Effective E2 operators: patterns and puzzles across the nuclear landscape

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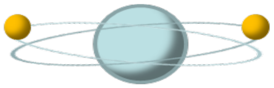
Outline: Focus on E2 effective charges

- Introduction: snapshot of general concepts
- An empirical survey based on B(E2)s in doubly magic nuclei ± 2 like nucleons
  - E0 transitions and shape mixing in ^{40}Ca – implications
  - [The ^{210}Po puzzle – consequence of octupole collectivity?]
- Concluding remarks

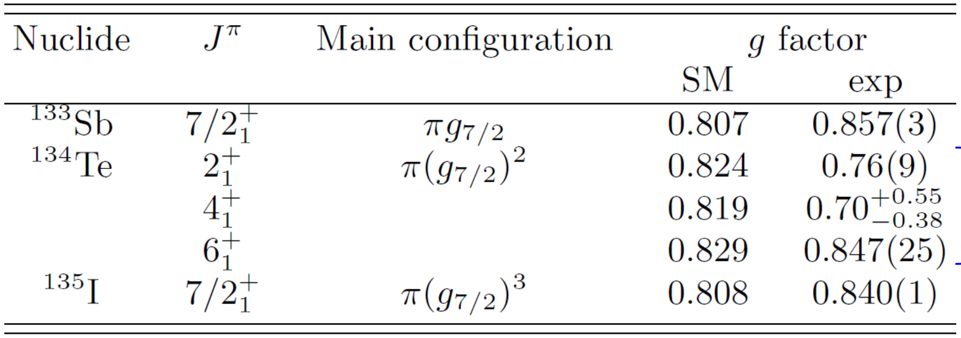
What I won’t cover:

- Theoretical approaches to evaluating effective charges (Summary in Jungclaus PRL 132, 222501(2004) see e.g. Ring and Schuck p.389; Bohr & Mottelson, etc.)
- EL transitions for L>3; see talk by AJ Mitchell on the E6 transition in ^{53}Fe
- DFT models for odd-A nuclei that do not use effective charges; see talk by Jacek Dobaczewski
  - M1 effective operators $\mu = (g_l + \delta g_l)l + (g_s + \delta g_s)s + g_p[Y_2, s]$

Will use g factors to illuminate the structure of excited states

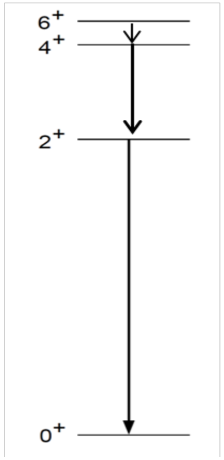


For a single-j orbit: $g(j^n) = g(j)$ independent of the total spin



Nuclide	J^π	Main configuration	g factor SM	g factor exp
^{133}Sb	$7/2_1^+$	$\pi g_{7/2}$	0.807	0.857(3)
^{134}Te	2_1^+	$\pi(g_{7/2})^2$	0.824	0.76(9)
	4_1^+		0.819	$0.70^{+0.55}_{-0.38}$
	6_1^+		0.829	0.847(25)
^{135}I	$7/2_1^+$	$\pi(g_{7/2})^3$	0.808	0.840(1)

2⁺, 4⁺ and 6⁺ have ≈ the same g factor



How I got here…

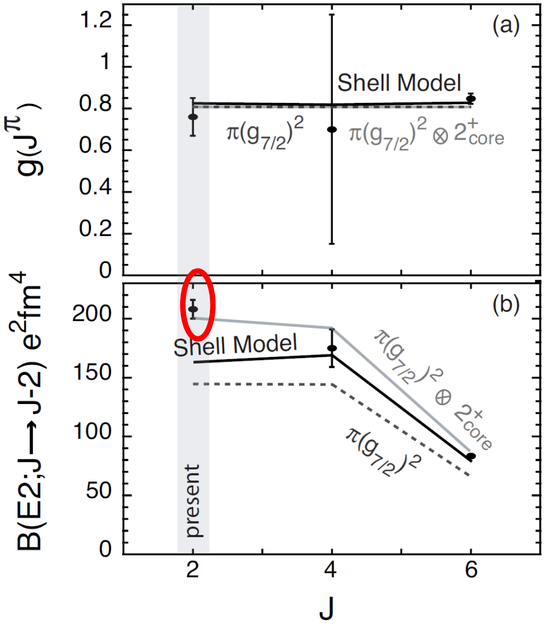
The last RIB measurement at HRIBF ORNL Feb 2012

RAPID COMMUNICATIONS

PHYSICAL REVIEW C **88**, 051304(R) (2013)

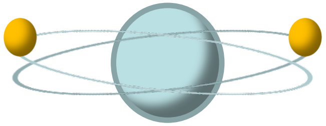
Electromagnetic properties of the 2_1^+ state in ^{134}Te: Influence of core excitation on single-particle orbits beyond ^{132}Sn

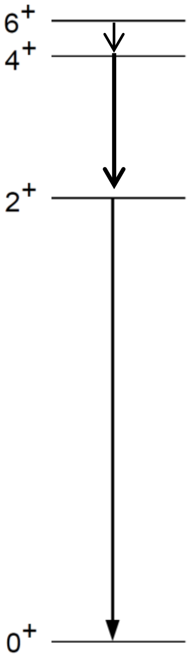
A. E. Stuchbery,[1] J. M. Allmond,[2] A. Galindo-Uribarri,[3,4] E. Padilla-Rodal,[5] D. C. Radford,[3] N. J. Stone,[4,6] J. C. Batchelder,[7] J. R. Beene,[3] N. Benczer-Koller,[8] C. R. Bingham,[3,4] M. E. Howard,[8] G. J. Kumbartzki,[8] J. F. Liang,[3] B. Manning,[8] D. W. Stracener,[3] and C.-H. Yu[3]



- ^{134}Te: 2 protons added to ^{132}Sn
- Lowest 0$^+$, 2$^+$, 4$^+$, 6$^+$ states are predominantly $\pi(0g_{7/2})^2$
- Extra collectivity in the 2$^+$ state can be explained by coupling the $\pi(0g_{7/2})^2$ configuration to the 2$^+$, 4041-keV excitation of the ^{132}Sn core

E2 transitions for the j^2 configuration:



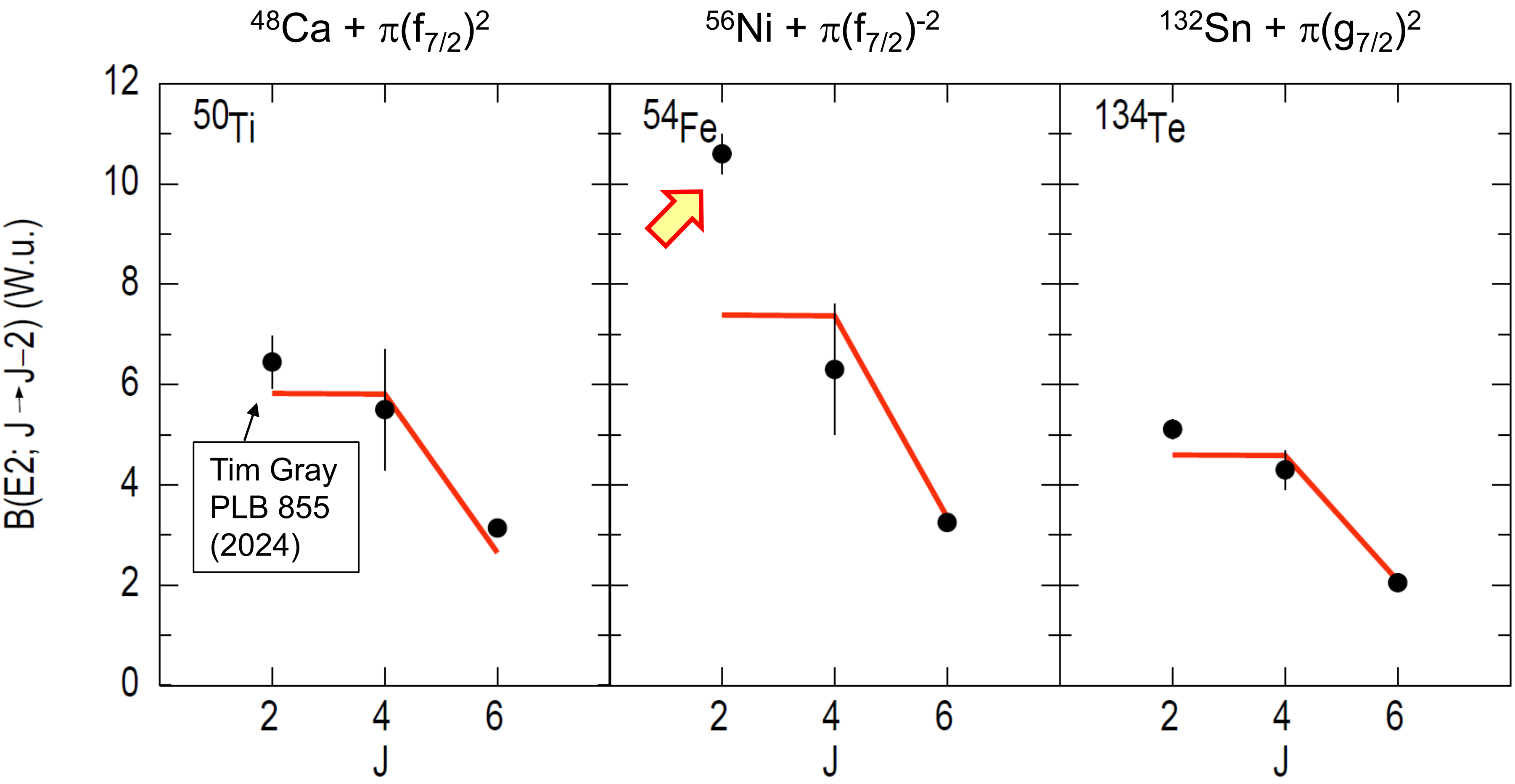


$$B(E2; J_i \to J_i-2) = 4(2J_i-3)\left\{\begin{array}{ccc} j & J_i-2 & j \\ J_i & j & 2 \end{array}\right\}^2 |\langle j||T(E2)||j\rangle|^2$$

$B(E2)$ ratios for the j^2 configuration and experimental B(E2) values in W.u.

j	J_i	$\frac{B(E2;J_i\to J_i-2)}{B(E2;2\to 0)}$	^{50}Ti	^{54}Fe	^{92}Mo	^{210}Po
7/2	6	0.4545455	3.14(13)	3.25(5)		
	4	0.9977324	5.5(12)	6.3(13)		
	2	1.000000	5.3(2)	10.6(4)		
9/2	8	0.3181818			1.311(22)	1.10(5)
	6	0.7946600			3.26(11)	3.00(12)
	4	1.148990			< 24	4.53(15)
	2	1.000000			8.4(5)	0.56(12)

“Doubly magic” + 2 protons with *j*=7/2

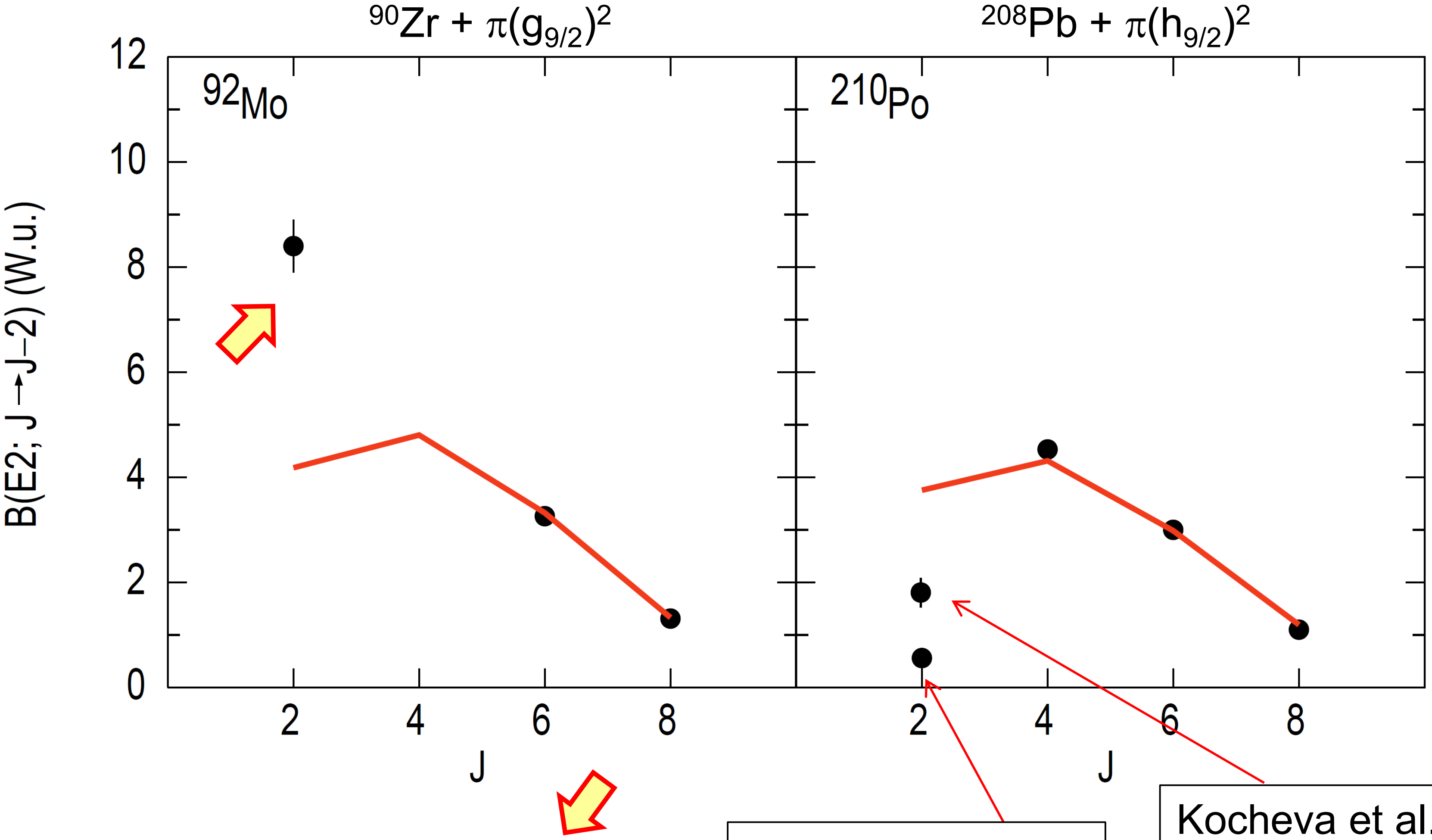


Tim Gray
PLB 855
(2024)

