Nuclear Shapes and Coexistence at the Islands of Inversion (with a Kumar twist)

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Outline

- The context; SM-CI
- Nuclear Shapes
- Coexistence at the Islands of Inversion

Basic elements of the SM-CI approach

- A valence space, computationally tractable, encompassing the targeted physics.
- An effective interaction for the valence space, that usually is expressed as a set of single particle energies and two body matrix elements (TBME)
- Shell Model codes to build and diagonalize the (huge dimensional) matrices involved, or other, approximated, mean field based methods (MCSM, DNO-SM, TAURUS, etc) to solve the secular problem.
- The present generation of SM codes includes, Antoine, Nushell, and K-shell, among others.

A game changer; the monopole multipole decomposition

The effective hamiltonian can be decomposed in two parts

- $H = \mathcal{H}_m + \mathcal{H}_M$. **Monopole and Multipole.**
- $\mathcal{H}_m$ **determines the spherical mean field and its evolution (aka shell evolution). Realistic two body interactions do not describe correctly this evolution. Hence it is necessary to modify empirically the two body centroids or the explicit inclusion of 3B forces to comply with experiment.**
- $\mathcal{H}_M$ **contains the terms responsible for the correlations** *i.e.* **pairing, quadrupole, etc. This part is correctly given by the realistic two body effective interactions.**

Nuclear Shape

- To assign a shape to the nucleus it is necessary to define an intrinsic reference frame, hence the rotational (and reflection) invariances must be broken. In addition, we usually rely on semiclassical models, liquid-drop like, to describe properties akin to the concept of shape.
- A surface in 3D can be expressed in the basis of the spherical harmonics $Y_{\lambda,\mu}(\theta,\phi)$. The coefficients of the development, $\alpha_{\lambda,\mu}$, are the shape parameters.

Nuclear Shape

- To characterise the quadrupole shapes in the intrinsic frame two parameters are used β and γ, and a large variety of recipes exist to relate them to the laboratory frame observables

Quadrupole Invariants

- The only rigorous method to relate the intrinsic parameters to laboratory-frame observables is provided by the so-called quadrupole invariants Q^n of the second-rank quadrupole operator Q_2 introduced by Kumar.
- The calculation of β and γ requires the knowledge of the expectation values of the second- and third-order invariants defined, respectively, by $\hat{Q}^2 = \hat{Q} \cdot \hat{Q}$ and $\hat{Q}^3 = (\hat{Q} \times \hat{Q}) \cdot \hat{Q}$ (where $\hat{Q} \times \hat{Q}$ is the coupling of $\hat{Q}$ with itself to a second-rank operator).

Fluctuations

Indeed, it is not very meaningful to assign effective (average) values to β and γ without also studying their fluctuations. Our aim is to go beyond the extraction of effective values of these intrinsic parameters and obtain their variances.

With this goal, we calculate:

$$\sigma(\hat{Q}^2) = (\langle \hat{Q}^4 \rangle - \langle \hat{Q}^2 \rangle^2)^{1/2}$$

and

$$\sigma(\hat{Q}^3) = (\langle \hat{Q}^6 \rangle - \langle \hat{Q}^3 \rangle^2)^{1/2}$$

Higher-order invariants

They can be calculated exactly using the Lanczos Projected Strength Function Method.

see, A. Poves, F. Nowacki, and Y. Alhassid, Phys. Rev. C 101, 054307 (2020) for the details.

Invariants and Shape parameters

The intrinsic quadrupole moment Q_0 and the effective (average) values of the Bohr-Mottelson shape parameters β and γ can be calculated from the expectation values of the second- and third-order invariants using

$$Q_0 = \sqrt{\frac{16\pi}{5}} \langle \hat{Q}^2 \rangle^{1/2} ,$$

$$\beta = \frac{4\pi}{3 r_0^2} \frac{\langle \hat{Q}^2 \rangle^{1/2}}{A^{5/3}} ,$$

$$\cos 3\gamma = -\sqrt{\frac{7}{2}} \frac{\langle \hat{Q}^3 \rangle}{\langle \hat{Q}^2 \rangle^{3/2}}$$

