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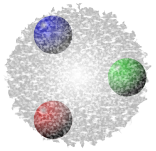
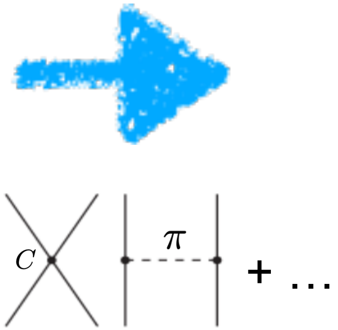
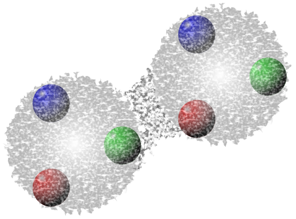
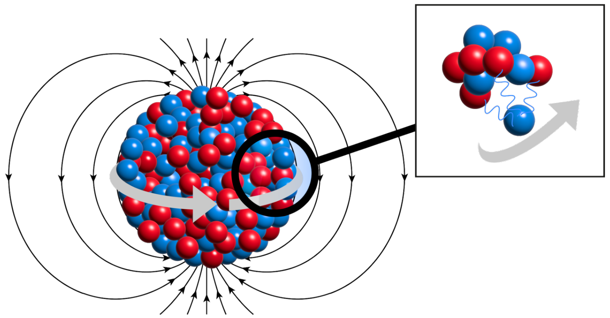
Effect of two-body current on magnetic dipole moments







$$H|\Psi\rangle = E|\Psi\rangle$$



Takayuki Miyagi

ERATO TOMOE

International Conferences on
Shapes and Symmetries on Nuclei:
from Experiment to Theory
SSNET @ IJCLab, France (Nov. 7, 2024)

Collaborators

- TU Darmstadt: C. Brase, K. Hebeler, A. Schwenk, R. Seutin
- TRIUMF: J. D. Holt
- University of Illinois: X. Cao
- Massachusetts Institute of Technology: R. G. Ruiz
- Johannes Gutenberg University of Mainz: S. Bacca
- University of Barcelona: J. Menendez

Motivations

- EM observables can be used
  - to investigate nuclear structure (shell structure, shape, …)
  - to test theories

- To test our theories, we need:
  - (precise) experimental data
  - reasonable starting nuclear Hamiltonian(s)
  - controllable many-body method(s)
  - higher-order contribution of EM operators (main focus of this talk)

$$H|\Psi\rangle = E|\Psi\rangle$$

$$O_{\rm EM}^{\rm exp.} \sim \langle\Psi|\mathcal{O}_{\rm EM}|\Psi\rangle$$

Motivations

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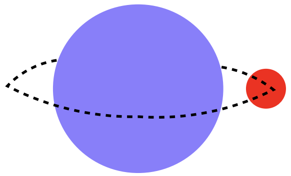
- Magnetic dipole moment: $$\langle \mu \rangle = \sqrt{\frac{J}{(J+1)(2J+1)}} \langle J || \mu || J \rangle$$

- Magnetic dipole operator: $$\boldsymbol{\mu} = \frac{e\hbar}{2m_p} \sum_i \left(g_i^l \boldsymbol{l}_i + g_i^s \boldsymbol{\sigma}_i \right)$$ Point-nucleon approximation

- Neighbors of doubly magic: $$|J\rangle \approx |\mathrm{Core}: 0^+\rangle \otimes |j_p\rangle, \; j_p = J$$



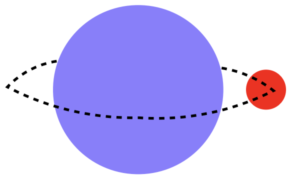
Schmidt limit

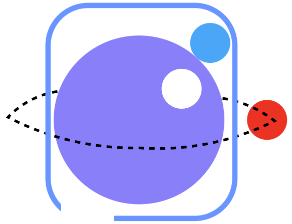
$$\langle \mu \rangle = \frac{e\hbar}{2m_p} \langle l_p j_p || g_i^l \boldsymbol{l}_i + g_i^s \boldsymbol{\sigma}_i || l_p j_p \rangle = j_p \left[g_l \mp (g_l - 2g_s) \frac{1}{2l_p + 1} \right], \; \left(j_p = l_p \pm \frac{1}{2} \right)$$

T. Schmidt 1937

Motivations

- Configuration mixing effect: $$|J\rangle \approx c_0[|\mathrm{Core}:0^+\rangle \otimes |j_p\rangle] + \sum_i c_i[[|j_h^{-1}\rangle \otimes |j_q\rangle]_{J_i} \otimes |j_p\rangle]_J$$





J_i Core polarization

- Arima and Horie computed c_i perturbatively: $$c \sim \frac{V}{\varepsilon}$$

NN interaction

SPE energy gap

A. Arima & H. Horie 1954

- Good agreement with data.
  - The deviation from the Schmidt value indicates how much the 0+ core is broken.

Motivations

- Ab initio IMSRG calculations
  - ✦ CP is included non-perturbatively!

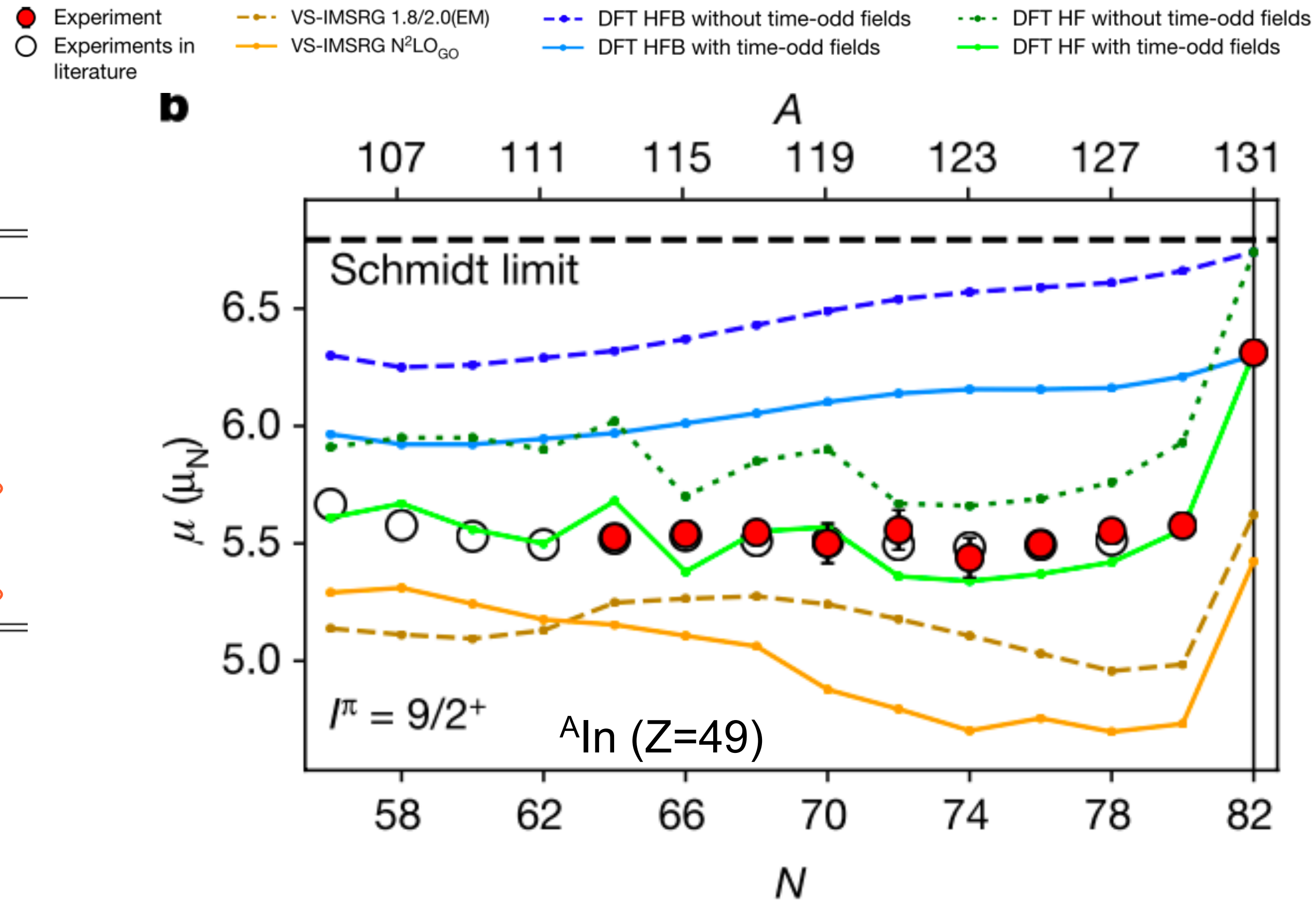
A. Klose et al., Phys. Rev. C 99, 061301 (2019).

^{39}Ca ^{39}K ^{37}Ca ^{37}Cl

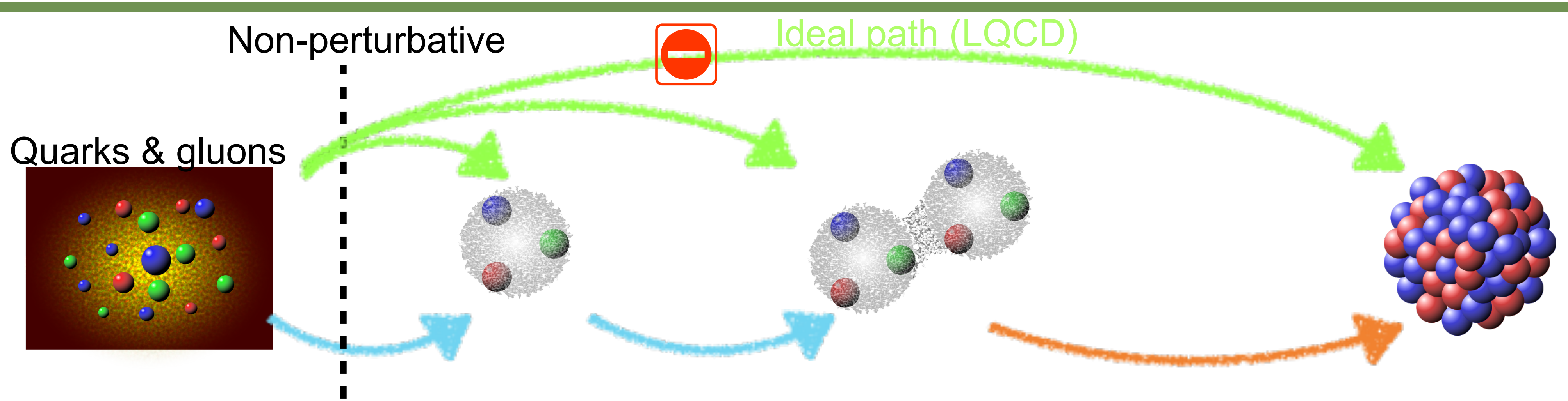
A		$Z = 20$	$N = 20$
	sp g^{free}	+1.148	+0.124
39	Expt.	+1.0217(1) [23]	+0.3915073(1) [24]
	sp g^{eff}	+0.930	+0.469
	VS-IMSRG	+1.349	−0.035
37	Expt.	+0.7453(72)	+0.6841236(4) [25]
	USDA-EM1	+0.770	+0.677
	USDB-EM1	+0.754	+0.675
	VS-IMSRG	+1.055	+0.290

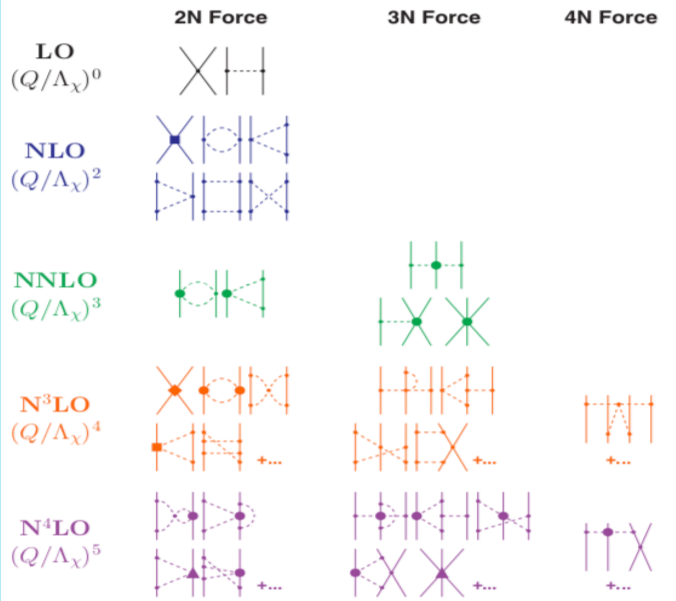
of ^{36}Ca. Compared to the USDA/B-EM1 calculations, the VS-IMSRG agrees with the dominance of the (620) partition for ^{36}Ca. However, the amount of the (522) partition that gives the core-polarization correction is a factor of 2 larger. The deviation is likely due to meson-exchange currents [39], which are not included in the present VS-IMSRG calculations, but are included indirectly through the effective g factors in the USDA/B-EM1 calculations.

