α knockout and transfer strengths in heavy nuclei

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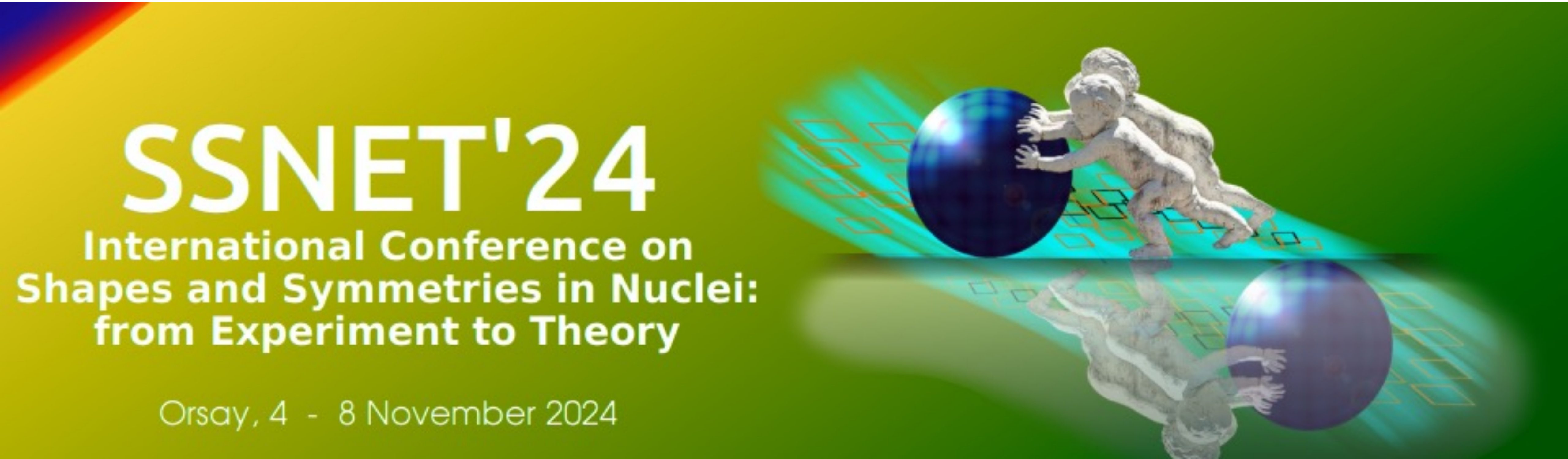
Takashi Nakatsukasa, Nobuo Hinohara
Center for Computational Sciences, University of Tsukuba



Contents

- Local α-removal strength function ⇒ α-knockout reaction
  - Distribution of α particles inside a nucleus

- In the former SSNET'22 [PRC 108, 014318 (2023)]
  - Local α-removal strength function in the HF+BCS calculation
  - Strength enhanced by the pairing correlation
  - Ground-ground transition strength consistent with experiments (Sn isotopes, $^{112\text{-}124}$Sn)

- Local α-addition strength function ⇒ α-transfer reaction
  - Distribution of open seats for α particles inside a nucleus
- Finite-size effect of the α particle
- Calculation of α-reduced width

Local α-removal strength function

$$S_\alpha^{(-)}(\boldsymbol{r},E) = \langle\Phi_0|\hat{\alpha}^\dagger(\boldsymbol{r})\delta\left(E-\hat{H}\right)\hat{\alpha}(\boldsymbol{r})|\Phi_0\rangle$$

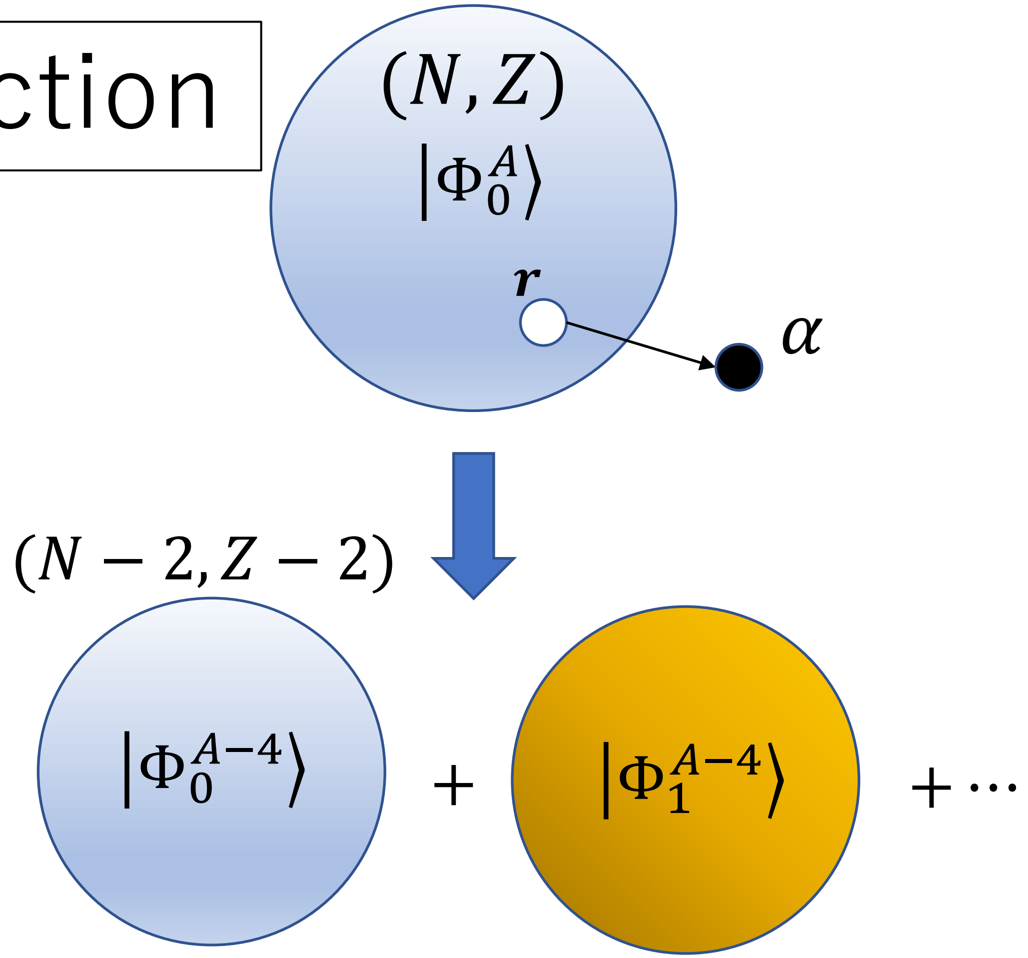
$$\hat{\alpha}(\boldsymbol{r}) = \int \phi_0^{\boldsymbol{r}}(\boldsymbol{x}_1)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_2)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_3)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_4)$$

$$\times\hat{\psi}_{n\uparrow}(\boldsymbol{x}_1)\hat{\psi}_{n\downarrow}(\boldsymbol{x}_2)\hat{\psi}_{p\uparrow}(\boldsymbol{x}_3)\hat{\psi}_{p\downarrow}(\boldsymbol{x}_4)d\boldsymbol{x}_1\cdots d\boldsymbol{x}_4$$

$$\phi_0^{\boldsymbol{r}}(\boldsymbol{x}) = \phi_{0s_{1/2}}(\boldsymbol{x}-\boldsymbol{r})$$

Mean-field (HFB) approximation

No correlations beyond mean field



$$\hat{\alpha}(\boldsymbol{r})|\Phi_0^A\rangle = \sum_n C_n(\boldsymbol{r})|\Phi_n^{A-4}\rangle$$

$$S_\alpha(\boldsymbol{r},E_n) = \sum_n |C_n(\boldsymbol{r})|^2\delta(E-E_n)$$

Local α-addition strength function

$$S_\alpha^{(+)}(\boldsymbol{r},E) = \langle \Phi_0 | \hat{\alpha}(\boldsymbol{r}) \hat{P}_c \delta(E-\hat{H}) \hat{P}_c \hat{\alpha}^\dagger(\boldsymbol{r}) | \Phi_0 \rangle$$

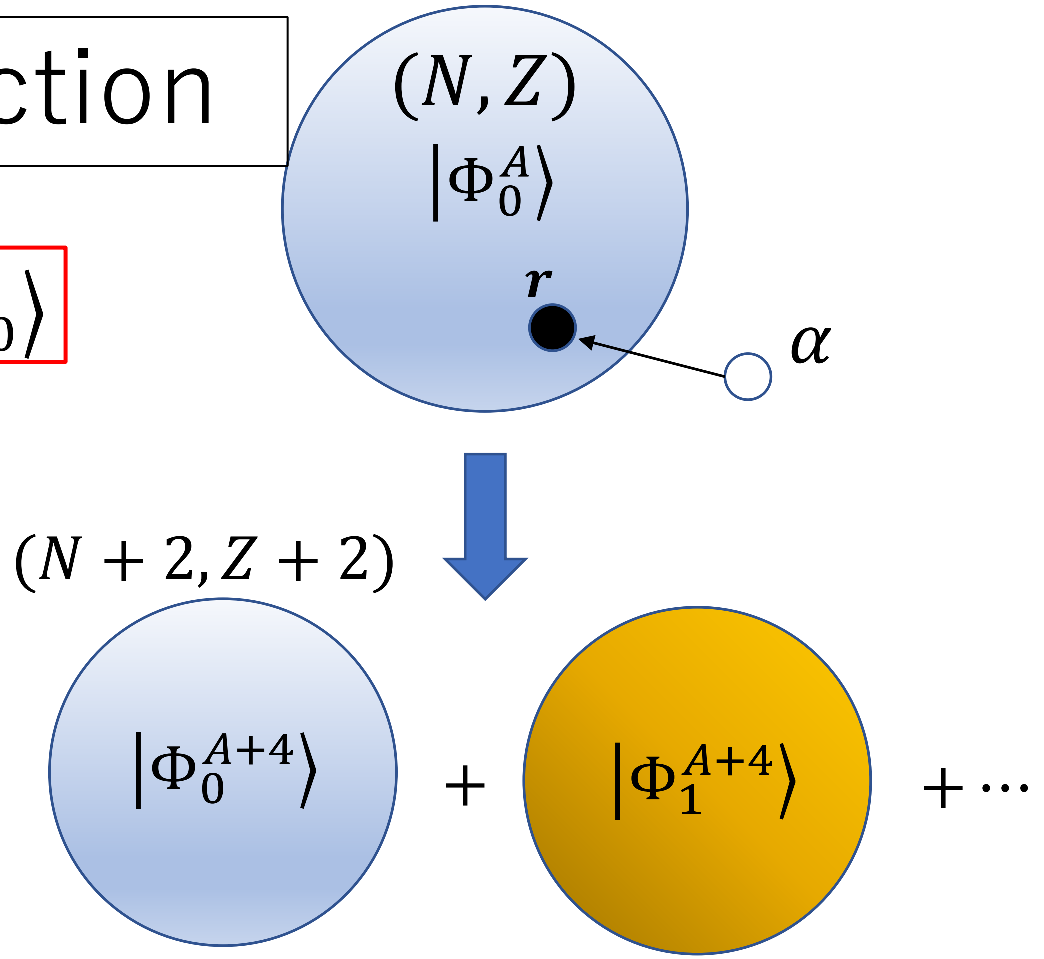
α-transfer reaction
without particle decay

$$\hat{P}_c = \theta(E_c - \hat{H})$$

Exclude unbound $U_i(\boldsymbol{r})$ wave functions

$E_i < |\lambda_\tau|$, λ_τ: chem. pot.