Fluctuations of β and γ

The variance of β is:

$$\frac{\Delta\beta}{\beta} = \frac{1}{2}\frac{\sigma\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle} \, .$$

and the variance of $\cos 3\gamma$:

$$\frac{\sigma^2(\cos 3\gamma)}{(\overline{\cos 3\gamma})^2} = \frac{\sigma^2\langle\hat{Q}^3\rangle}{\langle\hat{Q}^3\rangle^2} + \frac{9}{4}\frac{\sigma^2\langle\hat{Q}^2\rangle}{\langle\hat{Q}^2\rangle^2} - 3\frac{\langle\hat{Q}^5\rangle - \langle\hat{Q}^3\rangle\langle\hat{Q}^2\rangle}{\langle\hat{Q}^3\rangle\langle\hat{Q}^2\rangle} \, .$$

The spread of γ at 1σ is given by:

$$\cos^{-1}(\cos 3\gamma \pm \sigma(\cos 3\gamma))$$

Mind your words !!

The topic of this session is ”Nuclei without axial symmetry” so we better agree first on what are we talking about:

- Does axial means either γ=0° (prolate) or γ=60° (oblate) ?
- By the way, does spherical means β=0 ?
- How large are the fluctuations in γ that we are ready to accept before dropping out the concept of axial (or triaxial) shape?

A menagerie of shapes. Try to name them.

- ^{34}Si: 0_1^+, β = 0.18$\pm$ 0.10 γ= 40° span 0° – 60°
- ^{48}Ca: β = 0.15$\pm$ 0.05 γ= 33° span 0° – 60°
- ^{56}Ni: β = 0.21$\pm$ 0.07 γ= 40.5° span 13° – 60°
- ^{68}Ni: 0_1^+, β = 0.11$\pm$ 0.06 γ= 36° span 0° – 60°
- ^{34}Si: 0_2^+, β = 0.42$\pm$ 0.08 γ= 40° span 30° – 60°
- ^{48}Cr: β = 0.31$\pm$ 0.06 γ= 13° span 0° – 20°
- ^{68}Ni: 0_2^+, β = 0.19$\pm$ 0.05 γ= 38° span 23° – 60°
- ^{68}Ni: 0_3^+, β = 0.29$\pm$ 0.05 γ= 16° span 0° – 24°
- ^{64}Cr: β = 0.29$\pm$ 0.06 γ= 16° span 0° – 24°

If you can't, put the blame on Nature,
don't put the blame on me. (Rita Hayworth, Gilda, 1946)

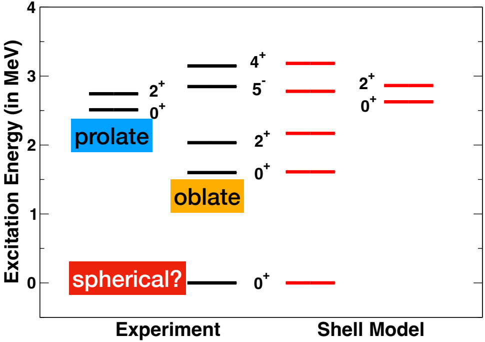
Only in the strict SU(3) limit
the variances of β and γ are zero

K-plots

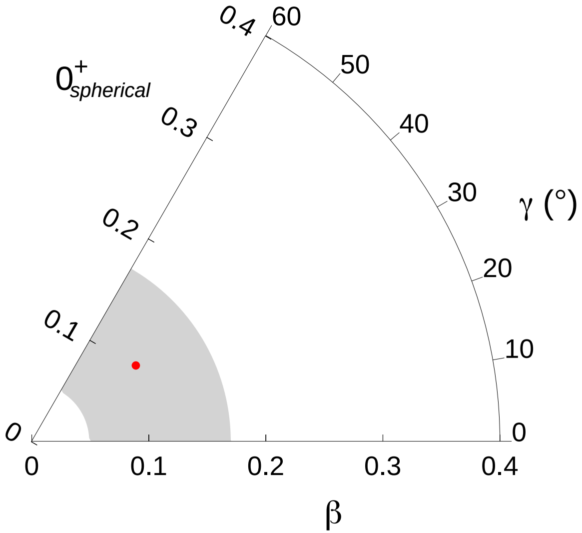
The K-plots are a representation in the (β, γ) sextant of their central values and the locus of their variances at 1σ.

They are a very useful tool to deal with shapes and shape coexistence. A few examples at or close to the IoI's follow.