A. R. Vernon et al., Nature 607, 260 (2022).



Nuclear ab initio calculation





Nuclear many-body problem

- Green's function Monte Carlo
- No-core shell model
- Nuclear lattice effective field theory
- Self-consistent Green's function
- Coupled-cluster
- In-medium similarity renormalization group
- Many-body perturbation theory
- …

Nuclear interaction from chiral EFT

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Weinberg, van Kolck, Kaiser, Epelbaum, Glöckle, Meißner, Entem, Machleidt, …

- Lagrangian construction
  - Chiral symmetry
  - Power counting
- Systematic expansion
  - Unknown LECs
  - Many-body interactions
  - Estimation of truncation error

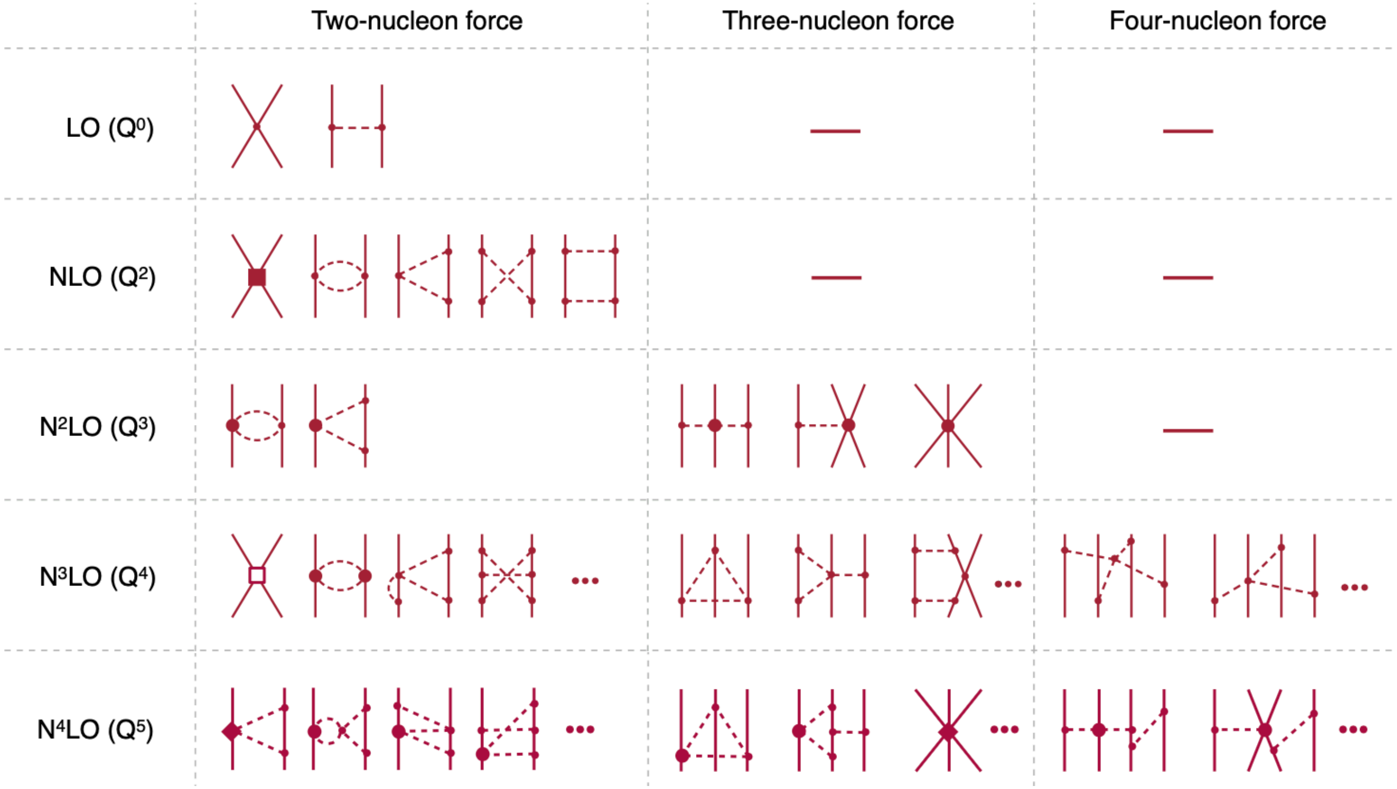
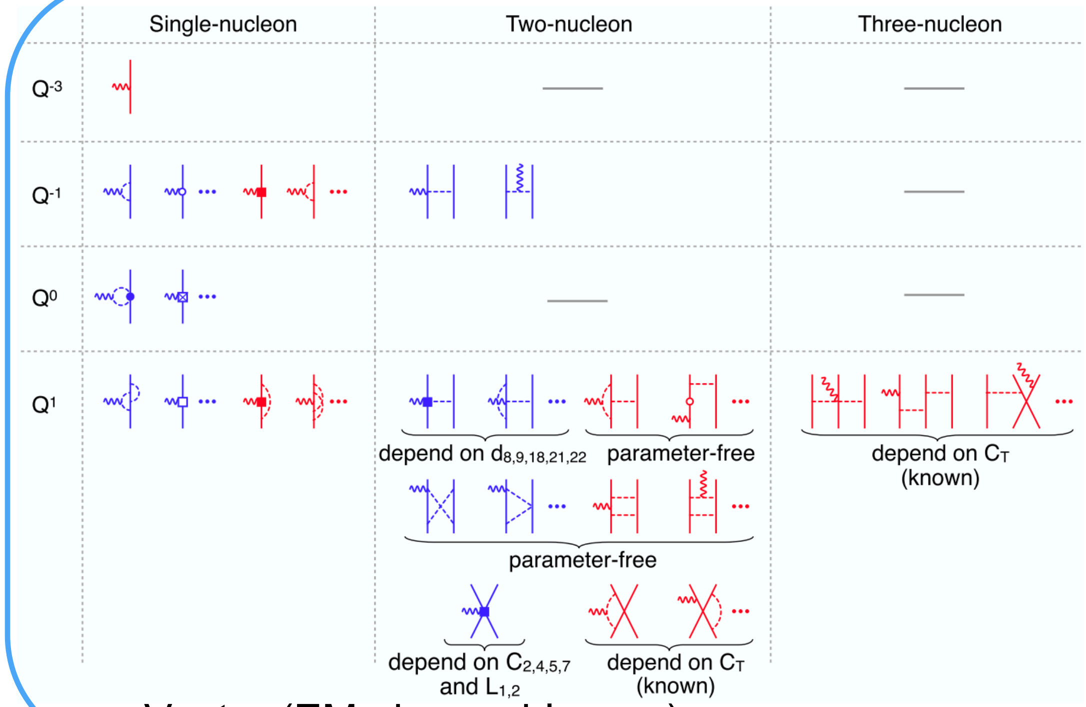


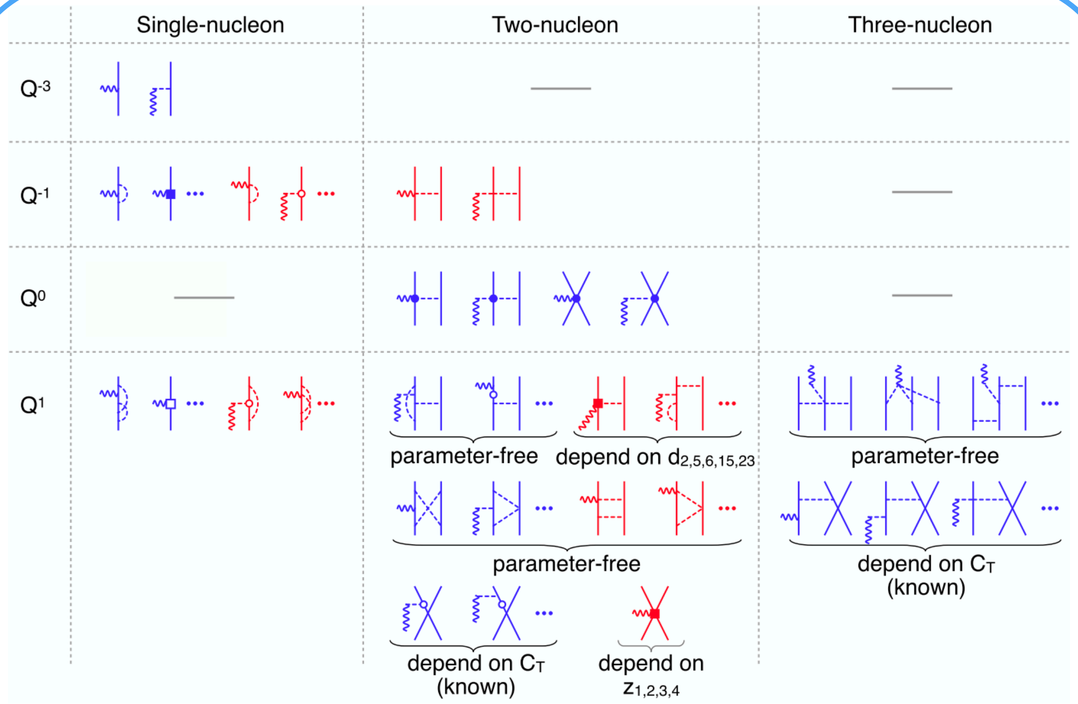
Figure is from E. Epelbaum, H. Krebs, and P. Reinert, Front. Phys. 8, 1 (2020).

Nuclear currents from chiral EFT

- Nuclear observables (EM properties, beta decay, …) are measured through the interaction between a nucleus and external field.
- Chiral EFT allows us a systematic expansion for charge and current operators.



Vector (EM observables, …)



Axial vector (GT transition, …)

E. Epelbaum, arXiv:1908.09349.

Nuclear currents from chiral EFT

- Nuclear observables (EM properties, beta decay, …) are measured through the interaction between a nucleus and external field.
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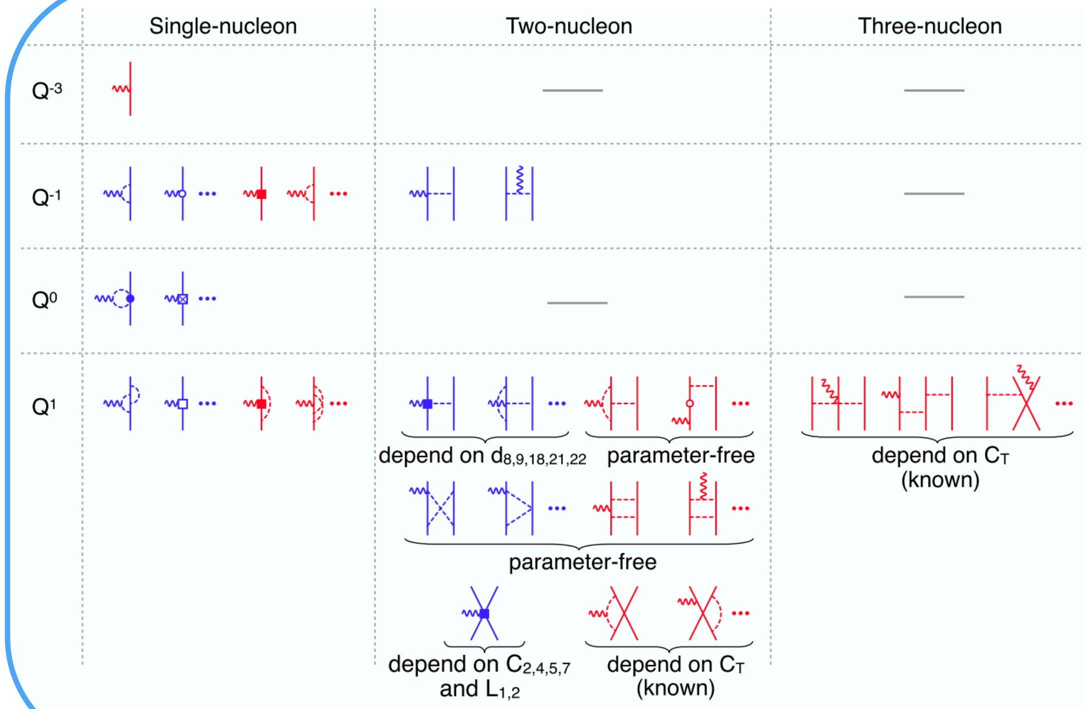
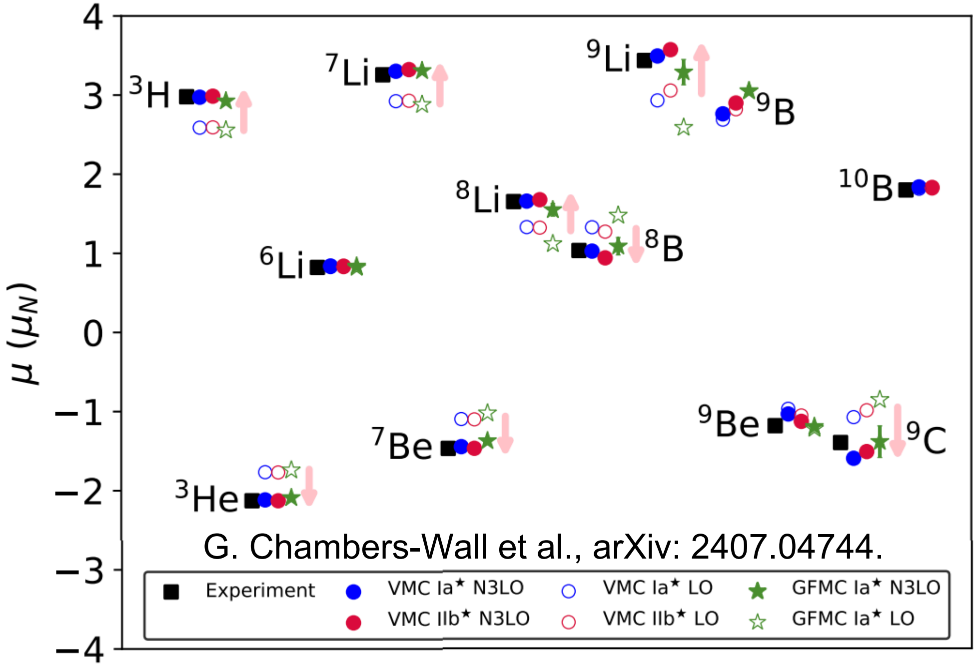