BCS approx.: Only bound orbitals



$$\hat{\alpha}^\dagger(\boldsymbol{r})|\Phi_0^A\rangle = \sum_n C_n(\boldsymbol{r})|\Phi_n^{A+4}\rangle$$

$$S_\alpha^{(+)}(\boldsymbol{r},E_n) = \sum_n |C_n(\boldsymbol{r})|^2 \delta(E-E_n)$$

Approximations & model

$$S_\alpha^{(-)}(\boldsymbol{r},E) = \langle \Phi_0 | \hat{\alpha}^\dagger(\boldsymbol{r}) \delta(E-\hat{H}) \hat{\alpha}(\boldsymbol{r}) | \Phi_0 \rangle$$

$$S_\alpha^{(+)}(\boldsymbol{r},E) = \langle \Phi_0 | \hat{\alpha}(\boldsymbol{r}) \hat{P}_c \delta(E-\hat{H}) \hat{P}_c \hat{\alpha}^\dagger(\boldsymbol{r}) | \Phi_0 \rangle$$

- Point-α approximation: $\phi_0^{\boldsymbol{r}}(\boldsymbol{x}) = \delta^3(\boldsymbol{x}-\boldsymbol{r})$

$$\hat{\alpha}(\boldsymbol{r}) = \hat{\psi}_{n\uparrow}(\boldsymbol{r})\hat{\psi}_{n\downarrow}(\boldsymbol{r})\,\hat{\psi}_{p\uparrow}(\boldsymbol{r})\,\hat{\psi}_{p\downarrow}(\boldsymbol{r})$$

- Hamiltonian H : Mean-field (HFB) Hamiltonian (qp excitations)
- No rearrangement of the mean fields

- SkM* EDF + monopole pairing (HF+BCS)
- Pairing gap determined by odd-even mass difference

HFB states and 2qp matrix elements

- Product of protons and neutrons

$$\left|\Phi^{(p)}\right\rangle \otimes \left|\Phi^{(n)}\right\rangle$$

- HFB vacuum

$$a_i \left|\Phi_0^{(\tau)}\right\rangle = 0, \qquad a_i\text{: quasiparticle annihilation operator}$$

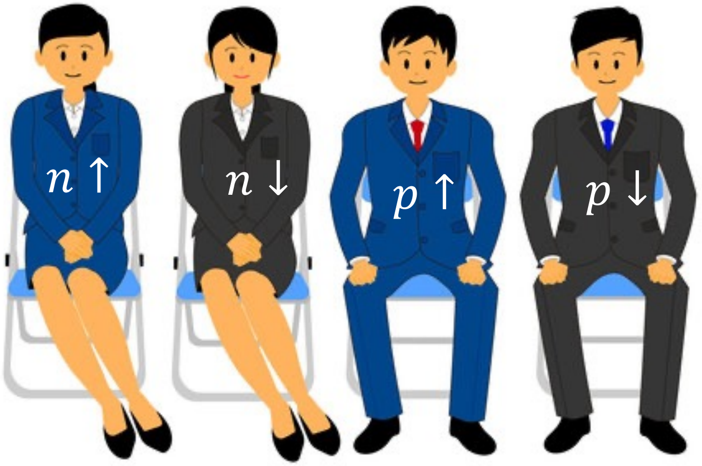
- Excited states in A-4 nucleus: π2qp-n2qp excitation

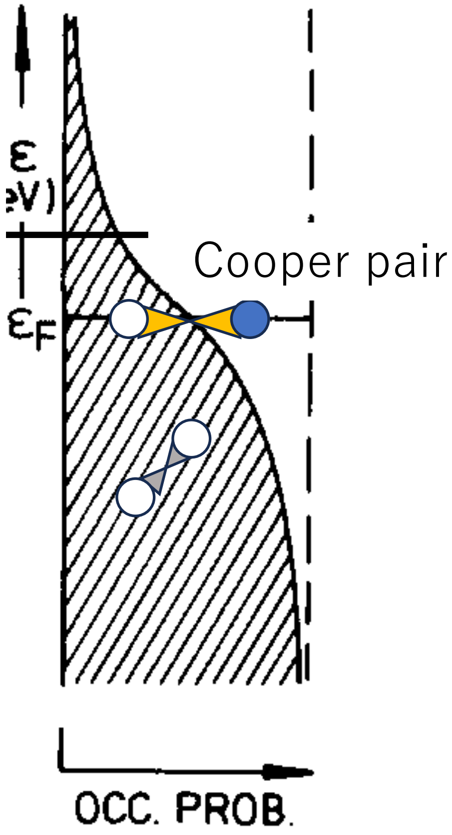
$$|ij,kl\rangle = a_i^\dagger a_j^\dagger \left|\Phi_0^{(p)}\right\rangle \otimes a_k^\dagger a_l^\dagger \left|\Phi_0^{(n)}\right\rangle$$

- 2qp matrix elements $\left\langle ij\middle|\hat{\psi}_\uparrow(\boldsymbol{r})\hat{\psi}_\downarrow(\boldsymbol{r})\middle|\Phi_0\right\rangle$

HFB wave functions

Looking for
Occupied 4 seats



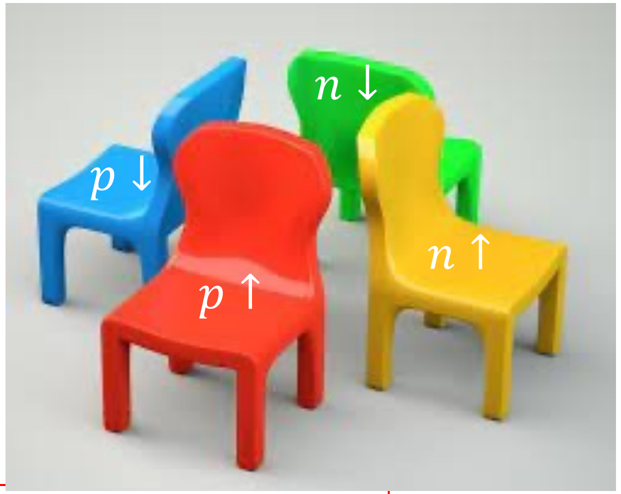


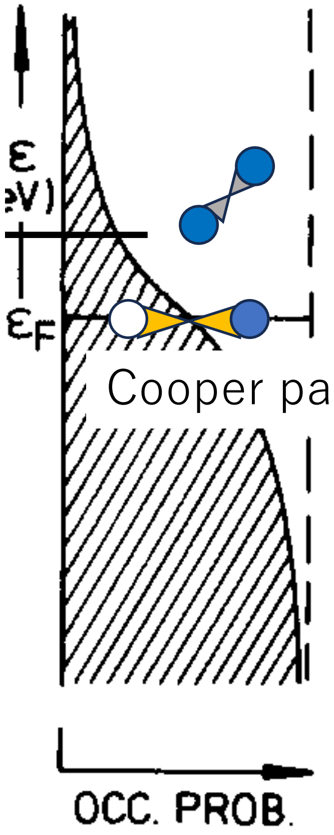
$$S_\alpha^{(-)}(\boldsymbol{r},E)=\sum_{k,k'}F_k^{(n)}(\boldsymbol{r})F_{k'}^{(p)}(\boldsymbol{r})\delta(E-E_{kk'})$$

$$F_k^{(q)}(\boldsymbol{r})=\begin{cases}\left|\kappa_q(\boldsymbol{r})\right|^2 & \text{for } k=0\\ \left|V_i^{(q)}(\boldsymbol{r}\uparrow)V_j^{(q)}(\boldsymbol{r}\downarrow)-V_j^{(q)}(\boldsymbol{r}\uparrow)V_i^{(q)}(\boldsymbol{r}\downarrow)\right|^2 & \text{for } k=(ij)_{2qp}\end{cases}$$

HFB wave functions

Looking for
Vacant 4 seats



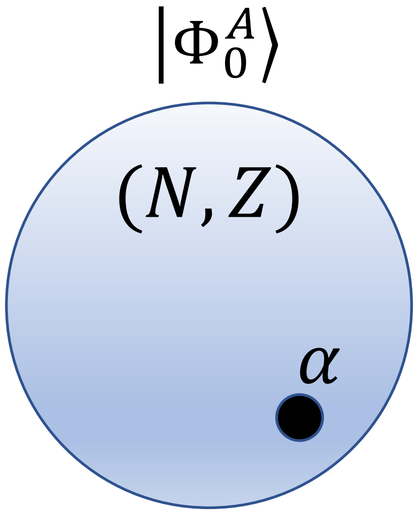


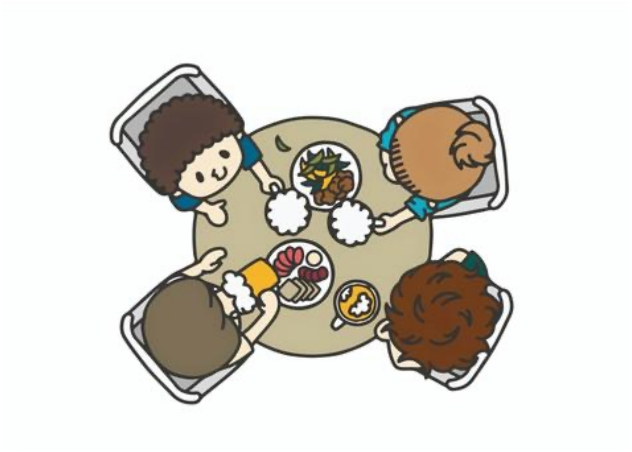
$$S_\alpha^{(+)}(\boldsymbol{r},E)=\sum_{k,k'} G_k^{(n)}(\boldsymbol{r})G_{k'}^{(p)}(\boldsymbol{r})\delta(E-E_{kk'})$$

Replacement: $V_i^*(\boldsymbol{r}) \leftrightarrow U_i(\boldsymbol{r})$

$$G_k^{(q)}(\boldsymbol{r})=\begin{cases} |\kappa_q(\boldsymbol{r})|^2 & \text{for } k=0 \\ \left|U_i^{(q)}(\boldsymbol{r}\uparrow)U_j^{(q)}(\boldsymbol{r}\downarrow)-U_j^{(q)}(\boldsymbol{r}\uparrow)U_i^{(q)}(r\downarrow)\right|^2 & \text{for } k=(ij)_{2qp} \end{cases}$$

