Shape evolution and coexistence in reflection asymmetric nuclei

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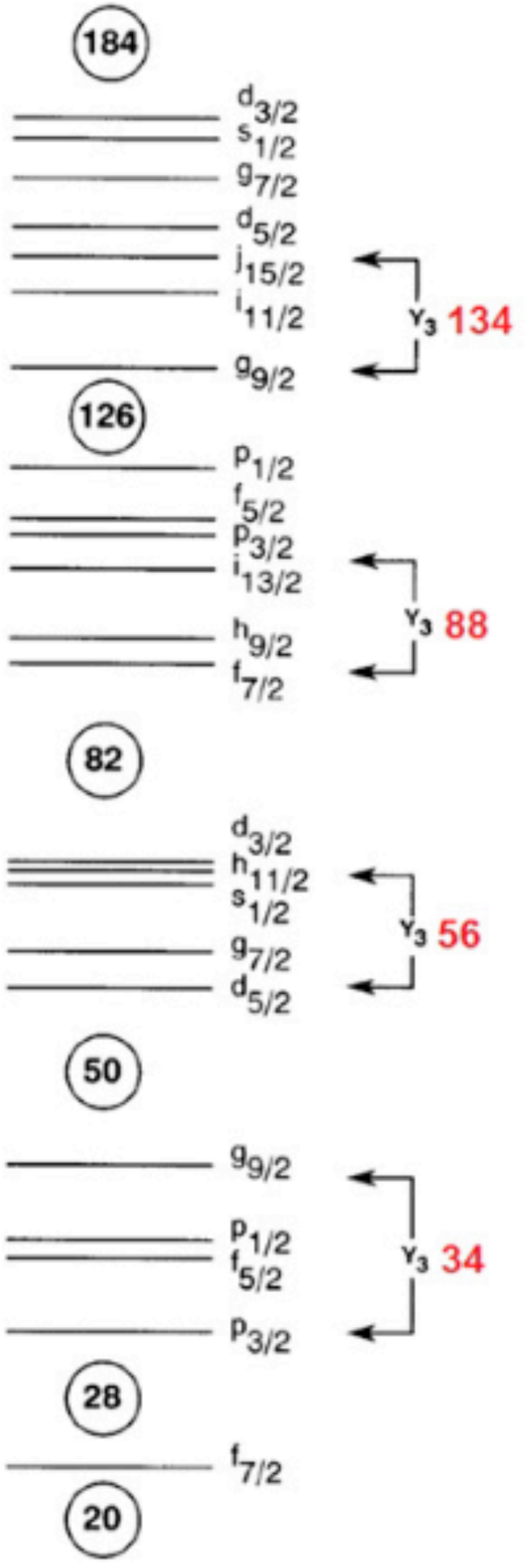
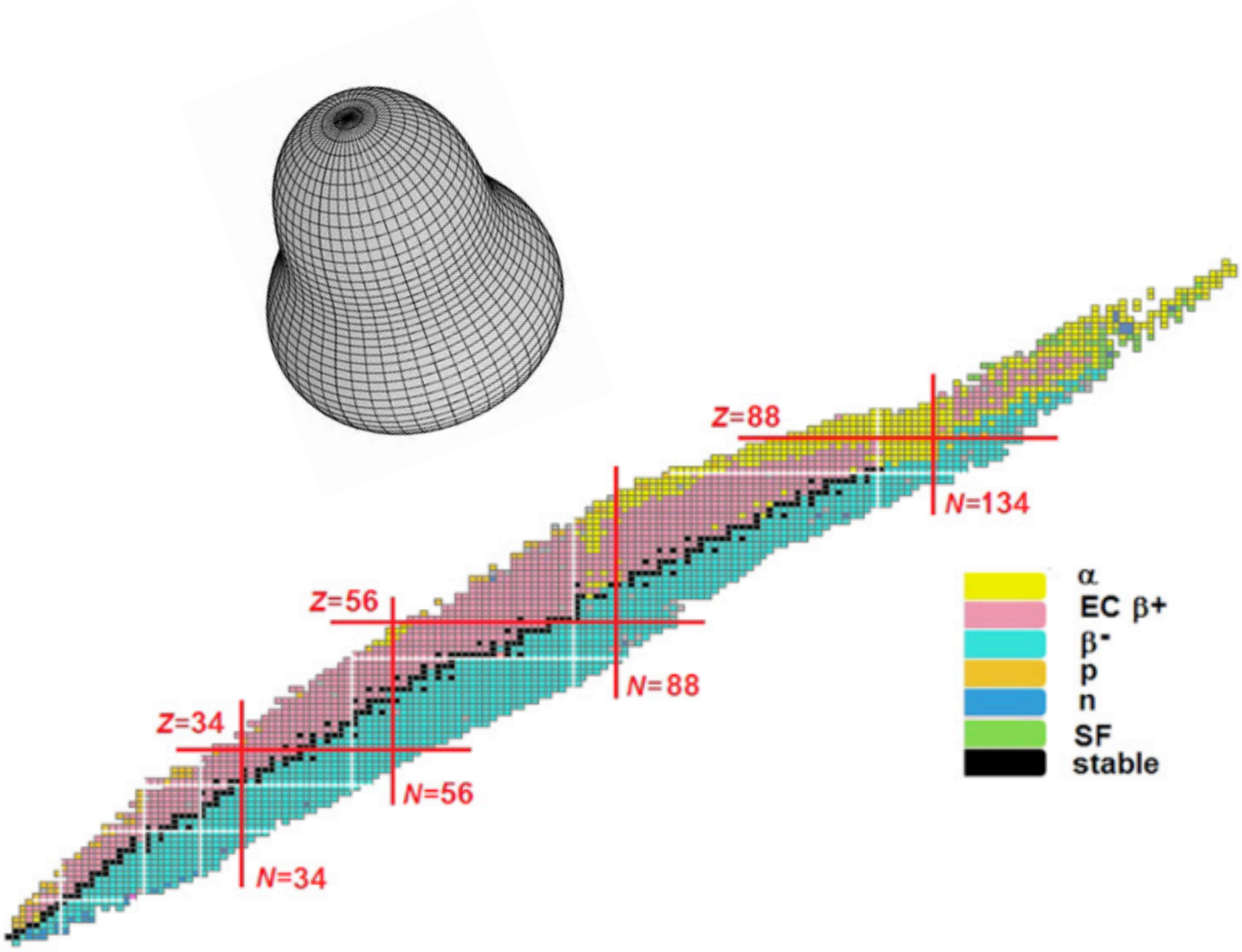
Outline

- Model for octupole deformed nuclei
- Octupole correlations in neutron-rich Xe, Ba, Ce, Nd
- Octupole correlations and shape coexistence in N~Z nuclei

- K. Nomura et al., Phys. Rev. C 106 (2021) 044324
- K. Nomura, Phys. Rev. C 106 (2022) 024330
- K. Nomura, Int. J. Mod. Phys. E (2023) 2340001

Search for pear shapes





Butler, Nazarewicz, RMP (1996); Butler, JPG (2016)

Octupole correlations

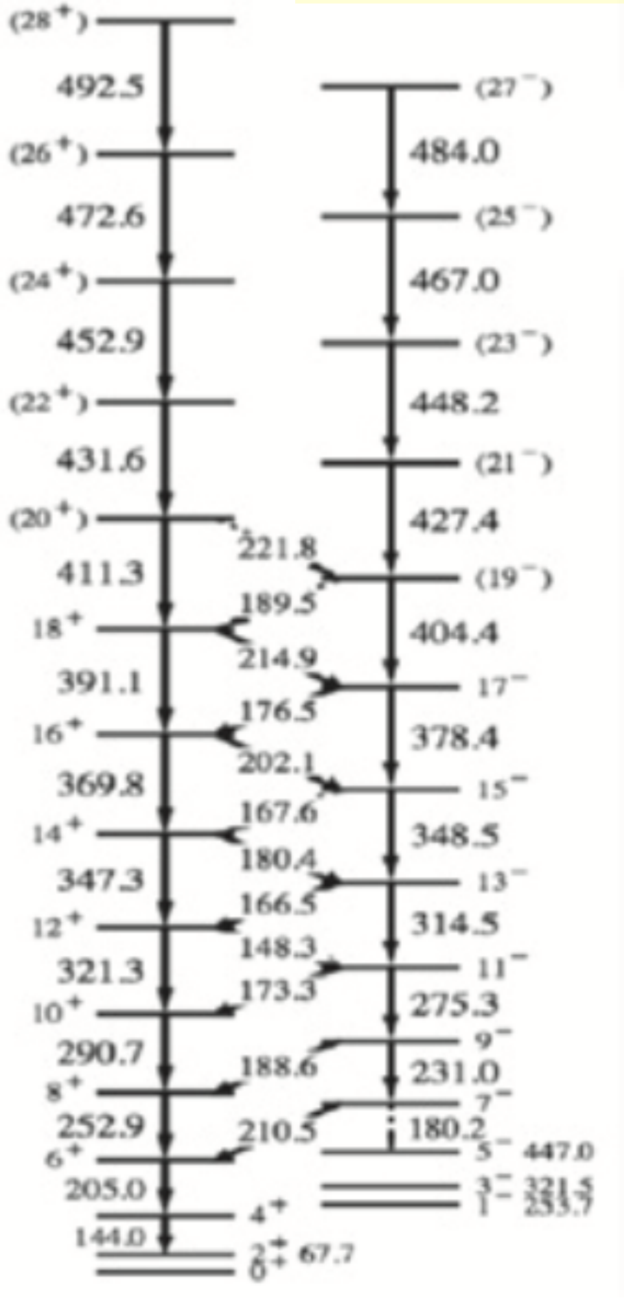
Empirical signatures:

- low-lying negative parity band
- enhanced E3 and E1 transitions
- experimental evidence:
  ^{220}Rn, ^{224}Ra (CERN), ^{144,146}Ba (ANL)

Theoretical descriptions:

