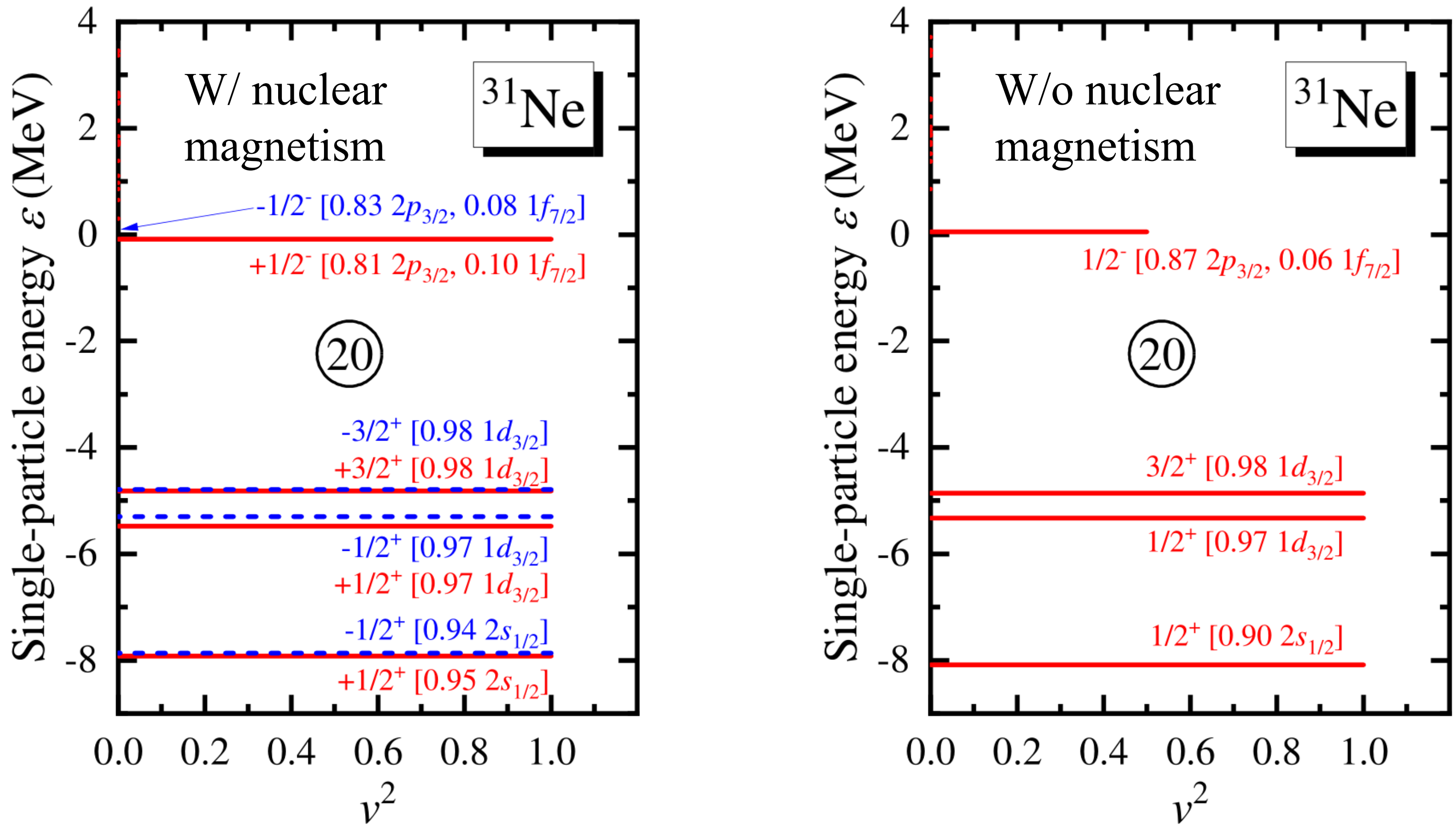
Nakamura *et al*., PRL 103, 262501 (2009)
Gaudefroy *et al*., PRL 109, 202503 (2012)
Nakamura *et al*., PRL 112, 142501 (2014)

- The DRHBc theory has been applied to study the halo structure in ^{31}Ne and the reaction cross section on carbon target.

Zhong *et al*., Sci. Chin. Phys. 65, 262011 (2022)

- Here, ^{31}Ne is calculated based on the TODRHBc + PC-PK1:
  - $S_n = 0.56$ MeV (including rotational correction)
  - $\beta_2 = 0.17$
  - By comparison with the DRHBc results, the influence of nuclear magnetism on binding energy is 90 keV.

