Collective Bohr Hamiltonian with effective Gogny interaction

SSNET24

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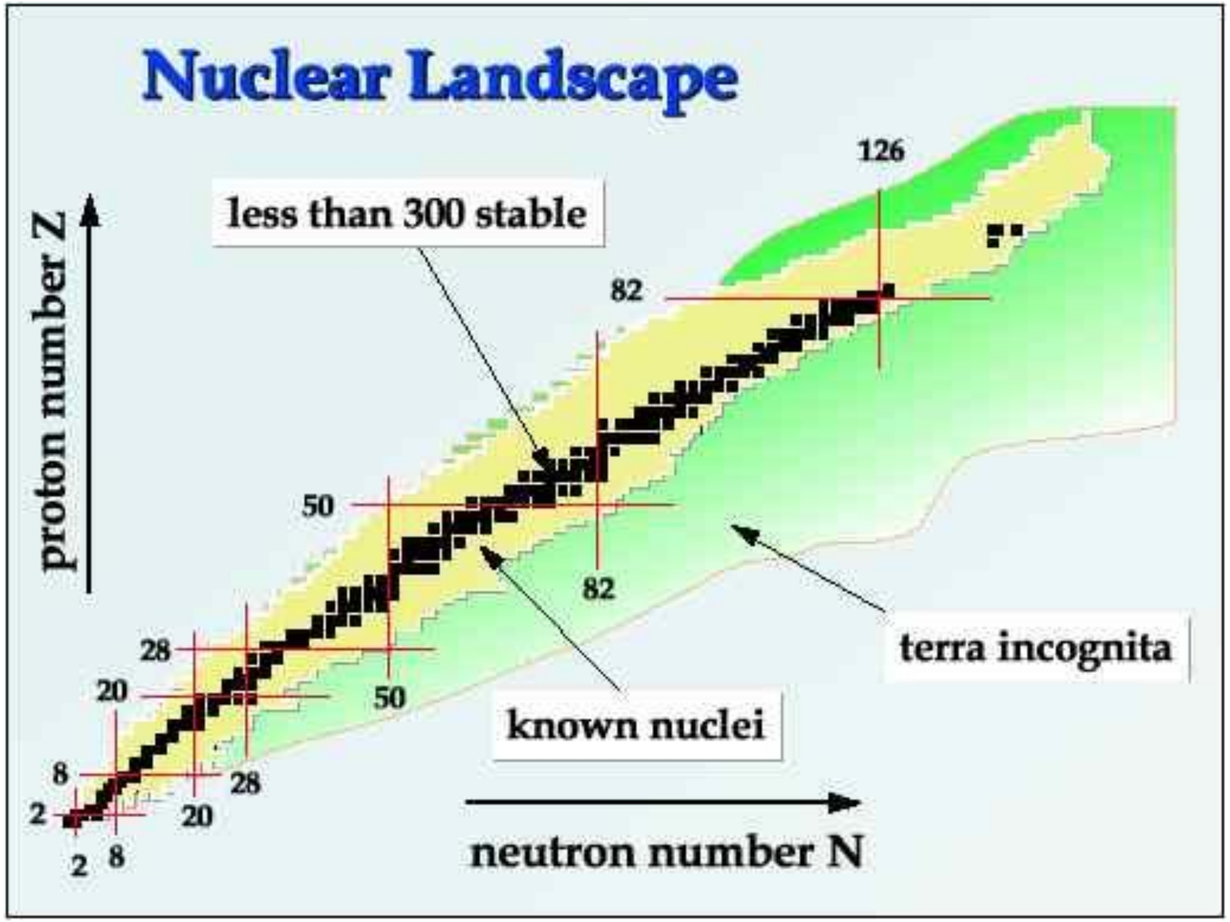
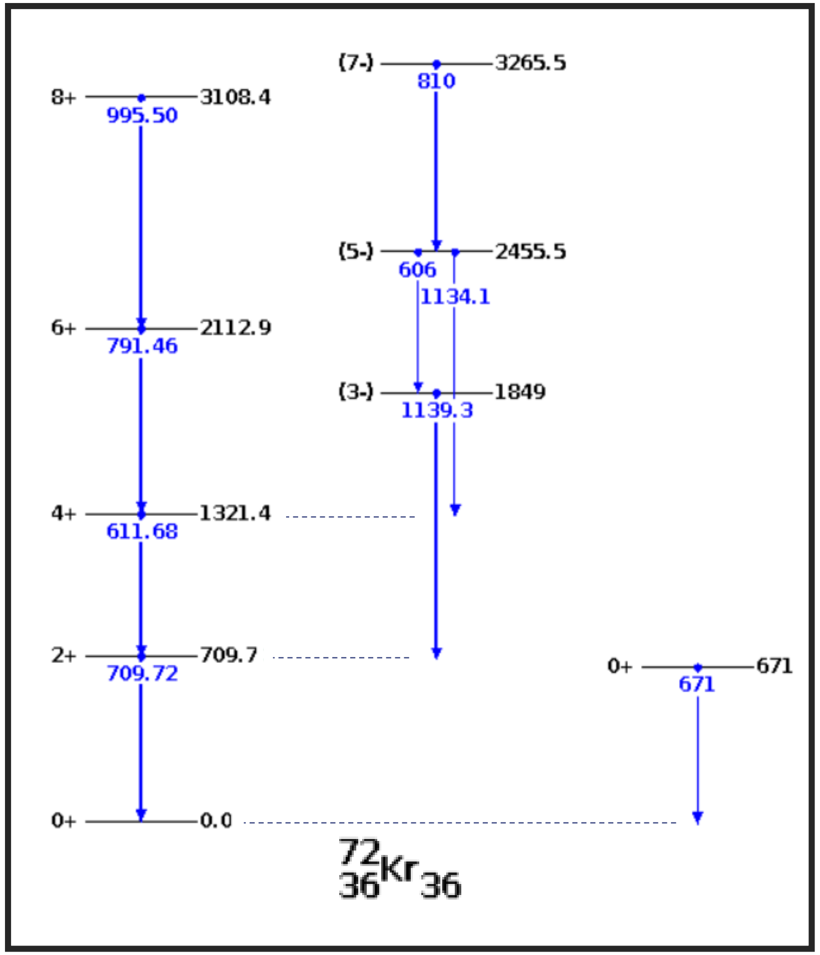
Covering all aspects of nuclear chart