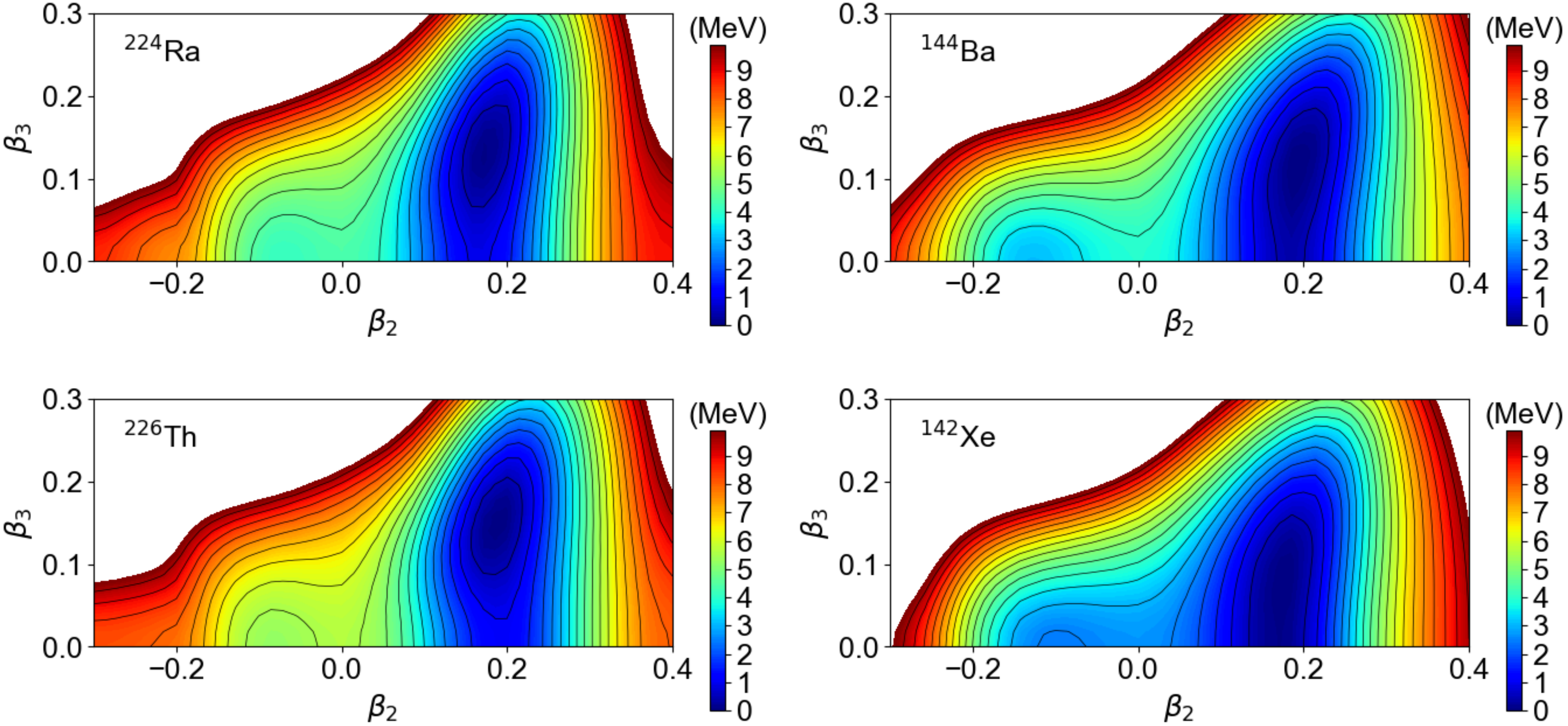
- **Interacting Boson Model**
- **Mean field and beyond** (Mic-Mac, Skyrme, Gogny, RMF energy density functionals: EDF)
- Geometrical models
- Cluster models
- Nuclear shell model

alternating-parity band



$^{226}_{88}$Ra

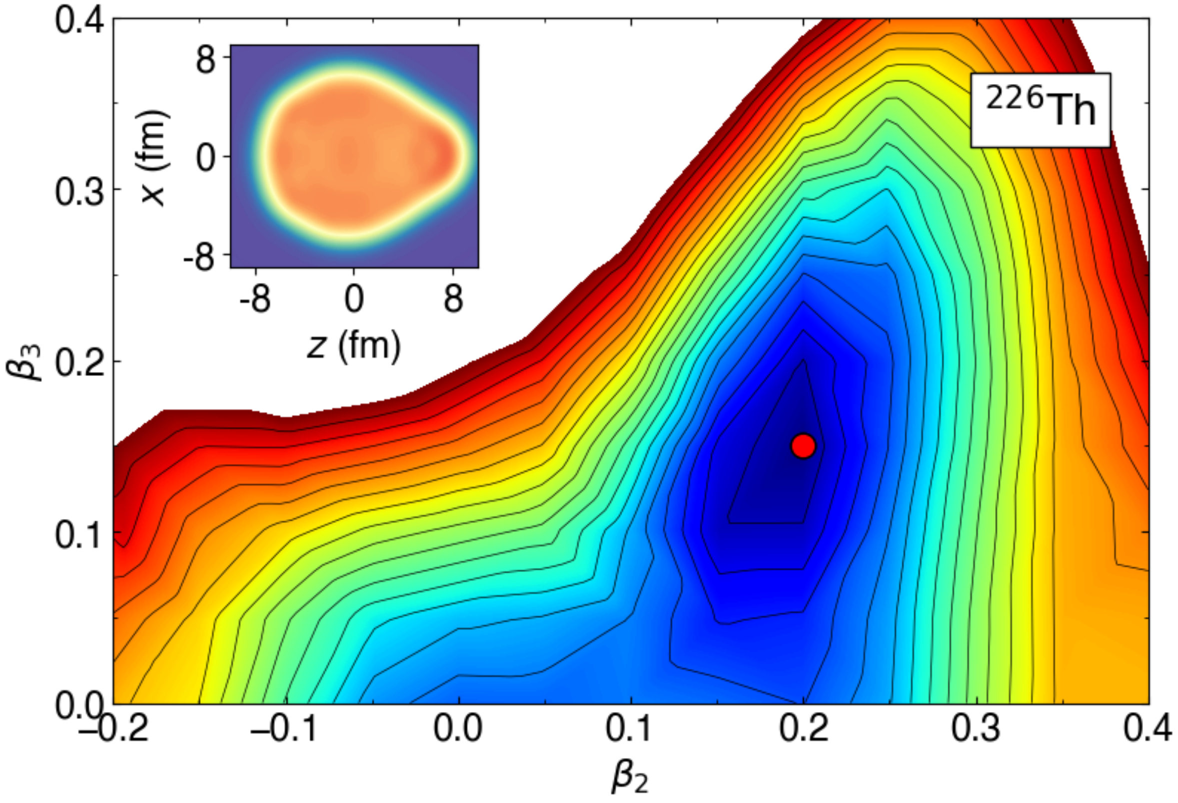
Potential energy surfaces (PESs)

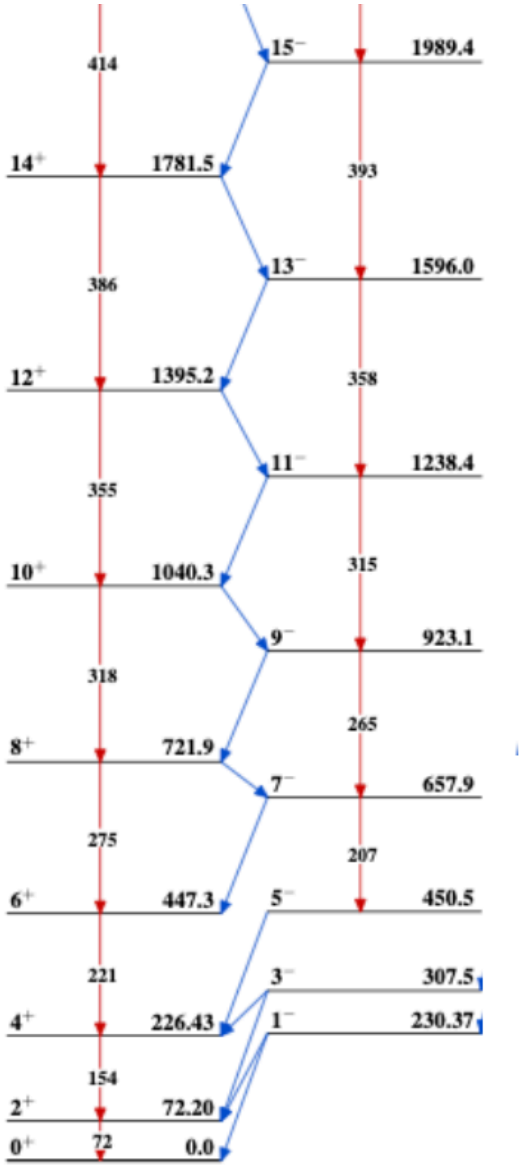


… from the axial Gogny-HFB mean-field calculations

Computing energy spectra

Intrinsic frame

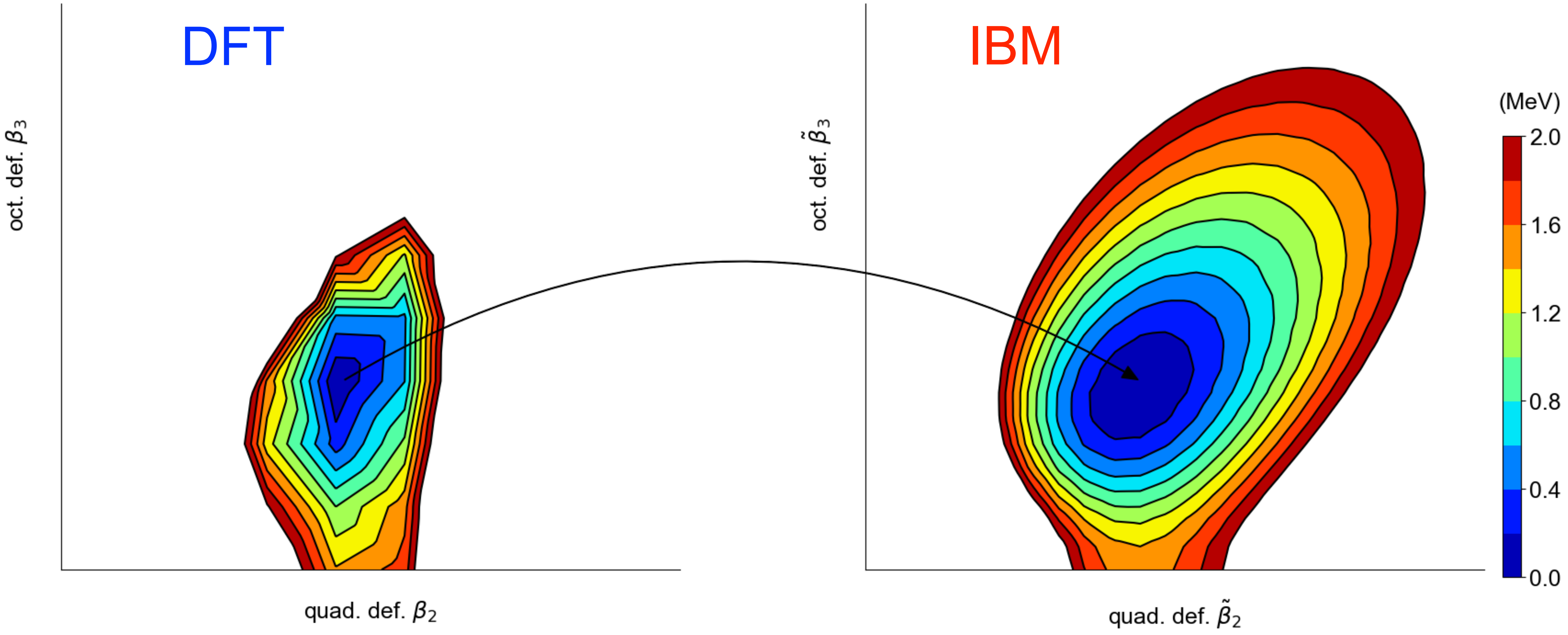




Beyond-mean-field treatments
- Symmetry projections, GCM
- Collective Hamiltonian
- Interacting Boson Model

Observables:
alternating parity band?
E1, E3?