Extra collectivity in 2$^+$ may reduce g factor towards Z/A ~ 0.4

$g(2_1^+)/g(6_1^+) = 0.93(11)$ $g(2_1^+)/g(6_1^+) = 0.77(5)$ $g(2_1^+)/g(6_1^+) = 0.90(11)$

“Doubly magic” + 2 protons with *j*=9/2

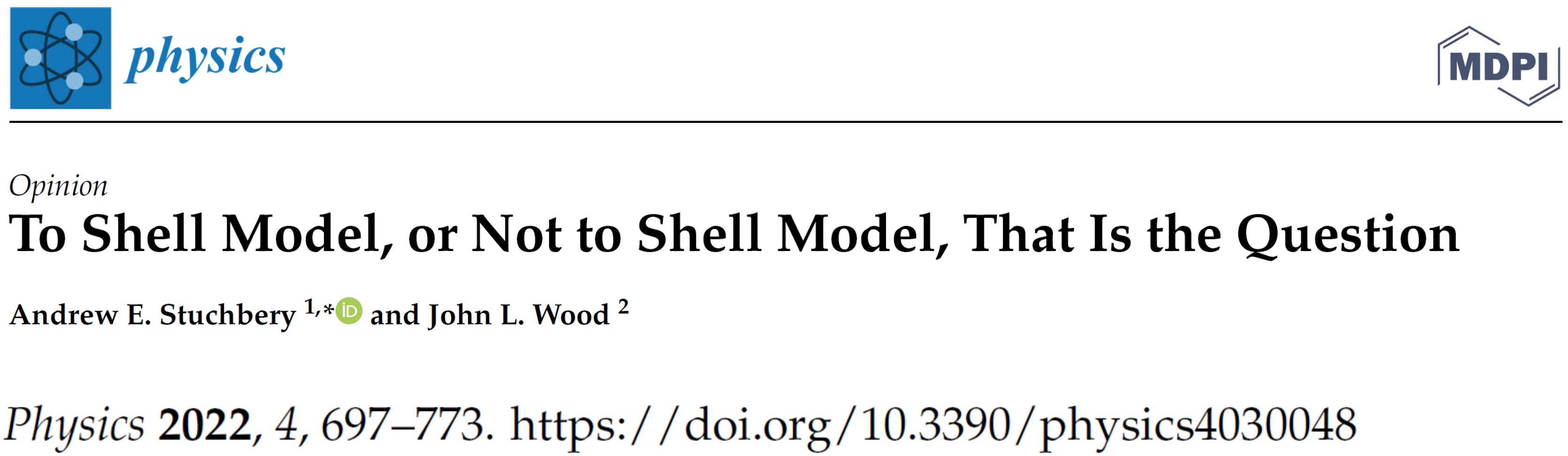


$$g(2_1^+)/g(8_1^+) = 0.81(11)$$

Unsafe Coulex on radioactive target (1973)

Kocheva et al. EPJ **53**, 175 (2017). DSAM - Still small

Further discussion & More details



physics

MDPI

Opinion

To Shell Model, or Not to Shell Model, That Is the Question

Andrew E. Stuchbery [1,*] and John L. Wood [2]

Physics **2022**, *4*, 697–773. https://doi.org/10.3390/physics4030048

Special Issue: "The Nuclear Shell Model 70 Years after Its Advent: Achievements and Prospects"

John Wood

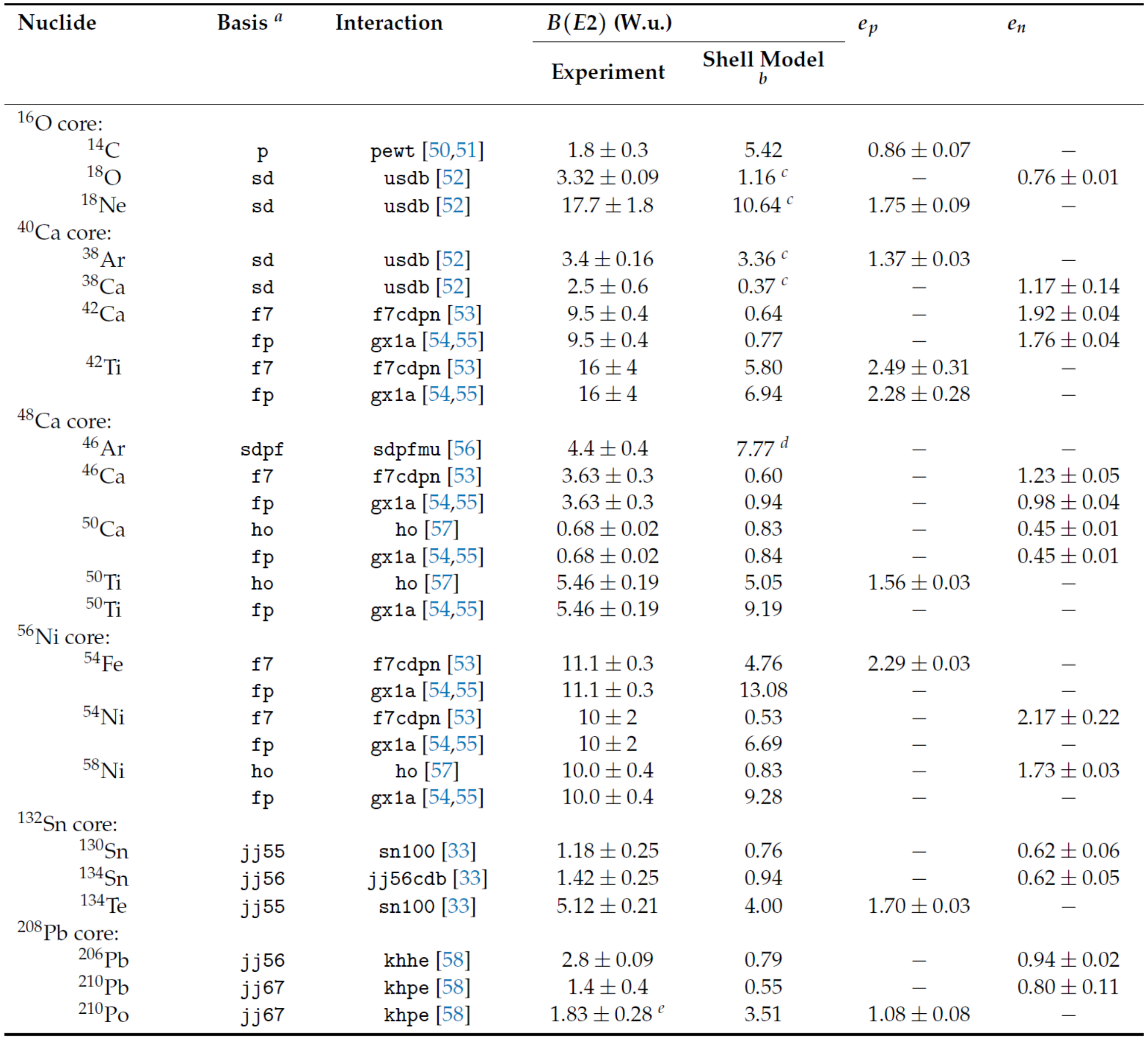
Hopkins Falls in background

November 2022

https://www.worldometers.info/maps/australia-physical-map-full/

e_{eff} doubly magic nuclei ± 2 like nucleons

Effective charges required to bring contemporary shell model calculations into agreement with experiment



Nuclide	Basis [a]	Interaction	$B(E2)$ (W.u.)		e_p	e_n
			Experiment	Shell Model [b]		
^{16}O core:						
^{14}C	p	pewt [50,51]	1.8 ± 0.3	5.42	0.86 ± 0.07	–
^{18}O	sd	usdb [52]	3.32 ± 0.09	1.16 [c]	–	0.76 ± 0.01
^{18}Ne	sd	usdb [52]	17.7 ± 1.8	10.64 [c]	1.75 ± 0.09	–
^{40}Ca core:						
^{38}Ar	sd	usdb [52]	3.4 ± 0.16	3.36 [c]	1.37 ± 0.03	–
^{38}Ca	sd	usdb [52]	2.5 ± 0.6	0.37 [c]	–	1.17 ± 0.14
^{42}Ca	f7	f7cdpn [53]	9.5 ± 0.4	0.64	–	1.92 ± 0.04
	fp	gx1a [54,55]	9.5 ± 0.4	0.77	–	1.76 ± 0.04
^{42}Ti	f7	f7cdpn [53]	16 ± 4	5.80	2.49 ± 0.31	–
	fp	gx1a [54,55]	16 ± 4	6.94	2.28 ± 0.28	–
^{48}Ca core:						
^{46}Ar	sdpf	sdpfmu [56]	4.4 ± 0.4	7.77 [d]	–	–
^{46}Ca	f7	f7cdpn [53]	3.63 ± 0.3	0.60	–	1.23 ± 0.05
	fp	gx1a [54,55]	3.63 ± 0.3	0.94	–	0.98 ± 0.04
^{50}Ca	ho	ho [57]	0.68 ± 0.02	0.83	–	0.45 ± 0.01
	fp	gx1a [54,55]	0.68 ± 0.02	0.84	–	0.45 ± 0.01
^{50}Ti	ho	ho [57]	5.46 ± 0.19	5.05	1.56 ± 0.03	–
^{50}Ti	fp	gx1a [54,55]	5.46 ± 0.19	9.19	–	–
^{56}Ni core:						
^{54}Fe	f7	f7cdpn [53]	11.1 ± 0.3	4.76	2.29 ± 0.03	–
	fp	gx1a [54,55]	11.1 ± 0.3	13.08	–	–
^{54}Ni	f7	f7cdpn [53]	10 ± 2	0.53	–	2.17 ± 0.22
	fp	gx1a [54,55]	10 ± 2	6.69	–	–
^{58}Ni	ho	ho [57]	10.0 ± 0.4	0.83	–	1.73 ± 0.03
	fp	gx1a [54,55]	10.0 ± 0.4	9.28	–	–
^{132}Sn core:						
^{130}Sn	jj55	sn100 [33]	1.18 ± 0.25	0.76	–	0.62 ± 0.06
^{134}Sn	jj56	jj56cdb [33]	1.42 ± 0.25	0.94	–	0.62 ± 0.05
^{134}Te	jj55	sn100 [33]	5.12 ± 0.21	4.00	1.70 ± 0.03	–
^{208}Pb core:						
^{206}Pb	jj56	khhe [58]	2.8 ± 0.09	0.79	–	0.94 ± 0.02
^{210}Pb	jj67	khpe [58]	1.4 ± 0.4	0.55	–	0.80 ± 0.11
^{210}Po	jj67	khpe [58]	1.83 ± 0.28 [e]	3.51	1.08 ± 0.08	–

$$B(E2) = \frac{\left(e_p A_p + e_n A_n\right)^2}{2I_i + 1}$$

From Physics 2022, **4**, 697

Effective charge and polarization effect

Textbook example: ^{17}O

$Q = -0.026$ b

$\Rightarrow$ Odd neutron in $d_{5/2}$ polarizes the ^{16}O core (to assume an oblate shape).

Origin: coupling to particle-hole excitations of the core, including GQR

The deformation of the core gives the odd-nucleon an effective charge:

$$e^{eff}(n) = (\delta e_n)e$$
$$e^{eff}(p) = \left(1 + \delta e_p\right)e$$

Only core protons can contribute, so a first estimate is

$$\delta e_p = \delta e_n \approx Z/A$$

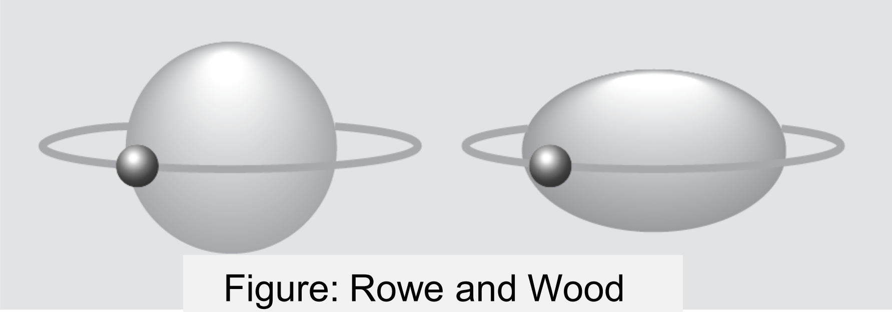
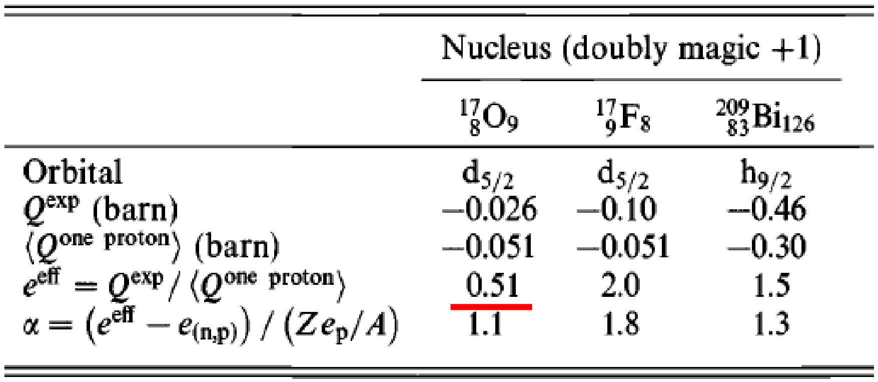


Figure: Rowe and Wood

Shapes and Shells, Nilsson and Ragnarsson



	Nucleus (doubly magic +1)		
	$^{17}_{8}O_9$	$^{17}_{9}F_8$	$^{209}_{83}Bi_{126}$
Orbital	$d_{5/2}$	$d_{5/2}$	$h_{9/2}$
Q^{exp} (barn)	−0.026	−0.10	−0.46
$\langle Q^{one\ proton}\rangle$ (barn)	−0.051	−0.051	−0.30
$e^{eff} = Q^{exp} / \langle Q^{one\ proton}\rangle$	0.51	2.0	1.5
$\alpha = (e^{eff} - e_{(n,p)}) / (Ze_p/A)$	1.1	1.8	1.3

$$B(E2{:}\,1/2^+ \rightarrow 5/2^+) \Longrightarrow \delta e_n = \underline{0.534(3)e}$$

BM: $\delta e = e(\frac{Z}{A} - 0.32\frac{N-Z}{A} + (0.32 - 0.3\frac{N-Z)}{A}\tau_z)$

Expect that $\delta e_n > \delta e_p$

The default effective charges in many shell model calculations are $\delta e_p = \delta e_n = 0.5$

But we expect that $\delta e_n > \delta e_p$ because only core protons contribute, so a valence proton can only interact via $T = 1$ interactions, whereas a valence neutron has $T = 0$ and $T = 1$ contributions *and* the $T = 0$ NN interaction is stronger than $T = 1$.