Challenges in nuclear physics

Need a model to explain all nuclear observables along the chart

Description of collective motion



ENSDF database, IAEA, https://www-nds.iaea.org/

GCM

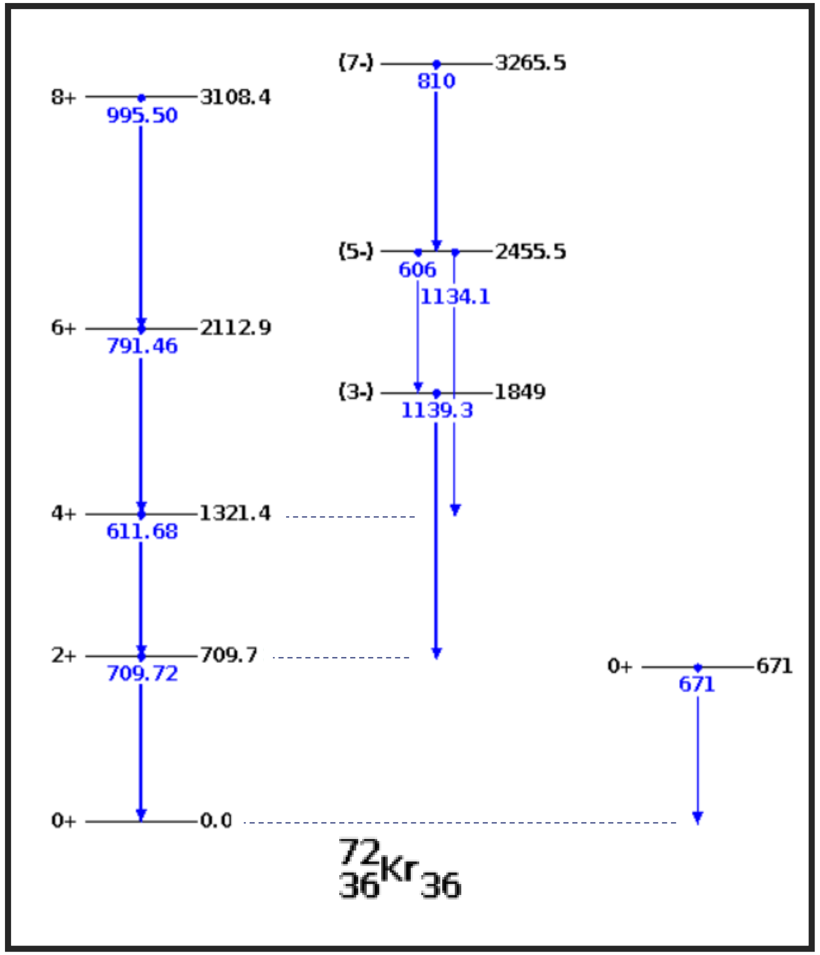
Rigid rotor

QRPA

...

Bohr Hamiltonian

Description of collective motion



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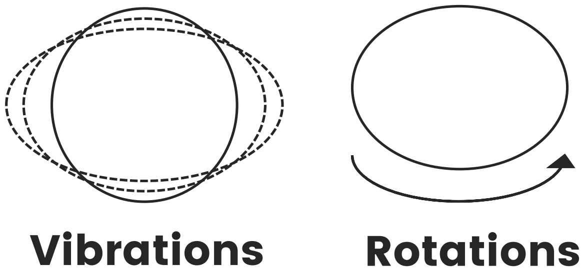
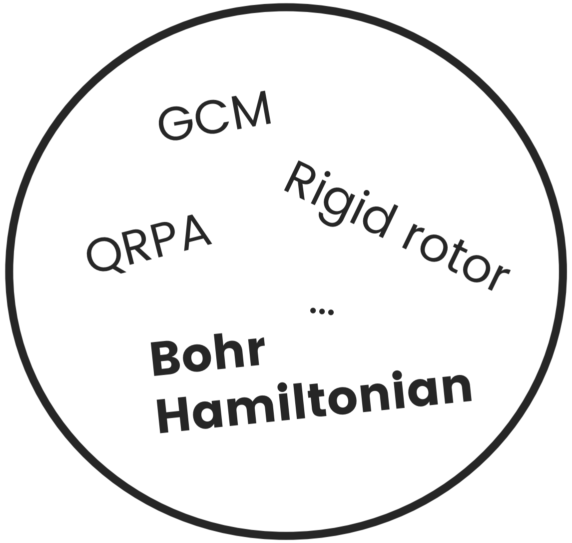
Low-energy collective effects :





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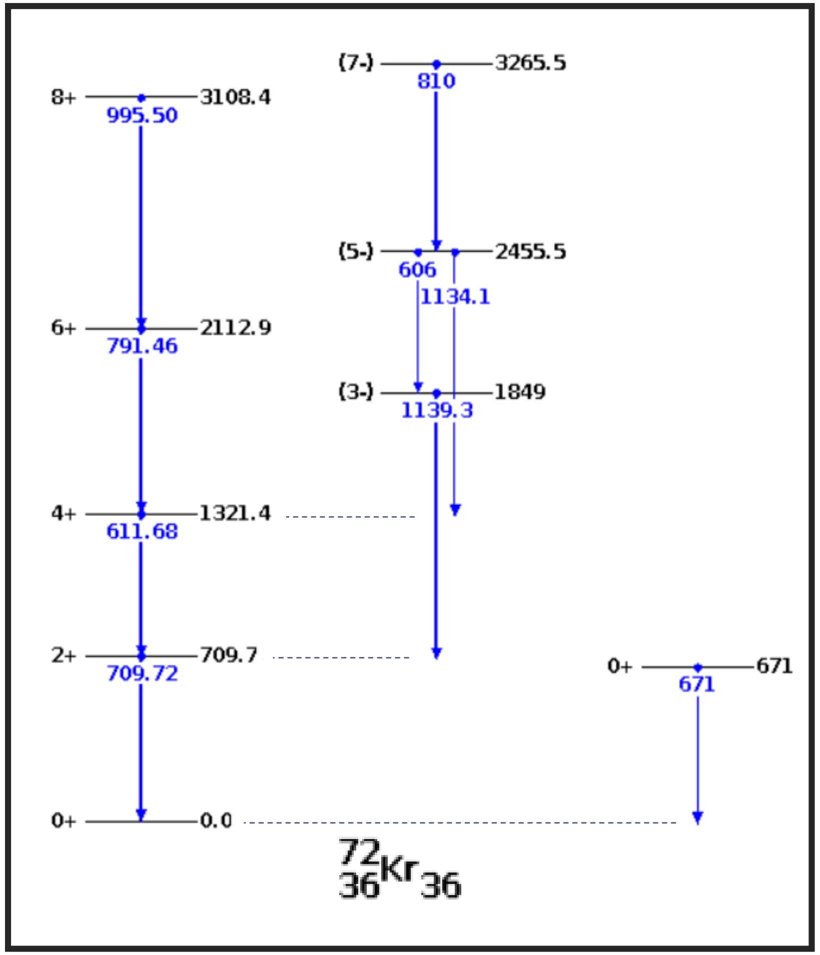

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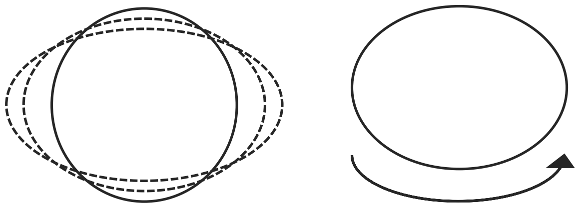
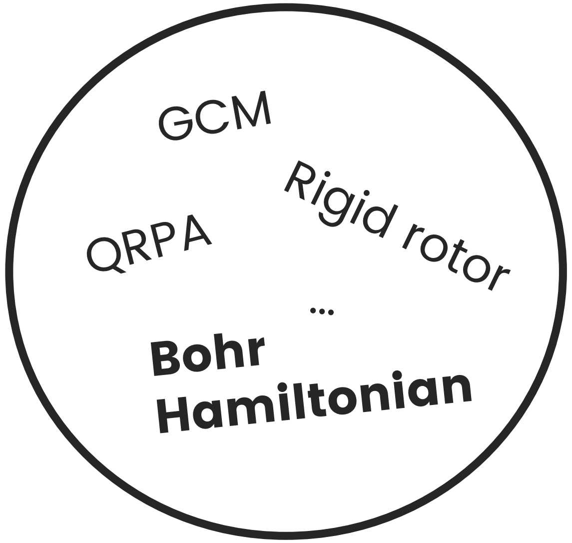


Vibrations **Rotations**

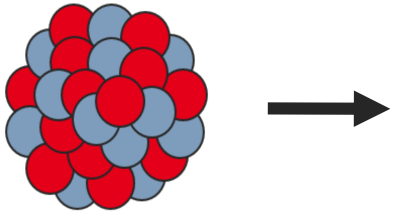


Quadrupole Bohr Hamiltonien ?