Mapping DFT onto IBM PESs



1) Mapping procedure specifies the IBM Hamiltonian (parameters)
2) Diagonalization of the mapped Hamiltonian yields energy spectra

- triaxial quadrupole: KN et al., PRL (2008); PRC (2011); PRL (2012)
- axial quadrupole-octupole: KN et al., PRC (2013); PRC (2014)

IBM for octupole states

- Building blocks:
  s, d, f bosons ~ J=0+, 2+ and 3- collective nucleon pairs

- Hamiltonian $\boxed{\hat{H}_{\mathrm{IBM}} = \epsilon_d \hat{n}_d + \epsilon_f \hat{n}_f + \kappa_2 \hat{Q}_2 \cdot \hat{Q}_2 + \kappa_3 \hat{Q}_3 \cdot \hat{Q}_3 + \rho \hat{L} \cdot \hat{L}}$

pairing-like

quadrupole-quadrupole

octupole-octupole

rotational term

$$\hat{Q}_2 = s^\dagger \tilde{d} + d^\dagger s + \chi_{dd}(d^\dagger \times \tilde{d})^{(2)} + \chi_{ff}(f^\dagger \times \tilde{f})^{(2)}$$

$$\hat{Q}_3 = s^\dagger \tilde{f} + f^\dagger s + \chi_{df}(d^\dagger \times \tilde{f} + f^\dagger \times \tilde{d})^{(3)}$$

$$\hat{L} = \sqrt{10}(d^\dagger \times \tilde{d})^{(1)} + \sqrt{28}(f^\dagger \times \tilde{f})^{(1)}$$

… with 8 parameters

Geometry of the IBM

Energy surface: $\boxed{E_{\mathrm{IBM}}(\beta,\gamma) = \langle\phi\,|\,\hat{H}_{\mathrm{IBM}}\,|\,\phi\rangle}$

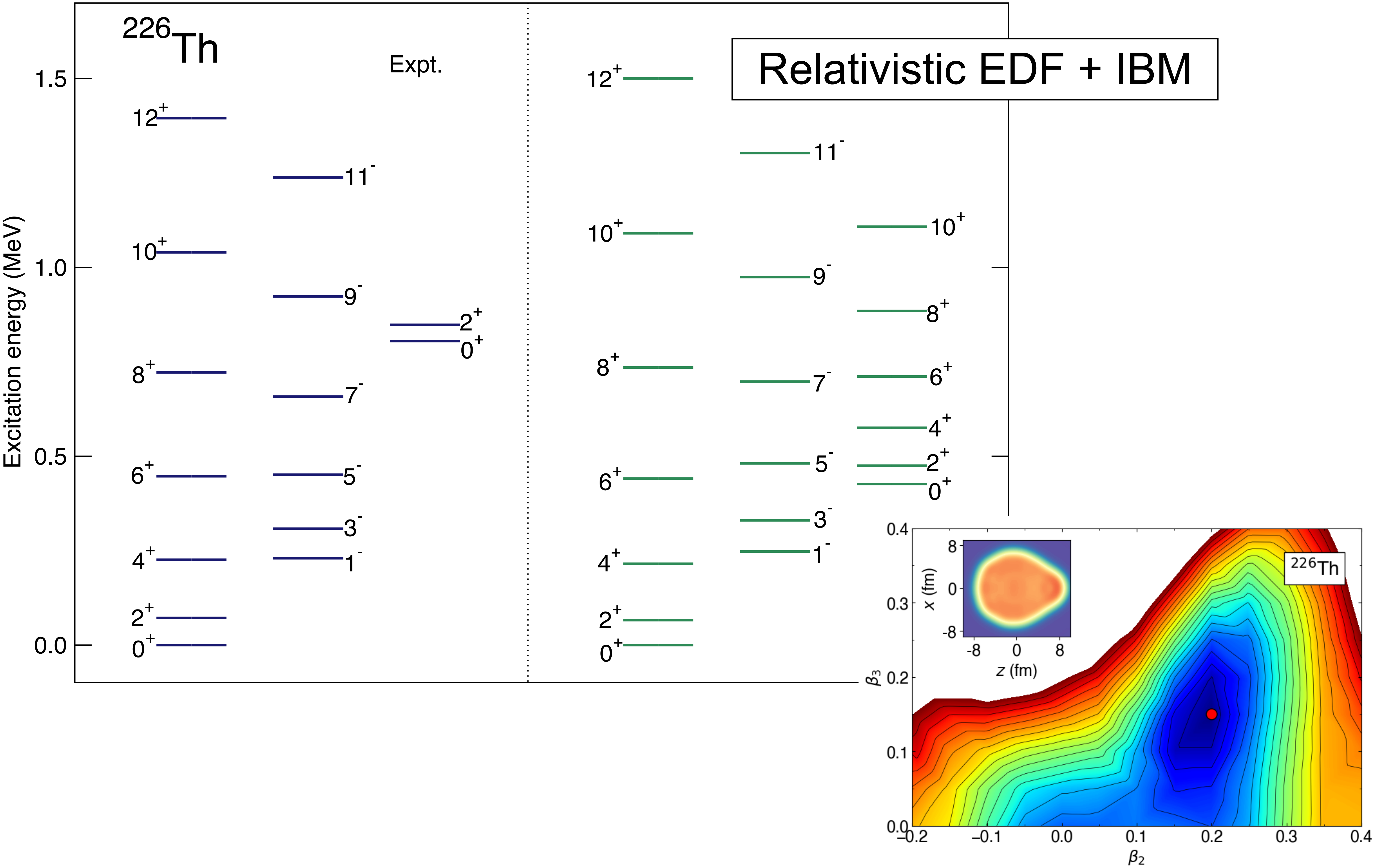
… with boson coherent state (axial symmetry)

$$|\,\phi\rangle \propto \left[s^{\dagger} + \beta_{20}d_0^{\dagger} + \beta_{30}f_0^{\dagger}\right]^N |\,0\rangle$$

IBM Hamiltonian is determined by

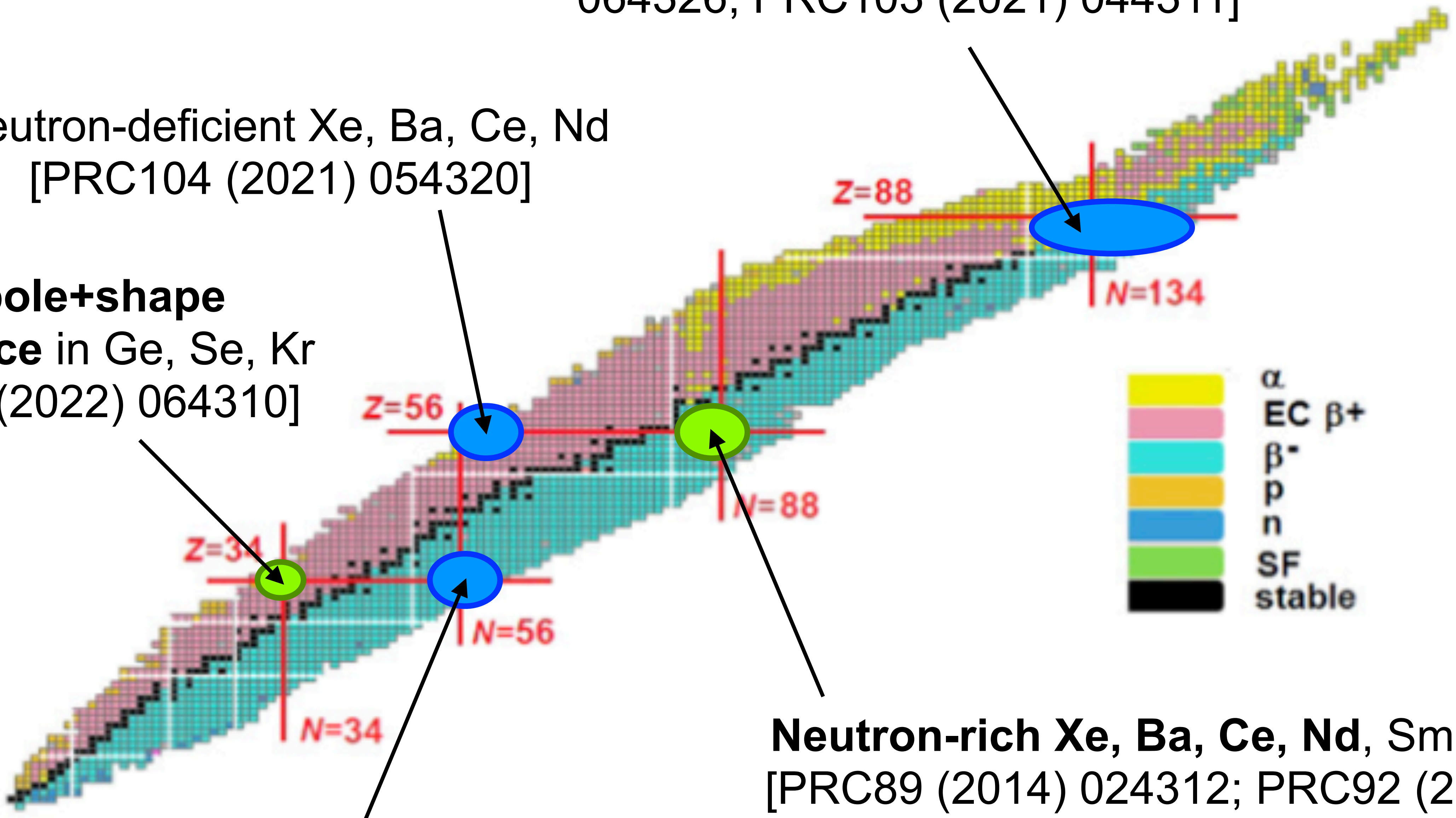
$$\boxed{E_{\mathrm{DFT}}(\beta_{20},\beta_{30}) \approx E_{\mathrm{IBM}}(\beta_{20},\beta_{30})}$$