Single-neutron levels



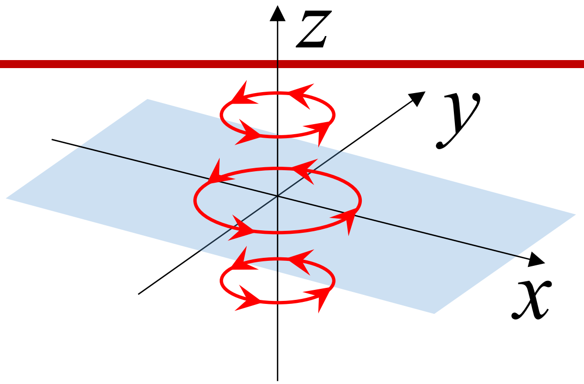
- TODRHBc: Valence neutron at $\varepsilon = -0.09$ MeV, weakly-bound.
- DRHBc: Valence neutron at $\varepsilon = 0.05$ MeV, unbound.

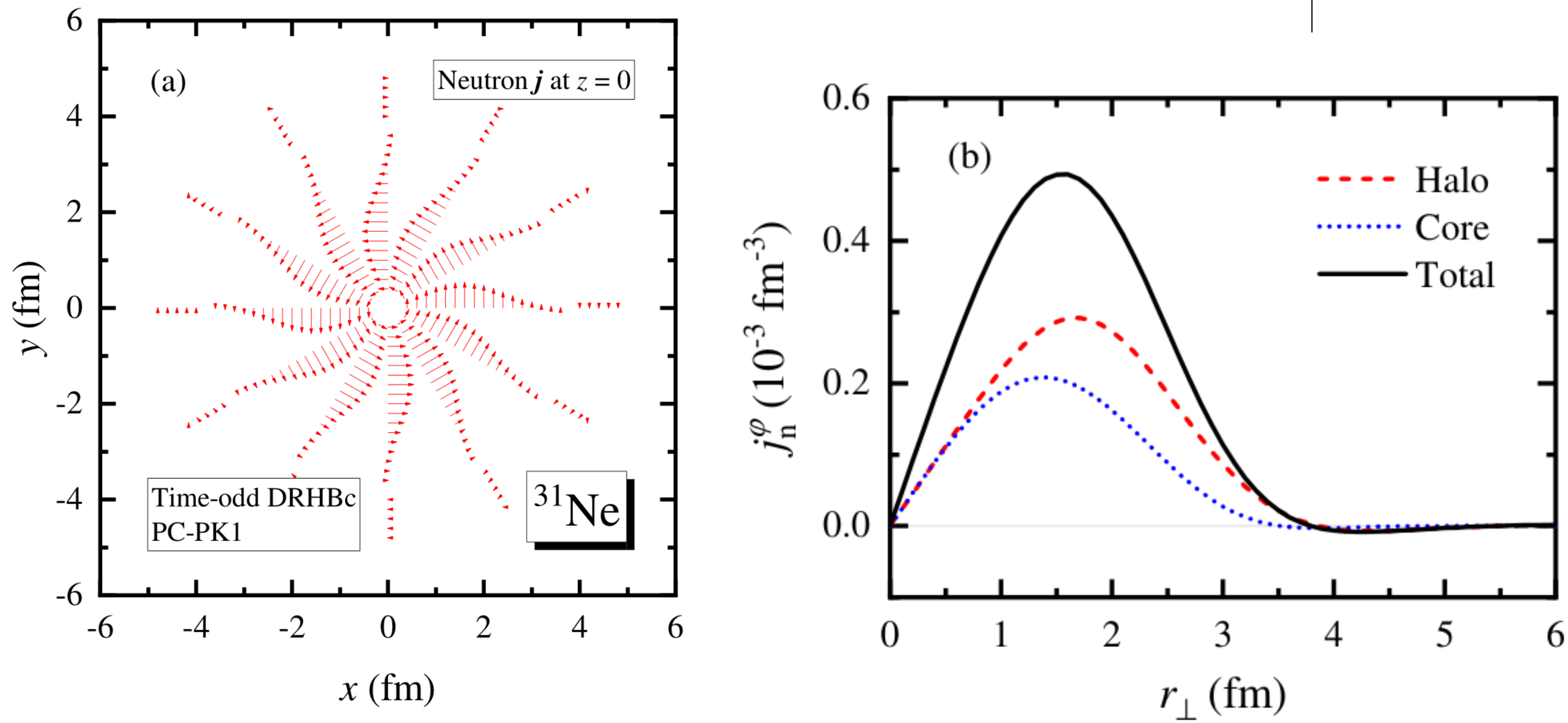
—— A mechanism that nuclear magnetism changes the drip-line location.

Neutron current

- Due to axial deformation, nucleon current has only azimuthal component: $\boldsymbol{j} = j^{\varphi}\mathbf{e}_{\varphi}$.

Hofmann *et al*., PLB 214, 307 (1988)





- The halo (valence neutron) always plays an important role in $\boldsymbol{j}$.