Total local $\boldsymbol{\alpha}$-removal strength

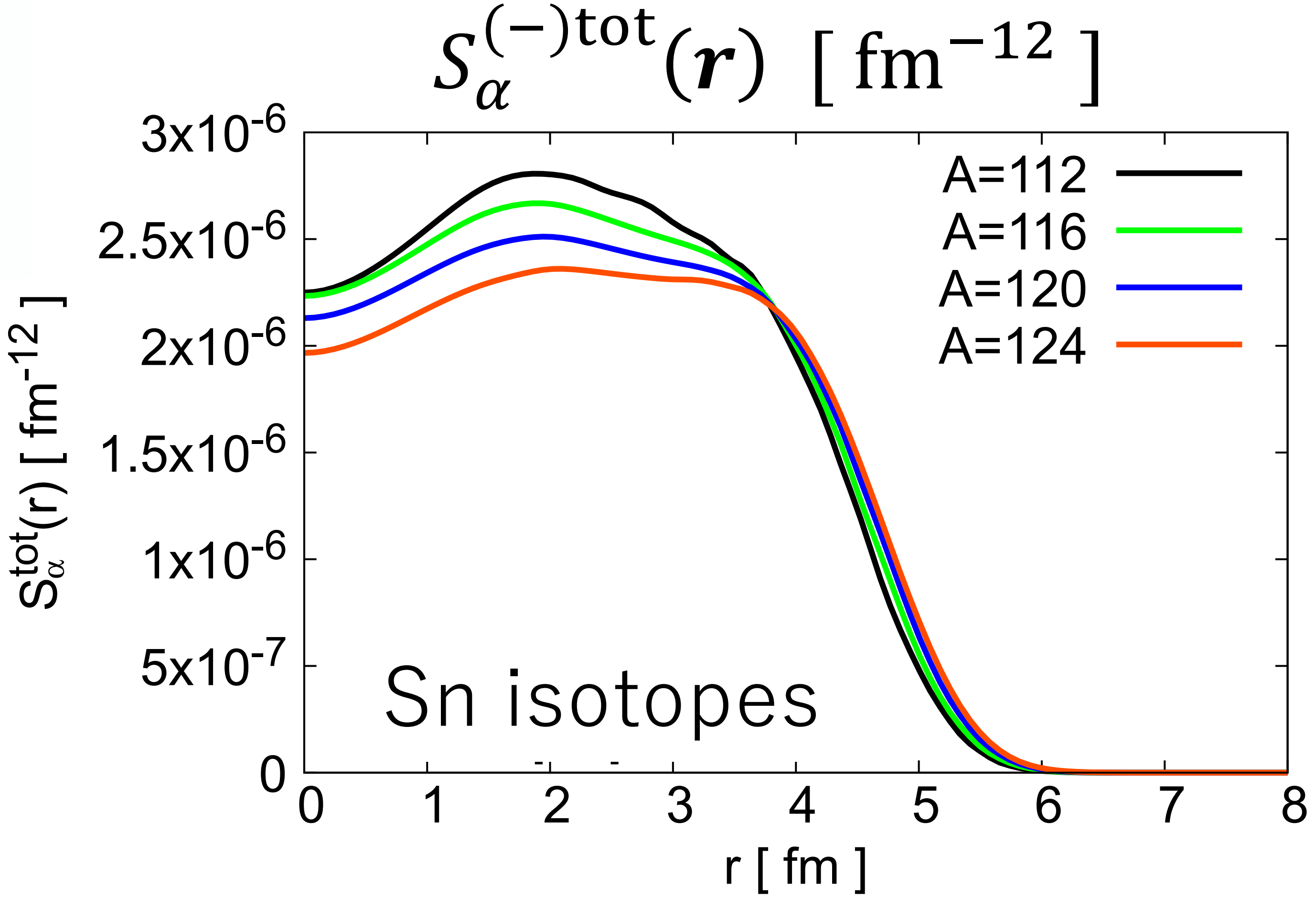




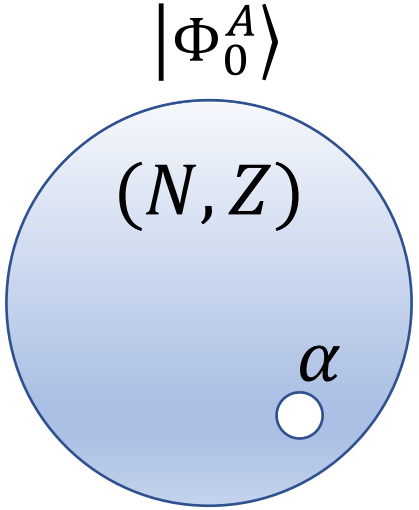
$$S_\alpha^{(-)\mathrm{tot}}(\boldsymbol{r}) \equiv \langle \Phi_0^A | \hat{\alpha}^\dagger(\boldsymbol{r}) \hat{\alpha}(\boldsymbol{r}) | \Phi_0^A \rangle$$

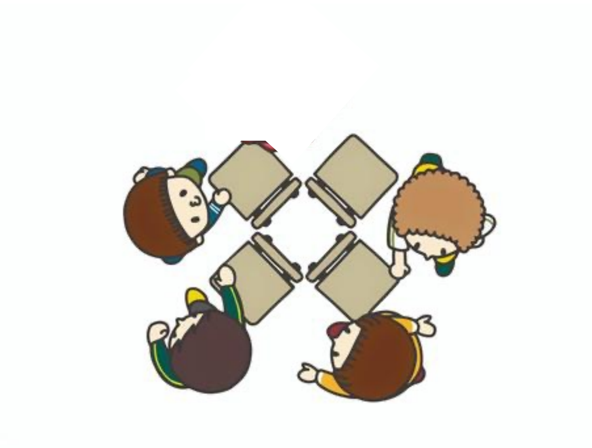
$$= \sum_n \left| \langle \Phi_n^{A-4} | \hat{\alpha}(\boldsymbol{r}) | \Phi_0^A \rangle \right|^2$$

$|\Phi_0^A\rangle$: HF+BCS state



Total local $\boldsymbol{\alpha}$-addition strength



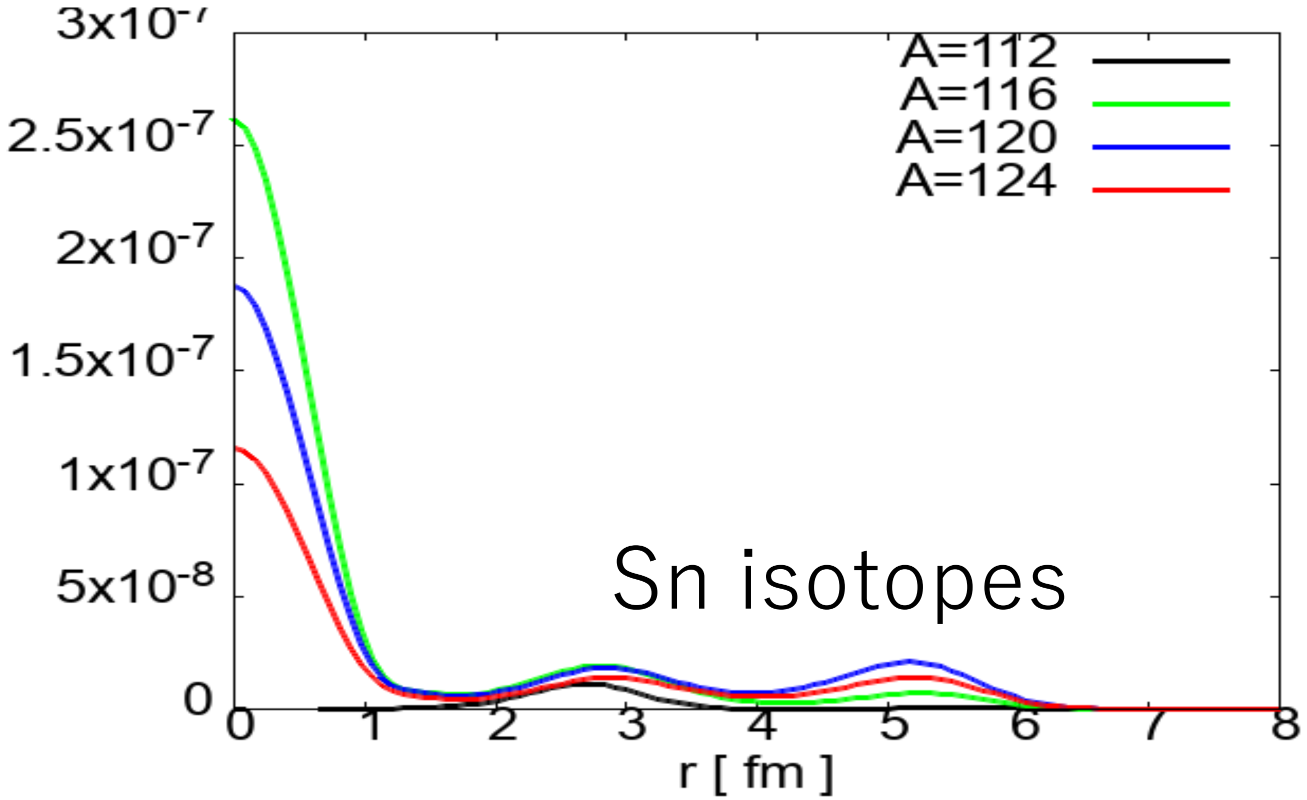


$$S_\alpha^{(+)\mathrm{tot}}(\boldsymbol{r}) \equiv \langle \Phi_0^A | \hat{\alpha}(\boldsymbol{r}) \hat{\alpha}^\dagger(\boldsymbol{r}) | \Phi_0^A \rangle$$

$$= \sum_n \left| \langle \Phi_n^{A+4} | \hat{\alpha}^\dagger(\boldsymbol{r}) | \Phi_0^A \rangle \right|^2$$

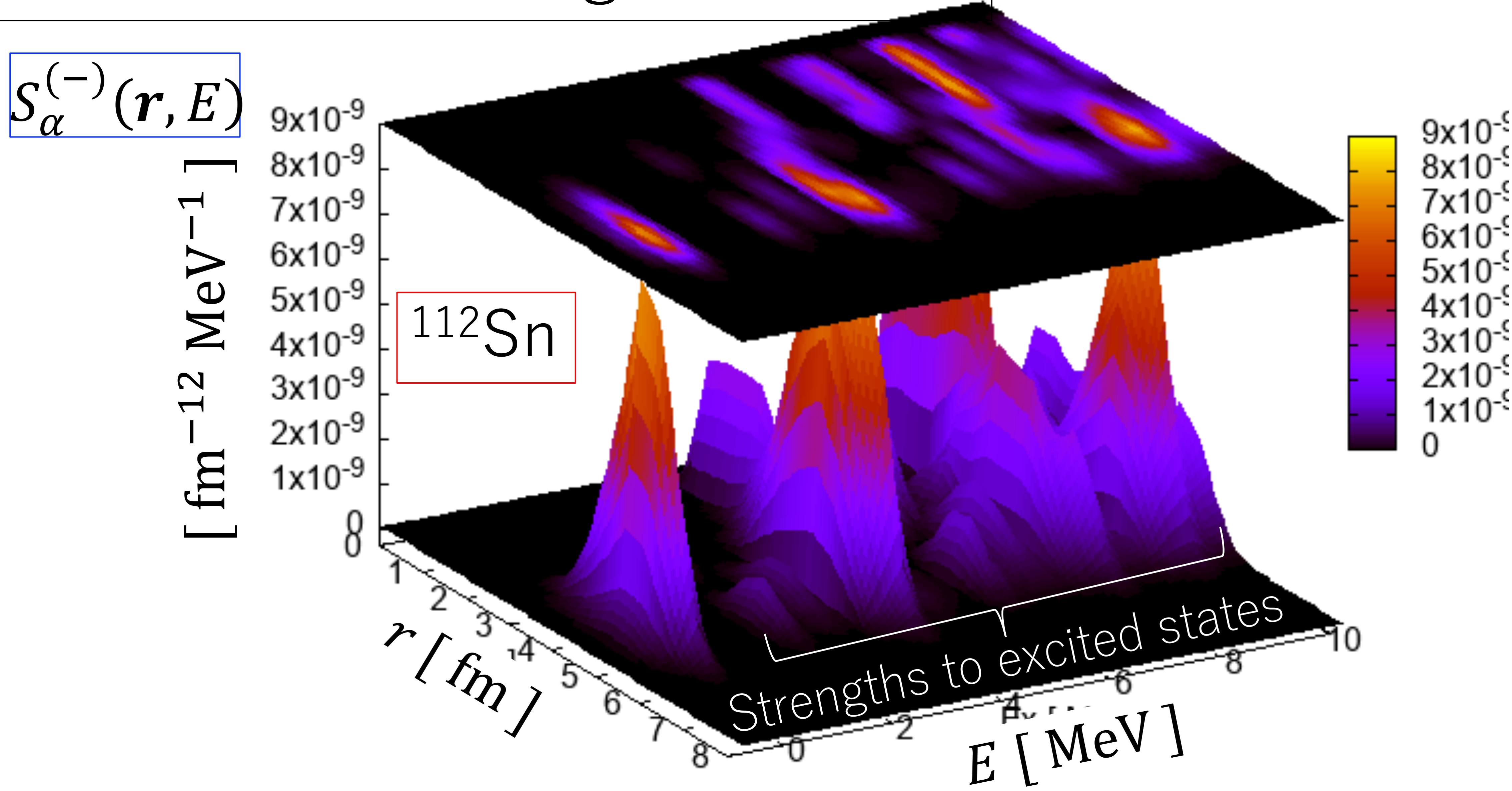
$|\Phi_0^A\rangle$: HF+BCS state

$S_\alpha^{(+)\mathrm{tot}}(\boldsymbol{r})$ [fm^{-12}]