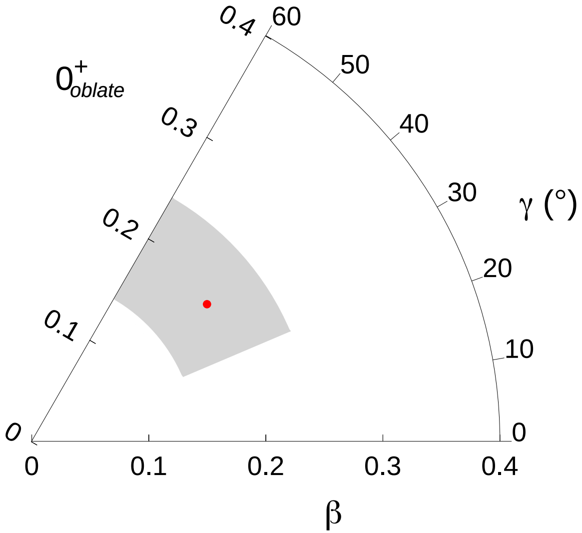
Shape Coexistence in ^{68}Ni



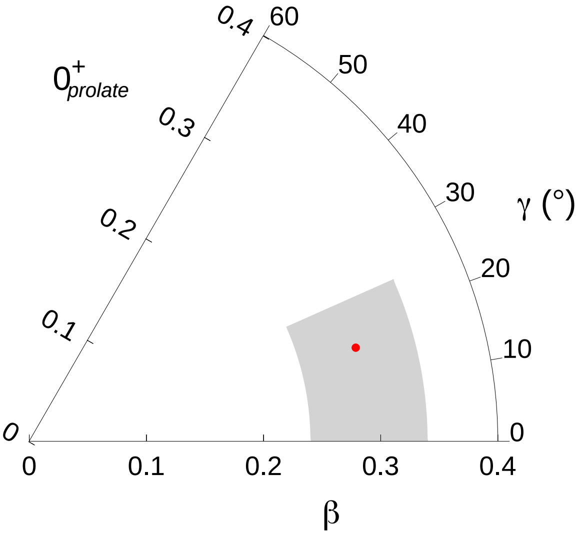
^{68}Ni; the doubly magic ground state



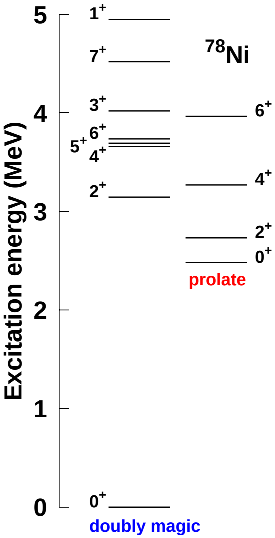
^{68}Ni; the first excited 0^+



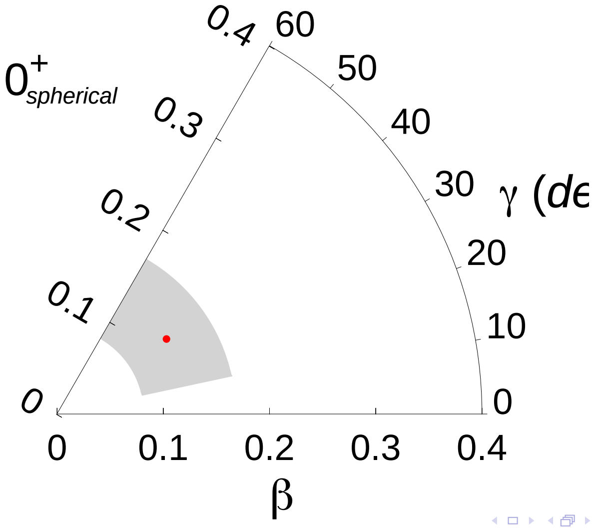
^{68}Ni; the second excited 0^+



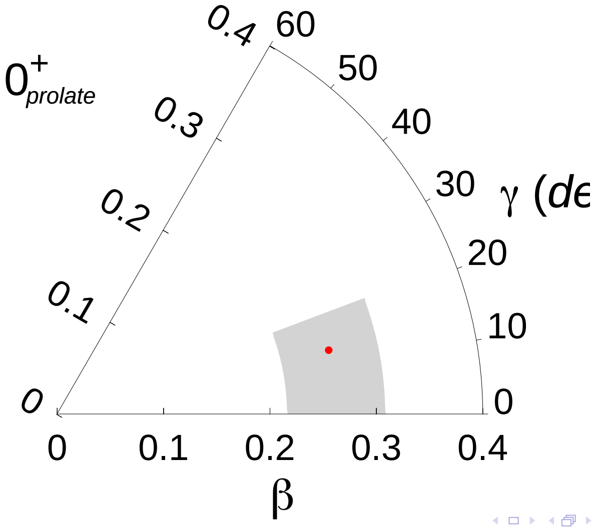
Shape Coexistence in ^{78}Ni



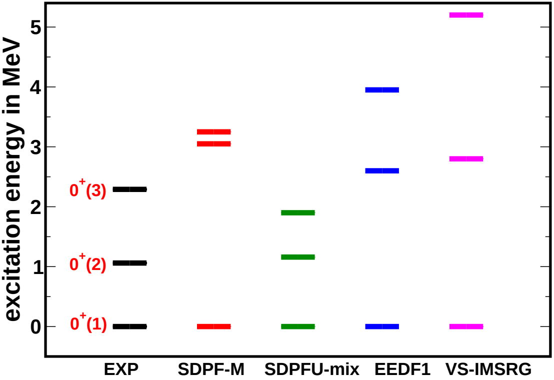