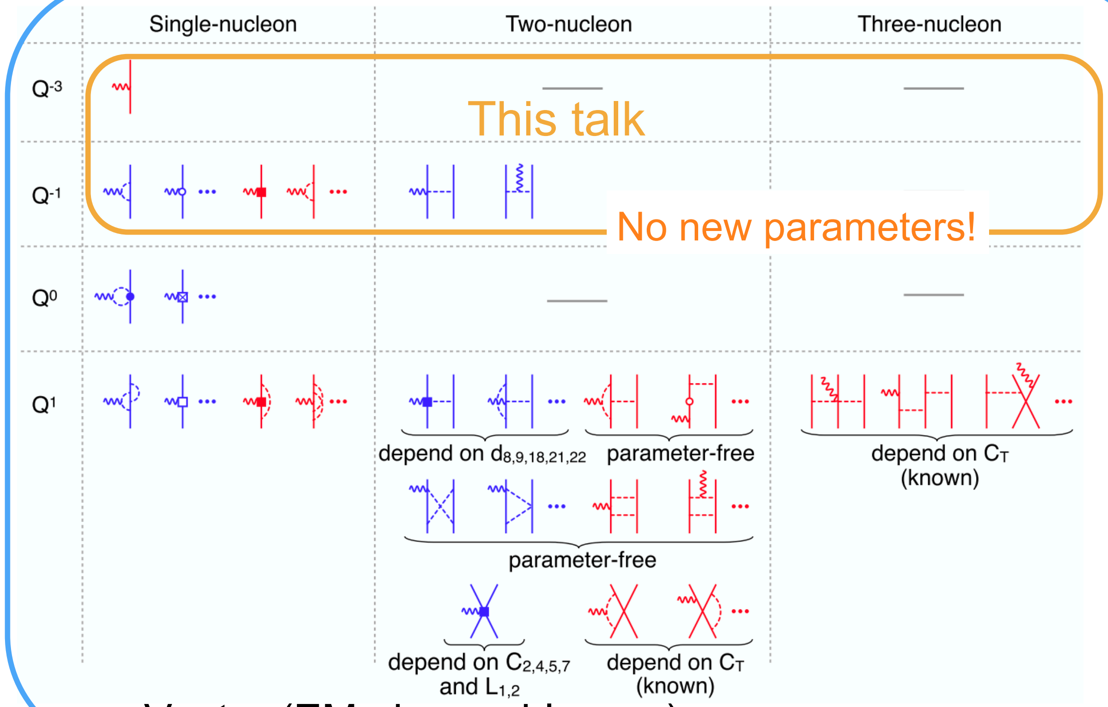
Vector (EM observables, …)



What about in heavier systems?

Nuclear currents from chiral EFT

- Nuclear observables (EM properties, beta decay, …) are measured through the interaction between a nucleus and external field.
- Chiral EFT allows us a systematic expansion for charge and current operators.



Vector (EM observables, …)

$$r_{ch}^2 = -\frac{6}{Z}\frac{1}{(4\pi)^{3/2}}\lim_{Q\to 0}\frac{d}{dQ^2}\int d\hat{\boldsymbol{Q}}\,\tilde{\rho}(\boldsymbol{Q})$$

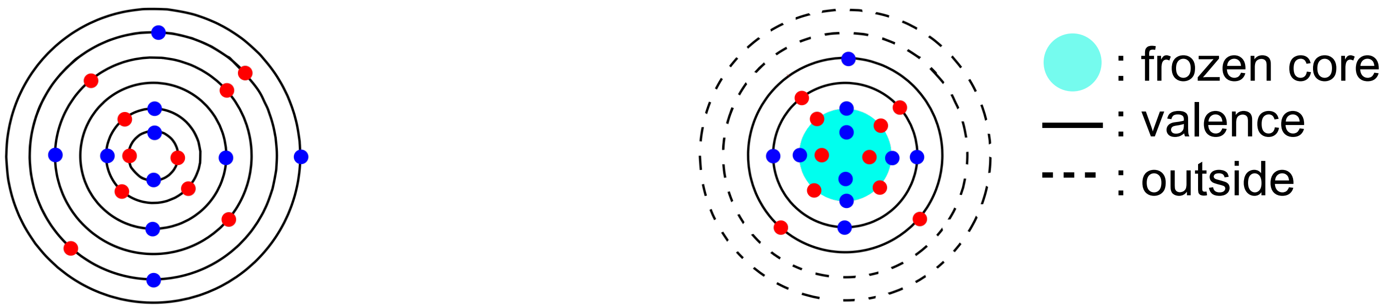
LO 2BC appear at Q^1 order (N^3LO)

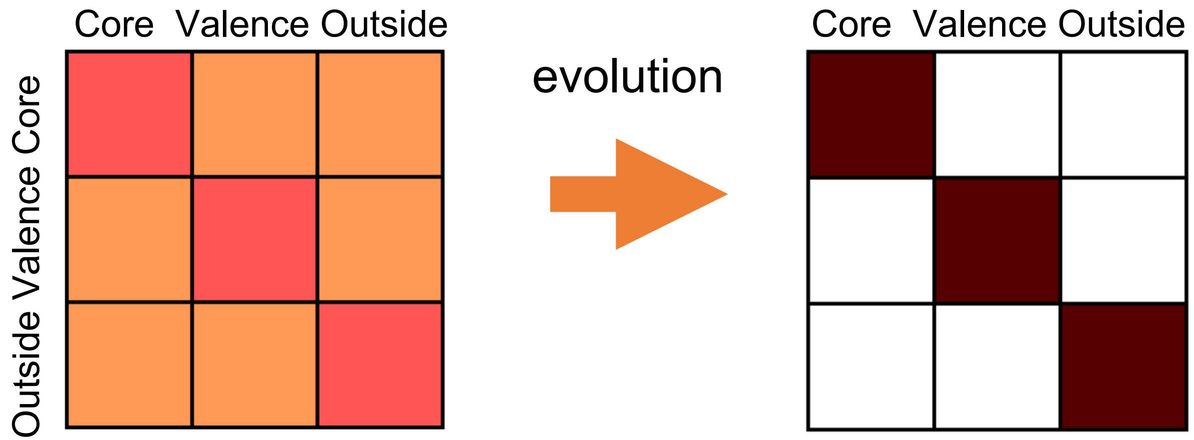
$$Q_{20} = -\frac{15}{8\pi}\lim_{Q\to 0}\frac{d^2}{dQ^2}\int d\hat{\boldsymbol{Q}}\,Y_{20}(\hat{\boldsymbol{Q}})\tilde{\rho}(\boldsymbol{Q})$$

$$M_{10} = -i\frac{3}{8\pi}\lim_{Q\to 0}\frac{d}{dQ}\int d\hat{\boldsymbol{Q}}\left\{[\boldsymbol{Q}\times\nabla_{\boldsymbol{Q}}]Y_{10}(\hat{\boldsymbol{Q}})\right\}\cdot\tilde{j}(\boldsymbol{Q})$$

or $\boldsymbol{M} = -\frac{i}{2}\sqrt{\frac{3}{4\pi}}\lim_{Q\to 0}\nabla_{\boldsymbol{Q}}\times\tilde{\boldsymbol{j}}(\boldsymbol{Q})$

Valence-space in-medium similarity renormalization group





Similarity transformation

$$H \qquad\qquad H(s) = e^{\Omega(s)} H e^{-\Omega(s)}$$

$$H(s) \approx E(s) + \sum_{12} f_{12}(s)\{a_1^\dagger a_2\} + \frac{1}{4}\sum_{1234} \Gamma_{1234}(s)\{a_1^\dagger a_2^\dagger a_4 a_3\}$$

s: flow parameter

$$\frac{d\Omega}{ds} = \eta(s) - \frac{1}{2}[\Omega(s), \eta(s)] + \cdots$$

$$\eta(s) = \sum_{12} \eta_{12}(s)\{a_1^\dagger a_2\} + \sum_{1234} \eta_{1234}(s)\{a_1^\dagger a_2^\dagger a_4 a_3\}$$

$$\eta_{12} = \frac{1}{2}\arctan\left(\frac{2f_{12}}{f_{11} - f_{22} + \Gamma_{1212} + \Delta}\right)$$

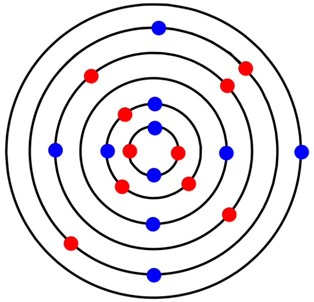
$$\eta_{1234} = \frac{1}{2}\arctan\left(\frac{2\Gamma_{1234}}{f_{11} + f_{22} - f_{33} - f_{44} + A_{1234} + \Delta}\right)$$

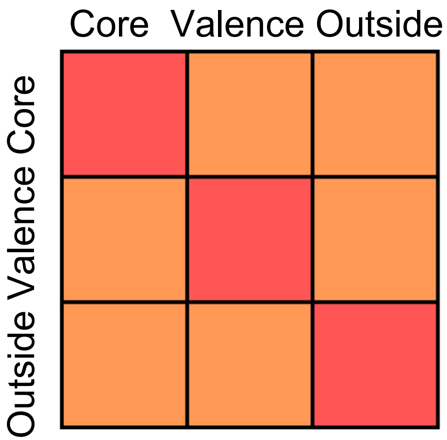
$$A_{1234} = \Gamma_{1212} + \Gamma_{3434} - \Gamma_{1313} - \Gamma_{2424} - \Gamma_{1414} - \Gamma_{2323}$$

f_{12}, Γ_{1234} : matrix element we want to suppress

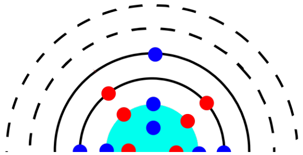
$$\mathcal{O}(s) = e^{\Omega(s)}\mathcal{O}e^{-\Omega(s)} \approx \mathcal{O}^{[0]}(s) + \sum_{12} \mathcal{O}^{[1]}_{12}(s)\{a_1^\dagger a_2\} + \frac{1}{4}\sum_{1234} \mathcal{O}^{[2]}_{1234}(s)\{a_1^\dagger a_2^\dagger a_4 a_3\}$$