Local α-removal strength function

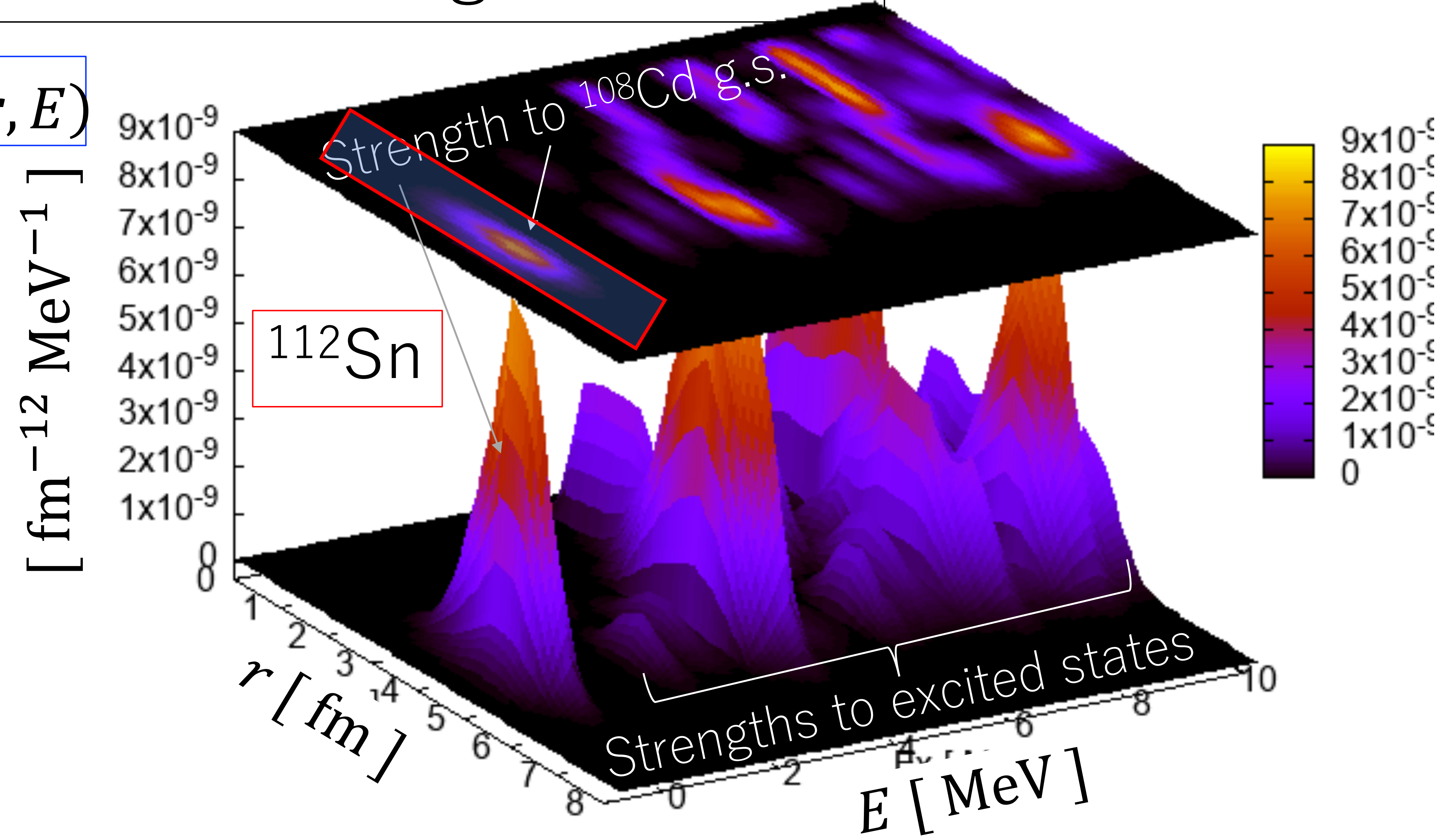
$^{A}Sn \rightarrow {}^{A-4}Cd$



Local α-removal strength function

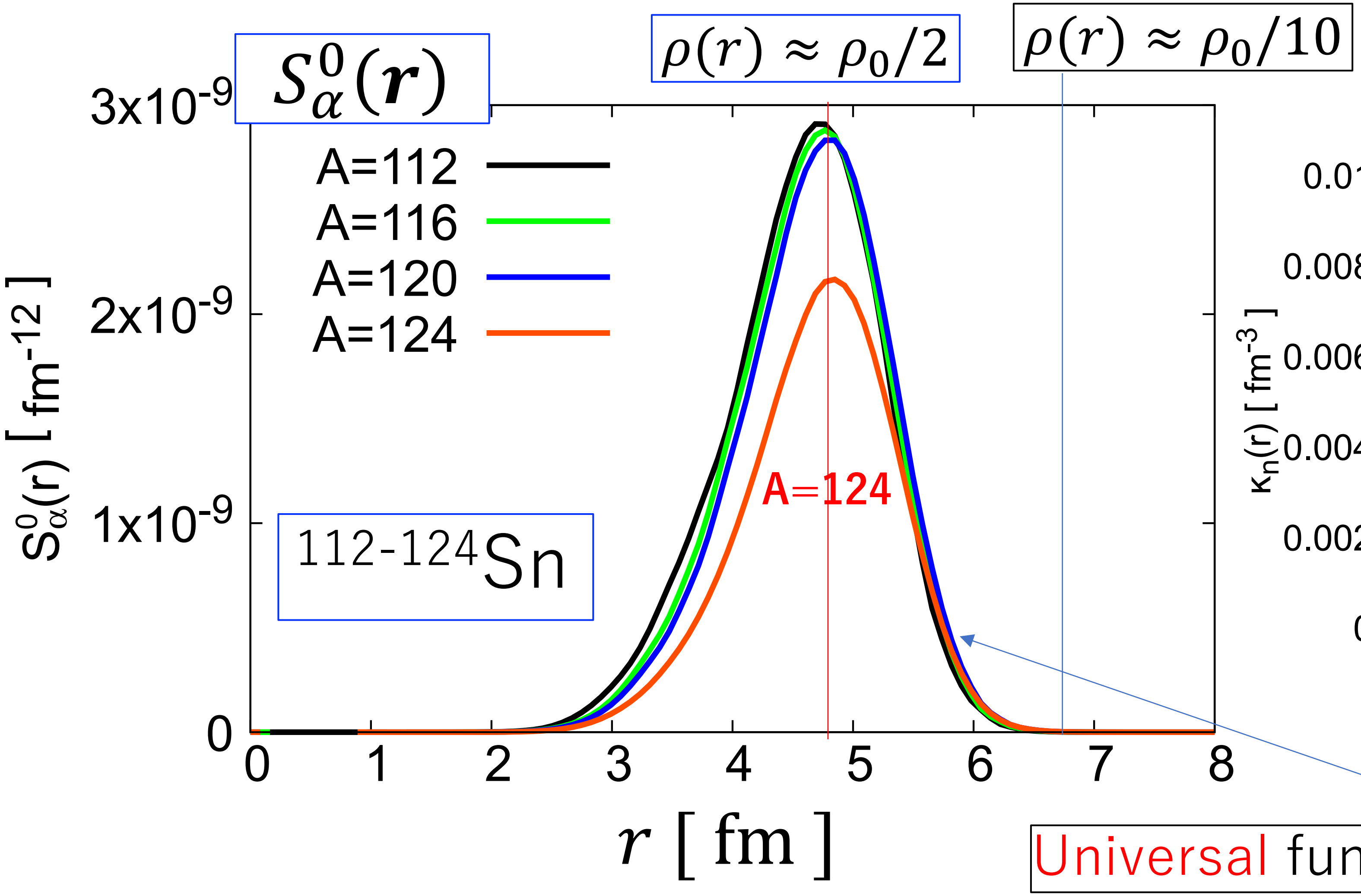
$^{A}Sn \rightarrow {}^{A-4}Cd$

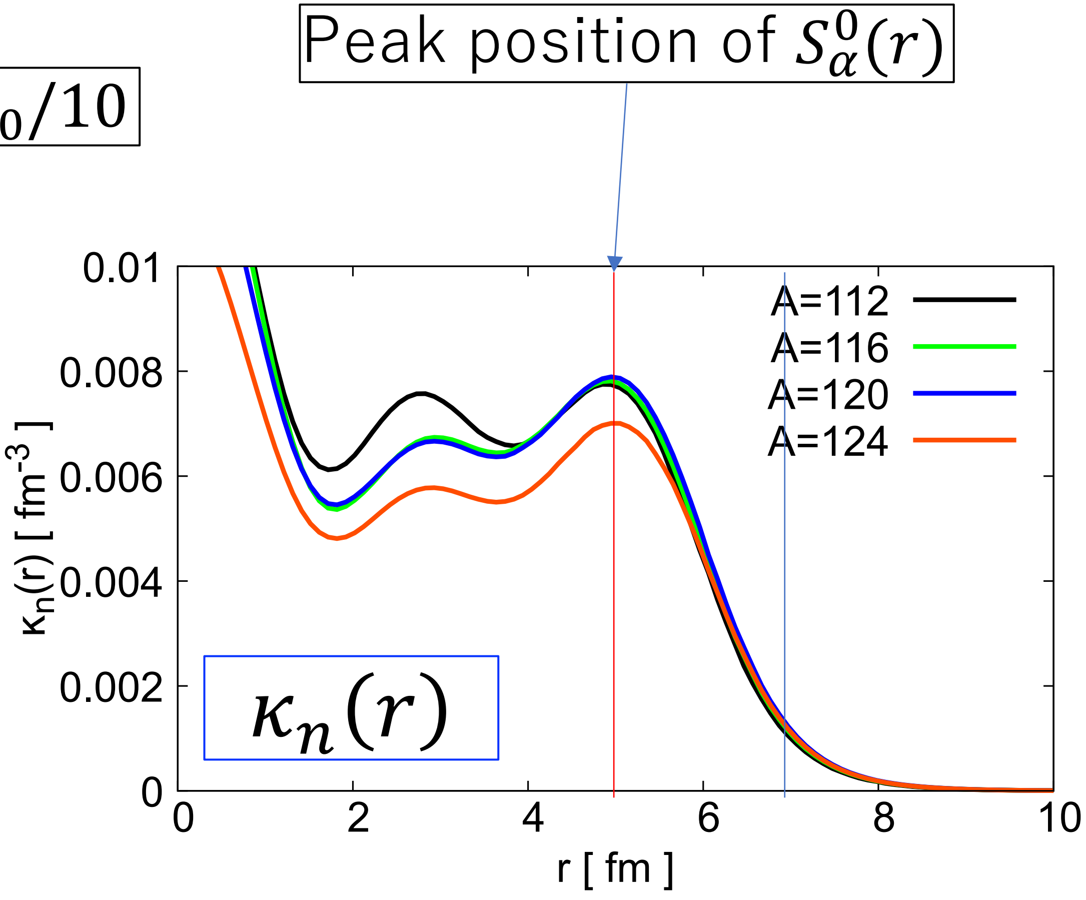
$S_{\alpha}^{(-)}(\boldsymbol{r}, E)$



Local α-removal strength: ^{A}Sn(g.s.) → $^{A-4}$Cd(g.s.)

$$S_\alpha^0(\boldsymbol{r}) \equiv \int_0^\epsilon S_\alpha(\boldsymbol{r},E)dE = \left|\langle\Phi_0^{A-4}|\hat{\alpha}(\boldsymbol{r})|\Phi_0^A\rangle\right|^2$$





Universal function $S_\alpha^0(r)$ at $r > 5.5$ fm

Normalized local α strength (g.s. → g.s.)

$$P_\alpha^0(\boldsymbol{r}) = \frac{S_\alpha^0(\boldsymbol{r})}{\rho_{n\uparrow}(\boldsymbol{r})\rho_{n\downarrow}(\boldsymbol{r})\rho_{p\uparrow}(\boldsymbol{r})\rho_{p\downarrow}(\boldsymbol{r})}$$