+ Vibrational and rotational
+ Fast

Collective quadrupole Hamiltonian

Bohr A *1952 K. Danske Vidensk. Selsk., Mat.-Fys. Medd.* 26 No 14



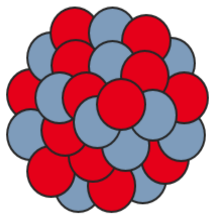
A nucleons

$$R(\theta, \Phi, t) = R_0\left(1 + \sum_{\lambda=0}^{\infty}\sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}(t) Y^*_{\lambda\mu}(\theta, \Phi)\right)$$

Surface

Collective quadrupole Hamiltonian

Bohr A *1952 K. Danske Vidensk. Selsk., Mat.-Fys. Medd.* 26 No 14



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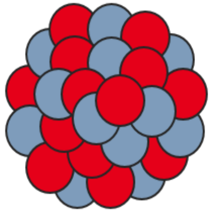

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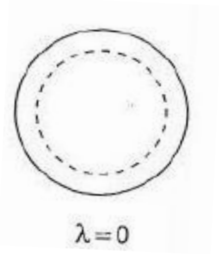
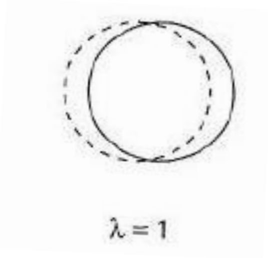
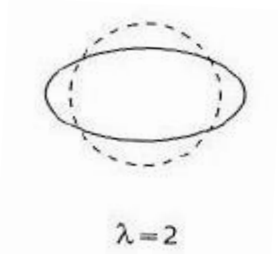


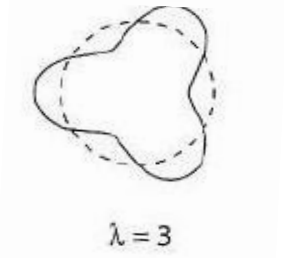
Monopole :
Volume variations



Dipole :
Center of mass



Quadrupole :
Elongation



Octupole :
Asymmetric deformation

Collective quadrupole Hamiltonian

Quadrupole

Laboratory frame

$$\alpha_2 = \begin{pmatrix} \alpha_{22} \\ \alpha_{21} \\ \alpha_{20} \\ \alpha_{2-1} \\ \alpha_{2-2} \end{pmatrix}$$

Collective quadrupole Hamiltonian

Quadrupole

Laboratory frame

$$\alpha_2 = \begin{pmatrix} \alpha_{22} \\ \alpha_{21} \\ \alpha_{20} \\ \alpha_{2-1} \\ \alpha_{2-2} \end{pmatrix}$$

Rotation
Euler angles
$\Omega=(\Phi,\theta,\Psi)$

Collective quadrupole Hamiltonian

Quadrupole

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Rotation
Euler angles
$\Omega=(\Phi,\theta,\Psi)$

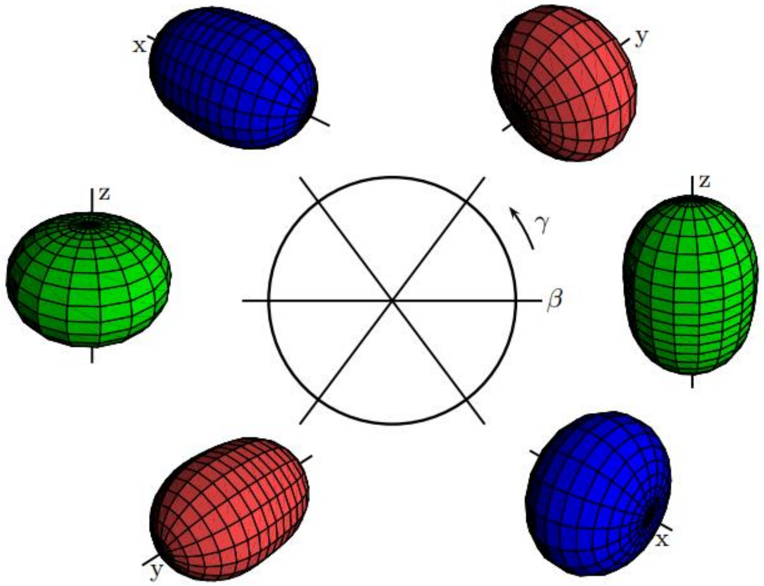
Intrinsic frame

$$\alpha_2 = \begin{pmatrix} a_2 \\ 0 \\ a_0 \\ 0 \\ a_2 \end{pmatrix}$$

$$a_0 = \beta \cos\gamma$$
$$a_2 = \beta \sin\gamma$$

$$\{\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\Phi}, \boldsymbol{\theta}, \boldsymbol{\Psi}\}$$

There are 48 ways to position the problem within the intrinsic axes of the nuclei, and defines the new reference frame.



Collective quadrupole Hamiltonian

$$\hat{H}_{coll} = \hat{H}_{vib} + \hat{H}_{rot} + V_{coll}$$

$$\hat{H}_{vib} = -\frac{\hbar^2}{2\sqrt{G}}\left[\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\sqrt{G}\frac{B_{\gamma\gamma}}{G_{vib}}\frac{\partial}{\partial\beta} + \frac{1}{\beta^2\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\sqrt{G}\frac{B_{\beta\beta}}{G_{vib}}\frac{\partial}{\partial\gamma} - \frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^3\sqrt{G}\frac{B_{\beta\gamma}}{G_{vib}}\frac{\partial}{\partial\gamma} - \frac{1}{\beta\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\sqrt{G}\frac{B_{\beta\gamma}}{G_{vib}}\frac{\partial}{\partial\beta}\right]$$

$$G = |\det(B)|$$

$$G_{vib} = B_{\beta\beta}B_{\gamma\gamma} - B_{\beta\gamma}^2$$

$$\hat{H}_{rot} = \frac{1}{2}\left[\frac{\hat{L}_x^2}{J_x} + \frac{\hat{L}_y^2}{J_y} + \frac{\hat{L}_z^2}{J_z}\right]$$

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➡ **Potential**
Mass parameters and inertia

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➡ **Potential**
Mass parameters and inertia

$$\Psi(\beta,\gamma,\Phi,\theta,\varphi) \;\Rightarrow\; <\Psi|\hat{H}_{coll}|\Psi>$$

Collective wave function basis

$$\Psi^{IM}_{Lmn}(\beta,\gamma,\Phi,\theta,\Psi) = e^{-\sigma\beta^2/2}\beta^n \begin{Bmatrix}\cos m\gamma \\ \sin m\gamma\end{Bmatrix} D^{I*}_{ML}(\Phi,\theta,\Psi)$$

Prochniak L *et al* . 1999 *Nucl. Phys.* A **648** 181–202
Libert JP *et al.* 1999 Phys. Rev. C 60 054301

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1 Symmetry constraints

Laboratory frame —**Rotation**→ **Intrinsic frame**

→ Intrisic wave functions must be invariants.

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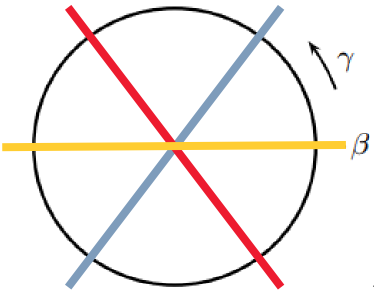
2 Element matrix of the Hamiltonian



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The inertia parameters J_k=0 on the $\gamma = \frac{k\pi}{3}$ axes.