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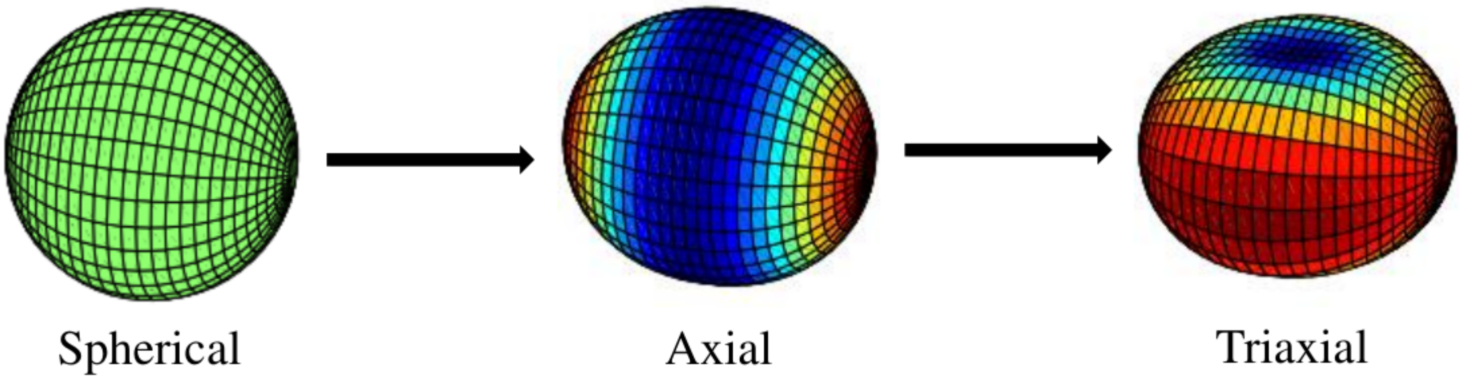
  Zhang, Pan and Wang, PRC 110, 014320 (2024)
- ☐ **Summary**

Triaxial deformation



- Triaxial deformation (non-axial deformation) is one of the basic deformation degrees of freedom in atomic nuclei, and closely related to rich nuclear phenomena.

Bohr and Mottelson, Nuclear Structure (1975)

- Starting from the framework of DRHBc and further considering the triaxial deformation, the triaxial relativistic Hartree-Bogoliubov theory in continuum (TRHBc) was developed, and ^{42}Al was predicted as a triaxial neutron-halo nucleus.

Zhang *et al.*, PRC 108, L041301 (2023)

$$f(\boldsymbol{r}) = \sum_{\lambda\mu} f_{\lambda\mu}(r) Y_{\lambda\mu}(\theta, \varphi) \quad (\lambda = 0, 2, 4, \dots, \lambda_{\max};\ \mu = -\lambda, -\lambda+2, \dots, \lambda\ )$$

Proton-halo candidate ^{22}Al

- In 2020, the experimental study on the first 1^+ excited state of ^{22}Al suggested a proton halo.

  Lee *et al.*, PRL 125, 192503 (2020)

- Recent experiments have measured the proton separation energy for ^{22}Al to be S_p = 100.4(8) keV.

  Sun *et al.*, CPC 48, 034002 (2024)
  Campbell *et al.*, PRL 132, 152501 (2024)

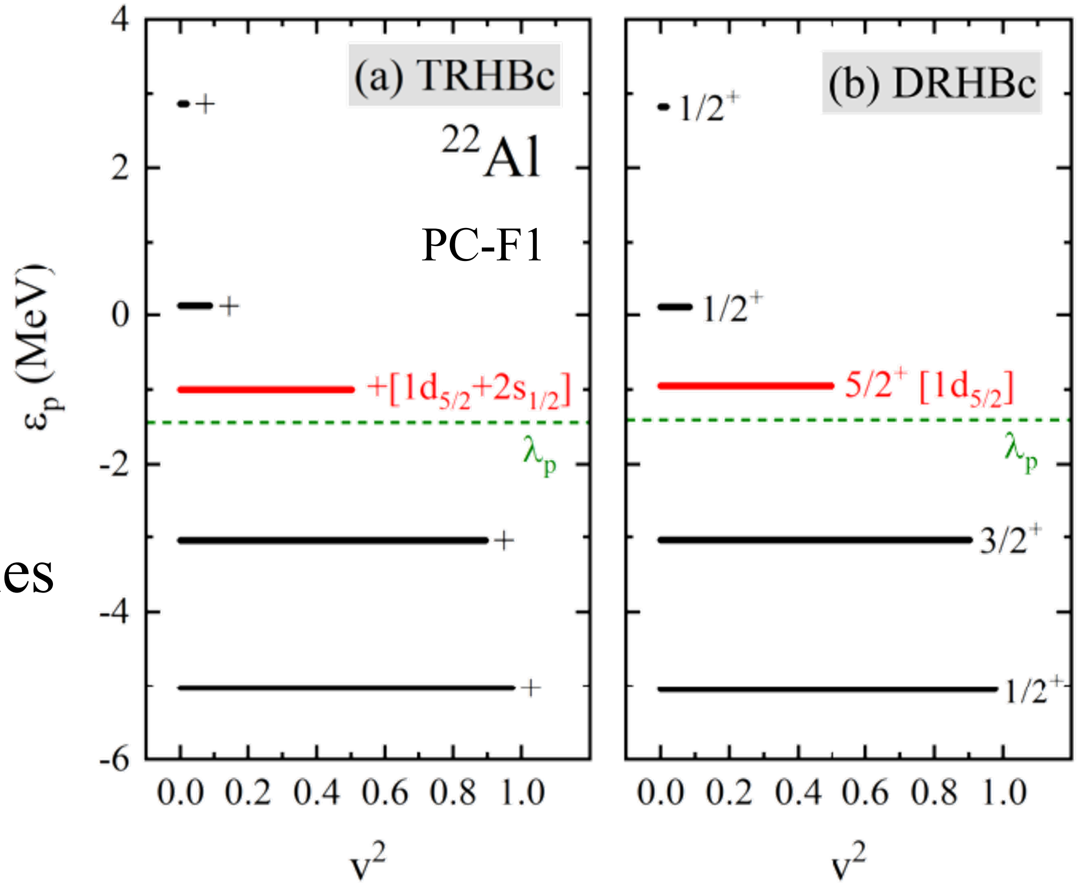
- Neither an enhanced interaction/reaction cross section nor a narrow momentum distribution of breakup fragments was observed for ^{22}Al.