Valence-space in-medium similarity renormalization group



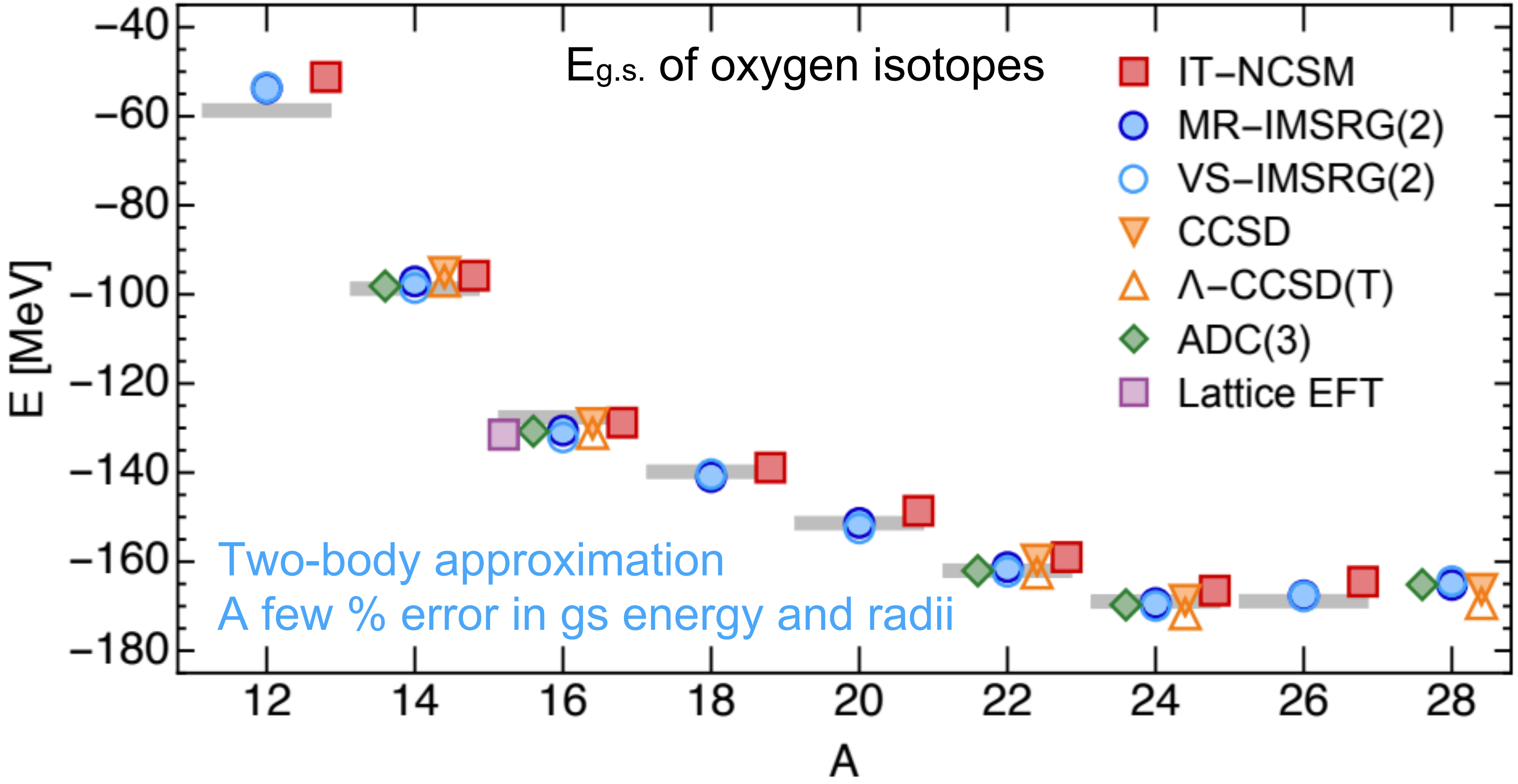


evolution



: frozen core

: valence



Similarity transformat

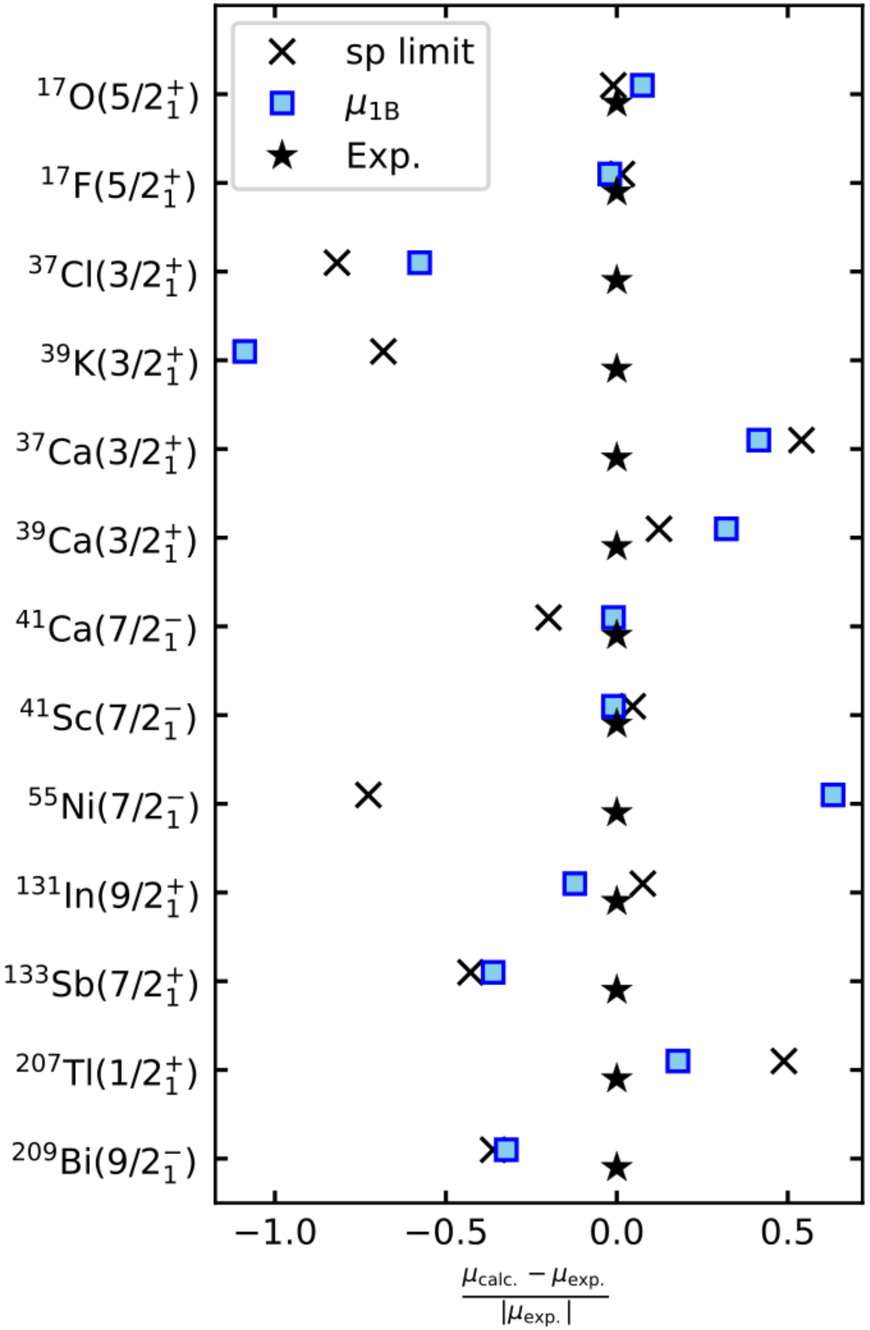
$$H$$

$$H(s) \approx E(s) + \sum_{12} f_{12}(s)\{a_1^\dagger a_2\} + \frac{1}{4} \sum_{1234}$$

s: flow parameter

Magnetic dipole moments

- Magnetic moment from IMSRG.
  - 1.8/2.0 (EM) interaction
- Single-particle analytical limits do not always explain the experimental data.
- IMSRG gives better agreements, but not perfect.
  - Suppression from many-body correlation



TM et al., Phys. Rev. Lett. **132**, 232503 (2024).

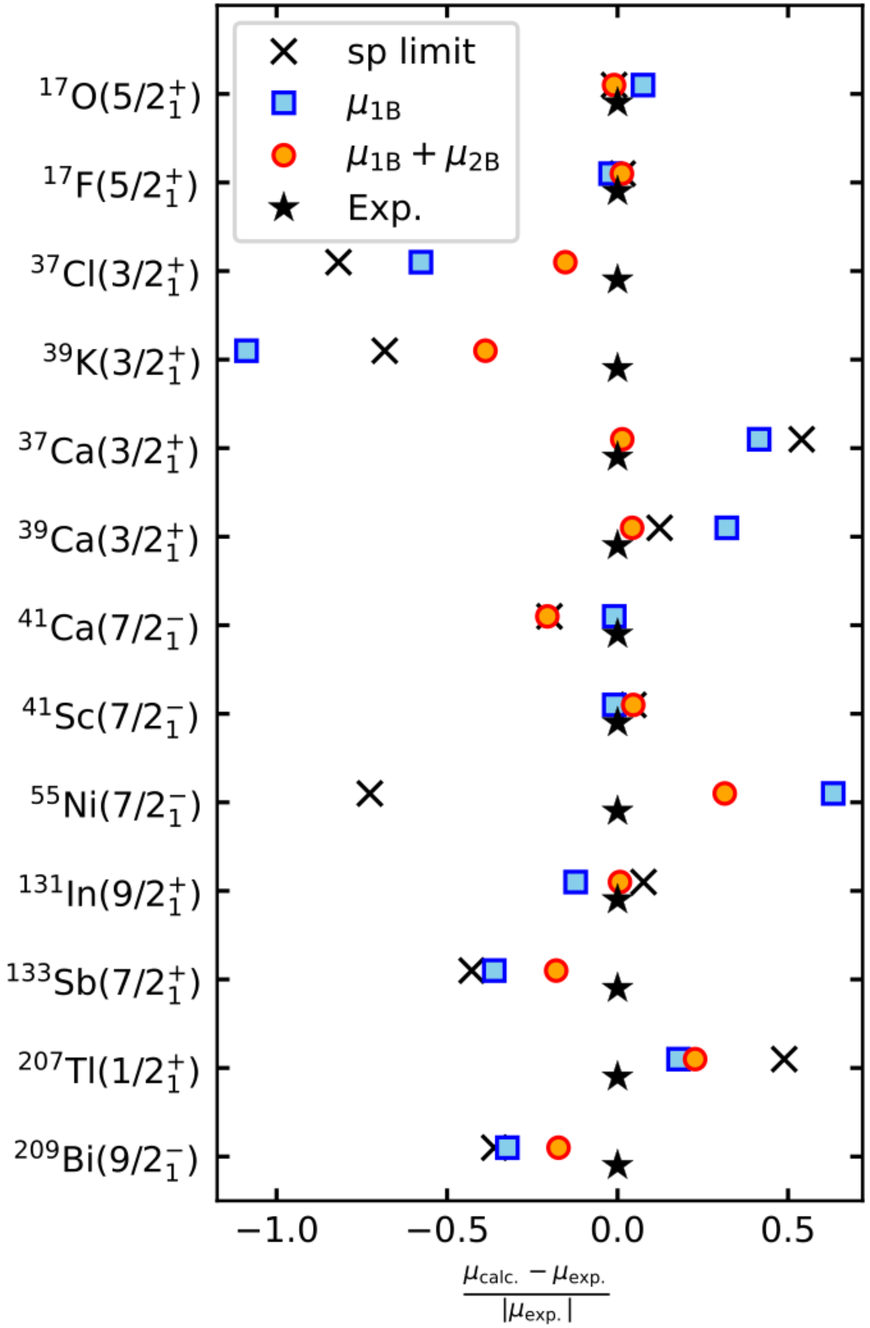
Magnetic dipole moments

- Magnetic moment from IMSRG.
  - ✦ 1.8/2.0 (EM) interaction

- Single-particle analytical limits do not always explain the experimental data.

- IMSRG gives better agreements, but not perfect

- 2BC globally improves the magnetic moments.
  - ✦ Enhancement from 2BC



TM et al., Phys. Rev. Lett. **132**, 232503 (2024).

Is ^{40}Ca magic?

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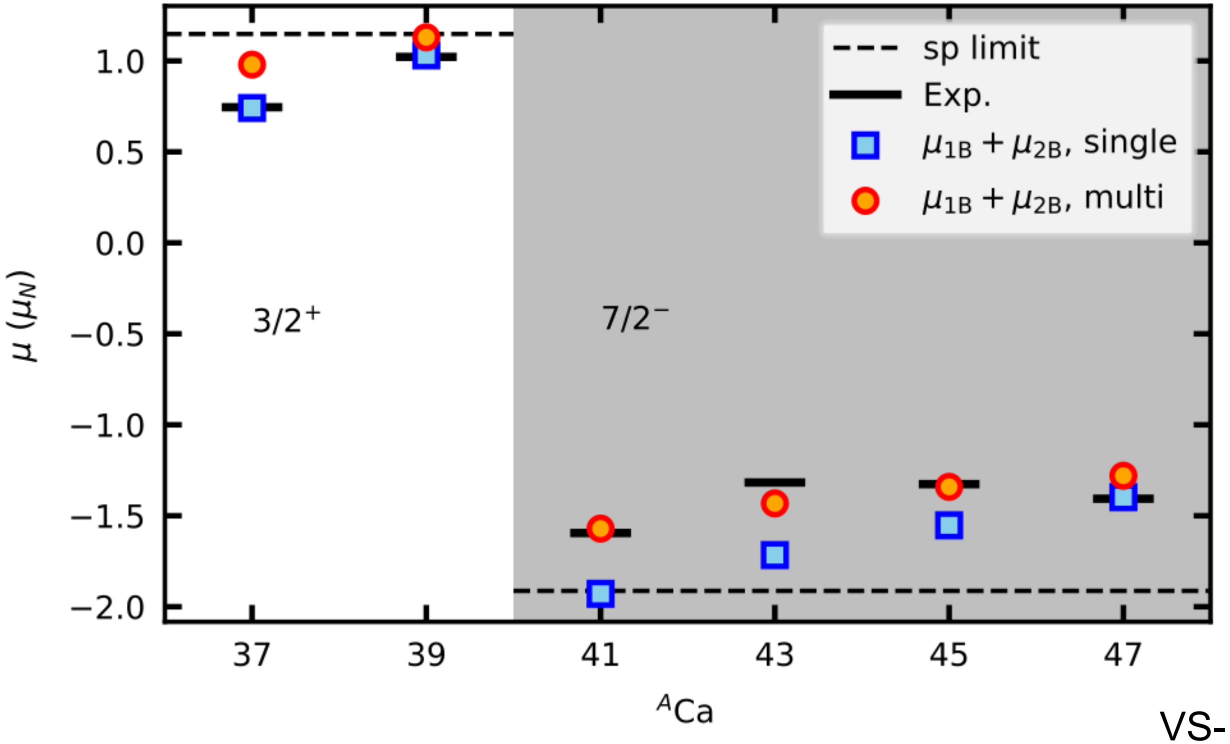
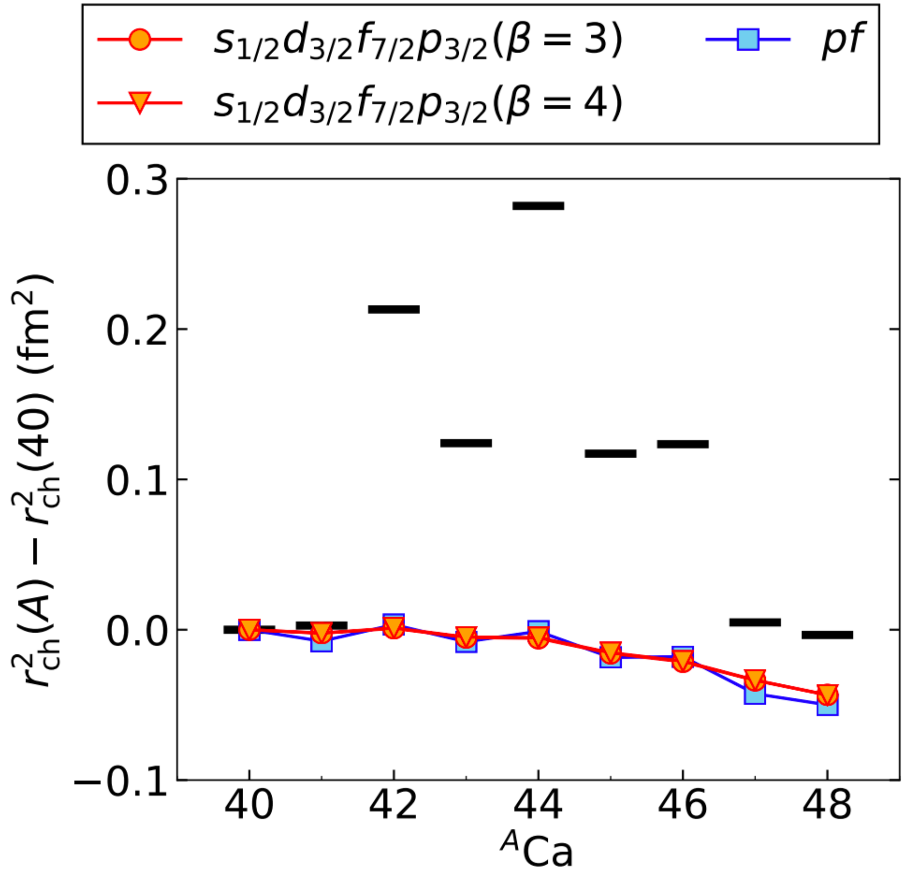
- 2BC makes agreement worse.
- Activating the ^{40}Ca core explains the magnetic moments better.
- The radii are not explained. Further investigations are needed!