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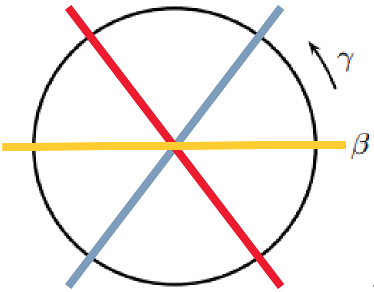

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3 Orthogonalization

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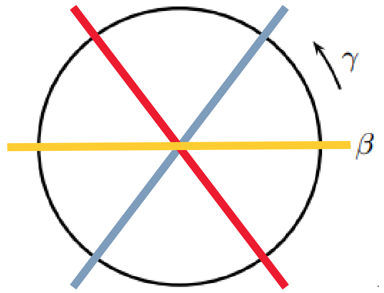

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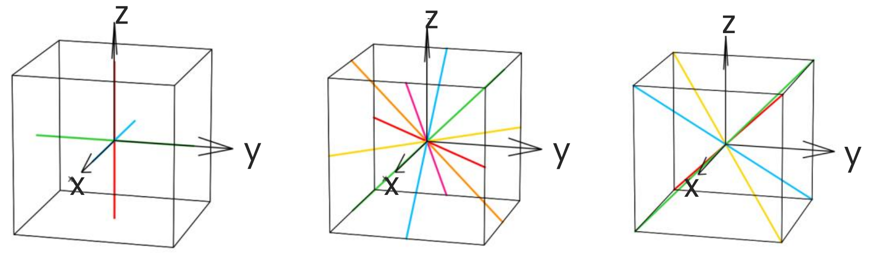


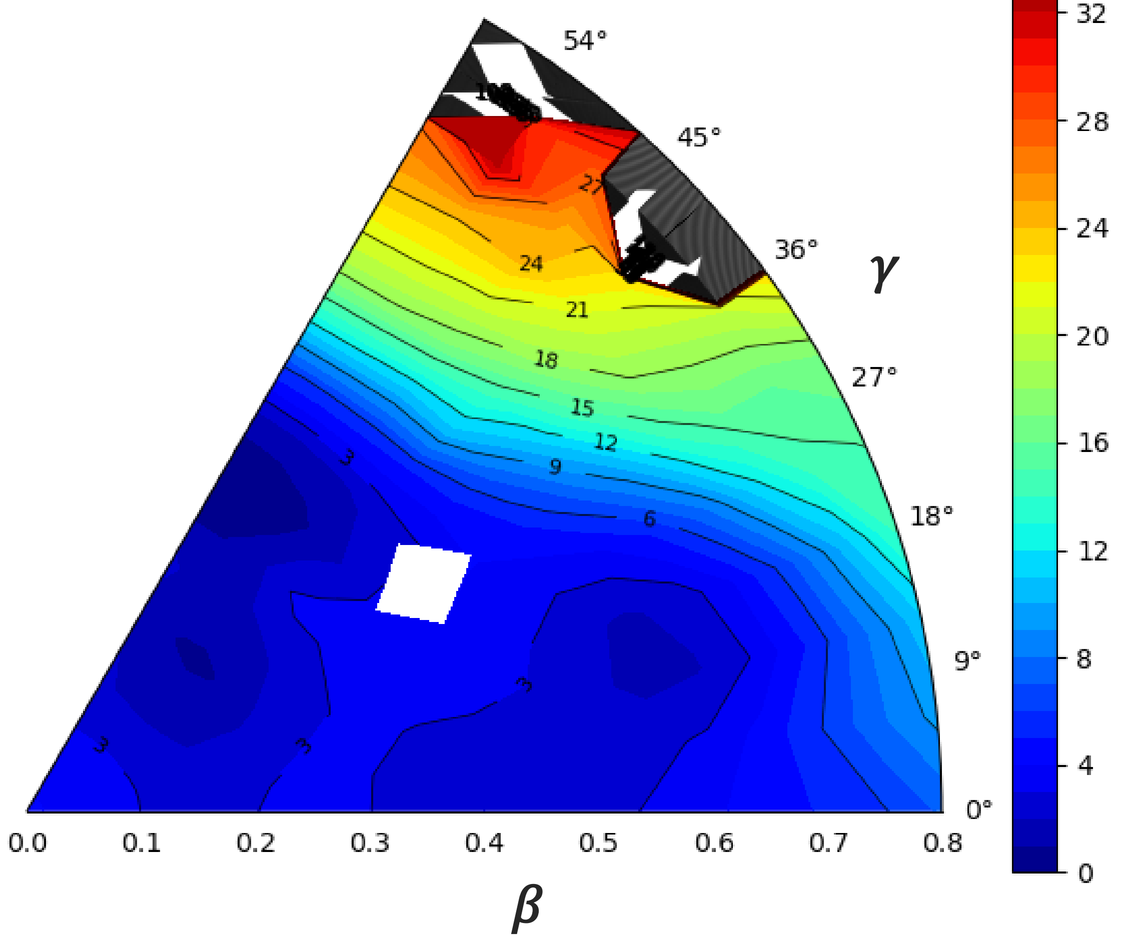
= Octahedral group O_h :

Microscopic calculations

$$B_{\beta\beta}\ ,\ B_{\beta\gamma}\ ,B_{\gamma\gamma}\ ,J_x\ ,\ J_y\ ,J_z\ ,V_{coll}$$

➡ Constrained Hartree-Fock Bogoliubov

PES (MeV)



^{72}Kr + Gogny D1S + HO basis 12 shells

Microscopic calculations

$B_{\beta\beta}$, $B_{\beta\gamma}$, $B_{\gamma\gamma}$, J_x , J_y , J_z , V_{coll}

➡ Constrained Hartree-Fock Bogoliubov

- Difficulties to converging

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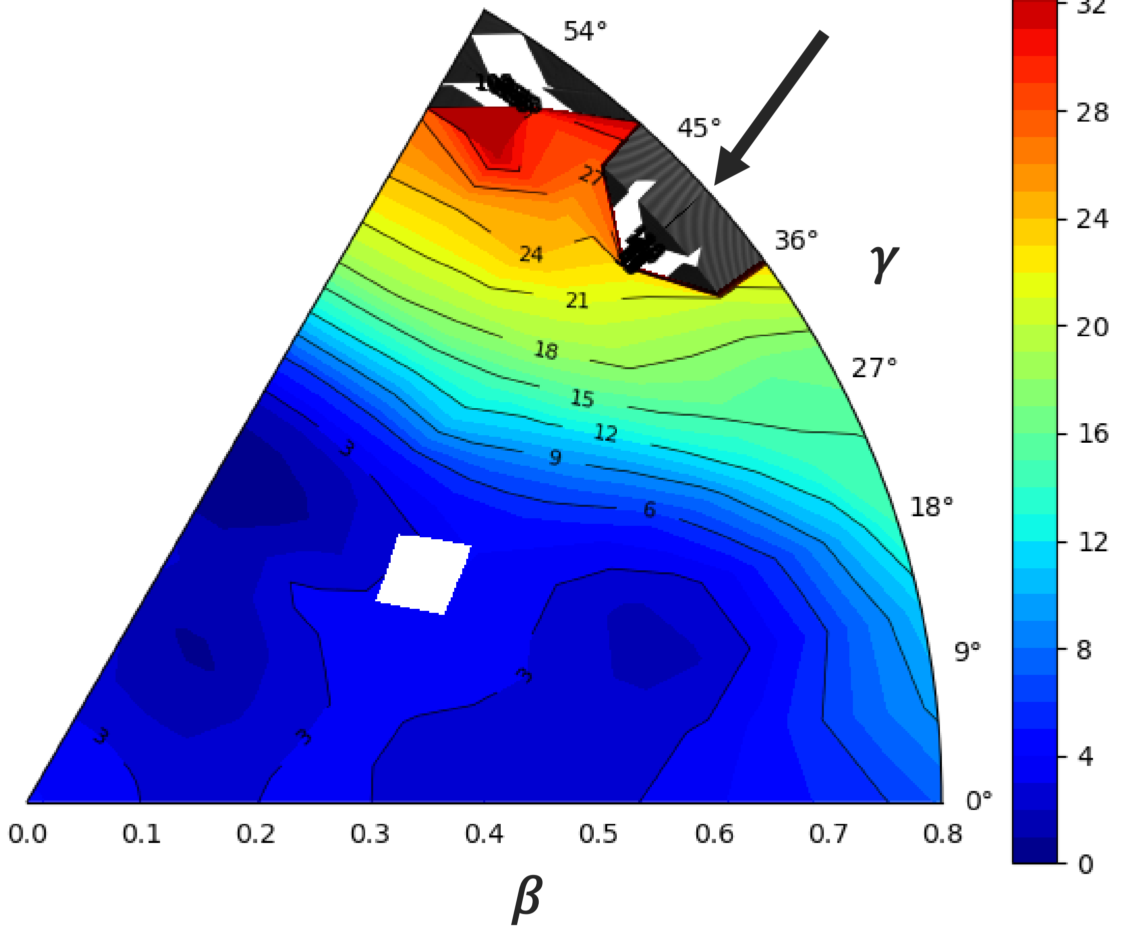