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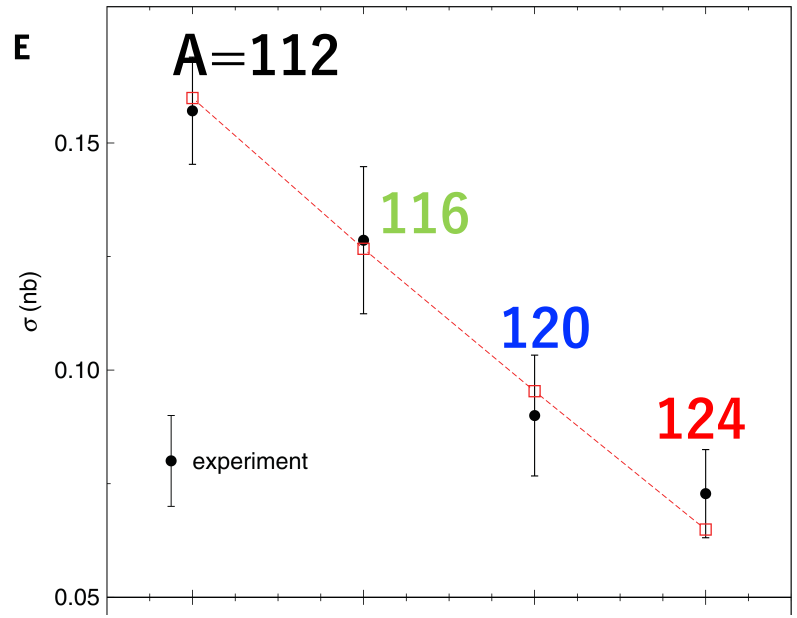
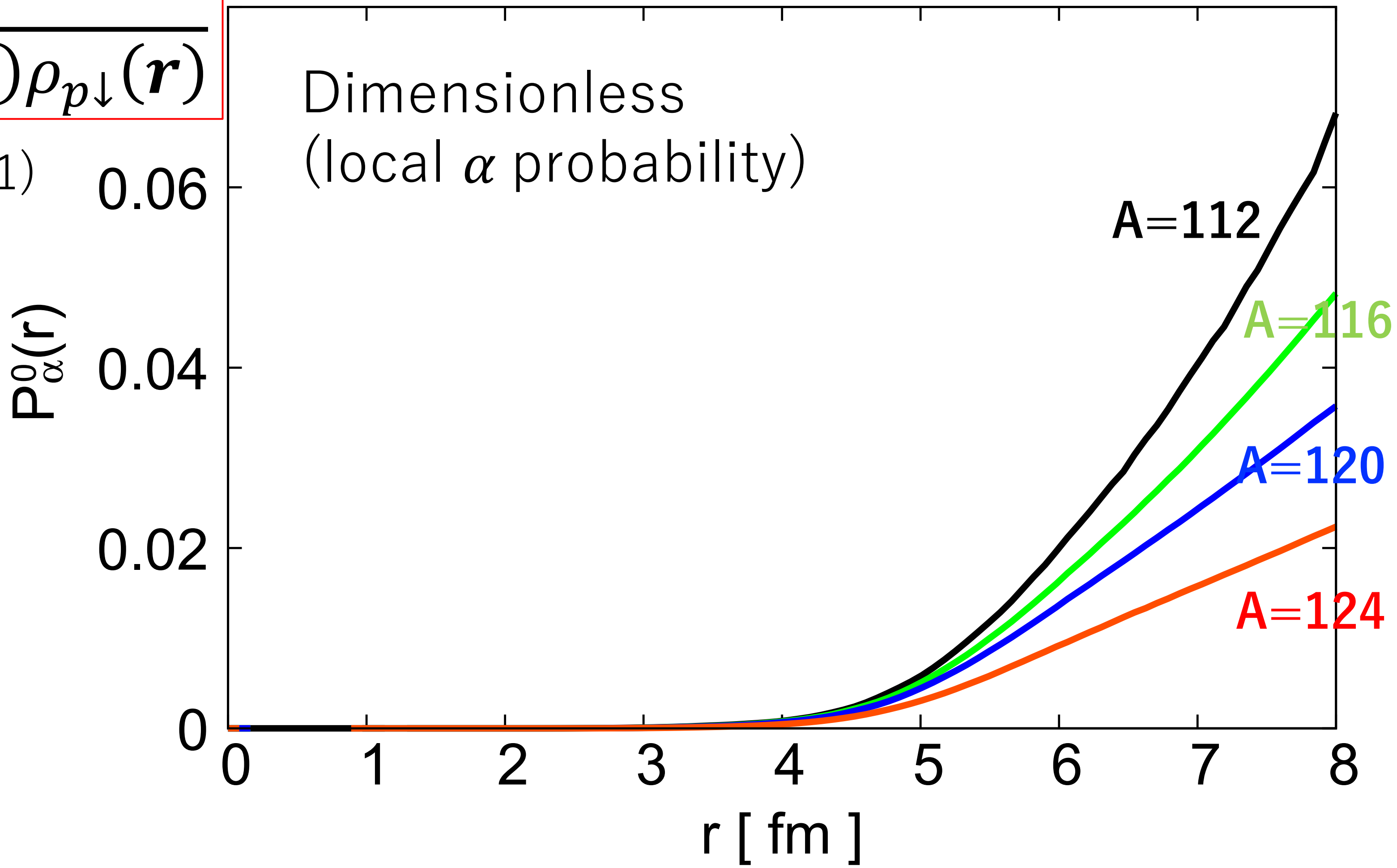


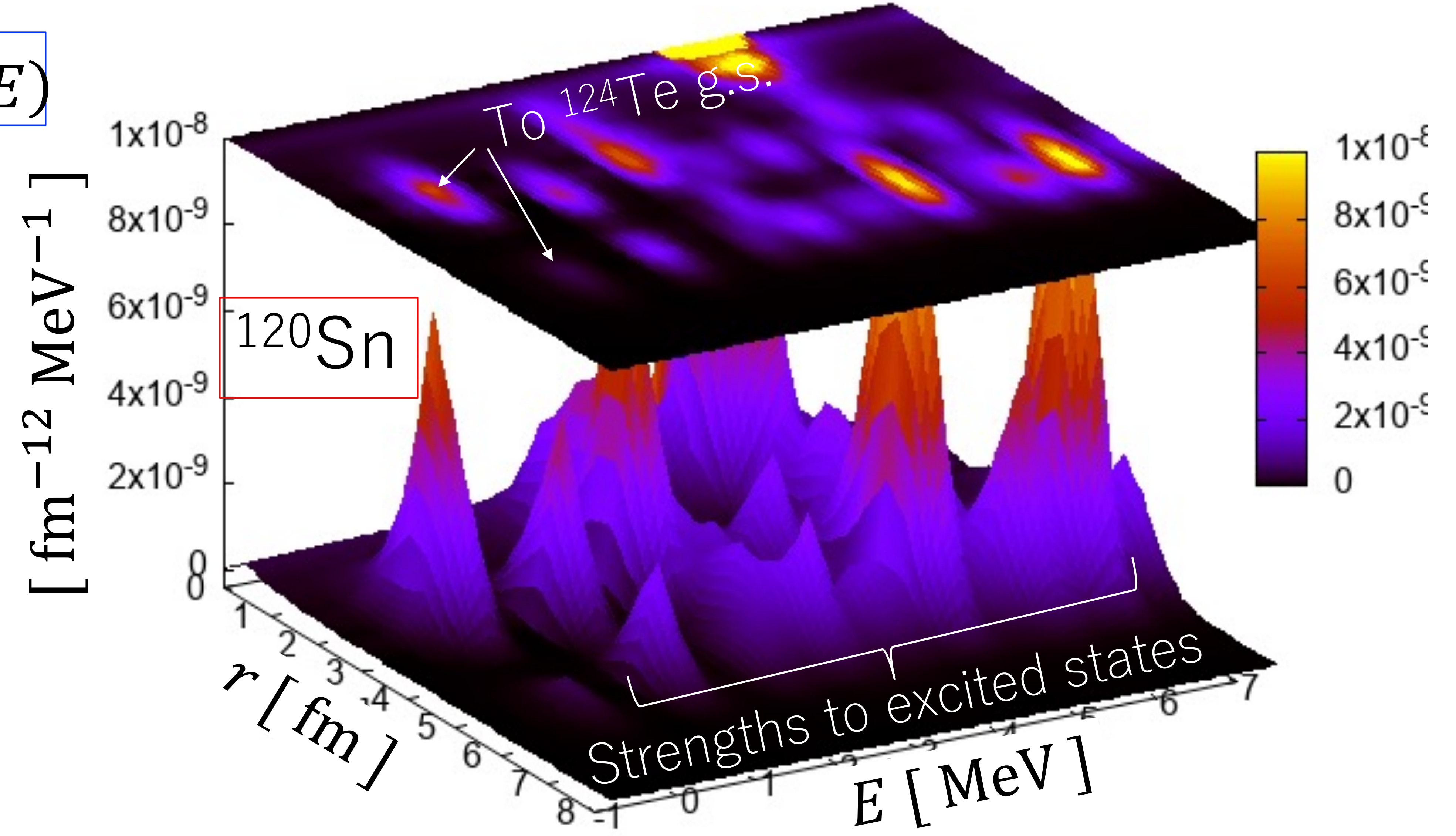
Consistent with exp.



