

Exploring quadrupole and octupole collectivity in ^{106}Cd via unsafe Coulomb excitation

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Where is the border between “safe” and “unsafe” Coulomb excitation?

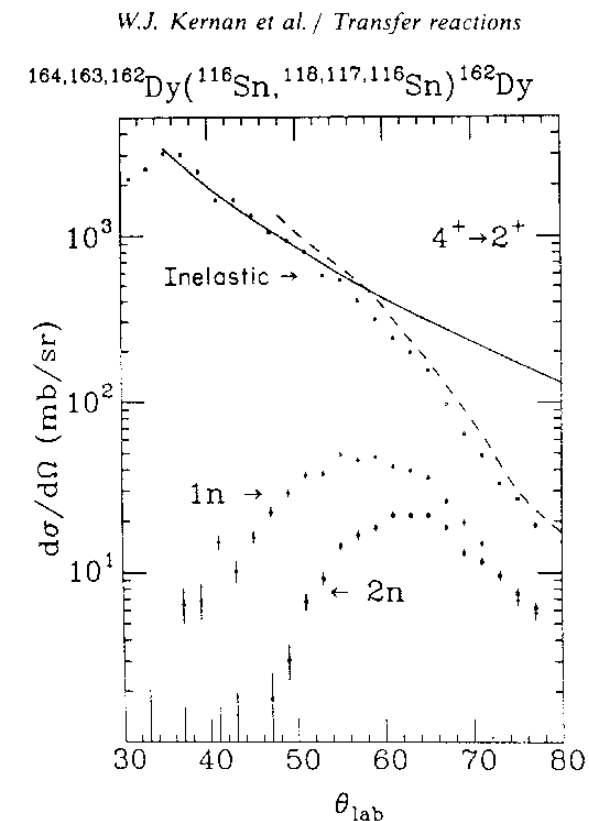
- **Cline’s “safe energy” criterion:** purely electromagnetic interaction if the distance between nuclear surfaces is greater than 5 fm

$$D_{\min} = 1.25 \cdot (A_p^{1/3} + A_t^{1/3}) + 5.0 \quad [\text{fm}]$$

- empirical criterion based on systematic studies of inelastic and transfer cross-sections at beam energies of few MeV/A (e.g. W.J. Kernan et al., Nucl. Phys. A 524, (1991) 344)

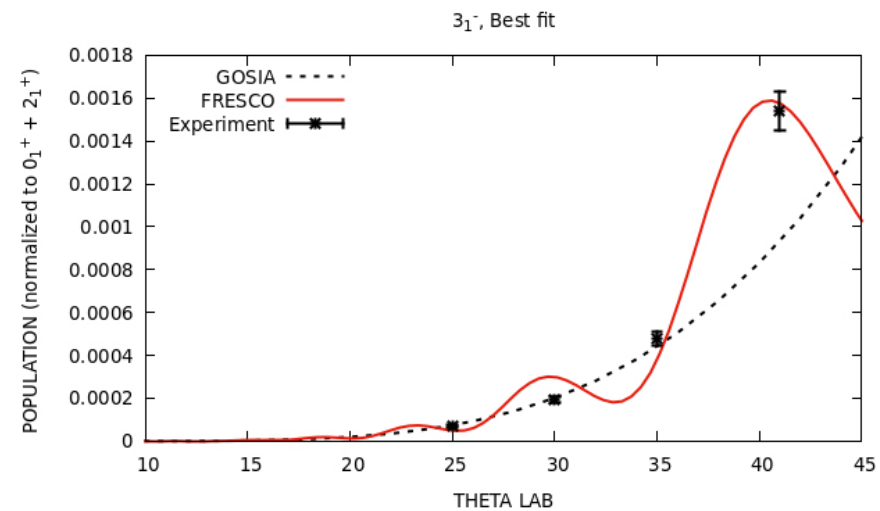
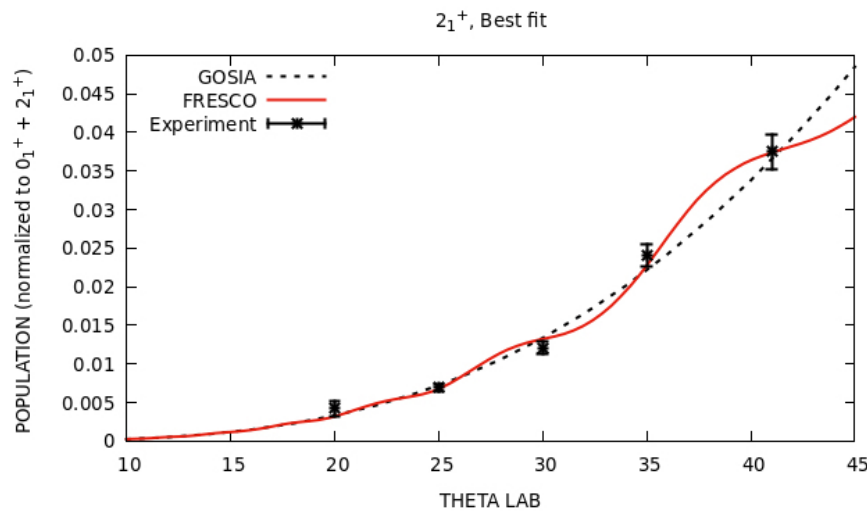
- one-neutron sub-barrier transfer recently observed in Coulomb excitation of ^{42}Ca on ^{208}Pb (K. Hadyńska-Klęk et al, PRC 97, 024326 (2018))

- for light reaction partners (^{12}C , ^{16}O ...) deviations from Cline’s criterion observed already at 6.5 fm separation



Why should we care?

- oscillatory behaviour around the pure Coulomb-excitation cross section due to the nuclear-electromagnetic interference
- deviation from the pure Coulomb-excitation cross section increases with the scattering angle
- multipolarity also plays an important role: much larger effect for E3 than E2
- large increase of the excitation cross section! possible application for RIB studies or higher-lying states?



FRESCO calculations: D. Kalaydjieva, N. Keeley.

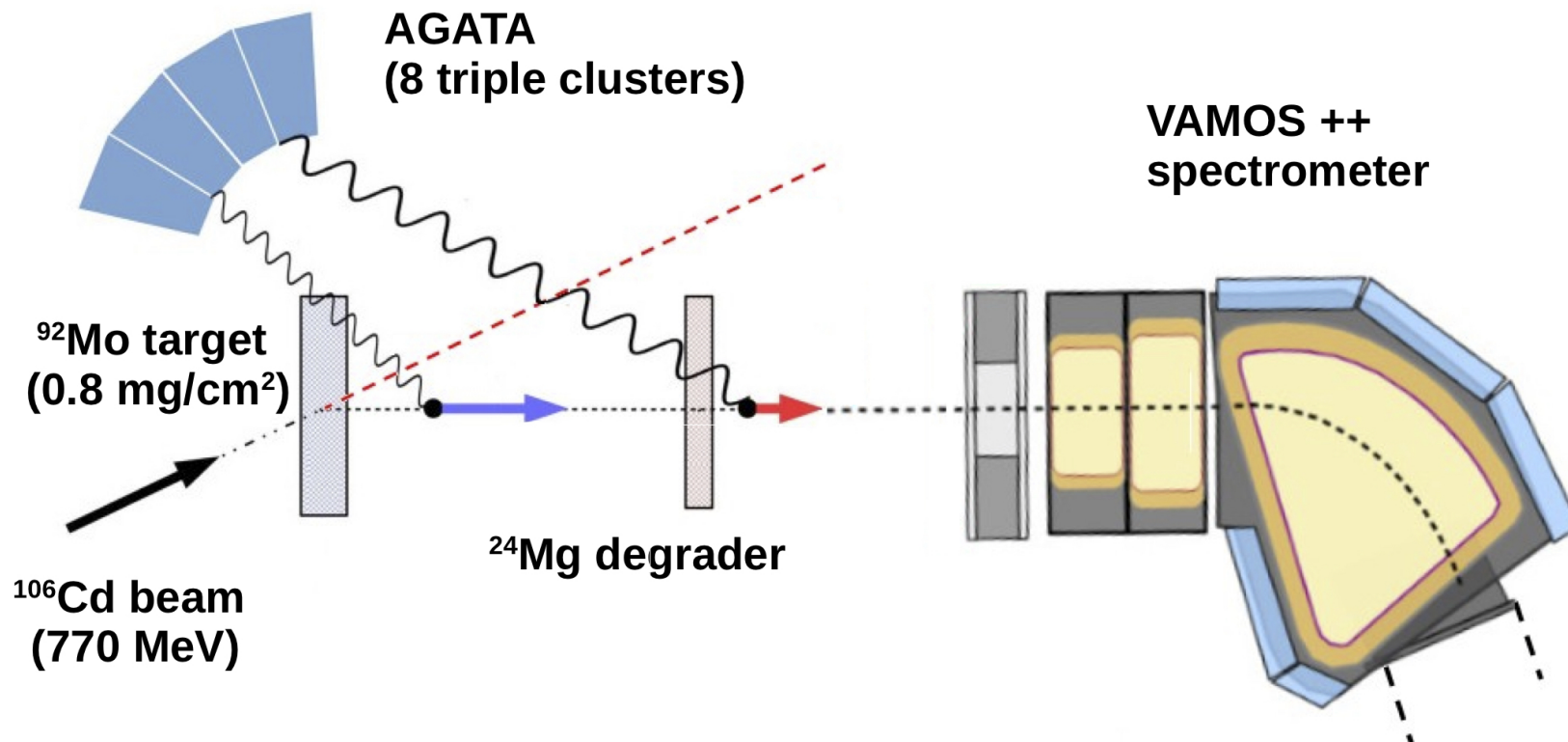
Data from P. Garrett, MZ et al, PRC 106, 064307 (2022) (¹⁰²Ru + ¹²C at 53 MeV)

Experiment

- inelastic scattering data on ^{106}Cd : byproduct of a RDDS lifetime measurement following multinucleon transfer in the $^{106}\text{Cd} + ^{92}\text{Mo}$ reaction at 7 MeV/A