Example



Relativistic EDF + IBM
^{226}Th
Expt.
Excitation energy (MeV)
0.0
0.5
1.0
1.5
12$^+$
10$^+$
8$^+$
6$^+$
4$^+$
2$^+$
0$^+$
11$^-$
9$^-$
7$^-$
5$^-$
3$^-$
1$^-$
2$^+$
0$^+$
12$^+$
10$^+$
8$^+$
6$^+$
4$^+$
2$^+$
0$^+$
11$^-$
9$^-$
7$^-$
5$^-$
3$^-$
1$^-$
10$^+$
8$^+$
6$^+$
4$^+$
2$^+$
0$^+$
^{226}Th
β_3
0.0
0.1
0.2
0.3
0.4
β_2
−0.2
−0.1
0.0
0.1
0.2
0.3
0.4
x (fm)
z (fm)
8
0
-8
-8
0
8

Applications of the mapped IBM



Light actinides Ra, Th, U, Pu, Cm, Cf [PRC88 (2013) 021303(R); PRC89 (2014) 024312, PRC102 (2020) 064326; PRC103 (2021) 044311]

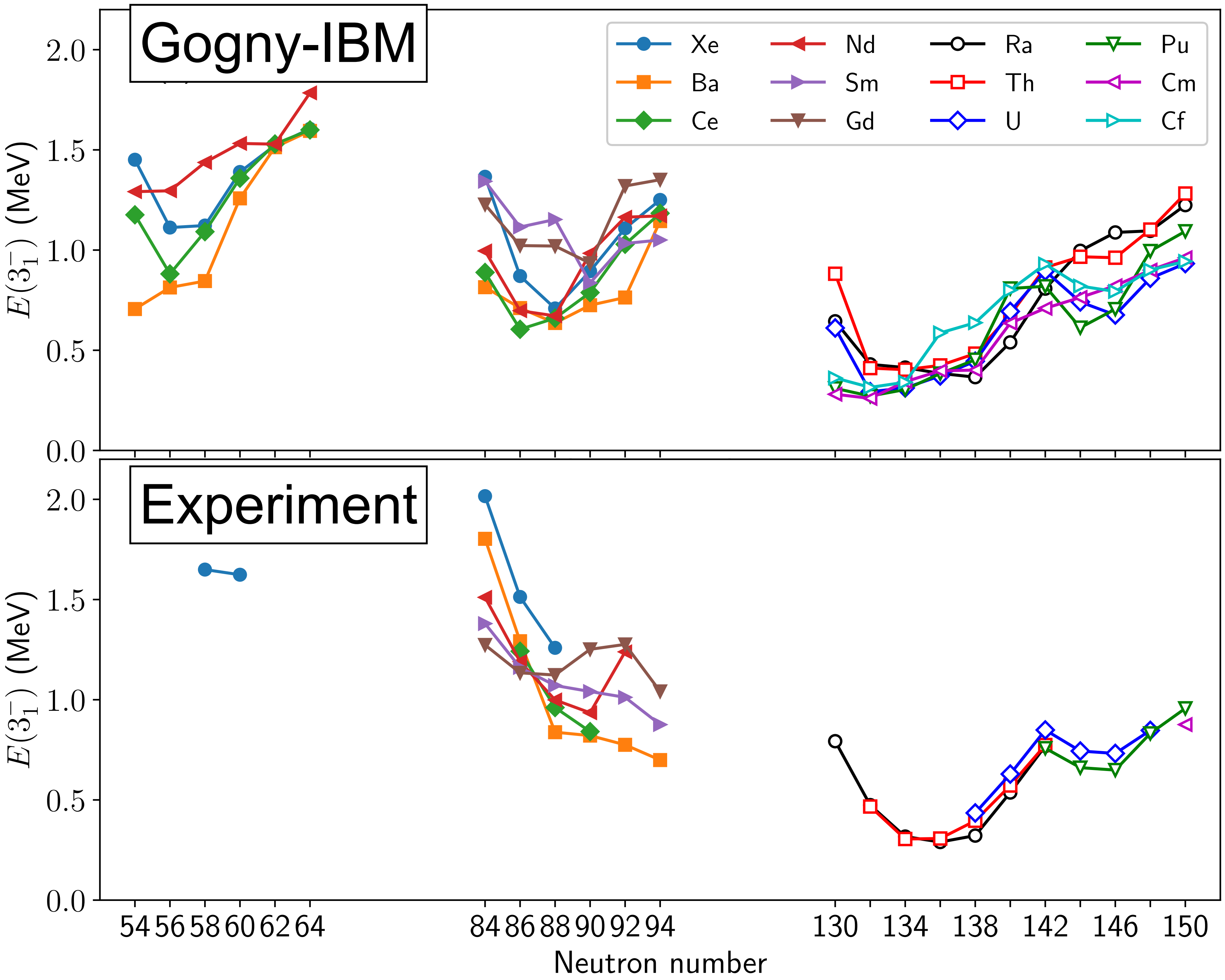
Neutron-deficient Xe, Ba, Ce, Nd [PRC104 (2021) 054320]

Octupole+shape coexistence in Ge, Se, Kr [PRC106 (2022) 064310]

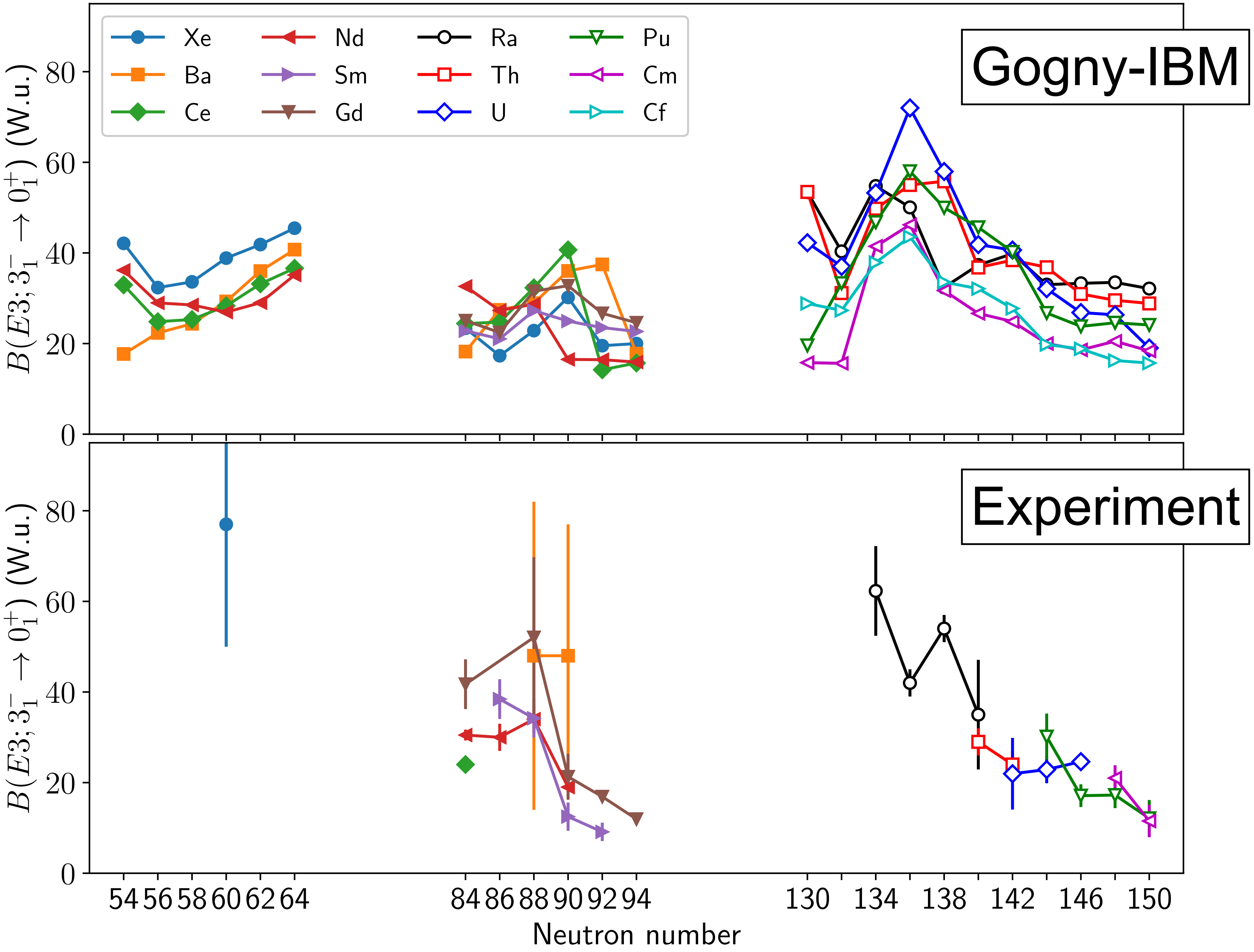
Neutron-rich Xe, Ba, Ce, Nd, Sm, Gd [PRC89 (2014) 024312; PRC92 (2015) 014312; PRC97 (2018) 024317; PRC104 (2021) 044324]

Neutron-rich Kr, Sr, Zr, Mo, Ru [PRC105 (2022) 054318]