Local α-addition strength function

$^{A}Sn \rightarrow {}^{A+4}Te$

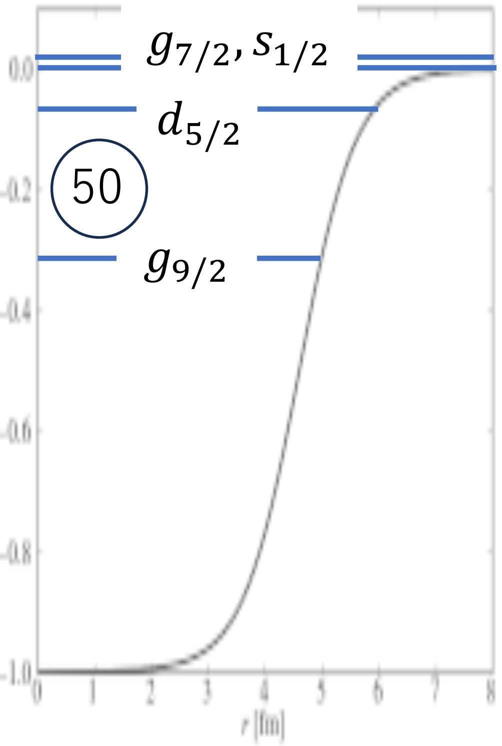
$S_{\alpha}^{(+)}(\boldsymbol{r}, E)$

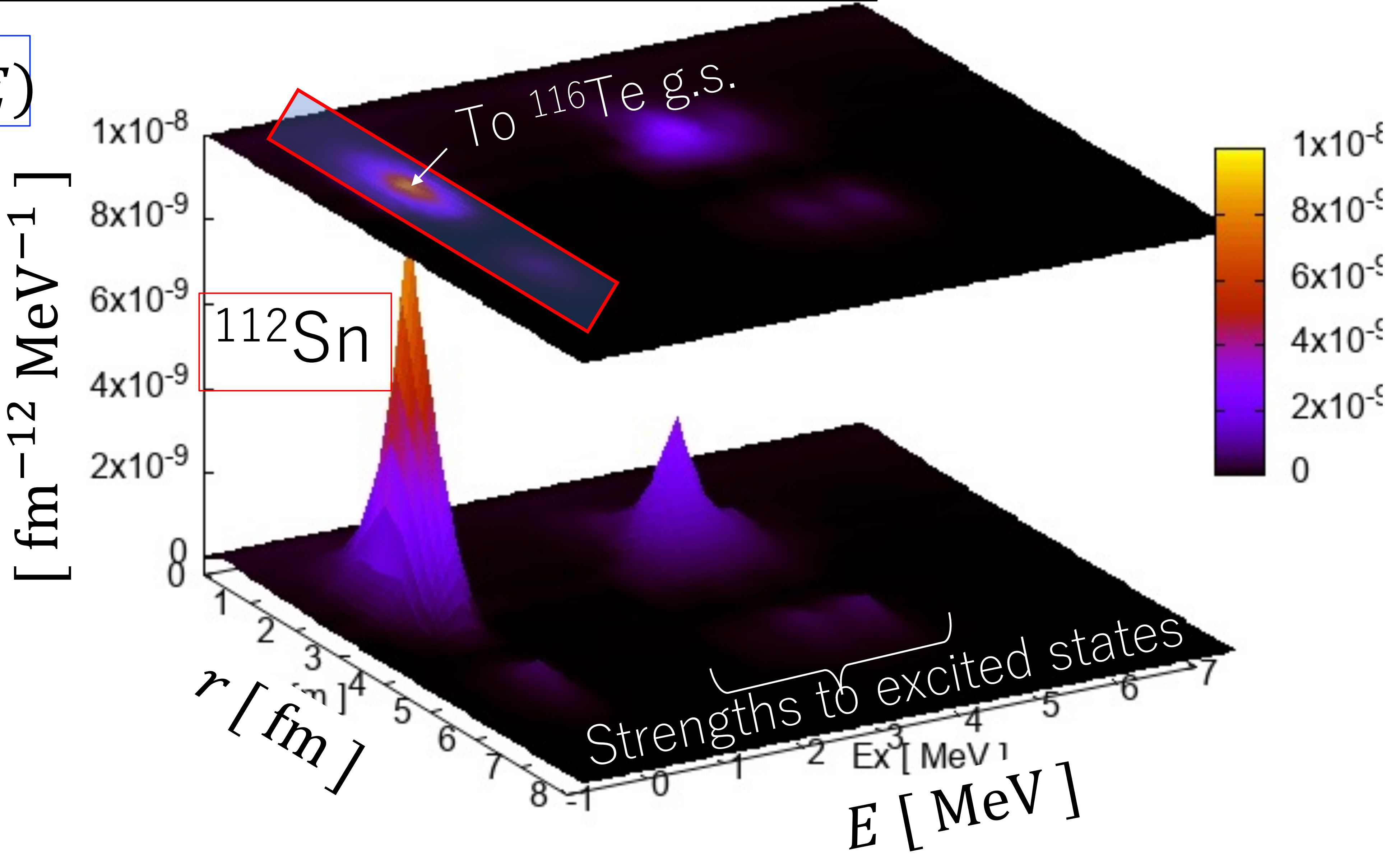


Local α-addition strength function

$^{A}Sn \rightarrow {}^{A+4}Te$

$$S_{\alpha}^{(+)}(\boldsymbol{r}, E)$$

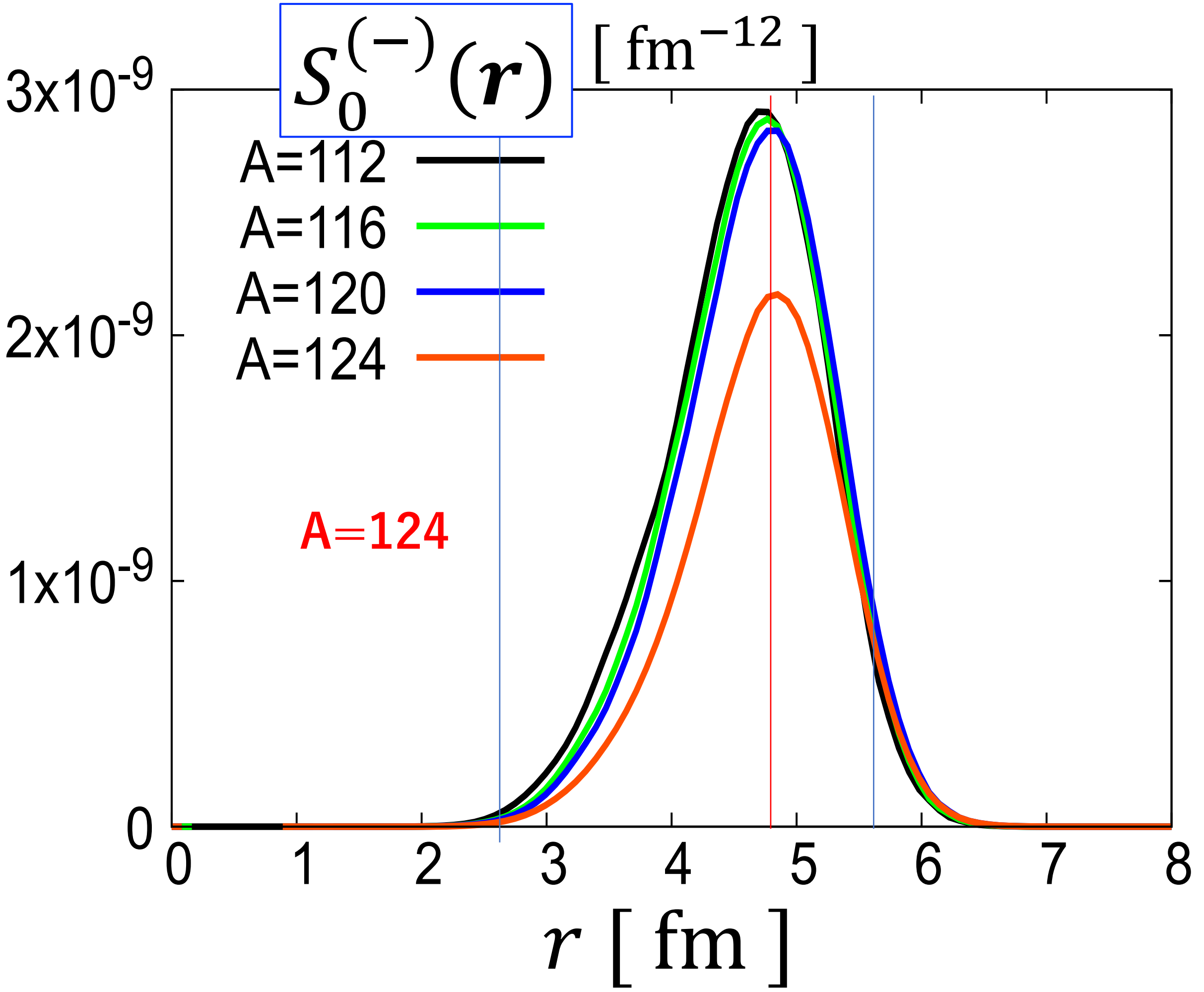


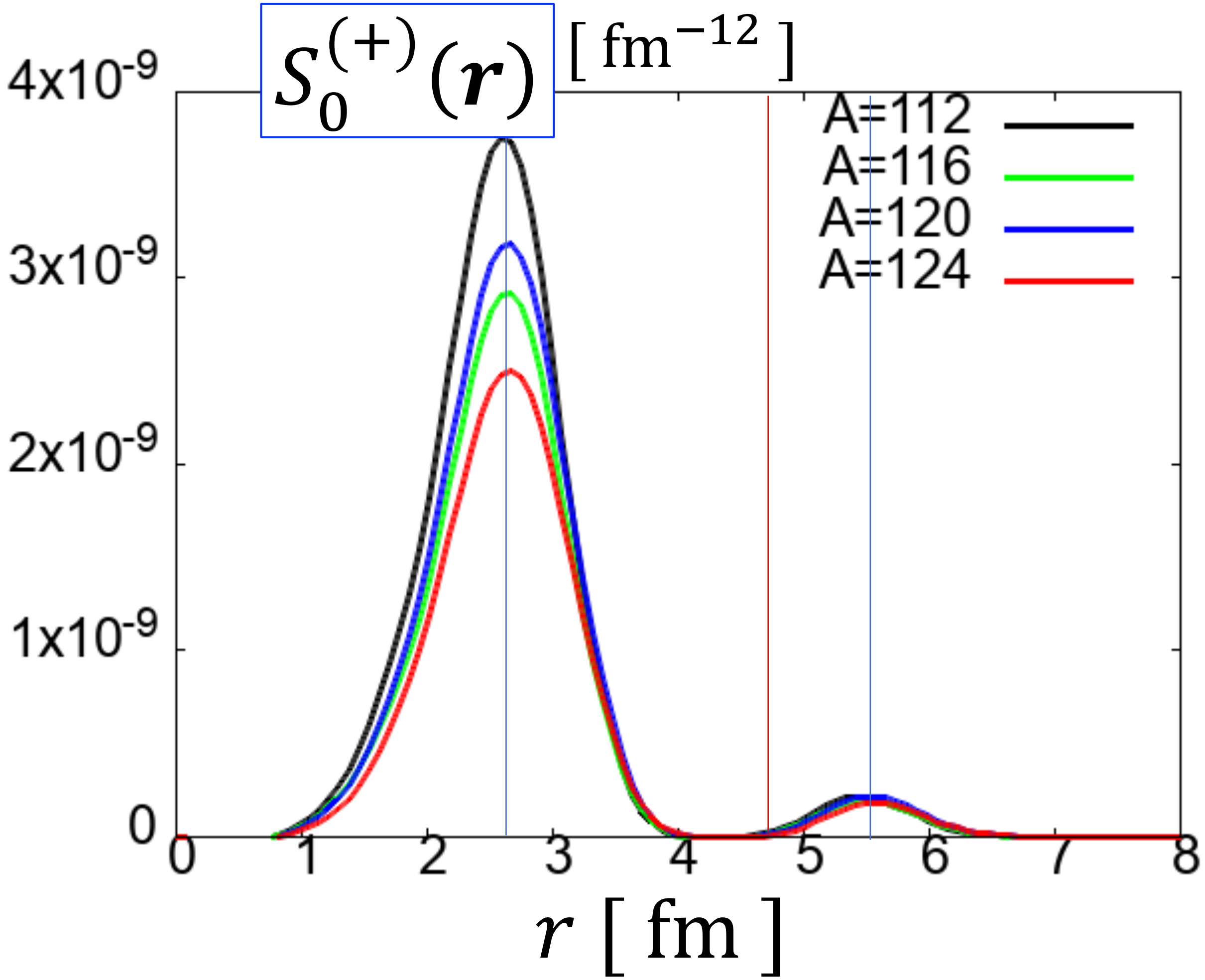


Local α-rem/add strength (g.s. → g.s.)

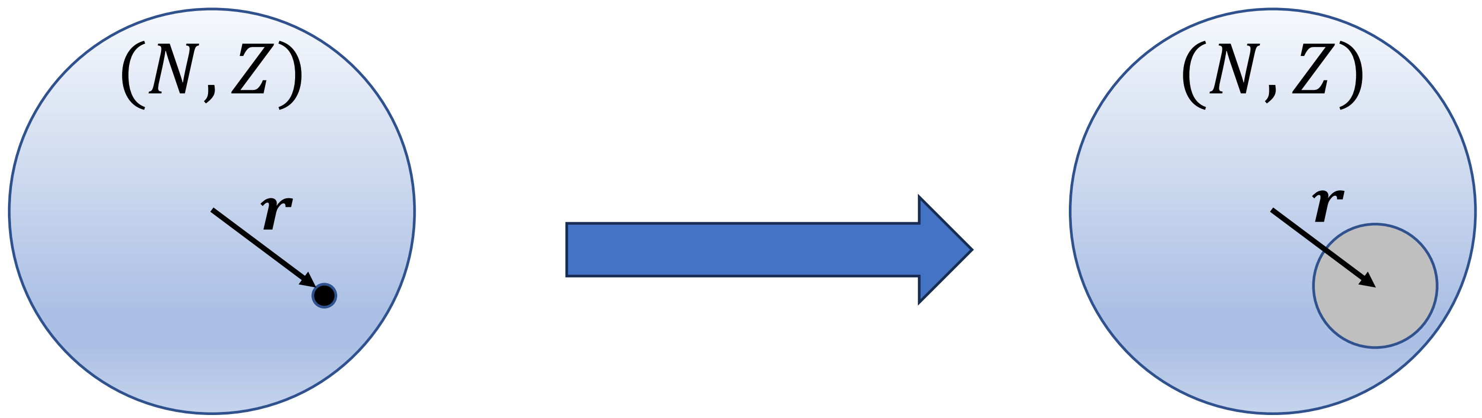
$^{112\text{-}124}$Sn

$$S_0^{(\pm)}(\boldsymbol{r}) \equiv \int_0^{\epsilon} S_\alpha^{(\pm)}(\boldsymbol{r},E)dE = \left|\langle \Phi_0^{A\pm4} \middle| \hat{\alpha}^{(\dagger)}(\boldsymbol{r}) \middle| \Phi_0^A \rangle\right|^2$$





Finite-size effect of $\boldsymbol{\alpha}$ particle



$$\phi_0^{\boldsymbol{r}}(\boldsymbol{x}) \equiv \left(\frac{\pi}{\nu}\right)^{\frac{3}{4}} \mathrm{e}^{-\frac{\nu(\boldsymbol{x}-\boldsymbol{r})^2}{2}}$$

$$\hat{\alpha}(\boldsymbol{r}) = \hat{\psi}_{n\uparrow}(\boldsymbol{r})\hat{\psi}_{n\downarrow}(\boldsymbol{r})\hat{\psi}_{p\uparrow}(\boldsymbol{r})\hat{\psi}_{p\downarrow}(\boldsymbol{r})$$

$$\hat{\alpha}(\boldsymbol{r}) = \int \phi_0^{\boldsymbol{r}}(\boldsymbol{x}_1)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_2)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_3)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_4)\hat{\psi}_{n\uparrow}(\boldsymbol{x}_1)\hat{\psi}_{n\downarrow}(\boldsymbol{x})\hat{\psi}_{p\uparrow}(\boldsymbol{x}_3)\hat{\psi}_{p\downarrow}(\boldsymbol{x}_4)d\boldsymbol{x}_1 \cdots d\boldsymbol{x}_4$$