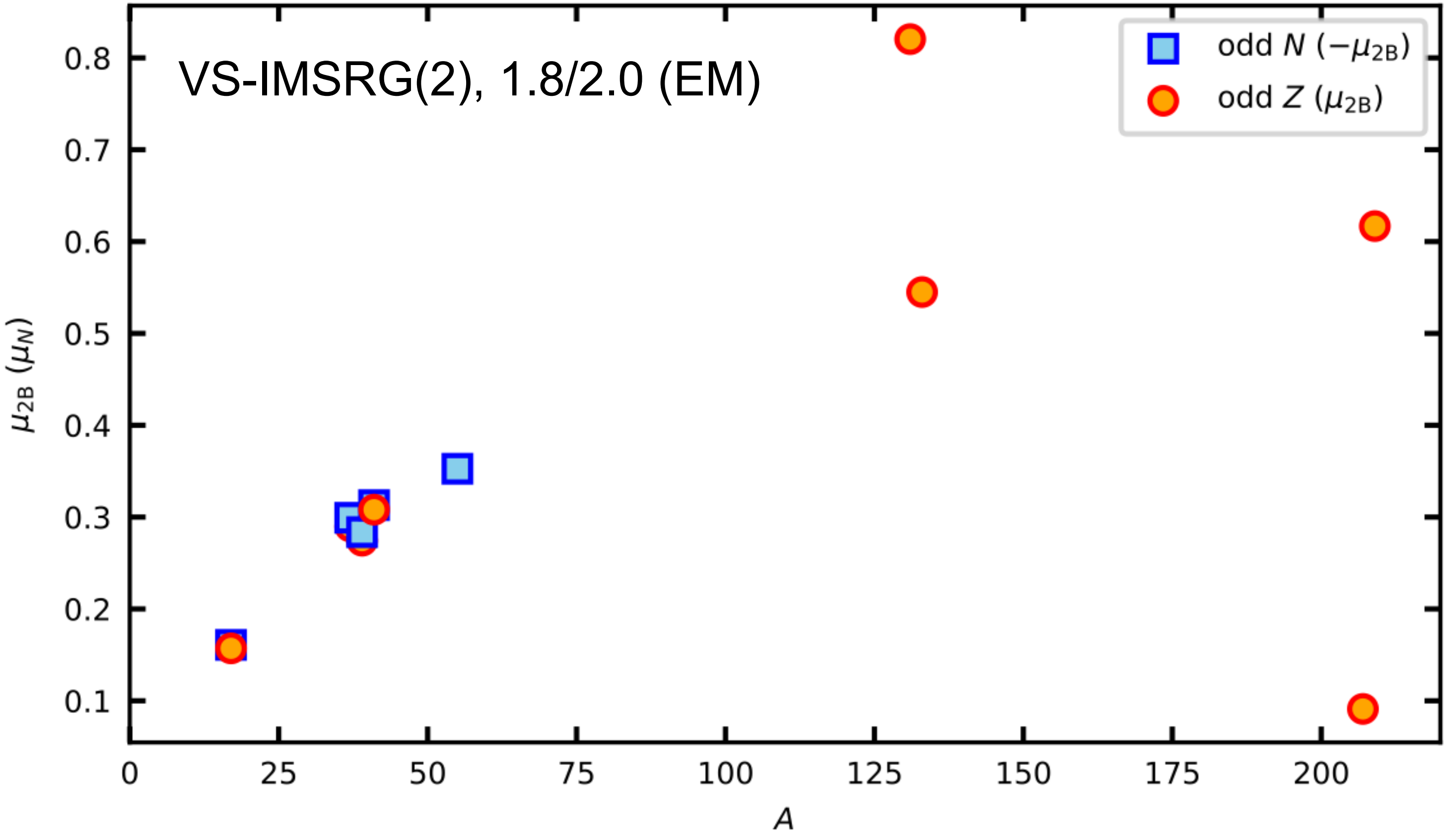
VS-IMSRG(2), 1.8/2.0 (EM)



TM et al., Phys. Rev. Lett. 132, 232503 (2024).

Mass dependence of 2B contribution

- The size of 2BC contribution is larger in heavier systems.



TM et al., Phys. Rev. Lett. 132, 232503 (2024).

Mass dependence of 2B contribution

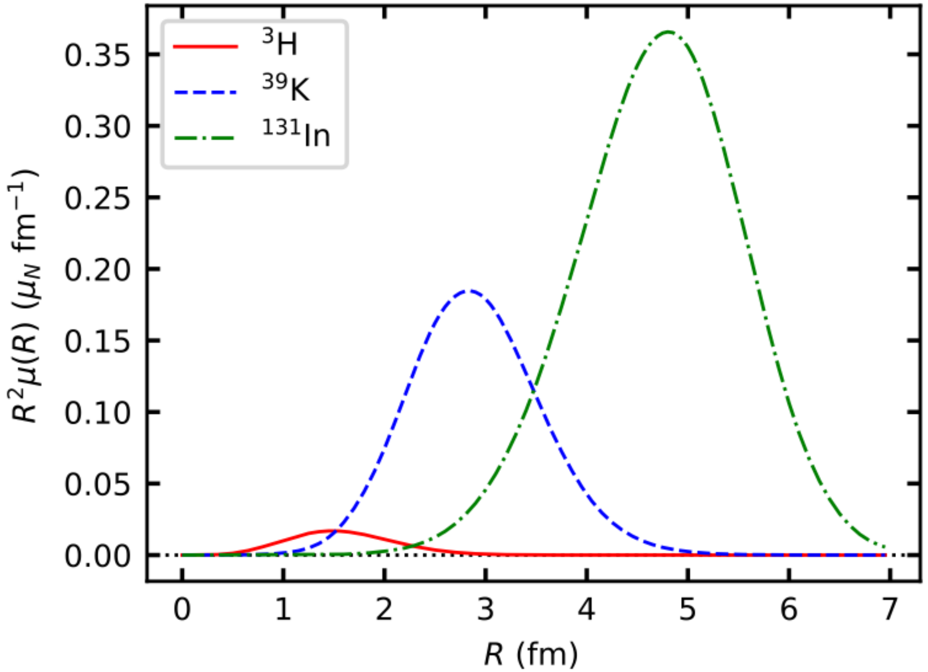
$$\mu^{\rm 2B} = \mu^{\rm 2B}_{\rm intr} + \mu^{\rm 2B}_{\rm Sachs} \qquad \mu^{\rm 2B}_{\rm Sachs} \propto \sum_{i<j} (\boldsymbol{R}_{ij} \times \boldsymbol{r}_{ij}) V^{\rm OPE}(r_{ij})$$

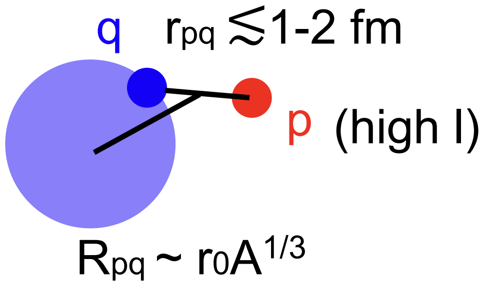
Dominant in heavy systems

- The size of 2BC contribution is larger in heavier systems.
- The simplest configuration limit is 0^+ core + 1 particle (or hole)

$$\langle J || \mu || J \rangle \sim \sum_{q \in {\rm core}} \sum_{I} f(j_p, j_q, I) \langle pq : I || \mu || pq : I \rangle$$

- $|r_p - r_q| \lesssim$ 1-2 fm because of pion-exchange potential





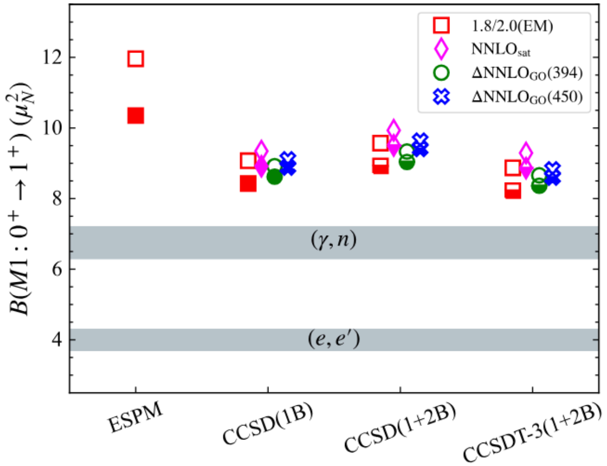
$$\boldsymbol{\mu}^{\rm Sachs}_{pq} \propto (\boldsymbol{R}_{pq} \times \boldsymbol{r}_{pq}) V^{\pi}(r_{pq})$$

The peak position moves to larger R for heavier systems.

TM et al., Phys. Rev. Lett. 132, 232503 (2024).

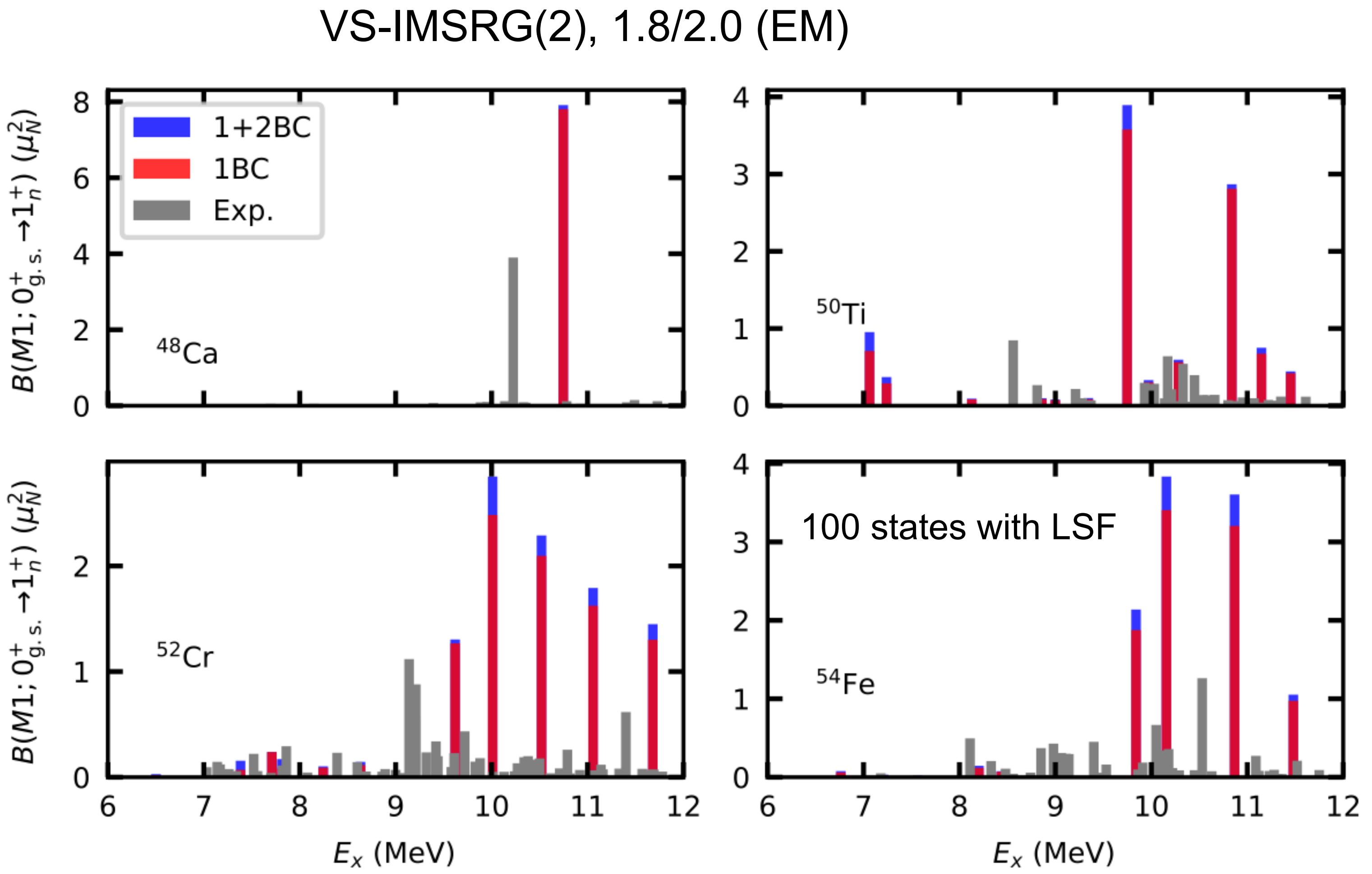
2BC effect on M1 transition

- M1 transition in pf-shell nuclei
- 2BC slightly enhances the major B(M1)'s.