$E(3^-)$ systematic

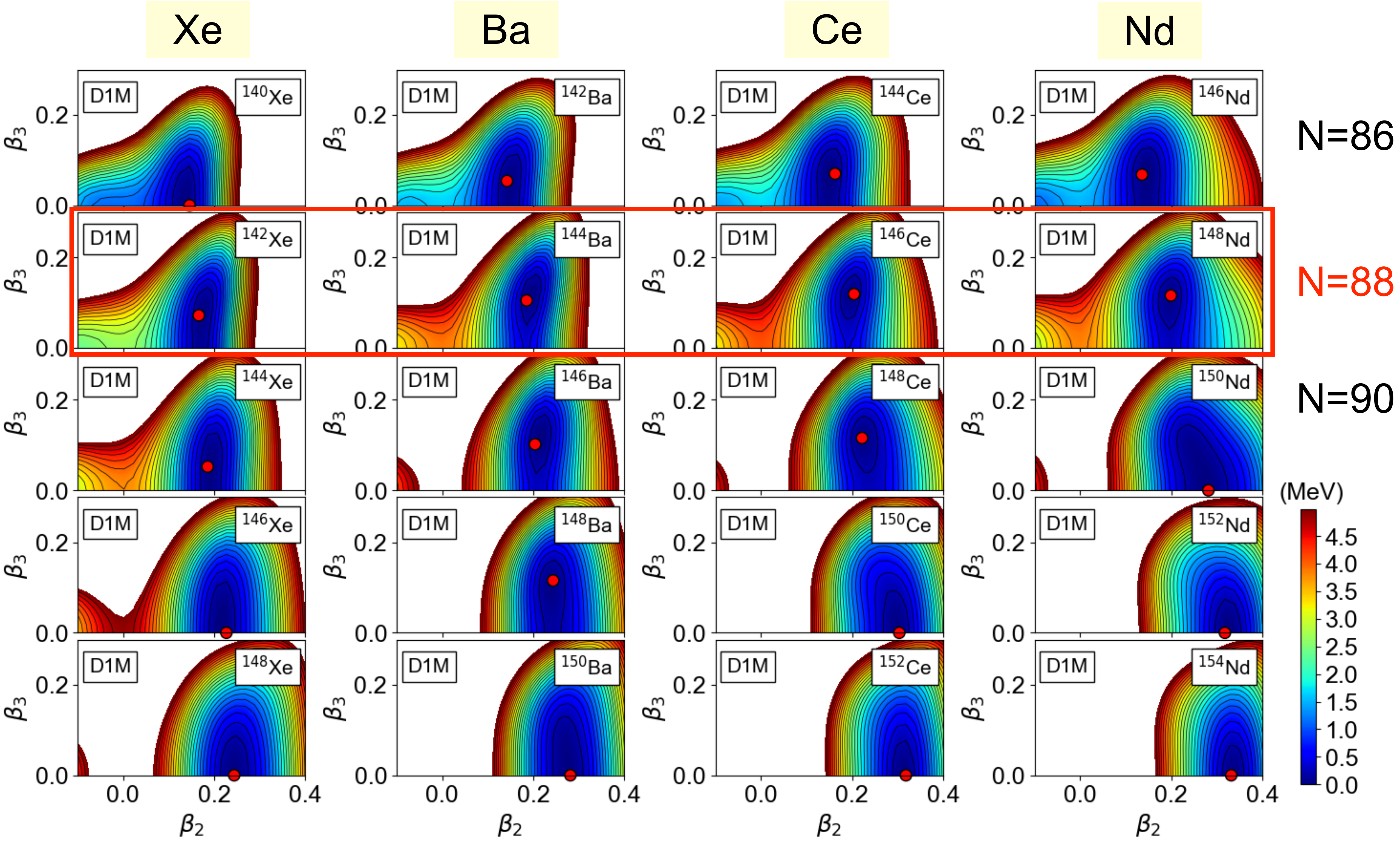


B(E3) systematic



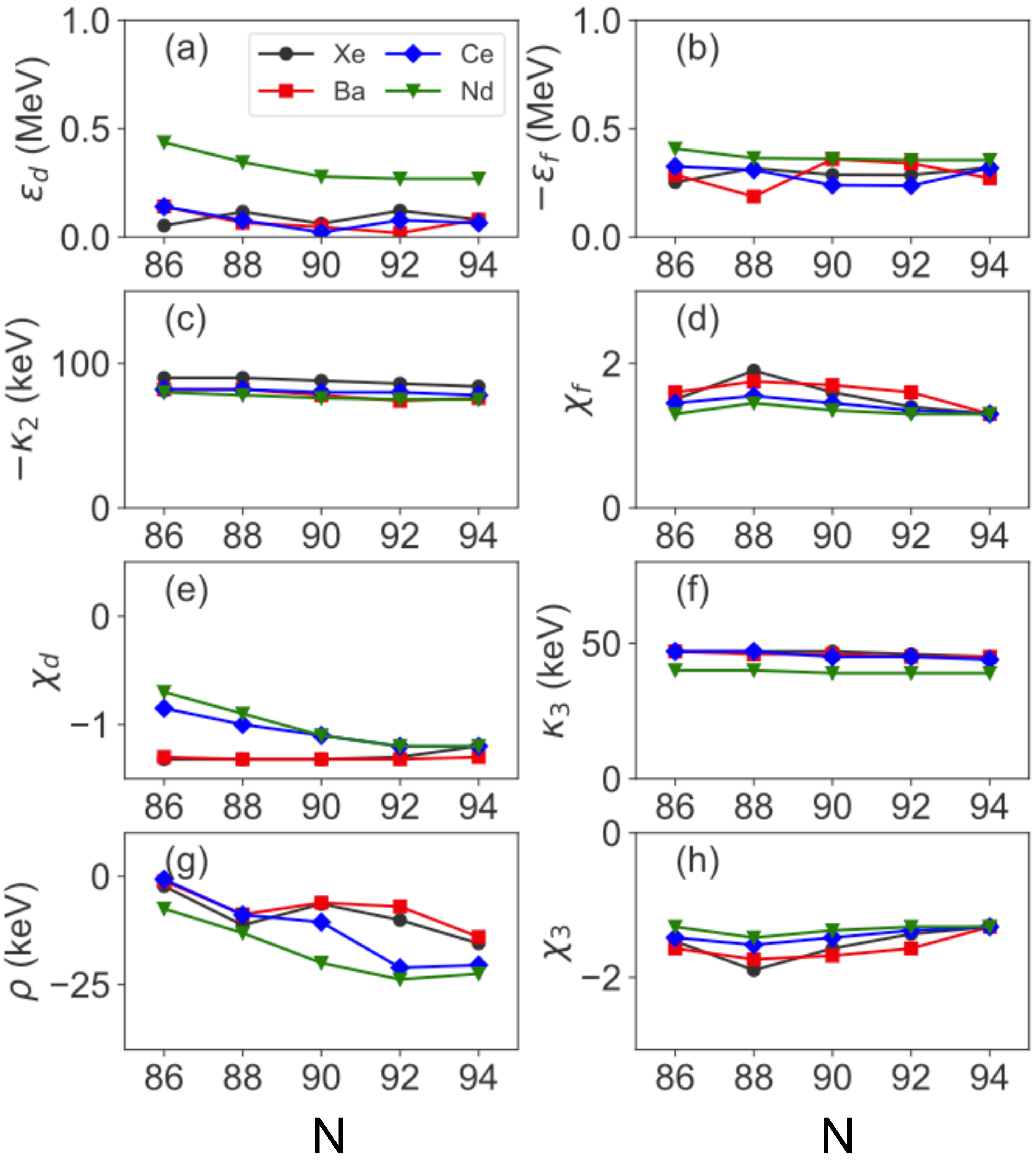
Octupole correlations in neutron-rich Xe, Ba, Ce, Nd