Finite-size effect of $\boldsymbol{\alpha}$ particle (BCS case)

$$\kappa^{\alpha}(\boldsymbol{r}) \equiv \int \phi_0^{r}(\boldsymbol{x}_1)\phi_0^{r}(\boldsymbol{x}_2)\langle\psi_{\uparrow}(\boldsymbol{x}_1)\psi_{\downarrow}(\boldsymbol{x}_2)\rangle d\boldsymbol{x}_1 d\boldsymbol{x}_2 = \sum_{i>0} u_i v_i \langle\phi_i|P_{\alpha}(\boldsymbol{r})|\phi_i\rangle$$

$$\sum_{\substack{i>j \\ \epsilon_i=\epsilon,\epsilon_j=\epsilon'}} F_{ij}(\boldsymbol{r}) = \frac{1}{2}\sum_{\substack{i,j \\ \epsilon_i=\epsilon,\epsilon_j=\epsilon'}} F_{ij}(\boldsymbol{r}) = \frac{1}{2}\sum_{\epsilon_i=\epsilon} v_i^2 \langle\phi_i|P_{\alpha}(\boldsymbol{r})P(\epsilon')P_{\alpha}(\boldsymbol{r})|\phi_i\rangle$$

Sum over the states with the same single-particle energy

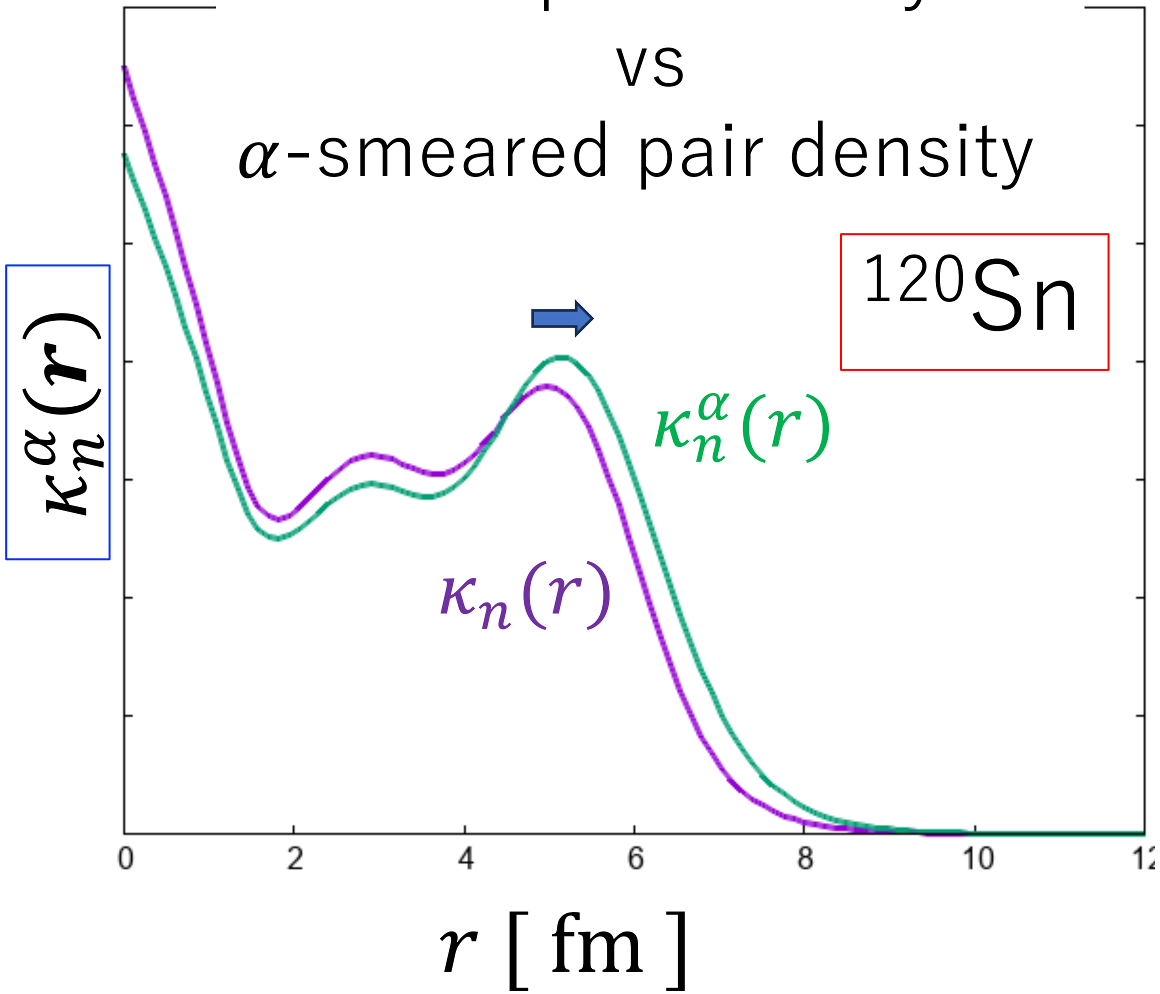
$$P(\epsilon') \equiv \sum_{\epsilon_j=\epsilon'} v_j^2 |\phi_j\rangle\langle\phi_j|$$

$$P_{\alpha}(\boldsymbol{r}) \equiv \sum_{\sigma=\uparrow,\downarrow} |\phi_{0\sigma}^{\boldsymbol{r}}\rangle\langle\phi_{0\sigma}^{\boldsymbol{r}}|$$

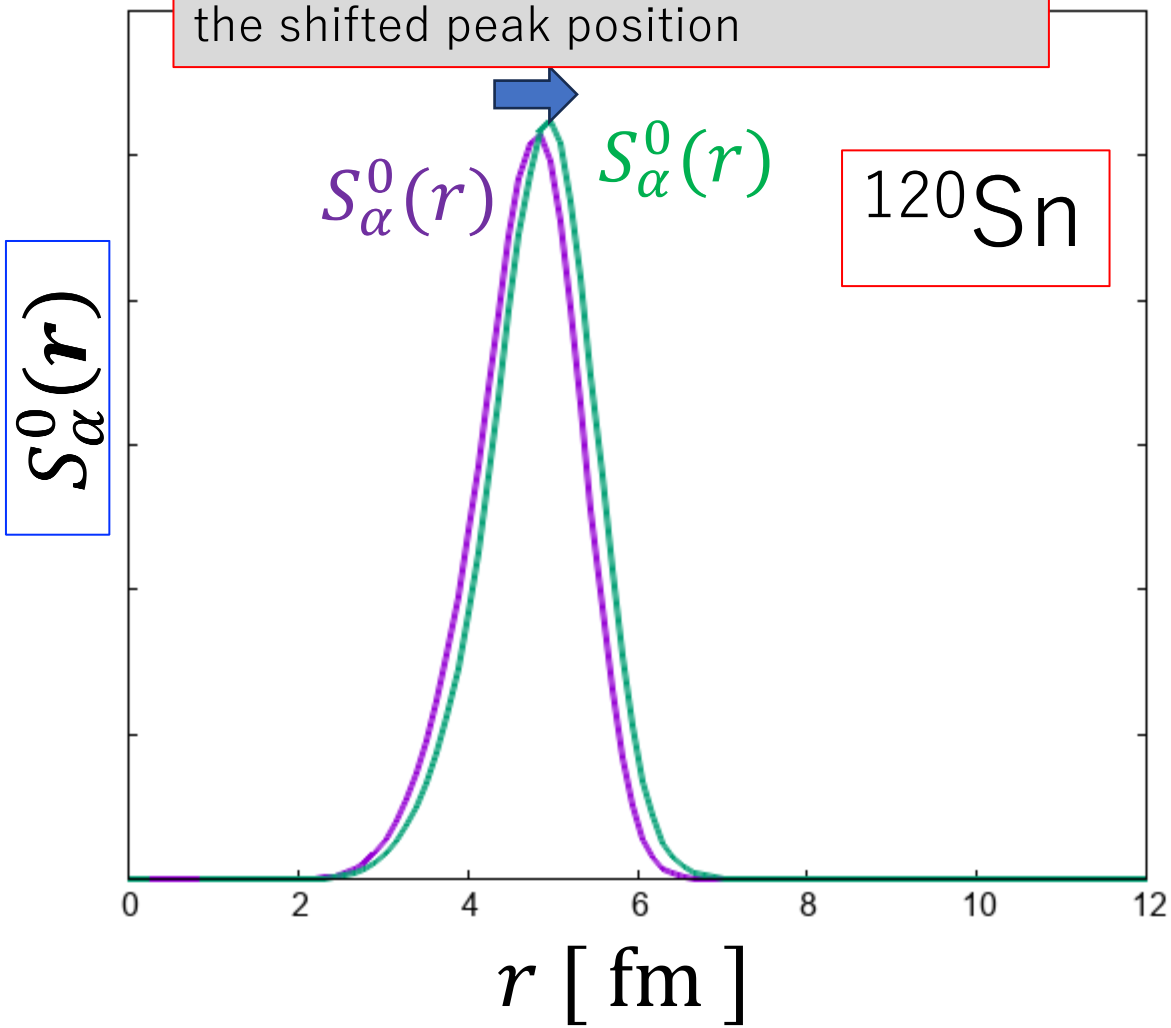
$$\phi_{0\sigma}^{\boldsymbol{r}}(\boldsymbol{x}) \equiv \left(\frac{\pi}{\nu}\right)^{\frac{3}{4}} \mathrm{e}^{-\frac{\nu}{2}(\boldsymbol{x}-\boldsymbol{r})^2}\,\chi_{\sigma}$$

Finite-size effect of $\boldsymbol{\alpha}$ particle

Neutron pair density
vs
α-smeared pair density



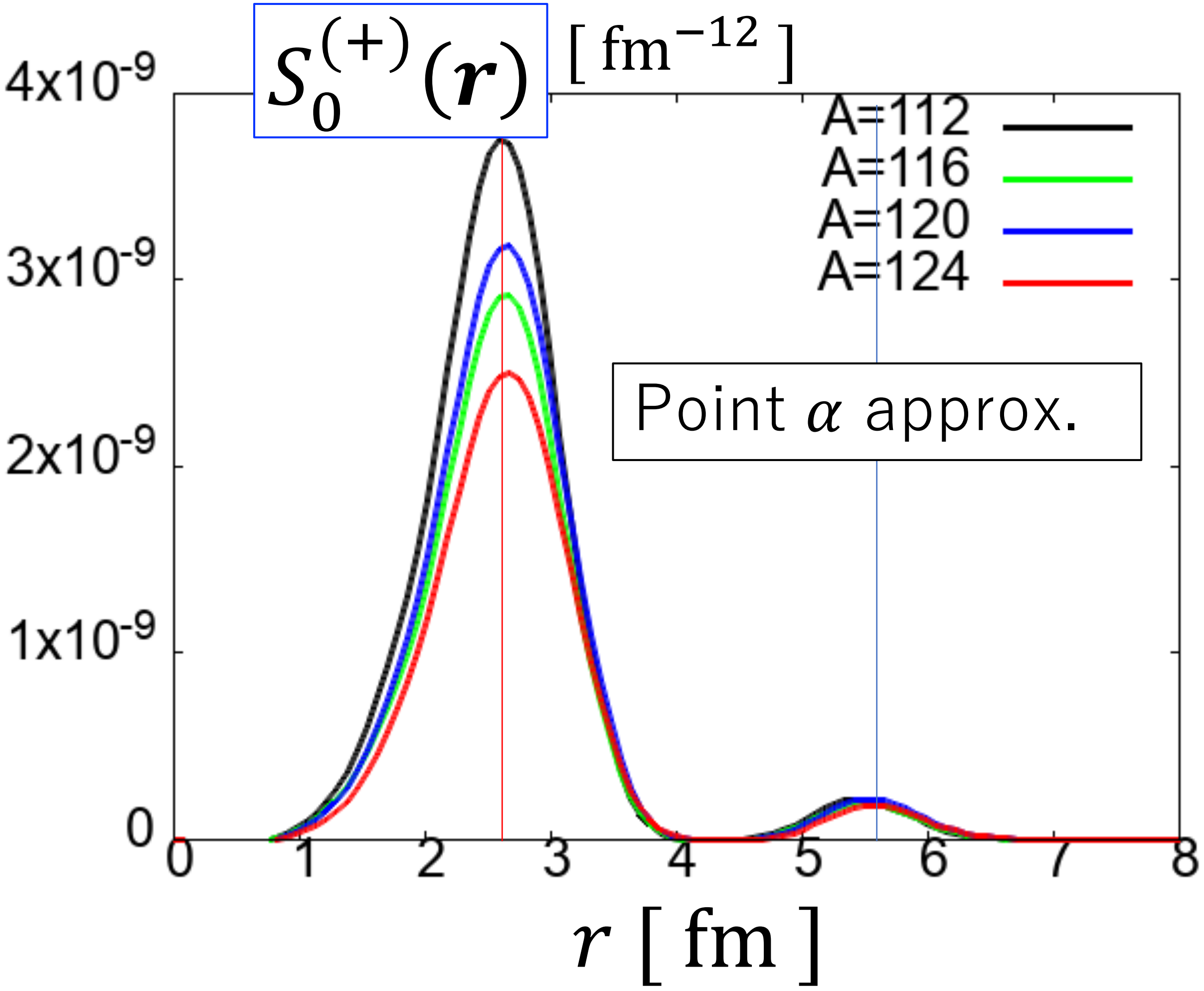
Qualitatively no change, except for the shifted peak position