Recent CC study found similar results

B. Acharya et al., Phys.Rev.Lett. 132, 232504 (2024).

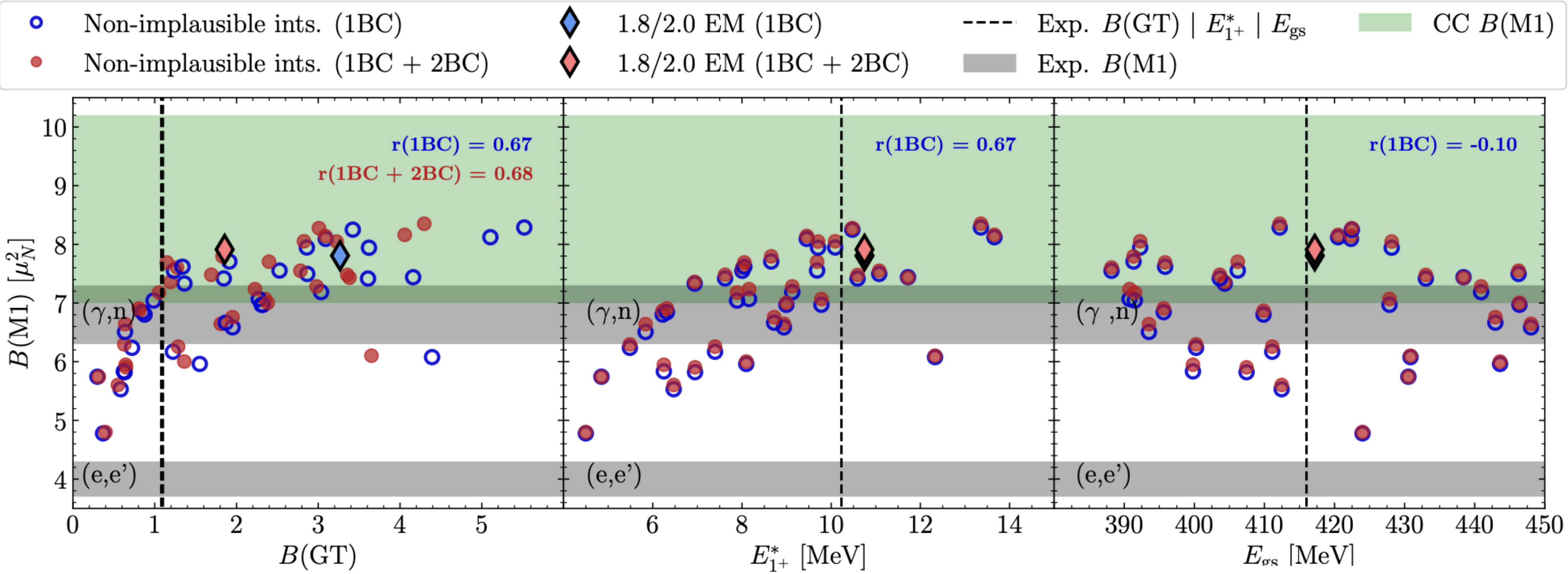


Exp. W. Steffen et al., Nucl. Phys. A 404, 413 (1983); D. I. Sober et al., Phys. Rev. C 31, 2054 (1985).

^{48}Ca M1 transition

C. Brase
TU Darmstadt

34 non-implausible interactions are from B. S. Hu et al., Nat. Phys. 18, 1196 (2022).



CC result is from B. Acharya et al., Phys.Rev.Lett. 132, 232504 (2024).

Exp.:
W. Steffen et al., Nucl. Phys. A 404, 413 (1983)
J. R. Tompkins et al., Phys. Rev. C 84, 044331 (2011).

Summary

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- Magnetic dipole moments
  - ✦ For most of doubly-closed shell nuclei +/- 1 systems, the 2BC improves the agreements.
  - ✦ 2BC effect becomes large for heavier systems due to the 2B CM dependence of the operator.

- M1 transitions
  - ✦ 2BC effects are small in all the test cases.
  - ✦ For ^{48}Ca B(M1 0+—> 1+; 10.23 MeV), the 34 NI interactions yield ~ 5 - 8 μ_N^2 .

*Uncertainty quantification is required to make a conclusion

- Future works:
  - ✦ 2BC effect with finite momentum transfer Q
  - ✦ Uncertainty quantification

Backup slides

Normal ordering wrt a single Slater determinant

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- Initial Hamiltonian is expressed with respect to nucleon vacuum

$$H = \sum_{pq} t_{pq} a_p^\dagger a_q + \frac{1}{4}\sum_{pqrs} V_{pqrs} a_p^\dagger a_q^\dagger a_s a_r + \frac{1}{36} V_{pqrstu} a_p^\dagger a_q^\dagger a_r^\dagger a_u a_t a_s$$

- Hamiltonian normal ordered with respect to a single Slater determinant

$$H = E_0 + \sum_{pq} f_{pq}\{a_p^\dagger a_q\} + \frac{1}{4}\sum_{pqrs} \Gamma_{pqrs}\{a_p^\dagger a_q^\dagger a_s a_r\} + \frac{1}{36} W_{pqrstu}\{a_p^\dagger a_q^\dagger a_r^\dagger a_u a_t a_s\}$$

$$E_0 = \sum_{pq} t_{pq}\rho_{pq} + \frac{1}{2}\sum_{pqrs} V_{pqrs}\rho_{pr}\rho_{qs} + \frac{1}{6}\sum_{pqrstu} V_{pqrstu}\rho_{ps}\rho_{qt}\rho_{ru}, \quad \Gamma_{pqrs} = V_{pqrs} + \sum_{tu} V_{pqtrsu}\rho_{tu}$$

$$f_{pq} = t_{pq} + \sum_{rs} V_{prqs}\rho_{rs} + \frac{1}{2}\sum_{rstu} V_{prsqtu}\rho_{rt}\rho_{su}, \quad W_{pqrstu} = V_{pqrstu}$$

- Normal ordered two-body (NO2B) approximation:

$$H \approx E_0 + \sum_{pq} f_{pq}\{a_p^\dagger a_q\} + \frac{1}{4}\sum_{pqrs} \Gamma_{pqrs}\{a_p^\dagger a_q^\dagger a_s a_r\}$$

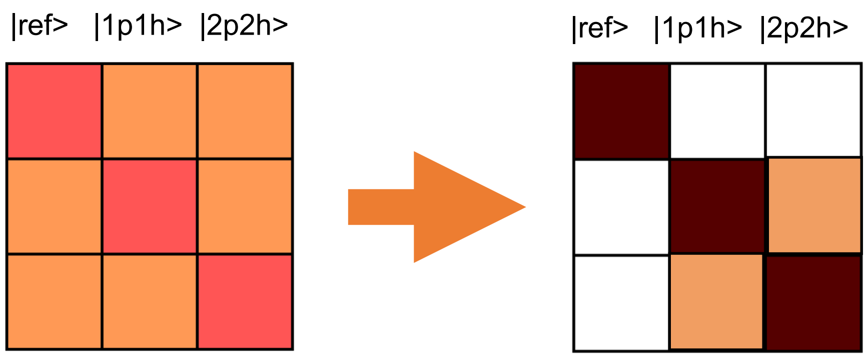
Many-body problem: similarity transformation methods

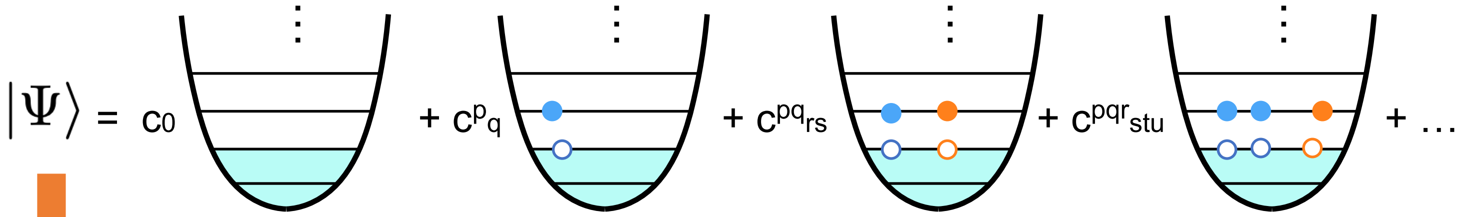
- Similarity transformation

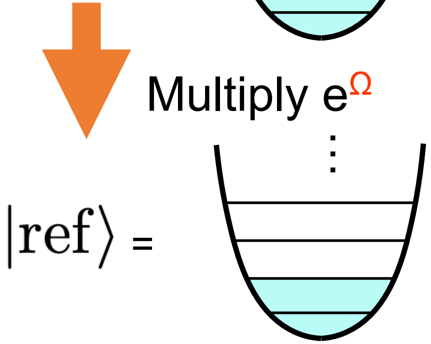
$$H|\Psi\rangle = E_{\mathrm{g.s.}}|\Psi\rangle$$

$$e^{\Omega}He^{-\Omega}e^{\Omega}|\Psi\rangle = E_{\mathrm{g.s.}}e^{\Omega}|\Psi\rangle \quad \text{Multiply } e^{\Omega} \text{ to both side}$$

$$\tilde{H}|\mathrm{ref}\rangle = E_{\mathrm{g.s.}}|\mathrm{ref}\rangle \quad \text{Similarity transformation}$$







All the complicated stuff is taken over by Ω.

- How can we find Ω operator?
  - Coupled-cluster method (CCM), in-medium similarity renormalization group (IMSRG), …

Model-space convergence

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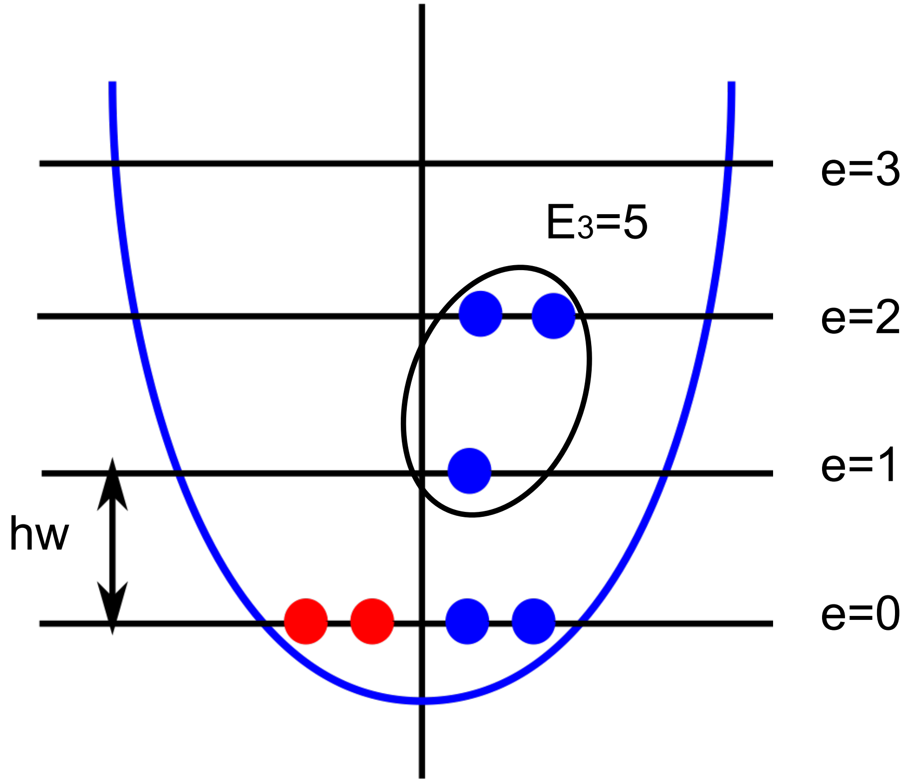
- NN+3N Hamiltonian (harmonic oscillator basis)

- Parameters:
  - ✦ hw
  - ✦ e_{max}=max(2n+l)*
  - ✦ E_{3max}=max(e_1+e_2+e_3).

- As e_{max} and E_{3max} increases, the observable should not depend on all the parameters.