M. Siciliano et al., Phys. Lett. B 806, 135474 (2020)

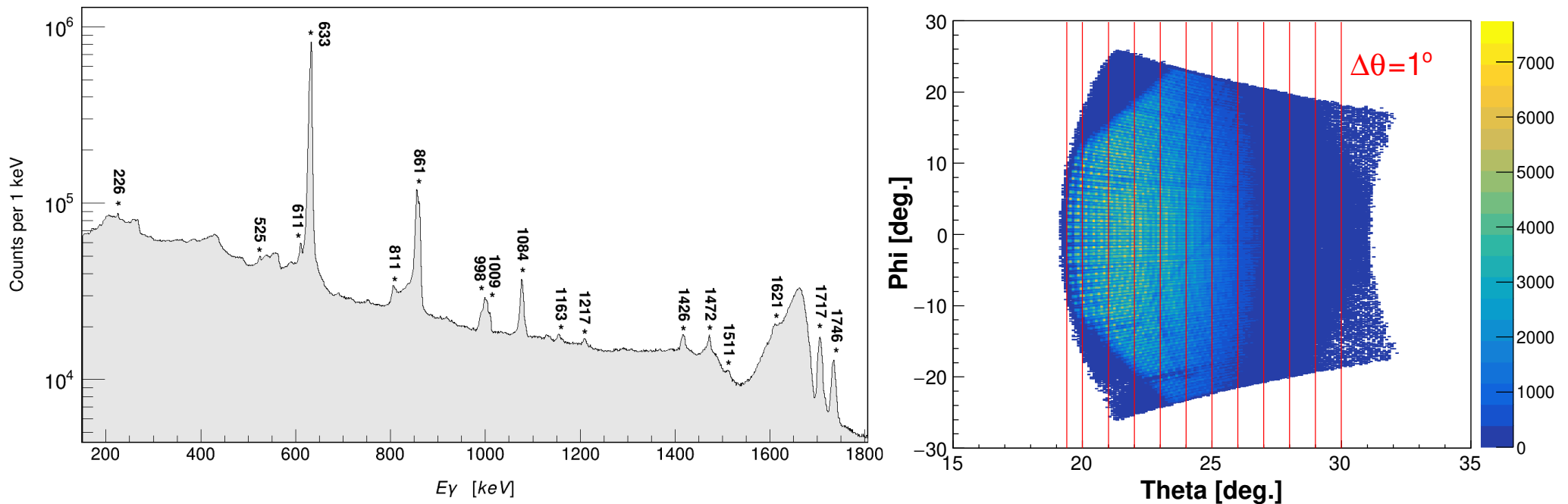
M. Siciliano et al., Phys. Rev. C 104, 034320 (2021)



- VAMOS at grazing angle (25°); lowest observed scattering angle (19.4°) corresponding to 107% of Cline's safe energy

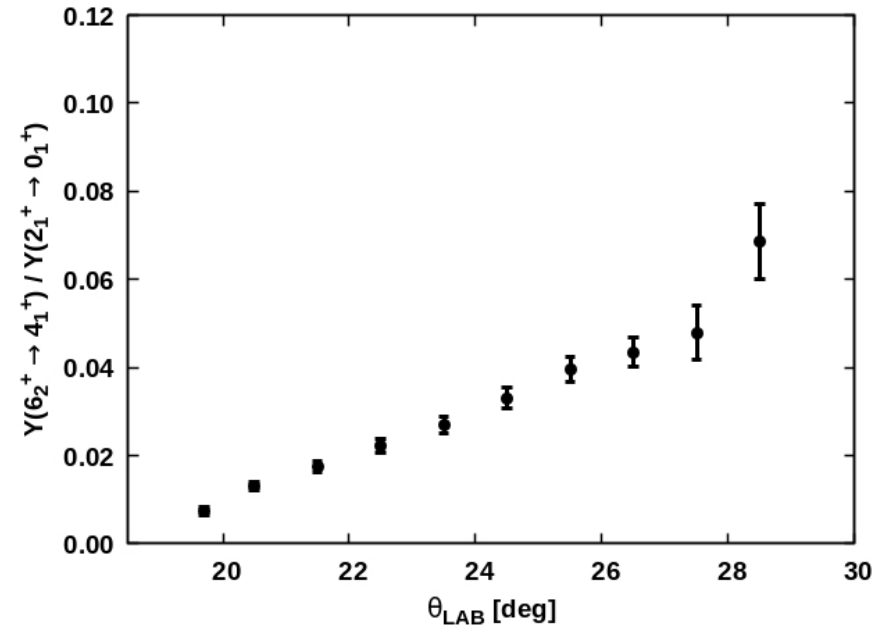
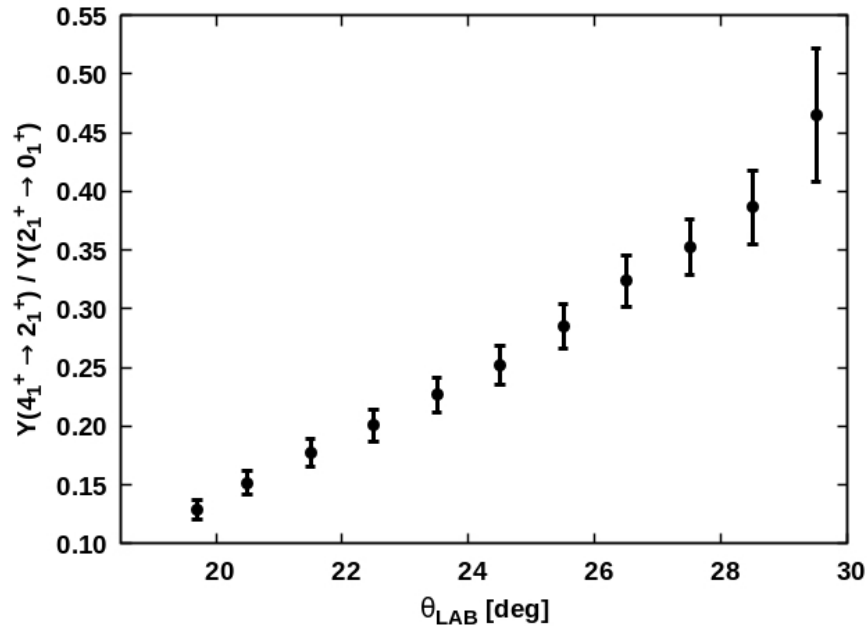
Experiment

- population of 21 excited states observed (up to spin 6^+) – comparable with Kasia's AGATA experiment on ^{110}Cd



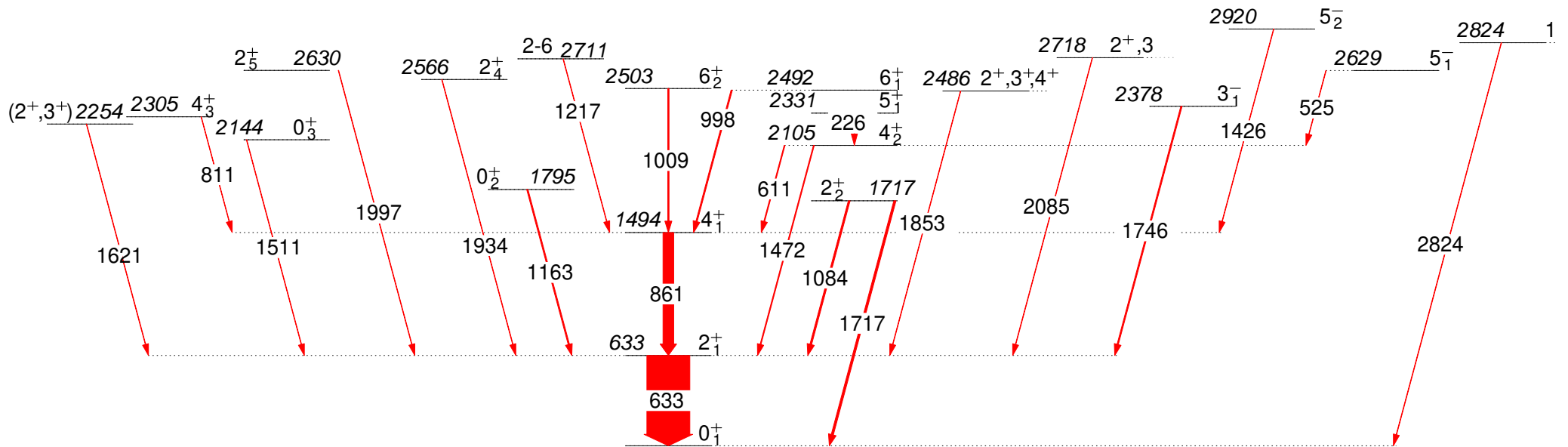
- ^{106}Cd ions identified in VAMOS with $19.4^\circ \leq \theta_{\text{LAB}} \leq 30^\circ$ (Cline's criterion fulfilled for $\theta_{\text{LAB}} \leq 18^\circ$)
- we apply gates on θ_{LAB} with 1° width to study the dependence of the excitation cross sections on scattering angle
- due to complicated acceptance of the spectrometer as a function of θ , we normalise the measured γ -ray intensities to that of the $2_1^+ \rightarrow 0_1^+$ transition

Sample results (strongly populated states)



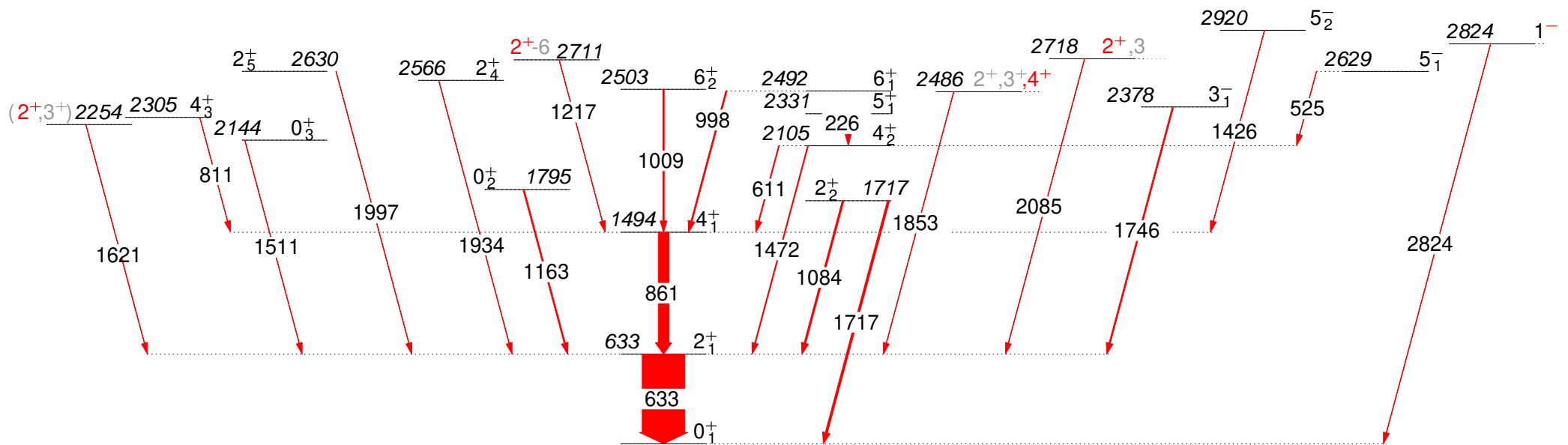
- where are the oscillations? the experimental points line up even for angles where the nuclear surfaces almost touch!
- let's try to assume pure Coulomb-excitation process and see if we can reproduce the measured γ -ray intensities using known spectroscopic data (lifetimes, branching and mixing ratios...)