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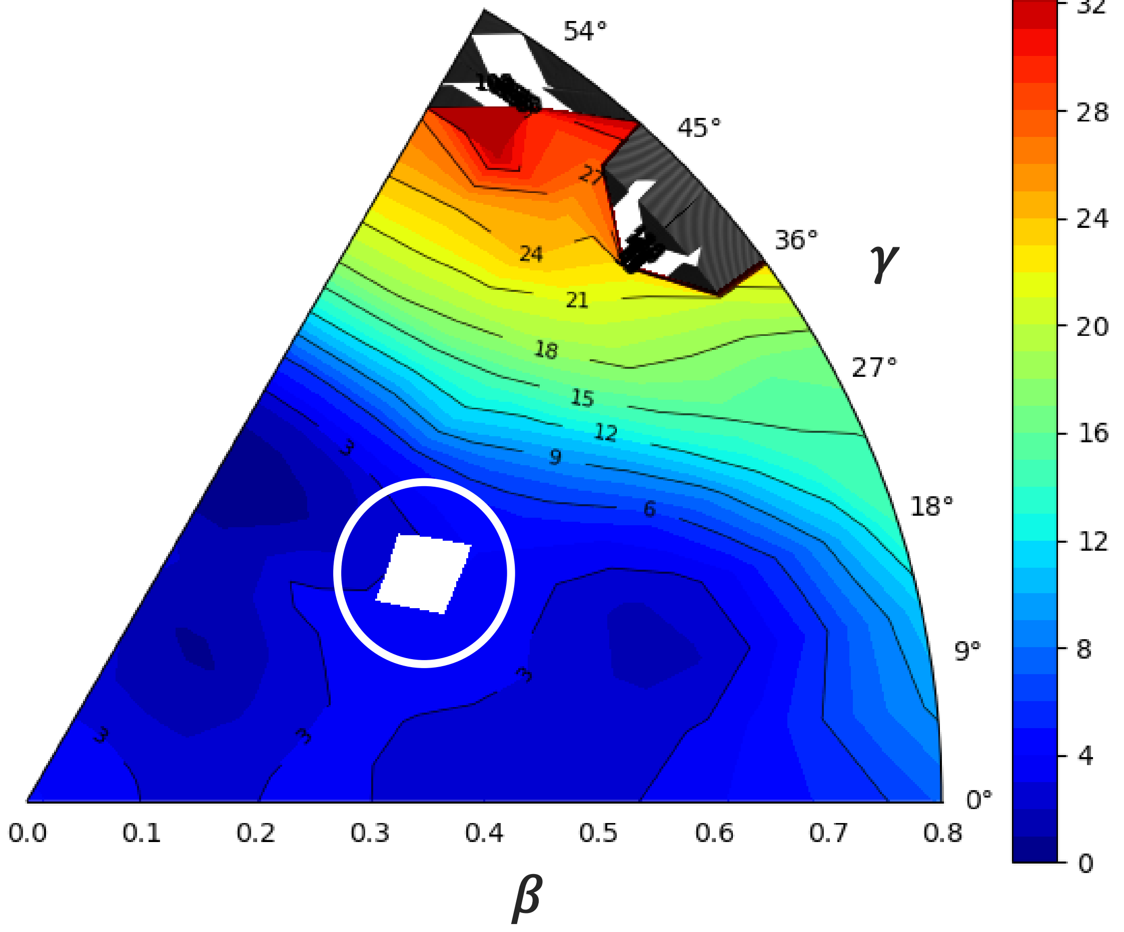

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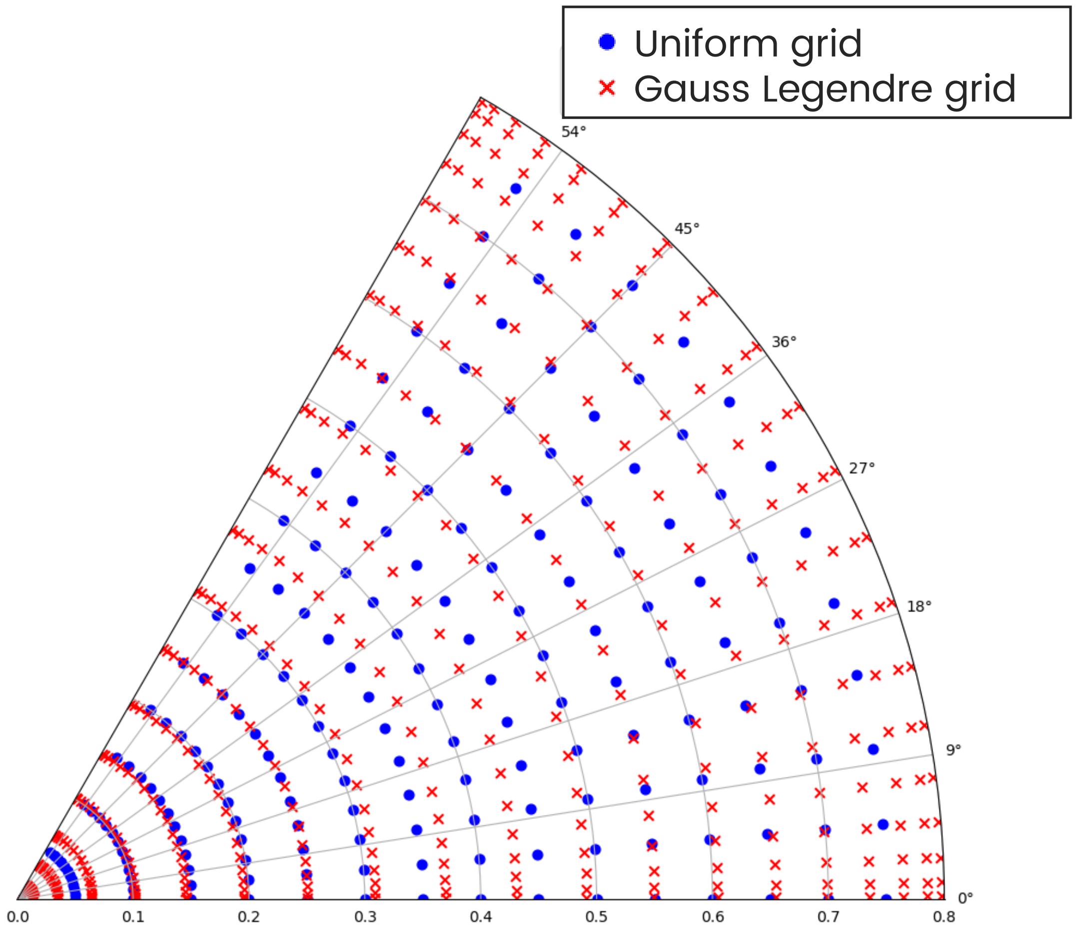
- Difficulties to converging
- Fail to converge
- Grid of calculation points