?

In our work, the possibility of a proton halo in the ground state of ^{22}Al is examined based on the DRHBc and TRHBc calculations.

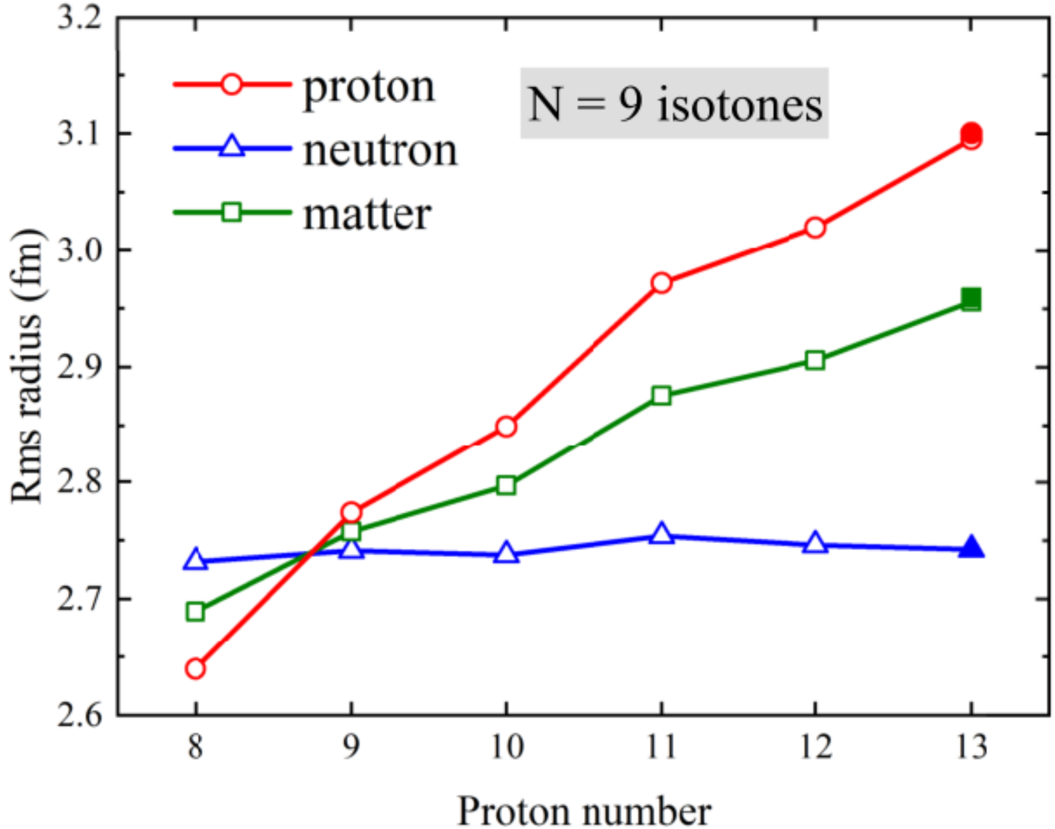
TRHBc and DRHBc results

- [Triaxial] TRHBc:
  - $\beta = 0.30$, $\gamma = 7.93^\circ$
  - $E_B =$ 149544.8 keV
    [Exp.: 149313.1(3) keV]
  - $S_p =$ 21.7 keV
    [Exp.: 100.4(8) keV]
  - The valence proton occupies a weakly-bound orbit with 93% $1d_{5/2}$ and 5% $2s_{1/2}$ components.



- [Axial] DRHBc:
  - The valence proton has $\Omega^\pi = 5/2^+$, which has no s-wave component, and does not contribute to halo.