Level scheme used in the analysis: observed transitions



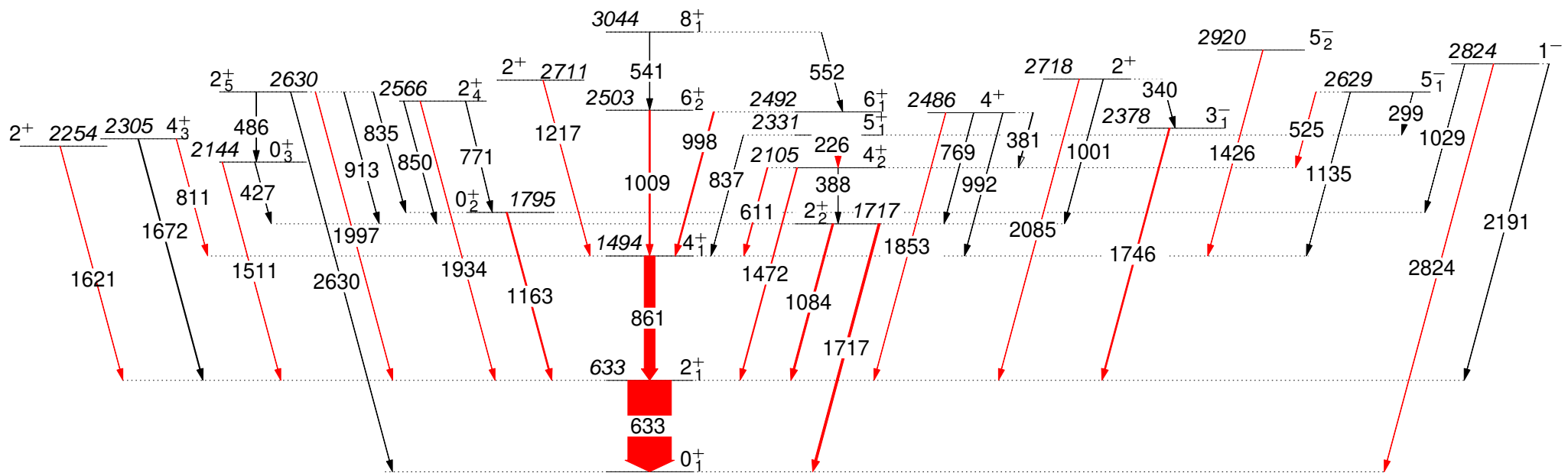
- level spin-parities taken from ENSDF
- assumptions required if there is no firm spin and/or parity assignment (2254 keV, 2486 keV, 2711 keV, 2718 keV, 2824 keV states)

Level scheme used in the analysis: observed transitions



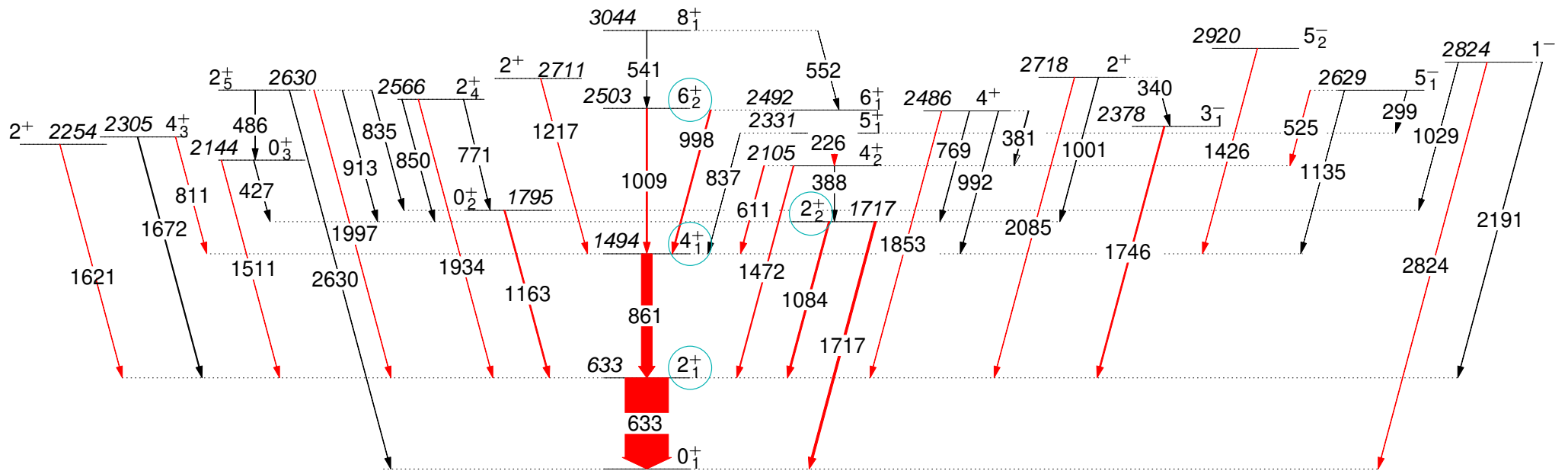
- mostly one- or two-step excitation
- placement of the 1217-keV transition in the level scheme taken from [A. Linnemann, PhD thesis, University of Cologne, 2005](#): in agreement with its observation in the present experiment and with the systematics of heavier Cd isotopes

Level scheme used in the analysis: additional spectroscopic data



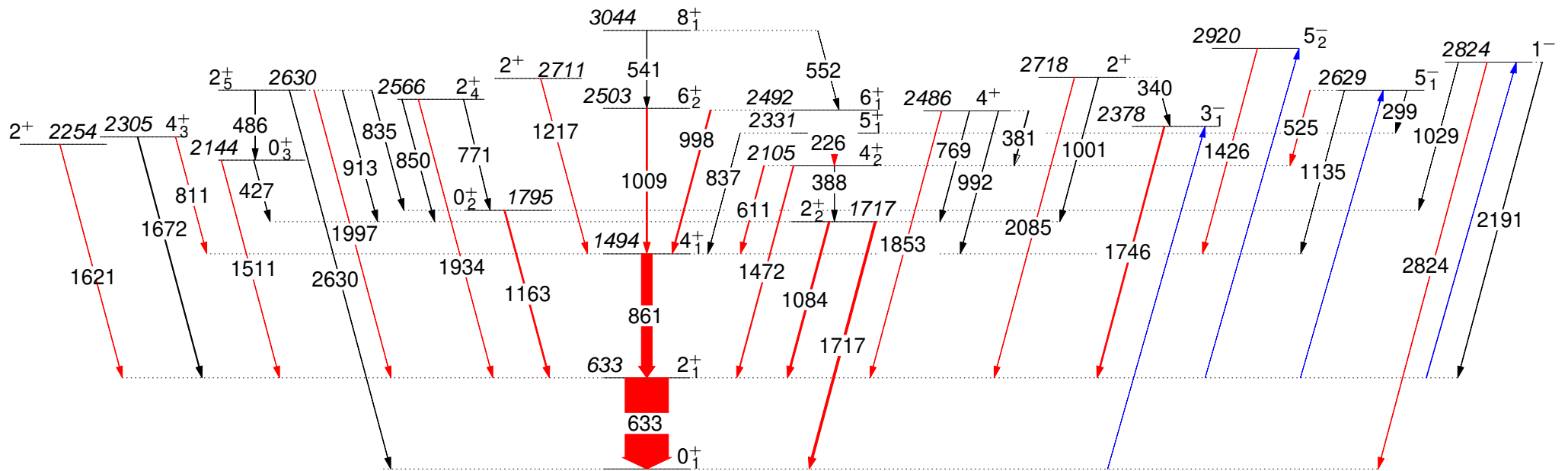
- branching ratios mostly taken from the most recent γ - γ coincidence measurement: (p,p' γ) T. Schmidt, PhD thesis, University of Cologne, 2019
- mixing ratios mostly taken from ENSDF; if they are missing for a $J^+ \rightarrow J^+$ transition – pure E2 assumed
- we note discrepancies in the literature for many branching and mixing ratios

Level scheme used in the analysis: additional spectroscopic data



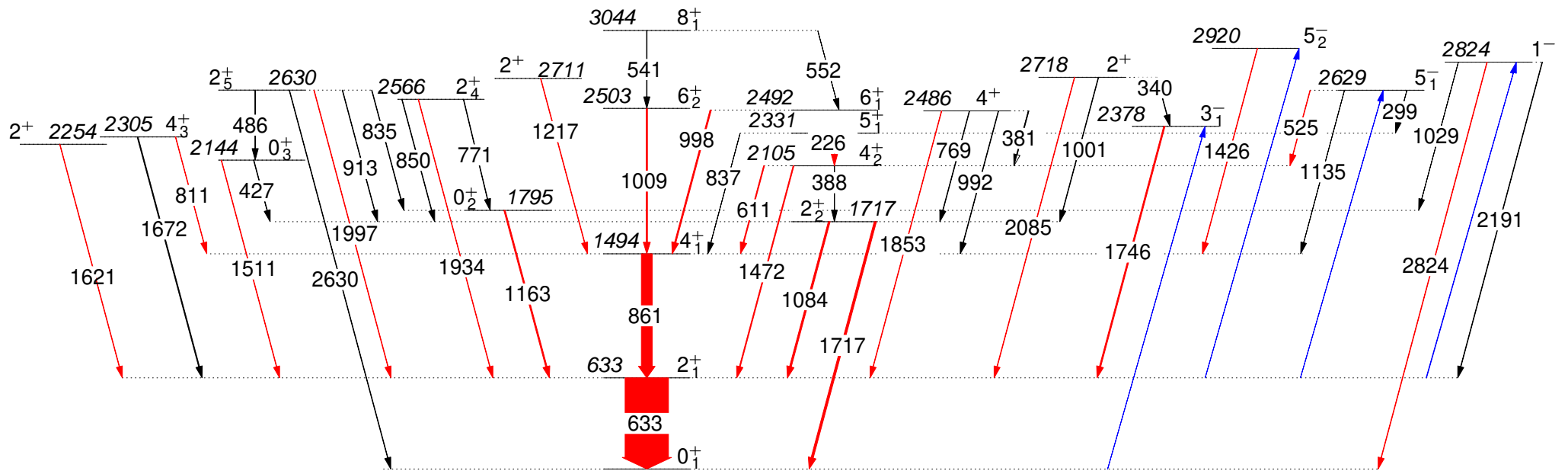
- quadrupole moments: weighted averages of results from D. Rhodes et al., PRC 103, L051301 (2021) and T.J. Gray et al., PLB 834 137446 (2021)