*Equivalent to (number of major shells)+1

E_{3max} convergence in heavy nuclei

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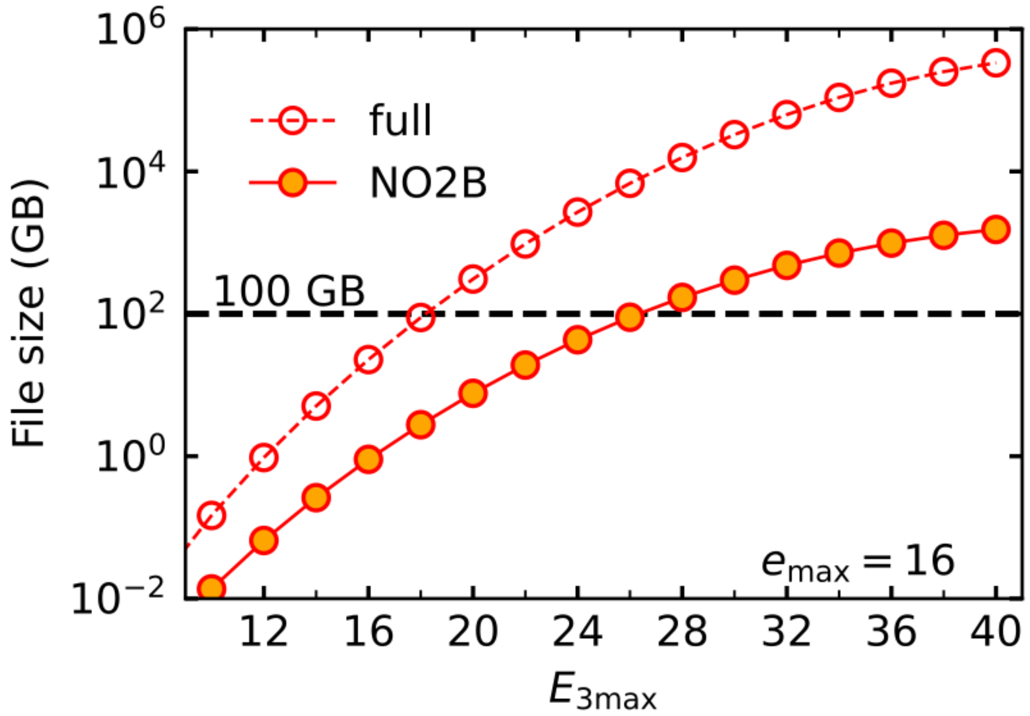
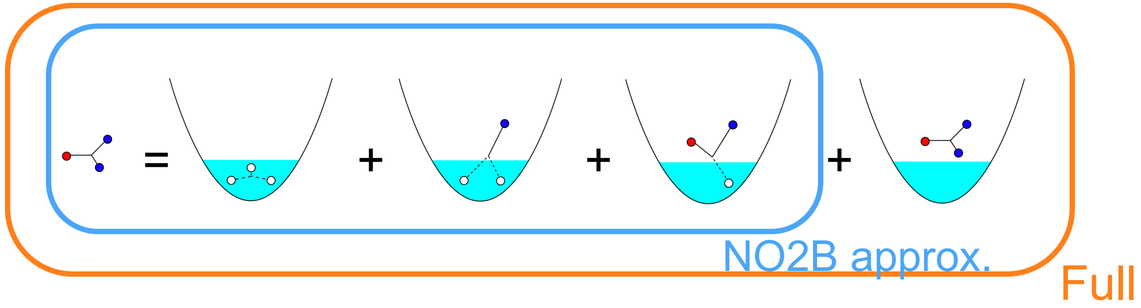
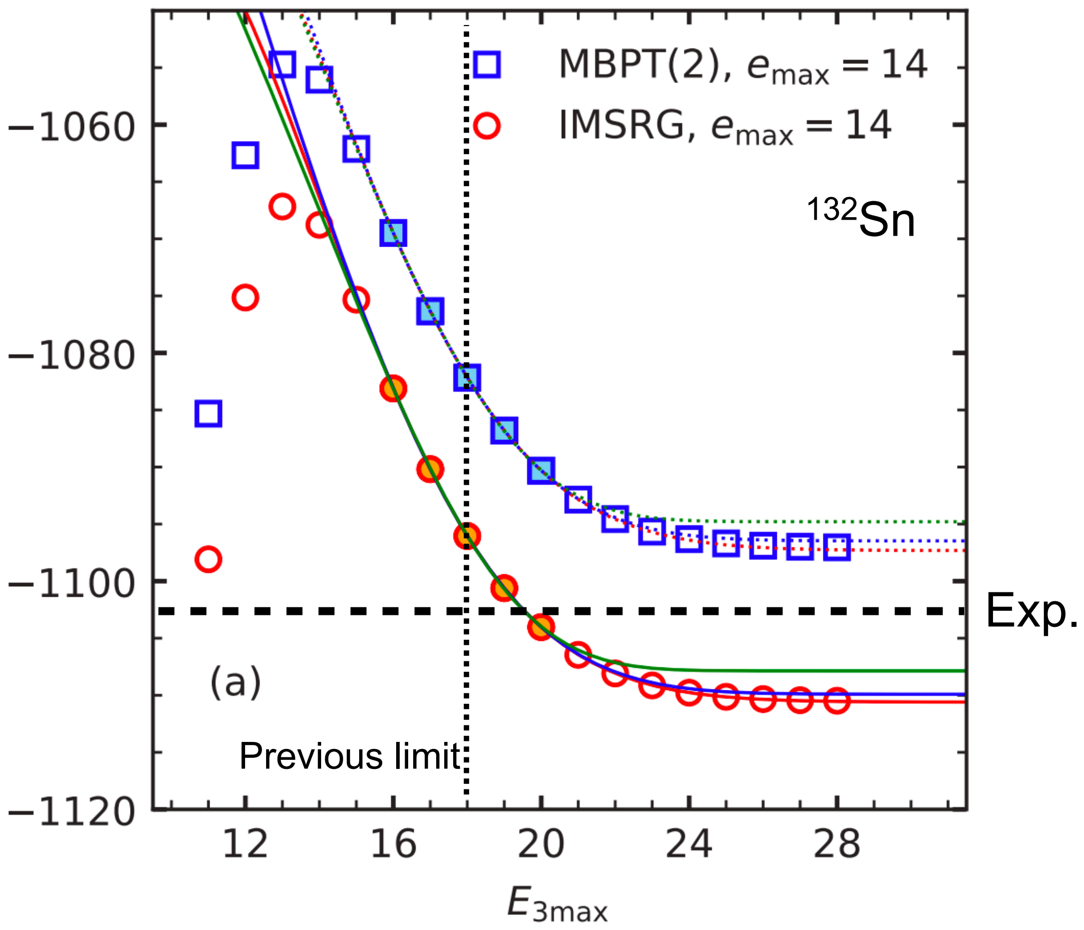




NO2B approximation error ~ a few %
[S. Binder et al., Phys. Rev. C 87, 021303 (2013).]

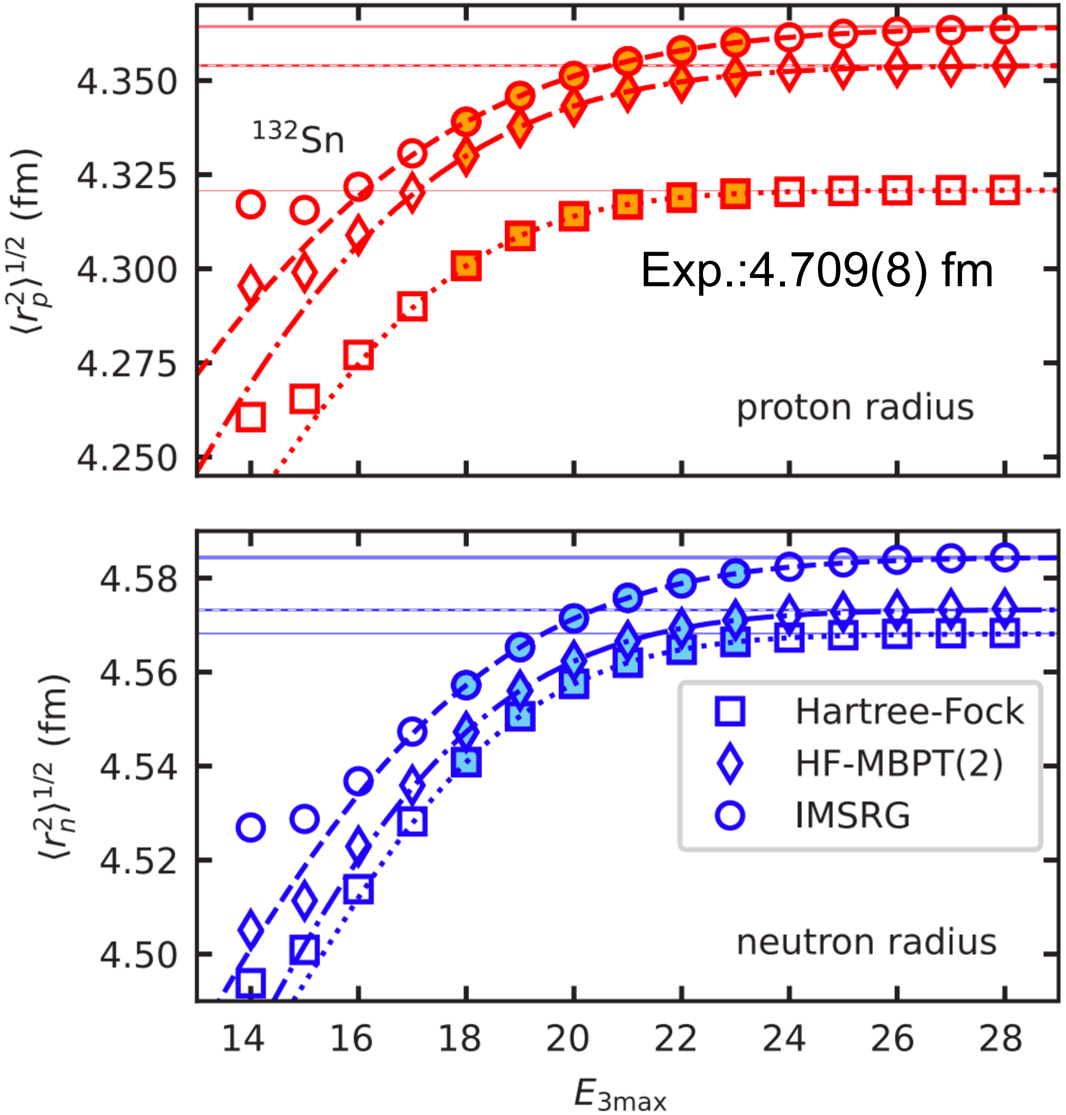
TM, S. R. Stroberg, P. Navrátil, K. Hebeler, and J. D. Holt, Phys. Rev. C 105, 014302 (2022).

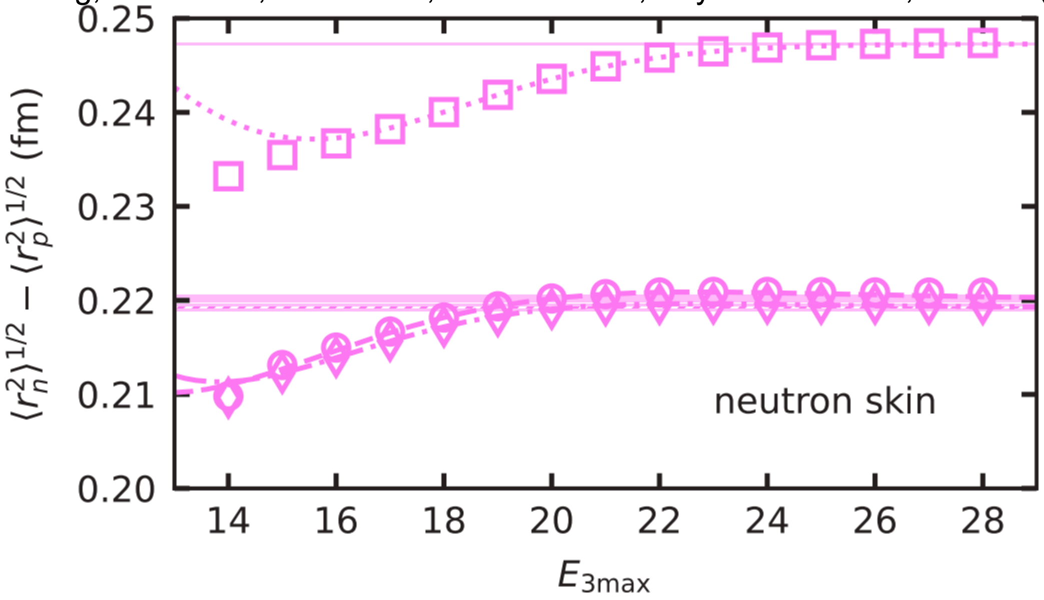


Asymptotic form: $E \approx A\gamma_{\frac{2}{n}}\left[\left(\frac{E_{3\mathrm{max}}-\mu}{\sigma}\right)^n\right] + E_\infty$

Radii

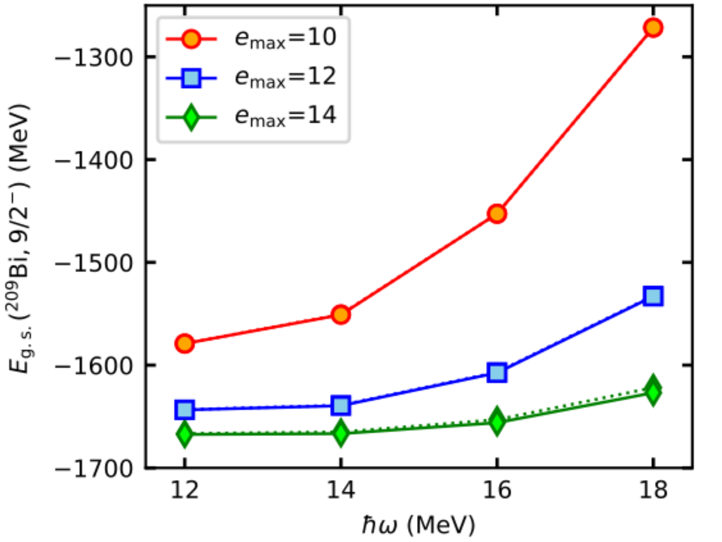
TM, S. R. Stroberg, P. Navrátil, K. Hebeler, and J. D. Holt, Phys. Rev. C 105, 014302 (2022).