Rms radii



- ● TRHBc [triaxial]
- ○ DRHBc [axial]

- The most significant increase in the proton radius occurs from $Z = 10$ to 11 due to deformation effects, rather than from $Z = 12$ to 13.
- For ^{22}Al, the small *s* component in TRHBc corresponds to a marginal increase in the proton radius, insufficient to form a discernible proton halo.

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Summary

- ^{31}**Ne** is studied with DRHBc and TODRHBc:
  - A possible mechanism is suggested that nuclear magnetism may make an unbound nucleus become bound.
  - The valence neutron plays a leading role in the neutron current.

Pan, Zhang and Zhang, PLB 855, 138792 (2024)

- ^{22}**Al** is studied with DRHBc and TRHBc:
  - Triaxial deformation brings about a small *s*-wave component for the valence proton, but it is still insufficient to form a discernible proton halo.
  - Halo-like characteristics may exist in the excited state of ^{22}Al, and such works are in progress.

Zhang, Pan, Wang, PRC 110, 014320 (2024)
Papakonstantinou, *et al.*, *in preparation* (2024)

- Based on the DRHBc theory and its extensions, more studies on halo nuclei are in progress.

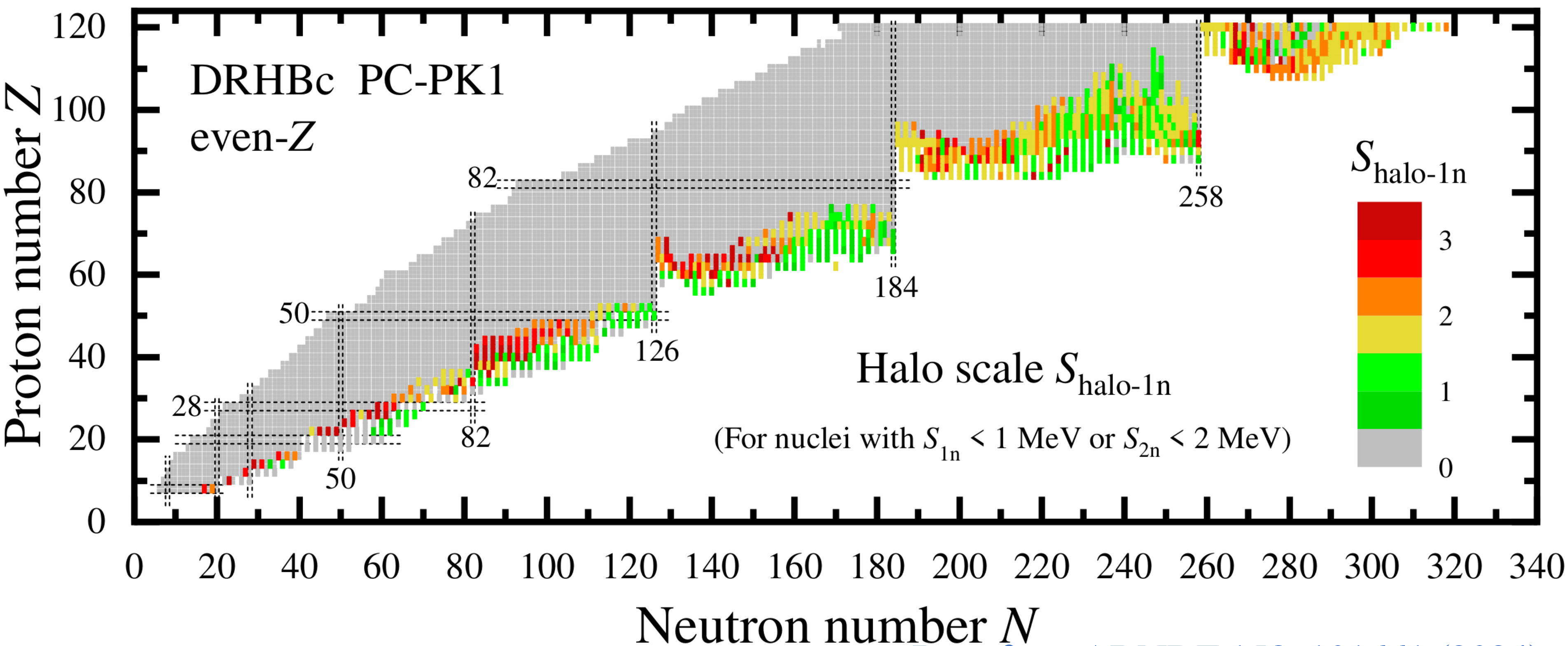
Halo scales

$$S^{\text{cal}}_{\text{halo-1n}} = \frac{\Delta R^{\text{cal}}_{1n}}{\Delta R^{\text{emp}}_{1n}} = \frac{R^{\text{cal}}_{n}(N+1) - R^{\text{cal}}_{n}(N)}{R^{\text{emp}}_{n}(N+1) - R^{\text{emp}}_{n}(N)}$$

Zhang *et al.*, PRC 108, L041301 (2023)

- An enhancement of S_{halo} might be regarded as a signal of the halo phenomenon.