Level scheme used in the analysis: E3 transitions



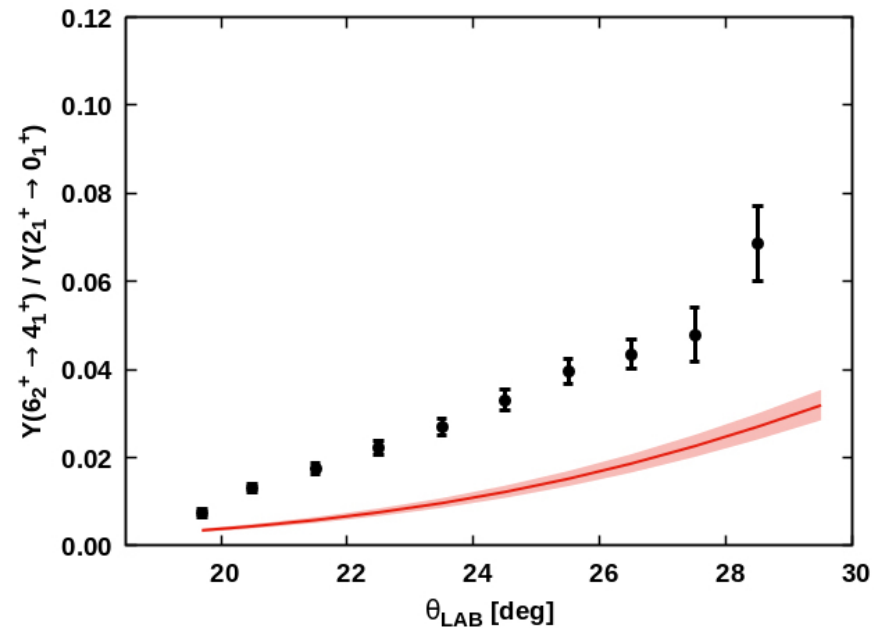
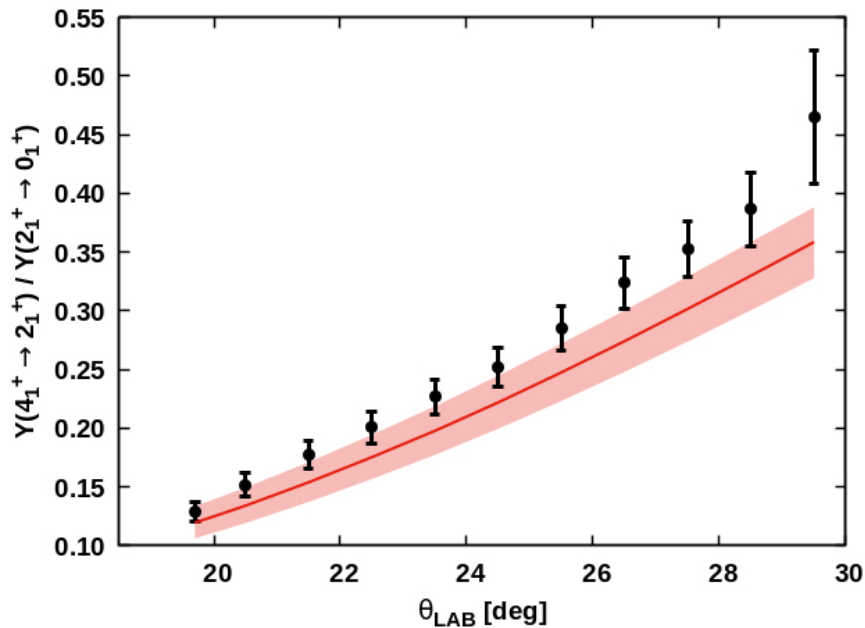
- initially we assume that only one E3 matrix element is responsible for population of each negative-parity state

Level scheme used in the analysis



- initially we assume that only one E3 matrix element is responsible for population of each negative-parity state
- now that we have a set of electromagnetic matrix elements corresponding to literature data, can we describe our measured transition intensities?

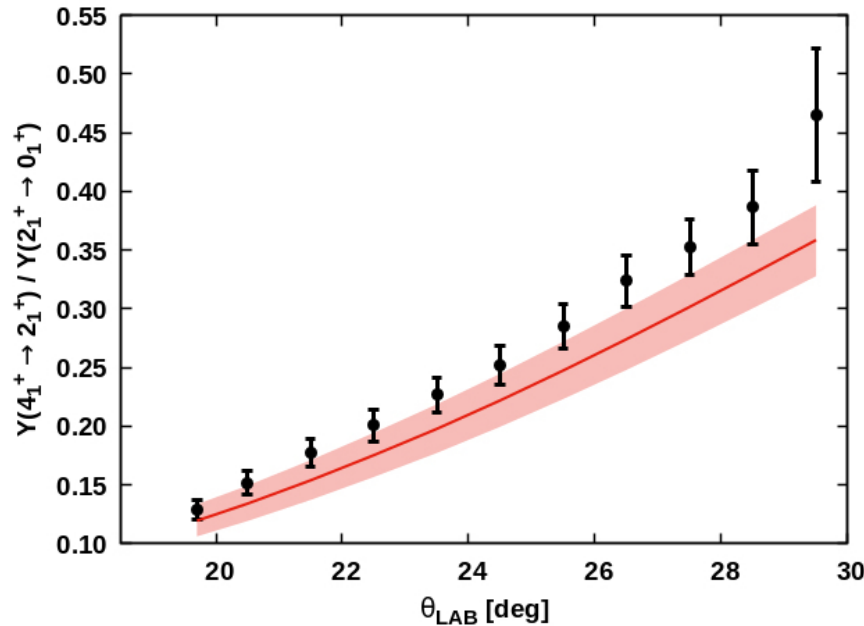
Sample results (strongly populated states)



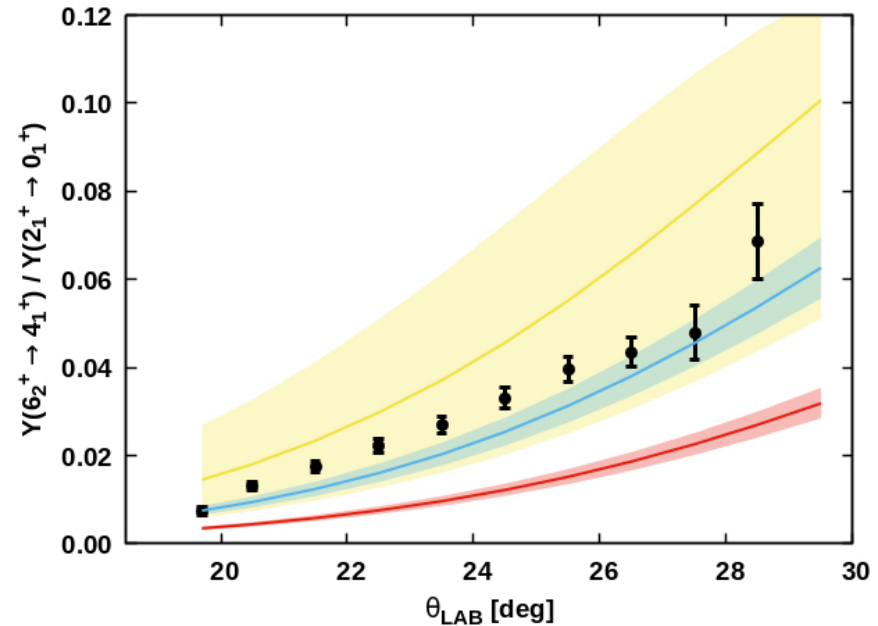
- reasonable agreement with literature data for 4_1^+ (weighted average of measured lifetimes)
- lifetime of the 6_2^+ state deduced from the same data as our transition intensities (M. Siciliano et al., Phys. Rev. C 104, 034320 (2021)) is not consistent with the measured intensity ratios

Sample results (strongly populated states)

$$4_1^+ \rightarrow 2_1^+$$

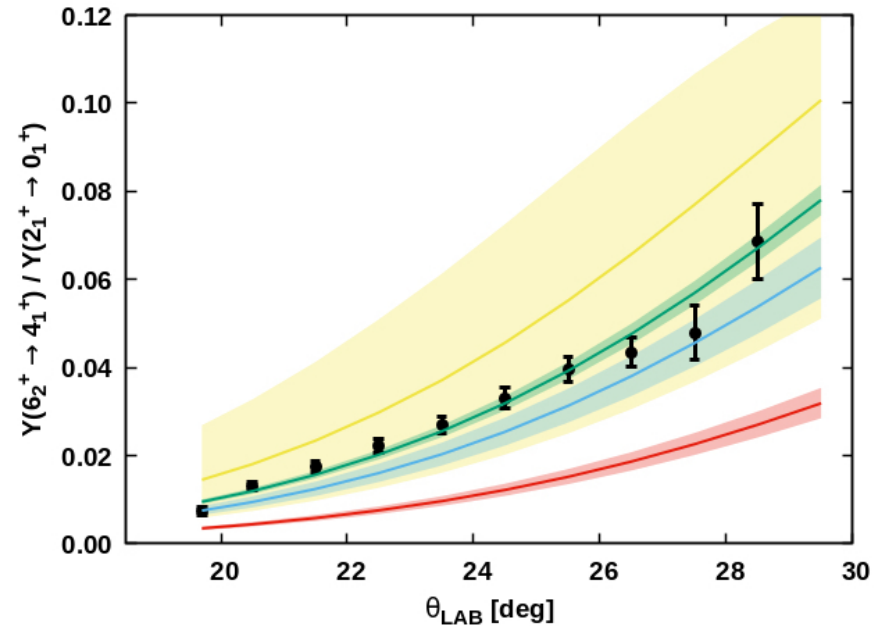
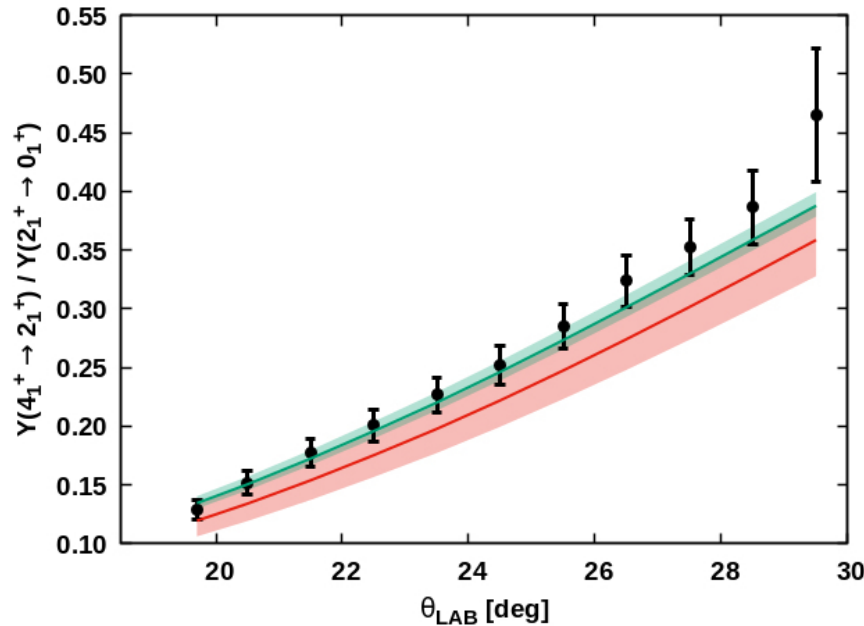


$$6_2^+ \rightarrow 4_1^+$$



- much better agreement for the 6_2^+ state if we assume:
 - $\langle 6_2^+ || E2 || 4_1^+ \rangle$ matrix element from Coulomb excitation (D. Rhodes et al., Phys. Rev. C 103, L051301 (2021))
 - or 6_2^+ lifetime from $(n, n'\gamma)$ (A. Linnemann, PhD thesis, University of Cologne, 2005 – but here the uncertainty is very large ($\tau = 0.26^{+0.44}_{-0.14}$ ps))