Examples from fits to data:

sd shell USDB interaction: $\delta e_p = 0.36$ and $\delta e_n = 0.45$

^{51}Fe and ^{51}Mn mirror pair: $\delta e_p \sim 0.15$ and $\delta e_n \sim 0.8$ Du Rietz et al. PRL 93, 222501 (2004)

^{132}Sn region (N < 82): $\delta e_p = 0.7$ and $\delta e_n = 0.8$ Gray et al., PRL 124, 032502 (2020)

^{98}Cd and ^{130}Cd: $\delta e_p \sim 0.2\ vs\ 0.35$ and $\delta e_n \sim 0.8\ vs\ 0.5$ Jungclaus et al., PRL 132, 222501 (2024)

Caveats and consequences

In principle the effective charges are orbit dependent

The effective charge changes with the number of valence nucleons (Andrea’s talk)

The effective charge depends on the size of the basis space

Empirical values of effective charges are a measure of what is missing in the model calculation

Polarization charge = “clean” polarizing effect of the odd nucleon on the core

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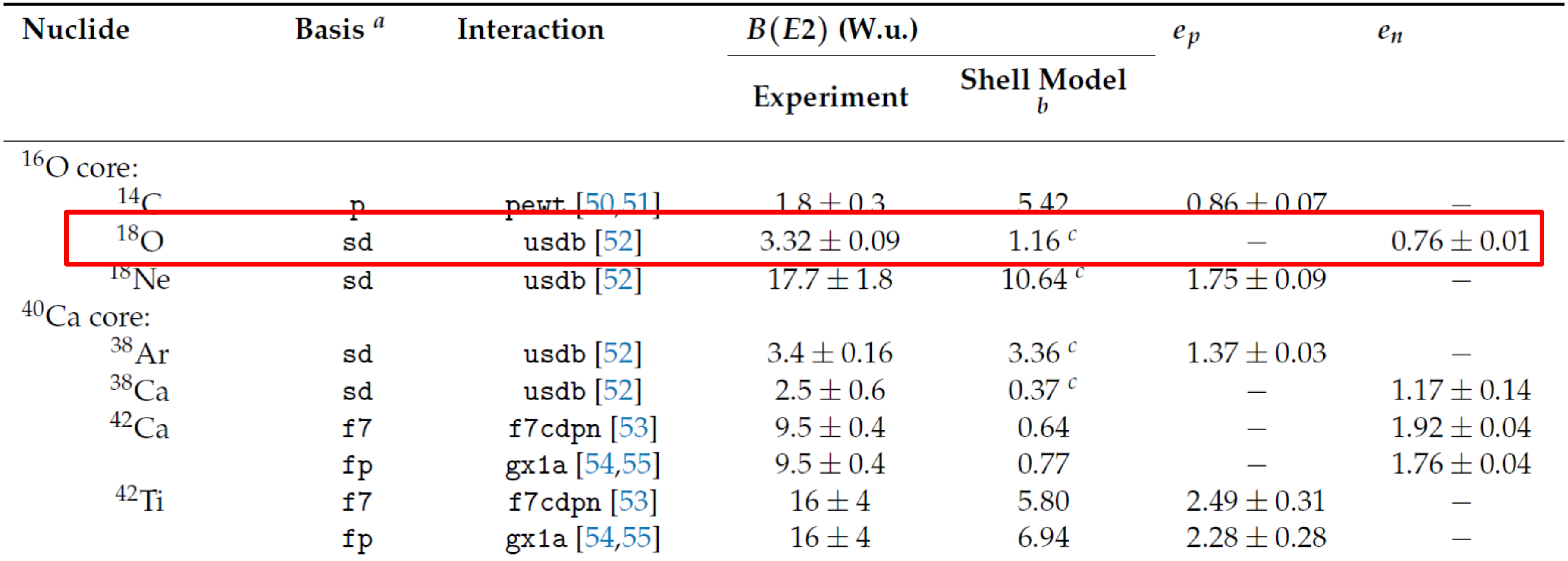
Effective charges near N=Z doubly magic nuclei

Effective charge $e_n \simeq 0.5$ handles ^{17}O but fails for ^{18}O due to additional correlations



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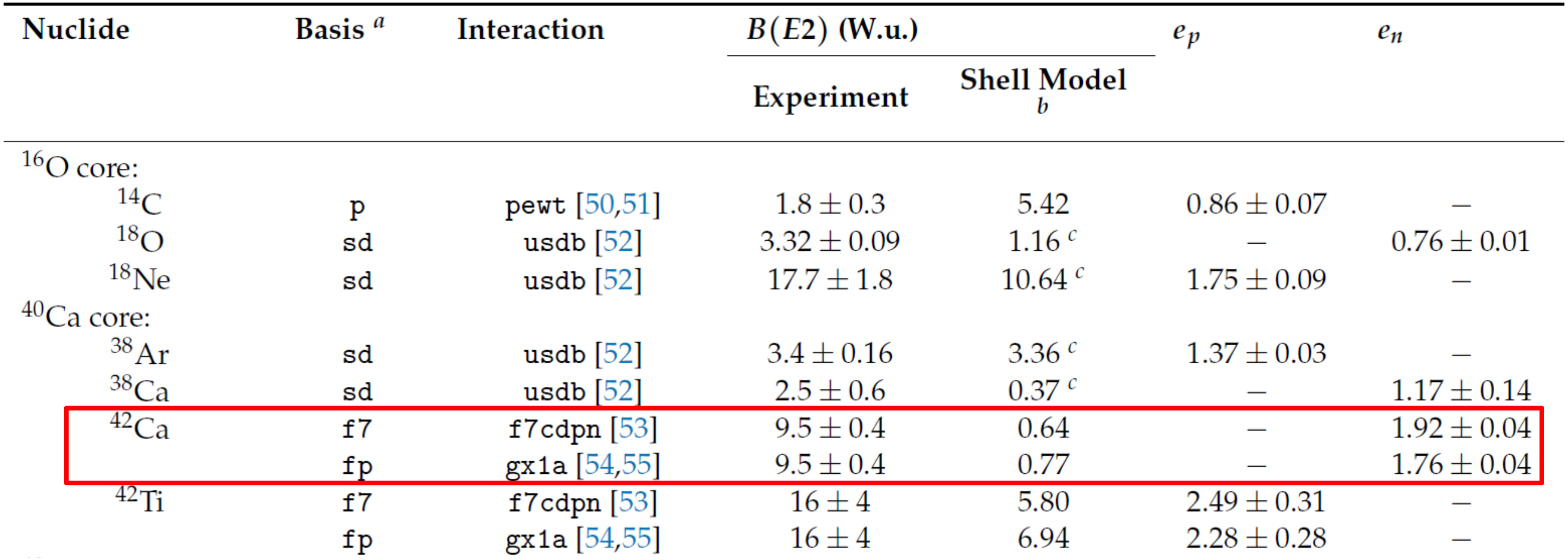
Similarly for ^{41}Ca and ^{42}Ca:

$$Q\left(7/2^- ; {}^{41}Ca\right) \Longrightarrow e_n = 0.58(2)$$

$$B(E2; 2 \longrightarrow 0; {}^{42}Ca) \Longrightarrow e_n \simeq 1.8(4)$$



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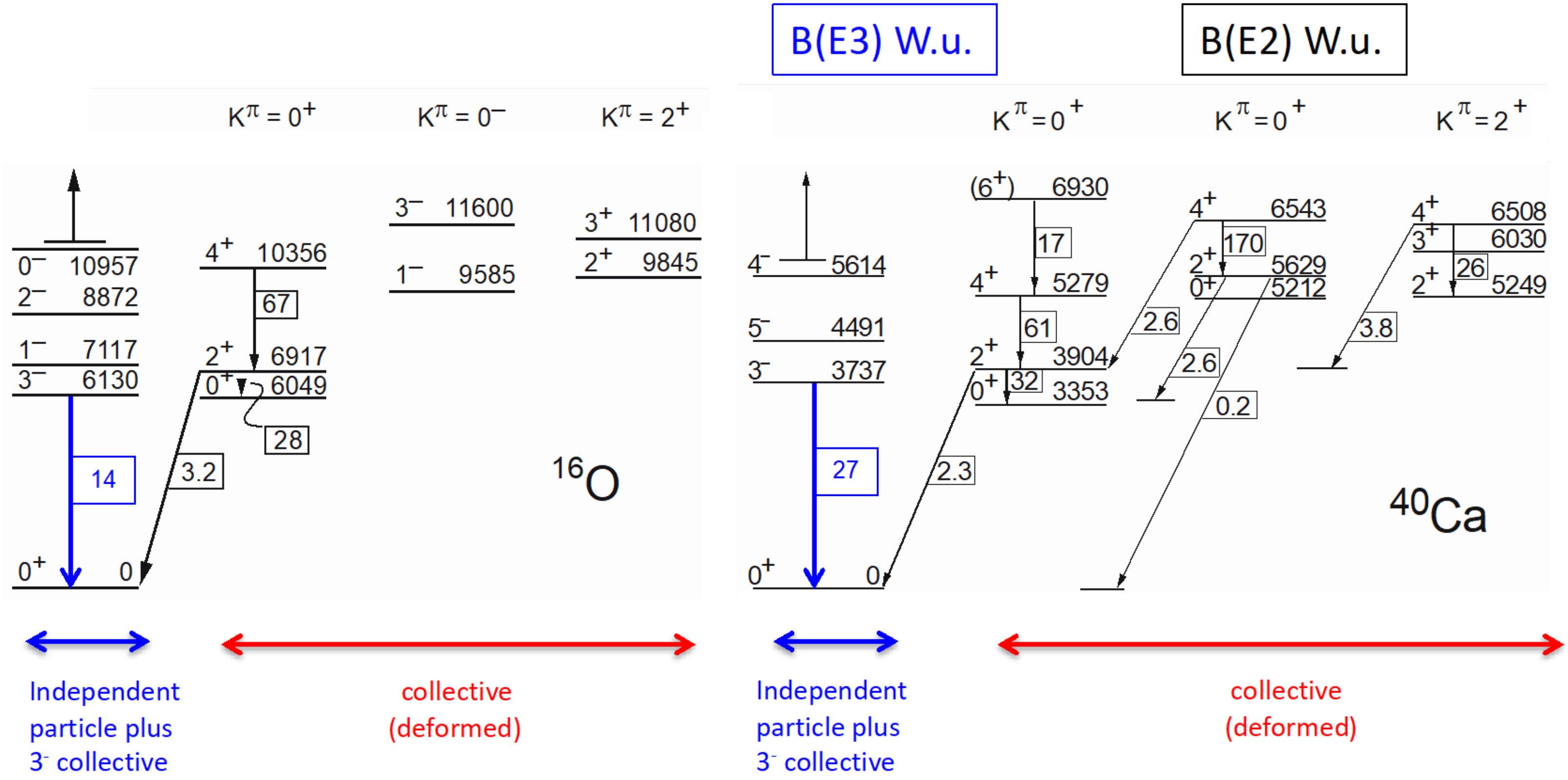
Excited states in ^{16}O and ^{40}Ca

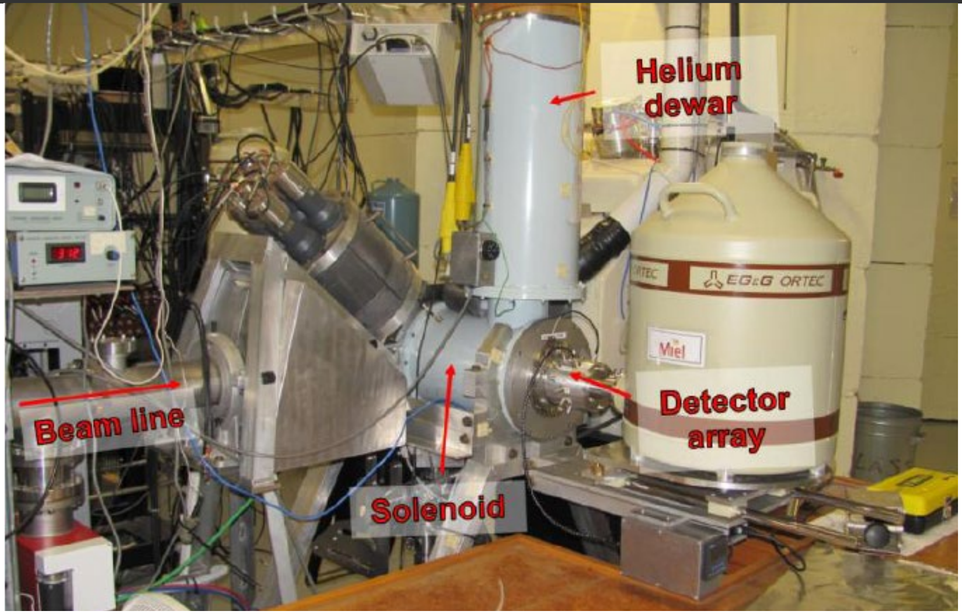


Both ^{16}O and ^{40}Ca have multiparticle-multihole deformed first-excited states