Quadrupole-octupole PESs

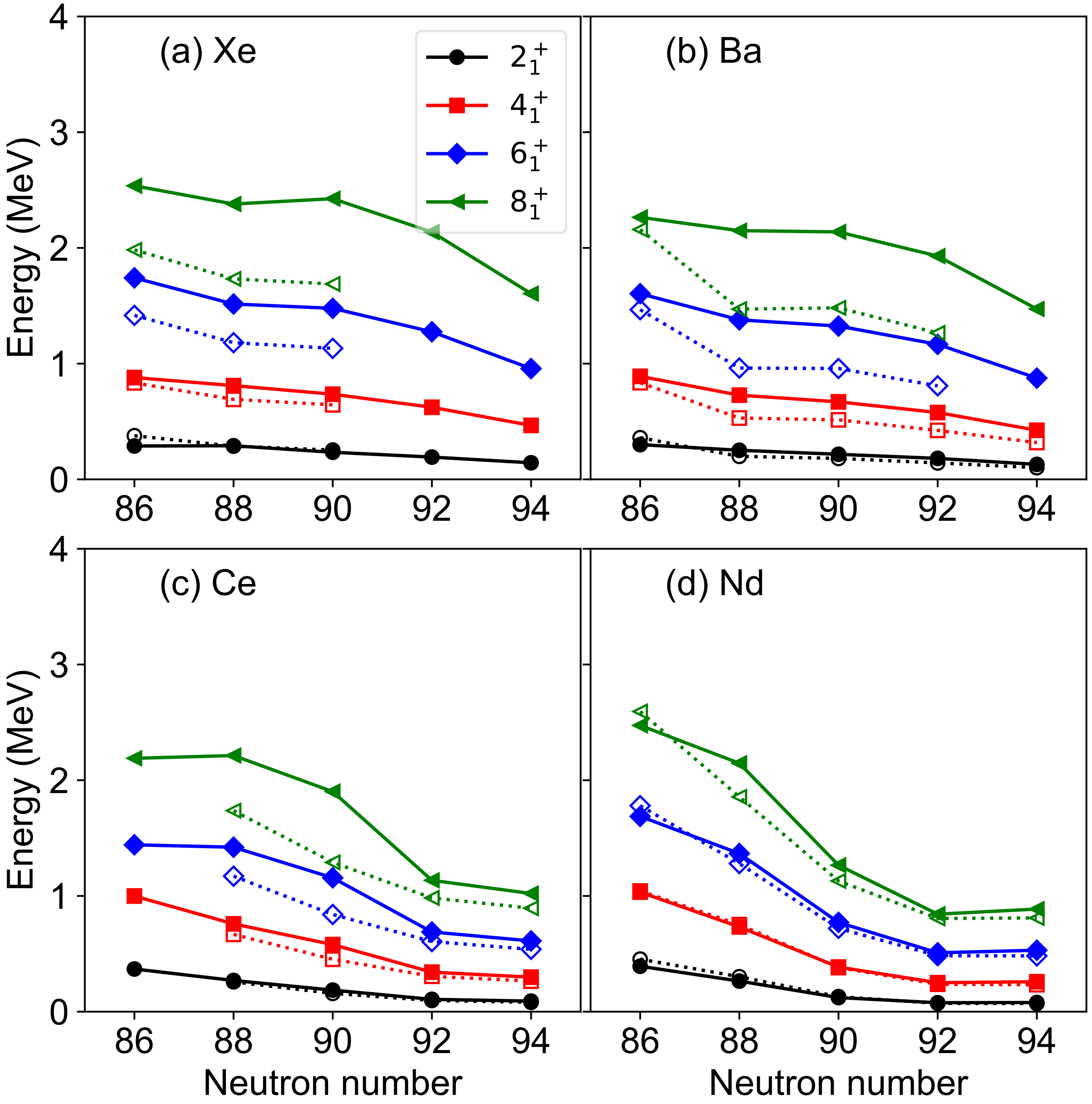


Gogny-D1M EDF

Derived IBM parameters

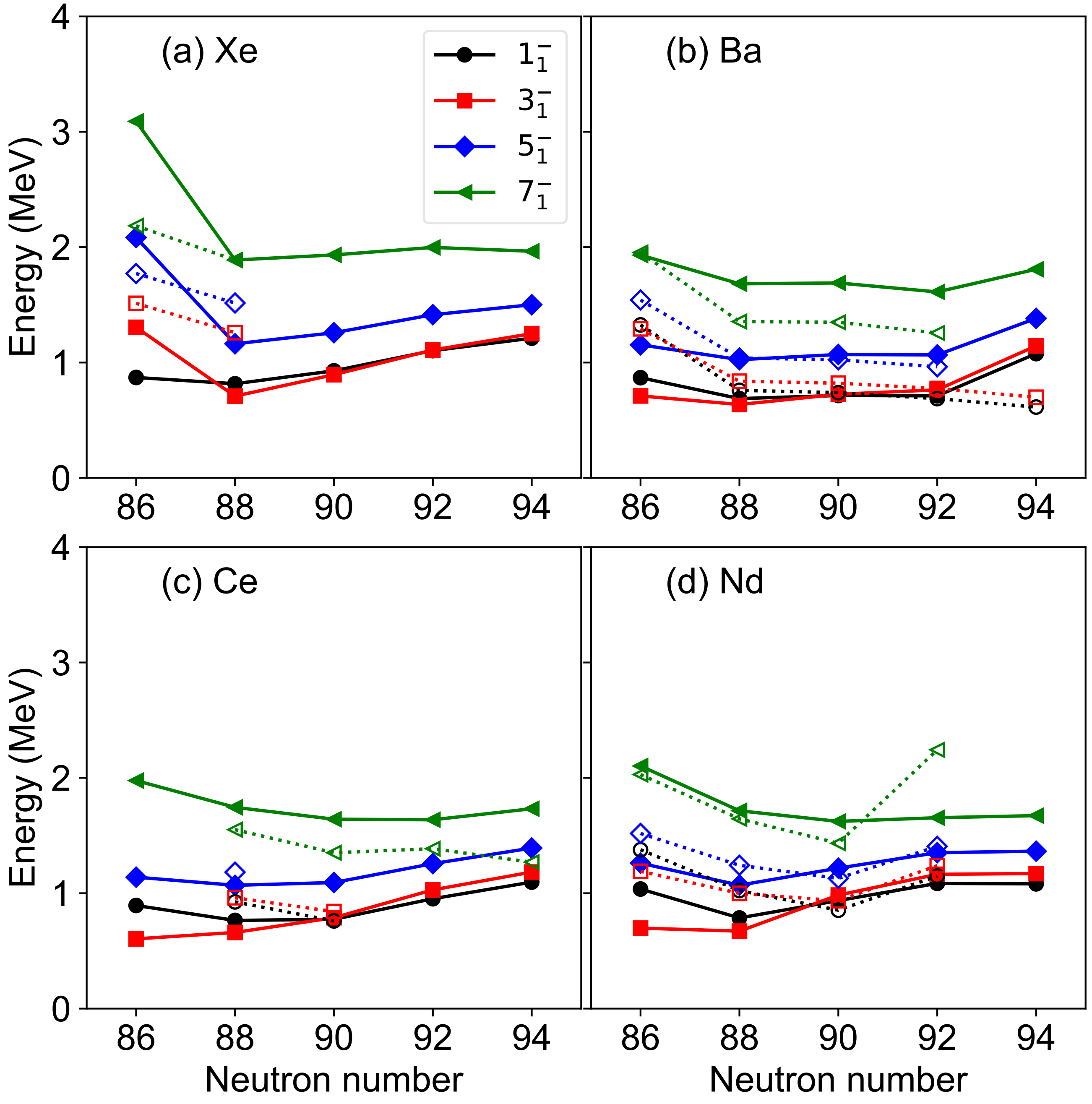


Positive-parity levels



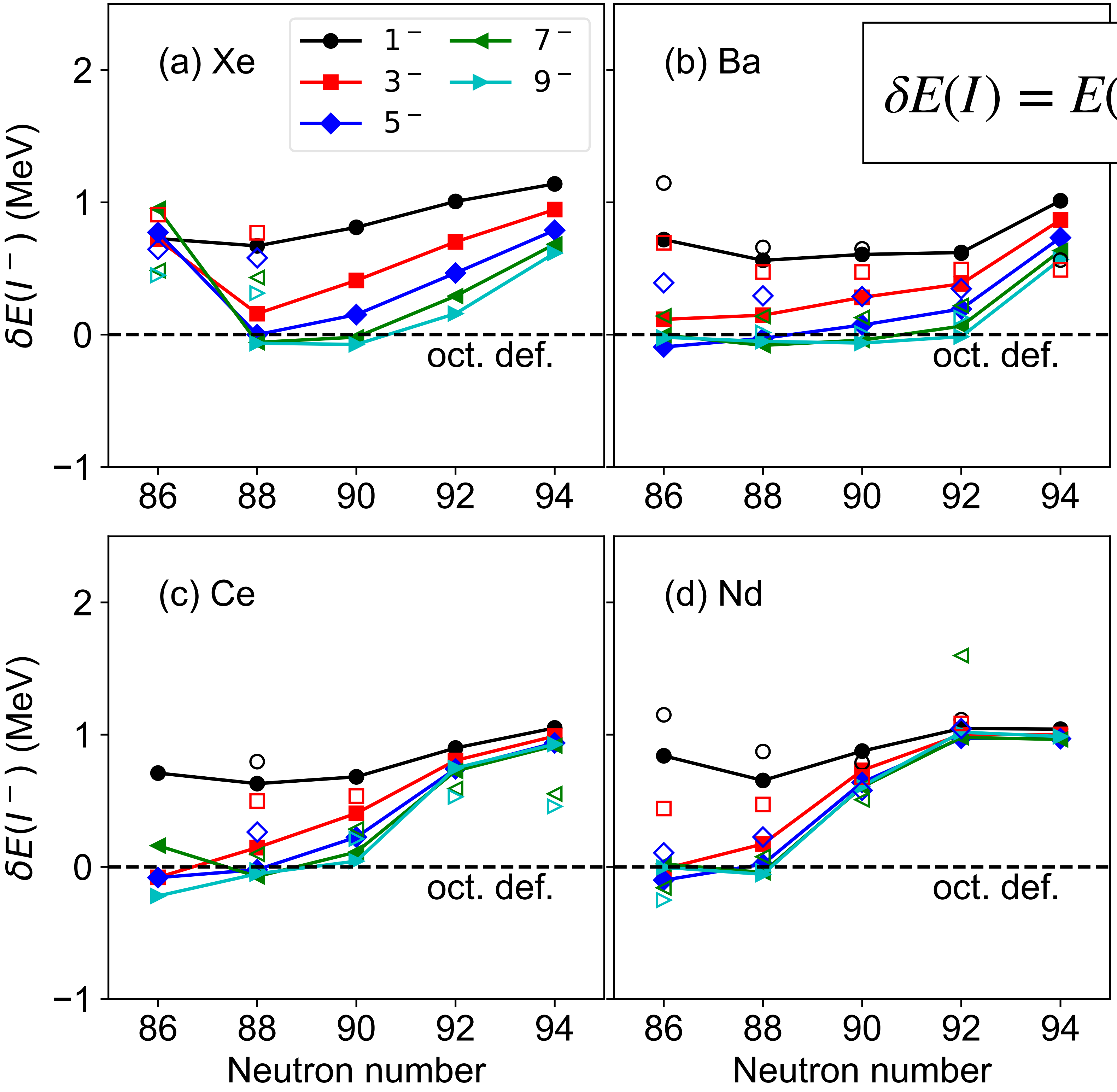
… gradual evolution of **quadrupole** collectivity with N

Negative-parity levels



… gradual evolution of **octupole** collectivity around **N=88**

A criteria for alternating-parity bands

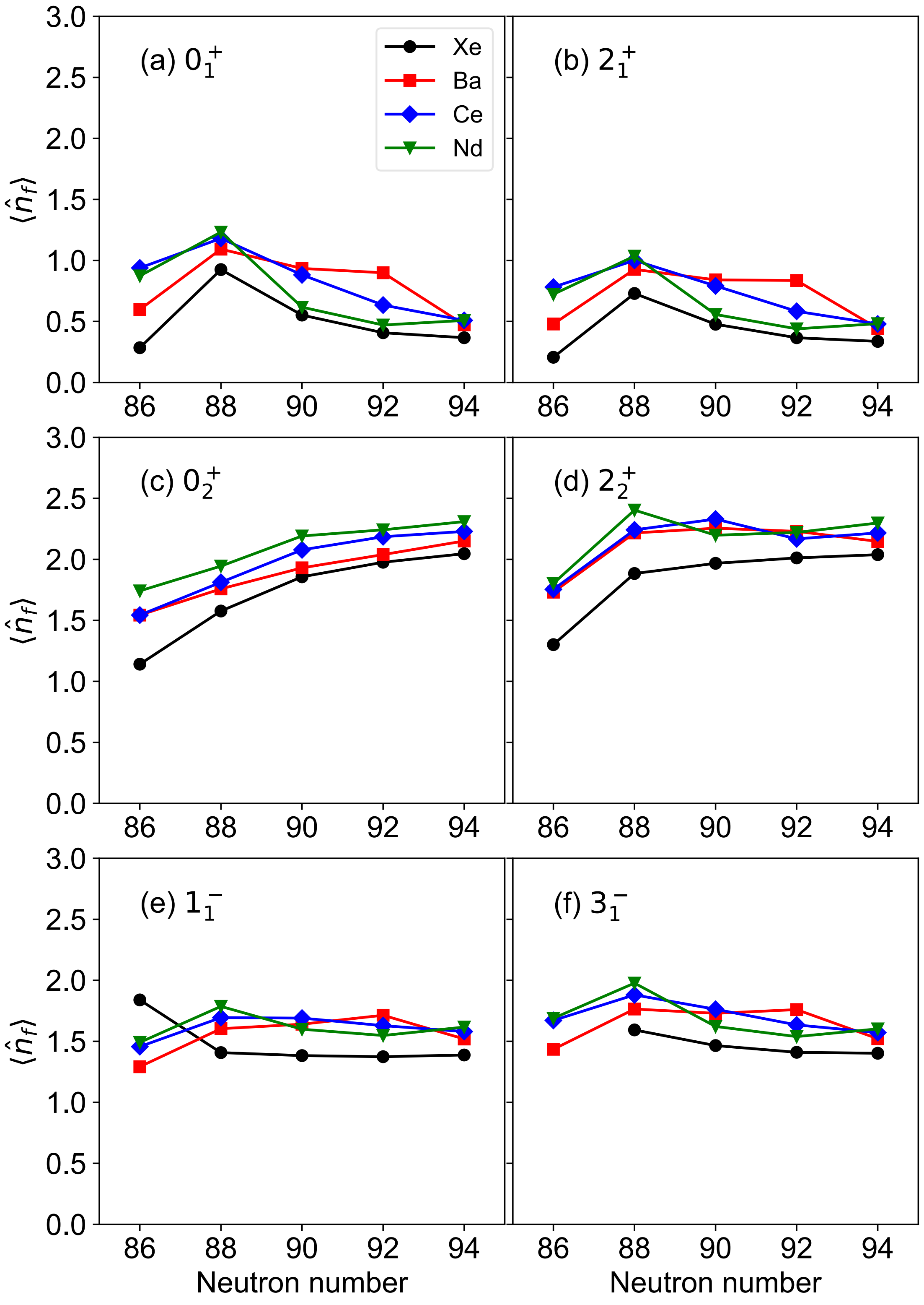


$$\delta E(I) = E(I^-) - \frac{E((I+1)^+) + E((I-1)^+)}{2}$$

$\delta E \approx 0$ for stable octupole deformation

Wave function contents

Average number of octupole (f) bosons: $\langle \hat{n}_f \rangle$



… significant octupole effects in **yrast states**

… 0_2^+ states mostly of **double octupole** phonon nature for deformed nuclei: $\langle \hat{n}_f \rangle \approx 2$