Sample results (strongly populated states)



- finally, we can try to fit a set of matrix elements to the first few points of the cross-section distribution, and compare the resulting lifetimes:

4_1^+ – GOSIA fit: 1.23(7) ps

weighted average of lifetimes:

1.32(12) ps

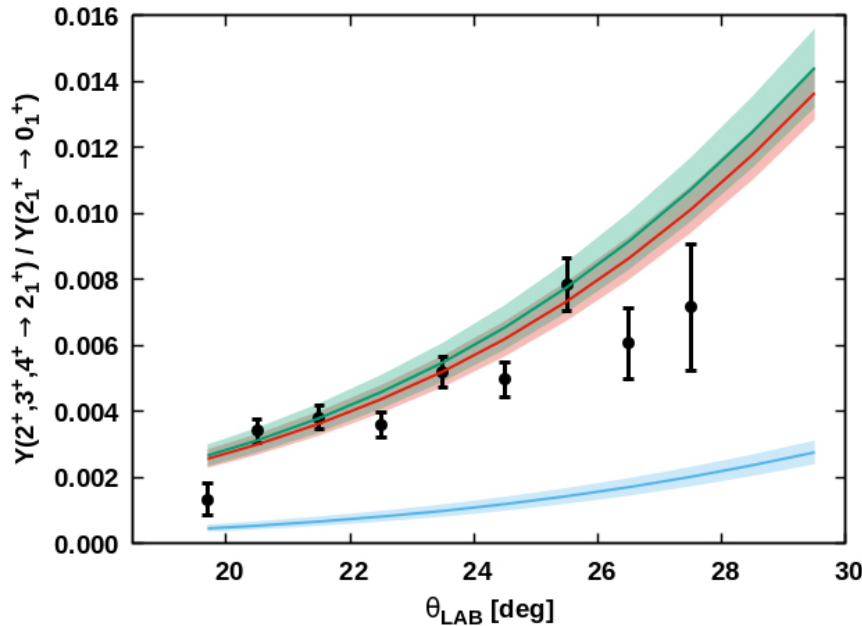
6_2^+ – GOSIA fit: 0.48(3) ps

M. Siciliano et al., Phys. Rev. C 104,
034320 (2021): 1.22(15) ps

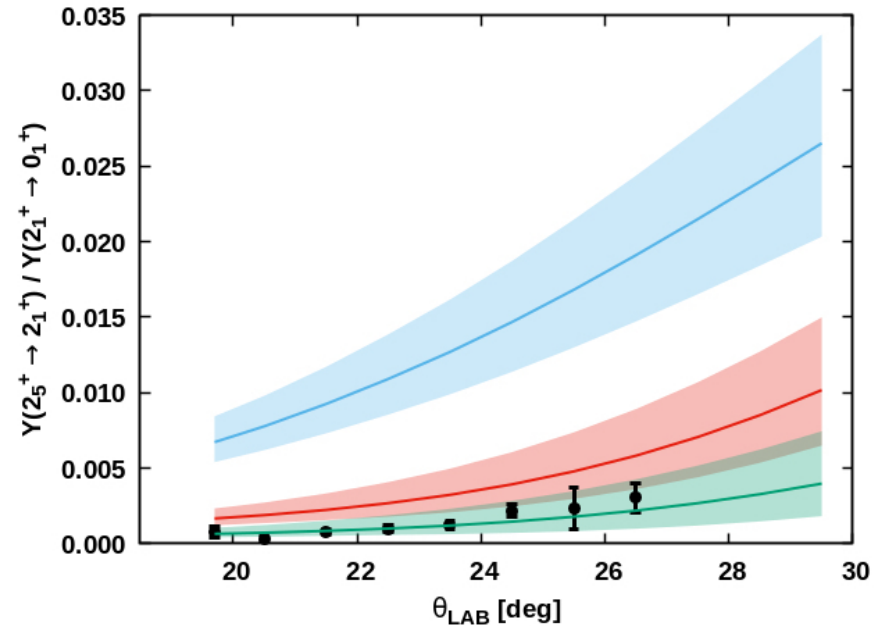
D. Rhodes et al., Phys. Rev. C 103,
L051301 (2021): 0.54(8) ps

Preference for certain branching or mixing ratios

$2^+, 3^+, 4^+ (2486 \text{ keV}) \rightarrow 2_1^+$



$2_5^+ (2630 \text{ keV}) \rightarrow 2_1^+$



different decay patterns in the literature:

ENSDF: 51% to 4_1^+ , 49% to 2_1^+

T. Schmidt, PhD thesis, University of Cologne, 2019:

63% to 2_1^+ , 25% to 4_1^+ , 9% to 2_2^+ , 3% to 4_2^+

lifetime: $2.12^{+0.21}_{-0.17}$ ps (GOSIA fit)

$2.34(17)$ ps (M. Siciliano, PRC 104, 034320 (2021))

two mixing ratios in ENSDF:

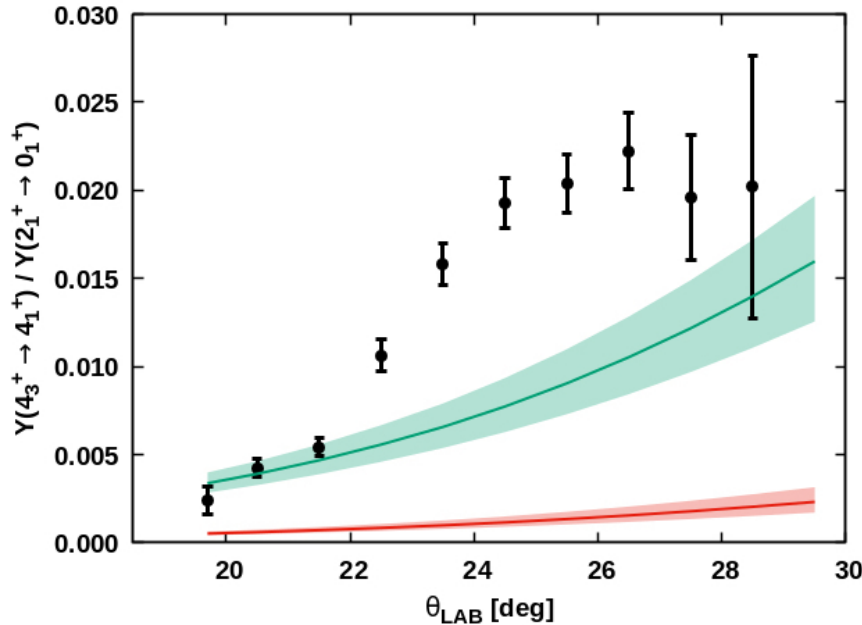
$\delta=3.2(4)$ and $\delta=-0.11(4)$

lifetime: $0.45^{+0.19}_{-0.14}$ ps (GOSIA fit)

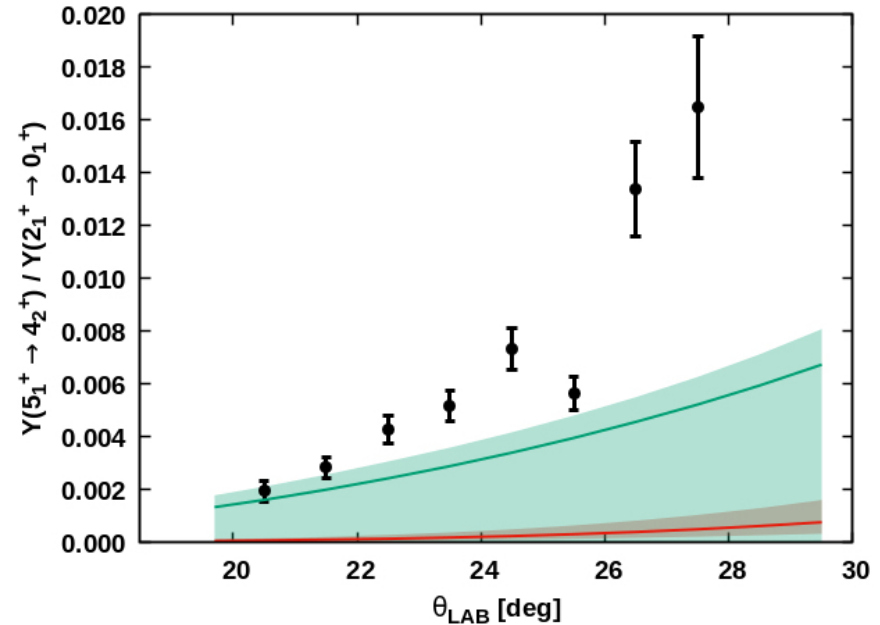
$0.19(3)$ ps (A. Linnemann PhD)

Things that did not work

$$4_3^+ (2305 \text{ keV}) \rightarrow 4_1^+$$



$$5_1^+ (2331 \text{ keV}) \rightarrow 4_2^+$$



can the divergence for $\theta > 22^\circ$ be due to direct population via E4? strong E4 in this mass region known from inelastic scattering: M.Pignanelli, NPA 540, 27 (1992).

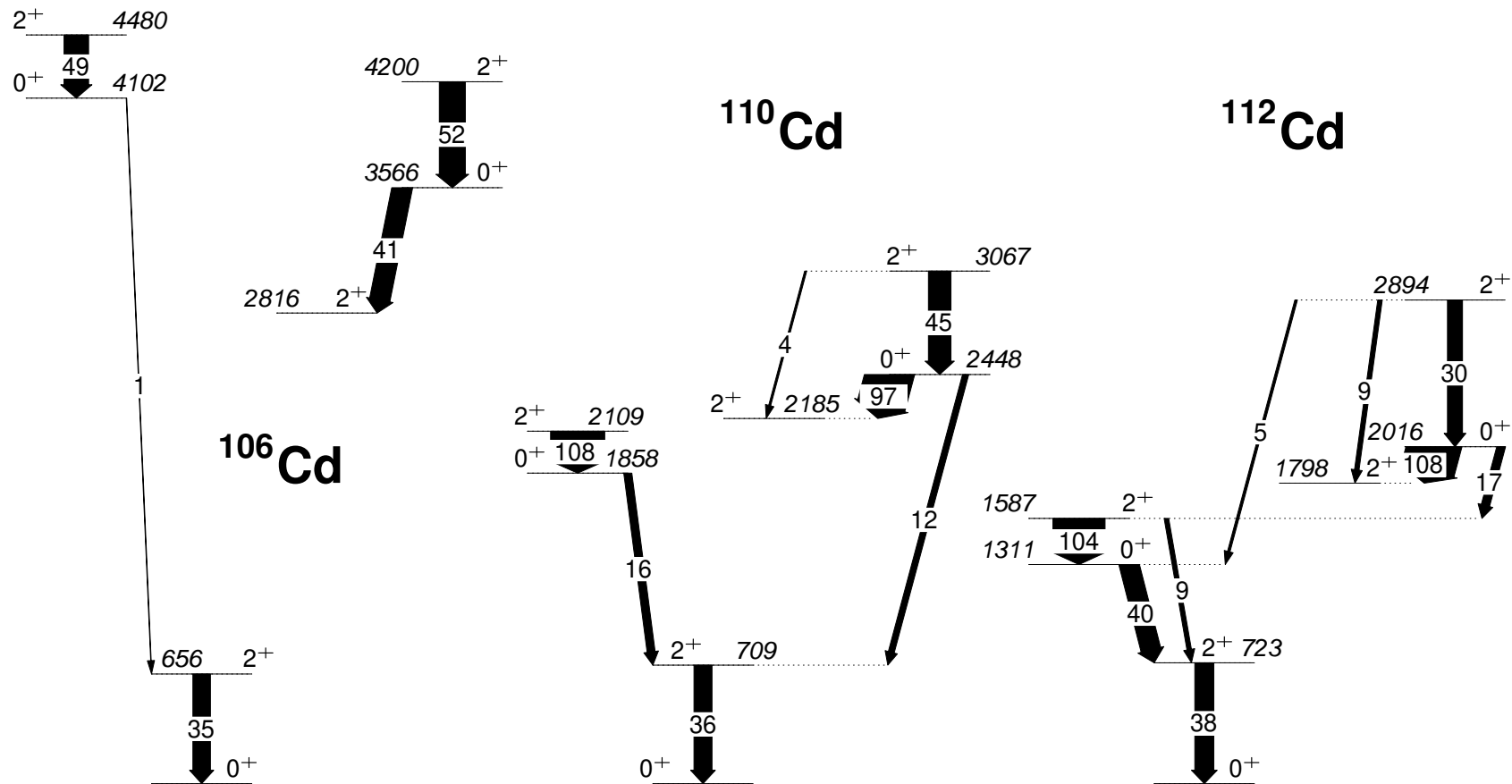
lifetime: 0.18(3) ps (GOSIA fit)
 1.1(1) ps (M. Siciliano, PRC 104, 034320)
 <0.36 ps ((n,n' γ), A. Linnemann PhD)