Probe this shape coexistence by E0 measurements

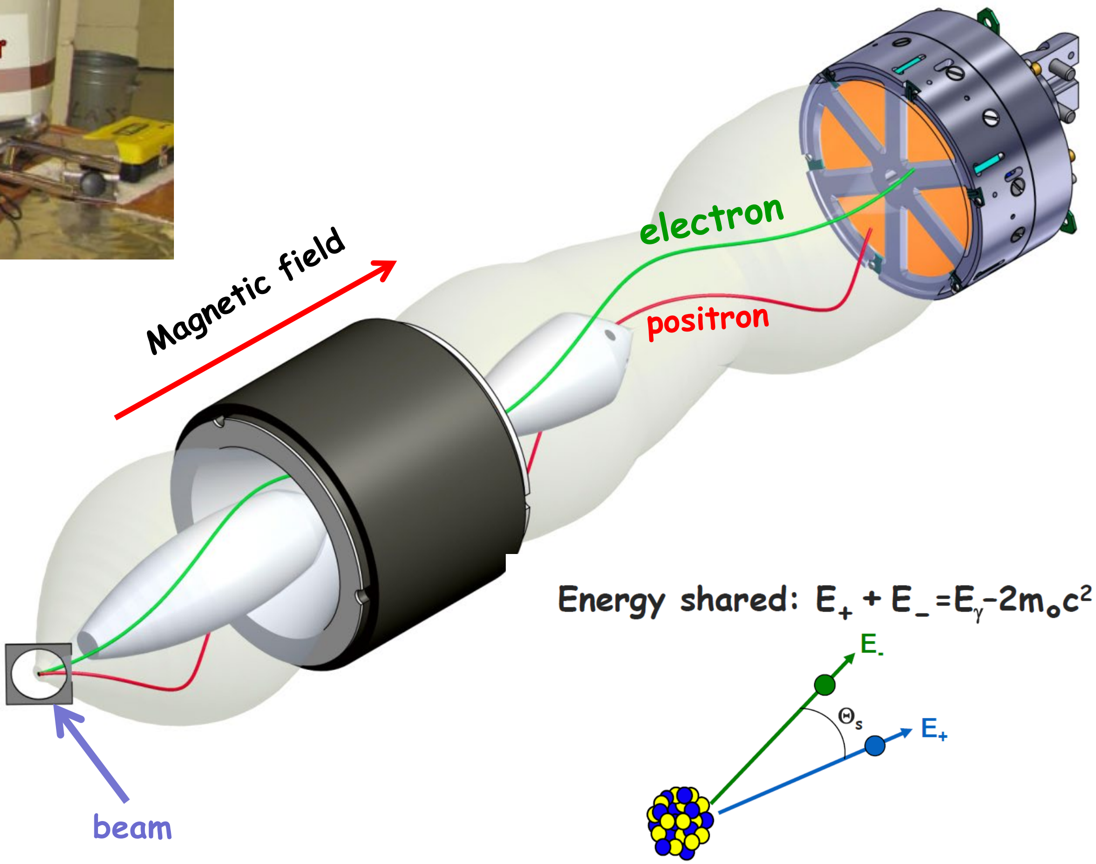
Can we get a clean measure of the polarization charge if shape-mixing is present?

Super-e: IC & Pair spectrometer @ ANU



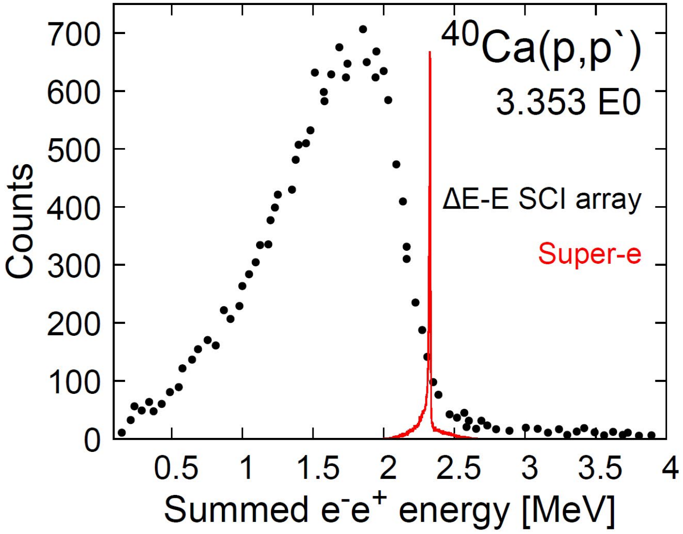
Si(Li) array of six detectors "Miel"
Thickness: 9 mm; Area: 236 mm^2
FWHM ≈ 2.5 keV; Semikon GmBh

Tibor Kibédi



Energy shared: $E_+ + E_- = E_\gamma - 2m_oc^2$

E0 measurements at ANU (Tibor Kibédi)



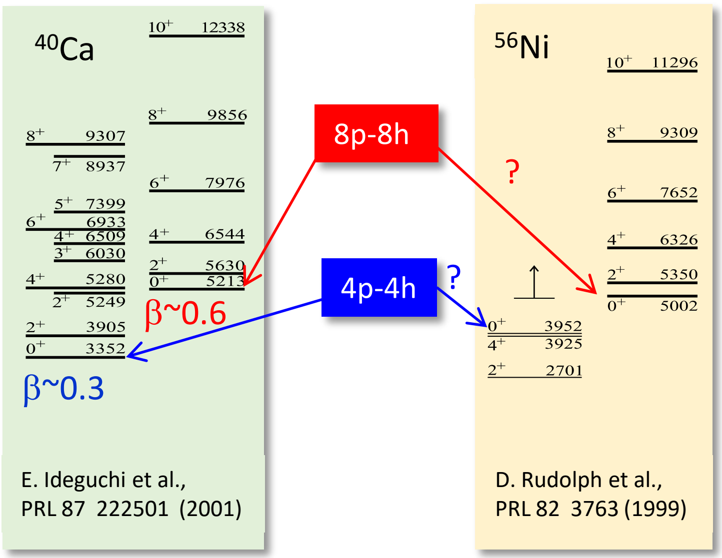


Figure from K. Heyde & J.L. Wood

- ☐ Few E0 previous observations
- ☐ ^{24}Mg, ^{40}Ca: observation of SD to GS E0
- ☐ ^{50,52}Cr, ^{54,56,58}Fe, ^{58,50,62}Ni

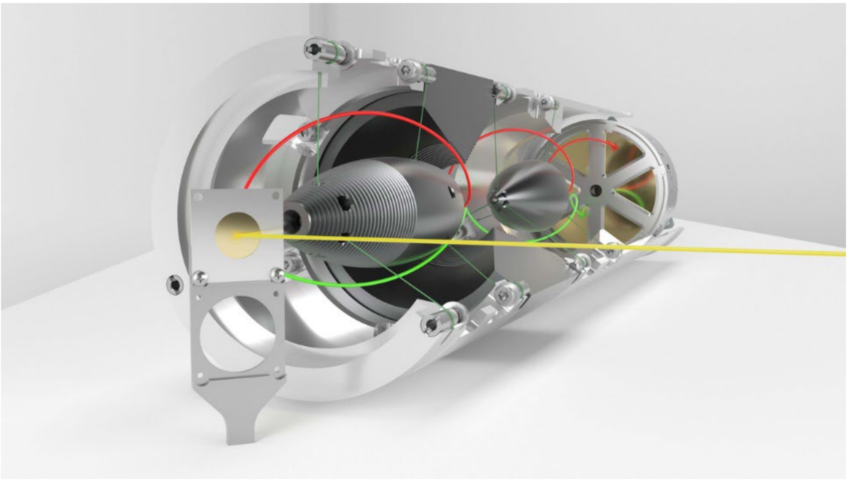
Evitts et al., PLB **779** (2018) 396; PRC **99** (2019) 024306

Eriksen et al., Proc. INPC2016 (2016) 069

Dowie et al., EPJ Web of Conferences **232** (2020) 1, PLB **811** (2020) 135855

Ideguchi et al., PRL **87** 222501 (2001

Electronic factors: Dowie et al., At. Data Nucl. Data Tab. **131** (2020) 101283



ANU Super-e pair spectrometer

Review: E0 transitions and shapes

Dimensionless E0 strength:

$$\rho(E0) = \frac{< f|M(E0)|i >}{eR^2}$$

Experimental E0 transition probability:

$$W(E0) = \frac{1}{\tau(E0)} = W_{ic}(E0) + W_{\pi}(E0)$$

Where:

$$W_{ic}(E0) + W_{\pi}(E0) = \rho^2(E0) \times [\Omega_{ic}(E0) + \Omega_{\pi}(E0)]$$

Internal conversion

Pair production

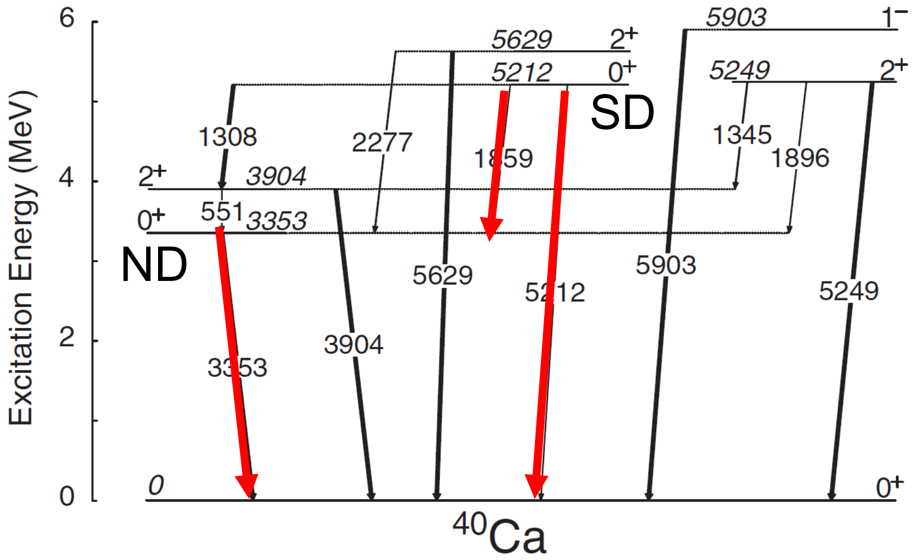
Electronic factors
Dowie et al., ADNDT **131** (2020) 101283

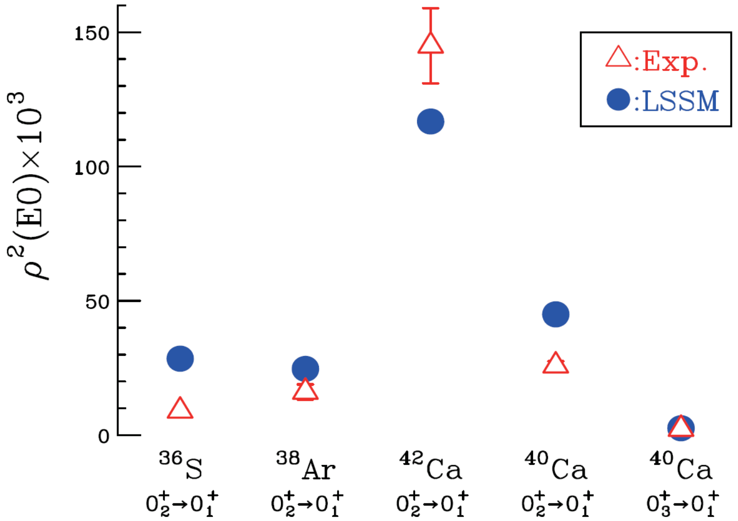
$$\rho_{if}^2 \cdot 10^3 = \alpha^2\beta^2(\Delta\langle r^2\rangle)^2 \cdot 10^3 \frac{Z^2}{R^4}$$

Strong E0 transitions ⇒ strong mixing between states with different shapes

See: Kibédi, Garnsworthy, Wood, Prog. Nucl. Part. Phys. **123** (2022) 103930

Shape co-existence in ^{40}Ca





LSSM with up to 10p-10h excitations across N,Z=20

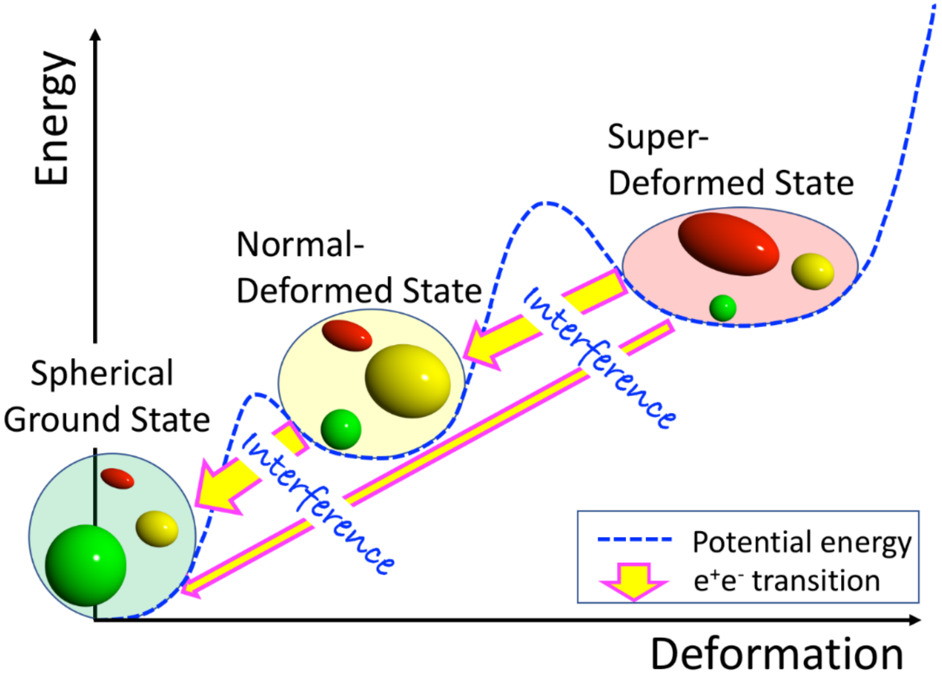
PHYSICAL REVIEW LETTERS **128,** 252501 (2022)