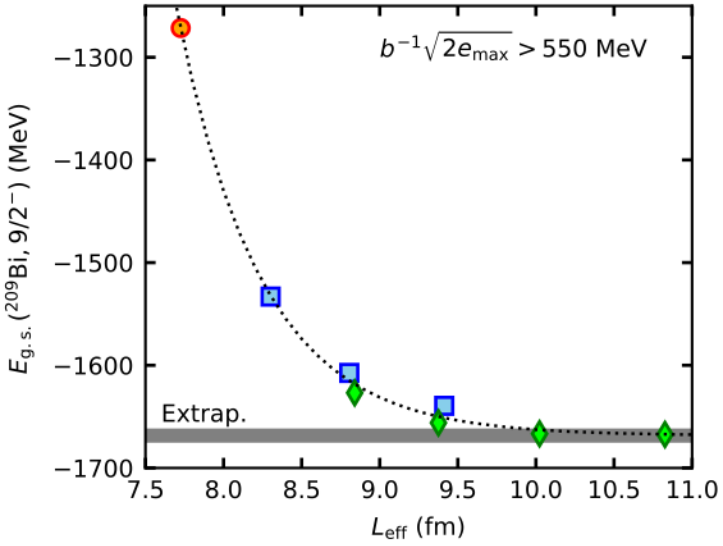




Asymptotic form: $\langle r^2\rangle \approx A\gamma_{\frac{2}{n}}\left[\left(\frac{E_{3\max}-\mu}{\sigma}\right)^n\right] + \langle r^2\rangle_\infty$

Convergence of ^{209}Bi





$$E(L_{\rm eff}) = E_\infty + A_\infty \exp(-2k_\infty L_{\rm eff})$$

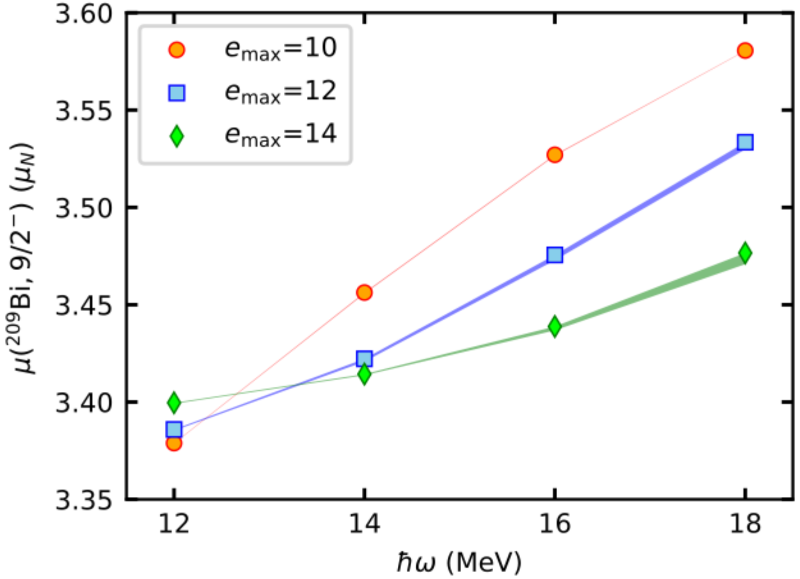
$$L_{\rm eff} = \sqrt{\frac{\sum_{nl} n_{nl}^{\rm occ} a_{nl}^2}{\sum_{nl} n_{nl}^{\rm occ} \kappa_{nl}^2}}, \quad \kappa_{nl}^2 \approx \frac{a_{nl}^2}{2b^2(N_l + 7/2)}$$

$$b^2 = \frac{\hbar}{m\omega}$$

$$N_l = \begin{cases} e_{\rm max} & e_{\rm max} + l \equiv 0 \pmod 2 \\ e_{\rm max} - 1 & e_{\rm max} + l \equiv 1 \pmod 2 \end{cases}$$

$n_{nl}^{\rm occ}$: occupation number of an orbit specified by n and l

a_{nl} : $(n+1)$-th zero of the spherical Bessel function



Magnetic moments of In isotopes

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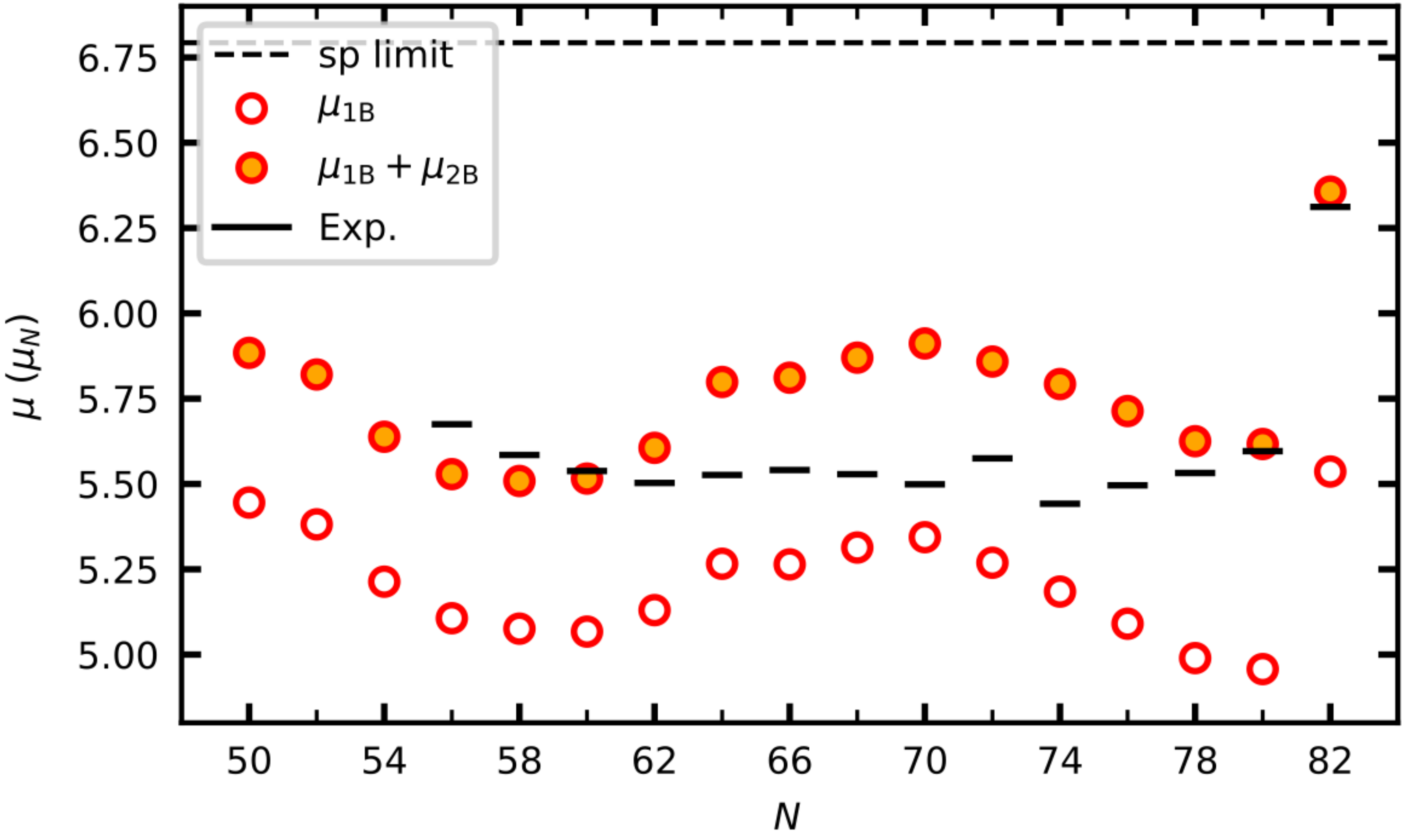
VS-IMSRG(2), 1.8/2.0 (EM), e_{max}=14, E_{3max}=24, hw = 16 MeV



2B contribution with the simplest limit

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- Expectation value: $\langle J||\mu||J\rangle$
- The simplest limit: $|JM\rangle = [|j_1 \dots j_{A-1} : 0^+\rangle \otimes |j_p m_p\rangle]\delta_{j_p J}\delta_{m_p M}$
- The expectation value depends a particle in the core and last unpaired particle.

$$\langle J||\mu||J\rangle \approx \delta_{J j_p} \sum_{q\in\text{core}} \langle p0 : j_p||\mu_{pq}||p0 : j_p\rangle$$

$$= \delta_{J j_p} \sum_{q\in\text{core}} \sum_{I} \frac{2I+1}{(2j_p+1)(2j_q+1)} \langle((pq)I, q : j_p||\mu_{pq}||(pq)I, q : j_p\rangle$$

$$= \delta_{J j_p} \sum_{q\in\text{core}} \sum_{I} \frac{2I+1}{2j_q+1} (-1)^{j_p+j_q+I+1} \begin{Bmatrix} j_p & I & j_q \\ I & j_p & 1 \end{Bmatrix} \langle pq : I||\mu||pq : I\rangle$$

2B contribution with the simplest limit

- The simplest limit: $|JM\rangle = [|j_1 \ldots j_{A-1} : 0^+\rangle \otimes |j_p m_p\rangle]\delta_{j_p J}\delta_{m_p M}$

- A simpler expression:

$$\langle \mu \rangle \sim \sum_{q \in \mathrm{core}} \langle pq|\bar{\mu}|pq\rangle$$

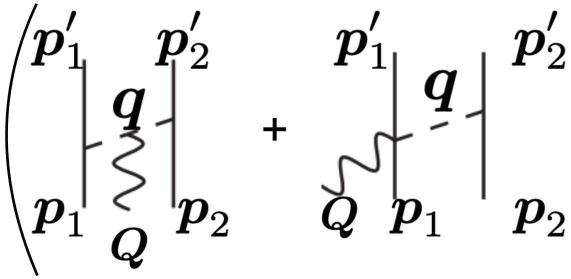
$$\langle pq|\bar{\mu}|pq\rangle = \delta_{J j_p}\sqrt{\frac{1}{2J+1}}\mathcal{C}^{J1J}_{J0J}\sum_I \frac{2I+1}{2j_q+1}(-1)^{j_p+j_q+I+1}\begin{Bmatrix} j_p & I & j_q \\ I & j_p & 1 \end{Bmatrix}$$

$$\times \frac{\sqrt{2I+1}}{\mathcal{C}^{I1I}_{J+m_q 0 J+m_q}}\left[\mathcal{C}^{j_p j_q I}_{J m_q J+m_q}\right]^2 \frac{1}{1+\delta_{n_p n_q}\delta_{l_p l_q}\delta_{j_p j_q}\delta_{t_{z,p} t_{z,q}}}\langle pq|\mu|pq\rangle$$

Magnetic dipole operator

$$\boldsymbol{\mu} = -\frac{i}{2}\nabla_{\boldsymbol{Q}} \times \left(\text{diagram} \right) \to \mu_N \sum_i \left(g_i^s \boldsymbol{\sigma}_i + g_i^l \boldsymbol{l}_i \right) \qquad (Q \to 0)$$

$$\boldsymbol{\mu} = -\frac{i}{2}\nabla_{\boldsymbol{Q}} \times \left(\text{diagram} + \text{diagram} \right) \to \sum_{i<j} \boldsymbol{\mu}_{ij}^{\text{intr}} + \boldsymbol{\mu}_{ij}^{\text{Sachs}} \qquad (Q \to 0)$$



$$\boldsymbol{\mu}_{ij}^{\text{intr}} = -\mu_N \frac{g_A^2 m_\pi m_p}{16\pi f_\pi^2} (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z \left\{ \left(1 + \frac{1}{x_{ij}}\right) \frac{[(\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j)\cdot \boldsymbol{x}_{ij}]\boldsymbol{x}_{ij}}{x_{ij}^2} - (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j) \right\} e^{-x_{ij}}$$

$$\boldsymbol{\mu}_{ij}^{\text{Sachs}} = -\mu_N \frac{g_A^2 m_\pi^2 m_p}{48\pi f_\pi^2} (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z (\boldsymbol{R}_{ij} \times \boldsymbol{x}_{ij}) V_{ij}(x_{ij})$$

$$V_{ij}(x_{ij}) = \left[S_{ij} \left(1 + \frac{3}{x_{ij}} + \frac{3}{x_{ij}^2} \right) + (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \right] \frac{e^{-x_{ij}}}{x_{ij}} - \frac{1}{x_{ij}^2} (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \delta(x_{ij})$$

$$\boldsymbol{x}_{ij} = m_\pi \boldsymbol{r}_{ij}$$