the observed population of the 5_1^+ state would require $B(E2; 5_1^+ \rightarrow 4_2^+)$ over 300 W.u. no clues from γ - γ where else this line could be placed

lifetime: 9(1) ps (GOSIA fit)
 870(290) ps (ENSDF)

Shape coexistence in Cd isotopes: BMF predictions

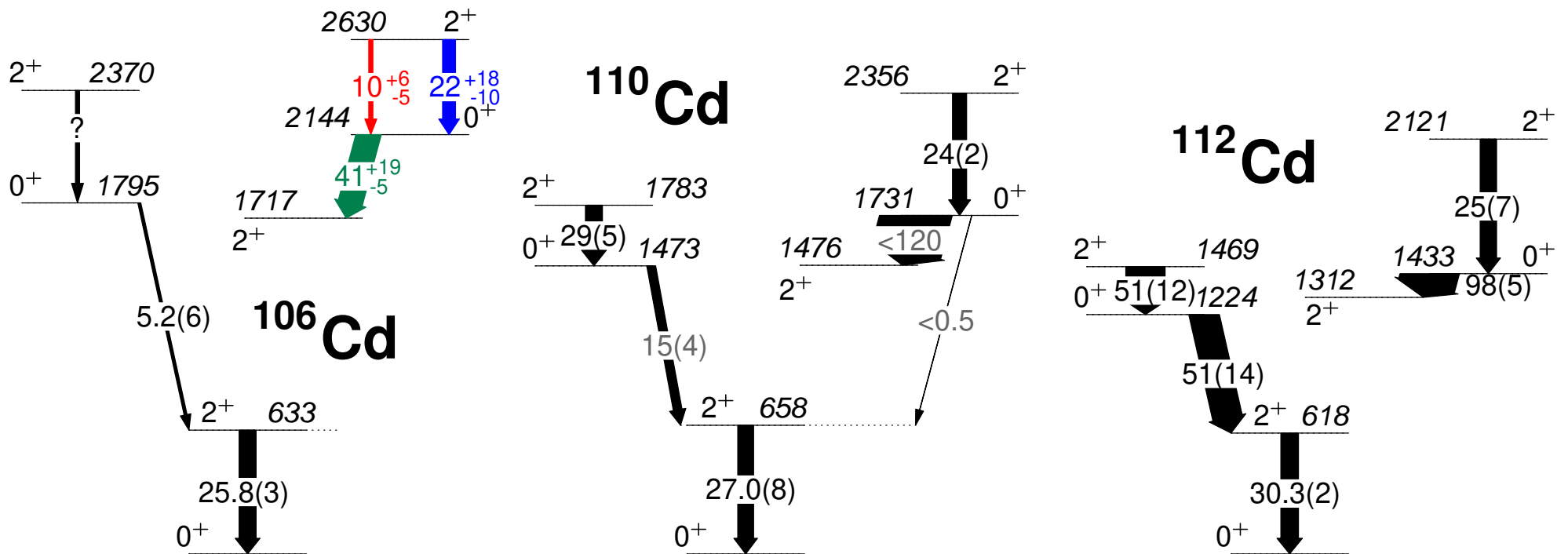
- similar shape-coexisting structures as in $^{110,112}\text{Cd}$ are predicted in ^{106}Cd
- in-band transition strength in the oblate structure predicted to increase with decreasing N, while the $B(E2; 0_3^+ \rightarrow 2_2^+)$ value decreases



SCCM calculations: T.R. Rodriguez

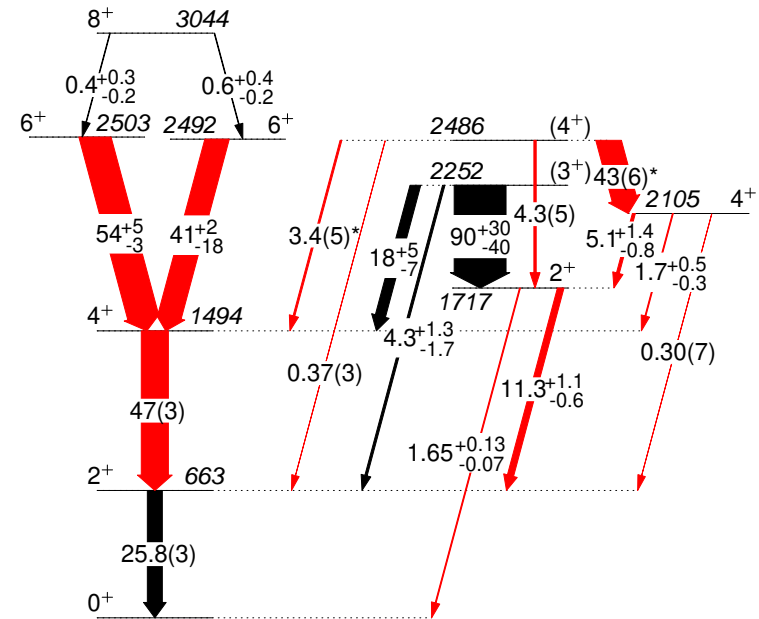
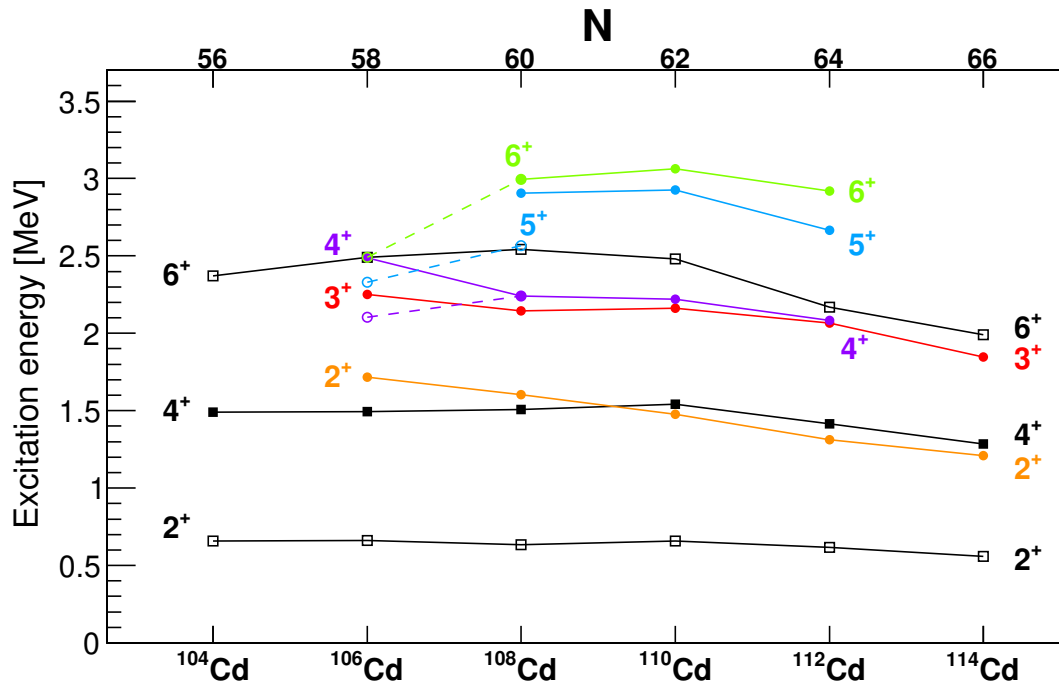
Shape coexistence in Cd isotopes: experimental results

- decay of the presumably oblate 0_3^+ state agrees well with the SCCM prediction, but the in-band transition strength has a very different trend
- larger $B(E2; 2_5^+ \rightarrow 0_3^+)$ (similar to that in the ground-state band) if the branching ratio from [A. Linnemann PhD \(Cologne, 2005\)](#) is assumed instead of the more precise value from [T. Schmidt PhD \(Cologne, 2019\)](#)