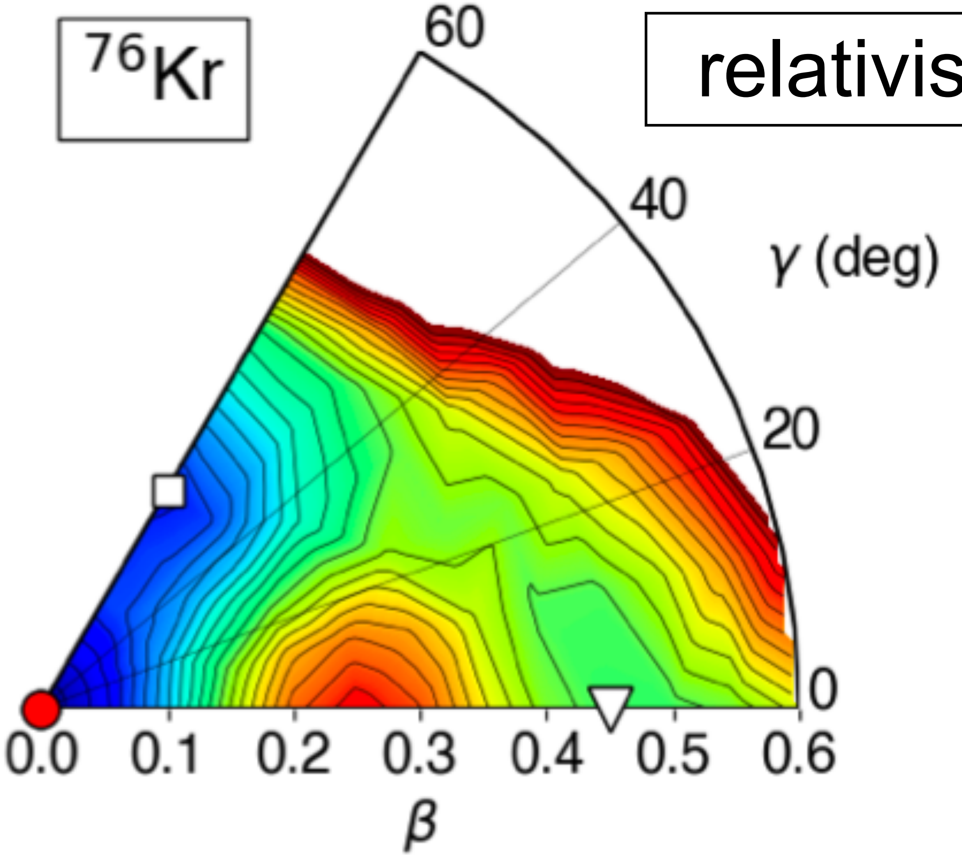
Octupole correlations and shape coexistence in N~Z nuclei

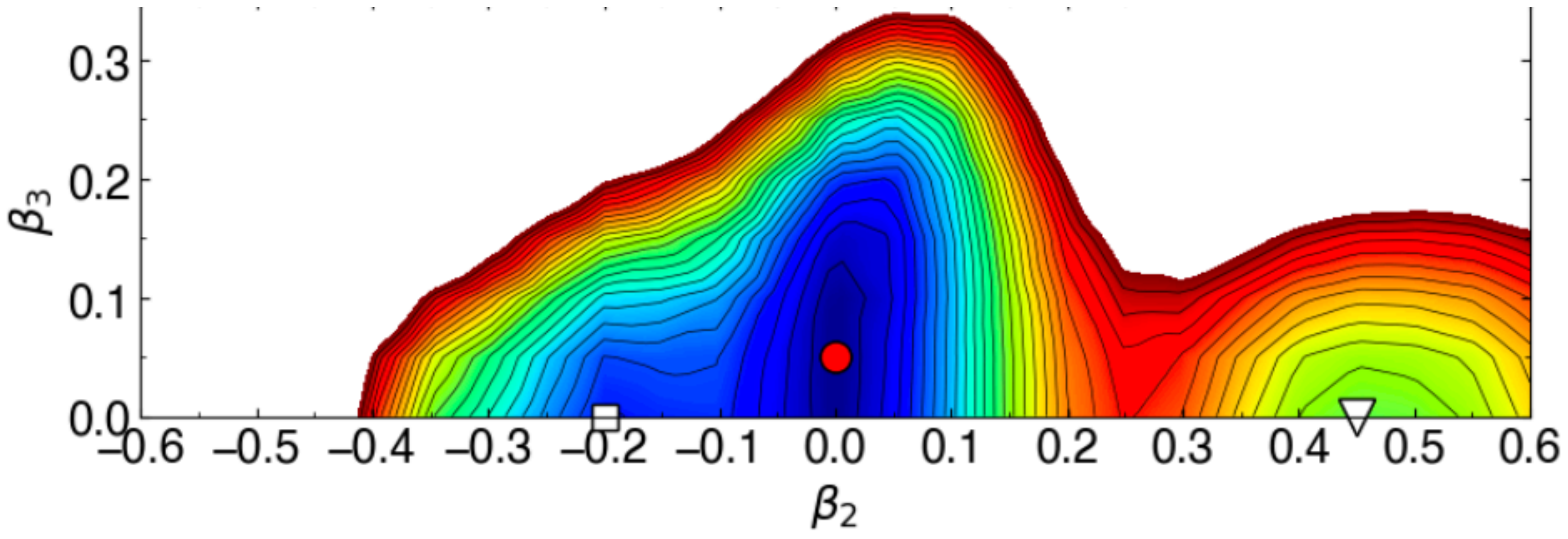
Shape coexistence in N~Z nuclei

$\beta_2 - \gamma$ (triaxial quadrupole)

$\beta_2 - \beta_3$ (axial quadrupole - octupole)

relativistic EDF

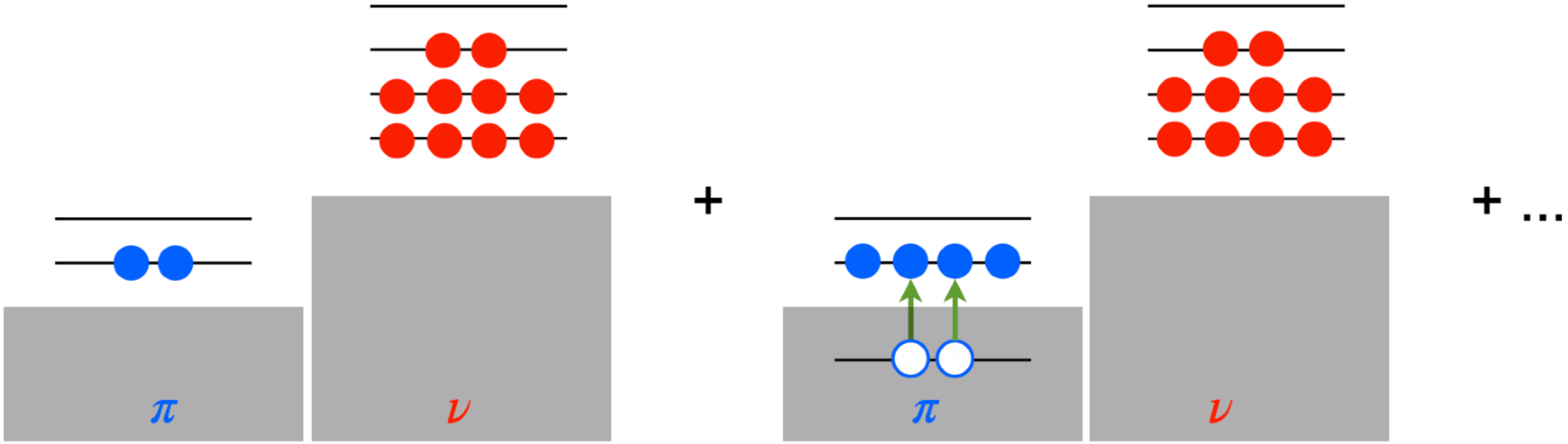




… mapped onto the sdf-IBM that includes **configuration mixing** of normal and intruder spaces (associated with three minima)

Configuration mixing in the IBM

Duval, Barrett (1981)



0p-0h (N bosons)

2p-2h (N+2 bosons)