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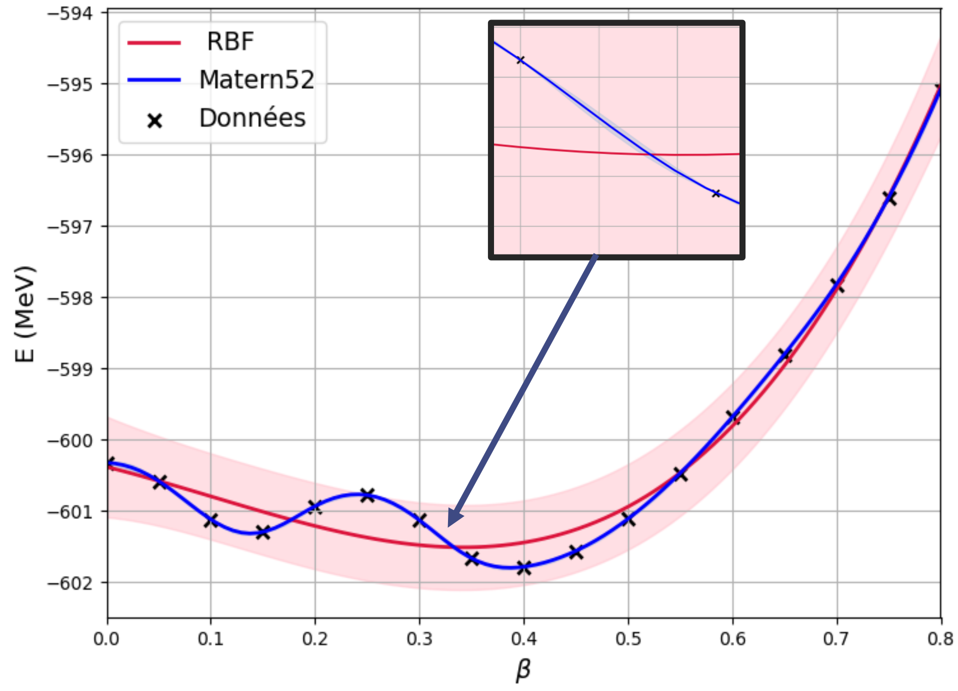
- Difficulties to converging
- Fail to converge
- Grid of calculation points

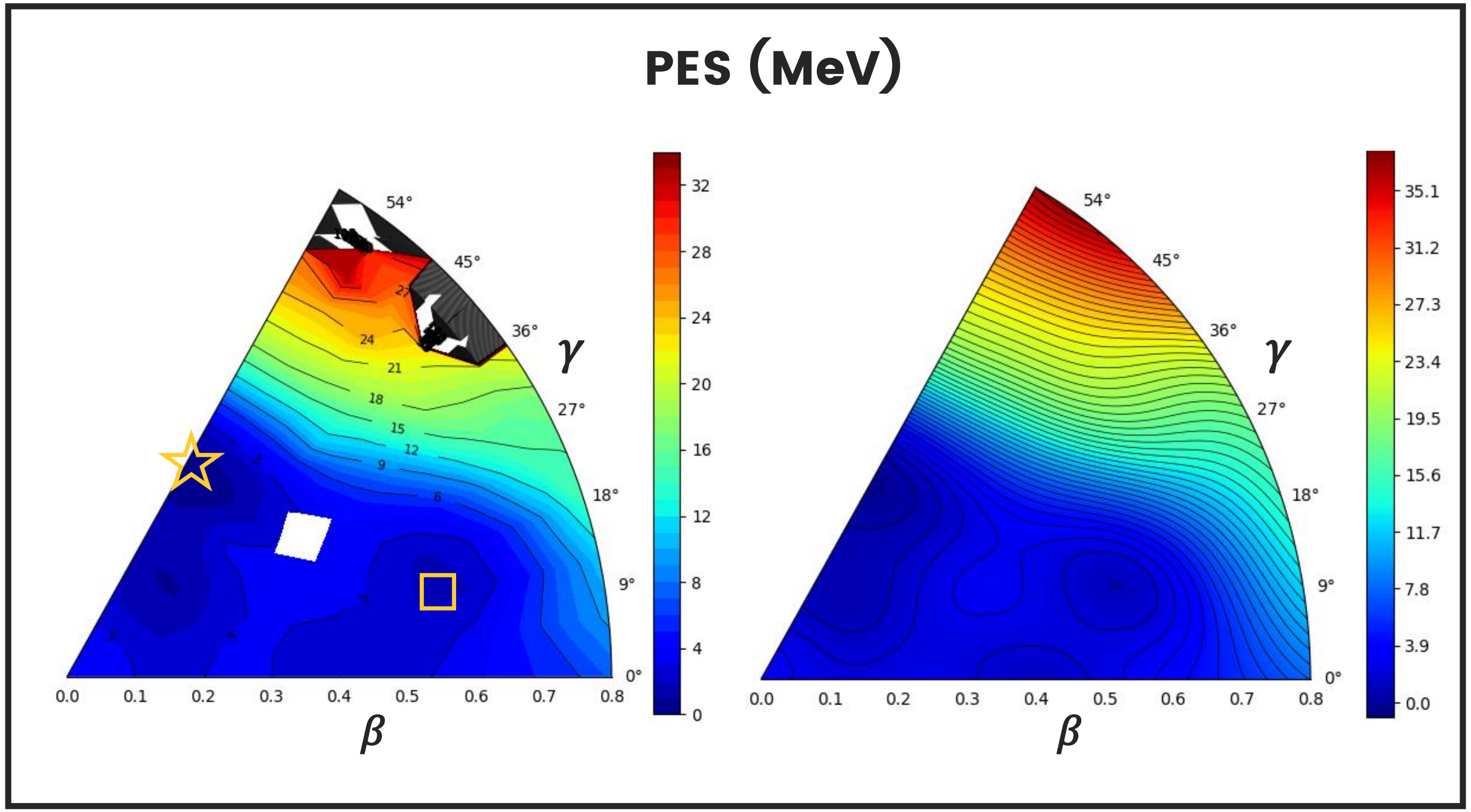
➡ **Gaussian process emulator**

Bayesian interpolation method for estimating unknown values based on a sample of data and the spatial structure of the observations.

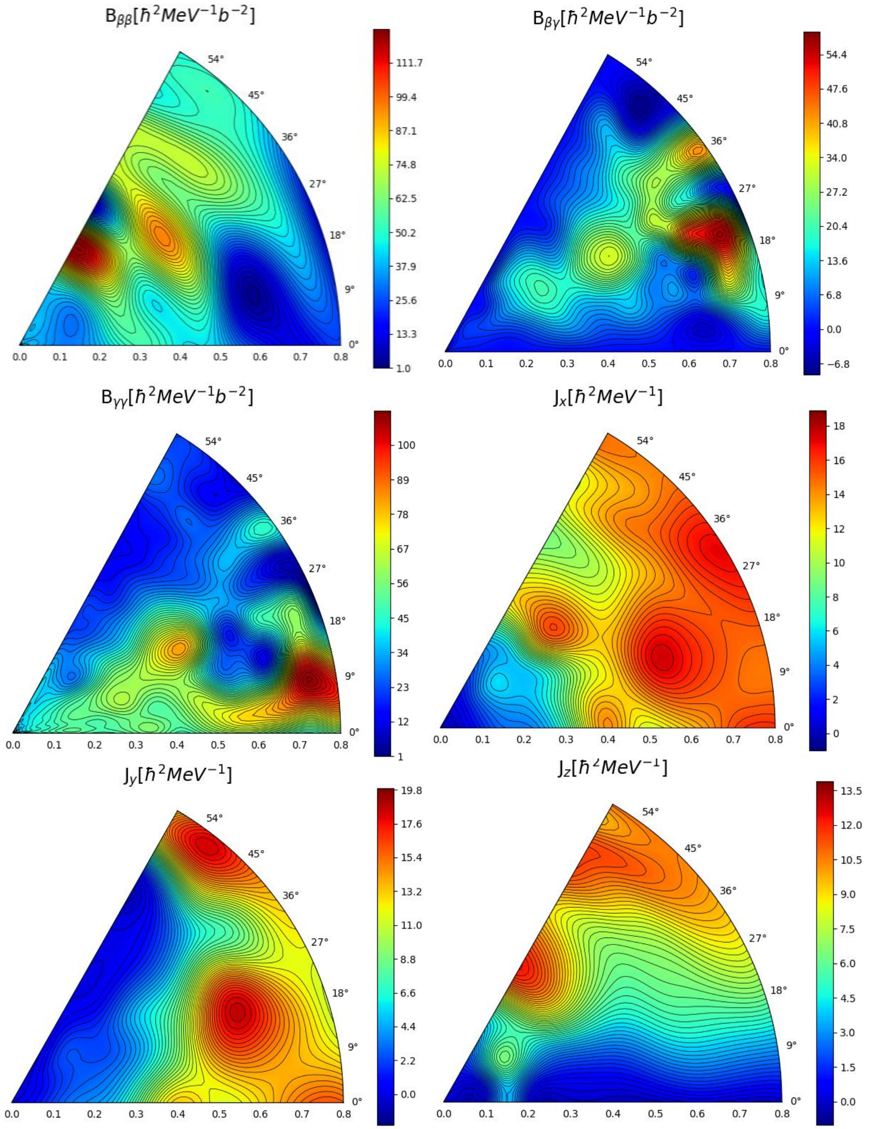
$$\{f_1(x_1), f_2(x_2), \ldots, f_n(x_n)\} \sim GP(\mu(x), k(x, x'))$$