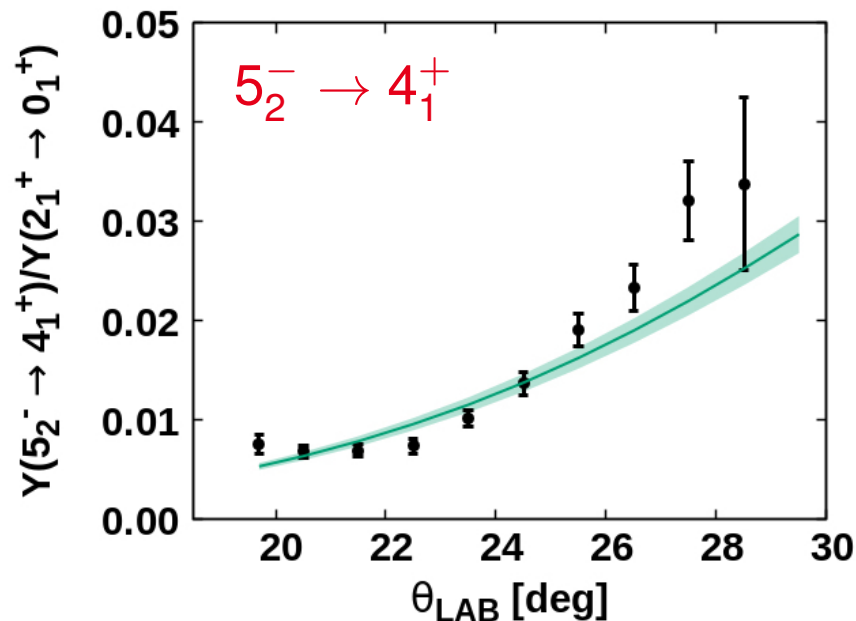
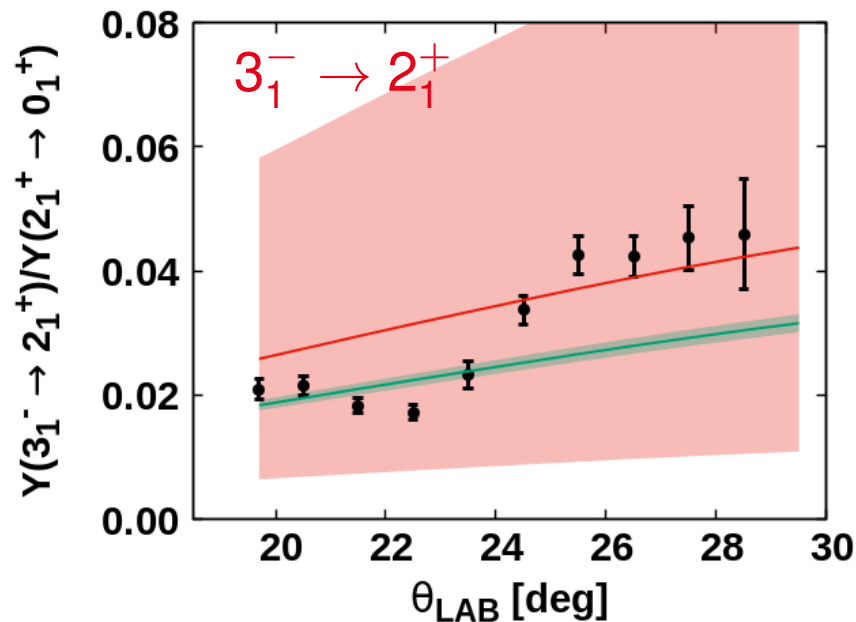
^{106}Cd : D. Kalaydjieva, PhD thesis, 2023; ^{110}Cd : preliminary values (K. Wrzosek-Lipska) in gray

Proposed reorganisation of the level scheme

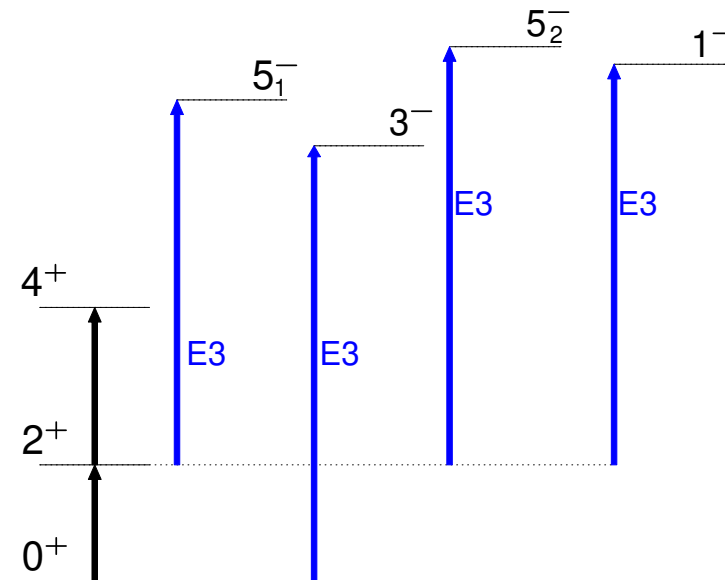


- new K=2 3⁺ and 4⁺ and K=4 4⁺ band members proposed that have expected decay patterns and excitation energies consistent with the systematics
- closely spaced 6⁺ states suggested to result from a strong mixing of the rotational band member with a seniority state
- non-observation of the 2252-keV state in the present data supports its 3⁺ spin-parity (Coulomb excitation of odd-spin positive parity states is strongly hindered)

Negative-parity states

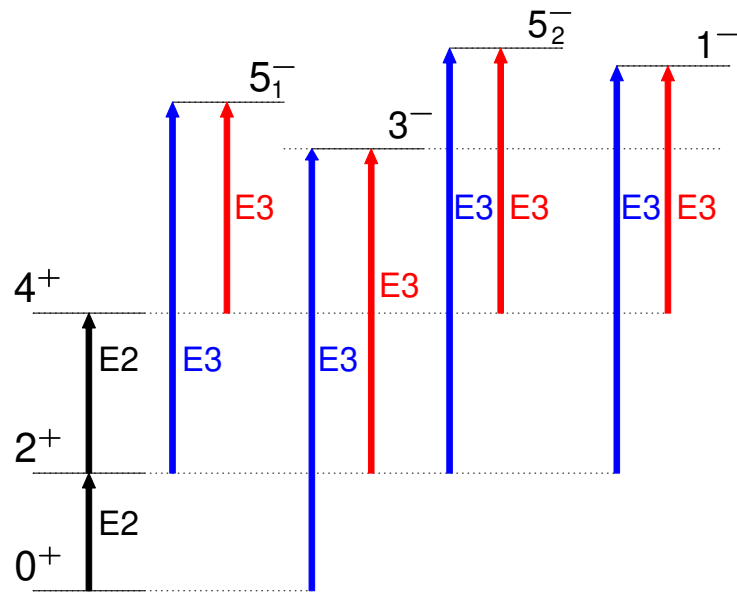


- oscillatory behaviour observed for the 3_1^- and 5_2^- excitation cross sections
- initially only a single E3 matrix element is assumed to be responsible for the population of each of the 3_1^- , 5_1^- , 5_2^- and 1_1^- states



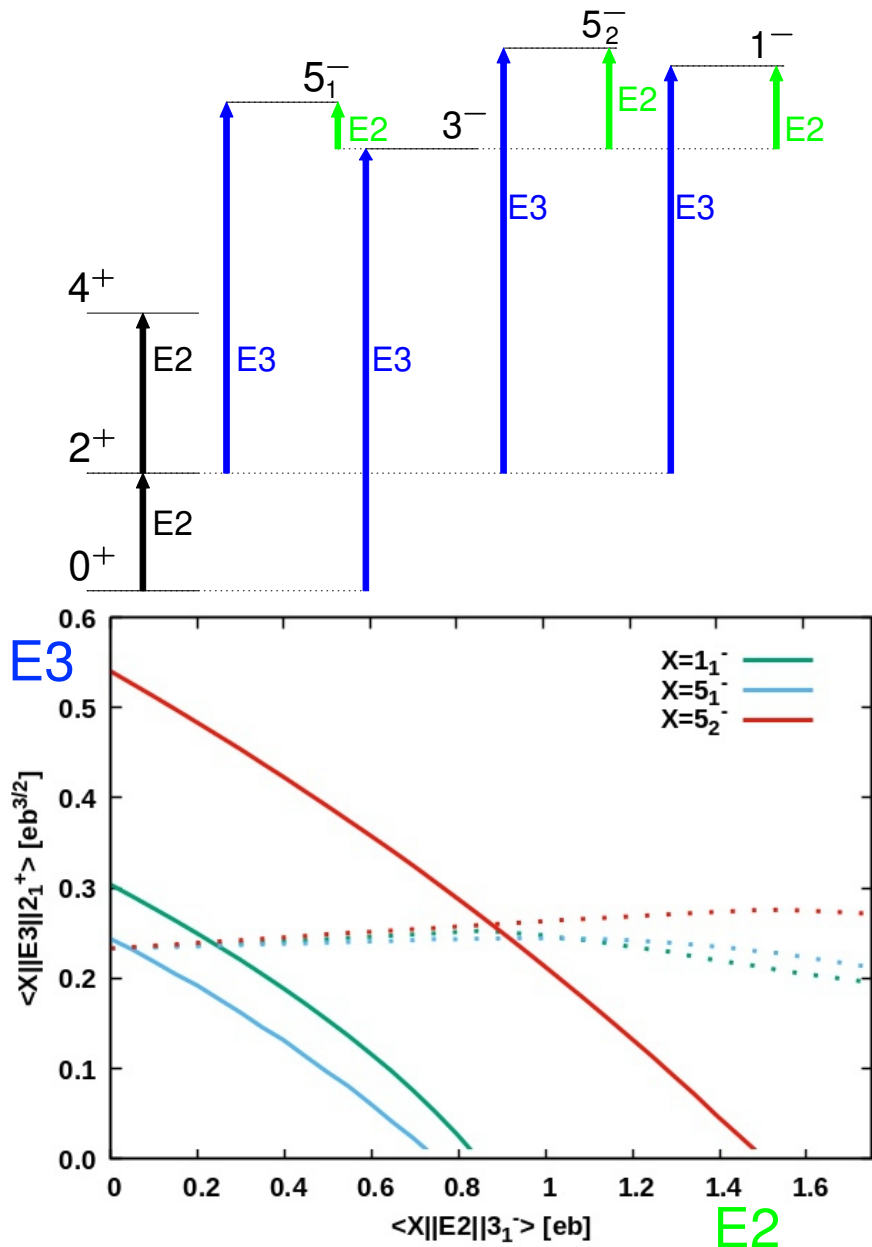
- Results:
 - $B(E3; 3_1^- \rightarrow 0_1^+) = 11.6(5) \text{ W.u.}$
 - $B(E3; 5_2^- \rightarrow 2_1^+) = 57(2) \text{ W.u. (a lot!)}$

Negative-parity states



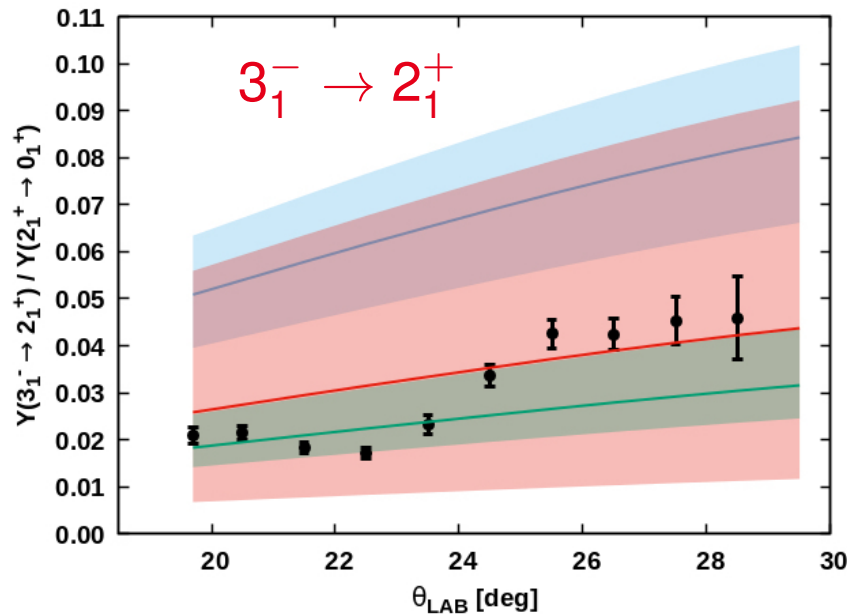
- it is necessary to introduce a more complicated coupling scheme to describe populations of the 5_1^- , 5_2^- and 1_1^- states
- **additional E3 transitions** change the populations by a few percent, even if values of many tens of W.u. are assumed