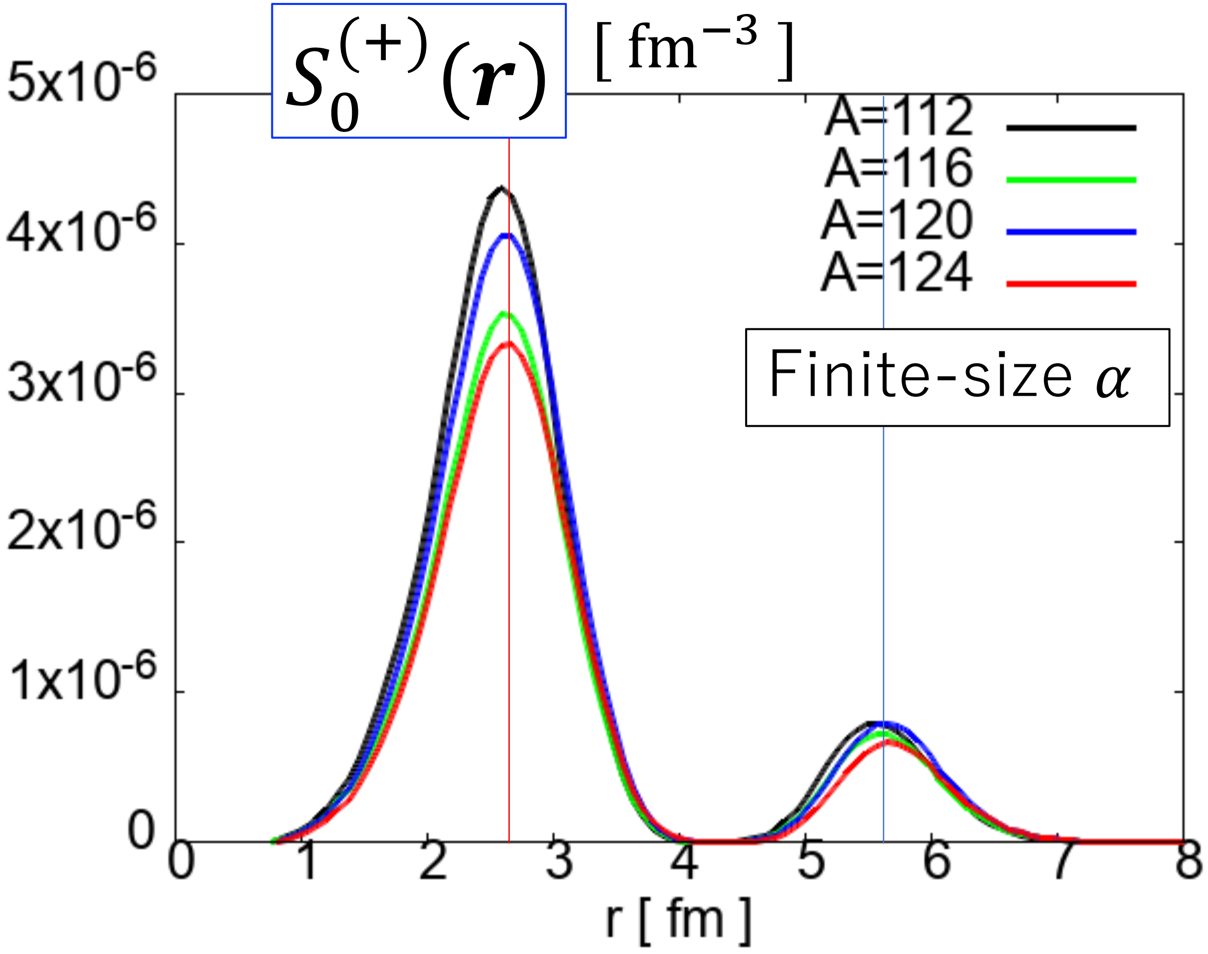


Local α-addition strength (g.s. → g.s.)

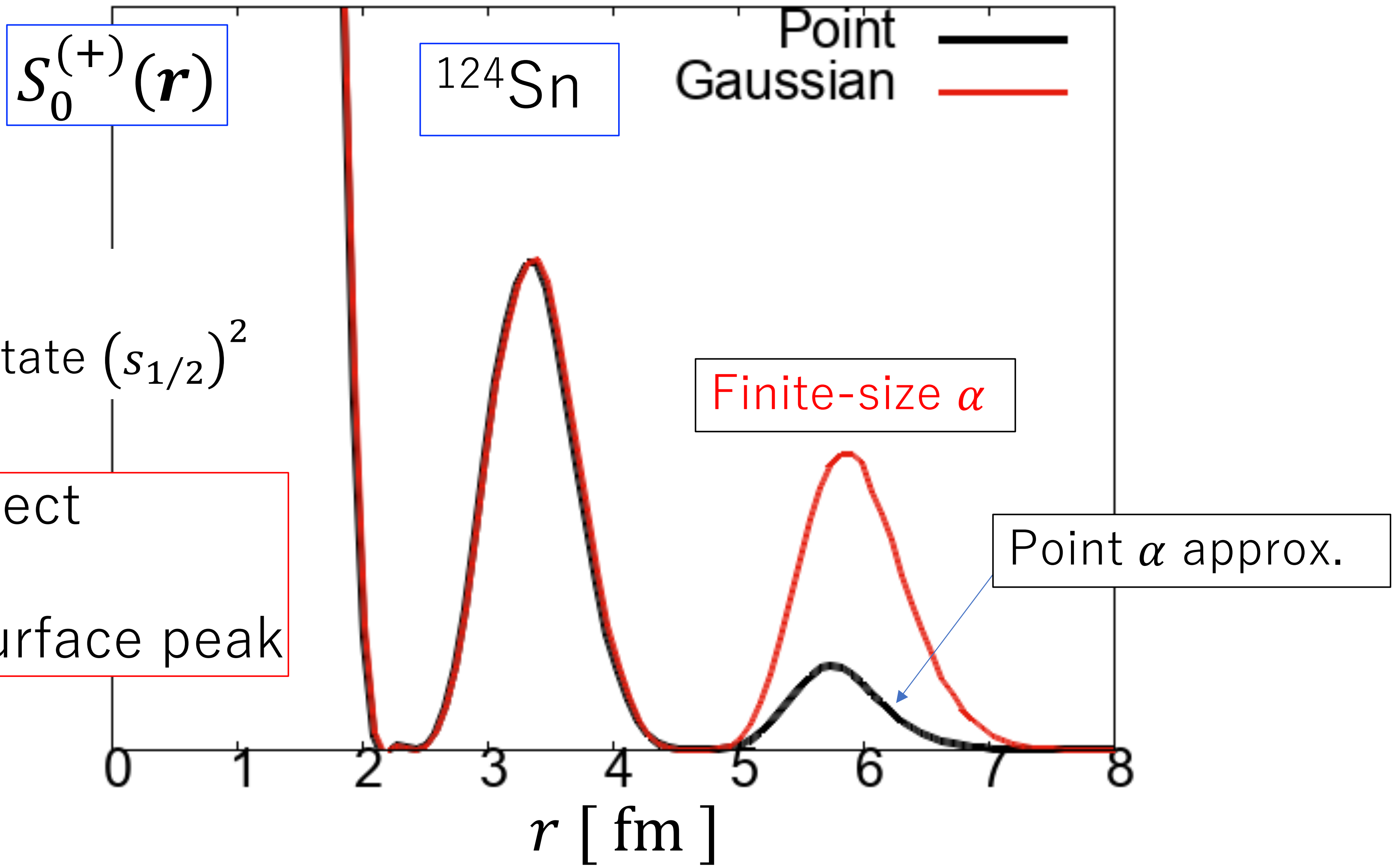
$^{112\text{-}124}$Sn

$$S_0^{(+)}(\boldsymbol{r}) \equiv \int_0^{\epsilon} S_\alpha^{(+)}(\boldsymbol{r}, E)\,dE = \left|\langle \Phi_0^{A+4} | \hat{\alpha}^\dagger(\boldsymbol{r}) | \Phi_0^A \rangle\right|^2$$





Local α-add. str.: ^{124}Sn(g.s.) → ^{128}Te($E_x = 5$ MeV)



Neutron g.s.

Proton two-particle state $(s_{1/2})^2$

Finite α effect

↓

Enhancement of surface peak

$\boldsymbol{\alpha}$ reduced width

$$\hat{\alpha}(\boldsymbol{r}) = \int \phi_0^{\boldsymbol{r}}(\boldsymbol{x}_1)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_2)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_3)\phi_0^{\boldsymbol{r}}(\boldsymbol{x}_4)\hat{\psi}_{n\uparrow}(\boldsymbol{x}_1)\hat{\psi}_{n\downarrow}(\boldsymbol{x})\hat{\psi}_{p\uparrow}(\boldsymbol{x}_3)\hat{\psi}_{p\downarrow}(\boldsymbol{x}_4)d\boldsymbol{x}_1\cdots d\boldsymbol{x}_4$$

$$= \int \Phi_{\mathrm{CM}}^{\boldsymbol{r}}(\boldsymbol{R})\ \hat{\alpha}^{\boldsymbol{R}}\, d\boldsymbol{R}$$

$$\hat{\alpha}^{\boldsymbol{R}} = \int \phi_{rel}^{\boldsymbol{r}}(\boldsymbol{\xi}_1,\boldsymbol{\xi}_2,\boldsymbol{\xi}_3)\delta\left(\boldsymbol{R} - \frac{1}{4}\sum_{k=1}^{4}\boldsymbol{x}_k\right)\hat{\psi}_{n\uparrow}(\boldsymbol{x}_1)\hat{\psi}_{n\downarrow}(\boldsymbol{x})\hat{\psi}_{p\uparrow}(\boldsymbol{x}_3)\hat{\psi}_{p\downarrow}(\boldsymbol{x}_4)d\boldsymbol{x}_1\cdots d\boldsymbol{x}_4$$

$$\mathcal{Y}_{mn}(\boldsymbol{r}) \equiv \langle\Phi_m^{A-4}|\hat{\alpha}^{\boldsymbol{r}}|\Phi_n^{A}\rangle \approx \frac{\langle\Phi_m^{A-4}|\hat{\alpha}(\boldsymbol{r})|\Phi_n^{A}\rangle}{\int \Phi_{\mathrm{CM}}^{\boldsymbol{r}}(\boldsymbol{R})\, d\boldsymbol{R}} = \left(\frac{\nu}{\pi}\right)^{3/4}\langle\Phi_m^{A-4}|\hat{\alpha}(\boldsymbol{r})|\Phi_n^{A}\rangle$$

Summary

- Local α-removal strength function: $S_\alpha^{(-)}(\boldsymbol{r}, E)$
  - HF+BCS calculation
  - $^{112\text{-}124}$Sn: g.s. → g.s.
    - Consistent with α-knockout experiment
    - Sensitive to pairing correlations
    - Finite-α effect: Peak shift to larger r
- Local α-addition strength function: $S_\alpha^{(+)}(\boldsymbol{r}, E)$
    - Strong isotopic dependence due to bound/unbound orbitals
    - Finite-α effect: Enhancement of surface peak
- Future perspectives
    - Deformed nuclei
    - Rearrangement effect
    - pn pairing
    - Other clusters (^{12}C, ^{16}O, etc.)

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Oakforest-PACS + Wisteria-O (Univ. of Tokyo)

ERATO

Three-nucleon forces project

Research Director: Kimiko Sekiguchi
(Professor, School of Science, Tokyo Institute of Technology)
Research Term: Oct 2023 - Mar 2029
Grant Number: JPMJER2304

TOMOE