Microscopic calculations

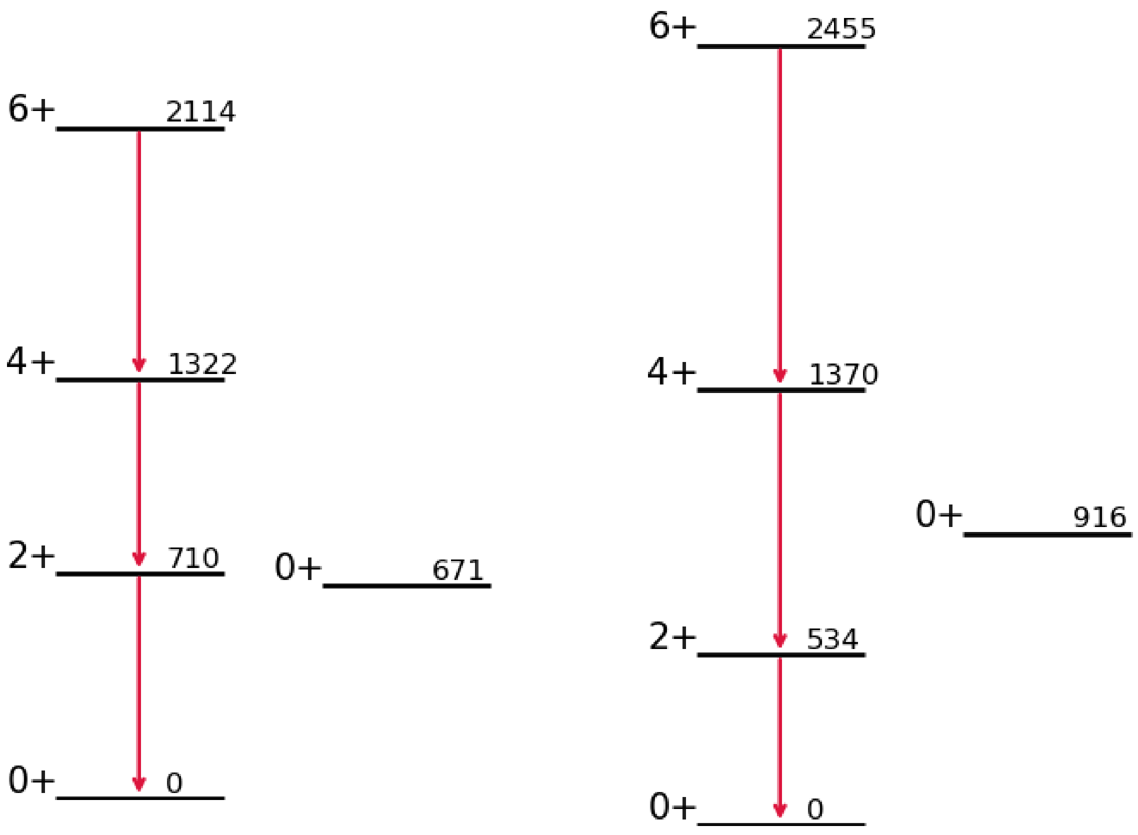


^{72}Kr + Gogny D1S + HO basis 12 shells



Libert J, Girod M and Delaroche JP 1999 *Phys. Rev.* C **60** 054301

^{72}Kr spectrum



Experiment **Theory**

ENSDF database, IAEA, https://www-nds.iaea.org/

What is missing to improve ?

- Zero point energy

Girod M, Delaroche JP, Görgen A, Obertelli A 2009 *Physics Letters B* **676**

- Calculate mass parameters using LQRPA

Washiyama K, Hinohara N, and Nakatsukasa T 2024 *Phys. Rev.* C **109** L051301

- Pairing degree of freedom

Xiang J, Li Z P, Nikšić T, Vretenar D, & Long W H 2020 *Physical Review C* ***101***(6) 064301

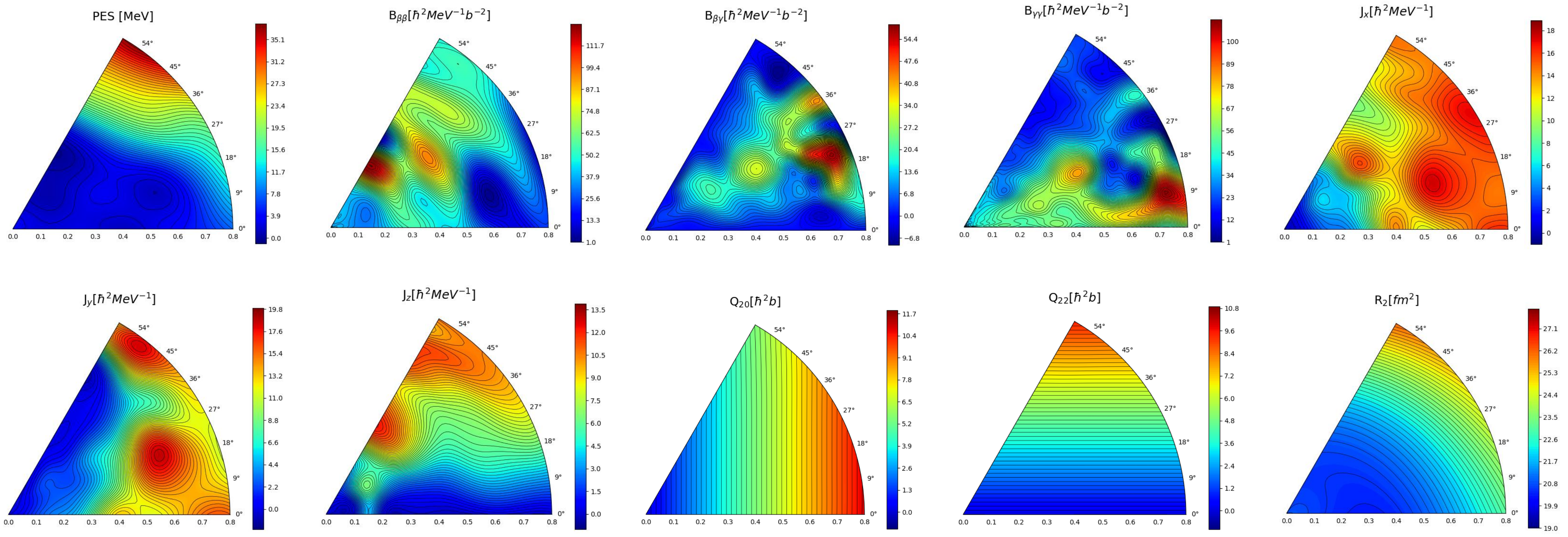
Conclusions and perspectives for the PhD

- ✓ We derived the full quadrupole Bohr Hamiltonian formalism.
- ✓ We perform HFB calculations using triaxial code with Gogny D1S interaction to calculate inertia and mass parameters for the Bohr Hamiltonian using Gaussian process emulator to interpolation.