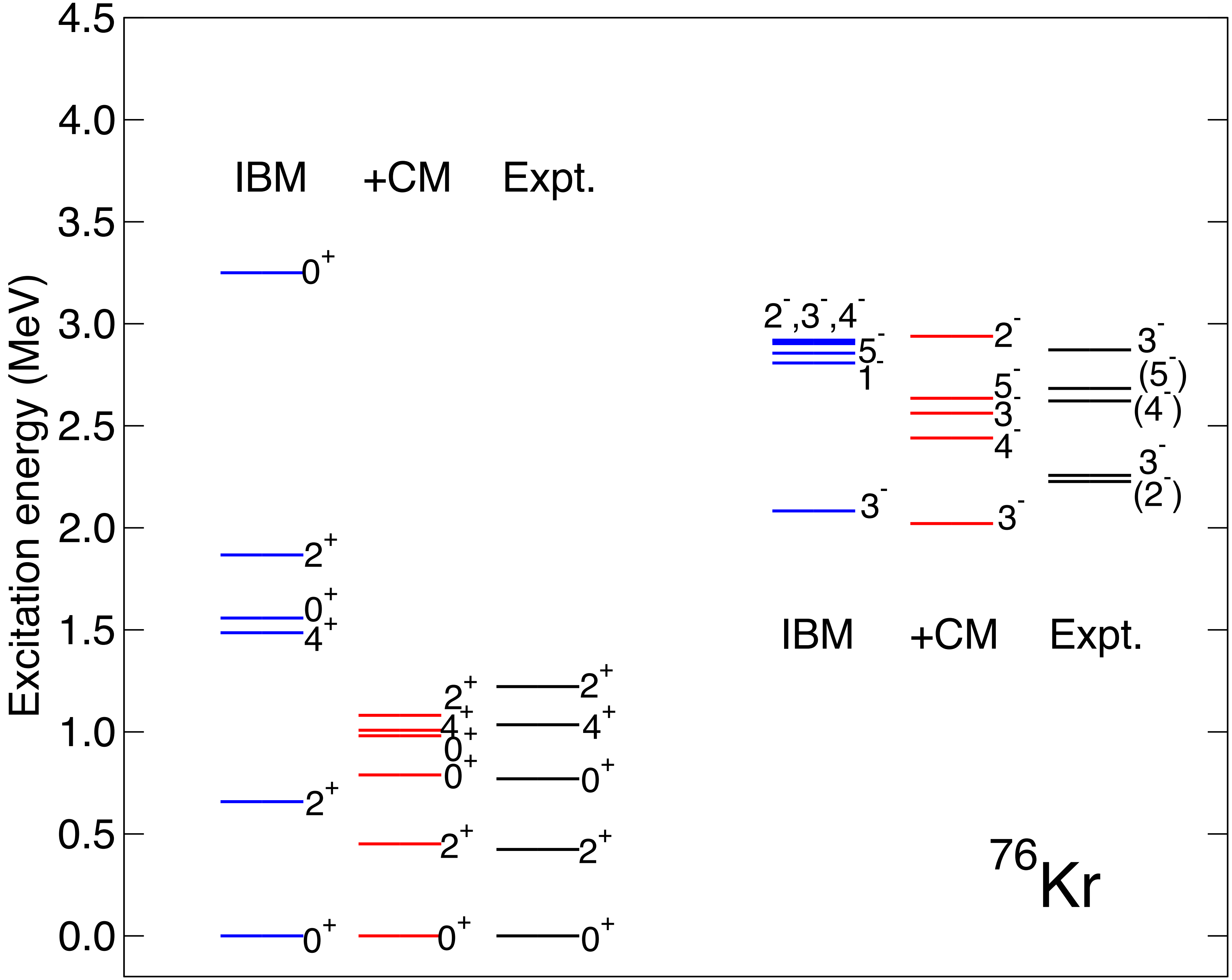
$$\boxed{\hat{H} = \hat{H}_{sdf}(N) + \left[\hat{H}_{sdf}(N+2) + \Delta\right] + \hat{V}_{mix}(N, N+2)}$$

Δ : energy to promote a boson

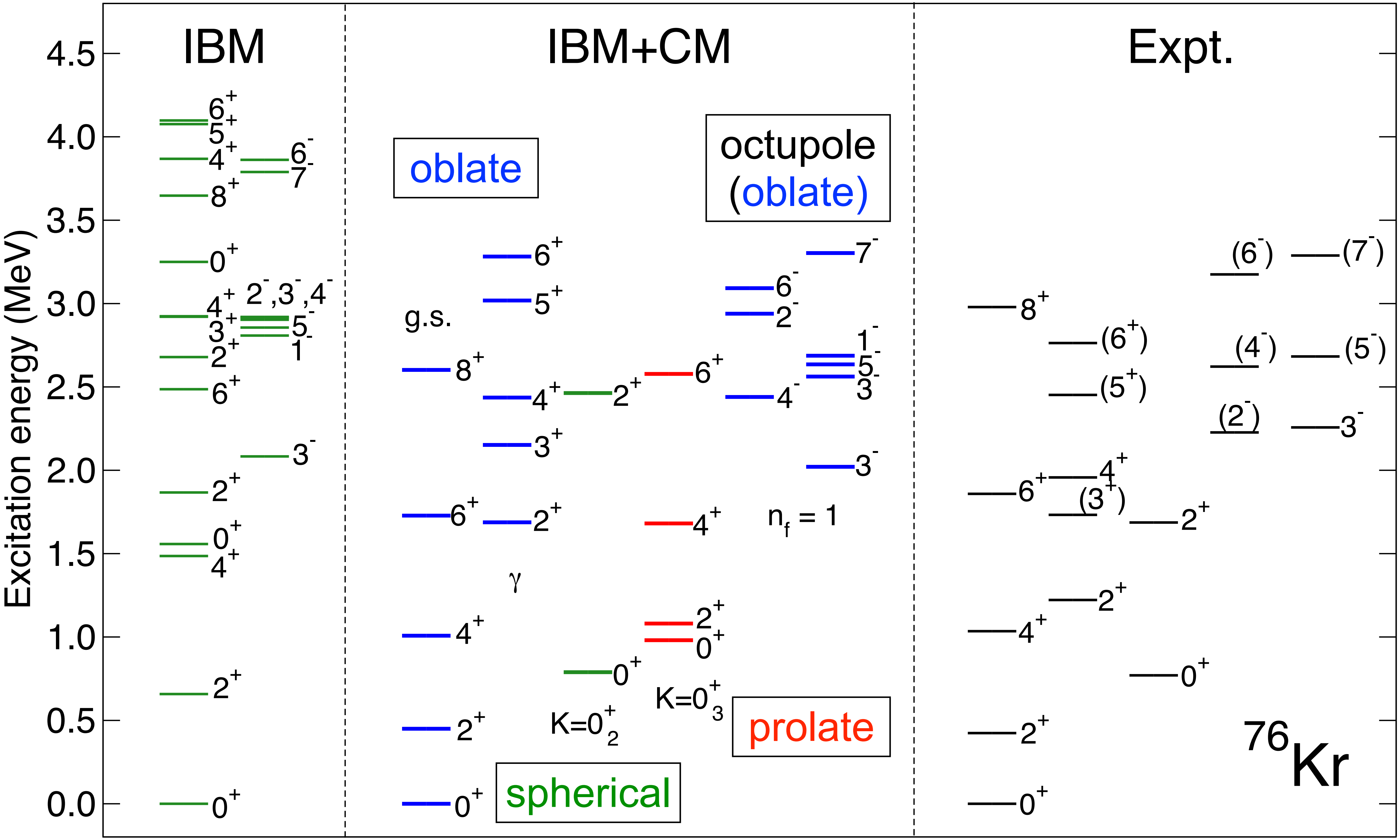
$\hat{V}_{mix}$: Mixing N - (N+2) spaces

Hilbert space: $[(sdf)^N \oplus (sdf)^{N+2}]$

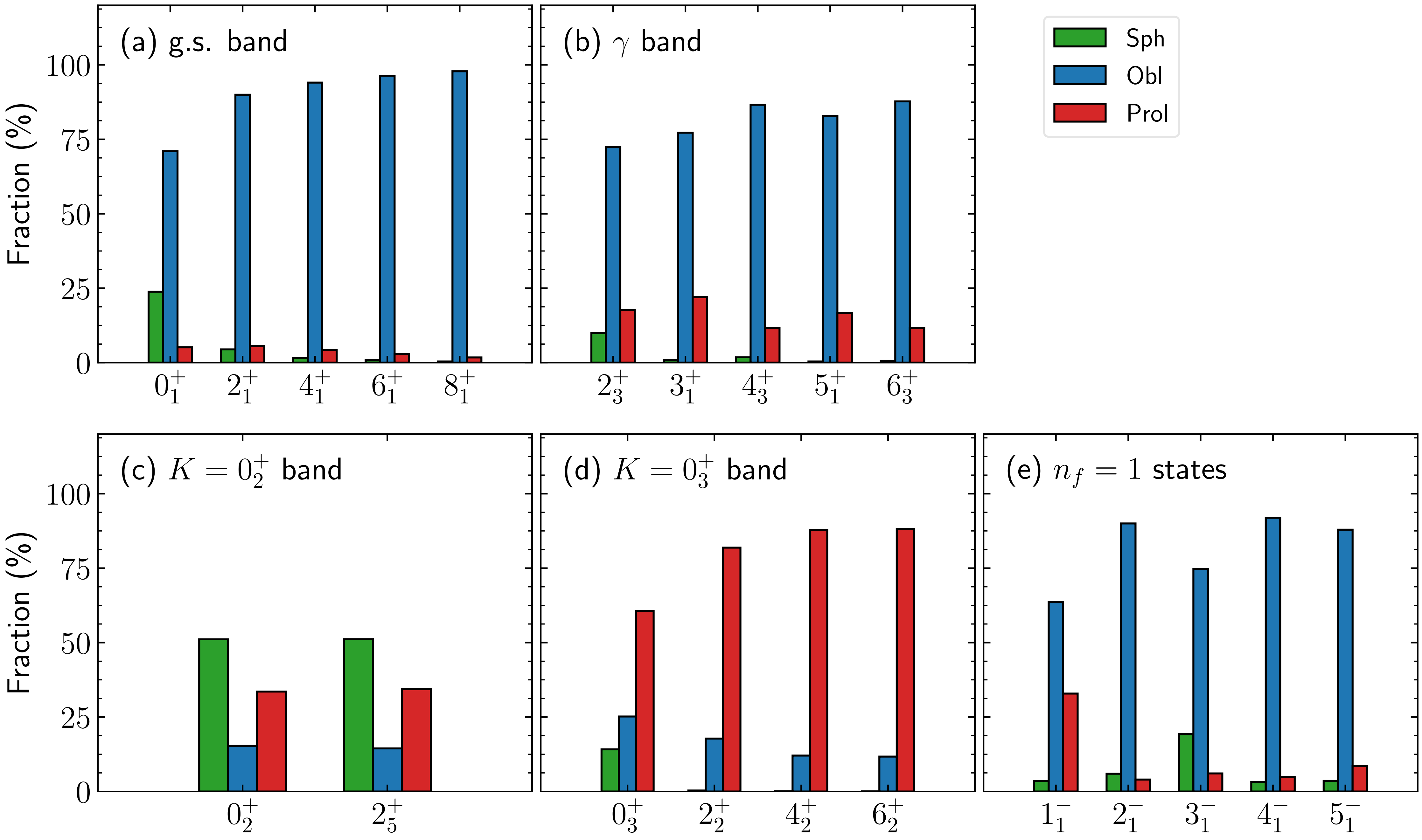
Effects of configuration mixing



Band structure



Decompositions of states



Summary

- Octupole deformations and collective excitations in heavy nuclei are becoming accessible
- New regions of octupole deformation?
- Relevance of triaxial and higher-order deformations?

Thank you