Pan, *et al.*, in preparation



Data from ADNDT 158, 101661 (2024)

Thanks for your attention!

Appendix

Numerical details for ^{31}Ne

- ✓ Nuclide: ^{31}Ne
- ✓ Effective interaction: PC-PK1 PRC 82, 054319 (2010)
- ✓ Pairing interaction: Density-dependent zero-range force
- ✓ Box size: $R_{box} = 20$ fm
- ✓ Step size: $\Delta r = 0.1$ fm
- ✓ Energy cutoff: $E_{cut} = 300$ MeV
- ✓ Angular momentum cutoff: $J_{max} = 19/2\ \hbar$
- ✓ Legendre expansion cutoff: $\lambda_{max} = 10$
- ✓ Pairing strength: -342.5 MeV fm^3 PRC 102, 024314 (2020) PRC 106, 014316 (2022)

DRHBc and TRHBc calculations

- DRHBc:

$$f(\boldsymbol{r}) = \sum_{\lambda} f_\lambda(r) P_\lambda(\cos\theta),$$

($\lambda = 0, 2, 4, \ldots, \lambda_{max}$)

- TRHBc:

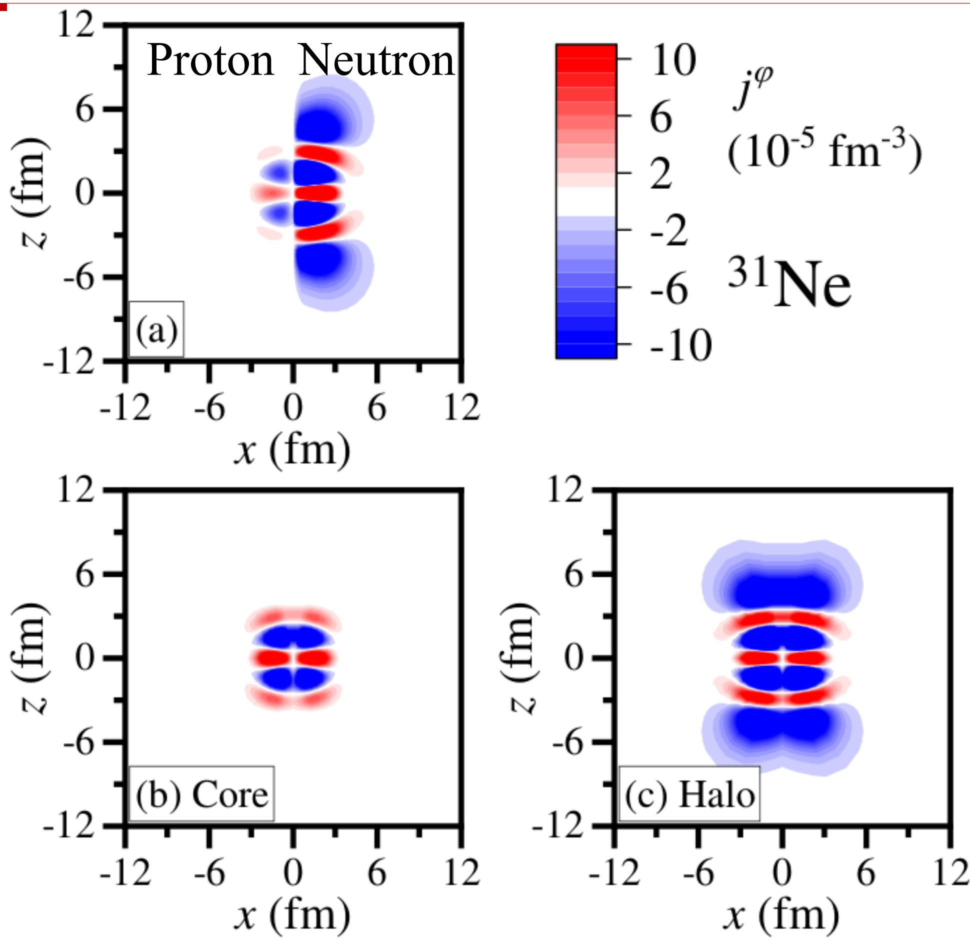
$$f(\boldsymbol{r}) = \sum_{\lambda\mu} f_{\lambda\mu}(\boldsymbol{r}) Y_{\lambda\mu}(\theta, \varphi)$$

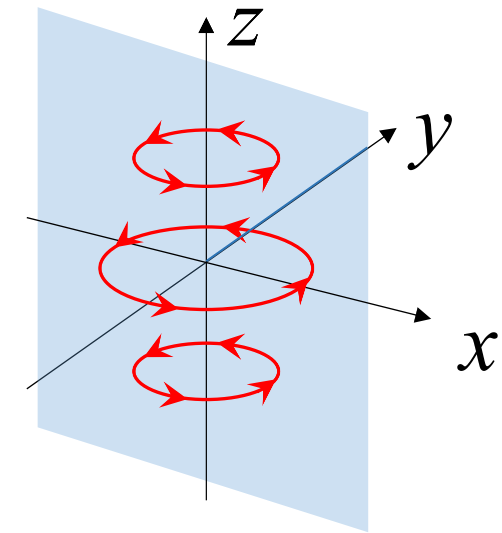
($\lambda = 0, 2, 4, \ldots, \lambda_{max}$;
$\mu = -\lambda, -\lambda+2, \ldots, \lambda$);