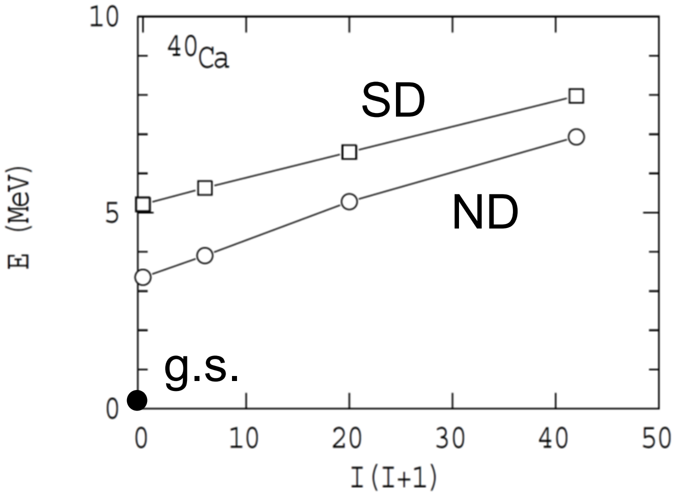
Electric Monopole Transition from the Superdeformed Band in ^{40}Ca

E. Ideguchi (井手口 栄治),[1,*] T. Kibédi,[2] J. T. H. Dowie,[2] T. H. Hoang,[1] M. Kumar Raju,[1,3] N. Aoi (青井 考),[1] A. J. Mitchell,[2] A. E. Stuchbery,[2] N. Shimizu (清水 則孝),[4,†] Y. Utsuno (宇都野 穣),[5,4] A. Akber,[2] L. J. Bignell,[2] B. J. Coombes,[2] T. K. Eriksen,[2] T. J. Gray,[2] G. J. Lane,[2] and B. P. McCormick[2]

Shape co-existence in ^{40}Ca

Figure credit: Eiji Ideguchi





- Significant mixing between 3 co-existing shapes
- Ground state of this doubly magic nucleus is not spherical?
- **How does this impact on empirical effective charges?**

3-state mixing in ^{40}Ca

Microscopic shell model calculations focused on E0 transition strengths (didn't calculate shapes)

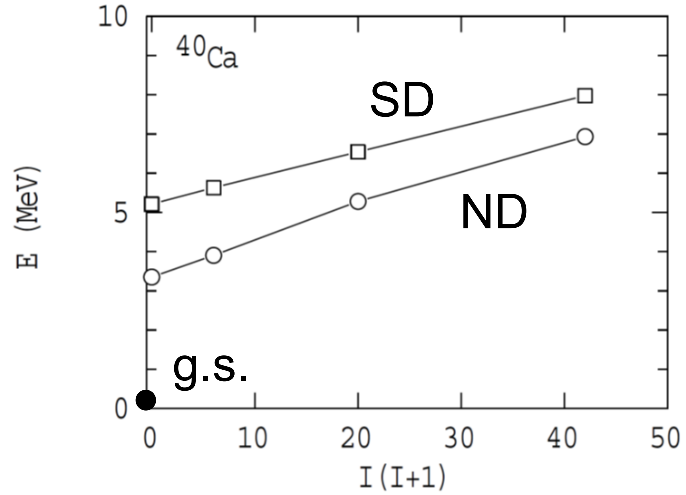
Schematic 3-state mixing to fit 0^+excitation energies and ρ^2values assuming $\beta_{sph} = 0$, $\beta_{ND} = 0.3$, $\beta_{SD} = 0.6$

Matrix: $\begin{bmatrix} 409 & 1105 & 70 \\ 1105 & 2990 & 277 \\ 70 & 277 & 5167 \end{bmatrix}$

Eigenvectors:

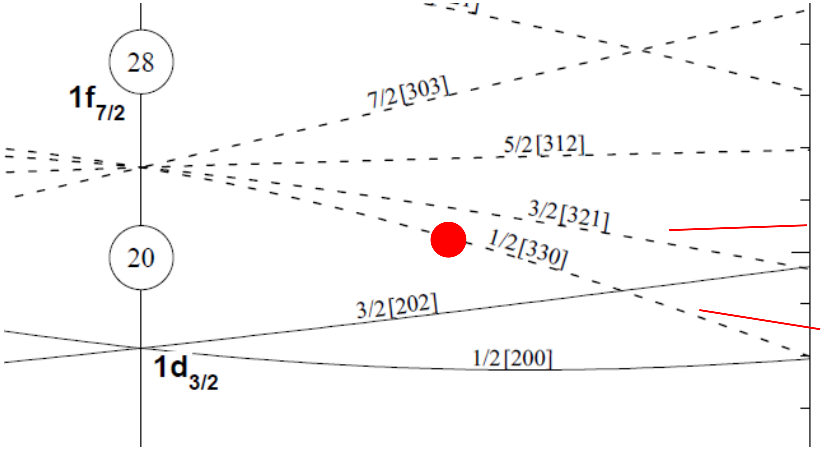
$$|0_1^+\rangle = 0.94|0_{sph}\rangle - 0.34\,|0_{ND}\rangle + 0.05|0_{SD}\rangle$$
$$|0_2^+\rangle = -0.35|0_{sph}\rangle - 0.93\,|0_{ND}\rangle + 0.15|0_{SD}\rangle$$
$$|0_3^+\rangle = 0.006|0_{sph}\rangle + 0.15\,|0_{ND}\rangle + 0.99|0_{SD}\rangle$$

$\Rightarrow \beta_{gs} \approx 0.037$ or $Q_0 \sim +0.09$ eb



^{40}Ca shape mixing & ^{41}Ca Q(g.s.)

- Microscopic shell model calculations focused on E0 transition strengths (didn't calculate shapes)
- But E0 transitions imply shape mixing in ^{40}Ca ground state making it *prolate*
- Empirically estimate $\beta \sim 0.04$, hence $Q_0 \sim +0.09$ eb $\Rightarrow Q(7/2^-;\, {}^{41}Ca) \sim +0.04$?
- No!! Experimentally $Q(7/2^+;\, {}^{41}Ca) = -0.066(2)$ eb
- $|{}^{41}Ca; 7/2^-\rangle \sim 0.94|\nu f_{7/2}\rangle - 0.34|\underbrace{\nu f_{7/2} \otimes deformed\ core}\rangle$



Lowest Nilsson orbits with f$_{7/2}$ parentage

3/2[321]

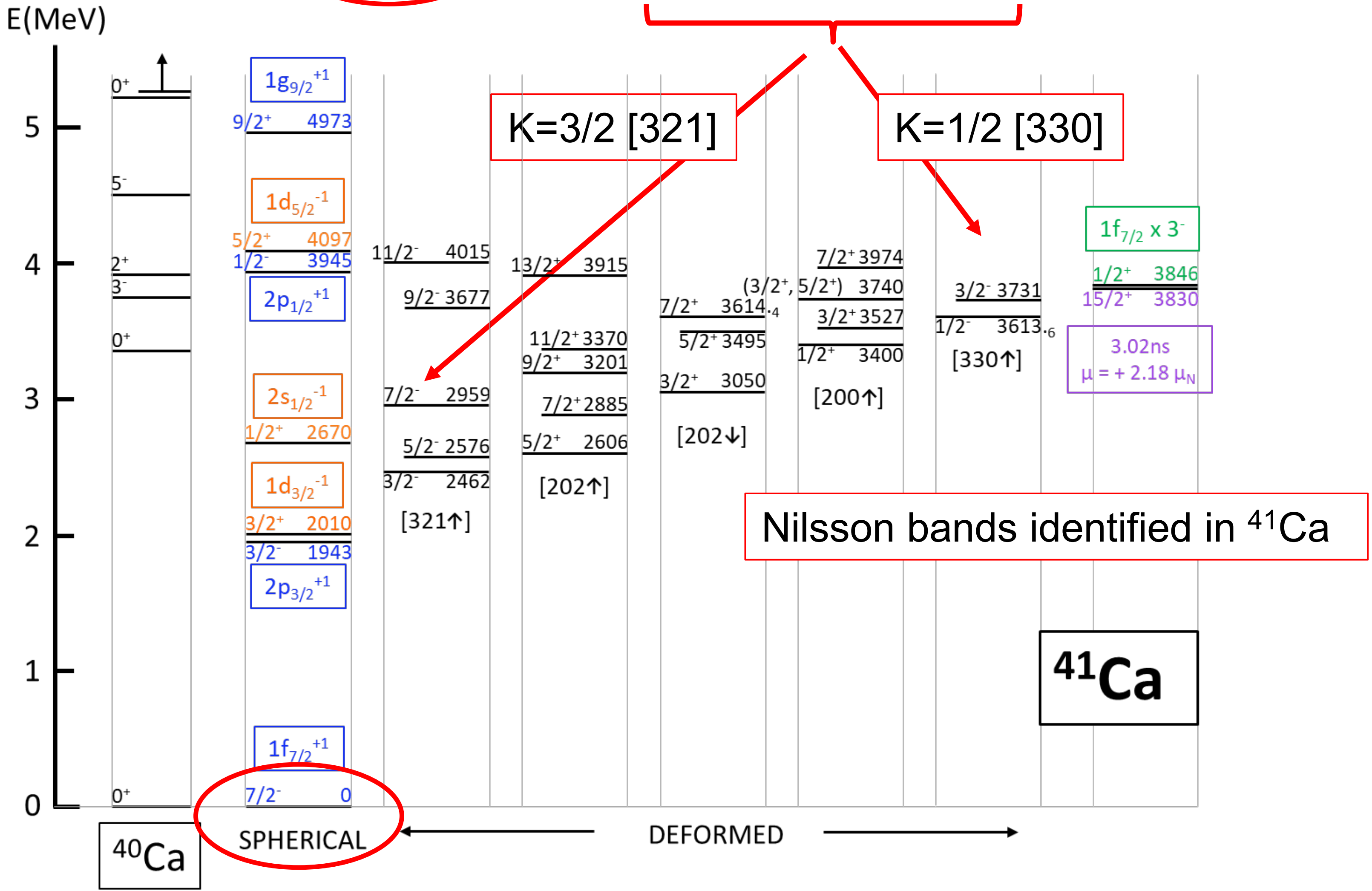
½[330]

$$Q(I) = \frac{3K^2 - I(I+1)}{(I+1)(2I+3)} Q_0$$

$I = 7/2$ and $K = 1/2$ or $K = 3/2$

^{40}Ca shape mixing & ^{41}Ca Q(g.s.)

- $|^{41}Ca;7/2^-\rangle \sim \boxed{0.94|\nu f_{7/2}\rangle} - 0.34|\nu f_{7/2} \otimes deformed\ core\rangle$



Nilsson bands identified in ^{41}Ca

Figure from Jenkins-Wood 3. Original classification: Röpke EPJ A 22, 213{230 (2004)

^{40}Ca shape mixing & ^{41}Ca Q(g.s.)

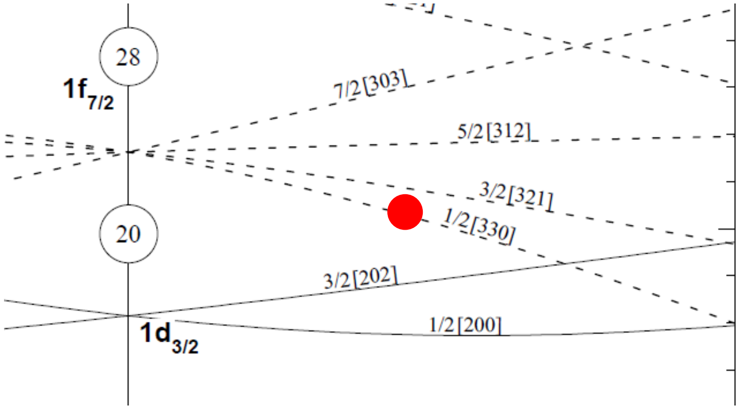
- Experimentally $Q(7/2^{+};{}^{41}Ca) = -0.066(2)$ eb
- $|^{41}Ca;7/2^{-}\rangle \sim 0.94|\nu f_{7/2}\rangle - 0.34|\underbrace{\nu f_{7/2} \otimes deformed\ core}\rangle$

Assume this looks like the $I = 7/2$ member of the ½$[330]$ Nilsson band.

For $\beta = 0.3$, $Q_0 \sim 0.78$ eb and hence $Q(7/2) = -Q_0/3 \sim -0.26$ eb.

Contribution to the g.s. moment could be about $0.34^2 \times -0.26 \sim \underline{-0.03}$ eb (**negative!**)

This is likely an overestimate as the mixing strength from ^{40}Ca is no longer applicable; mixing is with an excited band member – not the $K = 1/2$ band head!



^{40}Ca shape mixing & ^{41}Ca Q(g.s.)

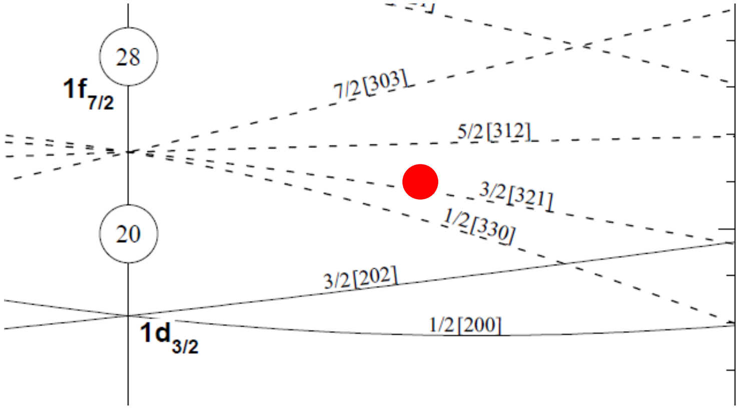
- Experimentally $Q(7/2^{+};{}^{41}Ca) = -0.066(2)$ eb
- $|{}^{41}Ca;7/2^{-}\rangle \sim 0.94|\nu f_{7/2}\rangle - 0.34|\underbrace{\nu f_{7/2} \otimes deformed\ core}\rangle$

Assume this looks like the $I = 7/2$ member of the $3/2[321]$ Nilsson band.