- ❑ We plan to improve our model to study odd parity states via octupole deformations.
- ❑ We plan to study the role of the interaction on structure of the spectrum by using the newly developed Gogny interactions.

Thank you for your attention !

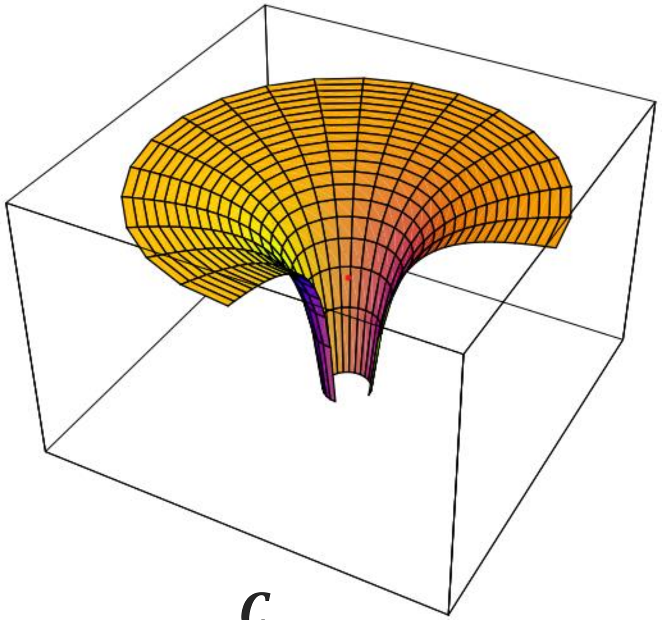
Microscopic calculations



Analytical solution

Solutions of the Bohr Hamiltonian, a compendium – L. Fortunato - 2005

Coulomb-like potential



$$\mathrm{V}(\boldsymbol{\beta}) = -\frac{\boldsymbol{C}}{\boldsymbol{\beta}}$$

$$\boldsymbol{B} = \boldsymbol{B}_{\beta\beta} = \boldsymbol{B}_{\gamma\gamma} = \boldsymbol{J}_x = \boldsymbol{J}_y = \boldsymbol{J}_z = \mathbf{0.5}$$
$$\boldsymbol{B}_{\beta\gamma} = \mathbf{0}$$

$$\Psi(\beta,\gamma,\Omega) = f(\beta)\Phi(\gamma,\Omega)$$

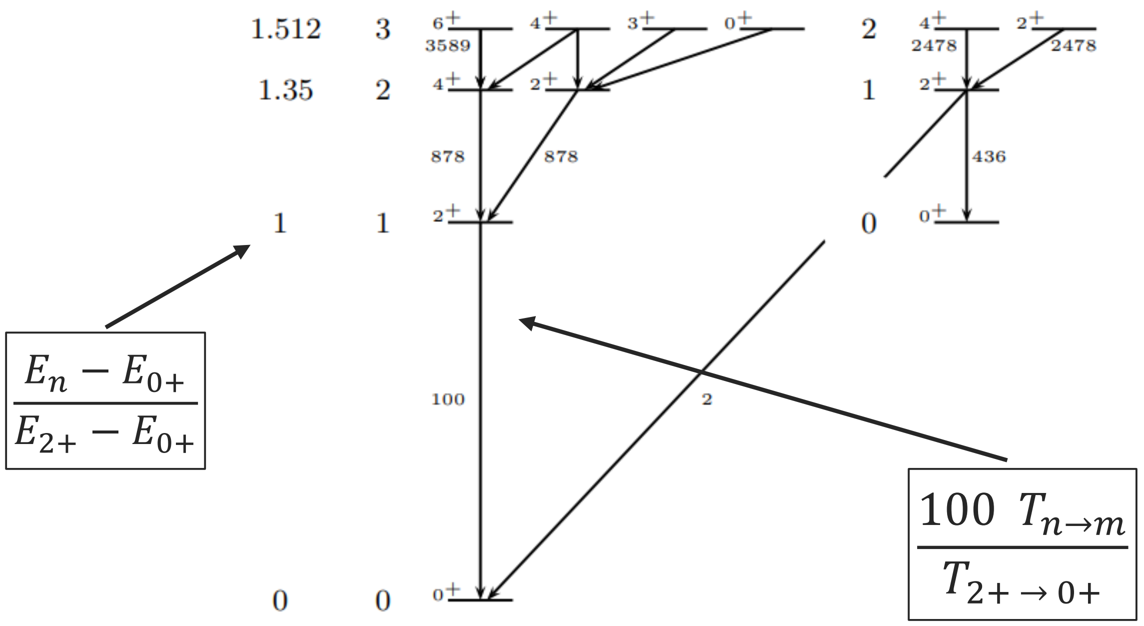
$$\left\{\frac{\hbar^2}{2B}\left(-\frac{1}{\beta^4}\frac{\partial}{\partial\beta}\beta^4\frac{\partial}{\partial\beta}+\frac{\Lambda^2}{\beta^2}\right)+V(\beta)\right\}f(\beta) = Ef(\beta)$$

$$\left\{-\frac{1}{\sin 3\gamma}\frac{\partial}{\partial\gamma}\sin 3\gamma\frac{\partial}{\partial\gamma}+\frac{1}{4}\sum_{k=1}^{3}\frac{\hat{L}_k^2}{\left(\hbar\sin(\gamma-\frac{2\pi}{3}k)\right)^2}\right\}\Phi(\gamma,\Omega) = \Lambda^2\Phi(\gamma,\Omega)$$

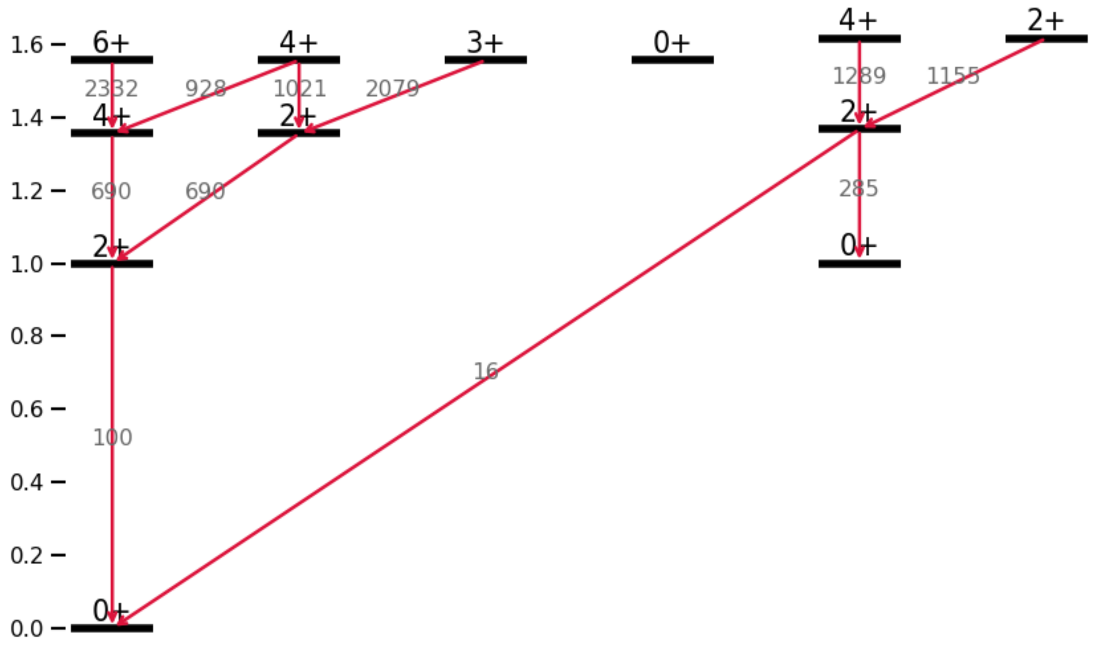
→ Analytical solution

Analytical solution

Analytical solution



Collective wave functions basis + diagonalization



Phenomenological potential+ constant mass parameters