- ✓ Density functional: PC-F1
- ✓ Pairing interaction: density-dependent zero-range force
- ✓ Box size: $R_{box} = 20$ fm
- ✓ Mesh size: $\Delta r = 0.1$ fm
- ✓ Energy cutoff: $E_{cut} = 300$ MeV
- ✓ Angular momentum cutoff: $J_{max} = 19/2\ \hbar$
- ✓ Legendre expansion cutoff: $\lambda_{max} = 6$
- ✓ Pairing strength: -342.5 MeV fm^3

PRC 102, 024314 (2020)
PRC 106, 014316 (2022)

Neutron current (*xz*)

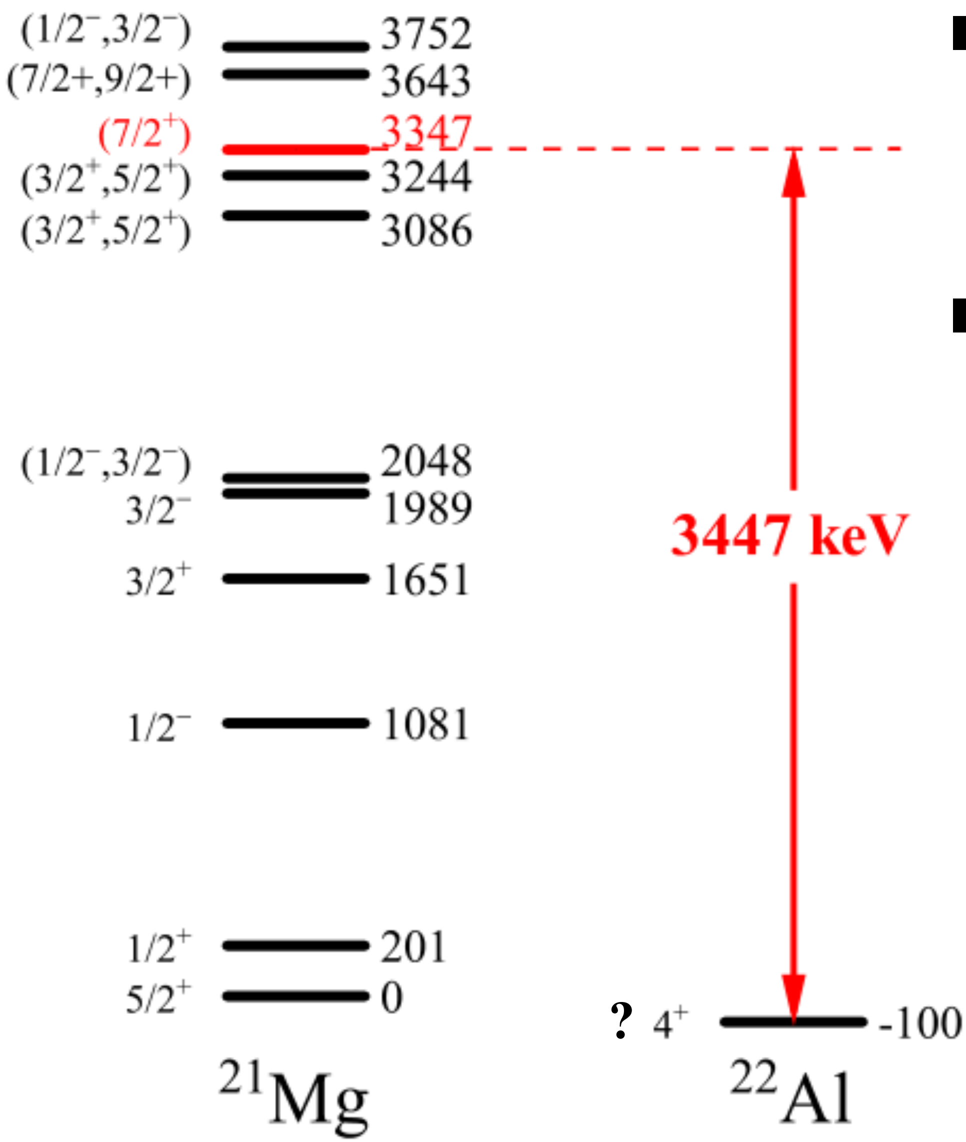




- $$j^\varphi = \sum_m \sum_{n\kappa} \sum_{n'\kappa'} \rho_{n\kappa m, n'\kappa' m'} \frac{1}{r^2} \left[G_{n\kappa} F_{n'\kappa'} (\mathrm{i} Y^{l\dagger}_{jm} \sigma_\varphi Y^{\tilde{l}'}_{j'm'}) - F_{n\kappa} G_{n'\kappa'} (\mathrm{i} Y^{\tilde{l}\dagger}_{jm} \sigma_\varphi Y^{l'}_{j'm'}) \right]$$

- A layered structure in ***j*** is found, which is dominated by the $2p_{3/2}$ and $1f_{7/2}$ components.