For $\beta = 0.3$, $Q_0 \sim 0.78$ eb and hence $Q(7/2) = -Q_0/5 \sim -0.16$ eb.

Contribution to the g.s. moment could be about $0.34^2 \times -0.16 \sim -0.02$ eb (**negative!**)

This is likely an overestimate as the mixing strength from ^{40}Ca is no longer applicable; mixing is with an excited band member – not the $K = 3/2$ band head!



^{40}Ca shape mixing & ^{41}Ca Q(g.s.)

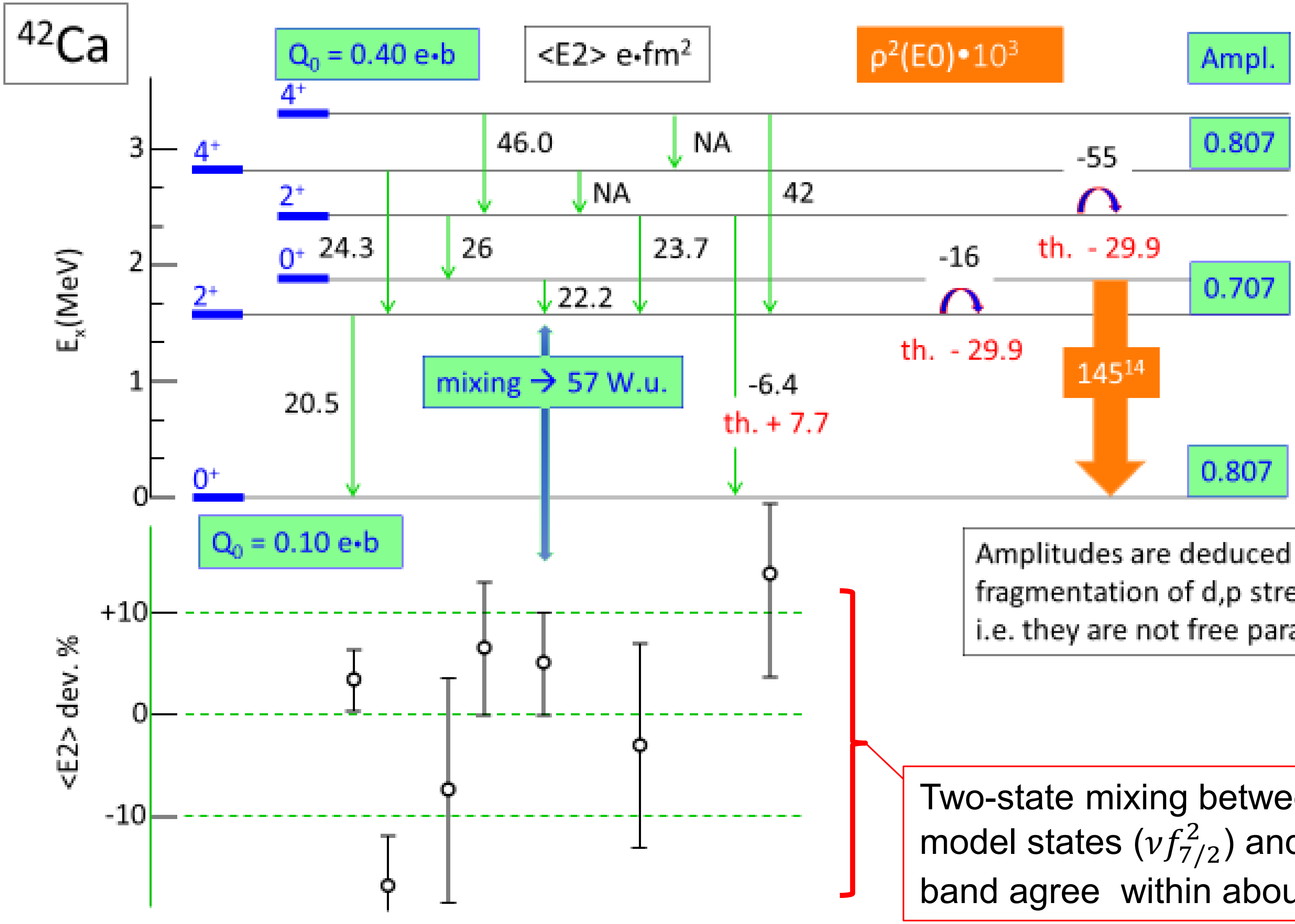
Need microscopic calculations of the quadrupole moment(s) and charge radii, etc. in ^{40,41}Ca treating the shape mixing on an equal footing.

Meanwhile observe that in ^{41}Ca the ground-state spectroscopic quadrupole moment can have a negative contribution from prolate shape-coexistence in the core

^{42}Ca next:

The level of mixing in ^{41}Ca is much reduced compared with that found in ^{42}Ca where the spherical $\nu f_{7/2}^2$ states and the deformed states have the same spin-parity sequence: 0^+, 2^+, 4^+, 6^+

Two-state mixing analysis E2s in ^{42}Ca



Note mixing amplitudes. 2$^+$ is 50/50 spherical/ deformed

Amplitudes are deduced from fragmentation of d,p strength, i.e. they are not free parameters.

Two-state mixing between the shell model states $(\nu f_{7/2}^2)$ and the deformed band agree within about ±10%

Stuchbery & Wood. Fig. 42

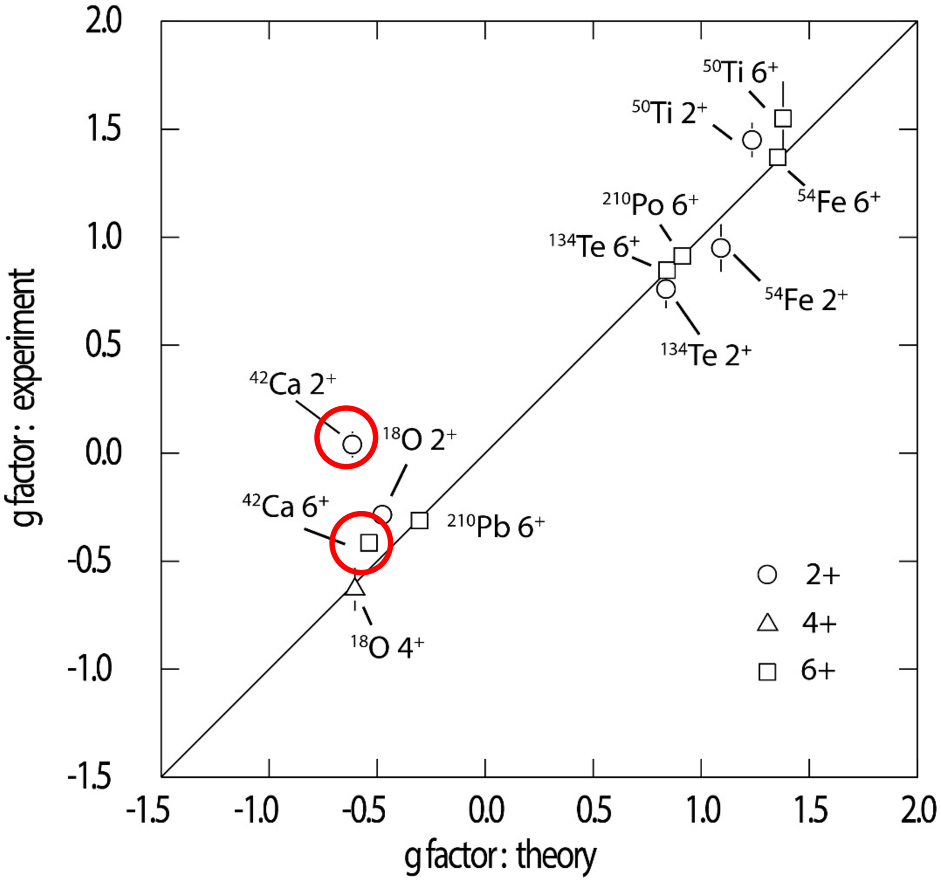
Data: mainly K. Hadyńska-Klęk *et al.* Phys. Rev. C **97**, 024326

Australian National University

^{42}Ca effective charges vs spin

Transition	δe_n
$2^+ \rightarrow 0^+$	1.76(4)
$4^+ \rightarrow 2^+$	1.60(12)
$6^+ \rightarrow 4^+$	0.70(1)

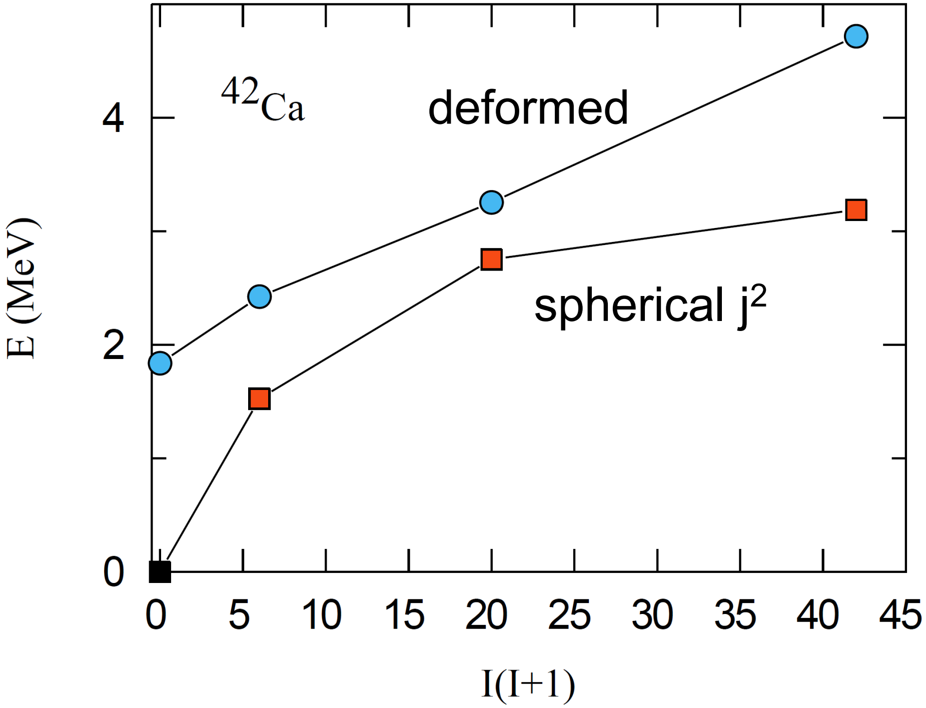
- Effective charge is smallest for highest member of the $\nu f_{7/2}^2$ multiplet
- Also, g factor is closest to shell model for that state

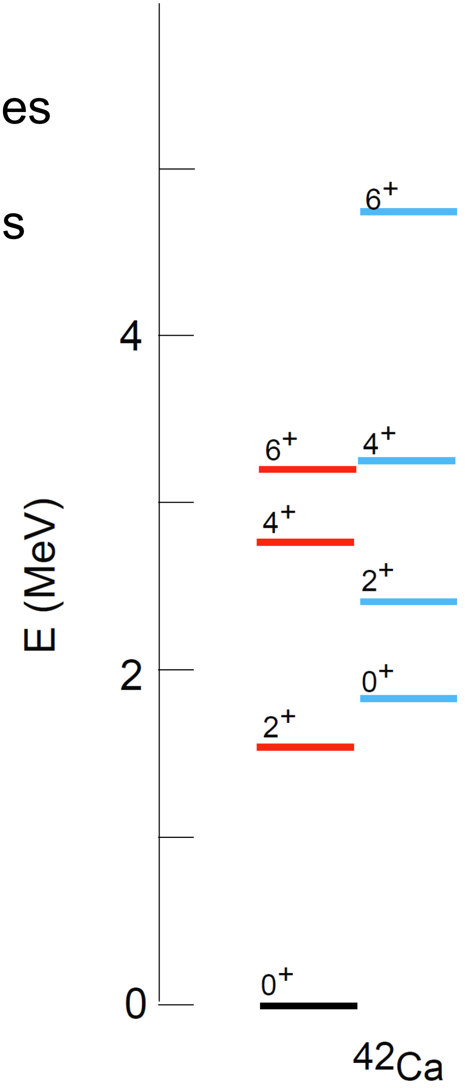


^{42}Ca levels: spherical and deformed

Transition	δe_n
$2^+ \rightarrow 0^+$	1.76(4)
$4^+ \rightarrow 2^+$	1.60(12)
$6^+ \rightarrow 4^+$	0.70(1)

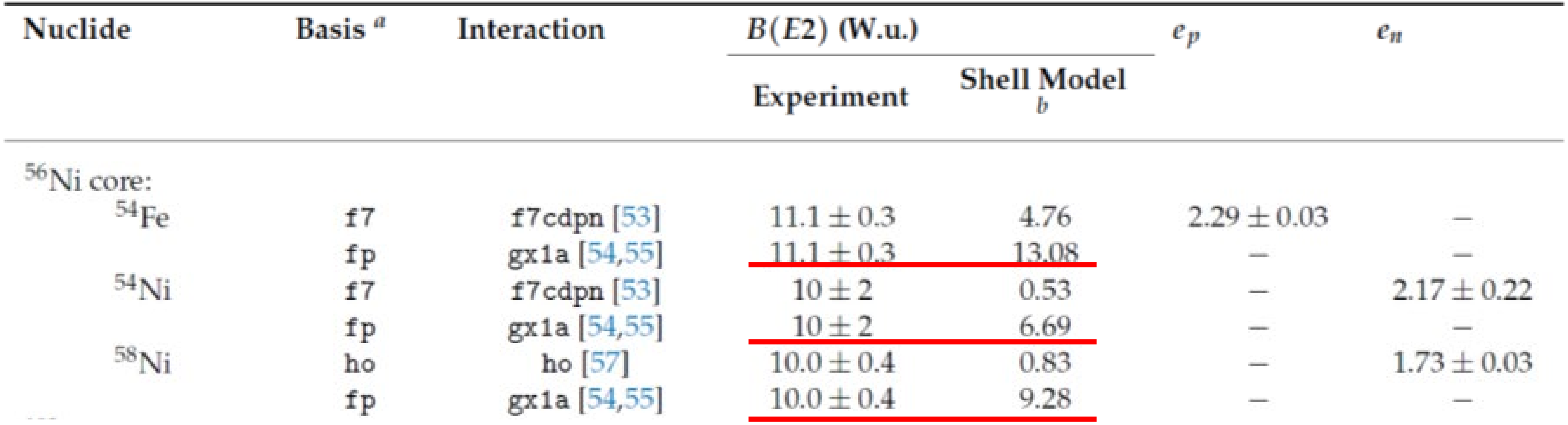
Energy separation increases (deceases) with spin for deformed (spherical) states