Negative-parity states

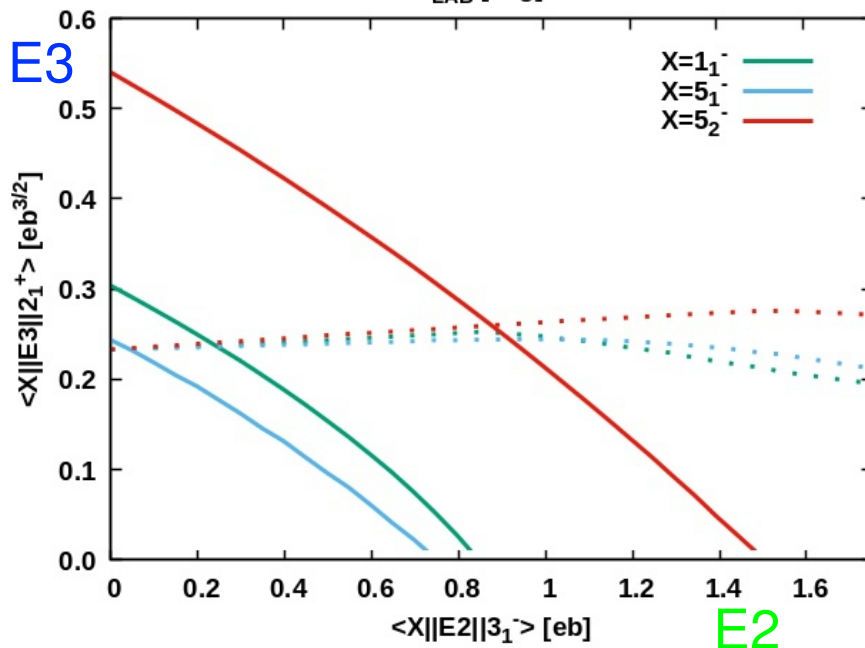


- it is necessary to introduce a more complicated coupling scheme to describe populations of the 5_1^- , 5_2^- and 1_1^- states
- **additional E3 transitions** change the populations by a few percent, even if values of many tens of W.u. are assumed
- **E2 transitions** to the 3_1^- state prove to be very important
- we do not know the relevant branching ratios, but we can extract the correlation between the **E3** and **E2** strength involved in the population of the negative parity states

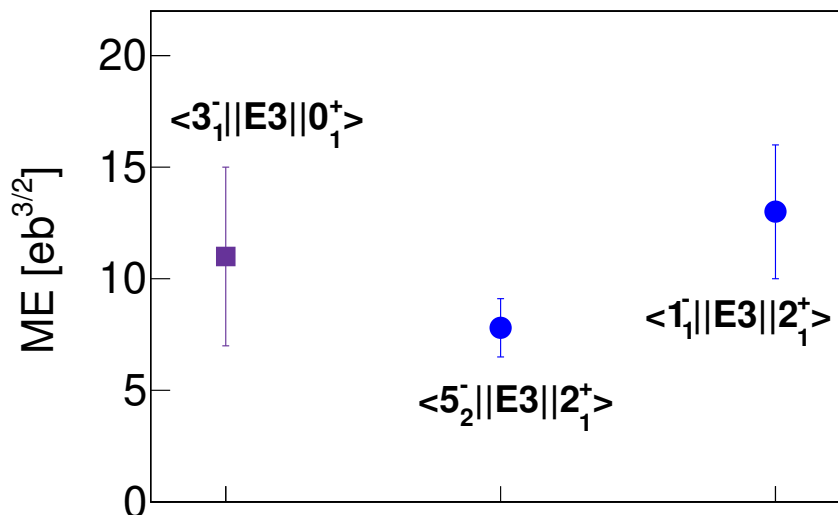
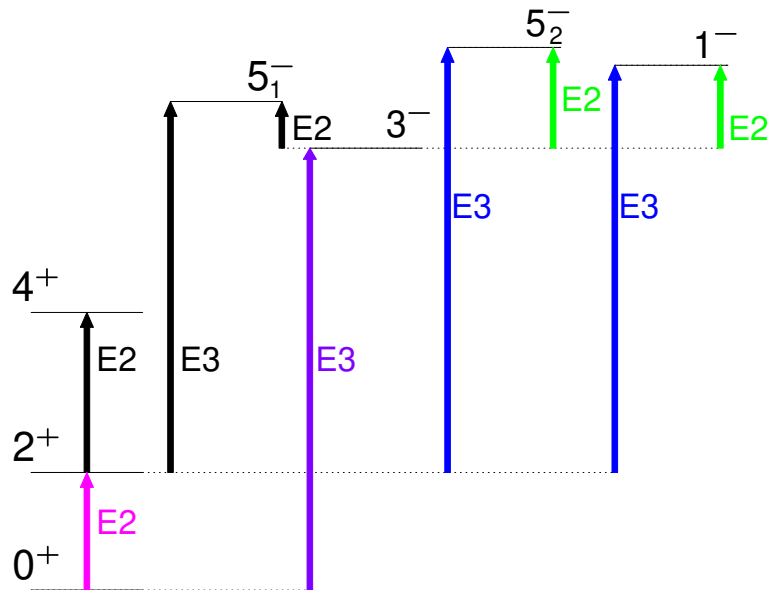
Negative-parity states: structure results



- $B(E3; 3_1^- \rightarrow 0_1^+) = 11(4)$ W.u. in agreement with D. Rhodes et al., PRC 103, L051301 (2021) (red) but in disagreement with the **evaluated value** of 34(9) W.u. (T. Kibedi and T. Spear, At. Data Nucl. Data Tables 80, 35 (2002))

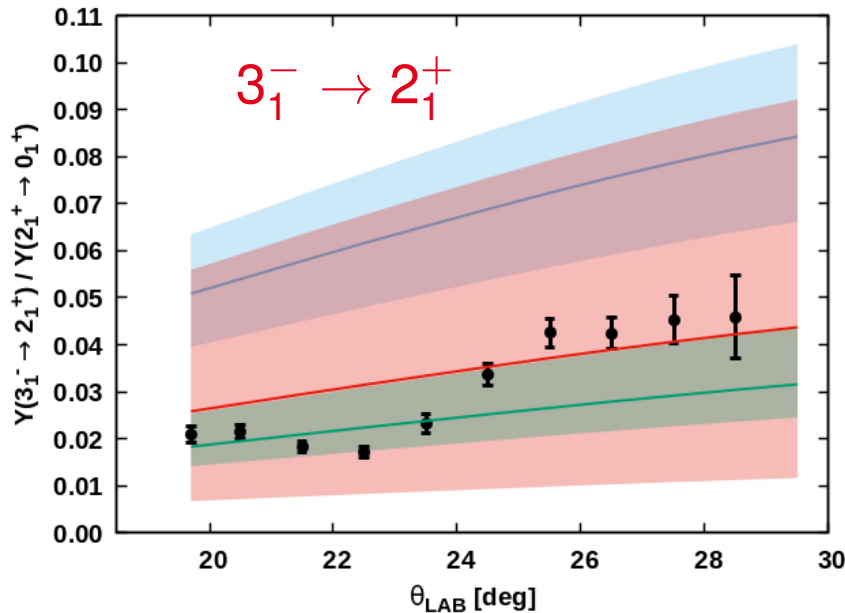


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- if one assumes $B(E2; 5_2^- \rightarrow 3_1^-) = B(E2; 1_1^- \rightarrow 3_1^-) = B(E2; 2_1^+ \rightarrow 0_1^+)$, we obtain $B(E3; 5_2^- \rightarrow 2_1^+) = 7.8(1.3)$ W.u. and $B(E3; 1_1^- \rightarrow 2_1^+) = 13(3)$ W.u, equal within error bars to $B(E3; 3_1^- \rightarrow 0_1^+)$
- 5_2^- seems to be either a member of a rotational structure built on the 3_1^- state or result from quadrupole-octupole vibrational coupling

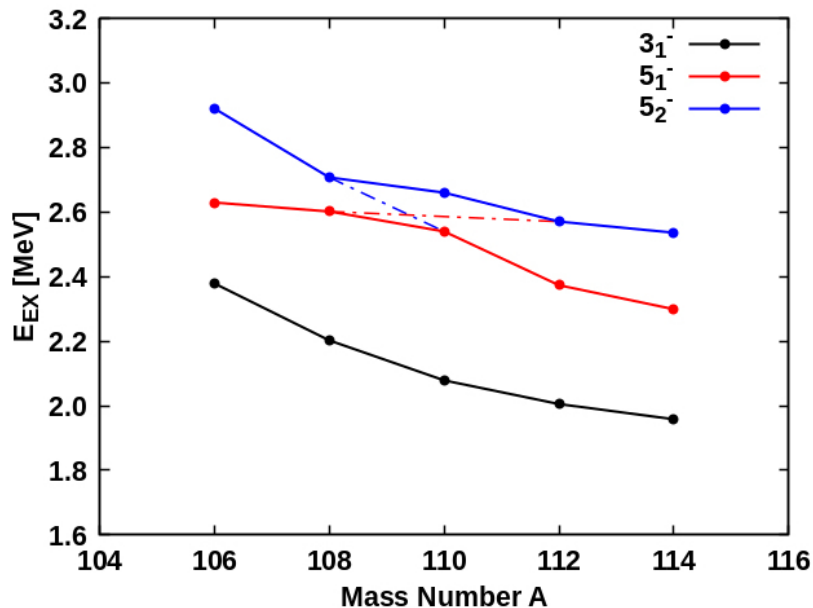
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- 5_2^- seems to be either a member of a rotational structure built on the 3_1^- state or result from quadrupole-octupole vibrational coupling, which is also consistent with energy systematics



Conclusions and outlook

- for the strongly populated states in ^{106}Cd we have shown that assuming pure Coulomb-excitation process we can well reproduce the experimentally measured transition intensities and extract E2 strengths that are in 1σ agreement with literature values
- there are numerous discrepancies in the level scheme of ^{106}Cd that make conclusions for higher-lying states more difficult: a new β -decay measurement approved at TRIUMF
- we obtained, in particular, new experimental information on the presumably oblate 0_3^+ and 2_5^+ states, as well as on the 3_1^- and 5_2^- states
- this method can be applied to analyse byproduct data of lifetime measurements using multinucleon transfer (in particular, to obtain information on states that were beyond sensitivity region of the lifetime measurement – here e.g. 2_4^+ , 2_5^+)
- the observed large increase in excitation cross section would also be beneficial for experiments with radioactive beams