Experimental level schemes



- ^{22}F (mirror nucleus) has a ground state 4^+ and an excited 3^+ state at 72 keV.
- For ^{22}Al:
  - If I^π(g.s.) = 4^+ ?
    Since proton halo has $s_{1/2}$, neutron should be $7/2^+$ or $9/2^+$,
    → a large S_p of this $s_{1/2}$ proton.
  - If I^π(g.s.) = 3^+?
    → A chance of a halo.
  - The β decay of ^{22}Al strongly suggests a 4^+ g.s.

Wu *et al*., PRC 104, 044311 (2021)

Data sources: NNDC; Campbell *et al*., PRL 132, 152501 (2024)

Basic logic presented in PRL125.192503(2020)

We now explore the physical origin of the large difference in mirror asymmetry for the first and second 1^+ states. The ft value of a pure GT transition relates to the nuclear matrix element through

$$ft = \frac{D}{(\frac{g_A}{g_V})^2_{\rm eff}|M_{\rm GT}|^2}, \quad (2)$$

with D and $(g_A/g_V)_{\rm eff}$ being coupling constants [2], which are fixed values in our calculation. The mirror asymmetry δ can then be expressed through nuclear matrix elements as

$$\delta = \frac{ft^+}{ft^-} - 1 = \frac{|M^-_{\rm GT}|^2}{|M^+_{\rm GT}|^2} - 1 = \frac{\Delta|M_{\rm GT}|^2}{|M^+_{\rm GT}|^2}, \quad (3)$$

where the deviation $\Delta|M_{\rm GT}|^2$ is defined as $\Delta|M_{\rm GT}|^2 = |M^-_{\rm GT}|^2 - |M^+_{\rm GT}|^2$. The quantity $\Delta|M_{\rm GT}|^2$, and therefore δ, should vanish if exact isospin symmetry holds.

β^+ : $^{22}\mathrm{Si} \rightarrow {}^{22}\mathrm{Al}$

β^- : $^{22}\mathrm{O} \rightarrow {}^{22}\mathrm{F}$

If isospin symmetry would be strictly held, a pair of mirror nuclei should have identical behavior. In Gamow-Teller (GT) transitions, the reduced transition probability ft^+ value of β^+ decay in a proton-rich nucleus should be identical to the ft^- value of β^- decay in its mirror partner nucleus. The extent of isospin-symmetry breaking can be quantified through the asymmetry parameter $\delta = ft^+/ft^- - 1$. A large mirror asymmetry in ft value of GT transitions is believed to be closely related to the structure of proton-halo nuclei due to the difference between the radial wave functions of the initial state $|i\rangle$ and the final state $|f\rangle$ in the nuclear matrix element $M^{\rm GT}_{fi} = \langle f|\tau\sigma|i\rangle$. This difference appears because, when moving from tightly bound stable systems to loosely bound exotic ones, modifications to the spacing and sequence of single-particle states, as well as the contents of the corresponding wave functions, are expected. Near the driplines, a general view of the density distributions in different single-particle orbitals suggests that those with longer tails have smaller separation energies [6]. Although the tail of proton halos is usually shorter than the tails of neutron halos for the same separation energy due to the Coulomb barrier, the slope of the distributions depends strongly on the orbital angular momentum.

Basic logic presented in PRL125.192503(2020)

β-delayed one-proton emissions of ^{22}Si, the lightest nucleus with an isospin projection $T_z = -3$, are studied with a silicon array surrounded by high-purity germanium detectors. Properties of β-decay branches and the reduced transition probabilities for the transitions to the low-lying states of ^{22}Al are determined. Compared to the mirror β decay of ^{22}O, the largest value of mirror asymmetry in low-lying states by far, with $\delta = 209(96)$, is found in the transition to the first 1^+ excited state. Shell-model calculation with isospin-nonconserving forces, including the $T = 1, J = 2, 3$ interaction related to the $s_{1/2}$ orbit that introduces explicitly the isospin-symmetry breaking force and describes the loosely bound nature of the wave functions of the $s_{1/2}$ orbit, can reproduce the observed data well and consistently explain the observation that a large δ value occurs for the first but not for the second 1^+ excited state of ^{22}Al. Our results, while supporting the proton-halo structure in ^{22}Al, might provide another means to identify halo nuclei.

Halo scales

$$S^{\rm cal}_{\rm halo\text{-}1n} = \frac{\Delta R^{\rm cal}_{1n}}{\Delta R^{\rm emp}_{1n}} = \frac{R^{\rm cal}_n(N+1) - R^{\rm cal}_n(N)}{R^{\rm emp}_n(N+1) - R^{\rm emp}_n(N)}$$

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