^{78}Ni; the doubly magic ground state



^{78}Ni; the excited deformed 0^+, the portal to the IoI

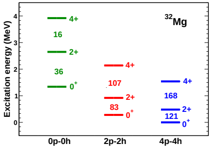


A landmark in the IoI's, ^{32}Mg



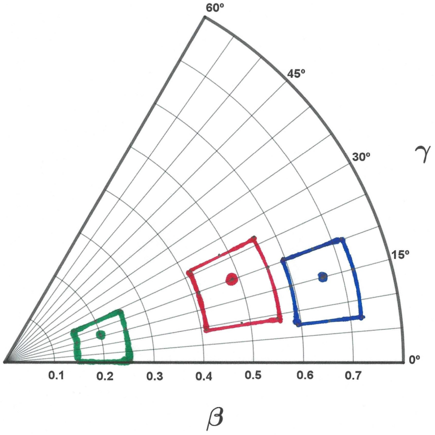
And a headache for the theorists !

K-plots for ^{32}Mg; the np-nh configulations

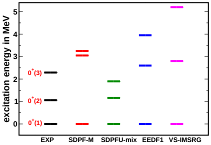


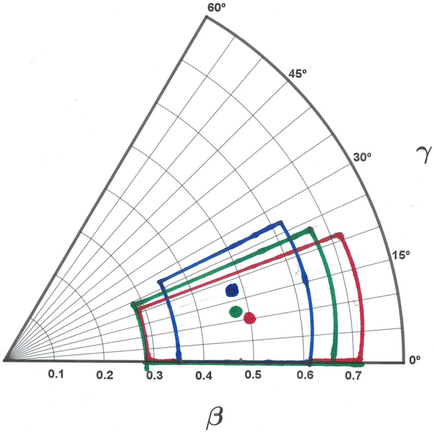
0^+_{0p-0h}
0^+_{2p-2h}
0^+_{4p-4h}

A neat case of shape coexistence !