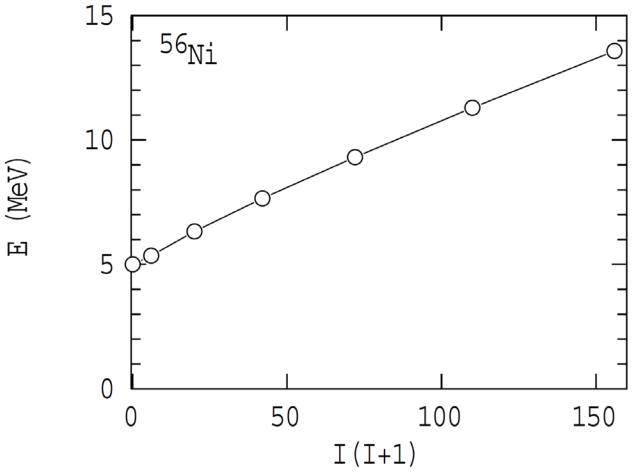
Other N=Z doubly magic nuclei: ^{56}Ni

^{56}Ni: core excitations included in fp model space



Nuclide	Basis [a]	Interaction	$B(E2)$ (W.u.) Experiment	Shell Model [b]	e_p	e_n
^{56}Ni core:						
^{54}Fe	f7	f7cdpn [53]	11.1 ± 0.3	4.76	2.29 ± 0.03	–
	fp	gx1a [54,55]	11.1 ± 0.3	13.08	–	–
^{54}Ni	f7	f7cdpn [53]	10 ± 2	0.53	–	2.17 ± 0.22
	fp	gx1a [54,55]	10 ± 2	6.69	–	–
^{58}Ni	ho	ho [57]	10.0 ± 0.4	0.83	–	1.73 ± 0.03
	fp	gx1a [54,55]	10.0 ± 0.4	9.28	–	–

Satisfactory agreement for gx1a interactions and $\delta e_p = \delta e_n = 0.5$

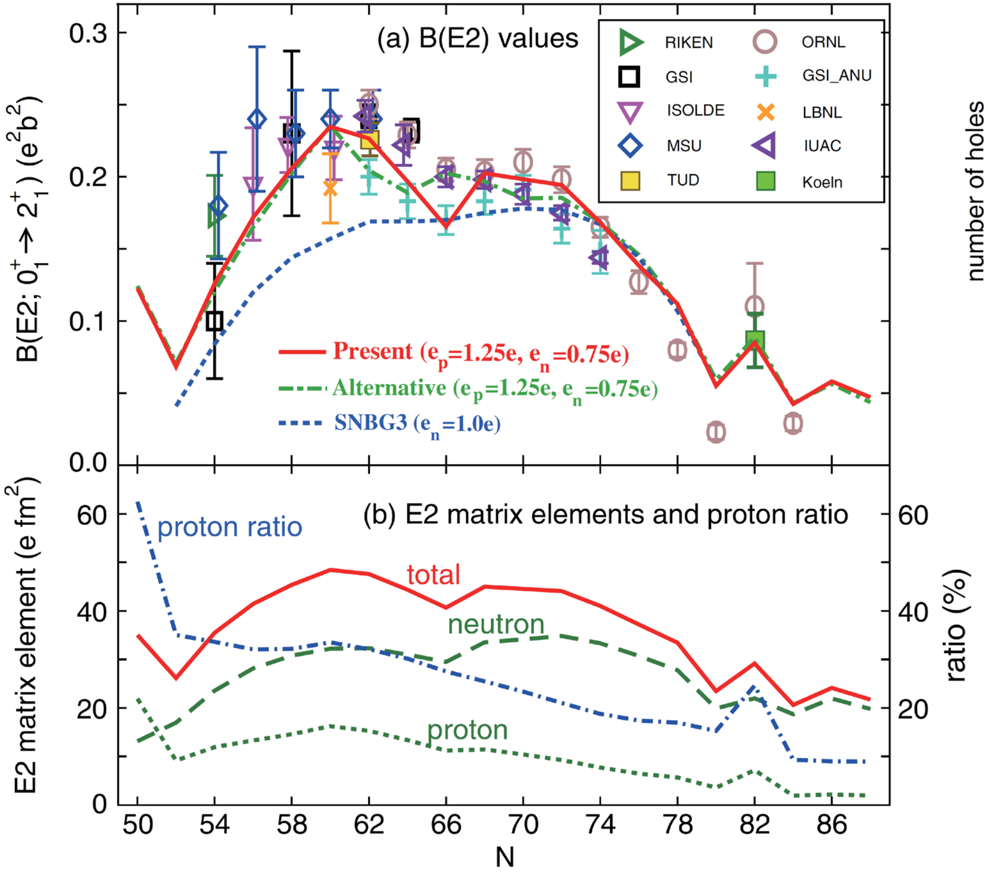


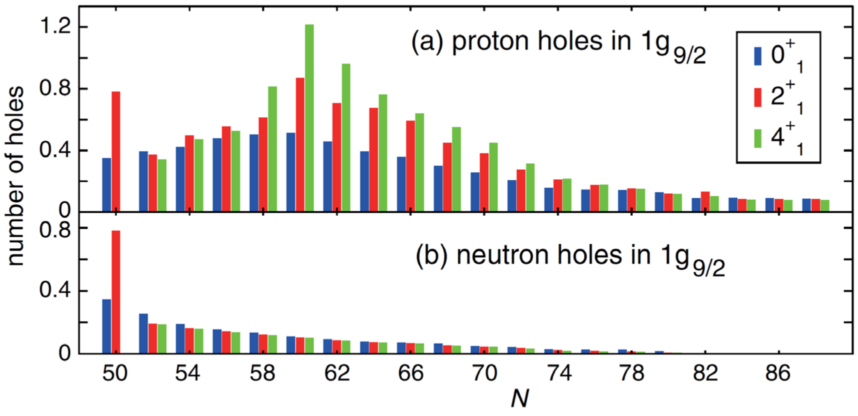
Deformed band in ^{56}Ni based on 3rd 0+ state

State	Ex exp (MeV)	Ex theory (MeV)
0_1^+	0	0
0_2^+	3.957	3.608
0_3^+	5.004	4.941 ⇦

Other N=Z doubly magic nuclei: ^{100}Sn

^{100}Sn: core excitations implied by B(E2) data and large basis MCSM calculations





Strategy:

- Use largest practical basis space
- Focus on high-spin nominally j^2 states

MCSM from PRL **121**, 062501 (2018)

Isoscalar and isovector effective charges

- Noted isoscalar and isovector polarization effects above
- Expect polarization charges $\delta e_n$and $\delta e_p$to vary with neutron number for given Z
- Define isoscalar and isovector polarization charges: $e_{IS} = \frac{1}{2}(\delta e_n + \delta e_p)$ and $e_{IV} = \frac{1}{2}(\delta e_n - \delta e_p)$
- Plot versus $(N - Z)/A$

c.f. Andrea’s talk – this session
^{98,130}Cd $\pi g_{9/2}^2$ isomers

- New results in n-rich fp shell from FRIB:

($^{49\text{-}51,55}$Ca, ^{51,54}Sc, ^{54}Ti)
Ogunbeku, Allmond, Gray et al.

- Emerging trend $e_{IV} = \frac{1}{2}(\delta e_n - \delta e_p) \rightarrow 0$ in

n-rich nuclei. Neutrons more
affected by shape coexistence near $N = Z$?

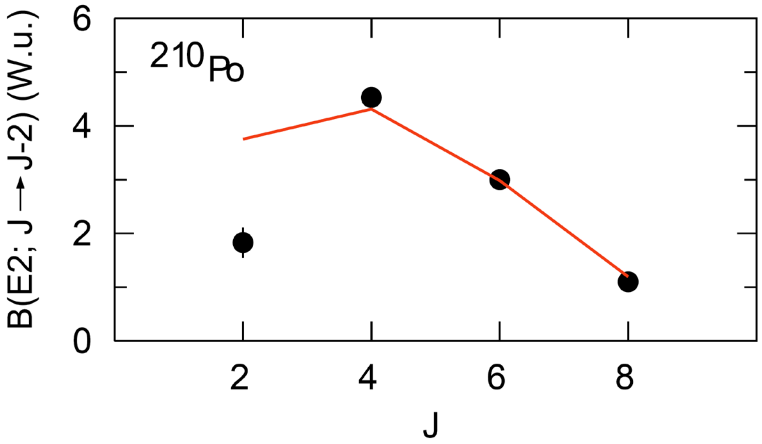
Preliminary data figure removed.

Ogunbeku, Allmond, Gray et al.

Outline: Focus on E2 effective charges

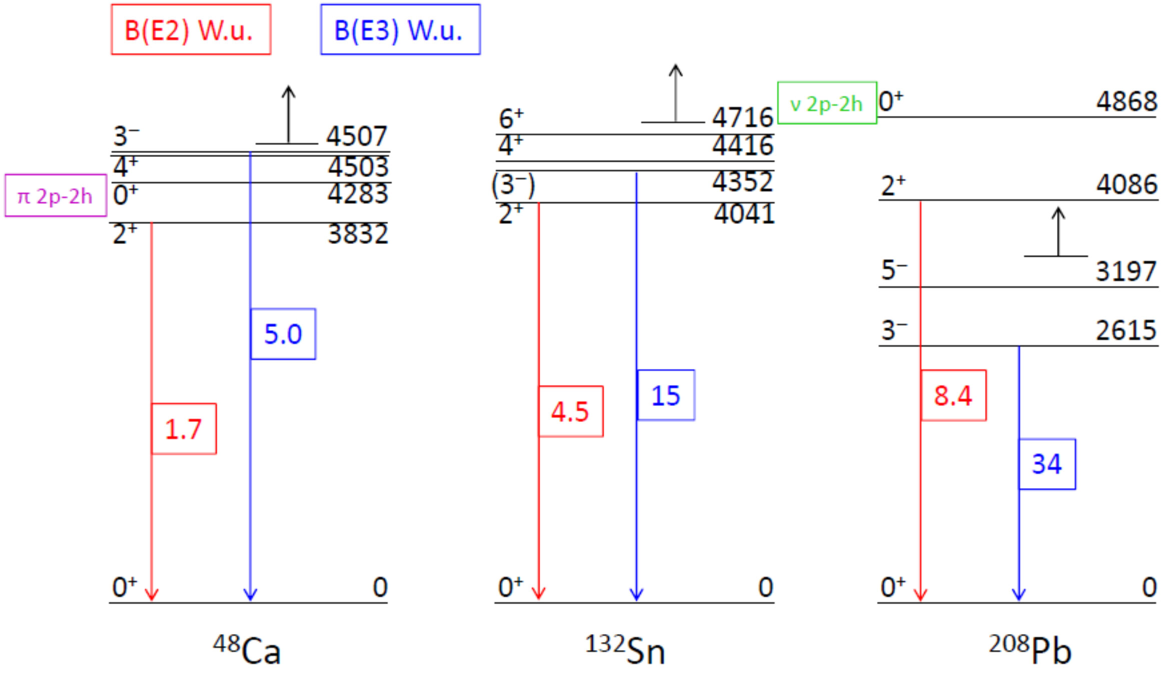
- Introduction: snapshot of general concepts
- An empirical survey based on B(E2)s in doubly magic nuclei ± 2 like nucleons
  - E0 transitions and shape mixing in ^{40}Ca – implications
  - The ^{210}Po puzzle – consequence of octupole collectivity?
- Concluding remarks

The ^{210}Po puzzle

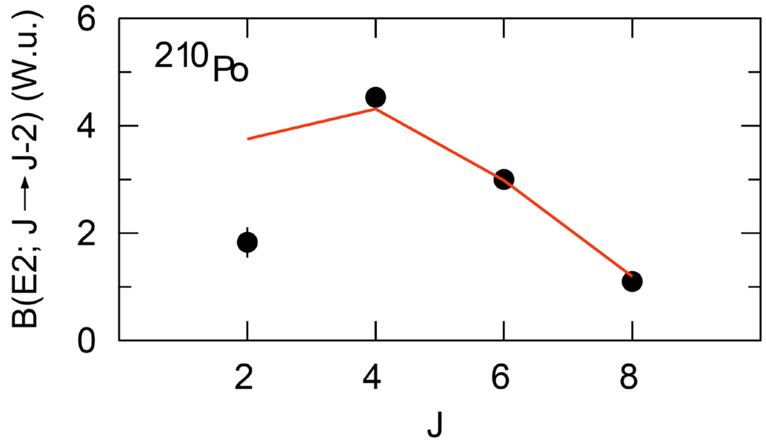


Octupole coupling?

“single-particle” states in ^{209}Bi are mixed:



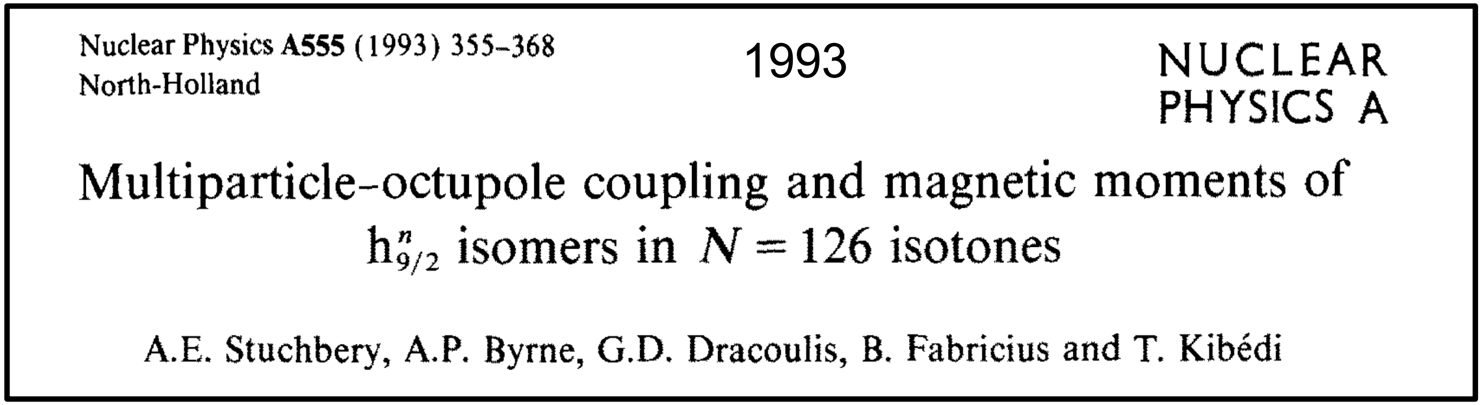
The ^{210}Po puzzle



Octupole coupling?