BUT, look at the three lowest (physical) 0^+ states





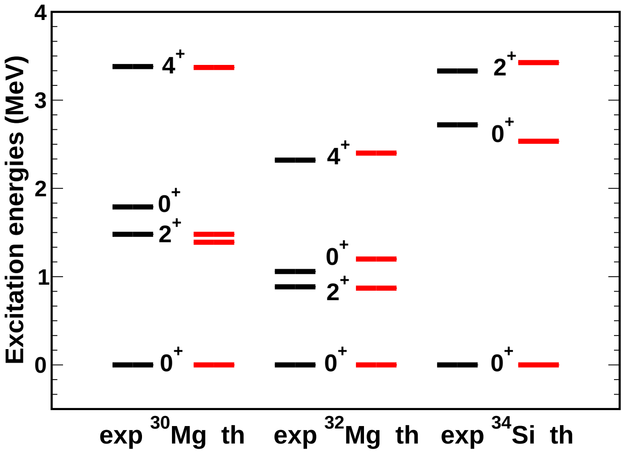
0_1^+
0_2^+
0_3^+

this is rather a case of shape mixing or entanglement

The remarkable structure of the three 0^+'s of ^{32}Mg

- **They are rather weird; the ground state is 9% 0p-0h, 54% 2p-2h, and 35% 4p-4h, thus, it is a mixture of deformed and superdeformed prolate shapes and it makes sense to speak of shape mixing.**
- **However, the first excited 0^+ (K. Wimmer's state) has 33% 0p-0h, 12% 2p-2h, and 54% 4p-4h, a surprising hybrid of semi-magic and superdeformed, whose direct mixing matrix element is strictly zero. Clearly, it is not a case of shape mixing, could it be an example of *shape entanglement*?**
- **Finally the second excited 0^+ turns out to be an even mixture of semi-magic, deformed and super-deformed. Quite exotic as well !**

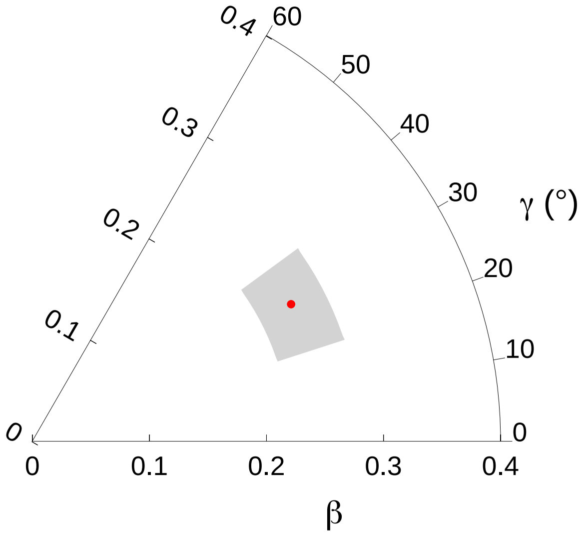
Shape Coexistence in ^{30}Mg and ^{34}Si



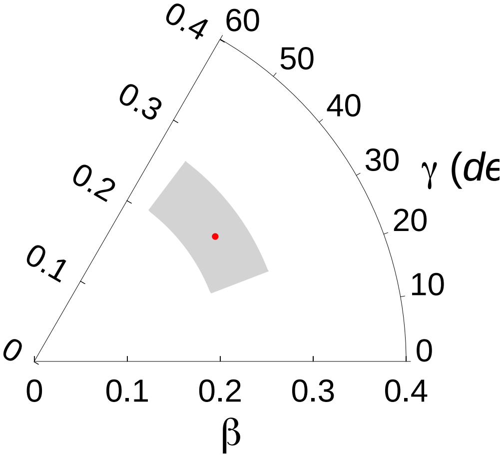
Rigid triaxiality in ^{76}Se and ^{76}Ge?

- **Shell Model Calculations in the r3g space with the jj44b interaction and standard effective charges**
- ^{76}**Ge:** $\beta = 0.17 \pm 0.02$ **and** γ **=** $(26^{+9}_{-9})^\circ$
- ^{76}**Se:** $\beta = 0.20 \pm 0.03$ **and** γ **=** $(31^{+17}_{-16})^\circ$
- **Shell Model Calculations in the LNPS space and standard effective charges**
- ^{76}**Ge:** $\beta = 0.25 \pm 0.03$ **and** γ **=** $(28^{+8}_{-10})^\circ$

^{76}Ge



^{76}Se



Finale

Thanks for your attention !

Work done in collaboration with Y. Alhassid, E. Caurier, S. M. Lenzi, F. Nowacki, K. Sieja and A. P. Zuker

More about these and other related topics in:
The neutron rich edge of the nuclear landscape, F. Nowacki, A. Obertelli, and A. Poves PPNP 120 (2021) 103866

Encore

**Whereof one cannot speak,
thereof one must be silent**

L. Wittgenstein, Tractatus logico-philosophicus, Proposition 7, Routledge and Kegan Paul eds., London (1922).

Quousque tandem abutere, Catilina, patientia nostra?

Marcus Tullius Cicero, Catilinarias (ca 60 BC)