“single-particle” states in ^{209}Bi are mixed:

$$|\widetilde{h_{9/2}}\rangle = 0.998|h_{9/2}\rangle - 0.06\ |i_{13/2} \otimes 3^-\rangle$$
$$|\widetilde{f_{7/2}}\rangle = 0.95|f_{7/2}\rangle + 0.31\ |i_{13/2} \otimes 3^-\rangle$$
$$|\widetilde{i_{13/2}}\rangle = 0.91|i_{13/2}\rangle + 0.37\ |f_{7/2} \otimes 3^-\rangle + 0.19|h_{9/2} \otimes 3^-\rangle$$



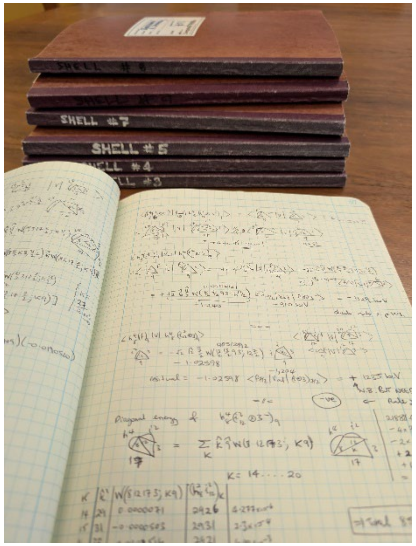
Nuclear Physics **A555** (1993) 355–368
North-Holland

1993

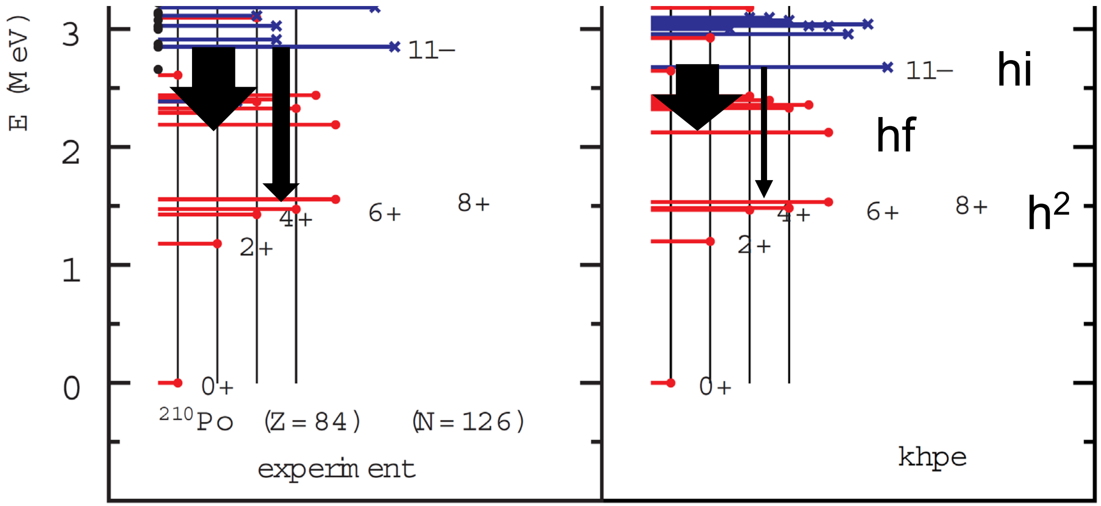
NUCLEAR
PHYSICS A

Multiparticle–octupole coupling and magnetic moments of $h^n_{9/2}$ isomers in $N = 126$ isotones

A.E. Stuchbery, A.P. Byrne, G.D. Dracoulis, B. Fabricius and T. Kibédi

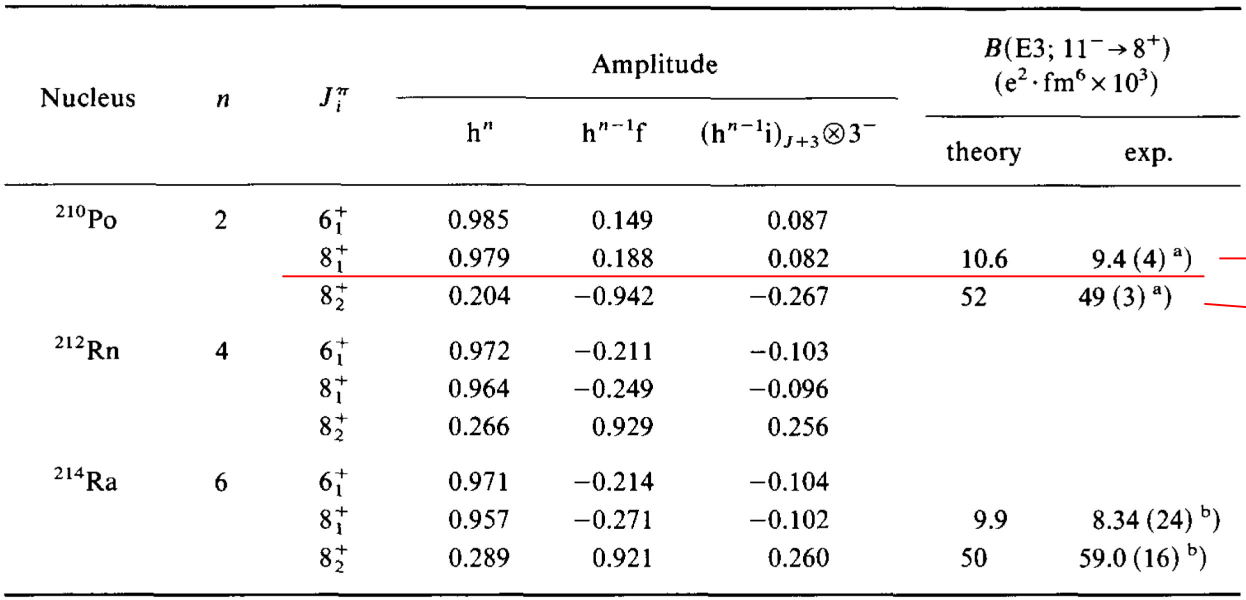


The ^{210}Po puzzle: octupole coupling & E3s



Need to explicitly include the octupole coupling to get the $11^- \rightarrow 8^+$ $B(E3)$ strengths correct, especially to the 8_1^+ state.

Wavefunctions and B(E3) rates for states in the even $N = 126$ isotones



Nucleus	n	J_i^π	Amplitude			$B(\mathrm{E3};\, 11^- \rightarrow 8^+)$ $(e^2 \cdot \mathrm{fm}^6 \times 10^3)$	
			h^n	$h^{n-1}f$	$(h^{n-1}i)_{J+3} \otimes 3^-$	theory	exp.
^{210}Po	2	6_1^+	0.985	0.149	0.087		
		8_1^+	0.979	0.188	0.082	10.6	9.4 (4) [a]
		8_2^+	0.204	−0.942	−0.267	52	49 (3) [a]
^{212}Rn	4	6_1^+	0.972	−0.211	−0.103		
		8_1^+	0.964	−0.249	−0.096		
		8_2^+	0.266	0.929	0.256		
^{214}Ra	6	6_1^+	0.971	−0.214	−0.104		
		8_1^+	0.957	−0.271	−0.102	9.9	8.34 (24) [b]
		8_2^+	0.289	0.921	0.260	50	59.0 (16) [b]

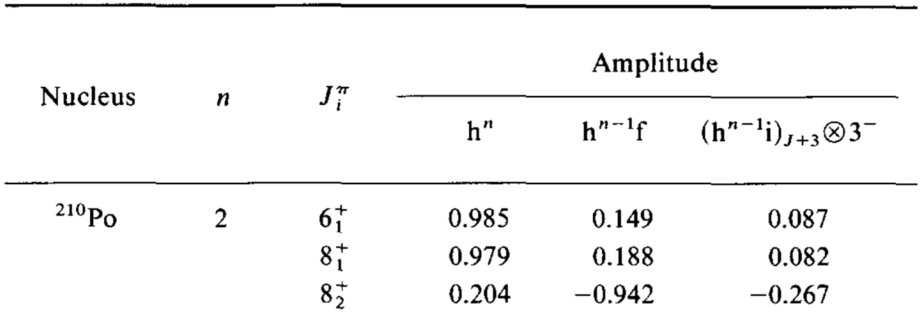
SM

$\delta e_p = 0.5$

0.90

21

The ^{210}Po puzzle: octupole coupling & E2s



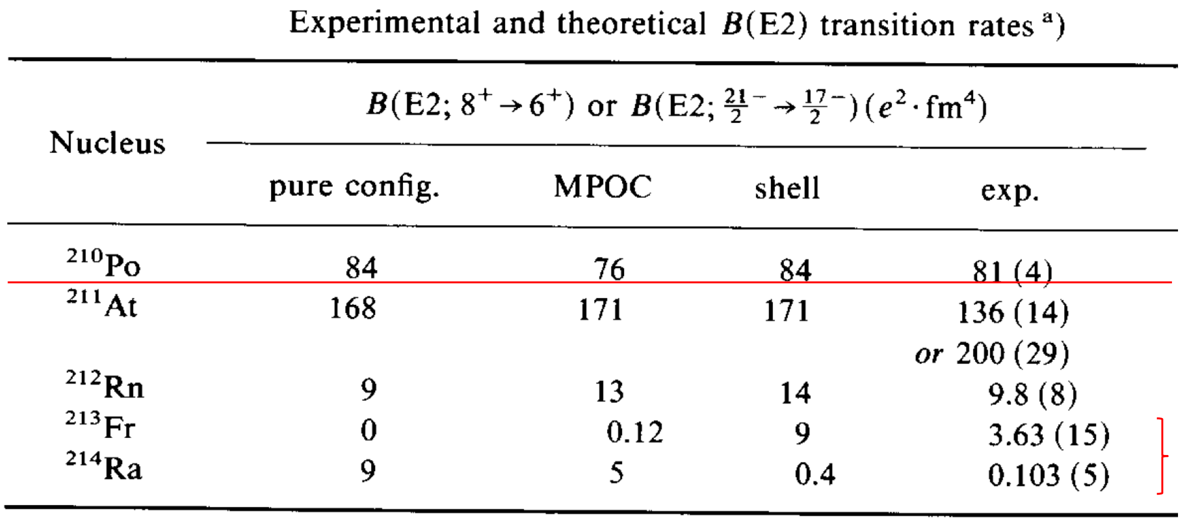
Nucleus	n	J_i^π	Amplitude h^n	Amplitude $h^{n-1}f$	Amplitude $(h^{n-1}i)_{J+3}\otimes 3^-$
^{210}Po	2	6_1^+	0.985	0.149	0.087
		8_1^+	0.979	0.188	0.082
		8_2^+	0.204	−0.942	−0.267

Octupole components have little effect on the $8^+ \rightarrow 6^+$ $B(E2)$.

They will have even less effect on the $2^+ \rightarrow 0^+$ $B(E2)$.

We need to check the experiment!

Coulex of a ^{210}Po beam.



Experimental and theoretical B(E2) transition rates [a])

Nucleus	$B(\mathrm{E2}; 8^+\rightarrow 6^+)$ or $B(\mathrm{E2}; \frac{21}{2}^-\rightarrow \frac{17}{2}^-)$ $(e^2\cdot\mathrm{fm}^4)$ pure config.	MPOC	shell	exp.
^{210}Po	84	76	84	81 (4)
^{211}At	168	171	171	136 (14) *or* 200 (29)
^{212}Rn	9	13	14	9.8 (8)
^{213}Fr	0	0.12	9	3.63 (15)
^{214}Ra	9	5	0.4	0.103 (5)

Mid-shell seniority cancellation: $h_{9/2}^5$

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Concluding observations

- The concept of the polarization charge is well defined… but…

- In practical terms the effective charge is a measure of what is missing in the model

- So getting a “clean” measure of the polarization charge is often a challenge.

- Some tips:
  - Study doubly magic nuclei $\pm$1 nucleon if possible
    - Be aware that prolate shape coexistence in the core can manifest as negative Q (oblate) if I is big and K is small

  - For nuclei with 2 valence nucleons (or holes) seek a pure high-*j* orbit and

    - Avoid the 2^+ state – it will couple to every 2^+ state of the core! If there is a low-excitation deformed intruder configuration the mixing will be strong.

    - Focus on the highest-spin states of the j^2 configuration. These are less likely to mix with collective core states

Selected experimental wish list

Two relevant approved experiments (ISOLDE):

- Coulex of ^{133}Sb (Effective charges near ^{132}Sn)
  G. Georgiev, Mitch Almond, AES

- Coulex of ^{210}Po (Resolve puzzling BE2 or find novel physics)
  G. Georgiev, G. Rainovski, AES

Experiments to characterize shape coexistence and shape mixing & impact on effective charges:

- Multistep Coulex of ^{38}Ar and ^{42}Ti – shape mixing like ^{42}Ca?

- E0 transitions in ^{42,44}Ca, ^{56}Ni

More to come on trends in n-rich nuclei

Emerging trend $e_{IV} = \frac{1}{2}(\delta e_n - \delta e_p) \rightarrow 0$ in n-rich nuclei.

Preliminary data figure removed.

Ogunbeku, Allmond, Gray et al. in preparation

Acknowledgments & Thanks

John Wood - discussions

Tibor Kibedi
Eiji Ideguchi
} Electron spectroscopy ^{40}Ca

Mitch Allmond – discussions and preliminary results from FRIB

END